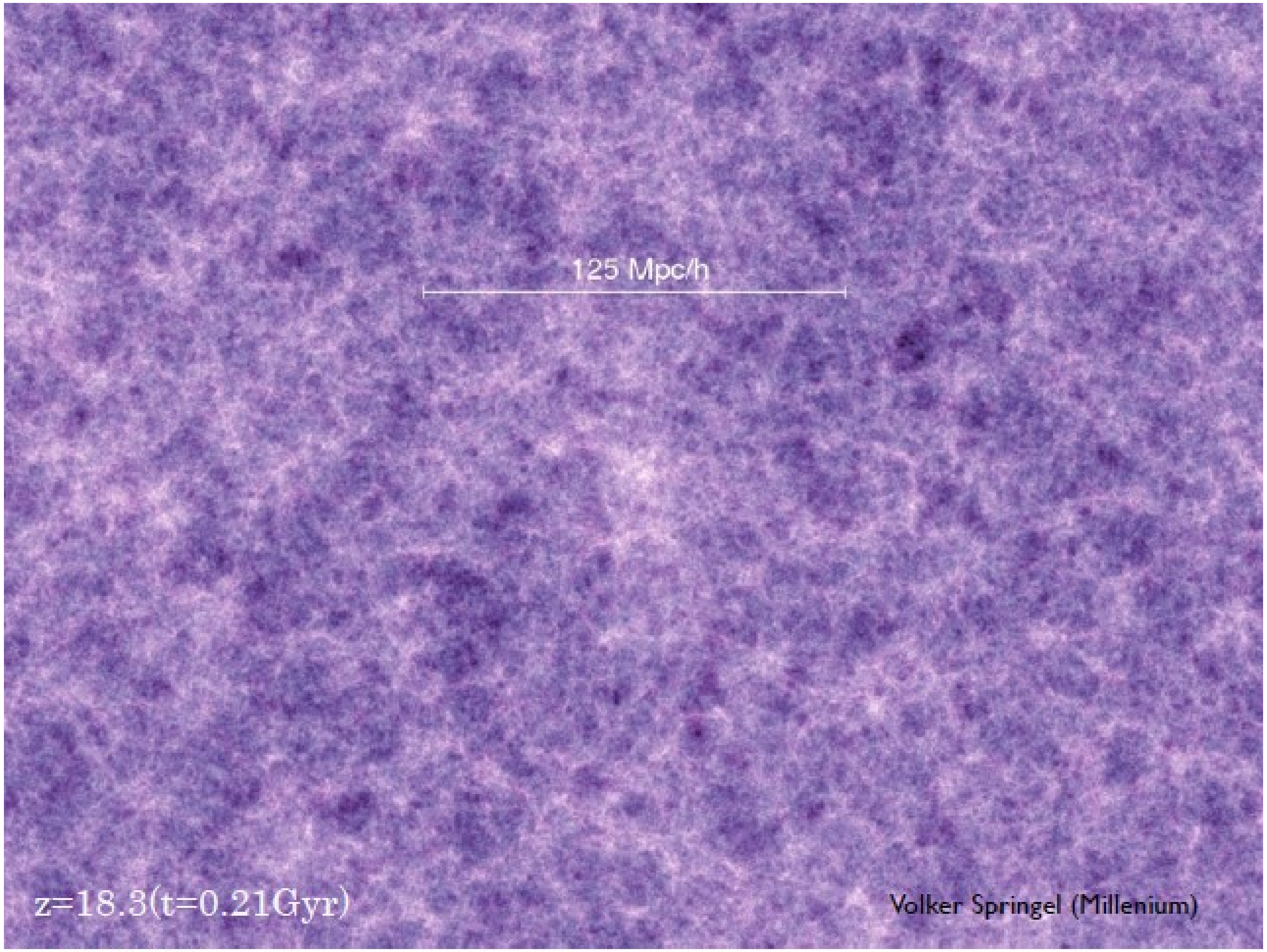
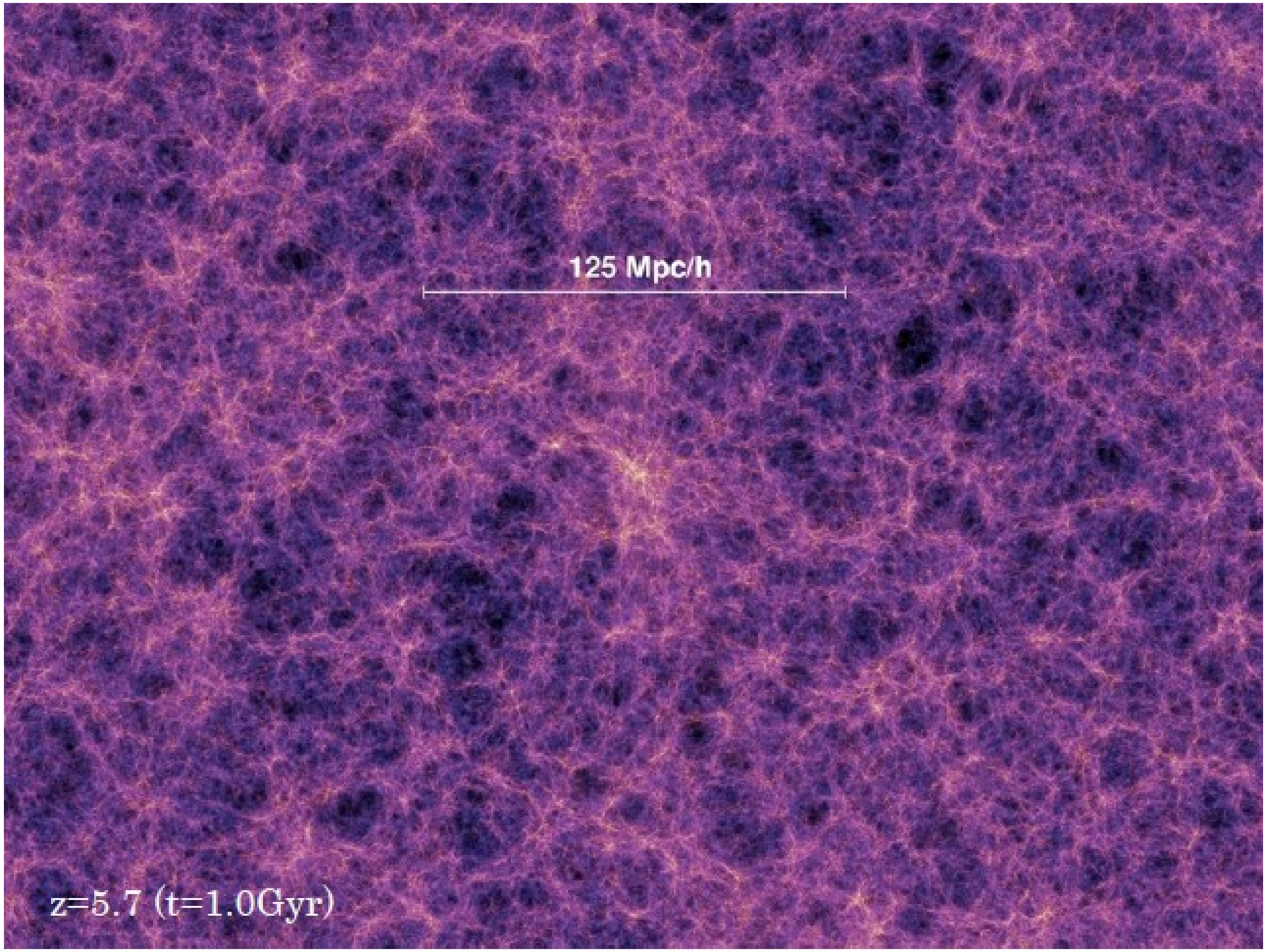
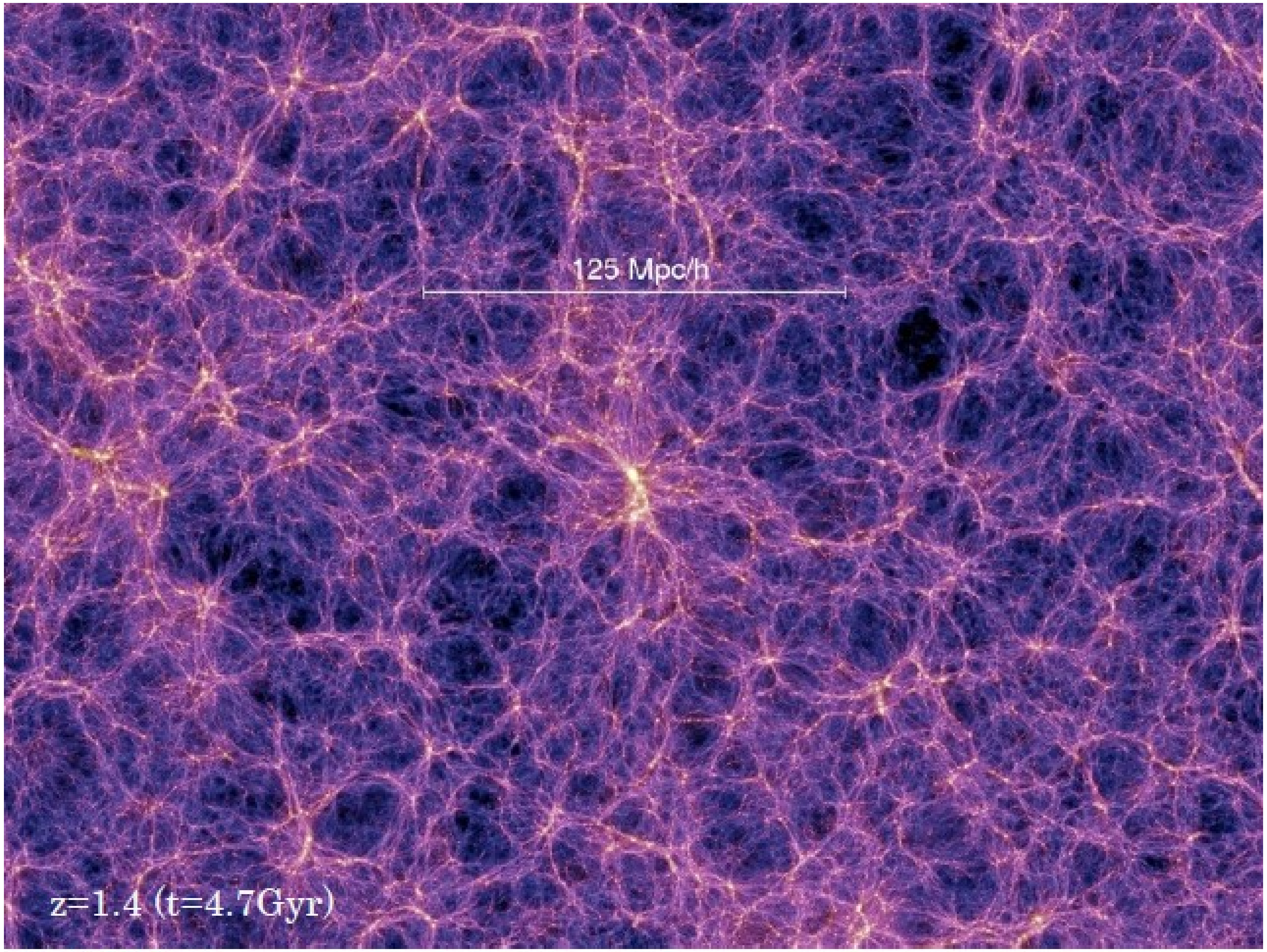
PBH Abundances

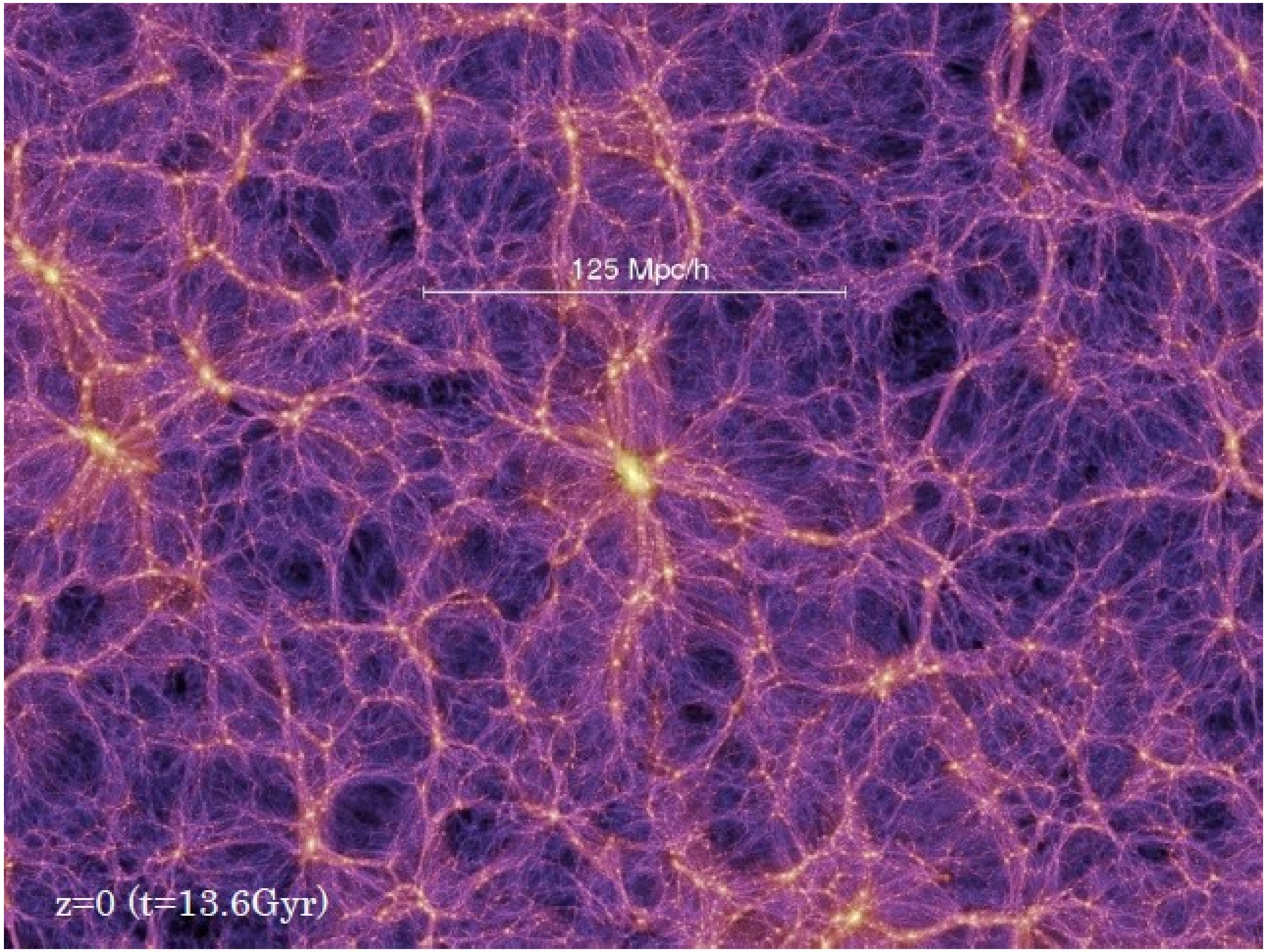
- ‘Press-Schechter’, peaks, excursion set peaks
- Universality?
- Non-linear, non-Gaussian, (Non-spherical)
- Non-delta function in mass, time, shape
- Examples
- Mergers

Ravi K Sheth (UPenn)
with C. Germani, A. Escriva

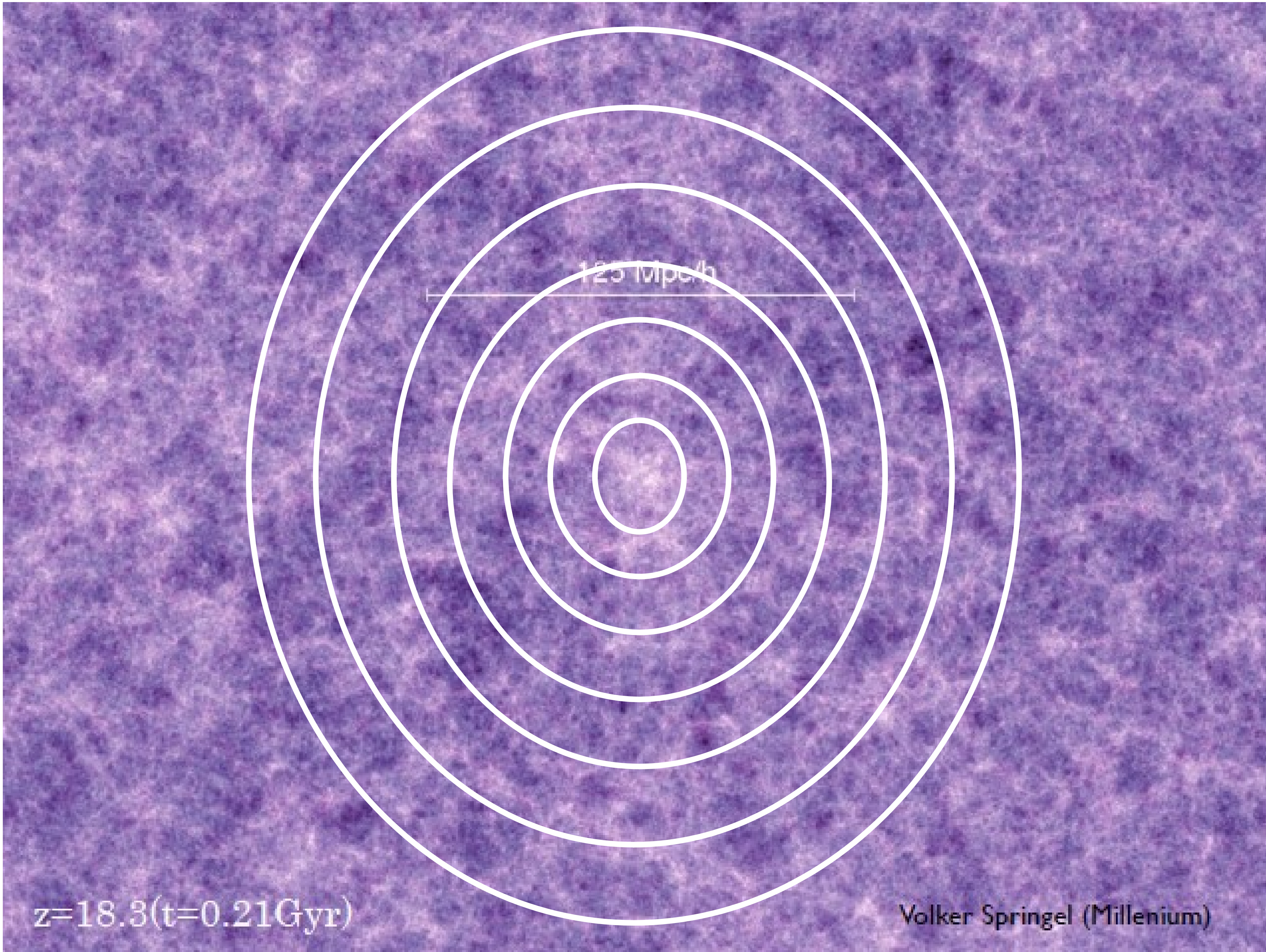

125 Mpc/h
z=18.3(t=0.21Gyr)
Volker Springel (Millenium)


125 Mpc/h
z=5.7 (t=1.0Gyr)

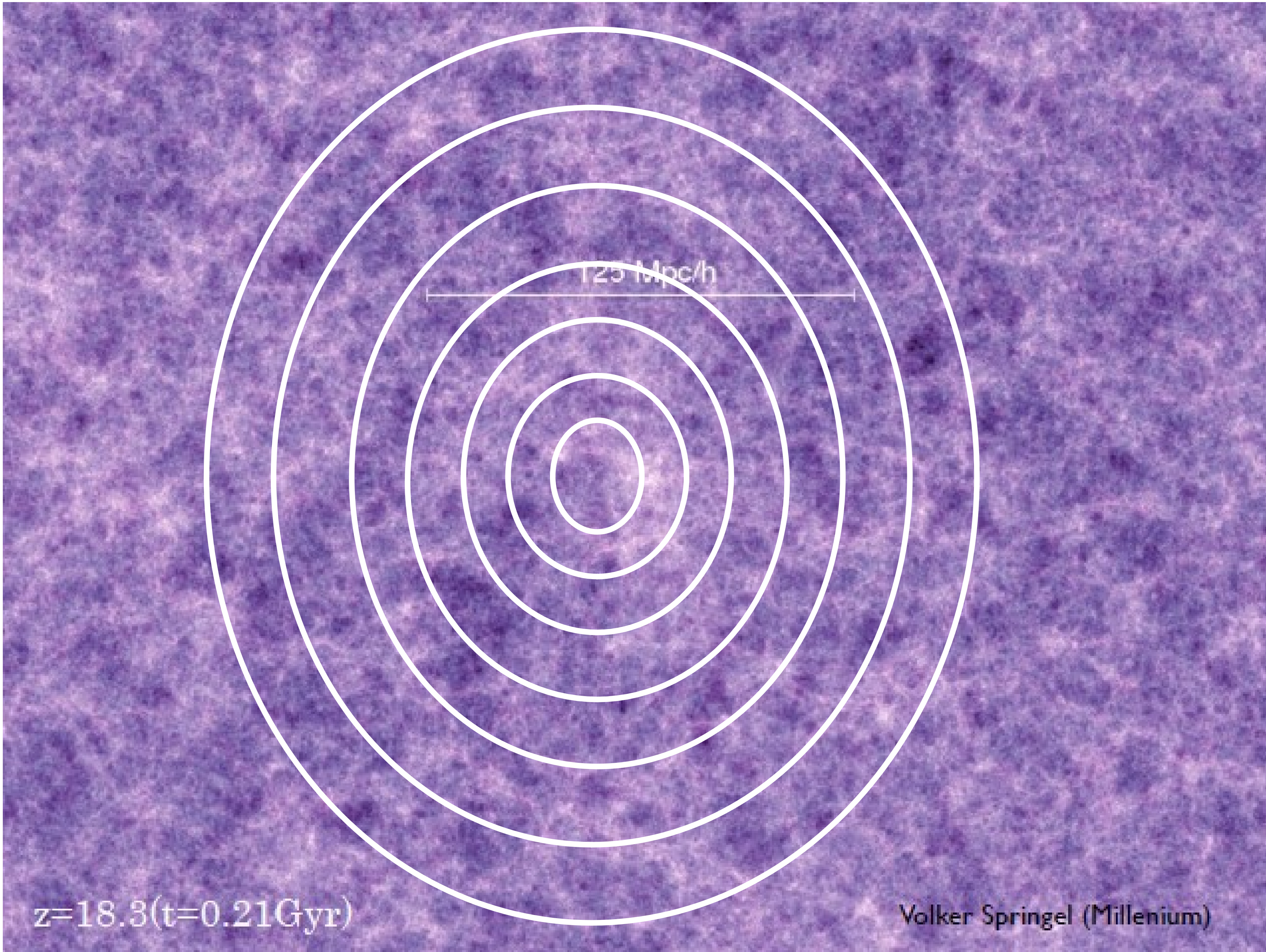

125 Mpc/h
z=1.4 (t=4.7Gyr)


125 Mpc/h
z=0 (t=13.6Gyr)

But wait…
We should be doing this in the INITIAL fluctuation field!
(no transfer function)


125 Mpc/h
z=18.3(t=0.21Gyr)
Volker Springel (Millenium)


125 Mpc/h
z=18.3(t=0.21Gyr)
Volker Springel (Millenium)


125 Mpc/h
z=18.3(t=0.21Gyr)
Volker Springel (Millenium)

Persistence of memory

Press-Schechter

We want largest scale on which random walk δ first crosses δ_c.

Assume $s(r) = \langle \delta_r^2 \rangle$, the variance on scale r, decreases monotonically with r (e.g. universe is homogeneous on large scales).

Let $f_1(S)$ denote probability that the density first exceeds δ_c on scale S.

Then $p(>\delta,s) = {}_0\int^s dS\, f_1(S)\, p(>\delta,s|\text{first cross on } S)$

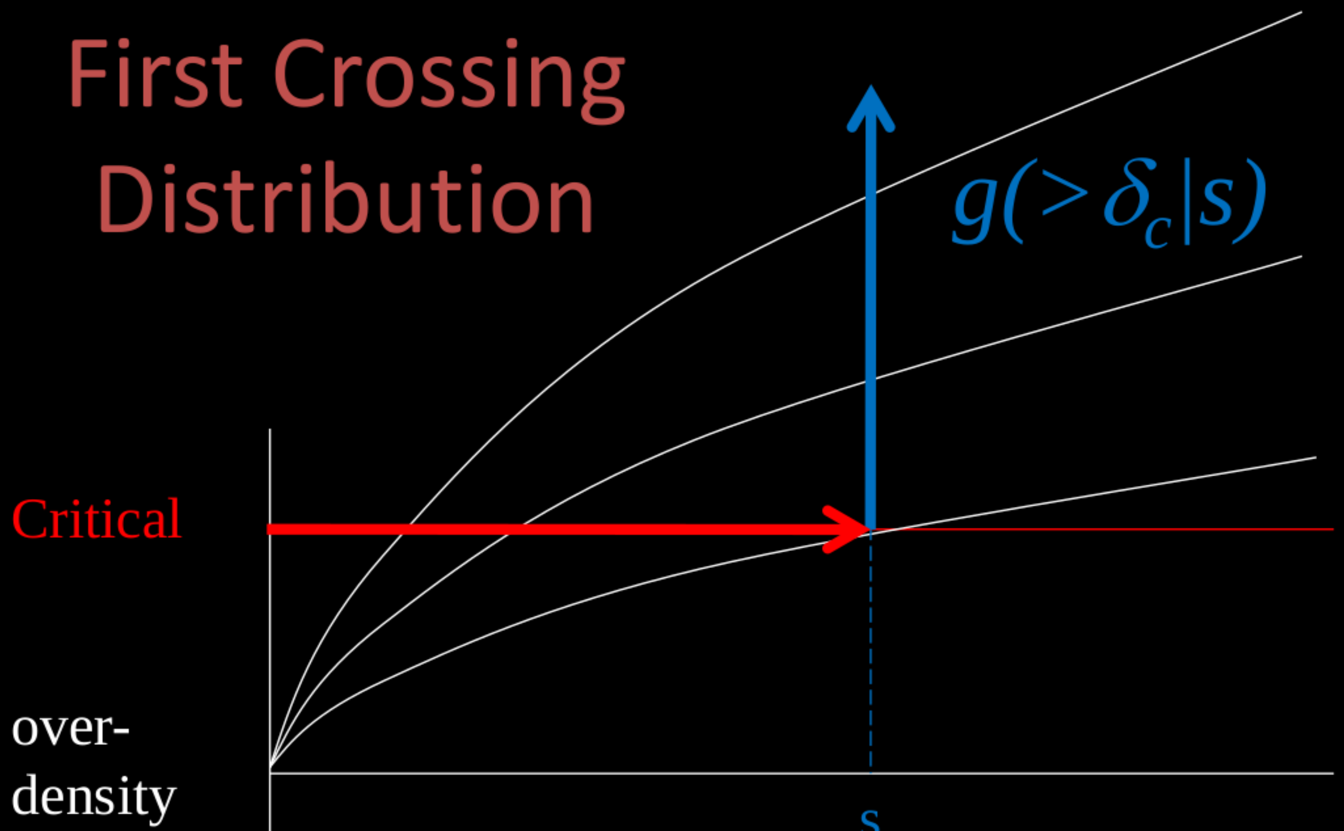
If walks increase monotonically (e.g. if $\delta_R = 2\sqrt{S}$ then $\delta_r = 2\sqrt{s}$), then

$p(>\delta,s|\text{first cross on } S) = 1$ and so $p(>\delta,s) = {}_0\int^s dS\, f_1(S)$.

This is the Press-Schechter expression.

Notice $p(>\delta,s)$ need not be Gaussian, though the assumption that walks are monotonic may be better for some cases than others.

First Crossing Distribution



$$_0\int^s dS\ f(S)\ p(>\delta_c\ ,s|\ \text{first}\ \delta_c\ \text{at}\ S)$$

$$\approx\ _0\int^s dS\ f(S)\ p(>\delta_c\ ,s|\ \delta_c\ ,S)$$

Press-Schechter (contd.)

$p(>\delta_c,s) = {}_0\int^s dS\, f_1(S)\, p(>\delta_c,s|\text{first cross on } S)$

If walks monotonic (e.g. if $\delta_R = 2\sqrt{S}$ then $\delta_r = 2\sqrt{s}$),

$p(>\delta_c,s|\text{first cross on } S) = 1.$

If walks noisy, Markov, then

$p(>\delta_c,s|\text{first cross on } S) = 1/2.$

(Neither assumption correct in general.)

So $p(>\delta_c,s) = {}_0\int^s dS\, f_1(S)$ or $p(>\delta_c,s) = {}_0\int^s dS\, f_1(S)/2.$

This difference is the origin of the 'Press-Schechter factor of 2'.

Of course, assumption that walks can be centered on random positions in space – implicit in direct use of PDF – is incorrect.

Conditions for PBH formation

Find that scale r where compaction C is maximum:

$C' = 0 \qquad C'' < 0$

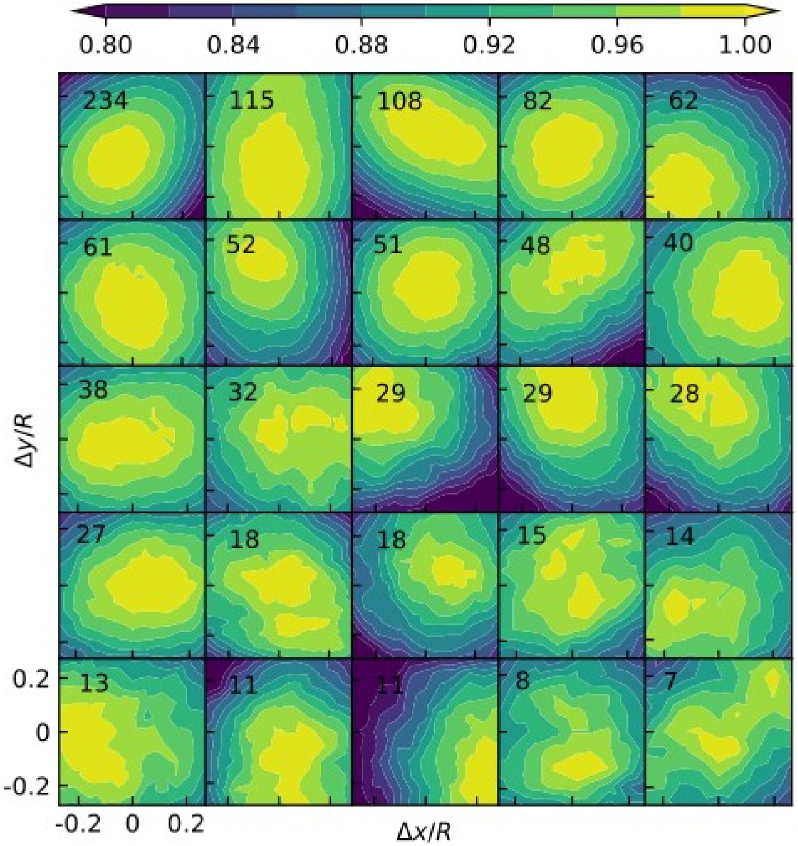
Require that, on that scale, C is also a local peak:

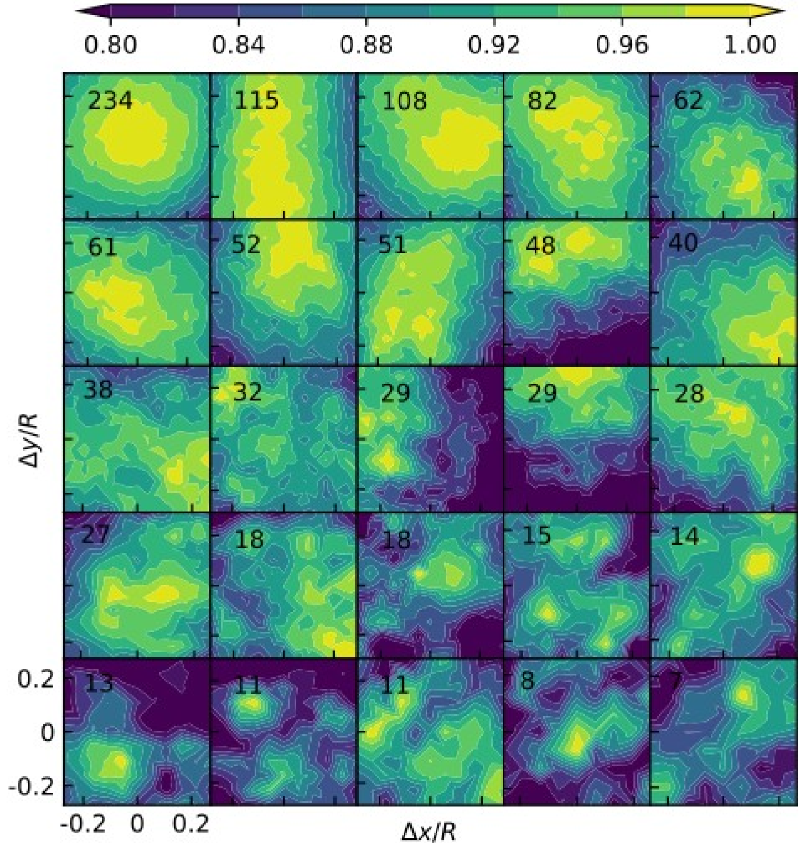
$\nabla C = 0 \qquad \nabla\nabla C < 0$

And, on that scale, C is above critical:

In principle, $C_{crit}(C', C'', C''', \ldots)$

Smoothed energy better (in matter domination)





Musso & Sheth 2021

$\varepsilon = \int d^3r\ [G\delta M(<r)/r]/V$

$\delta M(<R)/V$

Universal C_{crit}

Compaction function is C(r)

Let R be that scale where C' = dC/dr = 0.

Volume average is

$$\bar{C} = (3/R^3)\ \int_0^R dr\ r^2\ C(r)$$

Volume average is universal! $\bar{C} = 2/5$.

Use this to get $C_{crit}(C',C'',\ldots) = C_{crit}(C'')$.

(Escriva, Germani, Sheth 2020)

Conditions

Find that scale r where compaction C is maximum:

$C' = 0 \qquad C'' < 0$

Require that, on that scale, C is also a local peak:

$\nabla C = 0 \qquad \nabla\nabla C < 0$

And, on that scale, C is above critical:

$C = C_{crit}(C', C'', C''', \ldots) \sim C_{crit}(C'')$

C_{crit} from $(3/R^3)\ {}_0\!\int^R dx\, x^2\, C(x) = 2/5$ ~ 'universal'

(Escriva, Germani, Sheth 2020)

Variables

$C(r) = (2/3)\,[1 - (1 + d\zeta/d\ln r)]^2 = g_r\,(1 - g_r\,3/8)$

where $g_r = -(4/3)\,(d\zeta/d\ln r)$

If ζ is Gaussian, then so is g, but C is not

(‘nonlinear’ → ‘non-Gaussian’).

Work with $C, C', C'', \nabla\nabla C \rightarrow g,\ rg',\ r^2 g'',\ \nabla\nabla g$

$\rightarrow g,\ v,\ w,\ \chi$

$C' = g' - \tfrac{3}{4}\,gg' = g'\,(1 - g\,\tfrac{3}{4})$; $C'=0$ where $v=0$

Statistics

$dn_\bullet/d\ln r = \langle \Sigma_\bullet\, \delta_D(r-r_\bullet) \rangle = \langle \Sigma_\bullet\, \delta_D(r-r_p)\, \delta_D(r-r_m) \rangle$

$$= \langle |J|\, \delta_D(v)\, \delta_D(\nabla g)\, \vartheta(w)\, \vartheta(\nabla\nabla g)\, C_{crit} \rangle$$

So, eventually:

$$n_\bullet = \int_{r_{min}}^{r_{max}} \frac{dr}{r} \int_0^\infty dw\, w \int_{g_c(w)}^{\frac{4}{3}} dg \frac{f(\chi/\sigma_\chi)}{(2\pi r_*^2)^{3/2}}\, p(g, v=0, w)$$

Explicit sum over ‘formation’ times *r*, profile shapes *w*.

Mass density in PBH

'Weight' each PBH by its mass ...

$$\frac{d^2\rho_\bullet}{dr dM_\bullet} \sim \sum_{\vec{x}_0, r_m, M_\bullet, \mathcal{C}(r_m) > \mathcal{C}_c} \delta(\vec{x} - \vec{x}_0)\delta(r - r_m)\, \delta\left(M_\bullet - \frac{\mathcal{K}}{2H_r}\left[\mathcal{C}(g) - \mathcal{C}_c(w)\right]^{0.36}\right) M_\bullet$$

… and sum over all times *r*, shapes *w*, masses $M_\bullet$:

$$\rho_\bullet = \int_{r_{min}}^{r_{max}} \frac{dr}{r} \int_0^\infty dw\, w \int_{g_c(w)}^{\frac{4}{3}} dg \frac{f(\chi/\sigma_\chi)}{(2\pi r_*^2)^{3/2}} p(g, v=0, w) \frac{\mathcal{K}}{2H_r}\left[\mathcal{C}(g) - \mathcal{C}_c(w)\right]^{0.36}$$

Mass spectrum

Only sum over times and shapes:

$$\frac{dn(M_\bullet)}{dM_\bullet} = \int_{r_{\rm min}}^{r_{\rm max}} \frac{dr}{r} \int_0^\infty dw\, w \int_{g_c(w)}^{4/3} dg\, \frac{p(g,w,v=0)\, f(x)}{(2\pi r_*^2)^{3/2}}\, \delta_{\rm D}\Big(M_\bullet - \mathcal{K} M_{\rm H}[C(g) - C_c(w)]^{0.36}\Big)$$

Delta function generates power-law $M_\bullet^{1/0.36}$ at low masses

At higher masses, this slope modified/cutoff

Illustrative examples

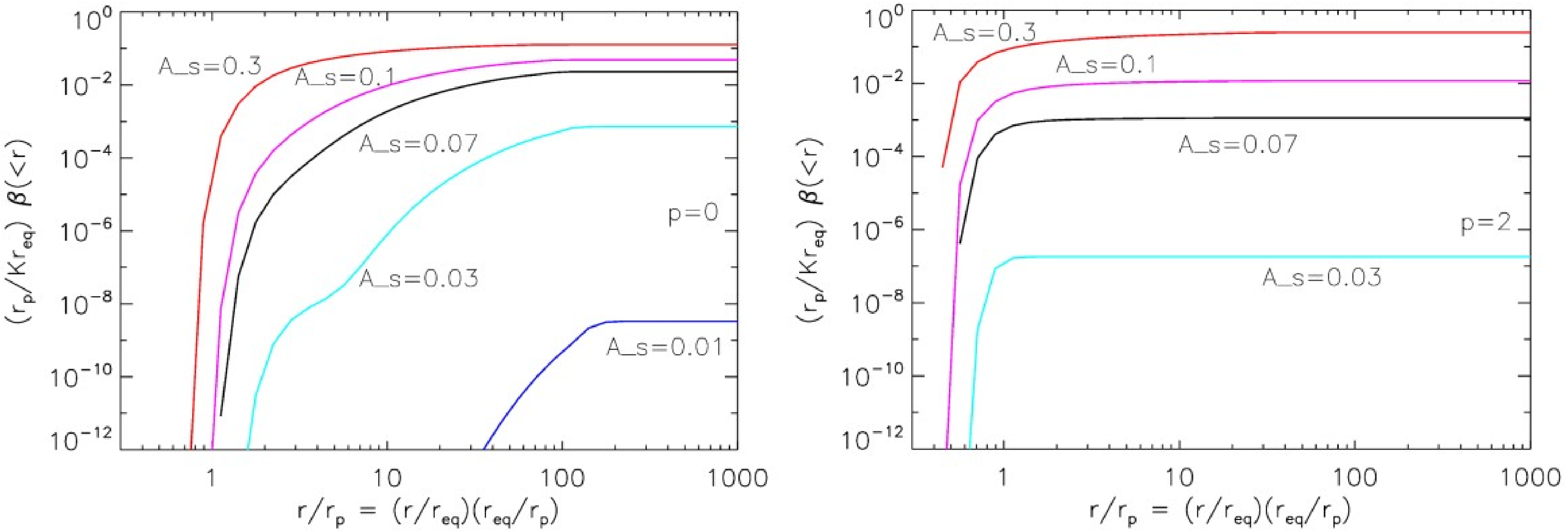
Broad $P(k) \sim A\,(kr_p)^p \exp(-kr_p)$

Narrower if p larger.

Joint PDF of variables depends on P(k).

E.g., for Gaussian case, only pairwise correlations matter, and these are given by (suitably weighted) integrals over P(k).

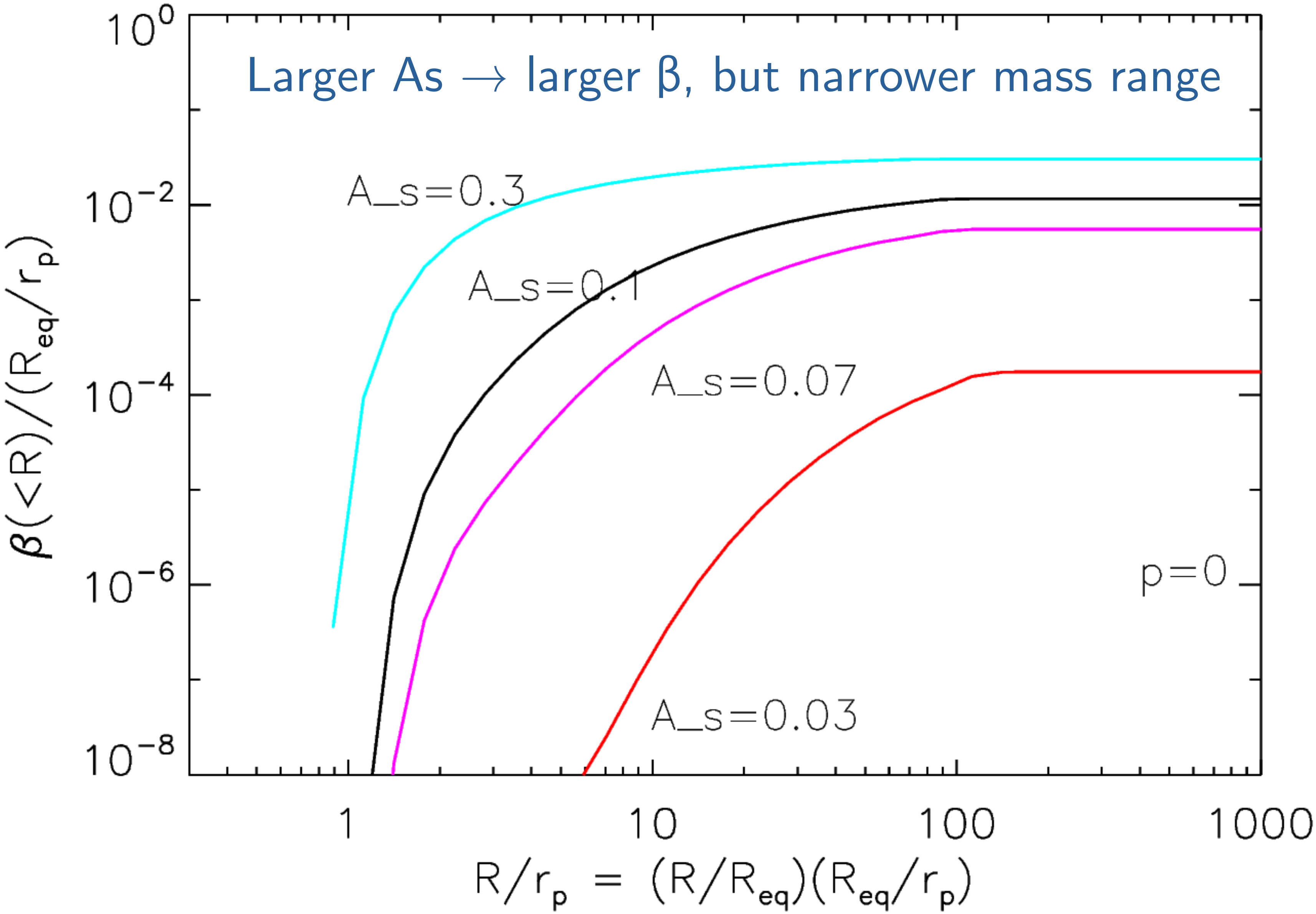
PBH mass fraction



Smaller amplitude, longer duration

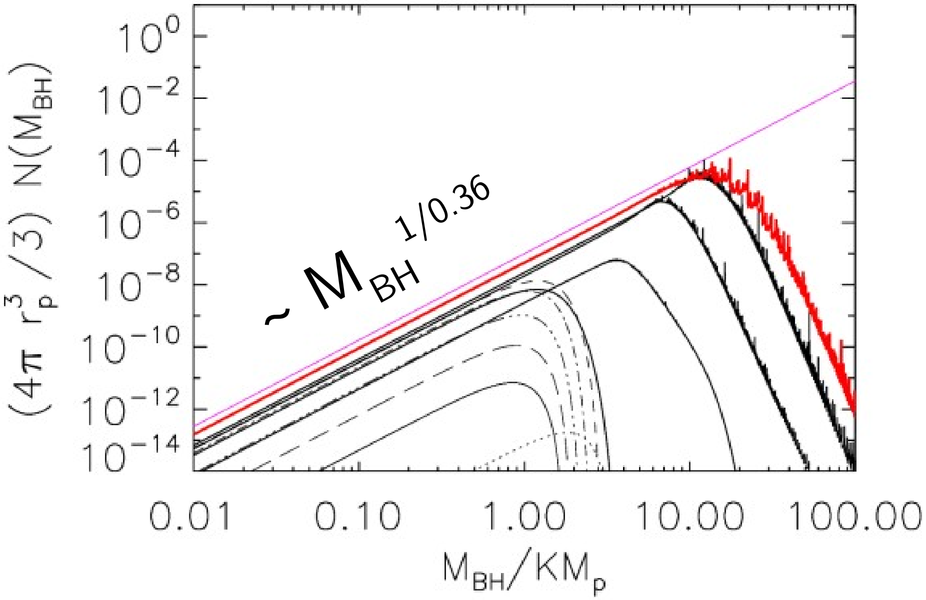
Larger p, narrower Pk, shorter duration of PBH formation

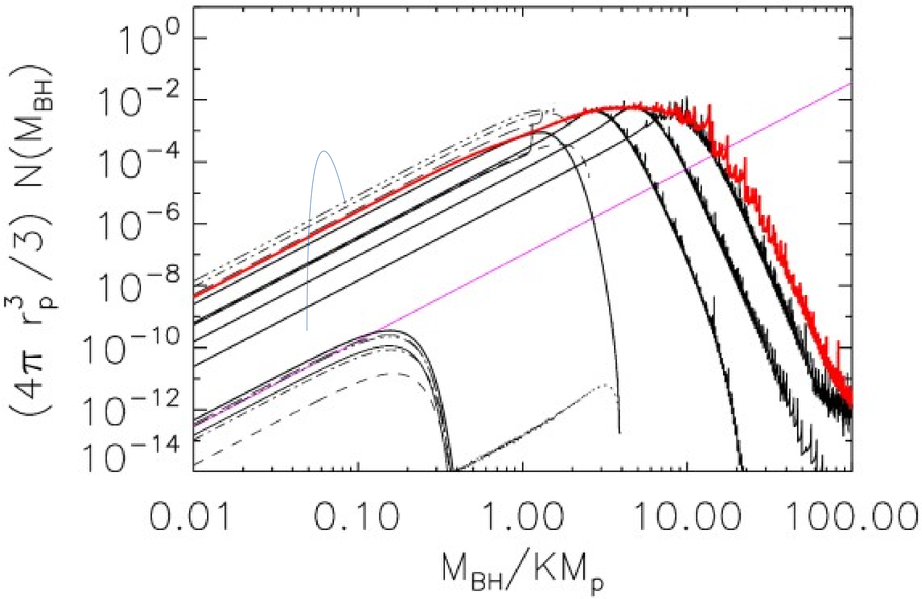
Require β=1 constrains A, r_p ~ sets formation times, masses



Larger As → larger β, but narrower mass range
A_s=0.3
A_s=0.1
A_s=0.07
A_s=0.03
p=0
10^0
10^{-2}
10^{-4}
10^{-6}
10^{-8}
1
10
100
1000
$\beta(<R)/(R_{eq}/r_p)$
$R/r_p = (R/R_{eq})(R_{eq}/r_p)$

Power law at low masses from scaling law

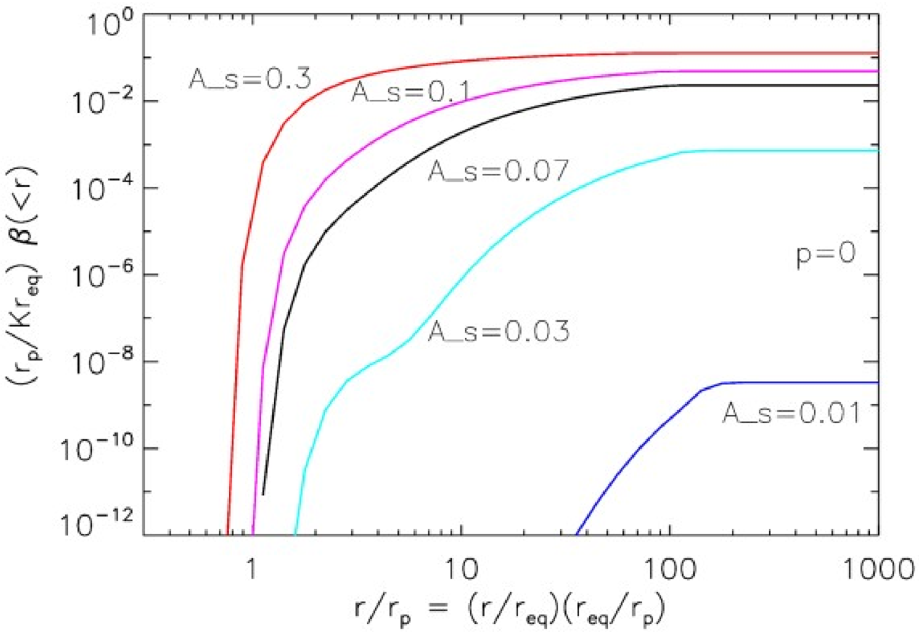


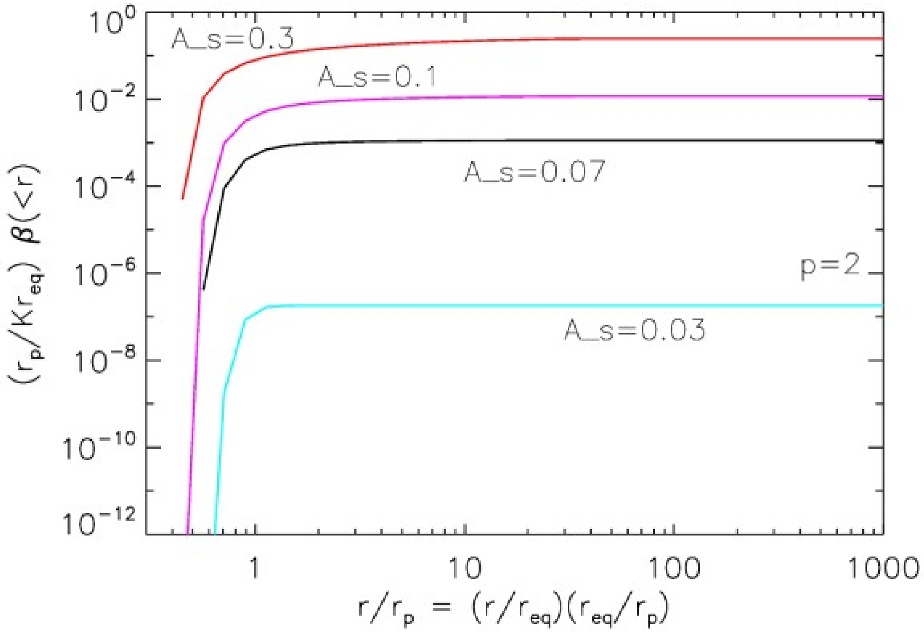


small r/r_p large r/r_p At fixed r, abundance ~ p(w)

Only small masses at early times

Work in progress





Require β=1 constrains A, r_p ~ sets formation times, masses

If pushed to small r_p, many small $M_\bullet \rightarrow$ evaporation?

Binary Mergers

$dn(M,t)/dt$ = Creation - Destruction

$= \frac{1}{2} \int_0^M dm\, n(m,t)\, n(M-m,t)\, B(m,M-m|t)$

$- n(M,t) \int dm\, n(m,t)\, B(m,M|t)$

If start from equal mass, and if

$B(m,M|t) = B(m,M) = m+M$

Then → Press-Schechter and spectrum of cluster sizes, distribution of merger times, known (Sheth 1996)

For BH, geometric cross-section ~ $R^2 \sim M^2$?