Contribution ID: **1** Type: **not specified**

Okounkov's conjecture via BPS Lie algebras

Monday, March 3, 2025 10:00 AM (1 hour)

Given a finite quiver Q, using their theory of stable envelopes and resulting R matrices, Maulik and Okounkov defined a new type of Yangian algebra. This algebra is defined as a subalgebra of the endomorphism algebra of the (equivariant) cohomology of all the Nakajima quiver varieties associated to Q.

If Q is an orientation of a type ADE Dynkin diagram, the Maulik-Okounkov algebra recovers the usual Yangian algebra, a deformation of the universal enveloping algebra of the current algebra of the associated ADE type Lie algebra. If Q is the one-loop quiver, their theory also recovers a well-known Yangian algebra, and the Grojnowski-Nakajima action of an infinite-dimensional Heisenberg algebra on the cohomology of Hilbert schemes of $\mathbb{C}^2$ is recovered as a part of the theory. For general quivers, the picture is less clear. Although, as in the cases above, there is a Lie algebra $\mathfrak{g}_{MO}$ which generates the whole of the Maulik-Okounkov algebra, even determining the dimensions of the graded pieces remained an open question until quite recently.

Okounkov's conjecture states that these dimensions are given by coefficients of Kac's polynomials, which count isomorphism classes of Q-representations over finite fields. I'll present a proof of this conjecture, which is joint work with Tomasso Botta. The proof proceeds by identifying the Maulik-Okounkov Lie algebra $\mathfrak{g}_Q$ with certain BPS Lie algebras (which I'll define during the talk).

Presenter: DAVISON, Ben (University of Edinburgh)