

# BPS cohomology, cohomological Hall induction and intrinsic DT theory

*Friday, March 7, 2025 2:00 PM (1 hour)*

In this talk, I will present a new foundation of the Donaldson—Thomas (DT) theory and its categorification that is intrinsic to moduli spaces. This framework, which we call the ‘intrinsic DT theory’, can be applied to the spaces which do not arise as the moduli spaces of objects in abelian categories, such as the stack of G-Higgs bundles and G-character stacks for real 3-manifolds. The key ingredient of the theory is the ‘component lattices for stacks’, which is a generalization of the cocharacter lattices for algebraic groups.

One of the main theorems in the intrinsic DT theory is the decomposition theorem (a.k.a. the cohomological integrality theorem), which provides a decomposition of the (critical or Borel—Moore) homology of the stacks into the induction of “cuspidal” pieces, which we call the ‘BPS cohomology’. The construction of the decomposition is based on the cohomological Hall induction, which is a non-linear generalization of the cohomological Hall algebra for abelian categories.

As an application of the intrinsic DT theory, I will propose a formulation of the topological mirror symmetry conjecture for the stack of G-Higgs bundles (in the style of Hausel—Thaddeus) and discuss its relation with the Langlands duality for 3-manifolds. Additionally, I will discuss how the ideas underlying the intrinsic DT theory led me and my collaborators to the construction of cohomological Hall algebras for 3-Calabi-Yau categories, whose existence was conjectured by Kontsevich—Soibelman and Joyce.

This talk is based on the following three papers:

- Cohomology of symmetric stacks (joint with Chenjing Bu, Ben Davison, Andrés Ibáñez Núñez and Tudor Pădurariu)
- Intrinsic Donaldson—Thomas theory. I. Component lattices of stacks (joint with Chenjing Bu, Daniel Halpern-Leistner and Andrés Ibáñez Núñez)
- Cohomological Hall algebras for 3-Calabi-Yau categories (joint with Hyeonjun Park and Pavel Safronov)

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