Program

Mar 3 – 7, 2025

Enumerative geometry, representation theory, and physics

Kavli IPMU, Lecture Hall

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Mon, March 3

9:50 AM – 10:00 AM

Registration

Break

10:00 AM – 11:00 AM

Okounkov's conjecture via BPS Lie algebras

Contribution | **Speaker:** Ben Davison (University of Edinburgh)

Description

Given a finite quiver Q, using their theory of stable envelopes and resulting R matrices, Maulik and Okounkov defined a new type of Yangian algebra. This algebra is defined as a subalgebra of the endomorphism algebra of the (equivariant) cohomology of all the Nakajima quiver varieties associated to Q.
If Q is an orientation of a type ADE Dynkin diagram, the Maulik-Okounkov algebra recovers the usual Yangian algebra, a deformation of the universal enveloping algebra of the current algebra of the associated ADE type Lie algebra. If Q is the one-loop quiver, their theory also recovers a well-known Yangian algebra, and the Grojnowski-Nakajima action of an infinite-dimensional Heisenberg algebra on the cohomology of Hilbert schemes of $ \mathbb{C}^2$ is recovered as a part of the theory. For general quivers, the picture is less clear. Although, as in the cases above, there is a Lie algebra $\mathfrak{g}_{MO}$ which generates the whole of the Maulik-Okounkov algebra, even determining the dimensions of the graded pieces remained an open question until quite recently.

Okounkov's conjecture states that these dimensions are given by coefficients of Kac's polynomials, which count isomorphism classes of Q-representations over finite fields. I'll present a proof of this conjecture, which is joint work with Tomasso Botta. The proof proceeds by identifying the Maulik-Okounkov Lie algebra $\mathfrak{g}_Q$ with certain BPS Lie algebras (which I'll define during the talk).

11:00 AM – 11:30 AM

Coffee Break

Break

11:30 AM – 12:30 PM

3D modularity revisited - enumerative invariants from 3D topology

Contribution | **Speaker:** Ioana Coman (Kavli IPMU, the University of Tokyo)

Description

A particular type of BPS q-series has been proposed recently as a topological invariant which provides a non-perturbative completion of complex Chern-Simons theory on closed 3-dimensional manifolds. This invariant has been defined mathematically from 3D topology, quantum groups and resurgence, and has key properties the integrality of the q-series and their behaviour under surgery. The q-series invariants have displayed relations to vertex operator algebras, and structurally, they have intriguing modular properties. Their mathematical formulation is nevertheless quite constrained, relying heavily on certain negative-definite conditions for the 3-manifolds, and much effort has been devoted to extending this definition. This operation is called “going to the other side”, with different interpretations from the physics, vertex algebra, and 3D topology perspectives. I will discuss different approaches to this challenge, in particular through modularity and resurgence, with implications for the related vertex operator algebras.

12:30 PM – 2:00 PM

Lunch Break

Break

2:00 PM

Hall algebras of quivers via coherent sheaves and D-modules

Contribution | **Speaker:** Tudor Pădurariu (CNRS and Sorbonne Université)

Description

I will report on joint work in progress with Yukinobu Toda, and with Sabin Cautis and Yukinobu Toda.
The Ringel (spherical) Hall algebra of a quiver is half of a quantum Kac-Moody algebra, and has a categorification (due to Lusztig) using D-modules on the stack of representations of the quiver. The K-theoretic preprojective Hall algebra of a quiver is expected to be half of a quantum affine algebra defined by Okounkov-Smirnov, and has a categorification (due to Porta-Sala) using categories of coherent sheaves on the cotangent of the stack of representations.
I will discuss a conjectural comparison between these two categorifications. The main objects in the study of (Porta-Sala) Hall algebras are quasi-BPS categories. In joint work with Yukinobu Toda, we compare quasi-BPS categories with categories of twisted coherent D-modules on the stack of representations of the quiver. These are the first examples of a more general construction for the cotangent of smooth stacks, called "limit categories", which we compare with categories of D-modules on the stack.

I will exemplify the various conjectures with results for the Jordan quiver, which are obtained in joint work (in progress) with Sabin Cautis and Yukinobu Toda.

3:00 PM

3:00 PM

Break

Break

4:00 PM

4:00 PM

Nekrasov’s gauge origami via DT4 theory

Contribution | **Speaker:** Woonam Lim (Yonsei University)

Description

The study of the classical instantons on the spacetime has led to many interesting mathematics through the associated partition functions. In a series of papers, Nekrasov introduced and studied the generalised ADHM equations, whose solutions are instantons on the “origami spacetime”. In this talk, I will explain how to interpret Nekrasov’s gauge origami via DT4 theory. Our main result determines the orientation in the Oh-Thomas localisation formula of the gauge origami partition function. After giving some applications, I will finish by presenting a conjectural sheaf-theoretic description of the gauge origami. This is joint work in progress with N. Arbesfeld and M. Kool.

5:00 PM

Tue, March 4

10:00 AM – 11:00 AM

Gopakumar-Vafa invariants via Wilson loops

Contribution | **Speaker:** Sheldon Katz (University of Illinois at Urbana-Champaign)

Description

A compact Calabi-Yau threefold X can be considered to approach a local Calabi-Yau threefold Y in certain limits of Kahler moduli. The enumerative geometry of Y is simpler and physical tools of 5-dimensional gauge theory apply, particularly the use of Wilson loops. A general proposal is made for the structure of the refined BPS numbers of X in terms of refined Wilson loops. Using low degree geometric computation and methods of physics including a refined holomorphic anomaly equation for the Wilson loop partition function, refined BPS numbers of elliptically fibered X are computed for many examples, with $h^{1,1}(X)$ as large as 5. In particular, the unrefined limit produces the Gopakumar-Vafa invariants of X. This talk is based on joint work with Minxin Huang, Albrecht Klemm, and Xin Wang.

11:00 AM – 11:30 AM

Coffee Break

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11:30 AM – 12:30 PM

Fourier analysis in GLSM quasimaps

Contribution | **Speaker:** Konstantin Aleshkin (Kavli IPMU, the University of Tokyo)

Description

Fourier analysis is a useful tool in the study of equivariant Gromov-Witten theory and quantum cohomology. In this talk, I will explain how it can be used to compute quasimap invariants of GIT quotients of vector spaces and Landau-Ginzburg pairs on such spaces, known as GLSM invariants. I also plan to discuss how it can be applied to study the variation of GIT quotients in curve counting theories. This talk is based on joint work in progress with Melissa Liu.

12:30 PM – 12:40 PM

Group photo

Break

12:40 PM – 2:00 PM

Lunch Break

Break

2:00 PM – 3:00 PM

D-equivalence conjecture for varieties of K3^[n]-type

Contribution | **Speaker:** Davesh Maulik (MIT)

Description

The D-equivalence conjecture of Bondal and Orlov predicts that birational Calabi-Yau varieties have equivalent derived categories of coherent sheaves. I will explain how to prove this conjecture for hyperkahler varieties of $K3^[n]$ type (i.e. those that are deformation equivalent to Hilbert schemes of K3 surfaces). This is joint work with Junliang Shen, Qizheng Yin, and Ruxuan Zhang.

3:00 PM – 4:00 PM

Break

Break

4:00 PM

Algebraic cycles on Hitchin fibrations

Contribution | **Speaker:** Junliang Shen (Yale University)

Description

Recent studies of the Hitchin fibration suggest that, from the cohomological perspective, it behaves like an abelian scheme. In this talk, I will discuss this phenomenon and provide supporting evidence. In particular, I will discuss a proof of the motivic decomposition conjecture of Corti-Hanamura for the Hitchin fibration, where the desired algebraic cycles are constructed using a combination of techniques from derived categories, K-theory, and Springer theory. The counter-part of this result for abelian schemes was established by Deninger and Murre over three decades ago. This work is based on joint work with Davesh Maulik and Qizheng Yin.

5:00 PM

Wed, March 5

10:00 AM

Path Integral Derivations of K-theoretic Donaldson Invariants

Contribution | **Speaker:** Jan Manschot (University of Dublin)

Description

I will discuss the formulation of 5-dimensional N=1 supersymmetric Yang-Mills theory on X times a circle S^1, with X a smooth, compact four-manifold. We include a partial topological twist on X. The 5-dimensional theory can then be reduced to a 4-dimensional Kaluza-Klein theory on X, or to a 1-dimensional theory on S^1. For either reduction, we demonstrate that correlation functions evaluate to K-theoretic Donaldson invariants, such as the Dirac index or holomorphic Euler characteristic of moduli spaces of instantons on X. Explicit evaluation demonstrates agreement with results for algebraic surfaces by Gottsche, Kool, Nakajima and Yoshioka. Based on work in progress with H. Kim, G. Moore, R. Tao, X. Zhang.

11:00 AM

11:00 AM

Coffee Break

Break

11:30 AM

11:30 AM

Inverse Hamiltonian reduction for affine W-algebras and factorization of divisors on Calabi-Yau threefolds

Contribution | **Speaker:** Dylan Butson (University of California Los Angeles)

Description

I will explain how certain enumerative invariants associated with divisors on toric Calabi-Yau threefolds behave nicely under the decomposition of divisors into their reduced, irreducible components, and moreover that this corresponds in representation theory to the existence of various embeddings of vertex algebras. In a particular family of examples this corresponds to inverse Hamiltonian reduction, a system of embeddings of affine W-algebras associated to smaller nilpotents into those associated with larger nilpotents in the same Lie algebra, recently proved for general type A in joint work with Sujay Nair.

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Lunch Break

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2:00 PM

2:00 PM

Free Afternoon

Break

5:00 PM

Thu, March 6

10:00 AM

Dimensional reduction and cohomological Hall algebras

Contribution | **Speaker:** Adeel A. Khan (Academia Sinica)

Description

In Donaldson-Thomas theory, dimensional reduction computes certain 3d invariants in terms of 2d ones. Conversely, I will explain how to upgrade 2d invariants to 3d ones using the technique of microlocalization (as generalized to derived algebraic geometry). For example, this allows one to upgrade the 2d cohomological Hall algebras of Kapranov-Vasserot to 3d cohomological Hall algebras for local surfaces. This is joint work with Tasuki Kinjo.

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Coffee Break

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The space of augmented stability conditions

Contribution | **Speaker:** Alekos Robotis (Cornell University)

Description

I will survey recent joint work with Daniel Halpern-Leistner which constructs a partial compactification of (a quotient of) the space of Bridgeland stability conditions on a triangulated category. The partial compactification is modular in the sense that its boundary points parameterise a new type of homological structure: an augmented stability condition.

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Lunch Break

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Gauge origami, vertex operators, and equivariant DT/PT count

Contribution | **Speaker:** Taro Kimura (Université de Bourgogne)

Description

We present a vertex operator formalism to study equivariant DT/PT invariants of Calabi-Yau 4-folds based on the underlying quiver description of the moduli space of sheaves. This system is associated with the so-called gauge origami, involving intersecting non-compact D-branes extending in various directions. We show that (1) the generating function of the equivariant invariants is given by a correlation function (conformal block) of the corresponding vertex operators, i.e., BPS/CFT correspondence, and (2) it gives rise to geometric realisations of module structures of quantum toroidal algebras. Mostly we focus on the primary example $\mathbb{C}^4$, and will mention how to deal with more generic toric varieties. Based on joint works with G. Noshita.

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Break

Break

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Intersection cohomology of moduli of vector bundles

Contribution | **Speaker:** Camilla Felisetti (Università di Modena e Reggio Emilia)

Description

Intersection cohomology is a topological notion adapted to the description of singular topological spaces, and the Decomposition Theorem for algebraic maps is a key tool in the subject. Motivated by the work of Mozgovoy and Reineke, in joint work with Andras Szenes and Olga Trapeznikova, we give a complete description of the intersection cohomology of the moduli space of vector bundles of any rank via a detailed analysis of the Decomposition Theorem applied to a certain map from parabolic bundles. We also give a new formula for the intersection Betti numbers of these moduli spaces, which has a clear geometric meaning.

5:00 PM

Fri, March 7

10:00 AM

Quiver Yangians as Coulomb branch algebras

Contribution | **Speaker:** Wei Li (Institute of Theoretical Physics, Chinese Academy of Sciences)

Description

For a 3D N=4 gauge theory, turning on the Omega-background deforms the Coulomb branch chiral ring into the quantum Coulomb branch algebra, generated by the 1/2-BPS monopoles together with the complex scalar in the vector-multiplet. We conjecture that for a 3D N=4 quiver gauge theory with unitary gauge group, the quantum Coulomb branch algebra can be formulated as the truncated shifted quiver Yangian $Y(\hat{Q},\hat{W})$ based on the triple quiver $\hat{Q}$ of the original quiver Q with canonical potential $\hat{W}$. The Hilbert spaces of vortices approaching different vacua at spatial infinity furnish different representations of the shifted quiver Yangian, and all the charge functions have only simple poles.

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Coffee Break

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Langlands duality for critical cohomology of local systems on the 3-torus

Contribution | **Speaker:** Sarunas Kaubrys (University of Edinburgh)

Description

In this talk I will explain a proof of Langlands duality of critical cohomology (cohomological DT invariants) of SLn/PGLn local systems on the 3-torus for prime n. This duality is expected to hold for all compact oriented 3 manifolds. From the physics point of view it arises as S duality of topological twists of 4D N=4 Yang-Mills theories and can be viewed as a Geometric Langlands type statement for 3-manifolds. The main tools of the proof are the use of an exponential map relating CoDT invariants of local systems to "additive" CoDT invariants, a decomposition result called cohomological integrality and a computation of BPS cohomology for the 3-torus. If time permits, I will also explain work in progress concerning applications of the exponential map construction to nonabelian Hodge theory.

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Lunch Break

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BPS cohomology, cohomological Hall induction and intrinsic DT theory

Contribution | **Speaker:** Tasuki Kinjo (RIMS)

Description

In this talk, I will present a new foundation of the Donaldson—Thomas (DT) theory and its categorification that is intrinsic to moduli spaces. This framework, which we call the ‘intrinsic DT theory’, can be applied to the spaces which do not arise as the moduli spaces of objects in abelian categories, such as the stack of G-Higgs bundles and G-character stacks for real 3-manifolds. The key ingredient of the theory is the `component lattices for stacks’, which is a generalization of the cocharacter lattices for algebraic groups.

One of the main theorems in the intrinsic DT theory is the decomposition theorem (a.k.a. the cohomological integrality theorem), which provides a decomposition of the (critical or Borel—Moore) homology of the stacks into the induction of “cuspidal” pieces, which we call the `BPS cohomology'. The construction of the decomposition is based on the cohomological Hall induction, which is a non-linear generalization of the cohomological Hall algebra for abelian categories.

As an application of the intrinsic DT theory, I will propose a formulation of the topological mirror symmetry conjecture for the stack of G-Higgs bundles (in the style of Hausel—Thaddeus) and discuss its relation with the Langlands duality for 3-manifolds. Additionally, I will discuss how the ideas underlying the intrinsic DT theory led me and my collaborators to the construction of cohomological Hall algebras for 3-Calabi-Yau categories, whose existence was conjectured by Kontsevich—Soibelman and Joyce.

This talk is based on the following three papers:

- Cohomology of symmetric stacks (joint with Chenjing Bu, Ben Davison, Andrés Ibáñez Núñez and Tudor Pădurariu)
- Intrinsic Donaldson–Thomas theory. I. Component lattices of stacks (joint with Chenjing Bu, Daniel Halpern-Leistner and Andrés Ibáñez Núñez)
- Cohomological Hall algebras for 3-Calabi-Yau categories (joint with Hyeonjun Park and Pavel Safronov)

3:00 PM