Spectrum Degeneracies in models with O(N) symmetry from Quantum Evanescence

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Outline

1. Motivation: Spectrum degeneracies in $O(N)$ model

2. Specialization rule for character of $O(N)$ irrep

3. Applications to continued partition function

4. Summary and Outlook

Critical $O(N)$ model

$O(N)$ symmetry is ubiquitous in physics

– Example: $O(N)$ conformal field theory (CFT)

	Physical application at $d = 3$
$N = 1$: Critical Ising model	Liquid-gas transition in water and carbon dioxide
$N = 2$: Critical XY model	The λ-line in helium and phase transition in XY ferromagnets
$N = 3$: Heisenberg model	Isotropic magnets

$$S = \int d^d x \left(\frac{1}{2}(\partial_\mu \phi^i)^2 + \frac{1}{2} m^2 \phi^i \phi_i + \frac{\lambda}{24}(\phi^i \phi_i)^2 \right)$$

For $d < 4$, the interaction is relevant

↓

$O(N)$ CFT

Assumption of Spectrum continuity

Spectrum continuity:
$O(N)$ CFT is assumed to be defined even for non-integer values of N and d.
The whole set of CFT data (the *spectrum* of operator and all their correlators) varies continuously with N and d.

— This assumption is observed (at least for low lying operators) by numerical simulations of the $O(N)$ lattice models at various N.

e.g. the critical exponents vary continuously with N.   Hasenbusch, 2112.03783

— How to make sense of the $O(N)$ symmetry for non-integer N? Binder & Rychkov, 1911.07895

Deligne Category $\overline{Rep}(O(N))$ "Categorical symmetry"

We will focus on *invalid* $O(N)$ irreps at integer N

$O(N)$ irreducible representation (irrep)

The conformal primary operators $\left\{ \begin{array}{l} O(N) \text{ irrep } \lambda \\ \text{Lorentz } SO(d) \text{ irrep L} \\ \text{Scaling dimension } \Delta(N, d) \text{ (spectrum)} \end{array} \right.$

- A valid $O(N)$ irreps are labelled by partitions $\lambda = (\lambda_1, \ldots, \lambda_l)$ with length l not exceeding the rank of the group $r = \lfloor N/2 \rfloor$

$()$: scalar irrep, (1): vector irrep,
(2): symmetric traceless irrep, $(1,1)$ antisymmetric irrep

Character: $\chi_\lambda^{O(N)}(x_1, \ldots, x_r) = \mathrm{Tr}_\lambda \; g, \quad$ where $g \in O(N)$

Partition function: $Z^{(N)} = \mathrm{Tr}_{\mathscr{H}} \; g e^{-\beta H} = \displaystyle\sum_{\lambda, l(\lambda) \leq r} \sum_{i=1}^{n(\lambda)} q^{\Delta_{\lambda,i}(N)} \chi_\lambda^{O(N)}$

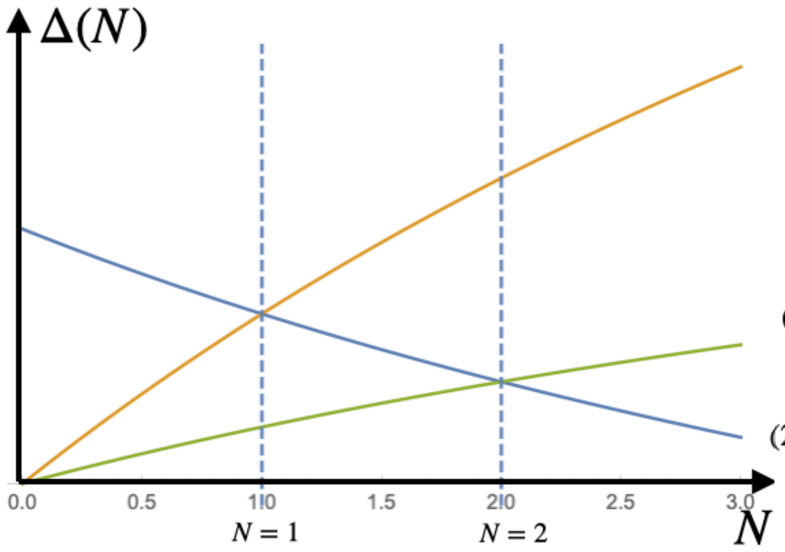
Spectrum degeneracies at Wilson-Fisher fixed point

Examples in the region $d = 4 - \varepsilon, \varepsilon \ll 1$

where the spectrum can be calculated perturbatively in power series of ε

Scaling dimension $\Delta(N)$ of Lorentz scalar $\partial^2\phi^4$ in different $O(N)$ irreps

Henriksson, 2201.09520



$O(N)$ irrep	Scaling dimension
()	$\Delta(N) = 6 - 2\epsilon + \frac{2(N+2)}{N+8}\epsilon$
(2)	$\Delta(N) = 6 - 2\epsilon + \frac{N+4}{N+8}\epsilon$
(2,2)	$\Delta(N) = 6 - 2\epsilon + \frac{6}{N+8}\epsilon$

$$N = 1, \frac{2(N+2)}{N+8} = \frac{6}{N+8} = \frac{2}{3}$$

$$N = 2, \frac{N+4}{N+8} = \frac{6}{N+8} = \frac{3}{5}$$

$(2{,}2)$ irrep is invalid for $N = 1{,}2$. How to make sense of the degeneracies?

Will the degeneracies still exist for higher orders?

Are there other degeneracies of this type? How to find them?

Evanescent operators

These degeneracies are related to *evanescent operators*

In literature, the evanescent operators are operators that appear away from $d = 4$ but vanish at $d = 4$

— Four fermion evanescent operators, vanishing in $d = 4$ for $n \geq 5$

$$\bar{\psi}\gamma^{[\mu_1}\ldots\gamma^{\mu_n]}\psi' \;\bar{\psi}\gamma_{[\mu_1}\ldots\gamma_{\mu_n]}\psi'$$

Dugan & Grinstein, *Phys.Lett.B* 256 (1991) 239-244

— Evanescent operators in scalar CFT and EFT

$$\delta^{\mu_1[\nu_1}\delta^{|\mu_2|\nu_2}\delta^{|\mu_3|\nu_3}\delta^{|\mu_4|\nu_4}\delta^{|\mu_5|\nu_5]}\left(\partial_{\mu_1}\partial_{\nu_1}\phi\right)\left(\partial_{\mu_2}\partial_{\nu_2}\phi\right)\left(\partial_{\mu_3}\partial_{\nu_3}\phi\right)\left(\partial_{\mu_4}\partial_{\nu_4}\phi\right)\left(\partial_{\mu_5}\partial_{\nu_5}\phi\right)$$

Hogervorst, Rychkov & van Rees, 1512.00013
Cao, Herzog, Melia & Roosmale Nepveu, 2105.12742

— Gluonic evanescent operators

$$\delta^{\mu_1[\nu_1}\delta^{|\mu_2|\nu_2}\delta^{|\mu_3|\nu_3}\delta^{|\mu_4|\nu_4}\delta^{|\mu_5|\nu_5]}\,\mathrm{Tr}\left(F_{\mu_1\nu_1}F_{\mu_2\nu_2}F_{\mu_3\nu_3}F_{\mu_4\nu_4}F_{\mu_5\nu_5}\right)$$

Jin, Ren, Yang & Yu, 2208.08976

We now relax the definition of evanescent operator as operators that vanish at certain integer values of parameter N $\quad \delta^{a_1[b_1}\delta^{|a_2|b_2]}\phi^{a_1}\phi^{b_1}\left(\partial^{\mu}\phi^{a_2}\right)\left(\partial_{\mu}\phi^{b_2}\right)$

Violation of unitarity

Evanescent operators contribute to *the violation of unitarity* at non-integer dimensions

At some integer values of N and d, the $O(N)$ CFT is tested to be unitary

e.g. the existence of unitary islands of the critical O(N) model in $d = 3$ dimensions with $N = 1,2,3$ has been confirmed in a bootstrap approach

El-Showk, Paulos, Poland, Rychkov, Simmons-Duffin &Vichi, 1203.6064
Kos, Poland, Rychkov, Simmons-Duffin &Vichi, 1603.04436

At non-integer dimension, evanescent operators can have *negative norms* and lead to complex anomalous dimensions at WF fixed point.

Hogervorst, Rychkov & van Rees, 1512.00013

We find that some evanescent operators gives *negative contributions* to the partition functions at some integer N

And the cancellation of these negative contributions leads to the degeneracies

Other Motivations for non-integer dimension

There are other reasons to consider non-integer dimension even if we are interested only in physics of integer N

— In $1/N$ expansion

Treat the variable of gauge group as a parameter

— Dimensional regularization

Treat the spacetime dimension as a parameter

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2. Specialization rule for character of $O(N)$ irrep

A proper definition of analytic continuation of $O(N)$ representation theory

(1) at the level of character theory and partition function

(2) the continuation will recover the ordinary partition function at every integer N

→ preserve the algebras of characters at each N

Decomposition of product of irreps: $\lambda_1 \otimes \lambda_2 = \oplus_\lambda\, m^\lambda_{\lambda_1\lambda_2}(N)\ \lambda$

Algebras of characters: $$\chi^{O(N)}_{\lambda_1}\chi^{O(N)}_{\lambda_2} = \sum_{\lambda, l(\lambda)\leq r} m^\lambda_{\lambda_1\lambda_2}(N)\ \chi^{O(N)}_\lambda$$

Newell-Littlewood numbers $$\bar{m}^\lambda_{\lambda_1\lambda_2}(N) = \lim_{N\to\infty} m^\lambda_{\lambda_1\lambda_2}$$

2. Specialization rule for character of $O(N)$ irrep

Example:

$$N \geq 4, \quad (1) \times (1) = (2) + (1,1) + ()$$

$$N = 3, \quad (1) \times (1) = (2) + (1) + ()$$

$$N = 2, \quad (1) \times (1) = (2) + 2\ ()$$

2. Specialization rule for character of $O(N)$ irrep

A proper definition of the continued character $\bar{\chi}_{\lambda}^{O(N)}$ should

(1) satisfy the algebra in the large N asymptotic limit

(2) and recover the algebra at every finite N

Continued character $\bar{\chi}_{\lambda}^{O(N)}$

(1) $\bar{\chi}_{\lambda_1}^{O(N)} \bar{\chi}_{\lambda_2}^{O(N)} = \sum_{\text{all } \lambda} \bar{m}_{\lambda_1 \lambda_2}^{\lambda} \, \bar{\chi}_{\lambda}^{O(N)}$

(2) Define a map from the continued character $\bar{\chi}_{\lambda}^{O(N)}$ to the valid character at given value of N. (Specialization rule)

K. Koike and I. Terada, Journal of Algebra 107, 466 (1987).

2. Specialization rule for character of $O(N)$ irrep

Specialization rule at $N = N_0$:

If λ is a valid irrep at N_0, $l(\lambda) \leq r$, $\bar{\chi}_\lambda^{O(N_0)} = \chi_\lambda^{O(N_0)}$

If λ is a invalid irrep at N_0, $l(\lambda) > r$, $\bar{\chi}_\lambda^{O(N_0)} = 0, \pm \chi_{\lambda'}^{O(N_0)}$

Where λ' is a valid irrep at N_0

$N \geq 4, \quad (1) \times (1) = (2) + (1,1) + ()$ $\qquad \bar{\chi}_{(1,1)}^{O(N \geq 4)} = \chi_{(1,1)}^{O(N \geq 4)}$

$N = 3, \quad (1) \times (1) = (2) + (1) + ()$ $\qquad \bar{\chi}_{(1,1)}^{O(3)} = \chi_{(1)}^{O(3)}$

$N = 2, \quad (1) \times (1) = (2) + 2()$ $\qquad \bar{\chi}_{(1,1)}^{O(2)} = \chi_{()}^{O(2)}$

2. Specialization rule for character of $O(N)$ irrep

I will give a practical way to compute the specialization of a character

Characters of vector irrep: $$\chi^{O(N)}_{(1)}(x_1,\ldots,x_r) = \frac{1-(-1)^N}{2} + \sum_{i=1}^{N}(x_i + x_i^{-1})$$

$$\bar{\chi}^{O(N)}_{(1)}(x_1,\ldots,x_r) = \chi^{O(N)}_{(1)}(x_1,\ldots,x_r), \quad \forall \text{integer } N \geq 1$$

The continued characters $\bar{\chi}^{O(N)}_{\lambda}$ is a functional of $\bar{\chi}^{O(N)}_{(1)}$

$$\bar{\chi}^{O(N)}_{\lambda} = \det[p_{\lambda_i-i+j} - p_{\lambda_i-i-j}]$$

K. Koike and I. Terada, Journal of Algebra 107, 466 (1987).

where p_n is the nth symmetric product of $\bar{\chi}^{O(N)}_{(1)}$ for $n \geq 0$ and $p_{n<0} = 0$

2. Specialization rule for character of $O(N)$ irrep

p_n is defined by the plethystic exponential

$$\sum_{n=0}^{\infty} u^n p_n = \exp\left[\sum_{k=1}^{\infty} \frac{1}{u^k} \bar{\chi}_V(x^k)\right], \quad \bar{\chi}_V \equiv \bar{\chi}_{(1)}^{O(N)}$$

$$p_0 = 1$$
$$p_1 = \bar{\chi}_V$$
$$p_2 = \frac{1}{2}\left(\bar{\chi}_V(x)^2 + \bar{\chi}_V(x^2)\right)$$
$$p_3 = \frac{1}{6}\left(\bar{\chi}_V(x)^3 + 3\bar{\chi}_V(x)\bar{\chi}_V(x^2) + 2\bar{\chi}_V(x^3)\right)$$

……

2. Specialization rule for character of $O(N)$ irrep

$$\chi^{O(N)}_{(1)}(x_1, \ldots, x_r) = \frac{1-(-1)^N}{2} + \sum_{i=1}^{N} (x_i + x_i^{-1})$$

For example $$\bar{\chi}^{O(N)}_{(1,1)} = \frac{1}{2}\left[\chi^{O(N)}_{(1)}(x)\right]^2 - \frac{1}{2}\chi^{O(N)}_{(1)}(x^2)$$

For $N \geq 4$, $$\bar{\chi}^{O(N\geq 4)}_{(1,1)}(x) = \chi^{O(N\geq 4)}_{(1,1)}(x)$$

For $N = 3$, $$\bar{\chi}^{O(3)}_{(1,1)} = 1 + x_1 + x_1^{-1} = \chi^{O(3)}_{(1)}(x)$$

For $N = 2$, $$\bar{\chi}^{O(2)}_{(1,1)} = 1 = \chi^{O(2)}_{()}(x)$$

$\overline{\chi}^{O(N)}_{\lambda}$	$N=2$	$N=3$	$N=4$	$N=5$	$N=6$
$(1,1)$	$\chi^{O(N)}_{()}$	$\chi^{O(N)}_{(1)}$	$\chi^{O(N)}_{(1,1)} \rightarrow$		

2. Specialization rule for character of $O(N)$ irrep

More examples:

$\overline{\chi}_\lambda^{O(N)}$	$N=2$	$N=3$	$N=4$	$N=5$	$N=6$
$(1,1)$	$\chi_{()}^{O(N)}$	$\chi_{(1)}^{O(N)}$	$\chi_{(1,1)}^{O(N)} \rightarrow$		
$(2,2)$	$-\chi_{(2)}^{O(N)}$	0	$\chi_{(2,2)}^{O(N)} \rightarrow$		
$(2,1,1)$	$-\chi_{()}^{O(N)}$	0	$\chi_{(2)}^{O(N)}$	$\chi_{(2,1)}^{O(N)}$	$\chi_{(2,1,1)}^{O(N)} \rightarrow$
$(2,2,1)$	$-\chi_{(1)}^{O(N)}$	$-\chi_{(2)}^{O(N)}$	0	$\chi_{(2,2)}^{O(N)}$	$\chi_{(2,2,1)}^{O(N)} \rightarrow$
$(2,2,2)$	$-\chi_{()}^{O(N)}$	$-\chi_{(2)}^{O(N)}$	$-\chi_{(2,2)}^{O(N)}$	0	$\chi_{(2,2,2)}^{O(N)} \rightarrow$

The negative sign lead to the degeneracies

Specialization rule for character of $SU(N)$ irrep

$$\overline{\chi}^{SU(N)}_{\lambda=(\lambda_1,\cdots,\lambda_l)} = \begin{cases} \chi^{SU(N)}_{\lambda} & N > l \\ \chi^{SU(N)}_{(\lambda_1-\lambda_l,\cdots,\lambda_{l-1}-\lambda_l)} & N = l \\ 0 & N < l \end{cases},$$

At $N = 3$, $\bar{\chi}^{SU(3)}_{(3,3,2)} = \chi^{SU(3)}_{(1,1)}$

Clipping rule:

1. Identify the West Coast;
2. Determine whether to clip;
3. Clip and repeat the steps.

$\overline{\chi}^{SU(N)}_{\lambda}$	$N=2$	$N=3$	$N=4$	$N=5$
$(1,1)$	$\chi^{SU(N)}_{()}$	$\chi^{SU(N)}_{(1,1)} \to$		
$(2,1)$	$\chi^{SU(N)}_{(1)}$	$\chi^{SU(N)}_{(2,1)} \to$		
$(3,1)$	$\chi^{SU(N)}_{(2)}$	$\chi^{SU(N)}_{(3,1)} \to$		
$(2,2)$	$\chi^{SU(N)}_{()}$	$\chi^{SU(N)}_{(2,2)} \to$		
$(1,1,1)$	0	$\chi^{SU(N)}_{()}$	$\chi^{SU(N)}_{(1,1,1)} \to$	
$(2,1,1)$	0	$\chi^{SU(N)}_{(1)}$	$\chi^{SU(N)}_{(2,1,1)} \to$	
$(1,1,1,1)$	0	0	$\chi^{SU(N)}_{()}$	$\chi^{SU(N)}_{(1,1,1,1)} \to$

Clipping rule for character of $O(N)$ irrep

The Clipping rule for $O(N)$ is more complicated

1. Identify the *East Coast* labelled by $i - j$

2. Identify the *clipping center*: $s = r = \lfloor N/2 \rfloor$

3. Identify the *clipping patch* on the coast

   n_B: the number of boxes strictly below the center

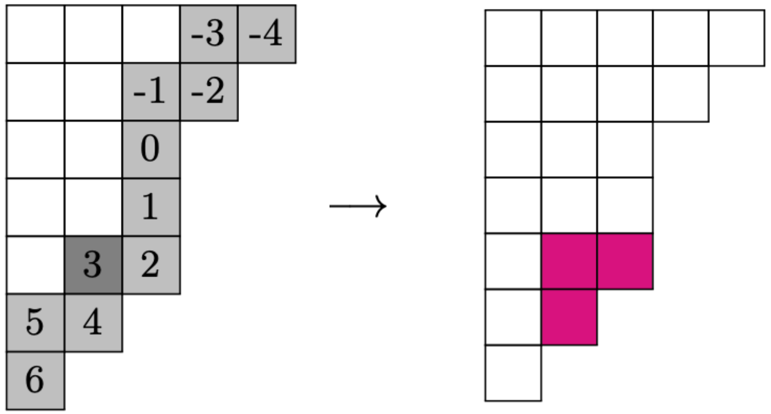
   $$n_A = n_B + \frac{1 + (-1)^N}{2}$$

   the number of boxes east or north from the center

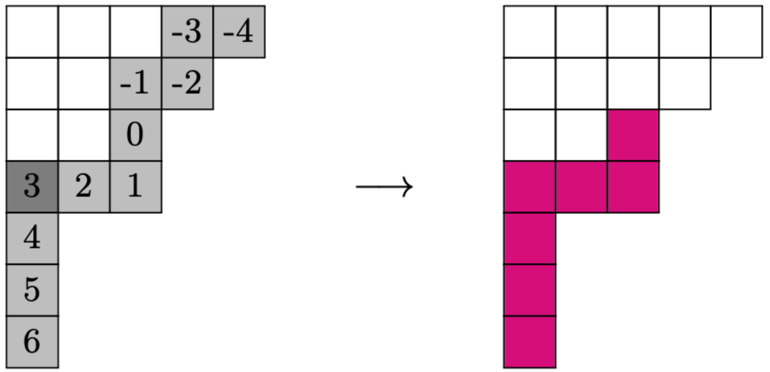
4. Clip and repeat steps

$$\bar{\chi}_{\lambda}^{O(N)} = (-1)^{n_{rows}+N} \bar{\chi}_{\lambda_{new}}^{O(N)}$$

$$\bar{\chi}^{O(7)}_{(5,4,3,3,3,2,1)} = -\bar{\chi}^{O(7)}_{(5,4,3,3,1,1,1)}$$



$$\bar{\chi}^{O(7)}_{(5,4,3,3,1,1,1)} = \bar{\chi}^{O(7)}_{(5,4,2)} = \chi^{O(7)}_{(5,4,2)}$$



$$\rightarrow \bar{\chi}^{O(7)}_{(5,4,3,3,3,2,1)} = -\chi^{O(7)}_{(5,4,2)}$$

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3. Applications to continued partition function

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3. Application to partition function

Following the continuation of the characters, one get the continued partition function.

$$\bar{Z} = \sum_{\text{all } \lambda} \sum_{i=1}^{n(\lambda)} q^{\Delta_{\lambda,i}(N)} \bar{\chi}_{\lambda}^{O(N)}$$

Specialize at give N_0,

$$\bar{Z}|_{N=N_0} = Z^{(N_0)}$$

$$\begin{aligned}
\bar{Z}|_{N=N_0} &= \sum_{\lambda, l(\lambda)\leq r} \sum_{i=1}^{n'(\lambda)} q^{\Delta_{\lambda,i}(N_0)} \bar{\chi}_{\lambda}^{O(N_0)} \\
&+ \sum_{\lambda^0} \sum_{i=1}^{n(\lambda^0)} q^{\Delta_{\lambda^0,i}(N_0)} \bar{\chi}_{\lambda^0}^{O(N_0)} + \sum_{\lambda^+} \sum_{i=1}^{n(\lambda^+)} q^{\Delta_{\lambda^+,i}(N_0)} \bar{\chi}_{\lambda^+}^{O(N_0)} + \sum_{\lambda^-} \sum_{i=1}^{n(\lambda^-)} q^{\Delta_{\lambda^-,i}(N_0)} \bar{\chi}_{\lambda^-}^{O(N_0)} \\
&= \sum_{\lambda, l(\lambda)\leq r} \chi_{\lambda}^{O(N_0)} \left(\sum_{i=1}^{n'(\lambda)} q^{\Delta_{\lambda,i}(N_0)} + \sum_{i=1}^{n(\lambda^+)} q^{\Delta_{\lambda^+,i}(N_0)} - \sum_{i=1}^{n(\lambda^-)} q^{\Delta_{\lambda^-,i}(N_0)} \right)
\end{aligned}$$

$$\boxed{\bar{\chi}_{\lambda^0}^{O(N_0)} = 0, \quad \bar{\chi}_{\lambda^\pm}^{O(N_0)} = \pm \chi_{\lambda'}^{O(N_0)}}$$

3. Application to partition function

$$\bar{Z}|_{N=(N_0)} = \sum_{\lambda, l(\lambda)\leq r} \chi_\lambda^{O(N_0)} \left(\sum_{i=1}^{n'(\lambda)} q^{\Delta_{\lambda,i}(N_0)} + \sum_{i=1}^{n(\lambda^+)} q^{\Delta_{\lambda^+,i}(N_0)} - \sum_{i=1}^{n(\lambda^-)} q^{\Delta_{\lambda^-,i}(N_0)} \right)$$

Compared with $$Z^{(N_0)} = \sum_{\lambda, l(\lambda)\leq r} \sum_{i=1}^{n(\lambda)} q^{\Delta_{\lambda,i}(N_0)} \chi_\lambda^{O(N_0)}$$

1.Direct evanescent operators

Operators with irrep λ^0 Null at N_0. No contribution to partition function.

2.Pair annihilation evanescent operators

Operators with irrep λ^- Negative contribution to the partition function.

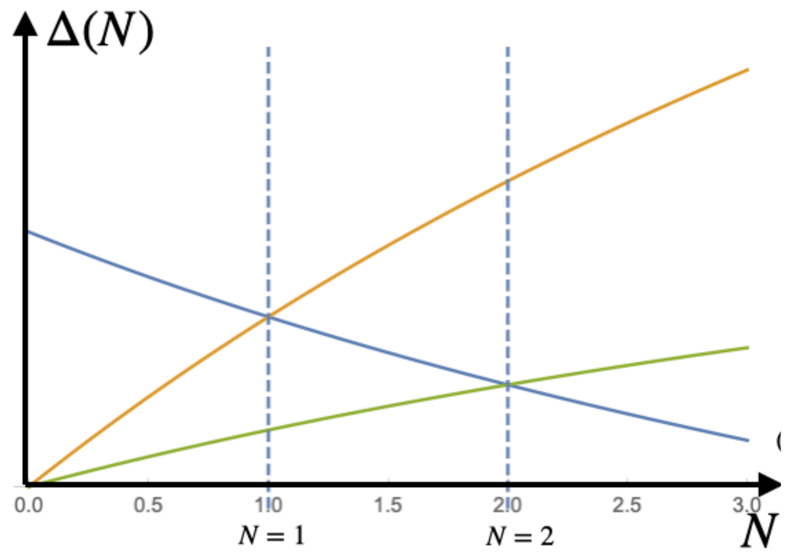
Must be cancelled. $\Delta_{\lambda^-,i}(N_0) = \Delta_{\lambda^+,j}(N_0)$ or $\Delta_{\lambda,k}(N_0)$

3. Application to partition function

$$\Delta_{\lambda^-,i}(N_0) = \Delta_{\lambda,k}(N_0)$$

Henriksson, 2201.09520

Lorentz irrep	$O(N)$ irrep λ	$\Delta(N)$
$()$	$()$	$6 - 2\epsilon + \frac{2(N+2)}{N+8}\,\epsilon + \mathcal{O}(\epsilon^2)$
$()$	$(2,2)$	$6 - 2\epsilon + \frac{6}{N+8}\,\epsilon + \mathcal{O}(\epsilon^2)$
$()$	(2)	$6 - 2\epsilon + \frac{N+4}{N+8}\,\epsilon + \mathcal{O}(\epsilon^2)$
(2)	$(2,2)$	$6 - 2\epsilon + \frac{14}{3(N+8)}\,\epsilon + \mathcal{O}(\epsilon^2)$
(2)	$()$	$6 - 2\epsilon + f(N)\,\epsilon + \mathcal{O}(\epsilon^2)$
$(1,1)$	$(2,2)$	$6 - 2\epsilon + \frac{14}{3(N+8)}\,\epsilon + \mathcal{O}(\epsilon^2)$
$(1,1)$	(2)	$6 - 2\epsilon + g(N)\,\epsilon + \mathcal{O}(\epsilon^2)$



$$\bar{\chi}^{O(2)}_{(2,2)} = -\chi^{O(2)}_{(2)}(x)$$

$$\bar{\chi}^{O(1)}_{(2,2)} = -\chi^{O(1)}_{()}(x)$$

$f(N) = (44 + 9N - \sqrt{624 - 8N + 9N^2})/(6(N+8))$ and $f(N=1) = 14/27 = 14/(3(N+8))\,|_{N=1}$

$g(N)$ is given as the root of a cubic equation, with $g(N=2) = 7/15 = 14/(3(N+8))\,|_{N=2}$

3. Application to partition function

One more example

Henriksson, 2201.09520

degeneracy at $N = 2$

$$\frac{5}{2} = \frac{4}{N+8} = \frac{N+2}{N+8}$$

$$\bar{\chi}^{O(2)}_{(2,1,1)} = -\chi^{O(2)}_{()}(x)$$

$$\bar{\chi}^{O(2)}_{(1,1)} = \chi^{O(2)}_{()}(x)$$

Lorentz irrep	$O(N)$ irrep λ	$\Delta(N)$
$(1,1)$	$(2,1,1)$	$6 - 2\epsilon + \frac{4}{N+8}\epsilon + \mathcal{O}(\epsilon^2)$
$(1,1)$	$(1,1)$	$6 - 2\epsilon + \frac{N+2}{N+8}\epsilon + \mathcal{O}(\epsilon^2)$

$$\Delta_{\lambda^-,i}(N_0) = \Delta_{\lambda^+,j}(N_0)$$

— Some λ^+ operators will remain and contribute to the physics spectrum

— All our examples are leading order results. Interesting to check it to higher order or non-perturbatively.

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Summary

— We identify new degeneracies in the energy spectrum $\{\Delta(N)\}$ of theories with a global $O(N)$ symmetry

where N is treated as a continuous parameter, and assuming spectrum continuity

— We also show that these degeneracies originate from the evanescent states of a global $O(N)$ that drops out of the spectrum in pairs

The contributions of two states to the partition function have to cancel, which in turn require to have equal energy

—We illustrate the findings in the $O(N)$ CFT in $d = 4 - \varepsilon$ and work at leading perturbative order of ε, but we emphasize that the degeneracies in general hold at the non-perturbative level and conformal symmetry is not a requisite.

Outlook

— a full set of degeneracies in the spectra of the $O(N)$ model

— verification of the degeneracies beyond leading order or non-perturbatively

— Will the degeneracies provide useful input/constraint to the bootstrap program?

— continuation of spinor irrep and degeneracies of fermionic theory

— study of other groups like $sp(2N)$

— Are there further hidden symmetry?

— study of $O(N)$ symmetry at non-integer N

— study of spacetime evanescent operators under a continuation of the spacetime dimension

Thank you!