Positive trajectories in EFT and large-N QCD

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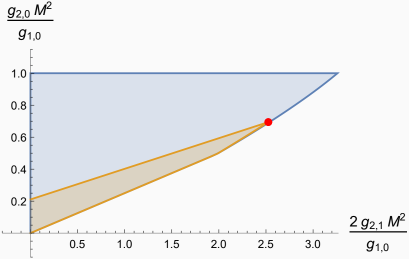
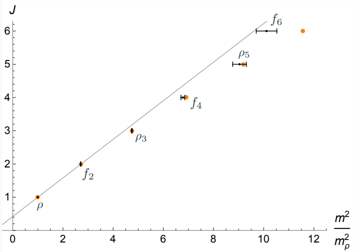
Work with Jan Albert, Leonardo Rastelli, Alessandro Vichi [2312.15013]

Summary: Positive trajectories in EFT and large-N QCD

Take-home messages

- Systematic methods to use bootstrap and semidefinite programming for bounds on scattering amplitudes
  For tree-level EFT — "positivity" ⇝ full set of constraints
- Suited for large-N QCD (theory of weakly interacting mesons)





[Albert, JH, Rastelli, Vichi 2023]

Plan

- Part 1. Pedagogical introduction to EFT bootstrap $\phi\phi \to \phi\phi$
- Part 2. Large-N QCD $\pi\pi \to \pi\pi$

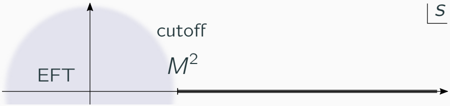
Part 1

Pedagogical introduction to EFT bootstrap

$$\phi\phi \to \phi\phi$$

$\mathcal{A}_{\phi\phi\to\phi\phi}(s,t,u)$

$u = -s - t$



Low-energy

- $\mathcal{L}_{\text{EFT}} = \frac{1}{2}(\partial\phi)^2 + \frac{\lambda}{24}\phi^4 + \frac{g_2}{2}(\partial\phi)^4 + \frac{g_3}{3}(\partial\phi)^2(\partial\partial\phi)^2 + \ldots$
- $\mathcal{A}_{\text{EFT}} = -\lambda + g_2(s^2+t^2+u^2) + g_3\, stu + \ldots$
- **Weakly coupled EFT**

High-energy

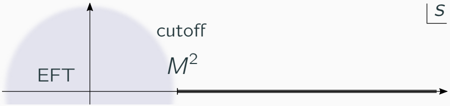
- Analyticity
- Unitarity
- P. w. expansion
- Regge bound $\lim_{|s|\to\infty} \frac{\mathcal{A}(s,t)}{s^2} = 0$

Goals: Derive rigorous bounds on EFT coefficients (dual bootstrap)

$$\frac{-\#}{M^2} \leqslant \frac{g_3}{g_2} \leqslant \frac{\#}{M^2}$$

Find interesting partial UV completions saturating the bounds

EFT bootstrap: Assumptions



Assumptions

- Analyticity + weakly coupled EFT

$$\text{cuts } s > M^2,\ s < -t - M^2$$

- Unitarity + Partial wave expansion $\rho_J(s) = \operatorname{disc} f_J(s),\ |1 + i f_J| \leqslant 1$

$$\operatorname{disc} \mathcal{A}(s,t) = \sum_J 16\pi(2J+1)\rho_J(s) P_J\left(1 + \frac{2t}{s}\right), \qquad \underbrace{0 \leqslant \rho_J(s)}_{\text{positivity}} \leqslant 2$$

- Regge bound

$$\lim_{|s|\to\infty} \frac{\mathcal{A}(s,t)}{s^2} = 0$$

Worked example: Setup

"Extremal Effective Field Theories" **[Caron-Huot & van Duong 2020]**

$$\mathcal{L}_{\text{EFT}} = \frac{1}{2}(\partial\phi)^2 + \frac{\lambda}{24}\phi^4 + \frac{g_2}{2}(\partial\phi)^4 + \frac{g_3}{3}(\partial\phi)^2(\partial\partial\phi)^2 + \dots$$

Most general low-energy amplitude ($s + t + u = 0$)

$$\mathcal{A}_{\text{EFT}}(s,t) = -\lambda + g_2(s^2 + t^2 + u^2) + g_3\, stu + g_4(s^2 + t^2 + u^2)^2 + \dots$$

1. Derive sum rules from dispersion relation

$$g_2 = \sum \int [\text{UV-data}] \qquad \text{etc.}$$

2. Numerical optimisation (semidefinite programming, SDPB)

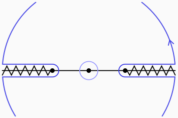
Let's try to get a dispersion relation for g_2

Worked example: Sum rules

$$\mathcal{A} = g_2(s^2 + t^2 + u^2) + \ldots$$

Let's try to get a dispersion relation for g_2

$$\lim_{|s|\to\infty} \frac{\mathcal{A}}{s^2} = 0$$



$$g_2 = \frac{1}{2}\oint \frac{ds}{2\pi i\, s}\, \frac{\mathcal{A}(s,t)}{s^2}\Big|_{t=0}$$

$$= \frac{1}{2}\int_{M^2}^{\infty} \frac{ds}{\pi s}\, \frac{\text{disc}\,\mathcal{A}(s,t)}{s^2}\Big|_{t=0} + \text{LH cut} + \overbrace{\text{arc at } \infty}^{=0}$$

$$= \frac{1}{2}\int_{M^2}^{\infty} \frac{ds}{\pi s} \sum_{J=0,2,\ldots} 16\pi(2J+1)\rho_J(s)\, \underbrace{\frac{P_J(1+\frac{2t}{s})}{s^2}\Big|_{t=0}}_{1/s^2} + \text{LH cut}$$

$$= \int_{M^2}^{\infty} \frac{ds}{s} \sum_{J=0,2,\ldots} 16(2J+1)\rho_J(s)\frac{1}{s^2}$$

$$\rho_J = \text{disc}\, f_J$$

Worked example: Positivity of g_2 from sum rule

$$g_2 = \underbrace{\sum_{J=0,2,\ldots} 16(2J+1) \int_{M^2}^{\infty} \frac{ds}{s} \rho_J(s)}_{\substack{\text{same for all observables} \\ \text{unknown} \\ \textbf{positive}}} \underbrace{\frac{1}{s^2}}_{\substack{\text{specific for } g_2 \\ \text{explicit}}}$$

"Bracket notation" [Caron-Huot & van Duong 2020]

$$g_2 = \left\langle \frac{1}{s^2} \right\rangle$$

$\langle \cdot \rangle$ is a positive transform. Conclusion $g_2 \geqslant 0$.

Need to check: $\frac{1}{s^2} \geqslant 0 \qquad \forall s \geqslant M^2$

$\forall J = 0, 2, 4, \ldots$

Equivalently [Adams, Arkani-Hamed, Dubovsky, Nicolis, Rattazzi 2006]

$$g_2 \propto \int_{M^2}^{\infty} ds\, s^{-2} \sigma_{\phi\phi\to\text{any}}(s) \geqslant 0$$

Worked example: Upper bound on ratio

Let's try something more advanced $\mathcal{A} = \ldots + g_3 stu + \ldots$

$$g_2 = \left\langle \frac{1}{s^2} \right\rangle$$

$$g_3 = -\oint \frac{ds}{2\pi i\, s} \left.\frac{\partial_t A(s,t)}{s^2}\right|_{t=0} = \ldots = \left\langle \frac{3 - 2J(J+1)}{s^3} \right\rangle$$

Assume we want to show that $Ag_2 - g_3 \geqslant 0$

$$Ag_2 - g_3 = \left\langle \frac{A}{s^2} - \frac{3}{s^3} + \frac{2J(J+1)}{s^3} \right\rangle$$

Need to check

$$\frac{A}{s^2} - \frac{3}{s^3} + \frac{2J(J+1)}{s^3} \geqslant 0 \qquad \forall s \geqslant M^2$$

$$\forall J = 0, 2, 4, \ldots$$

Satisfied for $A \geqslant A_* = \dfrac{3}{M^2}$

Conclusion

$$A_* g_2 - g_3 \geqslant 0 \qquad \Rightarrow \qquad \frac{g_3}{g_2} \leqslant A_* = \frac{3}{M^2}$$

Worked example: Null constraints

For lower bound on $\frac{g_3}{g_2}$, need null constraints [Caron-Huot & Van Duong 2020]

Two rules for the same coefficient $\qquad \mathcal{A} = \ldots + g_4(s^2+t^2+u^2)^2 + \ldots$

$$\frac{\mathcal{A}(s,t)}{s^4} \quad \Rightarrow \quad g_4 = \left\langle \frac{1}{2s^4} \right\rangle$$

$$\frac{\partial_t^2 \mathcal{A}(s,t)}{6s^2} \quad \Rightarrow \quad g_4 = \left\langle \frac{8-8J-7J^2+2J^3+J^4}{16s^4} \right\rangle$$

$$\text{subtract} \quad \Rightarrow \quad 0 = \left\langle \frac{J(J+1)(J^2+J-8)}{s^4} \right\rangle$$

Constraints on high-energy only

- Gives upper and lower bounds $\frac{-10.6124872}{M^2} \leqslant \frac{g_3}{g_2} \leqslant \frac{3}{M^2}$
- Introduces notion of numerical strength (larger N_{null}, stronger bounds)

See also [Bellazzini, Elias Miró, Rattazzi, Riembau and Riva 2020; Tolley, Wang, Zhou 2020; Arkani-Hamed, Huang, Huang 2020]

Worked example: A slide for the experts

Semi-definite problem semidefinite bootstrap solver SDPB [Simmons-Duffin 2015]

$$\begin{pmatrix} g_2 \\ g_3 \\ 0 \\ \vdots \end{pmatrix} = \begin{pmatrix} \left\langle \frac{1}{s^2} \right\rangle \\ \left\langle \frac{3-2J(J+1)}{s^3} \right\rangle \\ \left\langle \frac{J(J+1)(J^2+J-8)}{s^4} \right\rangle \\ \vdots \end{pmatrix} \Leftrightarrow \vec{L} = \left\langle \vec{V} \right\rangle$$

Search for functionals $\alpha[\vec{V}] = \vec{\alpha} \cdot \vec{V} \geqslant 0$ for all $s \geqslant M^2$, for all $J = 0, 2, \ldots$

Then $\langle \alpha[\vec{V}] \rangle \geqslant 0 \Rightarrow \alpha[\vec{L}] \geqslant 0$

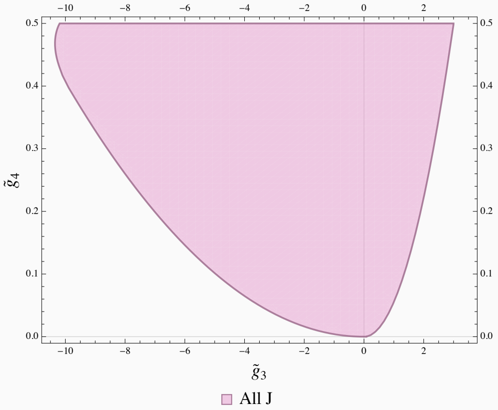
For lower bound: choose $\vec{\alpha} = (-B, 1, \alpha_3, \ldots)$. Max B keeping $\alpha[\vec{V}] \geqslant 0$

Result $B_* = -10.6124872$ ($N_{\text{Null}} = 1$)

$$-B_* g_2 + g_3 \geqslant 0 \quad \Rightarrow \quad \frac{B_*}{M^2} \leqslant \frac{g_3}{g_2}$$

Do try this at home!

Allowed regions



Pink: allowed

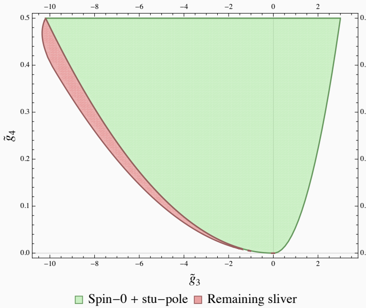
$$\tilde{g}_3 = \frac{g_3 M^2}{g_2}, \; \tilde{g}_4 = \frac{g_4 M^4}{g_2}$$

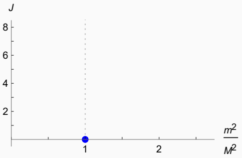
[Caron-Huot & Van Duong 2020]

Corners/kinks may be saturated by interesting extremal amplitudes: "partial UV completions"

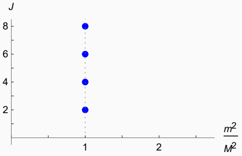
Populating the allowed regions

$\mathcal{A}_{stu}$ $\mathcal{A}_{\text{spin-0}}$





$$\mathcal{A}_{\text{spin-0}} = \frac{1}{M^2 - s} + \frac{1}{M^2 - t} + \frac{1}{M^2 - u}$$



$$\mathcal{A}_{stu} = \frac{M^4}{(M^2 - s)(M^2 - t)(M^2 - u)} - \frac{\ln 4}{3}\mathcal{A}_{\text{spin-0}}$$

Summary of part 1

$$\mathcal{L}_{\text{EFT}} = \tfrac{1}{2}(\partial\phi)^2 + \tfrac{\lambda}{24}\phi^4 + \tfrac{g_2}{2}(\partial\phi)^4 + \tfrac{g_3}{3}(\partial\phi)^2(\partial\partial\phi)^2 + \ldots$$

$$\mathcal{A}_{\text{EFT}}(s,t) = -\lambda + g_2(s^2+t^2+u^2) + g_3\, stu + g_4(s^2+t^2+u^2)^2 + \ldots$$

- λ inaccessible
- $g_2 \geqslant 0$ positivity of elastic amplitude at two subtractions
- Upper and lower bound on any $\dfrac{g_i}{g_2}$. *Puts dimensional analysis on a firm footing*

Charged scalars [McPeak, Venuti, Vichi], Photons [JH, McPeak, Russo, Vichi], gravitons [Caron-Huot, Li, Parra-Martinez, Simmons-Duffin], massless fields coupled to gravity [Caron-Huot, Mazáç, Rastelli, Simmons-Duffin], pions [Albert & Rastelli, . . .], massive vector [Bertucci, JH, McPeak, Ricossa, Riva, Vichi 2024], connection with non-perturbative bounds [Chen, Fitzpatrick, Karateev], etc. etc.

Part 2

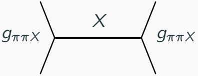
Large-N QCD

$\pi\pi \to \pi\pi$

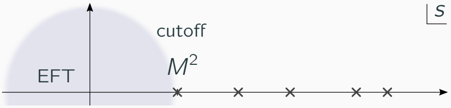
Large-N QCD

Planar limit ($N \to \infty$): $q\bar{q}$ mesons form a closed subsector **['t Hooft; Witten; ...]**

$\mathcal{A}_{\pi\pi\to\pi\pi}(s,t) \sim$



$\sim \frac{1}{N}$



Low-energy

- Chiral perturbation theory
- **Data:** $\{g_{0,1}, g_{2,0}, g_{2,1}, \ldots\} \leftrightarrow \{f_\pi, L_1, L_2, \ldots\}$
- Weak coupling assumptions satisfied everywhere

High-energy

- Meromorphic amplitude (stable mesons exchanged at tree-level)
- **Data:** meson masses and onshell couplings $\{m_X^2, g_{\pi\pi X}^2, \ldots\}$
- Cut $\rightsquigarrow$ poles $\rho_J(s) \sim \sum_X g_{\pi\pi X}^2 \delta(s - m_X^2)\delta_{JJ_X}$

Is there a unique set of meson masses m_i^2 and three-point couplings g_{ijk} consistent with crossing, unitarity, Regge limit, etc?

Would be large-N QCD

Planar $\pi\pi \to \pi\pi$ scattering amplitude

Disc amplitude $\mathcal{A}_{\pi^a\pi^b\pi^c\pi^d} = 4[\mathrm{Tr}(T_a T_b T_d T_c) + \mathrm{Tr}(T_a T_c T_d T_b)]M(s,t) + \dots$

- $M(s,t) = M(t,s)$ with no LH cut
- Pomeron suppressed $\Rightarrow$ improved Regge bound $M(s,t) \sim s^{\alpha_\rho(0)}$, $\alpha_\rho(0) < 1$

$$\lim_{|s|\to\infty} \frac{M(s,t)}{s} = 0$$

Parametrisation

$$M(s,t) = 0 + g_{1,0}(s+t) + g_{2,0}(s^2+t^2) + 2g_{2,1}st + \dots$$

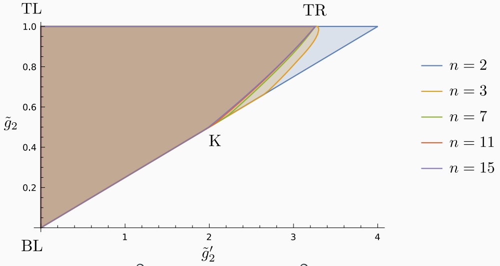
All EFT coefficients dispersive!

N-independent observables

$$\tilde{g}_2' = \frac{2g_{2,1}M^2}{g_{1,0}} \sim N^0, \qquad \tilde{g}_2 = \frac{g_{2,0}M^2}{g_{1,0}} \sim N^0$$

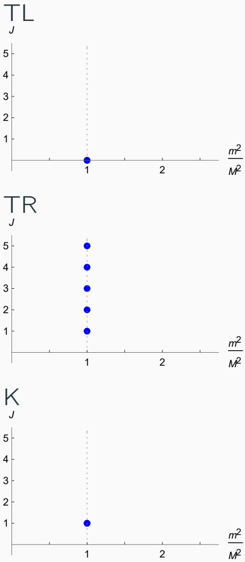
[Albert & Rastelli 2022]

$$M(s,t) = g_{1,0}(s+t) + g_{2,0}(s^2+t^2) + 2g_{2,1}st + \ldots$$



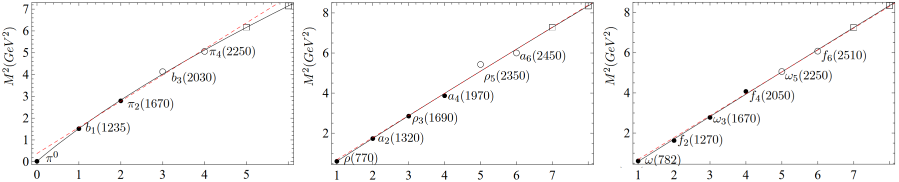
$$\tilde{g}_2' = \frac{2g_{2,1}M^2}{g_{1,0}}, \quad \tilde{g}_2 = \frac{g_{2,0}M^2}{g_{1,0}}$$

Revisited in [Fernandez, Pomarol, Riva, Sciotti 2022; Yue-Zhou Li 2023]



Physical mesons

Regge trajectories [Chen 2021]



Large-N selection rules:

- J even: f_J mesons ($I = 0$)
- J odd: ρ_J mesons ($I = 1$)

ρ meson $m_\rho^{\text{phys}} = 775$ MeV$/c^2$

f_2 meson $m_{f_2}^{\text{phys}} = 1275$ MeV$/c^2$

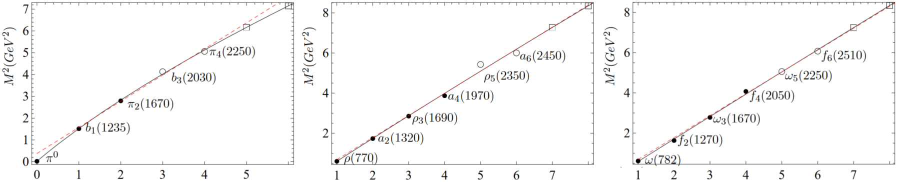
ρ_3 meson $m_{\rho_3}^{\text{phys}} = 1688$ MeV$/c^2$

$n^{2s+1}\ell_J$	J^{PC}	$I = 1$ $u\bar{d}$, $\bar{u}d$, $\frac{1}{\sqrt{2}}(d\bar{d} - u\bar{u})$	$I = 0$ f'	$I = 0$ f
1^1S_0	0^{-+}	π	η	$\eta'(958)$
1^3S_1	1^{--}	$\rho(770)$	$\phi(1020)$	$\omega(782)$
1^1P_1	1^{+-}	$b_1(1235)$	$h_1(1415)$	$h_1(1170)$
1^3P_0	0^{++}	$a_0(1450)$	$f_0(1710)$	$f_0(1370)$
1^3P_1	1^{++}	$a_1(1260)$	$f_1(1420)$	$f_1(1285)$
1^3P_2	2^{++}	$a_2(1320)$	$f_2'(1525)$	$f_2(1270)$
1^1D_2	2^{-+}	$\pi_2(1670)$	$\eta_2(1870)$	$\eta_2(1645)$
1^3D_1	1^{--}	$\rho(1700)$		$\omega(1650)$
1^3D_2	2^{--}			
1^3D_3	3^{--}	$\rho_3(1690)$	$\phi_3(1850)$	$\omega_3(1670)$
1^3F_4	4^{++}	$a_4(1970)$	$f_4(2300)$	$f_4(2050)$
1^3G_5	5^{--}	$\rho_5(2350)$		
2^1S_0	0^{-+}	$\pi(1300)$	$\eta(1475)$	$\eta(1295)$
2^3S_1	1^{--}	$\rho(1450)$	$\phi(1680)$	$\omega(1420)$
2^3P_1	1^{++}	$a_1(1640)$		
2^3P_2	2^{++}	$a_2(1700)$	$f_2(1950)$	$f_2(1640)$

[PDG]

Physical mesons

Regge trajectories [Chen 2021]



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ρ_3 meson $m_{\rho_3}^{\text{phys}} = 1688$ MeV$/c^2$

$n^{2s+1}\ell_J$	J^{PC}	I $= 1$ $u\bar{d}$, $\bar{u}d$, $\frac{1}{\sqrt{2}}(d\bar{d} - u\bar{u})$	I $= 0$ f'	I $= 0$ f
1^1S_0	0^{-+}	π	η	$\eta'(958)$
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1^1P_1	1^{+-}	$b_1(1235)$	$h_1(1415)$	$h_1(1170)$
1^3P_0	0^{++}	$a_0(1450)$	$f_0(1710)$	$f_0(1370)$
1^3P_1	1^{++}	$a_1(1260)$	$f_1(1420)$	$f_1(1285)$
1^3P_2	2^{++}	$a_2(1320)$	$f_2'(1525)$	$f_2(1270)$
1^1D_2	2^{-+}	$\pi_2(1670)$	$\eta_2(1870)$	$\eta_2(1645)$
1^3D_1	1^{--}	$\rho(1700)$		$\omega(1650)$
1^3D_2	2^{--}			
1^3D_3	3^{--}	$\rho_3(1690)$	$\phi_3(1850)$	$\omega_3(1670)$
1^3F_4	4^{++}	$a_4(1970)$	$f_4(2300)$	$f_4(2050)$
1^3G_5	5^{--}	$\rho_5(2350)$		
2^1S_0	0^{-+}	$\pi(1300)$	$\eta(1475)$	$\eta(1295)$
2^3S_1	1^{--}	$\rho(1450)$	$\phi(1680)$	$\omega(1420)$
2^3P_1	1^{++}	$a_1(1640)$		
2^3P_2	2^{++}	$a_2(1700)$	$f_2(1950)$	$f_2(1640)$

[PDG]

How to find more physical amplitudes?

Focus on new observables: masses and on-shell couplings

$$\{\, m_X^2 \,,\, g_{\pi\pi X}^2 \,\}$$

Minimal input to produce higher-spin particles/Regge trajectories?

Spin-J exchange: $\dfrac{P_J(1+\frac{2t}{M^2})}{s-M^2} + \dfrac{P_J(1+\frac{2s}{M^2})}{t-M^2} \sim s^J$

Regge bound $M(s,t) \lesssim s$

Logic:

- Spin-1 is marginally allowed $\left(M(s,t) = \frac{m_\rho^2+2t}{m_\rho^2-s}\frac{m_\infty^2}{m_\infty^2-t} + \frac{m_\rho^2+2s}{m_\rho^2-t}\frac{m_\infty^2}{m_\infty^2-s} \right)$
- Spin-2 can never appear alone — force presence by maximising spin-2 coupling!

Setup

Spin $J > 1$ cannot appear alone

Assumptions:

- Spin-1 state at $m = M = 1$ ($m_\rho^{\text{phys}} = 775$ MeV$/c^2$)
- Spin-2 state at $m = 1.65$ ($m_{f_2}^{\text{phys}} = 1275$ MeV$/c^2$)
- All other states above M' (free parameter)

Force presence by maximising $\dfrac{g_{\pi\pi f_2}^2}{g_{1,0}}$

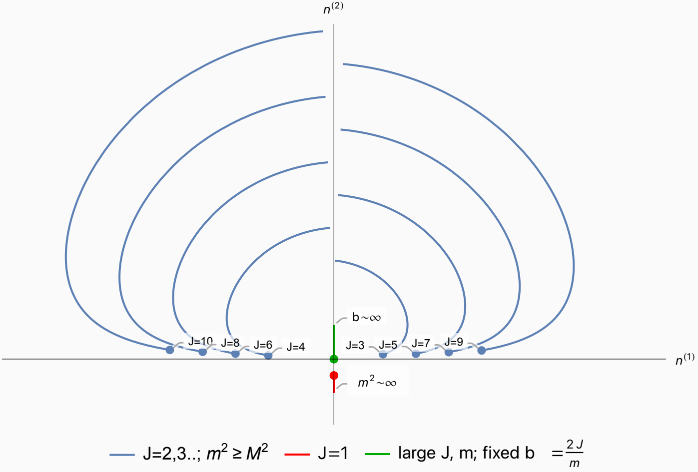
$$\vec{L} = \begin{pmatrix} g_{1,0} \\ 0 \\ \vdots \end{pmatrix} = g_{\pi\pi\rho}^2 \vec{V}(1, m_\rho^2) + g_{\pi\pi f_2}^2 \vec{V}(2, m_{f_2}^2) + \left\langle \vec{V}(J, s) \right\rangle_{M'}$$

Expect

- $M' \gg 1$ — nothing allowed
- $M' \sim m_{\rho_3}$ — extremal solution with all spins ($m_{\rho_3}^{\text{phys}} = 1688$ MeV$/c^2$)

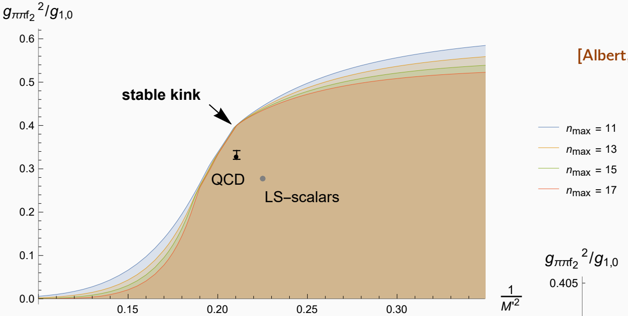
Aside: higher spins from sum rules

All physical states must sum to the zero vector in null-constraint space



[Albert, JH, Rastelli, Vichi 2023]

Results



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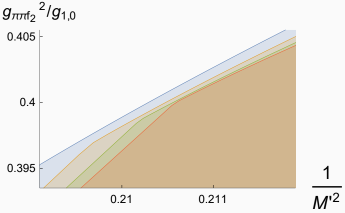
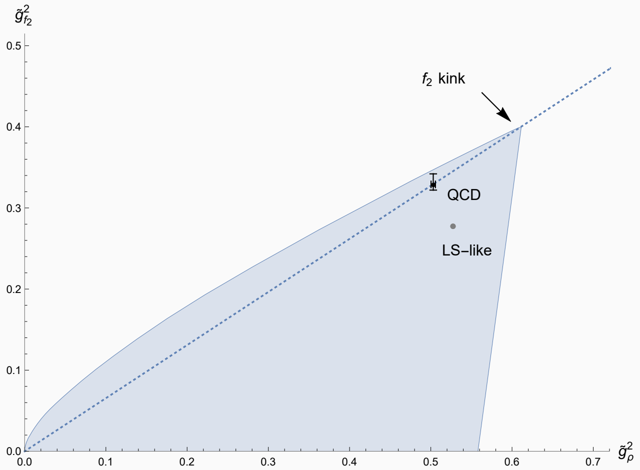


$$M_{\text{LS-scalars}} = -\frac{\Gamma(1/2 - s/2)\Gamma(1/2 - t/2)}{\Gamma(-s/2 - t/2)} - \text{scalars}$$

[Fernandez et. al. 2022]

What's the theory saturating the kink?

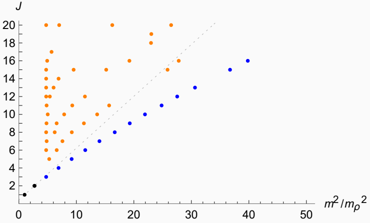


[Albert, JH, Rastelli, Vichi 2023]

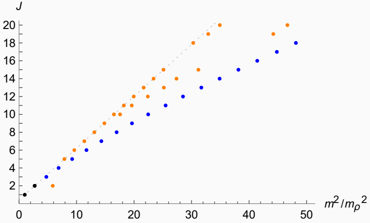
Extremal spectrum

Extremal functional method **[El-Showk & Paulos 2012]**

$m^2 \geqslant M'^2$



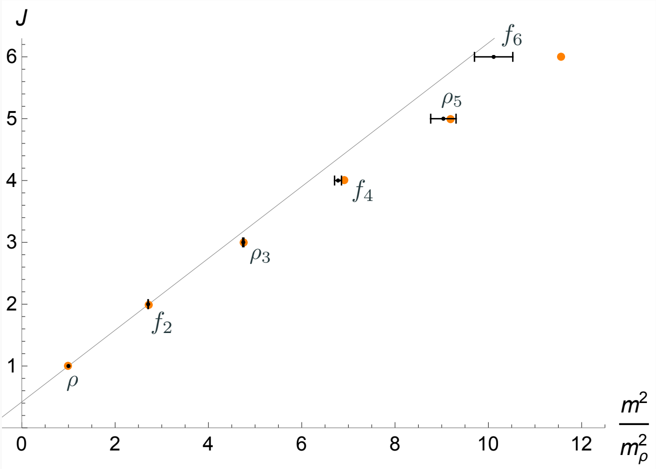
$m^2 \geqslant a + bJ$



	Dominant state		Other states					
J	m^2	$\tilde{g}_X^2$	m^2	$\tilde{g}_X^2$	m^2	$\tilde{g}_X^2$	m^2	$\tilde{g}_X^2$
3	4.747774	0.33527						
4	6.902792	0.28933						
5	9.181336	0.25334	5.278811	1.515×10^{-4}				
6	11.54579	0.22174	4.747774	3.297×10^{-6}	6.582414	1.773×10^{-4}		
7	14.01378	0.19857	4.835758	7.862×10^{-7}	7.581251	1.237×10^{-4}		
8	16.67318	0.18599	4.747774	8.771×10^{-8}	6.235041	1.265×10^{-6}	9.207674	1.352×10^{-4}
9	19.28674	0.16358	4.808180	1.895×10^{-8}	6.571938	4.367×10^{-7}	11.31167	2.537×10^{-4}
10	21.93016	0.14912	5.019793	7.308×10^{-9}	7.879411	3.754×10^{-7}	13.61458	4.242×10^{-4}
11	24.82063	0.11649	4.825621	6.643×10^{-10}	9.289181	1.875×10^{-6}	15.69828	1.554×10^{-4}
12	27.53345	0.10811	4.747774	8.380×10^{-11}	5.390215	7.235×10^{-11}	11.48907	7.067×10^{-6}

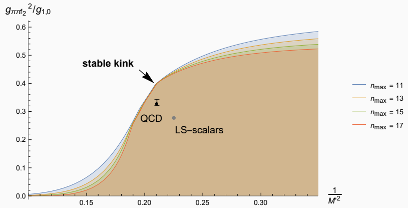
[Albert, JH, Rastelli, Vichi 2023]

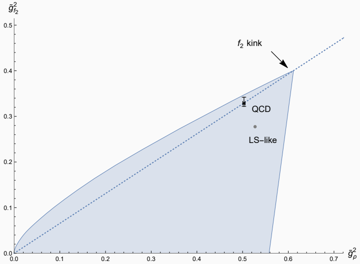
Comparison with real world

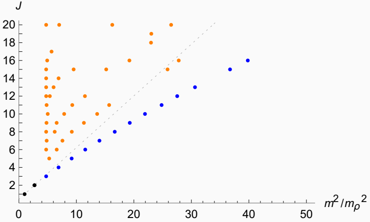


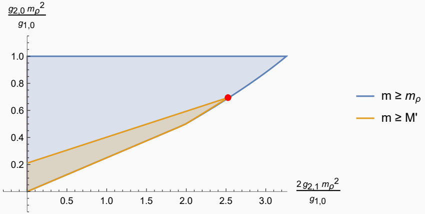
Black: [PDG], orange: our data [Albert, JH, Rastelli, Vichi 2023]

Results collected









[Albert, JH, Rastelli, Vichi 2023]

Is this large-N QCD?

What we have achieved:

- Spin 2 maximisation forces presence of Regge trajectory
- Interesting kink gives extremal amplitude
- Many observables are close to real-world QCD ($N = 3$)

Problems:

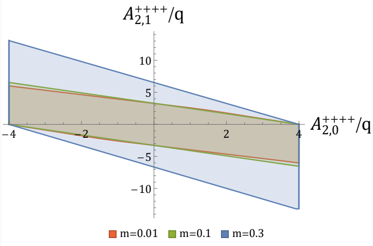
- No daughter trajectories
- Curved trajectory?
- Actual solution to crossing?
- Scarcity of large-N lattice data to compare with

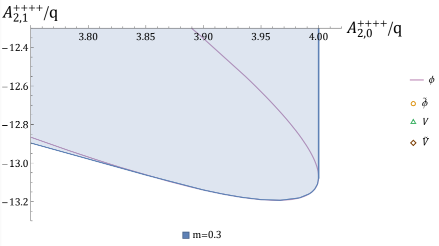
More to come!

Mixed π-ρ scattering amplitudes **[In progress with Albert, Rastelli, Vichi]**

$$M_{\pi\pi\pi\pi},\ M^{++}_{\rho\rho\pi\pi},\ M^{+0}_{\rho\rho\pi\pi},\ M^{+-}_{\rho\rho\pi\pi},\ M^{00}_{\rho\rho\pi\pi},\ M^{++++}_{\rho\rho\rho\rho},\ M^{+++0}_{\rho\rho\rho\rho},\ \ldots$$

Foundational work: Flavourless massive vector





[Bertucci, JH, McPeak, Ricossa, Riva, Vichi 2024]

Thank you for listening!