EFT of Cold Fermions near the Critical Point

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Many-body fermionic systems at quantum critical points have no quasiparticle description, ``Non-Fermi-Liquids"

- **Usually described by quasi-particles coupled to ``critical bosons" -**
- **No realistic expansion parameter (large N) -**

Consider Cold atomic gases, removes the need to treat the phonons

 $\frac{1}{\sqrt{2}}$ of the solution to the solution to the solution to the Schrodinger equation outside $\frac{1}{\sqrt{2}}$ **state at** $k = i/a$

e.g. a square well
$$
a = r_0 \left(1 - \frac{\tan(\kappa_0 r_0)}{\kappa_0 r_0}\right) \qquad \kappa_0 = \sqrt{mV_0}
$$

3 **E**ine tuning leads to a large scattering hound state F–0 with which can be seen by solving the Schrodinger equation. On the Schrodinger equ Fine tuning leads to a large scattering bound state E=0 with $\frac{2n-1}{\sqrt{n}}$ **infinite scattering length.** $\kappa_0 r_0 =$ $\frac{(2n-1)\pi}{\pi}$ $\frac{1}{2}$ $\frac{1}{2}$

- **- Scattering amplitude becomes independent of range as well as other UV** epenaent
CFT as a \boldsymbol{d} e becomes independent of range as well as other UV
- ter *k*!0 - parameters when a gets large (NRCFT as a diverges many body problem) gets large (ivrout i as a diverges many body problem). We can conclude the setting $\frac{d}{dt}$

Consider dilute Cold Fermionic gases whose interaction is tunable through Feshbach resonance. that we may ignore \sim may ignore (integrate out) all excitations of the particles theoremselves, so that they may be considered structurely be considered structure. Since the section is this induction the constant theory Consider dilute Cold Fermionic asses whose interaction is tunable through Feshbach resonance. this case we would expect scale invariance, and we should be able to see this by finding 1 + *^g*(*µ*0)(*µ*0+*ip*) a zero in the appropriate beta function. When we study the dimensionless \mathcal{L}_max *^A* ⁼ *^g*(*µ*0) **ider dilute Cold Fermionic ga**

UV Theory relativistic field theories. As opposed to NRQCD theory the mass of the mass of the mass of the particle is no
Separate is not the particle i cold atoms the Van der Waals scale, where the Van der Waals scale, where the theory breaks down, is the theory bre coupling scale in variance for the scale invariance for the scale function. However, in this case at \sim T this result could have been guessed with \mathbf{r} the exercise of the exe

has been integrated out. In the case of nucleons the case of nucleons this would be the pion mass, while for n
In this would be the pion mass, while for the pion mass, while for the pion mass, while for the pion mass, whil

Quantum Mechanics in zero Range approximations, valid for large scattering lengths (strong coupling) relevant. Indeed, since we are not concerned with relativistic corrections we may work in **Guantum Mechanic** tam moonamoo in zoro nango approximationo, vana
for large ecattering lengthe (strong coupling) **and inport in posed of the Candidae at the Cuantum Mechanics in zero.**

$$
L_0 = i\psi^{\dagger} \partial_0 \psi + \frac{1}{2} \psi^{\dagger} \vec{\nabla}^2 \psi.
$$

$$
L_I^{(1)} = g(\psi_+^{\dagger} \psi_+^{\dagger})(\psi_+ \psi_+)
$$

$$
g(\mu_0) = \left(\frac{4\pi}{-\mu_0 + 1/a}\right)
$$

$$
\hat{g}(\mu) = \frac{\mu}{4\pi} g(\mu),
$$

bry has a non-trivial UV fixed point which as a sense of $d_{\hat{a}} = \hat{a}(1+\hat{a})$ This theory has a non-trivial UV fixed point if we choose *point we have a homogeneous mediation* μ

 \mathbf{J} cold at the Van der Waals scale state scale scale. This scale $\mu \frac{d\mu}{d\mu} g = g(1 + g)$ *µ d* $\frac{\partial}{\partial \mu}\hat{g} = \hat{g}(1+\hat{g})$.

$$
g\thicksim a
$$

determining the counter-terms since we could have resummed the unrenormalized result

 α and α interaction in the Tuning to *d*4 *^x* ⇠ *^L*⁵ *,* ⇠ *^L*3*/*² *r* ing
∙ ⊏⊤ the *MS* scheme when we resum the series. The result for the scattering is then given by *g* ⇠ *a* **Tuning to infinite scattering** *^Asum* ⁼ *^g* 1 + *i pg* 4⇡ *d* \sim $\frac{3}{2}$ **length leads to an NRCFT We have a** *g a µ*⁰ *n n nime to infinite scattering to the scattering since the sinc*

Use this action in the context of a grand canonical ensemble, (*µ, T*)

At infinite scattering length we are sitting at a quantum critical point. We may like to think of the resulting Goldstones as being the ``critical bosons", and we are back to where we started in terms of our inability to keep calculational control w/o resorting to any unphysical limit.

However, given that we can solve the UV theory exactly, you would think we could utilize the short distance information such that we can gain predictive power.

``Anomaly Matching''

Away From the critical point we expect FLT to correctly describe the physics above Tc.

Basic Assumptions of FLT

1) In the UV there exists quasi-particle excitations

2) No long range forces

3) Rotational invariance: cold atoms as opposed to metals (here but but not in general)

Fermi Liquid Theory is the ``standard model'' of CMT

At temperatures above Tc it is described by a weakly coupled IR fixed point with one a marginal coupling functions. For a spherically symmetric Fermi surface we can decompose these function into an infinite number of couplings, generically called ``Landau Parameters"

$$
L_{int} = \sum_{l} g_{l} (\psi^{\dagger} \psi)^{2}
$$

Determine all of the Low Energy properties of the system, e.g. compressibility, quasi-particle width. *g*^{*l*}

EFT of Fermi Surface (Shankar/Polchinski)

Power Counting Parameter: $\lambda \equiv E/E_F$

 $[\psi(x), \psi^{\dagger}(0)] \sim \delta^{d}(x) \sim \lambda$ $\psi(x) \sim \lambda^{1/2}$

Field Scalings:

reached by using only the RG invariance and the Galilean building Action via power counting **Building Action via power counting**

$$
S_{int} = \frac{1}{2} \prod_i dt d^d p_i g(\vec{p}_i^{\parallel}) \psi_{\vec{p}_1}^{\dagger} \psi_{\vec{p}_2} \psi_{\vec{p}_3}^{\dagger} \psi_{\vec{p}_4} \delta^{d-1} (\sum \vec{p}_i^{\parallel}) \delta(\sum \vec{p}_i^{\perp})
$$

expanded around a point (permit surface) on the Fermi surface, permit surface, permit surface, permit surface, cial kinematic configurations does the delta function scal **Only for special kinematic configurations does the delta function scale**

Any new marginal or relevant interaction expected to obstruct FL behavior

It is important to understand that this theory is ``solvable" in the sense that response function can be calculated in terms of bubble sums up to power corrections even at strong coupling. At least under the assumption that one partial wave dominates.

When can we use FLT to study Cold Atomic System Near Unitarity

— At Unitarity Non-linearly realized Space-Time Symmetries Prohibit FL description

IZR, P. Shrivistava 1712.07797

— Can we do perturbation theory away from this conformal fixed (where FLT is expected to arise) and retain the predictive power that comes with the enhanced symmetry?

— Can we make first principles predictions by being near enough from the fixed point that we have approximate conformal symmetry but far enough away that we can still calculate using FLT?

To make progress we will consider FLT in the language of space-time symmetry breaking, as we will utilize the relevant Ward identities to gain predictive power.

All our predictive power will be based upon the Space-Time Symmetries of FLT stones [1–3] leading to a width which scale as ⇠ *^E*↵ with ↵ *<* 2. While relativistic dilatons can generate long range forces and, as such, their couplings are highly constrained[4]. The necessary conditions for the Goldstones in \mathbf{C}

Galilean Group:
$$
H, \vec{P}, \vec{L}, M, \vec{K}
$$

Spontaneously Broken: *K* \vec{K} ("Framid") (X: Broken Generators) δ pontaneously Broken: \vec{K} ("Framid") (X: Broken Generators)

$$
\sum_{n} \delta^{(d-1)}(\vec{p}_n) \left[\langle \Omega \mid j_0^X(0) \mid n \rangle \langle n \mid \phi(0) \mid \Omega \rangle e^{iE_nt} - \langle \Omega \mid \phi(0) \mid n \rangle \langle n \mid j_0^X(0) \mid \Omega \rangle e^{-iE_nt} \right] \neq 0
$$

ro momentum/energy state produced by current must exist in the spectrum Zero momentum/energy state produced by current must exist in the spectrum

Broken current can interpolate for a multi-particle state that saturates the Ward identity.

There is a general methodology which allows us to determine when Goldstone bosons need not appear in the spectrum via the ``inverse Higgs mechanism" (Volkov (73) Ogievetsky (74))

$$
[\bar{P},X]\propto X' \hspace{1cm} X,X'\in \hspace{1cm}\textbf{Generators} \\\hspace{1cm} \bar{P} \hspace{1.5cm} \textbf{unbroken} \\\hspace{1cm} \textbf{translation} \\\hspace{1.5cm}
$$

 X, X' coldstones are redundant *X, X'* **Then the associated Goldstones are redundant**

> In the case of Framids there is no inverse Higgs constraint since the only **broken generators are boosts. Where are the boost Goldstones?**

This is no puzzle, we've seen this before:

HQET is a Fermi liquid with N=1

Coset construction of HQET

$$
\mathcal{L}_{\phi} = \frac{i}{2} \left(\phi^{\dagger}(t, \vec{x}) (\partial_t + \vec{\eta} \cdot \vec{\partial} + \frac{i}{2} m \vec{\eta}^2) \phi(t, \vec{x}) - [(\partial_t + \vec{\eta} \cdot \vec{\partial} - \frac{i}{2} m \vec{\eta}^2) \phi^{\dagger}(t, \vec{x})] \phi(t, \vec{x}) \right) + \frac{c_1}{2m} ((\vec{\partial} + im\vec{\eta}) \phi^{\dagger}(t, \vec{x})) (\vec{\partial} - im\vec{\eta}) \phi(t, \vec{x}).
$$

$$
\vec{\eta} \to \vec{\eta} + \vec{v} \qquad \phi \to e^{\frac{i}{2}mv^2t + i\vec{v}\cdot\vec{x}}\phi
$$

But in this trivial vacuum there are no collective excitations. $\frac{1}{2}$ relat boost Goldstone as Lagrange multiplier that miposes **Treat boost Goldstone as Lagrange multiplier that imposes constraint leading to boost invariance.**

Nothing more than RPI constraints $c_1=1$

What happens in the case of FLT where Multi-particle states can saturate Ward identity (as opposed to HQET) one generalize the vacuum parameterization to include the unbroken translations (*P*¯*µ*) ⁷ such that The number of unbroken translations may be enhanced if the there exist in the exist internal translations in the exist internal translation of the exist internal translation of the exist internal translation of the exist i metrics (as opposed to the $\mathbf{1}$ What happens in the case of FLT where Multi-particle states can saturate Ward identity (as opposed to HQET) opposed to the bosonic cases.
Thus we have not the number of the number ogy certainly shows no signs of non-derivatively coupled Goldstone. One might be tempted to interpret zero sound as the boost Goldstone, however, the interaction between electrons due to the interaction
The interaction between electrons due to the interaction between electrons due to the interaction between elec $\frac{1}{2}$. Then is the Coset Construction of Fermi Little Effect with Rotational Symmetry: Type Interpretational Symmetry: Type Interpretational Symmetry: Type Interpretational Symmetry: Type Interpretational Symmetry: Ty

Space-time Coset construction: Space-time Coset $U = e^{i\bar{P}\cdot x} e^{i\pi \cdot X}$ Parameterizes G/H be considering such cases as we are interested in zero temperature ground states with delocalized pace and ecost $U = e^{2\pi i x} e^{2\pi i x}$ raidifferences g/n Space-time coset for $U = e^{\imath F \cdot x} e^{\imath \pi \cdot A}$ ogy certainly shows no signs of non-derivatively coupled Goldstone. One might be tempted to Choose time \mathbf{C} with broken boostruction: We construction: We can also be case of broken C **Parameterizes G/H**

 $U^{-1}\partial_\mu U = E^{\text{in}}_\mu (P_A + \nabla_A \pi^\alpha X^\alpha + A^\alpha_A T^\alpha)$ Extract: $E, A, \nabla \pi$ $U^{-1}\partial_\mu U = E^A_\mu (\bar{P}_A + \nabla_A \pi^a X^a + A^b_A T^b)$) **Extract:** E, A , \Box *b* = **E** F ^{*A*} $\nabla \pi$ **Extract:** $E, A, \nabla \pi$ $U^{-1}\partial_{\mu}U = E^{A}_{\mu}(\bar{P}_{A} + \nabla_{A}\pi^{a})$ $W = \frac{1}{\sqrt{2\pi}}$ μ^{\vee} but is broken boosts but unbroken rotations). We consider the constant invariance of broken L , Λ , V /($U^{-1}\partial_\mu U = E^A_\mu (\bar{P}_A + \nabla_A \pi^a X^a + A^b_A T^b)$

Subgroup 20 Sepace-to the Capital Roman: Hirreps Greek: Girreps Green Capital frame. In this work we will not Waynari nomant derivatives on the covalidatives on the matter of matter σ $\mathcal{L} = \mathcal{L} = \mathcal$ **Capital Roman: H irreps Greek: G irreps** ^r0⌘*ⁱ* = ˙⌘*ⁱ* ^r*i*⌘*^j* ⁼ @*i*⌘*^j*

AT^b

 $\overline{}$

 ψ

Covariant building blocks ensue $V A \psi = (E_A O_\mu + i \mathbf{1}^T A_A^T) \psi$ **We begin our investigation for the construction for the coset construction for the construction for the construc** with broken boosts but under the case of broken rotations). We consider the case of broken G

 $S_0 =$

$$
\nabla_A \psi \equiv (E^\mu_A \partial_\mu + i T^q A^q_A) \psi
$$

2

r*^A* ! *e* **FLT** The vacuum manifold is parameterized by

FLT

\n
$$
E_0^0 = 1 \quad E_i^j = \delta_i^j, \quad E_0^i = \eta^i, E_i^0 = 0
$$
\n
$$
U = e^{iP \cdot x} e^{-i\vec{K} \cdot \vec{\eta}(x)}
$$
\n**Only break boosts**

\n
$$
S_0 = \int d^d x dt \, \psi^\dagger \left[i(\partial_0 + \eta^i \partial_i) + \frac{1}{2} m \vec{\eta}^2 + \varepsilon (i\partial_i + m\eta_i) \right] \psi
$$

Calculation Condex boosts Ω nly bro **Only break boosts**

$$
\mathcal{S}_{int} = \prod_{i,a} \int a^{\cdot} \kappa_i a t \, g(\kappa_i + m\eta) \psi_{k_1}(t) \psi_{k_2}(t) \psi_{k_3}(t) \psi_{k_4}(t) \sigma^{\cdot}(\sum_i \kappa_i)
$$

Note: At this point we have not assumed anything about what scattering kinematics are allowed or not. [⌘*i j* $\frac{1}{2}$ **d** $\frac{1}{2}$ **d** Note: At this point we have not assumed anything about what scattering kinematics are allowed or not.

This is where things become interesting, as we now have a coupling function. I his is where things become interesting, as we now have a coupling functional control of the spatial dimension
I his is where things become interesting, as we now have a coupling functio Thie ie whara thinge hacoma interacting as wa now have a coupling function

> As we did in HQET, treat the framid as a L.M As we did in HQET, treat the framid as a L.M eliminate it from the theory, as will be discussed below. In three spatial dimensions this conclusion

 $\iota, a \quad J \quad i$

 ι

 i, a

$$
= \sum_{\vec{\mathbf{k}}(\theta)} \int d^d x dt \, \psi_{\vec{\mathbf{k}}(\theta)}^{\dagger}(x) \left[i \partial_0 - \vec{\eta}(x) \cdot \vec{v}_F(\theta) (m - m^{\star}) + i \vec{v}_F(\theta) \cdot \vec{\partial} \right] \psi_{\vec{\mathbf{k}}(\theta)}(x) + \dots
$$

$$
\frac{\partial \varepsilon}{\partial \vec{k}} = \vec{v}_F = \frac{\vec{k}_F}{m^{\star}}
$$

$$
S_{int} = \prod_i \int d^d k_i dt \sum_j \frac{m}{2} \vec{\eta} \cdot \frac{\partial g(k^j)}{\partial \vec{k}^j} \psi_{k_4}^{\dagger}(t) \psi_{k_3}^{\dagger}(t) \psi_{k_2}(t) \psi_{k_1}(t) \delta^d(\sum_i k_i)
$$

Constraint from treating framid as L.M.

2013Erved charges (thre independent) arguments in this section of the argument of arbitrary to generalize it w **Conserved charges (time independent) non-conserved**

$$
\langle \vec{k} \mid O_B | \vec{k} \rangle = 0.
$$

$$
k_i = m \frac{\partial \varepsilon(k)}{\partial k_i} + \frac{2m}{(2\pi)^d} \int d^d p \left(\frac{\partial g(k, p, p, k)}{\partial p_i} + \frac{\partial g(k, p, p, k)}{\partial k_i} \right) \theta(p_F - p)
$$

$$
\frac{k_F}{m} = v_F + \frac{2p_F}{(2\pi)^2} \int d\theta \cos\theta \sum_l g_l P_l(\cos\theta)
$$

$$
\frac{m^*}{m} = 1 + \frac{1}{3} \frac{2m^*}{(2\pi)^2} g_1
$$

 $\overline{1}$ AITErr
.. $\frac{2m^4}{(2\pi)^2}g_1$ relation. But we have only worked to **a**
1-loop? Alternative derivation of Landau 1-loop?

Constraint must be RG invariant: This implies that the beta function must vanish!

¹³In canonical EFT's one uses dimensional regularization exactly to avoid this mixing issue which complicates For generic kir Ear generic kinematice this will not he true hut it IS true for **For generic kinematics this will not be true, but it IS true for forward scattering!!**

 $\frac{1}{2}$ for sake of simplicity but the results are valid for arbitrary defined for arbitrary d. **We see that boost invariance ONLY allows for forward scattering *. This is an alternative derivation of the famous statement that FLT has an infinite number of conserved quantities [1712.07795](https://arxiv.org/abs/1712.07795) (IZR/Srivistava).**

> – 18 – – 18 – **This also shows that constraint hold to all order in PT.**

*** BCS is not ruled out because it does not contribute to the two point function (kinematically disallowed).**

The unitary limit in the UV theory point in the UV theory Now Suppose we tune to the fixed

Schrodinger Group mer growing strong in the IR leading to breaking of the particle number *U*(1) symmetry. However, when interac-

 $\bar{\bar{P}}$ *L,* $\bar{\bar{I}}$ *K,M, C, D* \bar{K}) ter G and \vec{F} and $\vec{F$ beings and approximations $(11, 1, 1, 2)$ and $(21, 1, 1, 2)$

If we tune the atomic interaction via Feschbach resonance to fixed point (diverging scattering length) then the Fermi sea will break

> $\vec K, C, D$ $\mathbf{f}_1, \mathbf{c}_2, \mathbf{r}_3$

Inverse Higgs constraint: particle width begins to develop the set of
Set of the set of the s $[H, C] = iD$

we consider small fluctuations are small fluctuations are small fluctuations are small fluctuations are small fluctuations around the S the broken scale invariance. The broken scale invariance in
the bookstone called the b the frame is necessary to write down a Galilean invariant down and the complete down and α **So in generating an action we only need to add the dilaton mode**

We will be interested in what happens as we perturb away from unitarity actly known) and the IR theory. Using this result, along with the fact that the dilaton coupling is fixed by symmetric is fixed by symmetric is fixed by symmetric is fixed by symmetric in the only scale in the theory is the Fermi energy *E^F* . To

As we move away from unitary the Dilaton gets gapped **(pseudo-Goldstone).** The unitary point can be written to unitary point can be written to be written to unitary point can be written to be unitary point can be unitary point can be written to be written to be written to un the fermion and the contact parameter. With this result \mathcal{L}_max in hand we then predict the value of the value of the value of the value of the comprehensive of

> **First Build action via coset construction:**

$$
U=e^{iHt}e^{-i\vec{P}.\vec{x}}e^{-i\vec{K}.\vec{\eta}}e^{-iD\phi}e^{-iC\xi}
$$

-Impose DIHC on framid (Landau Condition)

-Eliminate special conformal GB via IHC i complete and j and j

$$
S_{\psi} = \int dt d^{3}x e^{-\frac{5\phi}{\Lambda}} [\tilde{\psi}^{\dagger} (ie^{\frac{2\phi}{\Lambda}} \partial_{t} \tilde{\psi} - (\epsilon (e^{\frac{\phi}{\Lambda}} i \vec{\partial}) - \mu_{F}) \tilde{\psi})
$$

$$
S_{int} = \int d^{d}x dt e^{-5\phi/\Lambda} f_{0} (\tilde{\psi}^{\dagger} \tilde{\psi})^{2}
$$

$$
\tilde{\psi} = e^{\frac{3\phi}{2\Lambda}} \psi
$$

Only kept l=0 coupling for reasons which will become clear ton field in the exponential in the exponential. Under distributions, the distributions, **Expanding out to generate the Dilaton coupling** Expanding out to generate the Dilaten expensively better gonorate the breaten.
Coupling

$$
S_{\psi} = \int d^3x\,dt\,\,i\psi^{\dagger}\partial_t\psi + \psi^{\dagger}\vec{v}_F\cdot\vec{\partial}\psi - \frac{2\phi\mu_F}{\Lambda})\psi^{\dagger}\psi + \ldots
$$

the dilaton coupling (no quasi-particles). How let us scale homogeneously under a RG transformation in all the second terms in all the second in all the second in a
contribution in all the second in all
 We have no calculational control over this theory due to deform the UV theory away from criticality **We have no calculational control over this theory due to**

In the IR this will gap the Dilaton **In the IR this will gap the Dilaton**

$$
\delta L = \frac{1}{2} m_\phi^2 \phi^2
$$

For large enough scattering length we will have a light dilaton

$$
m_\phi \ll E_F
$$

Generate local interactions when $\;\;E\ll m_\phi$

 \sim μ_F^2 Λ^2 1 m_d^2 ϕ

Dominates scattering

 $\frac{2}{\phi}/E_F^2\ll 1$

We have two unknown parameters: m_ϕ, Λ

Dilaton mass can be fixed by "anomaly matching" Dilaton mass can he fixed by "anomaly m symmetry breaking scale. Thus if we are in the regime \mathbf{r} *m*² ² (8) scale homogeneously under an RG transformation in all all the set of the set of the set of the set of the set o @*µs^µ* = *m*² ⇤ (9) magnetic field such that the atomic system is sitting to a such that the atomic system is sitting in \sim as a spurion such that the action is invariant if we scale that the action is invariant if we scale the action n be fixed by "anon

$$
\partial_{\mu} s^{\mu} = m_{\phi}^{2} \Lambda \phi
$$
 (EFT)

$$
\partial_{\mu} s^{\mu} = (g(\mu) + \beta(g))(\chi^{\dagger}\chi)^2
$$
 (UV Theory)

$$
D^{0}(0) = \Lambda \int d^{3}x \ \pi(\vec{x}, 0) \qquad \int_{x} [D^{0}(0), \partial_{\mu}s^{\mu}(\vec{x}, 0)] = \int d^{3}x \ m_{\phi}^{2} \Lambda^{2} \qquad (EFT)
$$

$$
D^{0}(0) = \int d^{3}x \left(\frac{3}{2}\chi^{\dagger}(\vec{x},0)\chi(\vec{x},0) + \chi^{\dagger}(\vec{x},0)\vec{x}\cdot\vec{\partial}\chi(\vec{x},0)\right)
$$
 (UV Theory)

$$
\int_x [D^0(0), \partial_\mu s^\mu(\vec{x}, 0)] = 3 \int d^3x \left(g(\mu) + \beta(g) \right) (\chi^{\dagger} \chi)^2
$$

Now we equate the vacuum matrix elemerally commutators in full and EFT Now we equate the vacuum matrix elements of the
*COMMUTATORS in full and EFT d*³*x m*² is the dimensionless contact parameter which \mathbf{r} has been measured to be of order one (when *k^F a >* 1) conservation scales as 1*/* while the momentum space Now we equate the vacuum matrix elements of the $\frac{1}{\sqrt{2}}$ of $\frac{1}{\sqrt{2}}$, is an RG invariant, and the distribution of distribution of $\frac{1}{\sqrt{2}}$

$$
m_{\phi}^{2} \Lambda^{2} = \frac{3}{4\pi a} \langle g^{2} \chi_{\uparrow}^{\dagger} \chi_{\uparrow} \chi_{\downarrow}^{\dagger} \chi_{\downarrow} \rangle \equiv \frac{3}{4\pi a} C
$$

where we have now made the spin state explicit and α is the spin state explicit and α

⁹⇡*C*˜²

We integrate out the distribution to generate out the distribution of the distribu

which depend on the contact. Note that \mathcal{N} is still an un-still and un-still an un-s

 $(\mathbf{D}^2 + 2 \cdot 2 \cdot \mathbf{D}^2)$

 \mathcal{L}

*n*2

coupling, after repristing factors of f and the atomic f and the atomic f and the atomic f and the atomic f

 σ measure of the local pair density C Contact (Tan) parameter (controls *C* controls *thermodynamics* of system) states (1411) per amotor (consisted
thermodynamics of system) $\mathbf \sigma$ fermions and is independent of the RG scale $\frac{1}{2}$. For any $\frac{1}{2}$ where we have now made the spin state explicit and *C* 2*m*? tem) ntact (ran) parameter (controis
thermodynamics of system) ? the two-loop self-two-loop self-two-loop self-two-loop self-two-loop self-two-loop self-two-loop self-two-loop
The two-loop self-two-loop self-two-loop self-two-loop self-two-loop self-two-loop self-two-loop self-two-loop \mathcal{C} conta \mathcal{O} scale invariance. One can verify the distribution that the distribution of the distribution $t_{\rm H}$ the divergence of the divergence of the scale current and (Tan) parameter (controls are suppressed by powers \sim

Integrating out dilaton leads to	$L_{int} = (f_0 + \frac{8\pi a \mu_F^2}{3\mathcal{C}})(\psi^{\dagger}\psi)^2$
UV modes down by Ef,	$f_D \equiv \frac{8\pi a \mu_F^2 m}{3\hbar^4 k_F^4 \tilde{C}}$

re a prediction for the I=0 I andau parameter when $\cdot \quad \left(\frac{E_F}{\pm} \right)^2 \gg a k_f \gg 1.$ Thus we have a prediction $\frac{1}{\sqrt{1}}$ *E* Thus we have a prediction for the I=0 Landau parameter when : $\left(\frac{E_F}{E}\right)^2 \gg a k_f \gg 1.$ *^D*⁰(0) = ^Z *d*³*x*($\overline{}$ *†*(~*x,* 0)(~*x,* 0) + *†*(~*x,* 0)~*^x ·* @~ (~*x,* 0)) Thus we have a prediction for the I=0 $\sf L$ a au narameter when $\epsilon = \left(\frac{E_F}{\epsilon} \right) \gg a k_f \gg 1.$

the interaction, arising from integrating $\binom{E}{E}$ $\left(F_{F}\right)^2$ E_F *E* \setminus^2 $\gg ak_f \gg 1.$ in the cases of a transition ~ 2 triplet channel channel as $\left(\begin{array}{c} F \end{array} \right)$ as $\left(\begin{array}{c} F \end{array} \right)$

 $t\mathbf{r}^2$

¹ ¯*h*²*N^F ^f^D*

 $\overline{1}$ $\overline{2}$ $\overline{1}$

 $\overline{v}_\phi, \overline{m}_{\phi}/\overline{E}_F$)

 $\frac{2}{\phi}, m_\phi^2/E_F^2)$

 $\textrm{corrections:} \qquad \big(E^2/m_\phi^2, m_\phi^2/E_F^2\big)$ will correction $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{2}{2}$ $\frac{2}{2}$ $\frac{1}{2}$ $\,$ corrections: $\,$ (E^2/m_ϕ^2) s: $(E^2/m_\phi^2, m_\phi^2/E_F^2)$ mass is independent of the scale μ . The coupling and the coupling

has been measured to been measured to be one of the scale of the scale \mathbf{p} Ohservahles we can then calculate the \sim $\overline{}$

$$
\text{quasi-particle width}\quad \Gamma(E)\sim Im\quad \underbrace{\qquad \qquad}_{\text{A}_{\text{D}}}
$$

$$
\Gamma(E,T) = \frac{m}{9\pi \tilde{C}^2} \left(\sqrt{\frac{m}{m^{\star}}} \frac{a\mu_F^2}{\hbar E_F^2} \right)^2 (E^2 + (\pi kT)^2) \qquad \frac{\Delta\Gamma_T}{\Gamma} \sim O\left(\frac{1}{k_F a}\right) + O\left(k_F a \left(\frac{E^2}{E_F^2}\right)\right)
$$

2*m*? **Compressibility: Compressibility:**

$$
\kappa = \frac{V}{N} \left(\frac{\partial N}{\partial \mu} \right)_{T,V} = \lim_{q \to 0, \omega \to 0} \langle \rho(\omega, q) \rho(-\omega, -q) \rangle.
$$

$$
\angle g1\omega_{3}g1\omega_{3}=\frac{0}{1-\frac{1}{2}D}
$$

$$
\frac{\kappa}{\kappa_0} = \frac{1}{1 - \hat{f} + \frac{4}{3\pi} \frac{k_F a}{\tilde{C}} \left(\frac{m}{m^*} - 1\right)^2)}
$$

Short distance contribution (unknown) which is power suppressed

 $(\tilde{C}(a), m^{\star})$ **Measure quantities.**

prediction holds in the range -1<kf a<-10.

If we account for fact that f0 is independent of a, we can us the data at one point and fix f0, shifting the coupling such that $Out\lceil~|=$

We can also calculate the temperature dependence of the result Out[\circ]=

- Applications to other nearly critical systems

 \mathscr{A} is the final result at 1 GeV is the fi $\mathscr{A}(\varphi \to \pi^{\omega}(p_{1})\, \pi^{\omega}(p_{2}))$ (Voloshin+Zakharov): (Georgi, Manohar, Grinstein) in+Zakharov): (Georgi, Manohar, Grinstein) can be used at lower scales. The matrix elements can now be calculated using the $\mathscr{A}(\varphi \to \pi^a(\,p_{\,1})\ \pi^b(\,p_{\,2}))$ (Voloshin+Zakharov): (Georgi, Manohar, Grinstein)

$$
\mathscr{L}_{\text{eff}} = -\frac{n_h}{3b} \frac{\varphi}{v} \left(\frac{\beta(g)}{g} G^2 \right) \Big|_{\mu = 1 \text{ GeV}} \qquad \partial_{\mu} s^{\mu} = \frac{1}{2} \frac{\beta(g)}{g} G^2
$$

$$
\partial_{\mu} s^{\mu} = -\frac{f^2}{2} \operatorname{Tr} \partial_{\mu} \Sigma^{\dagger} \partial^{\mu} \Sigma - 2f^2 A (\operatorname{Tr} M \Sigma + \text{h.c.})
$$

$$
\mathscr{L}_{\text{eff}} = -\frac{2n_h}{3b} \frac{\varphi}{v} \partial_\mu s^\mu - \sum_i m_i \bar{\psi}_i \psi_i \frac{\varphi}{v} \left(1 - \frac{2n_h}{3b} \right)
$$

$$
\mathscr{L}_{\text{eff}} = \frac{n_h}{3b} \frac{\varphi}{v} f^2 \operatorname{Tr} \partial_\mu \Sigma^\dagger \partial^\mu \Sigma + \frac{f^2}{2} A (\operatorname{Tr} M \Sigma + \text{h.c.}) \left(1 + \frac{2n_h}{b} \right) \frac{\varphi}{v}
$$

$$
\mathscr{A}(\varphi \to \pi^a \pi^b) = -\left[\frac{2}{3v} \frac{n_h}{b} m_\varphi^2 + \frac{1}{v} m_a^2 \left(1 + \frac{2n_h}{3b}\right)\right] \delta_{ab}
$$

Relevant Side Note:

S.T. Goldstone bosons can lead to long range forces, i.e none-derivatively coupled.

 $[X_i, \bar{P}]$ $\ket{\neq 0}$ (Watanabe and Vishnawath 2014) **Sufficient criteria:** Sufficient $(\nabla \ \vec D) \not\equiv 0$ (Watanahe and Viehnawath 2014) it. Thus a necessary condition for non-derivative condition for α , i.e., i.

 $[\bar{P}^{\mu}, X] \neq 0$ (IZR and Srivistava 2017)

~ *^k*0![~] *k* h *k | X |* i!1*.* (3.2) which complete the explicit the explicit factor of the coupling process of the coupling. One may be coupled to **Notice that the framid (Boost GB) will be None-derivatively** coupled written threatens the Ferrin nguld picture. **coupled which threatens the Fermi liquid picture.**

We need to saturate the Boost Ward identities w/o any GB and no I.H.C the Hood to batalate the Boodt fraid identities the dry did and he mile **We need to saturate the Boost Ward identities w/o any GB and no I.H.C**

How can we stabilize Dilaton mass? C.C. Term is allowed

$$
S_{int} = \int d^d x dt e^{-5\phi/\Lambda} \Lambda
$$

 $d^dx dt e^{-5\phi/\Lambda} \Lambda$ Fine tuning needed keep dilaton light, $d^dx dt e^{-5\phi/\Lambda} \Lambda$ but that is exactly what is provided by **fine tuning the magnetic field**