Non-Invertible Peccei-Quinn Symmetry
and
the Massless Quark Solution
to the Strong CP Problem

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EFT in EFT at IPMU

Global Symmetries

Symmetry: most essential and powerful concept in the pursuit of fundamental physics

Every discovery of **new symmetry** or **new properties** of known symmetry led to radical leap in the progress of fundamental physics.

e.g. Meson spectrum from spontaneous symmetry breaking
CP violation of QCD from realization of anomalies
Unitarization of Standard Model from Higgs Mechanism
Understanding of phases of QFT by 't Hooft anomaly matching
Tachyon stabilization via supersymmetry

Generalized Global Symmetries!

Advent of Generalized Global Symmetries (GGS)

$\Rightarrow$ New symmetries: new ideas in QFT and many excitements

- New anomalies and deeper understanding of phases of QFT

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- GGS in Particle Physics

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(Q1) Are there generalized symmetries in (3+1)d QFTs relevant for particle physics?

(Q2) Can there be observable signals (even in principle) associated with (due to) the presence of those generalized symmetries?

(Q3) Can generalized symmetry provide novel or meaningful solutions to problems in particle physics?

Generalized Global Symmetries!

Problems in Particle Physics

I. Naturalness Problem

- Hierarchy Problem
- Strong CP Problem
- Naturally small neutrino mass
- Cosmological Constant Problem
- Flavor Structure/Hierarchy

II. Dark Matter

III. Baryon-antibaryon asymmetry

IV. H_0 and S_8

V. Global Structure of $G_{SM} = SU(3)_C \times SU(2)_L \times U(1)_Y/\Gamma, \quad \Gamma = 1, Z_{2,3,6}$

VI. Confinement of QCD

Generalized Global Symmetries!

Problems in Particle Physics

I. Naturalness Problem

- Hierarchy Problem
- **Strong CP Problem**
- **Naturally small neutrino mass**
- Cosmological Constant Problem
- **Flavor Structure/Hierarchy**

II. **Dark Matter (e.g. axion and axion domain wall problem)**

III. Baryon-antibaryon asymmetry

IV. H_0 and S_8

V. **Global Structure of** $\boldsymbol{G_{SM} = SU(3)_C \times SU(2)_L \times U(1)_Y/\Gamma, \quad \Gamma = 1, Z_{2,3,6}}$

VI. Confinement of QCD

Outline

I. Generalized Global Symmetries

I-1. Higher-form symmetry
I-2. Non-invertible symmetry

II. Strong CP Problem-I: IR to UV

II-1. Non-invertible Peccei-Quinn symmetry
II-2. Massless quark solution

III. Strong CP Problem-II: UV to IR

III-1. $SU(9)$ Color-Flavor unification
III-2. Flavor structure and CKM CPV phase

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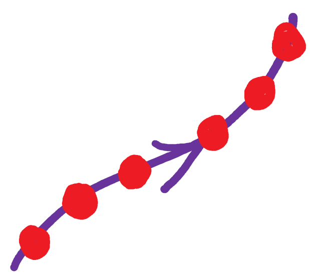
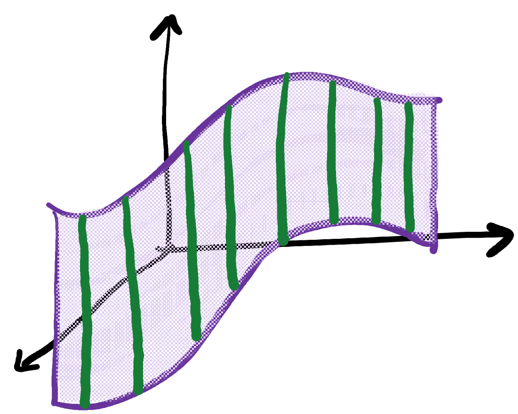
<u>Higher-form symmetries</u>

Various **extended objects** appear in broad class of theories.

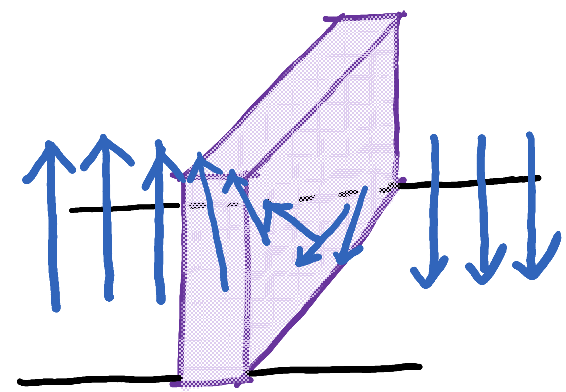
Local operator
e.g. particle
0-form symmetry



Line operator
e.g. Wilson line
't Hooft line
1-form symmetry



Surface operator
e.g. Cosmic string
2-form symmetry



Volume operator
e.g. Domain Wall
3-form symmetry

Higher-form symmetries

1. 0-form symmetry

Consider 4d two Weyl fermions Ψ_+, Ψ_- : $U(1)_+ \times U(1)_-$

$U(1)_V$: $\Psi_+ \to e^{i\alpha}\,\Psi_+$, $\Psi_- \to e^{-i\alpha}\,\Psi_-$ (can be gauged)

$U(1)_A$: $\Psi_+ \to e^{i\alpha}\,\Psi_+$, $\Psi_- \to e^{i\alpha}\,\Psi_-$ $\Rightarrow$ $d * j_A = \frac{K}{8\pi^2} F_2 \wedge F_2$

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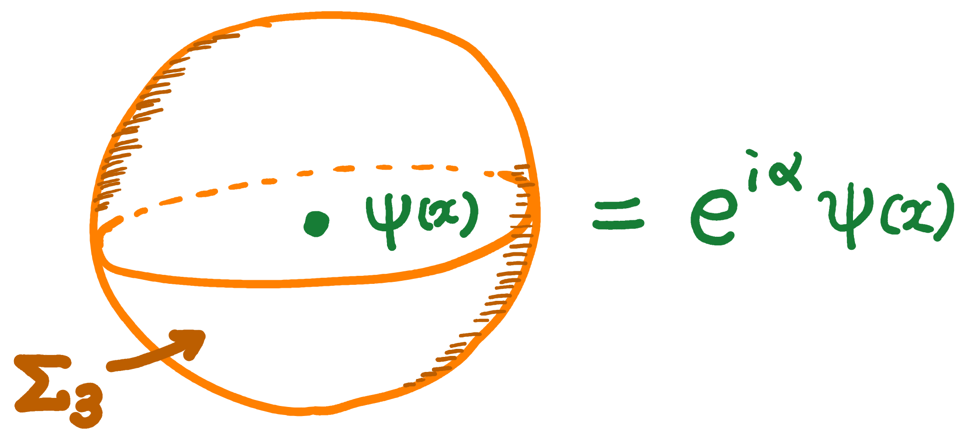
"Symmetry Defect Operator"

$$Q(\Sigma_3) = \int_{\Sigma_3} d^3x\, J^0 = \int_{\Sigma_3} * J_1$$

$$U(\alpha, \Sigma_3) = e^{i\alpha Q(\Sigma_3)}$$

$$\langle U(\alpha, \Sigma_3)\psi(x)\rangle \sim e^{i\alpha}\psi(x)$$



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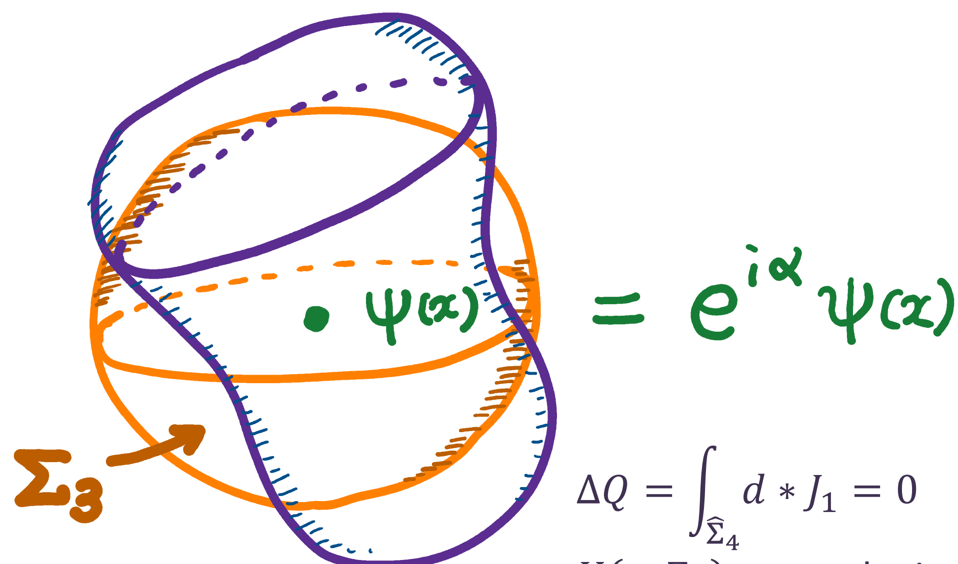

$$\Delta Q = \int_{\hat{\Sigma}_4} d * J_1 = 0$$

$U(\alpha, \Sigma_3) =$ topological

Higher-form symmetries

2. p-form symmetry

0-form $\leftrightarrow$ local op (particle)
0-form $\leftrightarrow$ j_1 (j_μ)
0-form $\leftrightarrow$ A_1 (A_μ)

$$S \supset \int d^4x\, A_\mu j^\mu = \int A_1 \wedge * j_1$$

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p-form $\leftrightarrow$ p-dim op
p-form $\leftrightarrow$ j_{p+1}
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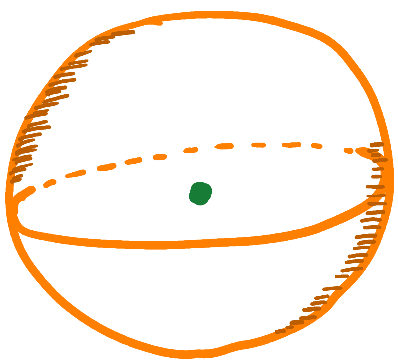
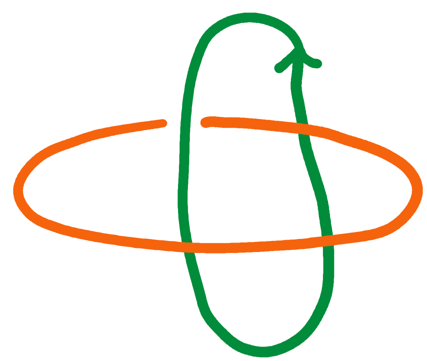
E.g.) 0- and 1-form symmetry in 3d





Higher-form symmetries

2. p-form symmetry

2-1. $U(1)_{EM}$ with Ψ_+, Ψ_-

EoM: $d * F_2 = j_\Psi \quad \left(d * F_2 = 0 \;\Rightarrow\; U(1)^{(1)}(e)\right)$

charged op: Wilson $W_1 = e^{i\oint A_1}$, SDO $U(\Sigma_2) = e^{i\oint * F_2}$

Bianchi id: $dF_2 = 0 \;\Rightarrow\; U(1)^{(1)}(m)$

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${U(1)^{(0)}}_A$: $\Psi_+ \rightarrow e^{i\alpha}\,\Psi_+$, $\quad \Psi_- \rightarrow e^{i\alpha}\,\Psi_- \;\Rightarrow\; d * j_A = \frac{K}{8\pi^2} F_2 \wedge F_2$

Higher-form symmetries

2. p-form symmetry

2-2. $SU(N)$ YM (either pure YM or with only adj matter)

$\exists\ Z_N^{(1)}(e)$: under 0-form center $\Psi \to e^{\frac{2\pi i}{N} * N}\ \Psi$

$\to$ Wilson line with charge $= 0, 1, \cdots, (N-1)$ not screened

$\nexists$ mag 1-form : $\Pi_1\left(SU(N)\right) = \emptyset$

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2-3. $PSU(N) = \frac{SU(N)}{Z_N}$: $Z_N^{(1)}(e)$ is gauged (electric states projected out)

$\nexists$ electric 1-form

$\exists\ Z_N^{(1)}(m)$: $\Pi_1\left(PSU(N)\right) = Z_N$ or "$N * \frac{1}{N} = 1$"

$\Rightarrow\ \oint G_2 = 2\pi/N$, $\int \mathrm{tr}(G_2 \wedge G_2) \supset \frac{N-1}{2N} \int w_2 \wedge w_2$

Fractional Instanton

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Non-Invertible Symmetry

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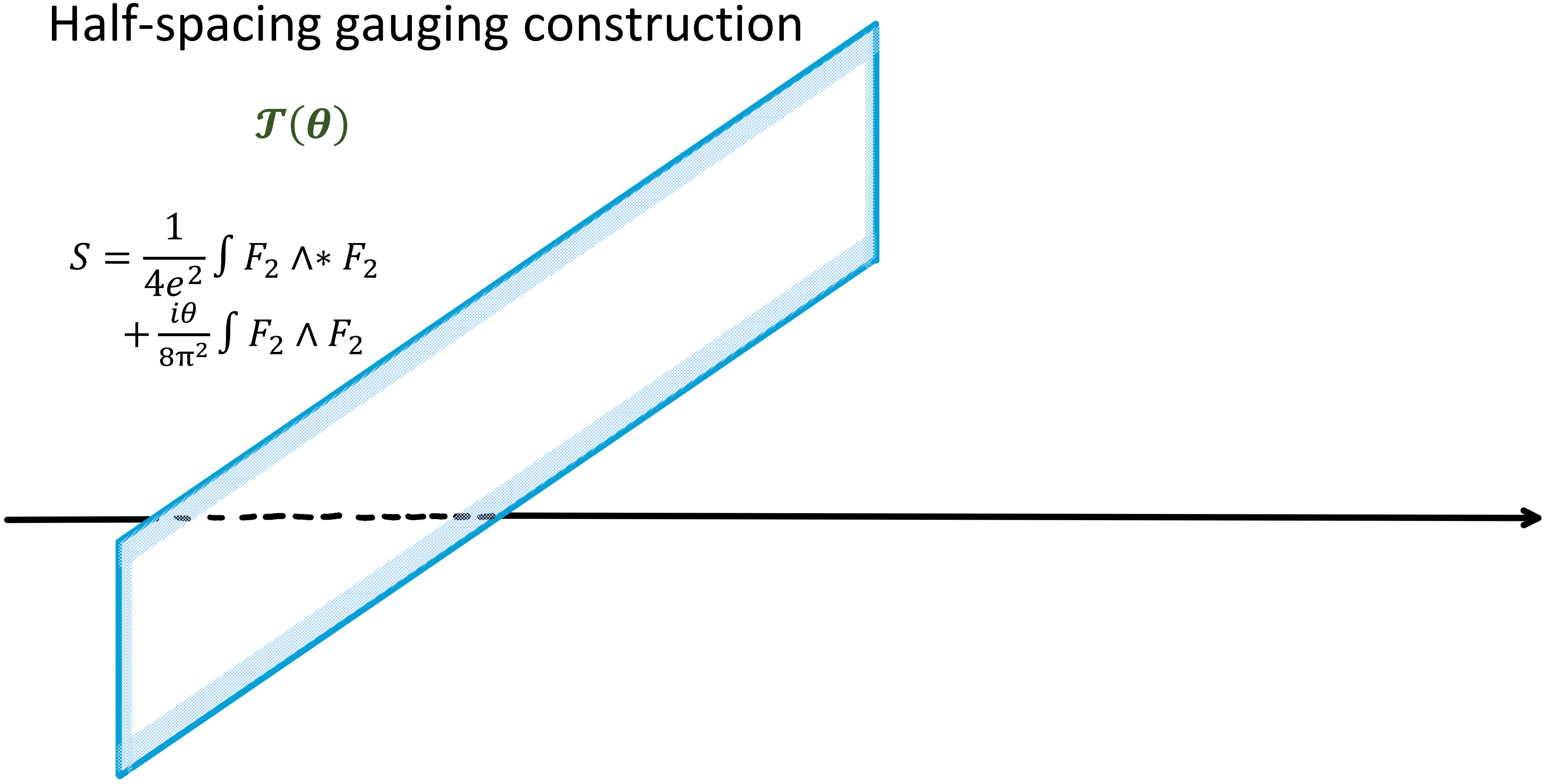
Half-spacing gauging construction



Non-Invertible Symmetry

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[Gauge $Z_N^{(1)}(m) \subset U(1)^{(1)}(m)$]

$\mathcal{T}(\boldsymbol{\theta})$

$$S = \frac{1}{4e^2}\int F_2 \wedge * F_2 + \frac{i\theta}{8\pi^2}\int F_2 \wedge F_2$$

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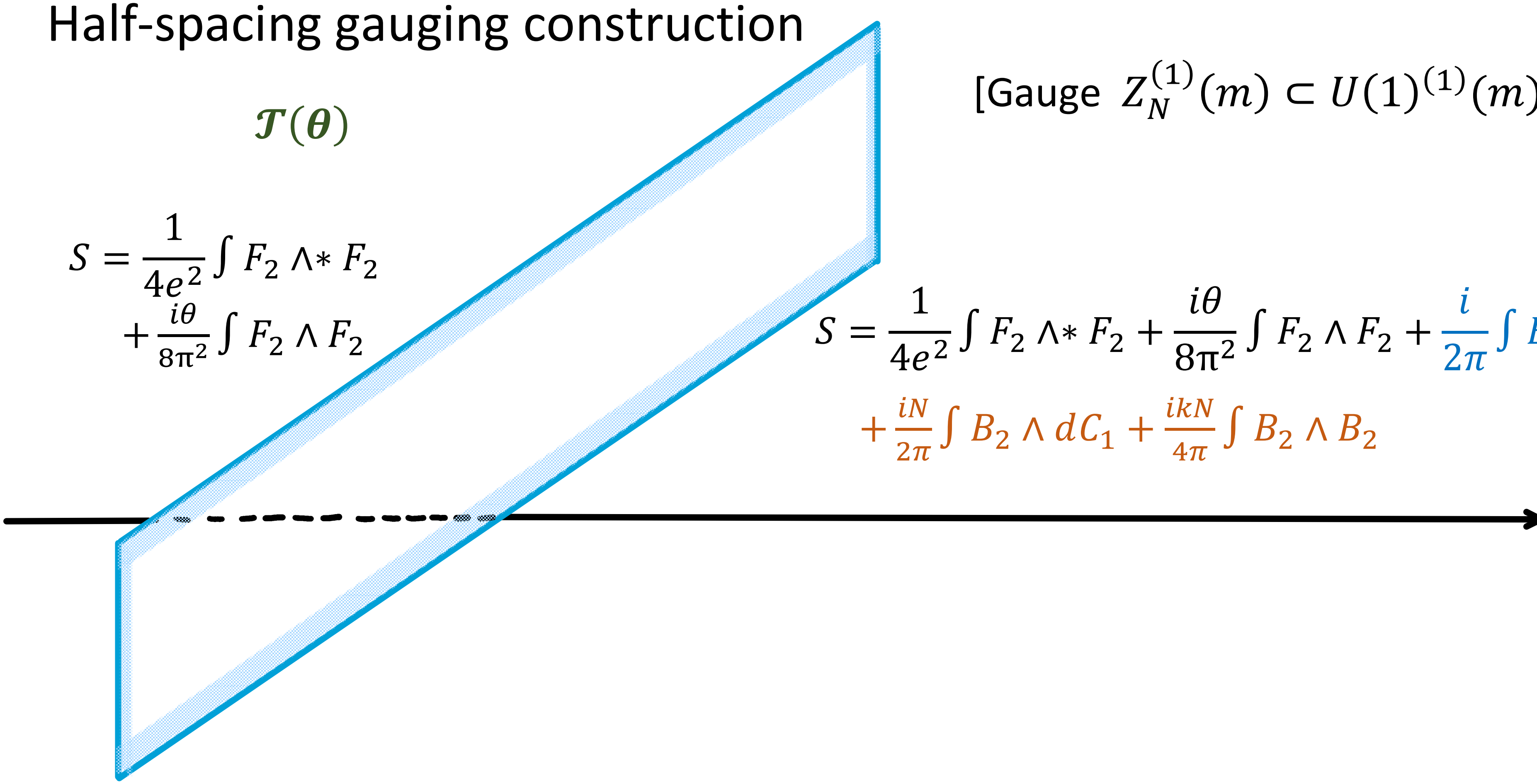

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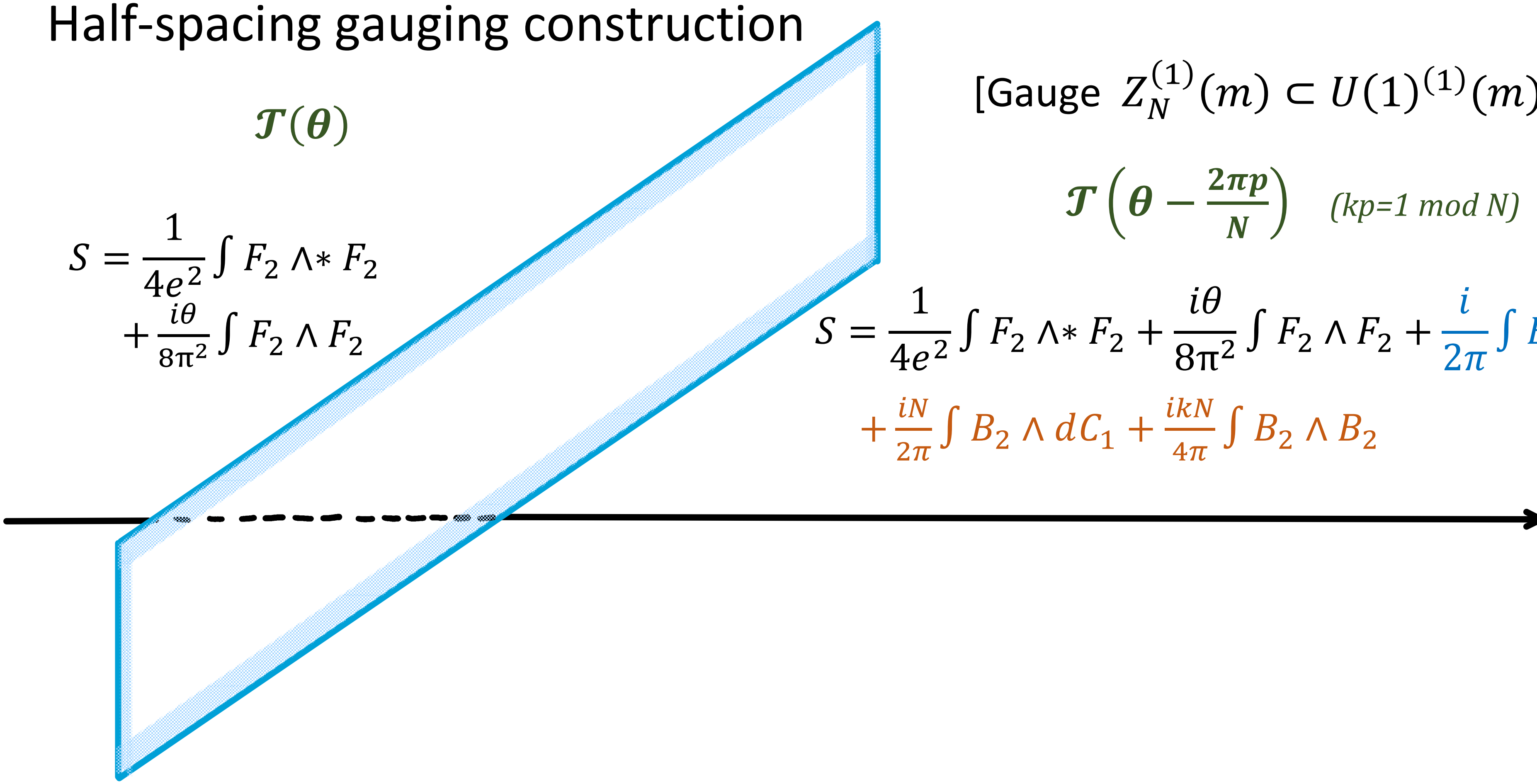

$\mathcal{T}(\theta)$

$$S = \frac{1}{4e^2}\int F_2 \wedge * F_2 + \frac{i\theta}{8\pi^2}\int F_2 \wedge F_2$$

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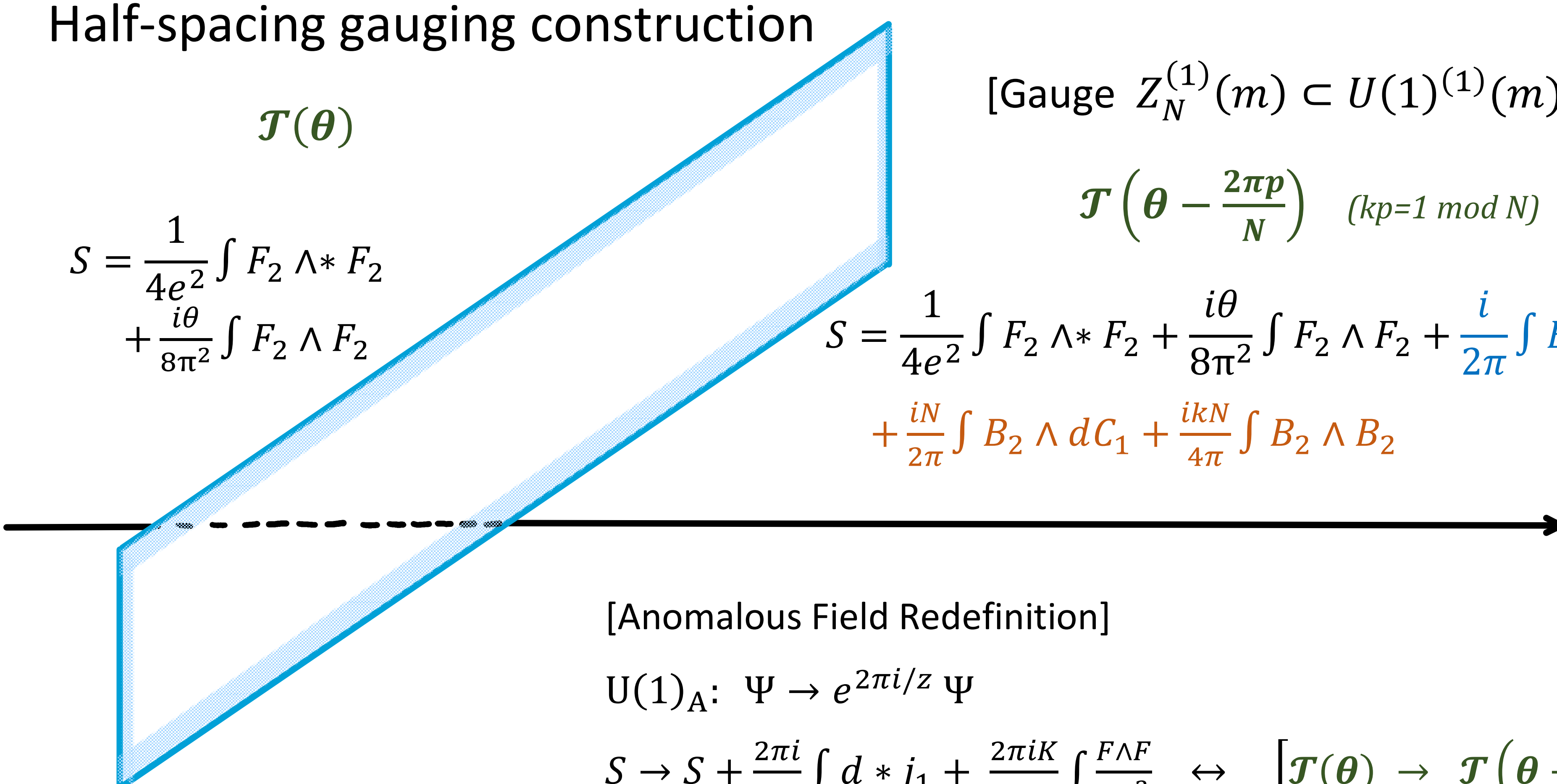

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[Anomalous Field Redefinition]

$U(1)_A$: $\Psi \to e^{2\pi i/z}\,\Psi$

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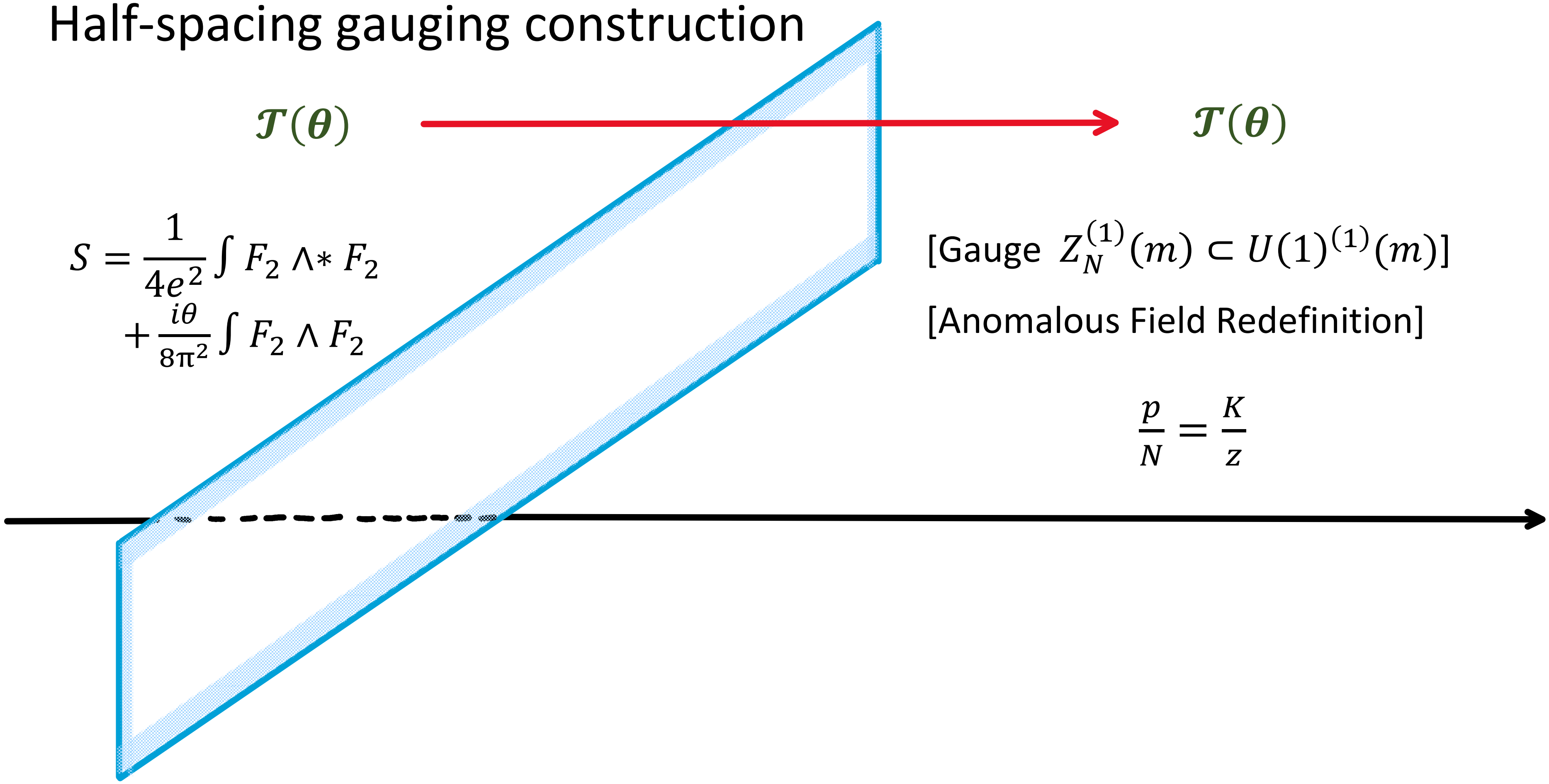

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$$\frac{p}{N} = \frac{K}{z}$$

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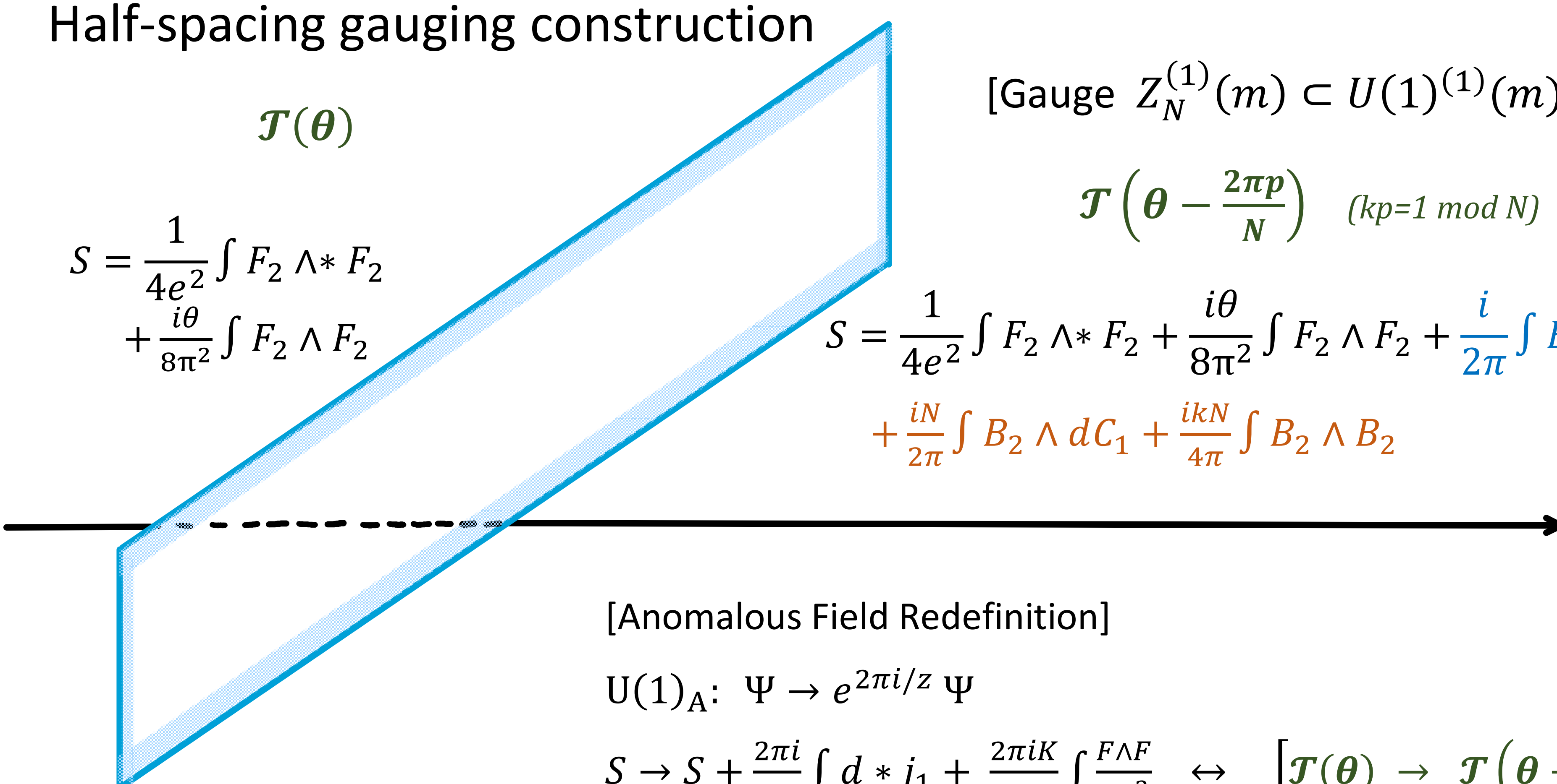

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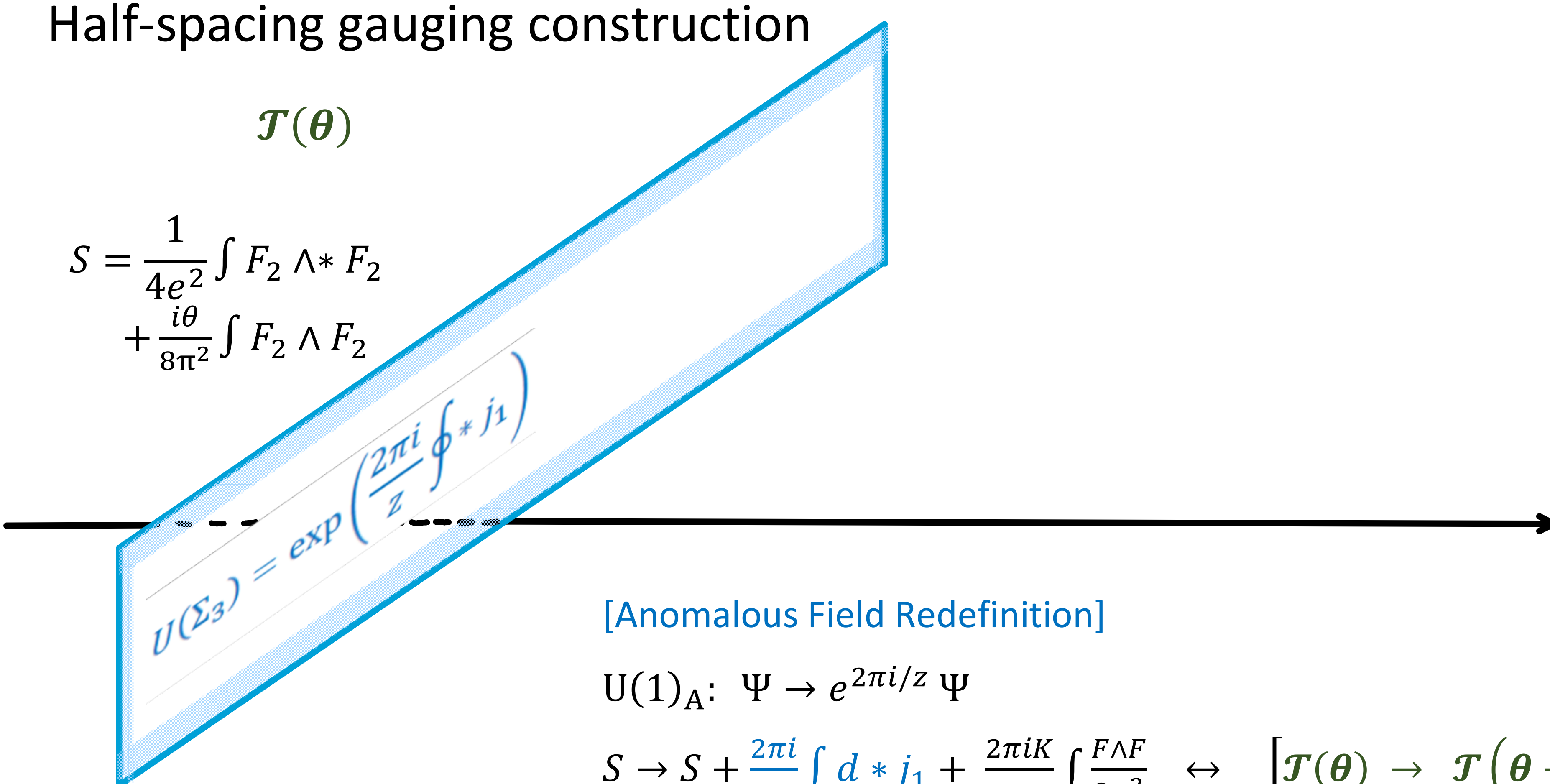

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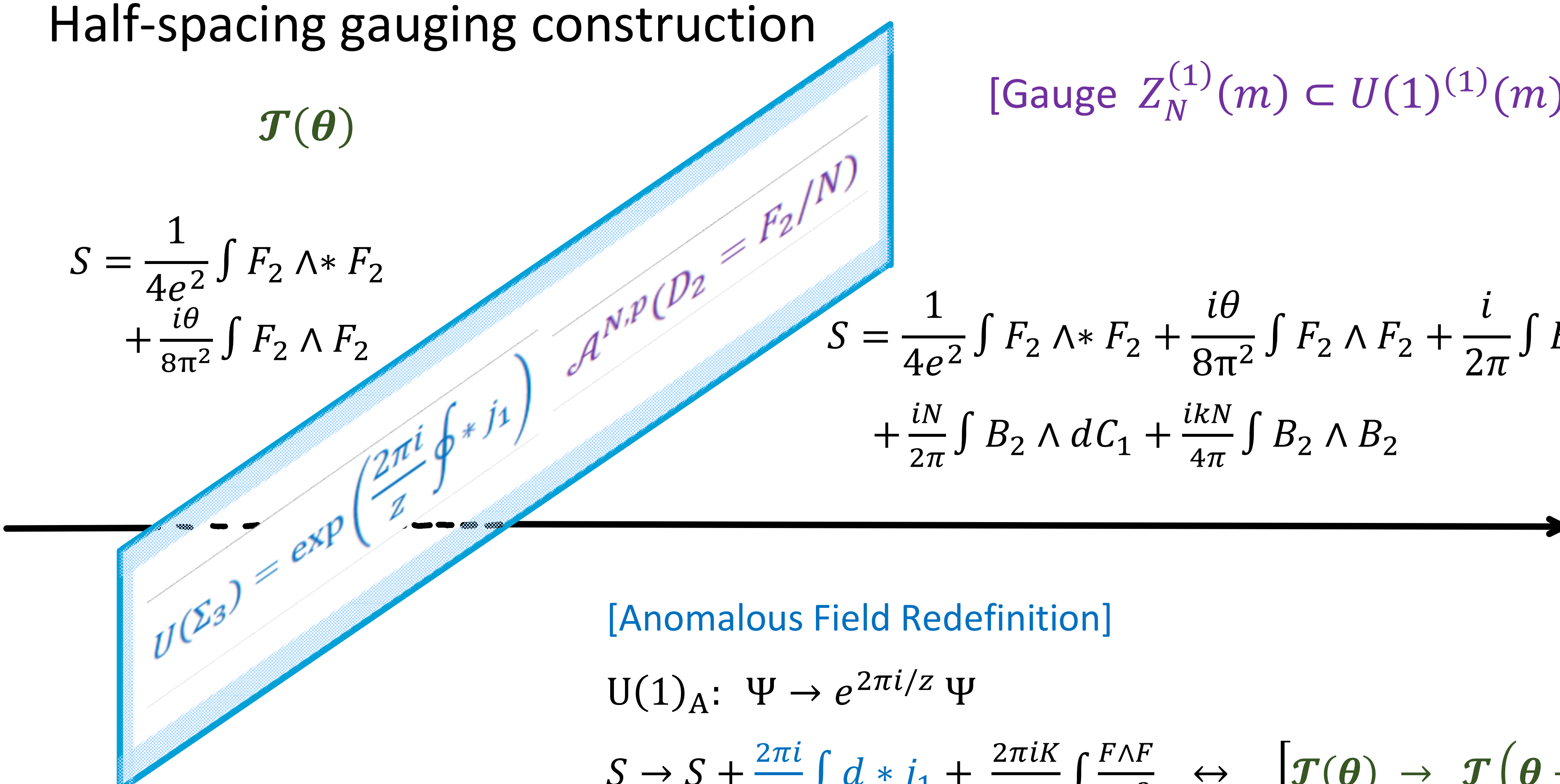

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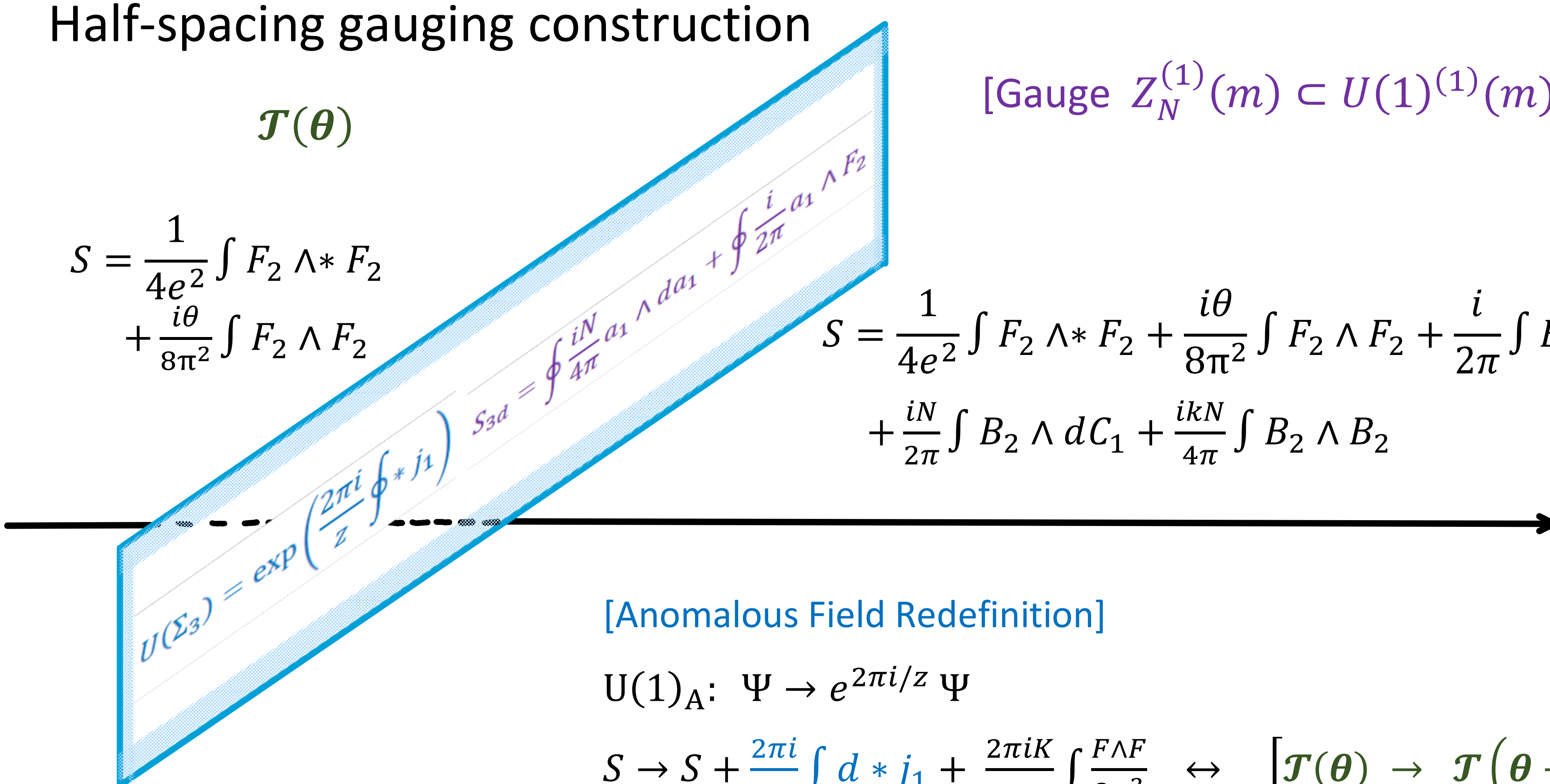

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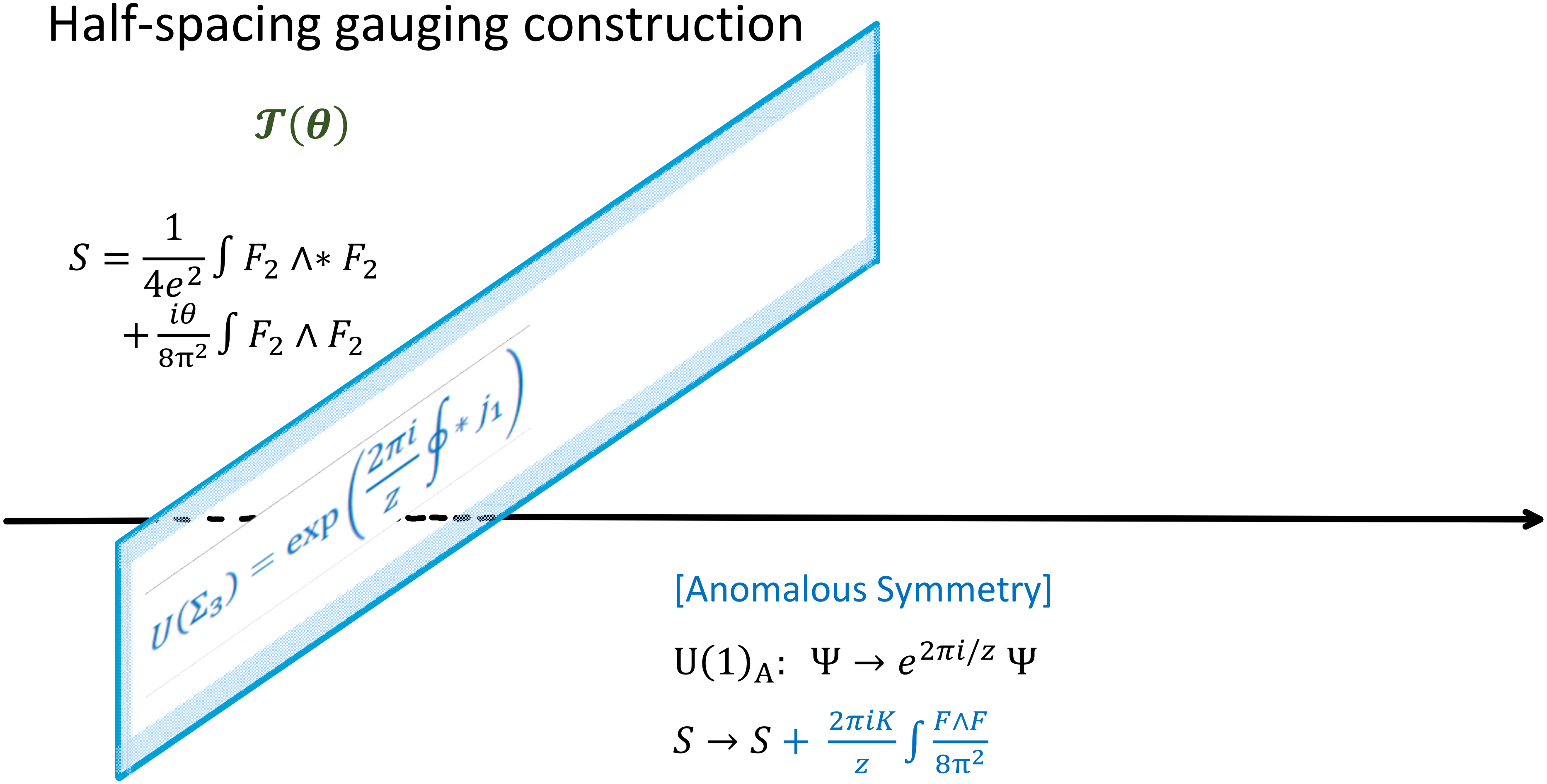

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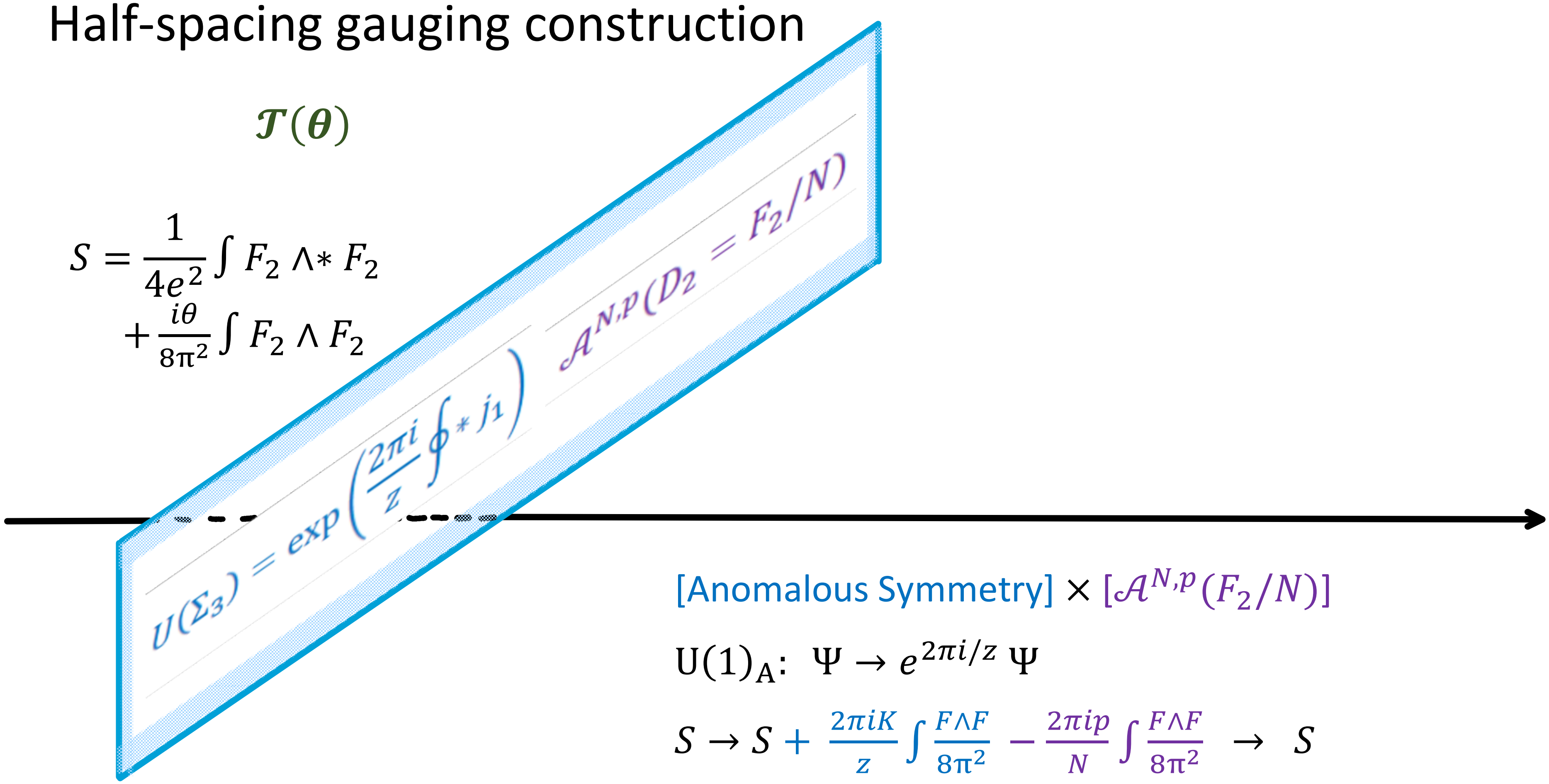


[Anomalous Symmetry] $\times$ [$\mathcal{A}^{N,p}(F_2/N)$]

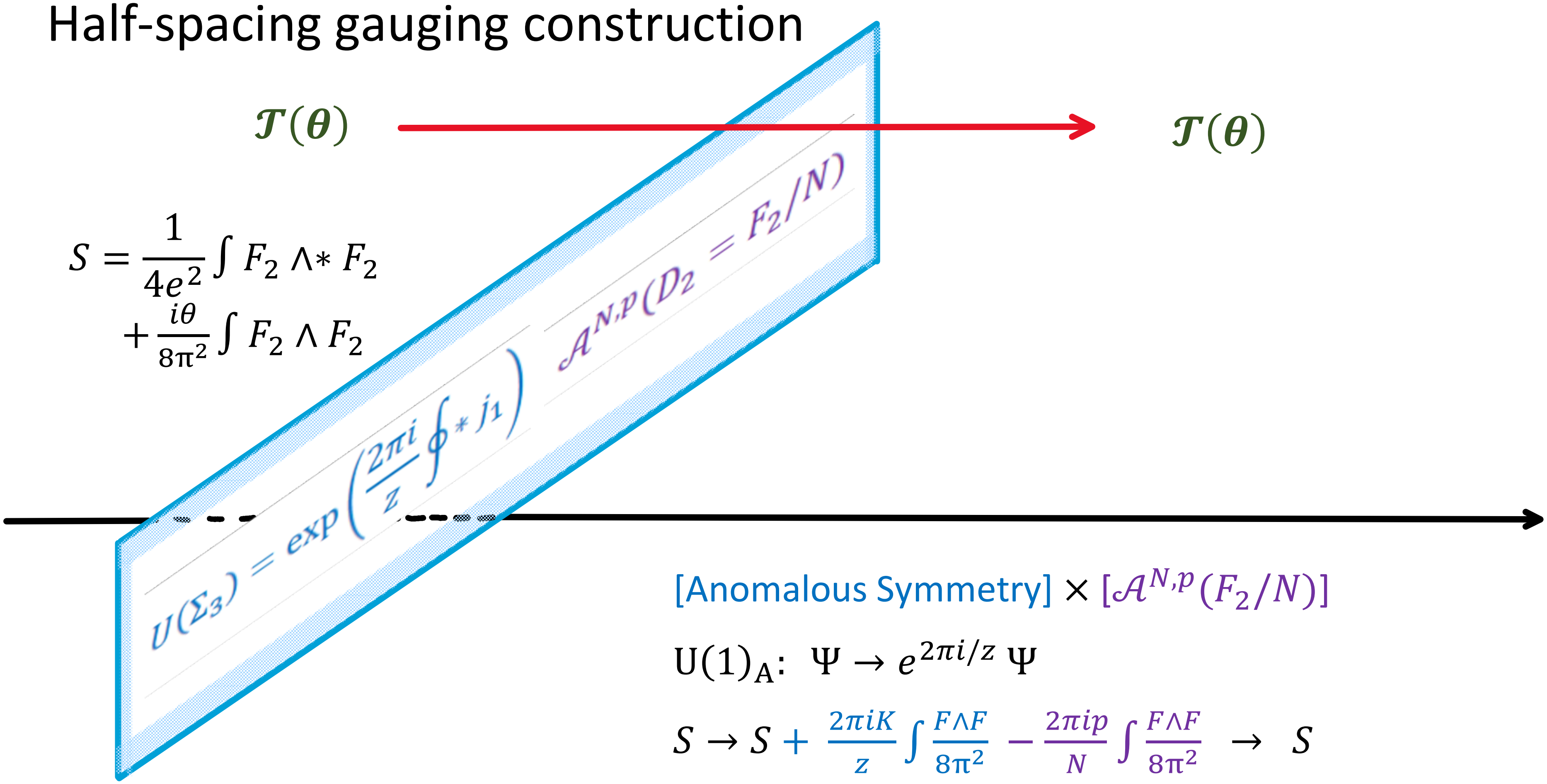
$U(1)_A$: $\Psi \to e^{2\pi i/z}\,\Psi$

$$S \to S + \frac{2\pi i K}{z}\int \frac{F\wedge F}{8\pi^2} - \frac{2\pi i p}{N}\int \frac{F\wedge F}{8\pi^2} \;\to\; S$$

Non-Invertible Symmetry

1. From $U(1)$ Instanton

Half-spacing gauging construction



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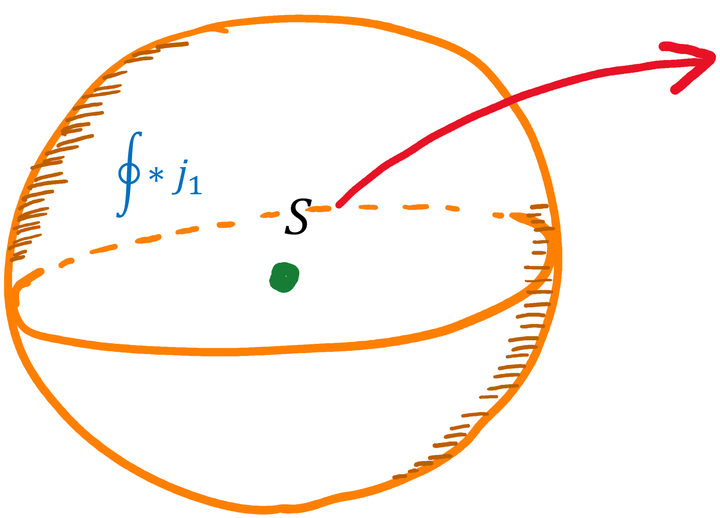
Non-Invertible Symmetry

1. From $U(1)$ Instanton

Summary:

$$U(1)^{(0)}{}_A : \quad \Psi_+ \to e^{i\alpha}\,\Psi_+ \,, \qquad \Psi_- \to e^{i\alpha}\,\Psi_- \quad \Rightarrow \quad d * j_A = \frac{K}{8\pi^2} F_2 \wedge F_2$$



$$S \to S + \frac{2\pi i K}{z} \int \frac{F_2 \wedge F_2}{8\pi^2}$$

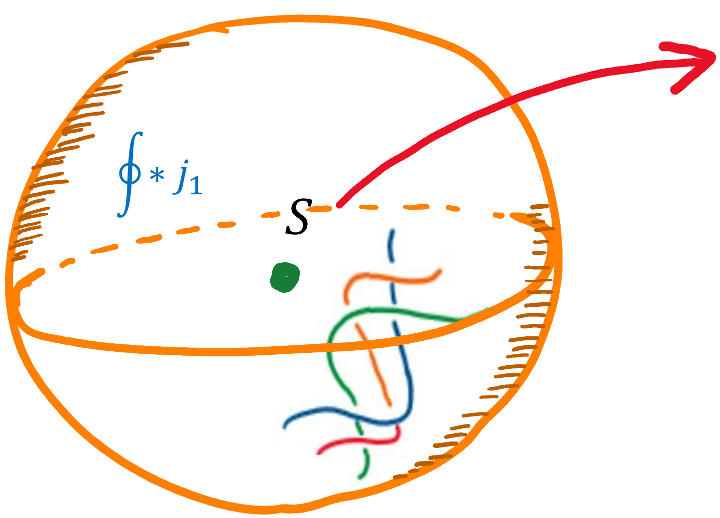
$$\underbrace{\exp\left(\frac{2\pi i}{z} \oint * j_1\right)}_{U\left(\frac{2\pi}{z},\, \Sigma_3\right)}$$

Non-Invertible Symmetry

1. From $U(1)$ Instanton

Summary:

$$U(1)^{(0)}{}_A : \quad \Psi_+ \to e^{i\alpha}\,\Psi_+ \ , \qquad \Psi_- \to e^{i\alpha}\,\Psi_- \quad \Rightarrow \quad d * j_A = \frac{K}{8\pi^2} F_2 \wedge F_2$$



$$S \ \to \ S + \frac{2\pi i K}{z} \int \frac{F_2 \wedge F_2}{8\pi^2} - \frac{2\pi i p}{N} \int \frac{F_2 \wedge F_2}{8\pi^2} \ \to \ S$$

$$\underbrace{\exp\left(\frac{2\pi i}{z} \oint * j_1\right)}_{U\left(\frac{2\pi}{z},\, \Sigma_3\right)} \times \mathcal{A}^{\mathrm{N,p}}(\mathrm{F_2/N})$$

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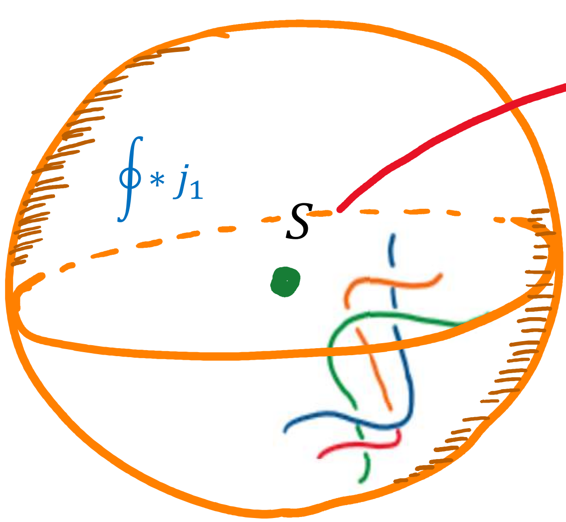

$$S \;\to\; S + \frac{2\pi i K}{z}\int \frac{F_2 \wedge F_2}{8\pi^2} - \frac{2\pi i p}{N}\int \frac{F_2 \wedge F_2}{8\pi^2} \;\to\; S$$

$$\mathcal{D}_{\frac{2\pi}{z}}(\Sigma_3) = \underbrace{\exp\left(\frac{2\pi i}{z}\oint * j_1\right)}_{U\left(\frac{2\pi}{z},\Sigma_3\right)} \times \mathcal{A}^{N,p}(F_2/N)$$

$$\mathcal{D}_k(\Sigma_3) \times \overline{\mathcal{D}}_k(\Sigma_3) \sim \sum_S \xi(S)\exp\left(\frac{i}{2\pi N}\int_S F_2\right) \neq 1$$

Non-Invertible Symmetry

2. From Fractional Instanton

e.g. $G = SU(N)$

electric 1-form: Z_N
magnetic 1-form: none

Non-Invertible Symmetry

2. From Fractional Instanton

e.g. $G = SU(N)/Z_L$

electric 1-form: $Z_{N/L}$

magnetic 1-form: Z_L

Non-Invertible Symmetry

2. From Fractional Instanton

e.g. $G = SU(N)/Z_L$

electric 1-form: $Z_{N/L}$

magnetic 1-form: Z_L

$U(1)_A$ with $\alpha = \frac{2\pi}{z}$, $S \to S + \frac{2\pi K i}{z} \boxed{\int_{M_4} \frac{G \wedge G}{8\pi^2}} + \frac{2\pi K i}{z} \boxed{\left(\frac{L-1}{L}\right) \int_{M_4} \frac{w_2 \wedge w_2}{2}}$

$\in Z$ $\in Z_L$

Non-Invertible Symmetry

2. From Fractional Instanton

e.g. $G = SU(N)/Z_L$

electric 1-form: $Z_{N/L}$

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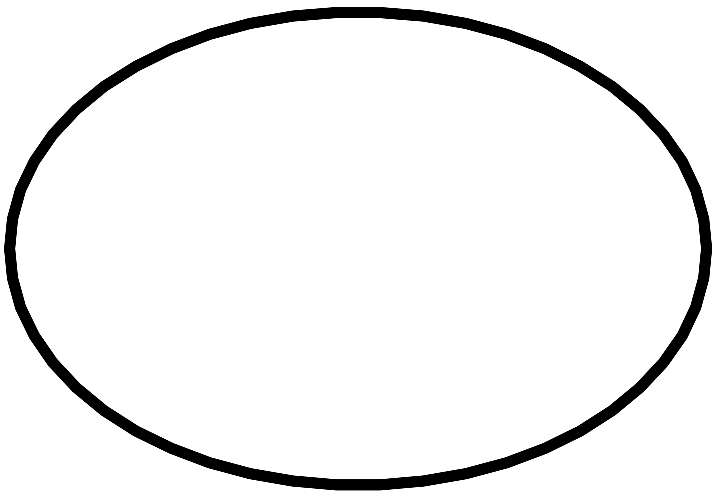
Global $U(1)_A$



Non-Invertible Symmetry

2. From Fractional Instanton

e.g. $G = SU(N)/Z_L$

electric 1-form: $Z_{N/L}$
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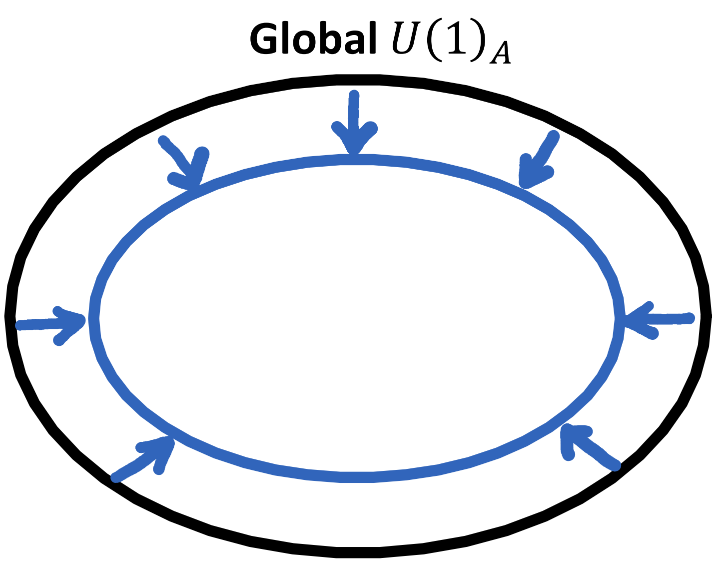
$U(1)_A$ with $\alpha = \frac{2\pi}{z}$, $\quad S \to S + \frac{2\pi K i}{z}\int_{M_4}\frac{G\wedge G}{8\pi^2} + \frac{2\pi K i}{z}\left(\frac{L-1}{L}\right)\int_{M_4}\frac{w_2\wedge w_2}{2}$

$\in Z$ $\qquad \in Z_L$



$\to$ Z **Instanton**

Non-Invertible Symmetry

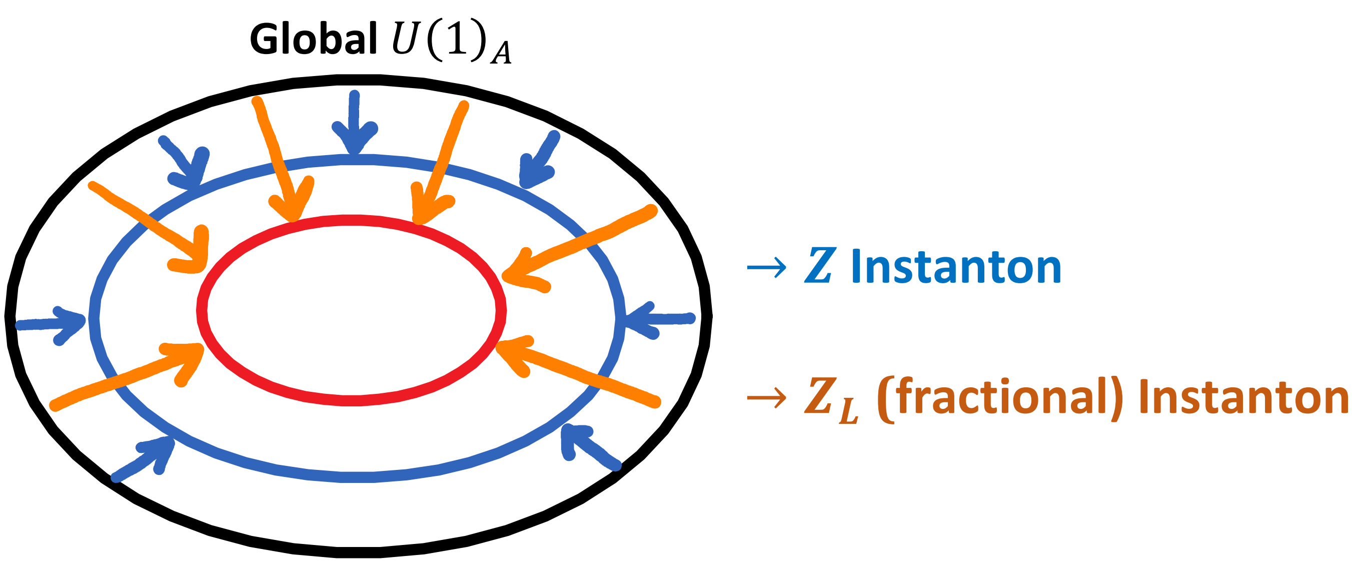
2. From Fractional Instanton

e.g. $G = SU(N)/Z_L$

electric 1-form: $Z_{N/L}$

magnetic 1-form: Z_L

$U(1)_A$ with $\alpha = \frac{2\pi}{z}$, $\quad S \rightarrow S + \frac{2\pi K i}{z} \underbrace{\int_{M_4} \frac{G \wedge G}{8\pi^2}}_{\in Z} + \frac{2\pi K i}{z} \underbrace{\left(\frac{L-1}{L}\right) \int_{M_4} \frac{w_2 \wedge w_2}{2}}_{\in Z_L}$



Non-Invertible Symmetry

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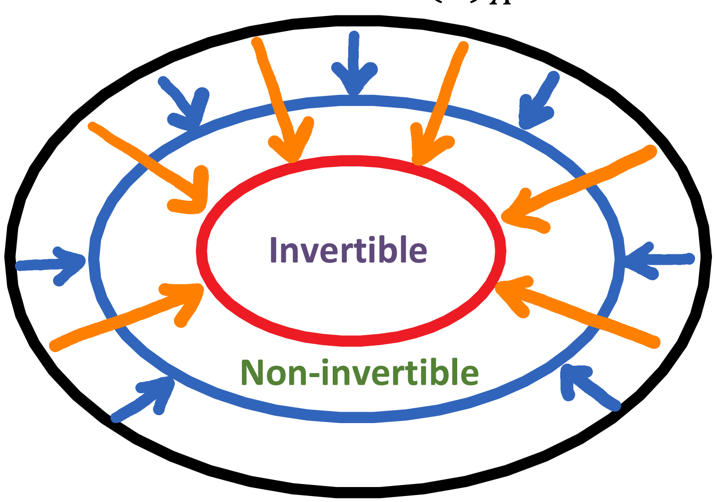
$\in Z$ $\qquad\qquad \in Z_L$

Global $U(1)_A$



$\to$ Z Instanton

$\to$ Z_L (fractional) Instanton

Non-Invertible Symmetry

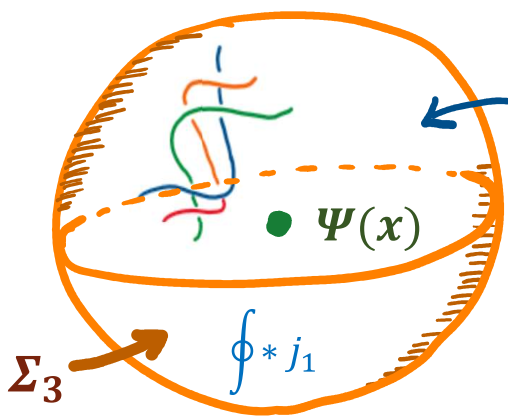
2. From Fractional Instanton

e.g. $G = SU(N)/Z_L$

electric 1-form: $Z_{N/L}$

magnetic 1-form: Z_L

$$U(1)_A \text{ with } \alpha = \frac{2\pi}{z}, \qquad S \to S + \frac{2\pi K i}{z} \underbrace{\int_{M_4} \frac{G \wedge G}{8\pi^2}}_{\in Z} + \frac{2\pi K i}{z} \underbrace{\left(\frac{L-1}{L}\right) \int_{M_4} \frac{w_2 \wedge w_2}{2}}_{\in Z_L}$$



$$S_{3d} = \frac{iN}{4\pi} \int_{\Sigma_3} a_1 \wedge da_2 + \frac{i}{2\pi} \int_{\Sigma_3} a_1 \wedge w_2$$

$$U\left(\frac{2\pi}{z}, \Sigma_3\right) \to D_z = U\left(\frac{2\pi}{z}, \Sigma_3\right) \times \mathcal{A}^{N,p}(w_2) \text{ with } \frac{p}{N} = \frac{K}{z}$$

Outline

I. Generalized Global Symmetries

I-1. Higher-form symmetry
I-2. Non-invertible symmetry

II. Strong CP Problem-I: IR to UV

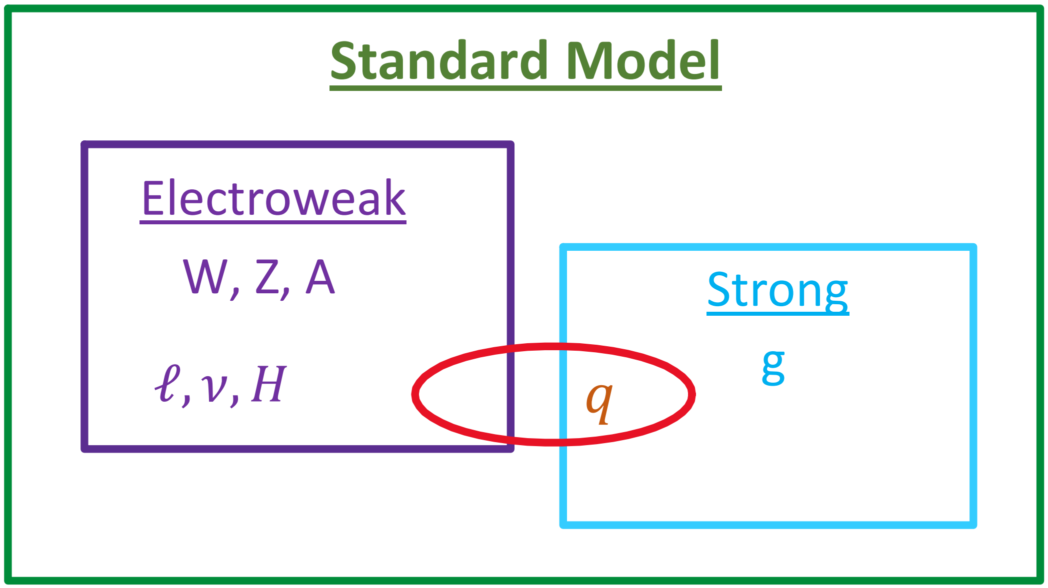
II-1. Non-invertible Peccei-Quinn symmetry
II-2. Massless quark solution

III. Strong CP Problem-II: UV to IR

III-1. $SU(9)$ Color-Flavor unification
III-2. Flavor structure and CKM CPV phase

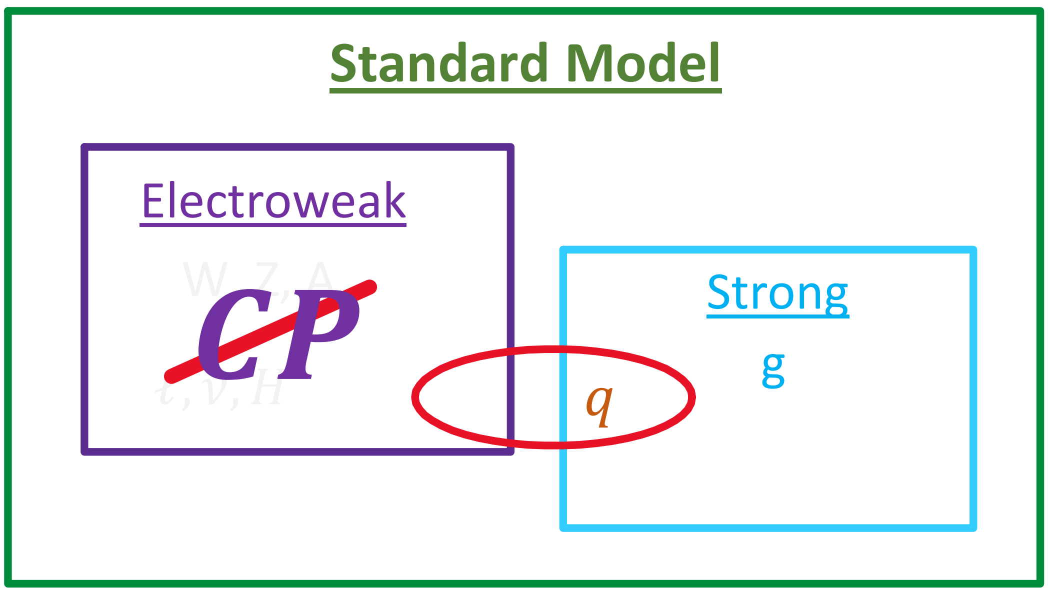
Strong-CP Problem

1. Strong CP Problem



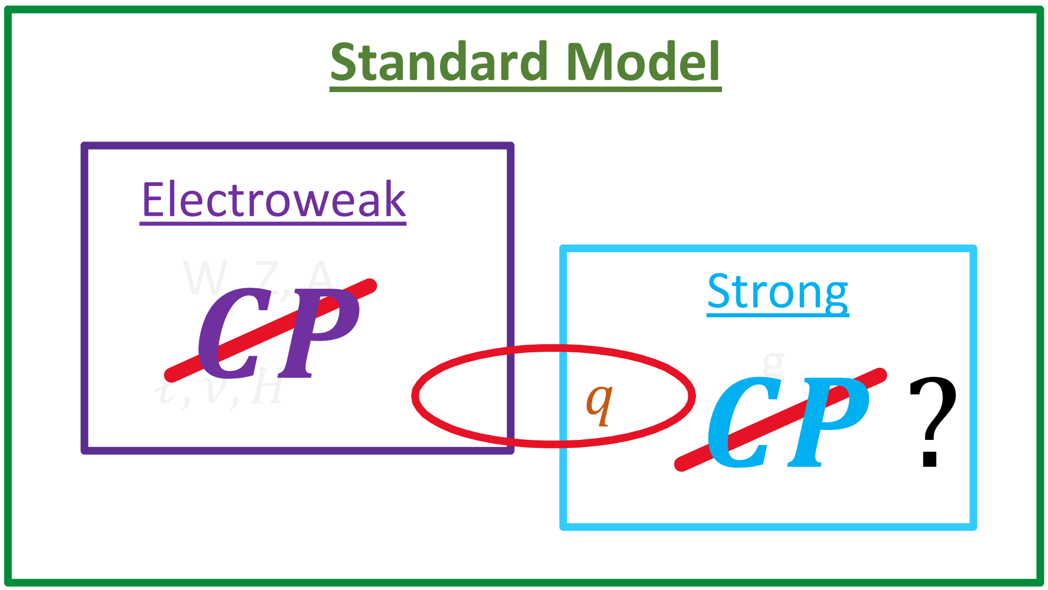
Strong-CP Problem

1. Strong CP Problem



Strong-CP Problem

1. Strong CP Problem



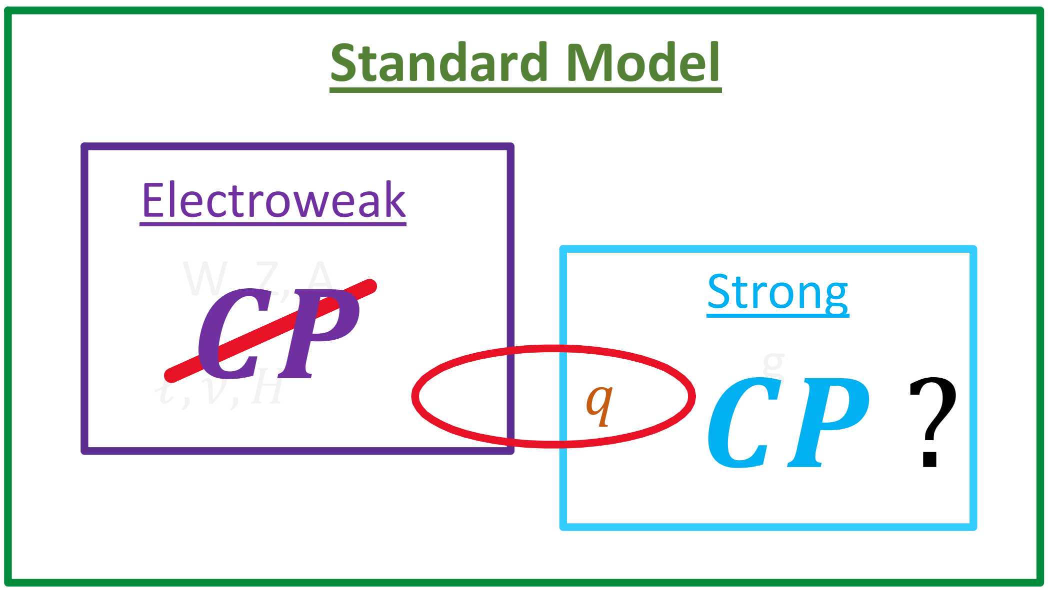
Expectation based on general rules of effective field theory

$$S_{QCD} \supset \frac{i\bar{\boldsymbol{\theta}}}{8\pi^2}\int Tr(G \wedge G)$$

$$\bar{\boldsymbol{\theta}} \sim \boldsymbol{O}(\mathbf{1})$$

Strong-CP Problem

1. Strong CP Problem



Expectation based on general rules of effective field theory

$$S_{QCD} \supset \frac{i\bar{\theta}}{8\pi^2}\int Tr(G \wedge G)$$

Neutron Electric Dipole Moment

$$d_n \sim 3 \times 10^{16}\,\bar{\theta} \quad \rightarrow \quad \bar{\theta} < 10^{-10}$$

Strong-CP Problem

2. Non-invertible Peccei-Quinn Symmetry

Conclusion:

We start with $\mathcal{L} \supset y_u \widetilde{H} Q \bar{u} + y_e H L \bar{e}$ but $\boldsymbol{y_d = 0}$

So, new symmetries appearing below are approximate symmetries

and $\boldsymbol{y_d}$ is the symmetry breaking spurion (parameter).

Strong-CP Problem

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So, new symmetries appearing below are approximate symmetries

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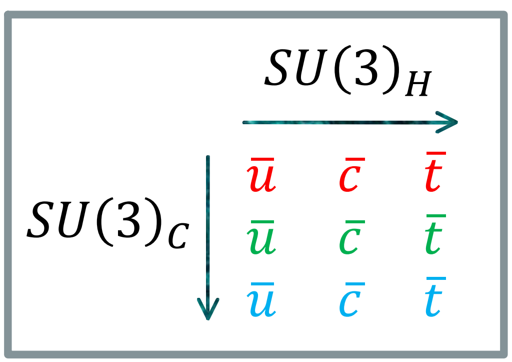
(1) $SU(3)_C \times SU(3)_H / Z_3$: $\boldsymbol{Z_3^{\bar{d}}}$ NIS

(2) $SU(3)_C \times U(1)_H / Z_3,\ H = B_1 + B_2 - 2B_3$: $\boldsymbol{Z_3^{Q-\bar{u}+\bar{d}}}$ NIS



$B_i \equiv Q_i - \bar{u}_i - \bar{d}_i$

Strong-CP Problem

2. Non-invertible Peccei-Quinn Symmetry

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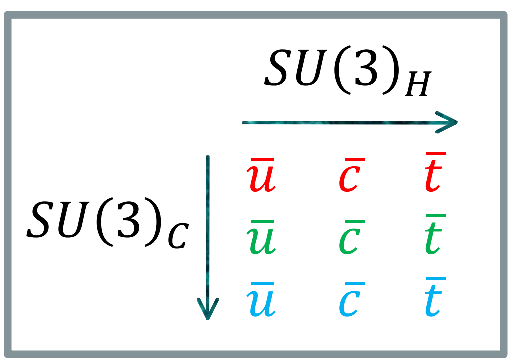
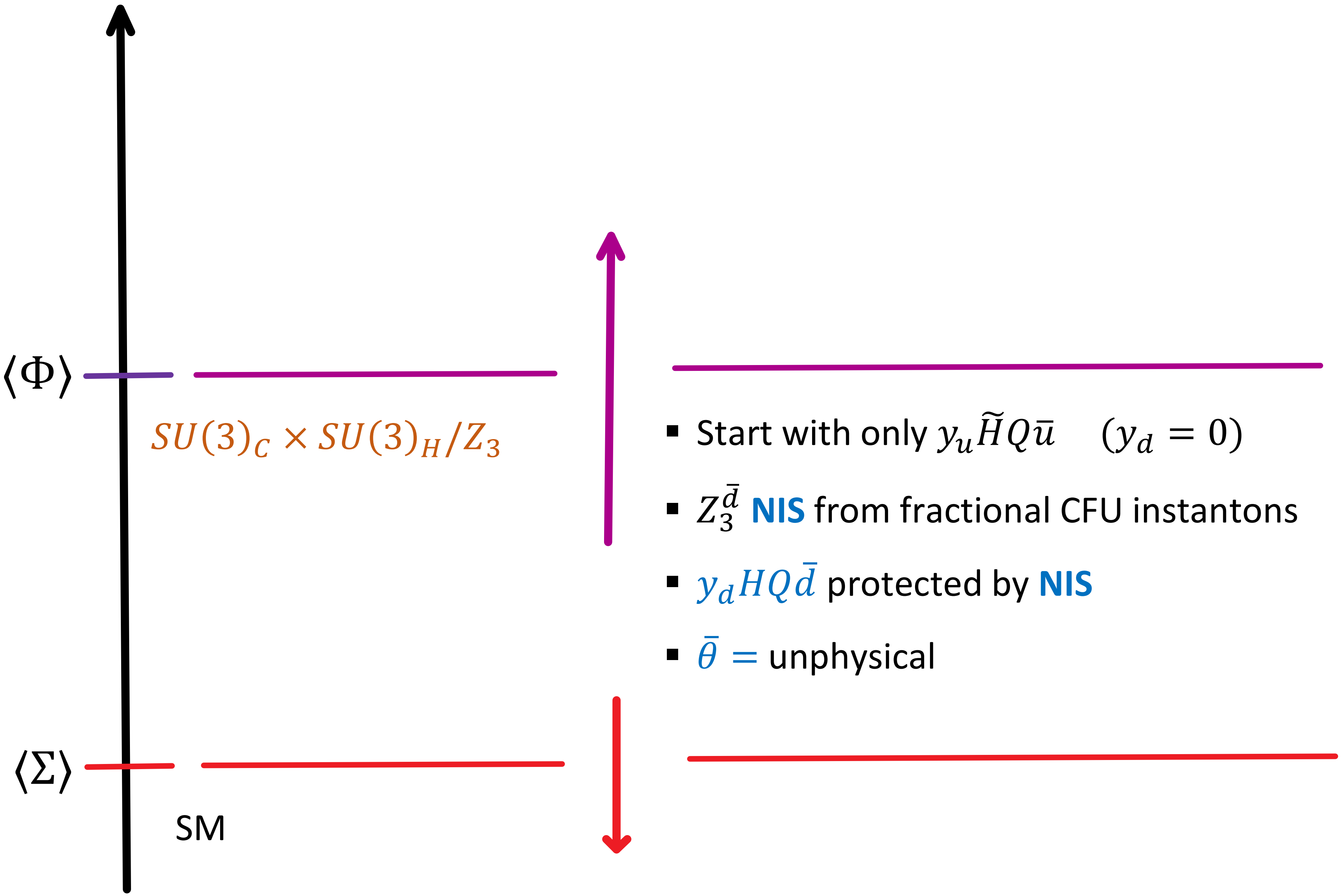


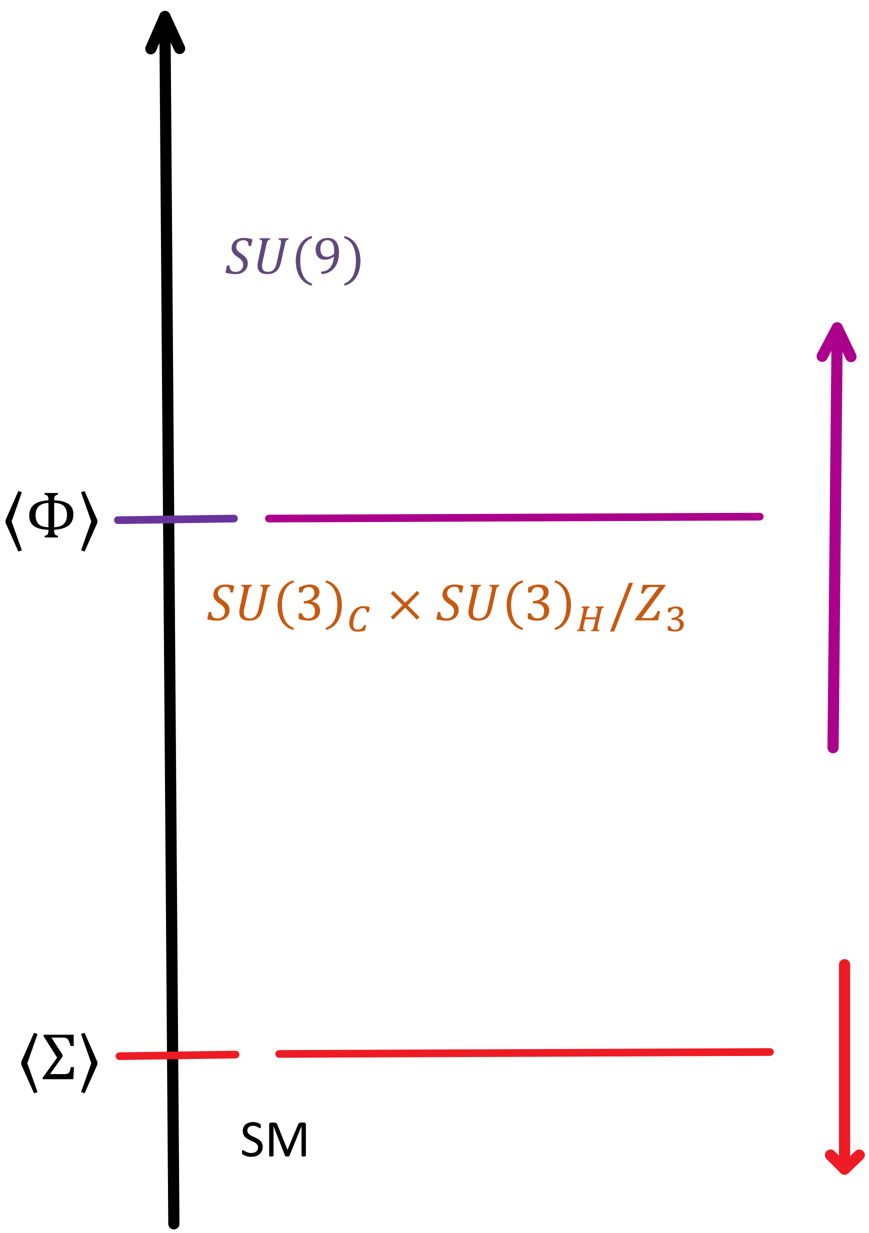
$B_i \equiv Q_i - \bar{u}_i - \bar{d}_i$

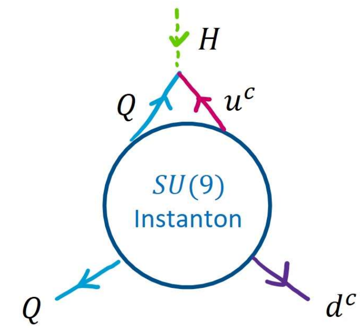
Solving Strong CP with Non-Invertible Symmetry



- Start with only $y_u \widetilde{H} Q \bar{u}$ $\quad (y_d = 0)$
- $Z_3^{\bar{d}}$ **NIS** from fractional CFU instantons
- $y_d H Q \bar{d}$ protected by **NIS**
- $\bar{\theta}$ = unphysical

Solving Strong CP with Non-Invertible Symmetry





$$\mathcal{L}_{UV} \supset y_u \tilde{H} Q \bar{u} + y_u^* e^{-\frac{2\pi}{\alpha_9}} H Q \bar{d} + \cdots$$

- Start with only $y_u \tilde{H} Q \bar{u}$ $\quad (y_d = 0)$
- $Z_3^{\bar{d}}$ **NIS** from fractional CFU instantons
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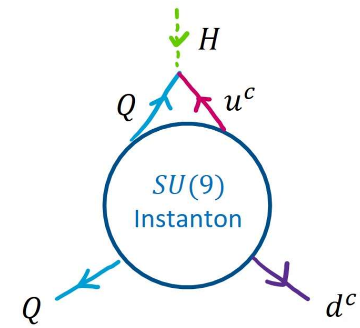
Solving Strong CP with Non-Invertible Symmetry

$SU(9)$



$$\mathcal{L}_{UV} \supset y_u \widetilde{H} Q \bar{u} + y_u^* e^{-\frac{2\pi}{\alpha_9}} H Q \bar{d} + \cdots$$

$\langle \Phi \rangle$

$SU(3)_C \times SU(3)_H / Z_3$

- Start with only $y_u \widetilde{H} Q \bar{u} \quad (y_d = 0)$
- $Z_3^{\bar{d}}$ **NIS** from fractional CFU instantons
- $y_d H Q \bar{d}$ protected by **NIS**
- $\bar{\theta} =$ unphysical

$\langle \Sigma \rangle$

SM

Yukawa texture and CKM CPV (+possible $\bar{\theta}$)

Strong-CP Problem

2. Non-invertible Peccei-Quinn Symmetry

(1) $SU(3)_C \times SU(3)_H / Z_3$: $\boldsymbol{Z_3^{\bar{d}}}$ NIS

	$SU(3)_C$	$SU(3)_H$	$U(1)_B$	$U(1)_{\bar{d}}$
$\boldsymbol{Q}$	3	3	$+1$	0
$\boldsymbol{\bar{u}}$	$\bar{3}$	$\bar{3}$	-1	0
$\boldsymbol{\bar{d}}$	$\bar{3}$	$\bar{3}$	-1	$+1$

$\mathcal{L} \sim y_u \tilde{H} Q^i \bar{u}_i$ (flavor-diagonal/universal)

$y_u = 1 \times 1$ number

Strong-CP Problem

2. Non-invertible Peccei-Quinn Symmetry

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$\mathcal{L} \sim y_u \widetilde{H} Q^i \bar{u}_i$ (flavor-diagonal/universal)

$y_u = 1 \times 1$ number

CFU Fractional Instanton (CFU=Color-(non-abelian)Flavor-U(1))

Quotient by Z_3: (i) $[Z_3 \in SU(3)_C] \equiv [Z_3 \in SU(3)_H]$

(ii) Under "diagonal" Z_3 entire fields are neutral, more magnetic states

(iii) $\exists Z_3$ magnetic 1-form: $\oint w_2(C) = \oint w_2(H) = 0,1,2 \ (\in Z_3)$

(iv) CFU instanton: $\mathcal{N}_C = \frac{1}{3}\int w_2(C) \wedge w_2(C)$, $\mathcal{N}_H = \frac{1}{3}\int w_2(H) \wedge w_2(H)$

Strong-CP Problem

2. Non-invertible Peccei-Quinn Symmetry

(1) $SU(3)_C \times SU(3)_H / Z_3$: $Z_3^{\bar{d}}$ NIS

	$SU(3)_C$	$SU(3)_H$	$U(1)_B$	$U(1)_{\bar{d}}$
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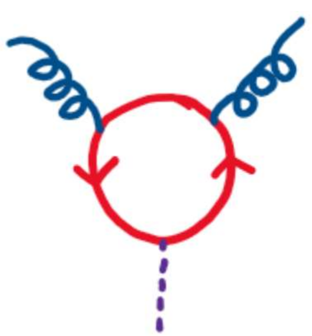

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$$\mathcal{A}_f = \sum_{\Psi_i} q_i I_{\Psi_i} = 3(\mathcal{N}_C + \mathcal{N}_H)(2q_Q + q_{\bar{d}} + q_{\bar{u}})$$

Strong-CP Problem

2. Non-invertible Peccei-Quinn Symmetry

(1) $SU(3)_C \times SU(3)_H / Z_3$: $\boldsymbol{Z_3^{\bar{d}}}$ NIS

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$\mathcal{L} \sim y_u \tilde{H} Q^i \bar{u}_i$ (flavor-diagonal/universal)

$y_u = 1 \times 1$ number

	$U(1)_B$	$U(1)_{\bar{d}}$
$[SU(3)_C]^2$	0	N_g
$[SU(2)_L]^2$	$N_c N_g$	0
$[U(1)_Y]^2$	$-18 N_c N_g$	$4 N_c N_g$
$[SU(3)_H]^2$	0	N_c
$[CH]$	0	2

Strong-CP Problem

2. Non-invertible Peccei-Quinn Symmetry

(1) $SU(3)_C \times SU(3)_H / Z_3$: $\boldsymbol{Z_3^{\bar{d}}}$ NIS

	$SU(3)_C$	$SU(3)_H$	$U(1)_B$	$U(1)_{\bar{d}}$
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	$U(1)_B$	$U(1)_{\bar{d}}$
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$[U(1)_Y]^2$	$-18 N_c N_g$	$4 N_c N_g$
$[SU(3)_H]^2$	0	N_c
$[CH]$	0	2

Symmetry

(i) Without including $[CH]$ instanton:

$$\frac{U(1)_B}{Z_3} \times U(1)_{\bar{d}} \rightarrow Z_3^B \times Z_3^{\bar{d}}$$

non-abelian instantons dominant → No NIS

Strong-CP Problem

2. Non-invertible Peccei-Quinn Symmetry

(1) $SU(3)_C \times SU(3)_H/Z_3$: $Z_3^{\bar{d}}$ NIS

	$SU(3)_C$	$SU(3)_H$	$U(1)_B$	$U(1)_{\bar{d}}$
$\boldsymbol{Q}$	3	3	$+1$	0
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$\mathcal{L} \sim y_u \widetilde{H} Q^i \bar{u}_i$ (flavor-diagonal/universal)

$y_u = 1 \times 1$ number

	$U(1)_B$	$U(1)_{\bar{d}}$
$[SU(3)_C]^2$	0	N_g
$[SU(2)_L]^2$	$N_c N_g$	0
$[U(1)_Y]^2$	$-18 N_c N_g$	$4 N_c N_g$
$[SU(3)_H]^2$	0	N_c
$[CH]$	0	2

Symmetry

(i) Without including $[CH]$ instanton:

$$\frac{U(1)_B}{Z_3} \times U(1)_{\bar{d}} \to Z_3^B \times Z_3^{\bar{d}}$$

non-abelian instantons dominant → No NIS

(ii) With $[CH]$ instanton:

$$Z_3^B \times Z_3^{\bar{d}} \;\to\; Z_3^B \times \emptyset$$

Strong-CP Problem

2. Non-invertible Peccei-Quinn Symmetry

(1) $SU(3)_C \times SU(3)_H / Z_3$: $\boldsymbol{Z_3^{\bar{d}}}$ NIS

	$SU(3)_C$	$SU(3)_H$	$U(1)_B$	$U(1)_{\bar{d}}$
$\boldsymbol{Q}$	3	3	$+1$	0
$\boldsymbol{\bar{u}}$	$\bar{3}$	$\bar{3}$	-1	0
$\boldsymbol{\bar{d}}$	$\bar{3}$	$\bar{3}$	-1	$+1$

$\mathcal{L} \sim y_u \tilde{H} Q^i \bar{u}_i$ (flavor-diagonal/universal)

$y_u = 1 \times 1$ number

	$U(1)_B$	$U(1)_{\bar{d}}$
$[SU(3)_C]^2$	0	N_g
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$\boldsymbol{Q}$	3	3	$+1$	0
$\boldsymbol{\bar{u}}$	$\bar{3}$	$\bar{3}$	-1	0
$\boldsymbol{\bar{d}}$	$\bar{3}$	$\bar{3}$	-1	$+1$

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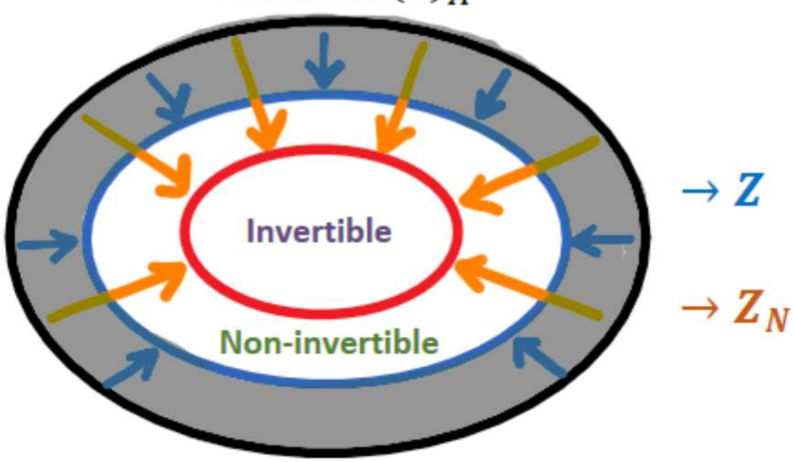

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(ii) With $[CH]$ instanton:

$$Z_3^B \times Z_3^{\bar{d}} \rightarrow Z_3^B \times \emptyset \quad \Rightarrow \quad \boldsymbol{Z_3^{\bar{d}}} \text{ NIPQ Symmetry}$$

Outline

I. Generalized Global Symmetries

I-1. Higher-form symmetry
I-2. Non-invertible symmetry

II. Strong CP Problem-I: IR to UV

II-1. Non-invertible Peccei-Quinn symmetry
II-2. Massless quark solution

III. Strong CP Problem-II: UV to IR

III-1. $SU(9)$ Color-Flavor unification
III-2. Flavor structure and CKM CPV phase

Strong-CP Problem

3. Massless Quark Solution to the Strong CP Problem

(1) $SU(3)_C \times SU(3)_H / Z_3$: $\boldsymbol{Z_3^{\bar{d}}}$ NIS

$\mathcal{L} \sim y_d H Q \bar{d}$ term is forbidden by $Z_3^{\bar{d}}$ non-invertible Peccei-Quinn symmetry

Down quarks (d, s, b) are massless if $Z_3^{\bar{d}}$ is exact.
($y_d =$ non-invertible $Z_3^{\bar{d}}$ breaking spurion (paramter))

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1. $$\mathcal{L}_{SM} \supset y_u \tilde{H} Q \bar{u} + y_d H Q \bar{d} + \frac{\theta}{32\pi^2} G \tilde{G}$$
$$= m_u e^{i\varphi_u} u \bar{u} + m_d e^{i\varphi_d} d \bar{d} + \frac{\theta}{32\pi^2} G \tilde{G}$$

$$u \to e^{i\alpha} u, \quad \varphi_u \to \varphi_u + \alpha, \quad \theta \to \theta - \alpha$$
$$d \to e^{i\alpha} d, \quad \varphi_d \to \varphi_d + \alpha, \quad \theta \to \theta - \alpha$$

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$$\boldsymbol{\bar{\theta} \equiv \arg e^{-i\theta} \det(y_u y_d)}$$

Neutron electric dipole moment $d_n \sim 3 \times 10^{16}\, \bar{\theta} \;\; \to \;\; \bar{\theta} < 10^{-10}$

Strong-CP Problem

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Massless Quark Solution:

2. In the presence of massless chiral fermion, e.g. , "$\boldsymbol{\bar{\theta}}$ is unphysical"

$$\mathcal{L}_{SM} \supset y_u \widetilde{H} Q \bar{u} + \frac{\theta}{32\pi^2} G\tilde{G} = m_u e^{i\varphi_u} u\bar{u} + \frac{\theta}{32\pi^2} G\tilde{G}$$

Field redefinition: $d \to e^{i\alpha} d \;\; \Rightarrow \;\; \delta S = \frac{i}{8\pi^2}(\bar{\theta} - \alpha)\int \mathrm{tr}(G_2 \wedge G_2)$

Strong-CP Problem

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Note: $M \equiv e^{-i\theta} \det(y_u y_d) \in \mathbb{C}$ and $CP\!: \mathrm{Im}(M) \to -\mathrm{Im}(M)$

M behaves smoothly as $|M| \to 0$

CP-invariance $\leftrightarrow$ $M \in \mathbb{R}_+$

Strong-CP Problem

3. Massless Quark Solution to the Strong CP Problem

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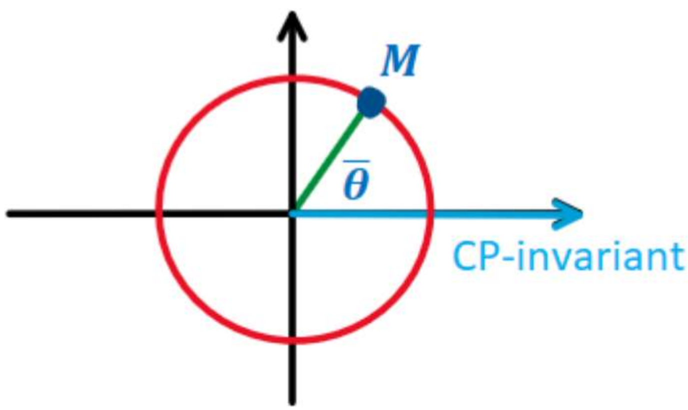

Strong-CP Problem

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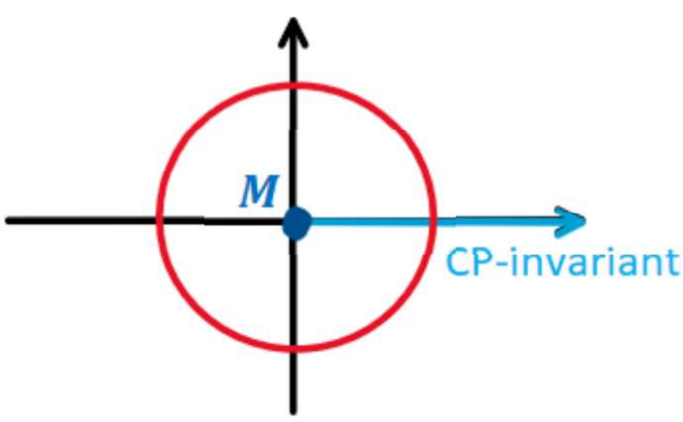

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Down quarks (d, s, b) are massless if $Z_3^{\bar{d}}$ is exact.
(y_d = non-invertible $Z_3^{\bar{d}}$ breaking spurion (paramter))

Massless Quark Solution:

3. In SM, "massless up quark solution" tried.

In nature, up quark seems to be massive

e.g. Chiral-PT + observed hadron mass : $m_u/m_d \sim 0.6$

QCD instanton calculation not under analytic control

Lattice QCD : QCD instanton **not sufficient** in size

Solving Strong CP with Non-Invertible Symmetry

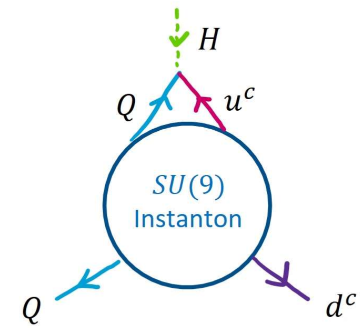
$SU(9)$

$\langle\Phi\rangle$

$SU(3)_C \times SU(3)_H/Z_3$

$\langle\Sigma\rangle$

SM



$$\mathcal{L}_{UV} \supset y_u \widetilde{H} Q \bar{u} + y_u^* e^{-\frac{2\pi}{\alpha_9}} H Q \bar{d} + \cdots$$

- Start with only $y_u \widetilde{H} Q \bar{u} \quad (y_d = 0)$
- $Z_3^{\bar{d}}$ **NIS** from fractional CFU instantons
- $y_d H Q \bar{d}$ protected by **NIS**
- $\bar{\theta} =$ unphysical

Yukawa texture and CKM CPV (+possible $\bar{\theta}$)

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Color-Flavor Unification

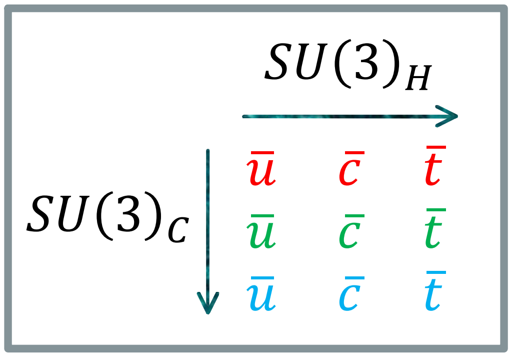
1. $SU(9)$ Unification and the Strong CP phase $\bar{\theta}$

$$SU(3)_H \rightarrow$$

$$SU(3)_C \downarrow \begin{matrix} \bar{u} & \bar{c} & \bar{t} \\ \bar{u} & \bar{c} & \bar{t} \\ \bar{u} & \bar{c} & \bar{t} \end{matrix} \quad = \; 9\;(\square)\;\text{ of }\; SU(9)$$

Color-Flavor Unification

1. $SU(9)$ Unification and the Strong CP phase $\bar{\theta}$

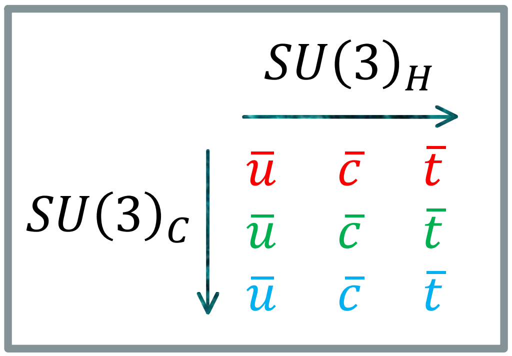


$= 9\ (\square)$ of $SU(9)$

	$SU(9)$	$U(1)_{Q-\bar{u}}$	$U(1)_{\bar{d}}$
$\boldsymbol{Q} = (\boldsymbol{u}, \boldsymbol{d})^{\boldsymbol{t}}$	9	$+1$	0
$\bar{\boldsymbol{u}}$	$\bar{9}$	-1	0
$\bar{\boldsymbol{d}}$	$\bar{9}$	0	$+1$
H	1	0	0

Color-Flavor Unification

1. $SU(9)$ Unification and the Strong CP phase $\bar{\boldsymbol{\theta}}$



$= 9\ (\square)$ of $SU(9)$

	$SU(9)$	$U(1)_{Q-\bar{u}+\bar{d}}$
Φ	165 (3S)	0
$\Sigma_{1,2}$	80 (adj)	0
ρ	9	-1
χ	1	0

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H	1	0	0

Color-Flavor Unification

1. $SU(9)$ Unification and the Strong CP phase $\bar{\theta}$

$$SU(3)_C \downarrow \overset{SU(3)_H \rightarrow}{\begin{array}{ccc} \bar{u} & \bar{c} & \bar{t} \\ \bar{u} & \bar{c} & \bar{t} \\ \bar{u} & \bar{c} & \bar{t} \end{array}} = 9\ (\square)\ \text{of}\ SU(9)$$

	$SU(9)$	$U(1)_{Q-\bar{u}}$	$U(1)_{\bar{d}}$
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$\bar{\boldsymbol{u}}$	$\bar{9}$	−1	0
$\bar{\boldsymbol{d}}$	$\bar{9}$	0	+1
H	1	0	0

$$\mathcal{L}_0 = y_t \tilde{H} \boldsymbol{Q} \bar{\boldsymbol{u}} + h.c. + \frac{i\theta_9}{32\pi^2} \int F\tilde{F}$$

$$y_t \sim O(1)$$

y_d perturbatively protected by $U(1)_{\bar{d}}$

Color-Flavor Unification

1. $SU(9)$ Unification and the Strong CP phase $\overline{\theta}$

$$\overset{SU(3)_H \;\longrightarrow}{SU(3)_C \downarrow \begin{matrix} \bar{u} & \bar{c} & \bar{t} \\ \bar{u} & \bar{c} & \bar{t} \\ \bar{u} & \bar{c} & \bar{t} \end{matrix}} \;=\; 9\,(\square)\ \text{of}\ SU(9)$$

	$SU(9)$	$U(1)_{Q-\bar{u}}$	$U(1)_{\bar{d}}$
$\boldsymbol{Q} = (\boldsymbol{u}, \boldsymbol{d})^t$	9	$+1$	0
$\overline{\boldsymbol{u}}$	$\bar{9}$	-1	0
$\overline{\boldsymbol{d}}$	$\bar{9}$	0	$+1$
H	1	0	0

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$$y_t \sim O(1)$$

y_d perturbatively protected by $U(1)_{\bar{d}}$

$$U(1)_{Q-\bar{u}}[SU(9)]^2 = U(1)_{\bar{d}}[SU(9)]^2 = 1$$

$\Rightarrow$ $[AF]$ $U(1)_{B=Q-\bar{u}-\bar{d}}$

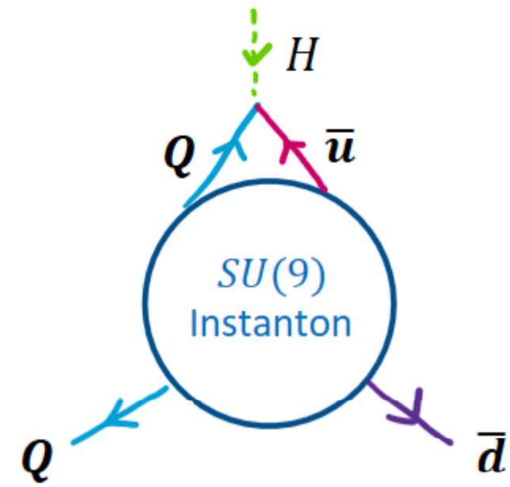
[Anomalous] $U(1)_{Q-\bar{u}+\bar{d}}$ or $U(1)_{\bar{d}}$

Color-Flavor Unification

1. $SU(9)$ Unification and the Strong CP phase $\overline{\theta}$

$SU(3)_H$ (→), $SU(3)_C$ (↓):

$$\begin{matrix} \bar{u} & \bar{c} & \bar{t} \\ \bar{u} & \bar{c} & \bar{t} \\ \bar{u} & \bar{c} & \bar{t} \end{matrix} = 9\ (\square)\ \text{of}\ SU(9)$$



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$\bar{\boldsymbol{d}}$	$\bar{9}$	0	$+1$
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$$\mathcal{L}_0 = y_t \widetilde{H} \boldsymbol{Q}\bar{\boldsymbol{u}} + h.c. + \frac{i\theta_9}{32\pi^2} \int F\widetilde{F}$$

$$+ \, y_t^* e^{i\theta_9} e^{-\frac{2\pi}{\alpha_9}} H \boldsymbol{Q} \bar{\boldsymbol{d}}$$

$$y_d \sim y_t^* e^{i\theta_9} e^{-\frac{2\pi}{\alpha_9(\Lambda_9)}}$$

$$U(1)_{Q-\bar{u}}[SU(9)]^2 = U(1)_{\bar{d}}[SU(9)]^2 = 1$$

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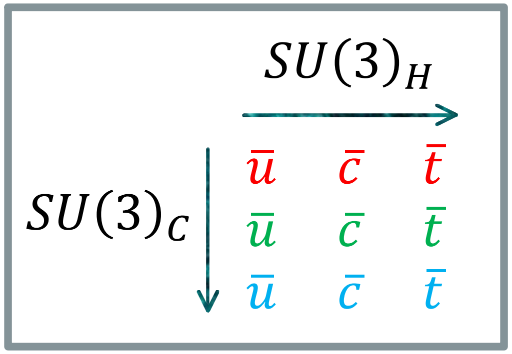
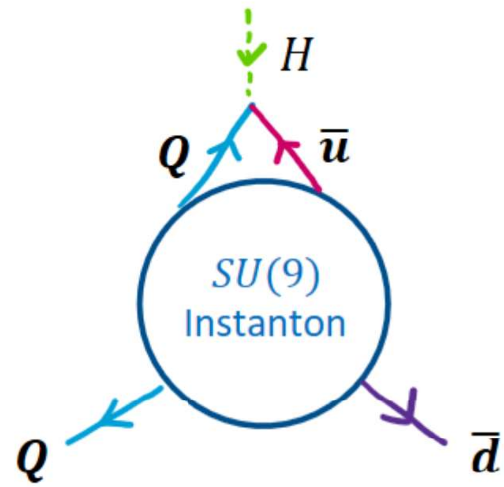


$= 9\,(\square)$ of $SU(9)$



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$$+\, y_t^* e^{i\theta_9} e^{-\frac{2\pi}{\alpha_9}} H \boldsymbol{Q}\bar{\boldsymbol{d}}$$

$$y_d \sim y_t^* e^{i\theta_9} e^{-\frac{2\pi}{\alpha_9(\Lambda_9)}}$$

$$\bar{\theta} = \arg e^{-i\theta_9}\det(y_u y_d) = -\theta_9 + \arg|y_t|e^{i\theta_9} = 0 \quad \checkmark$$

Color-Flavor Unification

2. $SU(9) \rightarrow SU(3)_C \times SU(3)_H / Z_3$

	$SU(9)$	$U(1)_{Q-\bar{u}+\bar{d}}$
Φ	165 (3S)	0
$\Sigma_{1,2}$	80 (adj)	0
ρ	9	-1
χ	1	0

(i) SSB by $\langle \Phi^{\mathrm{ABC}} \rangle = \Lambda_9 \epsilon^{abc} \epsilon^{ijk}$

(ii) $9(\mathrm{Q}, \bar{u}, \bar{d}, \rho) \rightarrow (3,3)$

(iii) Z_3 Quotient: $Q \rightarrow g_C\, Q\, g_H^\dagger$

(iv) $165 \rightarrow (10,10) + (\mathbf{8},\mathbf{8}) + (1,1)$
$(\Phi_{\{ai,bj,ck\}} \sim \phi_{\{abc\}} \cdot \tilde{\phi}_{\{ijk\}} + \phi_{[ab]c} \cdot \tilde{\phi}_{[ij]k} + \phi_{[abc]} \cdot \tilde{\phi}_{[ijk]})$

(v) $80 \rightarrow (8,8) + (8,1) + (1,8)$

Color-Flavor Unification

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$$\to y_t \widetilde{H} \boldsymbol{Q}\bar{\boldsymbol{u}} + y_t^* e^{i\theta_9} e^{-\frac{2\pi}{3\alpha_S(\Lambda_9)}} H \boldsymbol{Q}\bar{\boldsymbol{d}} + \frac{i3\theta_9}{32\pi^2} \int \left(G\tilde{G} + K\tilde{K}\right)$$ (Yukawa$\propto \mathbb{I}_3$, Flavor-diag)

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(i) SSB by $\langle \Phi^{ABC} \rangle = \Lambda_9 \epsilon^{abc} \epsilon^{ijk}$

(ii) $9(Q, \bar{u}, \bar{d}, \rho) \to (3,3)$

(iii) Z_3 Quotient: $Q \to g_C \, Q \, g_H^\dagger$

(iv) $165 \to (10,10) + (\mathbf{8},\mathbf{8}) + (1,1)$

$(\Phi_{\{ai,bj,ck\}} \sim \phi_{\{abc\}} \cdot \tilde{\phi}_{\{ijk\}} + \phi_{[ab]c} \cdot \tilde{\phi}_{[ij]k} + \phi_{[abc]} \cdot \tilde{\phi}_{[ijk]})$

(v) $80 \to (8,8) + (8,1) + (1,8)$

$$\mathcal{L}_0 = y_t \widetilde{H} \boldsymbol{Q}\bar{\boldsymbol{u}} + y_t^* e^{i\theta_9} e^{-\frac{2\pi}{\alpha_9}} H \boldsymbol{Q}\bar{\boldsymbol{d}} + \frac{i\theta_9}{32\pi^2} \int F\tilde{F}$$ (Yukawa=single number)

$$\to y_t \widetilde{H} \boldsymbol{Q}\bar{\boldsymbol{u}} + y_t^* e^{i\theta_9} e^{-\frac{2\pi}{3\alpha_S(\Lambda_9)}} H \boldsymbol{Q}\bar{\boldsymbol{d}} + \frac{i3\theta_9}{32\pi^2} \int \left(G\tilde{G} + K\widetilde{K}\right)$$ (Yukawa$\propto \mathbb{I}_3$, Flavor-diag)

$$\bar{\theta} = -3\theta_9 + \arg\det|y_t|^2 e^{i\theta_9} = -3\theta_9 + 3\theta_9 = 0$$ ✓

Color-Flavor Unification

2. $SU(9) \to SU(3)_C \times SU(3)_H / Z_3$

	$SU(9)$	$U(1)_{Q-\bar{u}+\bar{d}}$
Φ	165 (3S)	0
$\Sigma_{1,2}$	80 (adj)	0
ρ	9	-1
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(i) SSB by $\langle \Phi^{ABC} \rangle = \Lambda_9 \epsilon^{abc} \epsilon^{ijk}$

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$$\bar{\theta} = -3\theta_9 + \arg\det|y_t|^2 \, e^{i\theta_9} = -3\theta_9 + 3\theta_9 = 0 \qquad \checkmark$$

From now on, we set $\bar{\boldsymbol{\theta}}_9 = \boldsymbol{0}$ and take **real yuakwas.**

Outline

I. Generalized Global Symmetries

I-1. Higher-form symmetry
I-2. Non-invertible symmetry

II. Strong CP Problem-I: IR to UV

II-1. Non-invertible Peccei-Quinn symmetry
II-2. Massless quark solution

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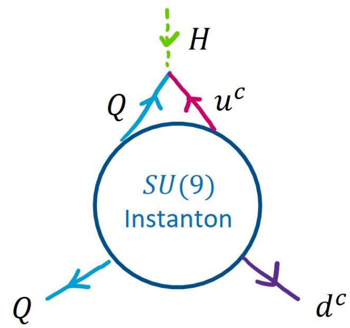
III-1. $SU(9)$ Color-Flavor unification
III-2. Flavor structure and CKM CPV phase

Solving Strong CP with Non-Invertible Symmetry



$SU(9)$

$$\mathcal{L}_{UV} \supset y_u \widetilde{H} Q \bar{u} + y_u^* e^{-\frac{2\pi}{\alpha_9}} H Q \bar{d} + \cdots$$

$\langle \Phi \rangle$

$SU(3)_C \times SU(3)_H / Z_3$

- Start with only $y_u \widetilde{H} Q \bar{u}$ $\quad (y_d = 0)$
- $Z_3^{\bar{d}}$ **NIS** from fractional CFU instantons
- $y_d H Q \bar{d}$ protected by **NIS**
- $\bar{\theta} =$ unphysical

$\langle \Sigma \rangle$

SM

Yukawa texture and CKM CPV (+possible $\bar{\theta}$)

Color-Flavor Unification

3. $SU(3)_C \times SU(3)_H/Z_3 \to SU(3)_C$: Flavor Structre and CKM CPV

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$\Sigma_{1,2}$	80 (adj)	0
ρ	9	-1
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(i) Two $SU(3)_H$ adjoint scalar $\Sigma_{1,2}$: $SU(3)_H \to \emptyset$

(ii) Textures of y_u, y_d generated by structure of $\langle\Sigma_{1,2}\rangle$

(iii) Required CKM CPV phase from $V(\Sigma)$

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Combine $\Sigma = \Sigma_1 + i\Sigma_2$

Consider a simple case with Z_4 invariant potential $V(\Sigma)$
(our mechanism works regardless of this simplifying assumption)

$$V(\Sigma) = \eta_1 \mathrm{Tr}(\Sigma^4) + \eta_2 \left(\mathrm{Tr}(\Sigma^2)\right)^2 + h.c. + \xi \mathrm{Tr}\left(\Sigma^\dagger \Sigma\right)^2 + \cdots \text{(terms with real coeffs)}$$

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Field redefinition invariant CPV : $\boldsymbol{\eta_1^\dagger \eta_2}$

$$\Sigma \to e^{-i\varphi_1/4}\Sigma: \quad |\eta_1| e^{i\varphi_1} \mathrm{Tr}(\Sigma^4) + |\eta_2| e^{i\varphi_2} \left(\mathrm{Tr}(\Sigma^2)\right)^2 \to |\eta_1| \mathrm{Tr}(\Sigma^4) + |\eta_2| e^{i(\varphi_2 - \varphi_1)} \left(\mathrm{Tr}(\Sigma^2)\right)^2$$

Color-Flavor Unification

3. $SU(3)_C \times SU(3)_H / Z_3 \to SU(3)_C$ **: Flavor Structre and CKM CPV**

(i) Two $SU(3)_H$ adjoint scalar $\Sigma_{1,2}$: $SU(3)_H \to \emptyset$

Complete breaking $\Leftrightarrow$ $[\Sigma_1, \Sigma_2] = \frac{[\Sigma^\dagger, \Sigma]}{2i} \neq 0$

(ii) Textures of y_u, y_d generated by structure of $\langle \Sigma_{1,2} \rangle$

Color-Flavor Unification

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Generate complete 3×3 y_u, y_d but in a way that $\boldsymbol{\bar{\theta} = 0}$ is maintained.

$\Rightarrow$ **Our mechanism:** generate **Hermitian Yukawas**

(I) all CPV in scalar sector

(II) CPV transferred to SM fermions via bosonic mediation

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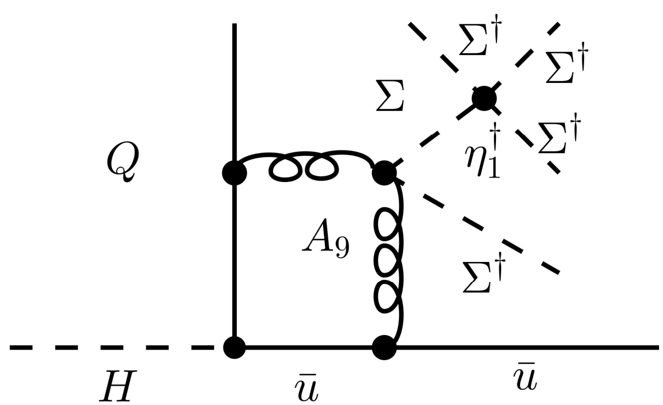

+ diagram with $h.c.(\eta \leftrightarrow \eta^\dagger, \Sigma \leftrightarrow \Sigma^\dagger)$

Color-Flavor Unification

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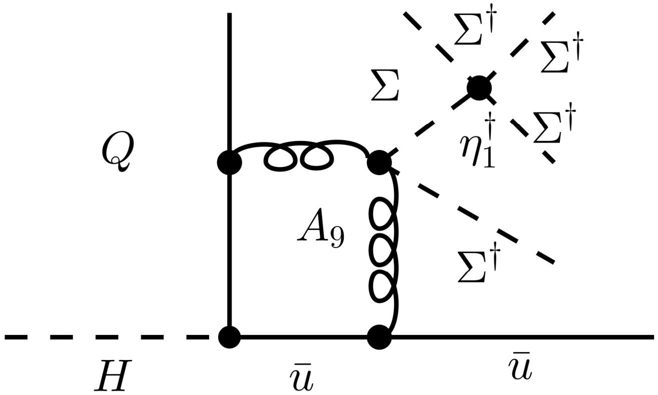

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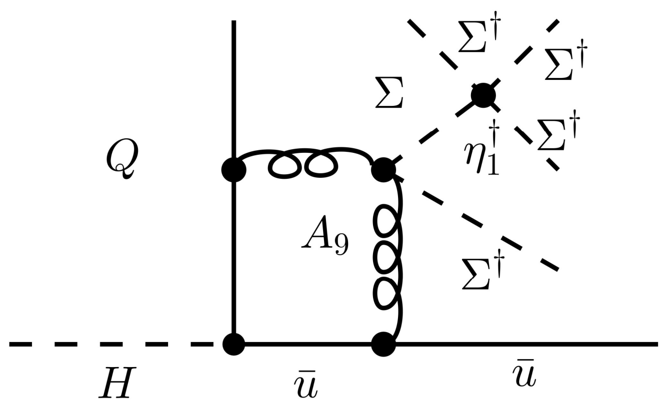

+ diagram with $h.c.(\eta \leftrightarrow \eta^\dagger, \Sigma \leftrightarrow \Sigma^\dagger)$

$$(y_u)^i_j \sim y_t \left(1 + \frac{\alpha_9}{4\pi} \frac{\{\Sigma^\dagger, \Sigma\}}{2\Lambda_9^2} + \left(\frac{\alpha_9}{4\pi} \frac{\eta_1^\dagger (\Sigma^\dagger)^4 + \eta_2^\dagger (\Sigma^\dagger)^2 (\Sigma^\dagger)^2}{\Lambda_9^4} + h.c. \right) \right)^i_j$$

Hermitian **Real** **Complex**

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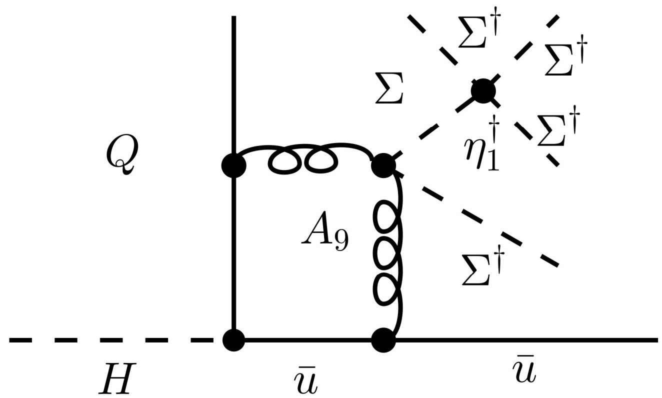

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Hermitian **Real** **Complex**

$y_u, y_d \Rightarrow$ real eigenvalues $\Rightarrow$ $\bar{\theta} = \arg e^{-i\theta} \det(y_u y_d) = \arg \det(y_u y_d) = 0$ ✓

Color-Flavor Unification

3. $SU(3)_C \times SU(3)_H / Z_3 \to SU(3)_C$: Flavor Structre and CKM CPV

(iii) Generate $O(1)$ CKM CPV phase δ_{CKM} (without destabilizing $\boldsymbol{\bar{\theta} = 0}$)

Field-redefinition invariant definition of CKM CPV phase

$\tilde{J} = \mathrm{Im}\,\det\left[y_u^\dagger y_u, y_d^\dagger y_d\right] \propto \sin\delta_{CKM}$ "Jarlskog invariant"

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"Jarlskog invariant"

"misalignment" of y_u and y_d

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"misalignment" of y_u and y_d

So far, we have

$$y_u \sim y_t\left(1 + \{\Sigma^\dagger, \Sigma\} + (\eta\Sigma^4 + h.c.) + \cdots\right)$$

$$y_d \sim y_t e^{-\frac{2\pi}{\alpha_9}}\left(1 + \{\Sigma^\dagger, \Sigma\} + (\eta\Sigma^4 + h.c.) + \cdots\right)$$

Color-Flavor Unification

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$\boldsymbol{y_d \propto y_u}$ as a matrix

$\Rightarrow \boldsymbol{\tilde{J} = 0}$

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$\boldsymbol{y_d \propto y_u}$ as a matrix

$\Rightarrow\ \boldsymbol{\tilde{J} = 0}$

We need extra ingredients to **misalign $\boldsymbol{y_d}$ vs $\boldsymbol{y_u}$** : **'down-philic' interactions**

Color-Flavor Unification

3. $SU(3)_C \times SU(3)_H/Z_3 \to SU(3)_C$: Flavor Structre and CKM CPV

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	$SU(9)$	$U(1)_{Q-\bar{u}+\bar{d}}$
Φ	165 (3S)	0
$\Sigma_{1,2}$	80 (adj)	0
ρ	9	-1
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$$\mathcal{L}_{\rho\chi} \sim \lambda_d \bar{d}\rho\chi + \rho\left(c_1\Sigma^\dagger\Sigma + c_2\Sigma\Sigma^\dagger\right)\rho^\dagger$$

$$+ a_1\rho\Sigma\rho^\dagger + a_2\rho\Sigma\Sigma\rho^\dagger + h.c.$$

Color-Flavor Unification

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$$+\, a_1\rho\Sigma\rho^\dagger + a_2\rho\Sigma\Sigma\rho^\dagger + h.c.$$

- Use χ rotation to set $\lambda_d \in \mathbb{R}$
- $c_{1,2} \in \mathbb{R}$
- $a_{1,2} \in \mathbb{C} \;\rightarrow\; a_1^2 a_2^\dagger, \eta_1^\dagger a_2^2$: new CPV source
- $a_{1,2} = 0$ if Z_4^Σ is imposed

  (again, our mechanism works regardless)

Color-Flavor Unification

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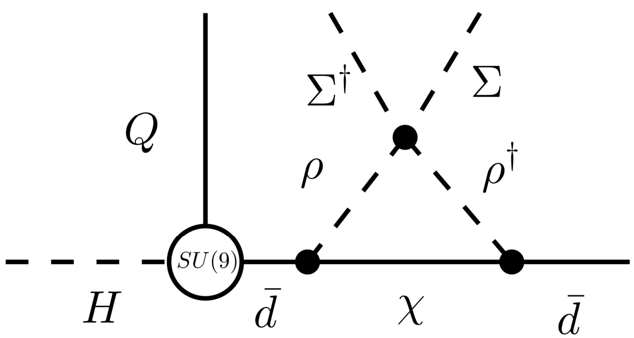

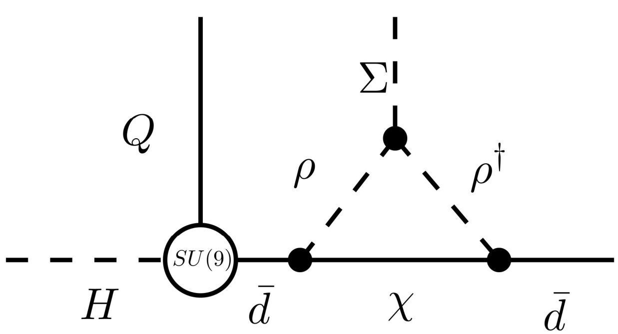

Without "down-philic" interactions

$$y_u \sim y_t\left(1 + \{\Sigma^\dagger, \Sigma\} + (\eta\Sigma^4 + h.c.) + \cdots\right)$$

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$$\tilde{J} = 0$$

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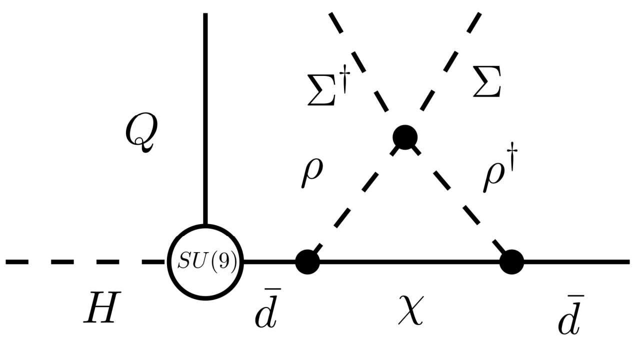
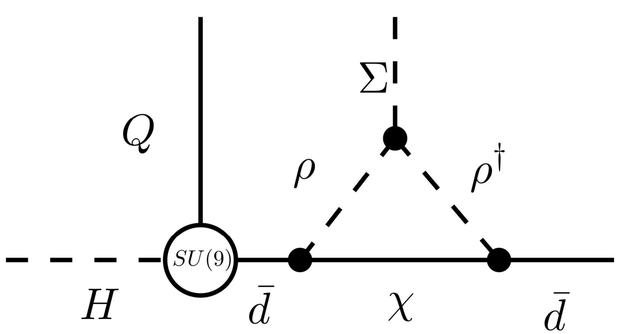


With "down-philic" interactions ($a_{1,2} = 0$)

$$y_u \sim y_t\left(1 + \{\Sigma^\dagger, \Sigma\} + (\eta\Sigma^4 + h.c.) + \cdots\right)$$

$$y_d \sim y_t e^{-\frac{2\pi}{\alpha_9}}\left(1 + \{\Sigma^\dagger, \Sigma\} + (\eta\Sigma^4 + h.c.) + \cdots + c\Sigma^\dagger\Sigma\right)$$



$$\tilde{J} \sim \mathrm{Im}\det\left(4r^2[\eta\Sigma^4 + \eta^\dagger\Sigma^{\dagger 4}, c\Sigma^\dagger\Sigma]\right), \qquad r \sim e^{-\frac{2\pi}{\alpha_9}}$$

$$\propto \mathrm{Im}\det\left(\eta\begin{pmatrix}[\Sigma,\Sigma^\dagger]\Sigma^4 + \Sigma[\Sigma,\Sigma^\dagger]\Sigma^3 + \Sigma^2[\Sigma,\Sigma^\dagger]\Sigma^2 \\ +\Sigma^3[\Sigma,\Sigma^\dagger]\Sigma\end{pmatrix} - h.c.\right)$$

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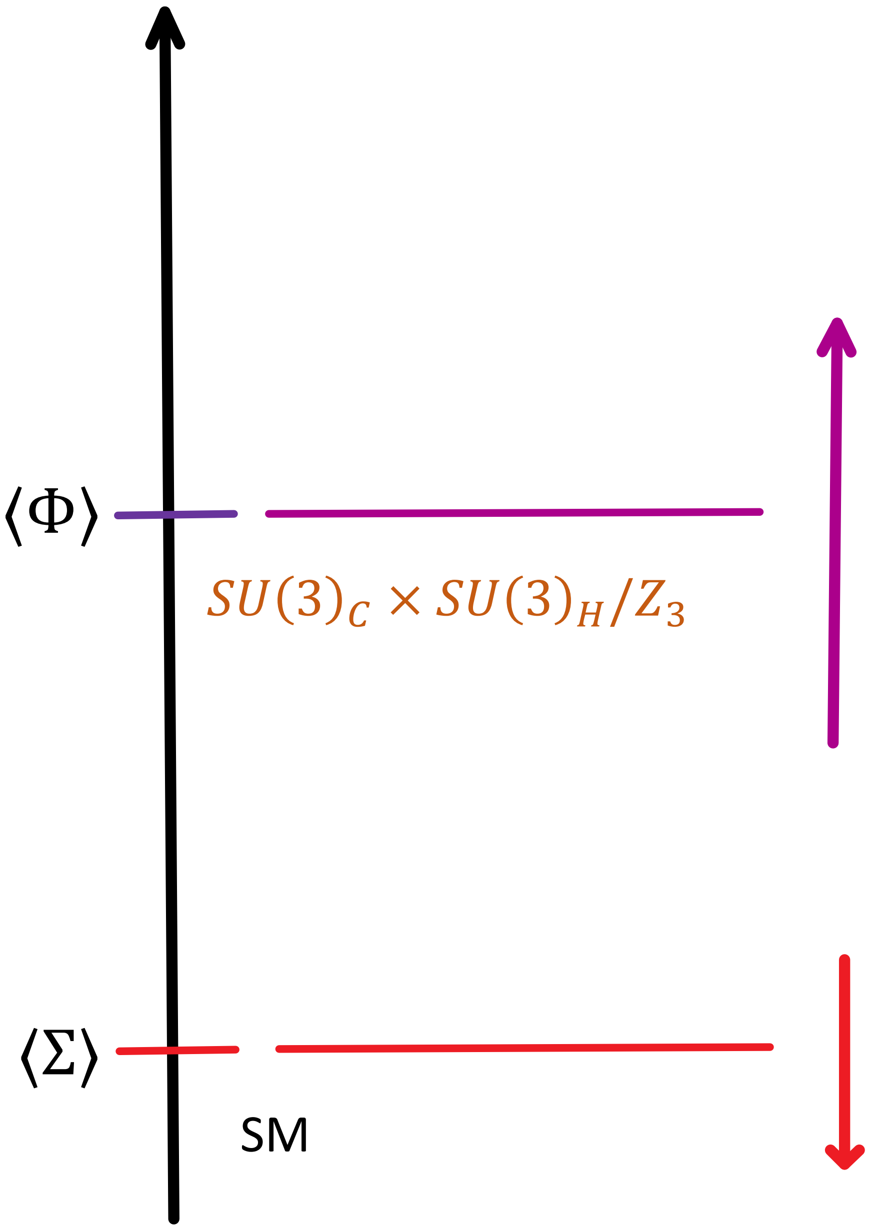
Solving Strong CP with Non-Invertible Symmetry



- Start with only $y_u \widetilde{H} Q \bar{u} \quad (y_d = 0)$
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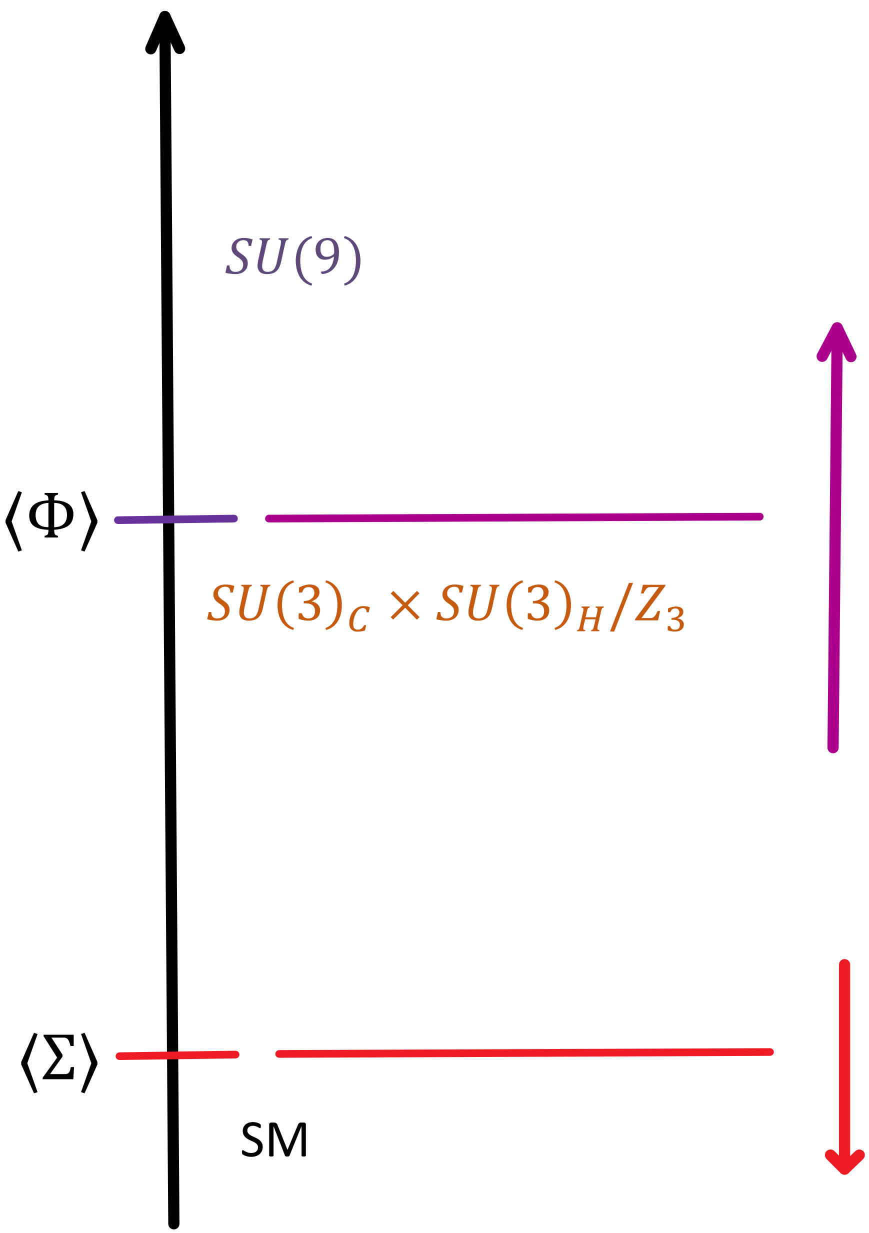

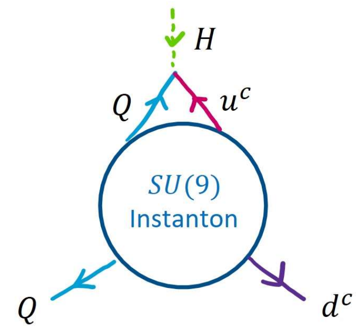

$$\mathcal{L}_{UV} \supset y_u \tilde{H} Q \bar{u} + y_u^* e^{-\frac{2\pi}{\alpha_9}} H Q \bar{d} + \cdots$$

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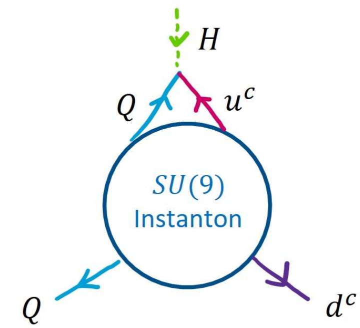

$SU(9)$

$$\mathcal{L}_{UV} \supset y_u \widetilde{H} Q \bar{u} + y_u^* e^{-\frac{2\pi}{\alpha_9}} H Q \bar{d} + \cdots$$

$\langle \Phi \rangle$

$SU(3)_C \times SU(3)_H / Z_3$

- Start with only $y_u \widetilde{H} Q \bar{u}$ $\quad (y_d = 0)$
- $Z_3^{\bar{d}}$ **NIS** from fractional CFU instantons
- $y_d H Q \bar{d}$ protected by **NIS**
- $\bar{\theta} =$ unphysical

$\langle \Sigma \rangle$

SM

Yukawa texture and CKM CPV (+possible $\bar{\theta}$)
(Hermitian Yukawa + down-philic interactions)

THANK YOU

FOR

YOUR ATTENTION!