

$(1, 2)$ -symmetric subgroups of $\mathrm{SL}(4, \mathbb{C})$

Thursday, 21 December 2023 17:10 (50 minutes)

The topic is finite diagonal subgroups $A \subset \mathrm{SL}(4, \mathbb{C})$ and their A -Hilbert schemes. As a dimension reducing assumption, I impose the additional $(1, 2)$ -symmetric condition. The case to bear in mind is $\frac{1}{r}(1, 1, a, b)$ with $r = a + b + 2$. The “junior end and all-even” conditions for the quotient $X = \mathbb{A}^4/A$ to have a crepant resolution are known from Sarah Davis’s thesis [D].

Studying the A -Hilbert scheme $A\text{-Hilb}\mathbb{A}^4$ in the general $(1, 2)$ -symmetric case is interesting in its own right, and provides more detailed insight into case of the crepant resolution. The variety $Y = A\text{-Hilb}\mathbb{A}^4$ is toric, a union of affine pieces corresponding to monomial ideals $I \subset k[\mathbb{A}^4] = k[x, y, z, t]$, and can be constructed by my 2009 computer algebra routine [M]. In very many cases Y is nonsingular, and is a resolution $Y \rightarrow X$ with exceptional divisors of discrepancy 0 or 1.

The calculation of $A\text{-Hilb}\mathbb{A}^4$ mirrors the classical construction of Nakamura [A] and Craw–Reid [CR], with some remarkable modifications.

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