McKay correspondence, Tilting theory and related topics

Report of Contributions

Contribution ID: **3** Type: **not specified**

TF equivalence, silting theory and canonical decompositions

Friday, December 22, 2023 11:40 AM (50 minutes)

This talk is based on joint work with Osamu Iyama. The representation theory of a finite dimensional algebra A deals with the category $\mathsf{mod}A$ of finitely generated A-modules. One of the main topics is torsion pairs in $\mathsf{mod}A$. Functorially finite torsion pairs have been well-studied, but they are too few among all torsion pairs. Thus, we are now studying a wider class called semistable torsion pairs introduced by Baumann-Kamnitzer-Tingley associated to elements of the real Grothendieck group $K_0(\mathsf{proj}A)_{\mathbb{R}}$ of the category of finitely generated projective A-modules, which is identified with the Euclidean space whose canonical basis is given by the isoclasses of indecomposable projective A-modules. By using semistable torsion pairs, I (Asai) introduced an equivalence relation called TF equivalence on $K_0(\mathsf{proj}A)_{\mathbb{R}}$. A typical example of TF equivalence classes is the silting cone $C^\circ(U)$ generated by the g-vectors of indecomposable direct summands of each 2-term presilting complex U. On the other hand, there can be other TF equivalence classes. To study them, we have found that canonical decompositions introduced by
Derksen-Fei is useful. I would like to explain some important properties of these notions.

Presenter: ASAI, Sota (Tokyo)

Contribution ID: **4** Type: **not specified**

Nc resolutions of finite dimensional commutative algebras via smooth surface blow-ups

Thursday, December 21, 2023 3:30 PM (50 minutes)

I will explain how to get noncommutative resolutions of some finite dimensional commutative algebras via birational morphisms of smooth surfaces. The resolution is given by the null category which happens to be highest weight category, whose projective generator is the discrepancy sheaf.

Presenter: BONDAL, Alexey (Steklov/Tokyo)

Contribution ID: **5** Type: **not specified**

On recent developments in quasi-hereditary structures

Friday, December 22, 2023 10:00 AM (50 minutes)

The notion of quasi-hereditary algebras were introduced by Cline-Parshall-Scott, and there is an abundance of examples arising in algebraic Lie theory and non-commutative resolution of singularities. This notion is defined with respect to a poset structure on the set of simple modules. In this talk, we will survey some recent developments in enumerating these structures, and in particular, their relation with the (po)set of tilting modules. This talk contains joint works with Takahide Adachi, Yuta Kimura, and Mayu Tsukamoto.

Presenter: CHAN, Aaron (Nagoya)

Contribution ID: **6** Type: **not specified**

Hilbert schemes and the Le Bruyn-Procesi theorem

Monday, December 18, 2023 3:30 PM (50 minutes)

For a positive integer n and a finite subgroup \Gamma in SL(2,C), I'll describe work in preparation with Ryo Yamagishi which shows that the Hilbert scheme of n-points on C^2/\Gamma is reduced. In fact, it's isomorphic to a Nakajima quiver variety, so it has symplectic singularities and it admits a unique crepant resolution. This strengthens previous joint work of mine with Gammelgaard, Gyenge and Szendroi. The main tool is a generalisation of the Le Bruyn-Procesi theorem which describes the invariant algebra for the natural action of the product of general linear groups on the space of representations of a quiver for a given dimension vector.

Presenter: CRAW, Alastair (Bath)

Contribution ID: **7** Type: **not specified**

Exceptional surfaces in 3-folds and derived symmetries

Wednesday, December 20, 2023 3:30 PM (50 minutes)

Crepant resolutions of 3-fold singularities may contain elaborate configurations of exceptional surfaces. Using toric cases as a guide, I review some known contributions of these configurations to the derived autoequivalence group of the resolution, in particular from work of Seidel-Thomas, and discuss work in progress with Luyu Zheng.

Presenter: DONOVAN, Will (Tsinghua)

Contribution ID: **8** Type: **not specified**

An example of crepant resolution in characteristic 2 and its duality

Monday, December 18, 2023 5:10 PM (30 minutes)

In positive characteristic, few examples of crepant resolutions of modular quotient singularities are known. In this talk, I will introduce a crepant resolution of the quotient singularity given by the permutation action of the alternating group of degree 4 in characteristic 2. In addition, I will present some special properties of this resolution by considering its algebraic and geometric structure.

Presenter: FAN, Linghu (Tokyo)

Contribution ID: **9** Type: **not specified**

Reflexive modules and Auslander-Gorenstein rings

Friday, December 22, 2023 4:30 PM (30 minutes)

Motivated by the theory of non-commutative resolutions and the results on Auslander correspondence, we study the category of reflexive modules over (commutative or non-commutative) Noetherian rings. One well-established sufficient condition for this category to behave well is that the ring should be (commutative) normal. We will explain that these nice behaviors are governed by the Auslander-type conditions which are some requirements on the minimal injective resolution of the ring.

Presenter: HANIHARA, Norihiro (Tokyo)

Contribution ID: **10** Type: **not specified**

Spherical objects in dimension two and three

Wednesday, December 20, 2023 11:00 AM (30 minutes)

During this talk we discuss the classification problem of spherical "like"objects in various geometric settings including the minimal resolution of an ADE surface singularity and a 3-fold flopping contraction. The classification of spherical objects is related to questions about the autoequivalence groups or Bridgeland stability conditions, but in 3-fold settings this is not always a correct problem to ask. During the talk, we discuss what kind of objects should be classified, and then, a sketch of the proof will be explained. Our new technique also can be applied to the heart of a bounded t-structure, and classifies all t-structures of the associated null category. As a corollary, the connectedness of the space of stability conditions follows. This is all joint work with Michael Wemyss.

Presenter: HARA, Wahei (Tokyo)

Contribution ID: **11** Type: **not specified**

Towards derived Reid's recipe for dimer models

Thursday, December 21, 2023 11:40 AM (50 minutes)

Reid's recipe is an equivalent of the McKay correspondence in dimension three. It marks interior line segments and lattice points in the fan of the G-Hilbert scheme with characters of irreducible representations of G. In joint work with Craw and Tapia Amador, we generalise this by marking the toric fan of a crepant resolution of any affine Gorenstein singularity, in a way that is compatible with both the G-Hilbert case and its categorical counterpart known as Derived Reid's Recipe. To achieve this, we foray into the combinatorial land of quiver moduli spaces and dimer models. In this talk I will discuss connections between combinatorial and derived Reid's recipe and recent progress concerning low-valency vertices in the quiver. This is joint work with Alastair Craw.

Presenter: HEUBERGER, Liana (Bath)

Contribution ID: **12** Type: **not specified**

Mutations of noncommutative crepant resolutions in geometric invariant theory

Monday, December 18, 2023 11:40 AM (50 minutes)

For a generic quasi-symmetric representation X of a reductive group G, Halpern-Leistner and Sam show that the derived category of coherent sheaves on a GIT (stacky) quotient of X is equivalent to magic windows, which are certain triangulated subcategories of the derived categories of coherent sheaves on the quotient stack [X/G]. In this talk, we explain that the equivalences of magic windows, which correspond to wall-crossings in a hyperplane arrangement, correspond to derived equivalences of noncommutative crepant resolutions induced by tilting modules, and these tilting modules are obtained by certain operations of modules, which we call exchanges of modules. This talk is based on joint work with Wahei Hara.

Presenter: HIRANO, Yuki (Tokyo U. Agri & Tech)

Contribution ID: **13** Type: **not specified**

Hyperpolygon spaces and their crepant resolutions

Thursday, December 21, 2023 11:00 AM (30 minutes)

Hyperpolygon spaces are a family of symplectic singularities in all even dimensions generalising the D4 surface singularity. We present a work in which we describe the Cox rings of crepants resolutions of hyperpolygon spaces and give a method for enumerating ALL such crepant resolutions (including non-projective resolutions).

Presenter: HUBBARD, Austin (Bath)

Contribution ID: **14** Type: **not specified**

The Donovan-Wemyss Conjecture via the Derived Auslander-Iyama Correspondence

Thursday, December 21, 2023 2:00 PM (50 minutes)

The Donovan-Wemyss Conjecture predicts that the isomorphism type of an isolated compound Du Val singularity R that admits a crepant resolution is completely determined by the derived-equivalence class of any of its contraction algebras. Crucial results of August, Hua-Keller and Wemyss reduced the DW conjecture to a problem closely related the question of uniqueness of enhancements of the singularity category of R. I will explain, based on an observation by Bernhard Keller, how the DW conjecture follows from a recent theorem of Fernando Muro and myself that we call the Derived Auslander-Iyama Correspondence.

Presenter: JESSO, Gustavo (Lund)

Contribution ID: **15** Type: **not specified**

On derived McKay correspondence between non-commutative deformations

Tuesday, December 19, 2023 3:30 PM (50 minutes)

We consider an example of derived McKay correspondence between non-commutative deformations in the case of surface singularities of type An.
We construct a versal NC deformation of the commutative crepant resolution and compare it with the versal NC deformation of the non-commutative crepant resolution. We show the derived McKay correspondence in the case n=1.

Presenter: KAWAMATA, Yujiro (Tokyo)

Contribution ID: **16** Type: **not specified**

The invariant Hilbert scheme of the Cox realization

Monday, December 18, 2023 11:00 AM (30 minutes)

The invariant Hilbert scheme is a moduli space of schemes which are stable under an action of a reductive algebraic group. By a suitable choice of the parameter, it becomes a candidate for a resolution of singularities of an affine quotient variety via the so-called Hilbert-Chow morphism. In this talk, we will focus on the Cox realization as a way to represent an affine singularity as a quotient variety and consider the associated invariant Hilbert scheme.

Presenter: KUBOTA, Ayako (Waseda)

Contribution ID: **17** Type: **not specified**

McKay correspondence and perverse schobers

Wednesday, December 20, 2023 4:30 PM (30 minutes)

I will explain how most of the results we know so far about the McKay correspondence in dim = 2 and 3 can be conjecturally packaged up into a single mathematical object known as a perverse schober. These were proposed by Kapranov and Schechtman in 2014 as a categorification of an earlier notion of a perverse sheaf by Beilinson, Bernstein, and Deligne.

Presenter: LOGVINENKO, Thimothy (Cardiff)

Contribution ID: **18** Type: **not specified**

Non-commutative crepant resolutions of a special family of stable set rings

Thursday, December 21, 2023 4:30 PM (30 minutes)

The existence of non-commutative crepant resolutions (NCCRs) for certain classes is one of the most well-studied problems. In this talk, we discuss the construction of NCCRs of toric rings using their conic divisorial ideals and we give an NCCR of a special family of stable set rings, which are toric rings arising from graphs.

Presenter: MATSUSHITA, Koji (Osaka)

Contribution ID: **19** Type: **not specified**

Tilting theory and g-fans

Friday, December 22, 2023 2:00 PM (50 minutes)

The notion of tilting objects is basic to study the structure of a given derived category. The class of silting objects gives a completion of the class of tilting objects from the point of view of mutation, and they correspond bijectively with other important objects in the derived category. The subset of 2-term silting complexes enjoys especially nice properties, which is closely related to τ-tilting theory and cluster theory. In this talk, we discuss the notion of g-simplicial complexes, g-polytopes and g-fans, which is defined from 2-term silting complexes. We study several properties of these three objects. In particular, we give tilting theoretic interpretations of the h-vectors and Dehn-Sommerville equations of the g-simplicial complex. Moreover, we discuss the convexity of the g-polytope and its dual polytope.
This is a joint work with Aoki-Higashitani-Iyama-Kase.

Presenter: MIZUNO, Yuya (Osaka Metropolitan)

Contribution ID: **20** Type: **not specified**

Distribution of bricks and one-parameter families of stable modules

Wednesday, December 20, 2023 11:40 AM (50 minutes)

Let A be a finite dimensional associative algebra A over an algebraically closed field k. A (left) A-module M is called a brick if the endomorphism algebra of M over A is isomorphic to k. Bricks (also known as Schur representations) play decisive roles in the algebraic and geometric aspects of representation theory of algebras, including in the stability conditions, wall-and-chamber structures, (tau-)tilting theory and wide subcategories. In this talk, I will focus on the behavior of the 1-parameter families of stable modules (in the sense of King) and discuss some new results on a (still open) conjecture that I first posed in 2019. When restricted to the setting of tame algebras, the aforementioned conjecture states that A admits infinitely many non-isomorphic bricks if and only if there exists a 1-parameter family of stable modules under a fixed stability condition. I will discuss the conjecture over arbitrary algebras, present a reduction theorem in the general case, and then prove it for some important families of tame algebras. This talk is partially based on my joint work with Charles Paquette.

Presenter: MOUSAVAND, Kaveh (OIST)

Contribution ID: **21** Type: **not specified**

The E10 diagram, symmetric polytopes and Enriques surfaces

Tuesday, December 19, 2023 10:00 AM (50 minutes)

The affine E8 diagram parametrizes the irreducible representations of the binary icosahedral groups by McKay. The next diagram E10, or T237, parametrizes a set of symmetric polyhedra consisting of octa, dodeca, icosa and several zonohedra. This was observed in studying polarization types of Enriques surfaces. In this talk I will present type III degenerations of Enriques surfaces corresponding to such symmetric polytopes as many as possible.

Presenter: MUKAI, Shigeru (Kyoto)

Contribution ID: **22** Type: **not specified**

Projective crepant resolutions of toric cDV singularities and dimer combinatorics

Friday, December 22, 2023 3:30 PM (50 minutes)

It is known that any projective crepant resolution of a three-dimensional Gorenstein toric singularity can be described as the moduli space of representations of the quiver associated to a consistent dimer model for some stability parameter. The space of stability parameters has the wall-and-chamber structure, that is, it is decomposed into chambers separated by walls. The moduli spaces associated to stability parameters contained in the same chamber are isomorphic, but a stability parameter contained in another chamber would give a different moduli space. Thus, it is important to detect the wall-and-chamber structure of the space of stability parameters to understand the relationships among projective crepant resolutions of three-dimensional Gorenstein toric singularities.
In this talk, I observe the wall-and-chamber structure for a particular class of three-dimensional Gorenstein toric singularities called toric compound Du Val (cDV) singularities. In particular, I will show that zigzag paths of a consistent dimer model associated to a toric cDV singularity control the wall-and-chamber structure. This talk is based on the preprint arXiv:2309.16112.

Presenter: NAKAJIMA, Yusuke (Kyoto Sangyo)

Contribution ID: **23** Type: **not specified**

Cotilting complexes over commutative noetherian rings

Monday, December 18, 2023 4:30 PM (30 minutes)

We give a nontrivial example of a cotilting complex that induces a compactly generated t-structure in the unbounded derived category of a commutative noetherian ring, and explain its relation with big Cohen-Macaulay modules and Cohen-Macaulay approximations. This talk is partly based on joint work with Michal Hrbek and Jan Stovicek (arXiv:2207.01309).

Presenter: NAKAMURA, Tsutomu (Mie)

Contribution ID: **24** Type: **not specified**

Explicit examples of trihedral G-Hilbert schemes

Tuesday, December 19, 2023 4:30 PM (30 minutes)

In this talk I will show how to construct examples of G-Hilb when G is a trihedral group in SL(3,C). The calculations are based on the connection between trihedral boats and representations of the McKay quiver, allowing us to compute explicitly the exceptional locus of the crepant resolution G-Hlib C^3 \to C^3/G and Reid's recipe for small cases.

Presenter: NOLLA DE CELIS, Alvaro (Madrid)

Contribution ID: **25** Type: **not specified**

L-genera and applications

Wednesday, December 20, 2023 10:00 AM (50 minutes)

My goal in this talk is to give an accessible discussion of an ongoing research project with David Kazhdan. In this work, we define L-function genera and use them in the spectral analysis of Eisenstein series and in related problems of enumerative geometry.

Presenter: OKOUNKOV, Andrei (Columbia)

Contribution ID: **26** Type: **not specified**

$(1,2)$-symmetric subgroups of SL$(4,\mathbb{C})$

Thursday, December 21, 2023 5:10 PM (50 minutes)

The topic is finite diagonal subgroups $A\ \subset$SL$(4,\mathbb{C})$ and their \hbox{A-Hilbert} schemes. As a dimension reducing assumption, I impose the additional $(1,2)$-symmetric condition. The case to bear in mind is $\frac{1}{r}(1,1,a,b)$ with $r=a+b+2$. The "junior end and all-even" conditions for the quotient $X=\mathbb{A}^4/A$ to have a crepant resolution are known from Sarah Davis's thesis [D].

Studying the A-Hilbert scheme A-Hilb$\mathbb{A}^4$ in the general $(1,2)$-symmetric case is interesting in its own right, and provides more detailed insight into case of the crepant resolution. The variety $Y=A$-Hilb$\mathbb{A}^4$ is toric, a union of affine pieces corresponding to monomomial ideals $I\subset k[\mathbb{A}^4]=k[x,y,z,t]$, and can be constructed by my 2009 computer algebra routine [M]. In very many cases Y is nonsingular, and is a resolution $Y\to X$ with exceptional divisors of discrepancy 0 or 1.

The calculation of A-Hilb$\mathbb{A}^4$ mirrors the classical construction of Nakamura [A] and Craw–Reid [CR], with some remarkable modifications.

Presenter: REID, Miles (Warwick)

Contribution ID: **27** Type: **not specified**

Classifying torsionfree classes of the category of coherent sheaves and their Serre subcategories

Friday, December 22, 2023 11:00 AM (30 minutes)

The classification of subcategories is one of the long-studied topics in the representation theory of algebras. The most classical result is Gabriel's classification of Serre subcategories (i.e., subcategories closed under taking subobjects, quotients, and extensions). He classified the Serre subcategories of the category of coherent sheaves on a noetherian scheme by using specialization-closed subsets of the scheme. In the case of the module category over a commutative noetherian ring, various subcategories such as torsion classes (= subcategories closed under taking quotients and extensions) and torsion-free classes (= subcategories closed under subobjects and extensions) are classified.
In this talk, we will talk about an extension of these classification results to the category of coherent sheaves. A naive extension does not hold even in the case of projective lines. Therefore, we consider subcategories closed under tensoring with line bundles and show that by imposing this condition, various subcategories of the category of coherent sheaves can be classified in the same way as in the case of the module category. Using these classifications, we also classify the Serre subcategories of the category of torsion-free sheaves over a reduced projective curve.

Presenter: SAITO, Shunya (Nagoya)

Contribution ID: **28** Type: **not specified**

The Euler characteristic of the Fujiki-Oka resolution via continued fractions

Tuesday, December 19, 2023 11:00 AM (30 minutes)

Let G be a finite subgroup of SL(n, C). If a quotient variety C n/G has a crepant resolution, then its Euler characteristic is equal to the number of conjugacy classes of G, which is a weak version of the McKay correspondence. In this talk, we generalize this correspondence to a finite cyclic group of GL(n, C). We construct this correspondence using certain toric resolutions obtained through continued fractions.

Presenter: SATO, Yusuke (Kogakuin)

Contribution ID: **29** Type: **not specified**

McKay correspondence for Hilb^n(C^3), categorical DT theory and geometric Langlands

Monday, December 18, 2023 10:00 AM (50 minutes)

The McKay correspondence for Hilb^n(C^2) is its derived equivalence with C^{2n}/S_n, proven by Bridgeland-King-Reid and Haiman. In this talk, I will explan how to give its version for Hilb^n(C^3) using categorical DT theory and its categorical wall-crossing formula. It involves semiorthogonal decomposition with factors categorical Hall products of quasi-BPS categories, which we conjecture to be equivalent to the category of matrix factorizations over C^{3n}/S_n with zero potential. I explain that how (a variant of) the above conjecture is implied by Betti geometric Langlands conjecture. This is a part of my series of joint works with Tudor Padurariu.

Presenter: TODA, Yukinobu (Tokyo)

Contribution ID: **30** Type: **not specified**

Invariant theory of skew polynomial algebras

Tuesday, December 19, 2023 11:40 AM (50 minutes)

Traces are a classical tool in (commutative) invariant theory. When studying invariant subalgebras of noncommutative algebras, traces also play an essential role. In this talk, for skew polynomial algebras, we provide a generalization of the classical formula that expresses the trace series of an automorphism as the reciprocal of the reverse characteristic polynomial of it. We then use this generalization to study 'good' automorphisms and 'good' invariant subalgebras of skew polynomial algebras. This talk is based on joint work with W. Frank Moore.

Presenter: UEYAMA, Kenta (Shinshu)

Contribution ID: **31** Type: **not specified**

3d McKay correspondence and 5d SCFTs

Wednesday, December 20, 2023 2:00 PM (50 minutes)

In theoretical physics, a large class of superconformal field theories (SCFTs) can be constructed by putting superstring/M-theory on canonical singularities. In this talk, I'm going to mainly discuss the cases of M-theory on C3 orbifold singularities, which lead to a large class of 5d SCFTs. Many physical properties of the SCFT, such as rank, flavor rank and 1-form symmetry can be read off from 3d McKay correspondence. We have also studied new crepant resolutions of non-abelian C3 orbifolds. The cases of C4 orbifolds will also be shortly mentioned in the end.

Presenter: WANG, Yi-Nan (Peking)

Contribution ID: **32** Type: **not specified**

(Derived) Deformations of Crepant Curves

Thursday, December 21, 2023 10:00 AM (50 minutes)

Motivated by various contraction conjectures, I will describe the full A_infty structure associated to a general (-3,1)-curve C inside a smooth CY 3-fold. As a corollary, the noncommutative deformation theory of C can be described as a superpotential algebra derived from what we call free necklace polynomials, establishing a suitably interpreted string theory prediction due to Ferrari, Aspinwall-Katz and Curto-Morrison. This is joint work with Gavin Brown.

Presenter: WEMYSS, Michael (Glasgow)

Contribution ID: **33** Type: **not specified**

Birational geometry of quiver moduli through Cox rings

Tuesday, December 19, 2023 2:00 PM (50 minutes)

Moduli spaces of quiver representations appear in various contexts of mathematics. These spaces depend on the choice of stability parameters and, in many cases, variation of the parameters induces birational transformations such as flops. In this talk I will introduce a new method to investigate such transformations using the Cox rings. We will then see that this method works well especially in the context of the McKay correspondence and that it is also useful in the study of a Craw-Ishii type problem.

Presenter: YAMAGISHI, Ryo (Bath)

Contribution ID: **34** Type: **not specified**

F-blowups of quotient singularities and toric singularities

Monday, December 18, 2023 2:00 PM (50 minutes)

The F-blowup gives a caonical way to construct a birational transform of a singular variety in positive characteristic. There is also a non-commutative counterpart of this construction. There are several natural questions concering the F-blowup. Does this blowup give resolution of singularities? If not, does it improve singularities? Is it close to be a resolution of singularities in a certain sense? In this talk, I would like to present some recent results for quotient singularities by finite group scheme actions and for toric singularities in this direction, which were obtained in my joint works with C. Liedtke, E. Chavez-Martinez and D. Duarte.

Presenter: YASUDA, Takehiko (Osaka)

Contribution ID: **35** Type: **not specified**

Combinatorial actions on quivers and geometric interpretations

Wednesday, December 20, 2023 5:10 PM (50 minutes)

The Coulomb branch of 3d N=4 gauge theory is a new construction of symplectic singularities. This talk will cover operations on quivers that start with a given symplectic singularity and ends with a new symplectic singularity, while the relation between the two is understood geometrically. These operations include
—Quiver subtraction where the resulting singularity is a degeneration of a bigger singularity, thus revealing the stratification of symplectic leaves.
—Quiver subtraction where the resulting singularity is a HK quotient.
These combinatorial operations extend to orthosymplectic quivers and few examples of this type will be discussed.
It is hoped that such operations can be further extended to Gorenstein singularities.

Presenter: HANANY, Amihay