Towards Standard Model at large charge

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Large charge expansion

Hellerman, Orlando, Reffert, Watanabe 2015

EFT approach for strongly coupled CFTs.
Conformal dimension of the lowest-lying primary with large charge Q:

$$\Delta_Q = Q^{\frac{d}{d-1}} \left[\alpha_1 + \alpha_2 Q^{\frac{-2}{d-1}} + \alpha_3 Q^{\frac{-4}{d-1}} + \dots\right] + Q^0 \left[\beta_0 + \beta_1 Q^{\frac{-2}{d-1}} + \dots\right] + \mathcal{O}\left(Q^{-\frac{d}{d-1}}\right)$$

What about UV complete models?

What about UV complete models?

Badel, Cuomo, Monin, Rattazzi 2019

Wilson-Fisher FP in U(1) model in $4-\epsilon$

$$L = \partial_\mu \bar{\phi} \partial^\mu \phi + \frac{\lambda}{4} \left(\bar{\phi}\phi\right)^2$$

Double scaling limit….

$$\lambda \to 0 \qquad Q \to \infty \qquad \lambda Q = fixed$$

<<1 — Superfluid interacts with light radial mode

>>1 — Radial mode decouples

Reproduce diagramatics

Reproduce EFT results

Semiclassical method

Badel, Cuomo, Monin, Rattazzi 2019

Double scaling limit : $\lambda \to 0, \quad Q \to \infty, \quad \lambda Q = fixed$

- Tune QFT to the (perturbative) fixed point (WF or BZ type)
- Map the theory to the cylinder $\mathbb{R}^d \to \mathbb{R} \times S^{d-1}$
- Exploit operator/state correspondence for the 2-point function to relate anomalous dimension to the energy

$$\langle \bar{\phi}^Q(x_f)\phi^Q(x_i)\rangle_{CFT} = \frac{1}{|x_f - x_i|^{2\Delta_{\phi^Q}}} \qquad E = \Delta_{\phi^Q}/R$$

- To compute this energy, evaluate expectation value of the evolution operator in an arbitrary state with fixed charge Q

Identify the operator

Perturbatively it is $\phi^Q(x)$

- Adding derivatives increases dimension in the free theory limit
- Level crossing involves non-analyticities

Towards realistic theory

To use large charge expansion technology to compute anomalous dimensions in the SU(3)xSU(2)xU(1) Standard Model with quarks, leptons and the Higgs we need, at least, to:

- Generalise the construction to the local symmetry
- Find the way to identify the operator
- Add fermions

SU(3)xSU(2)xU(1)   (for SU(3) see F.Sannino's talk)

Can we apply this semiclassical method to 4d local U(1) model?

$$S = \int d^D x \left(\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + (D_\mu \phi)^\dagger D_\mu \phi + \frac{\lambda (4\pi)^2}{6} (\bar{\phi}\phi)^2 \right)$$

$$D_\mu \phi = (\partial_\mu + ieA_\mu)\phi$$

Both $\langle \phi^Q(x) \rangle$ and $\langle \bar{\phi}^Q(x_f) \phi^Q(x_i) \rangle$

are not gauge-invariant and vanish due to Elitzur's theorem (1975)

→ our generalisation should correspond to <u>gauge-invariant correlator</u> in flat space

But which one? The choice is not unique

Kleinert et al 05

Schwinger proposal:

Wilson line on the shortest path connecting x and x'

$$\langle\bar{\phi}(x')\exp\Big[-ie\int_{x}^{x'} dx^{\mu}A_{\mu}(x)\Big]\phi(x)\rangle$$

Dirac proposal:

$$G_D = \langle\bar{\phi}(x_f)\exp\left(-i\ e\int d^D x J^{\mu}(x)A_{\mu}(x)\right)\phi(x_i)\rangle$$

$J_{\mu}(x)$ is the background current

Dirac proposal:

$$G_D = \langle \bar{\phi}(x_f) \exp\left(-i\ e \int d^D x J^\mu(x) A_\mu(x) \right) \phi(x_i) \rangle$$

$$\partial^\mu J_\mu = \delta(x - x_f) - \delta(x - x_i) \qquad\qquad \partial^2 J_\mu = 0$$

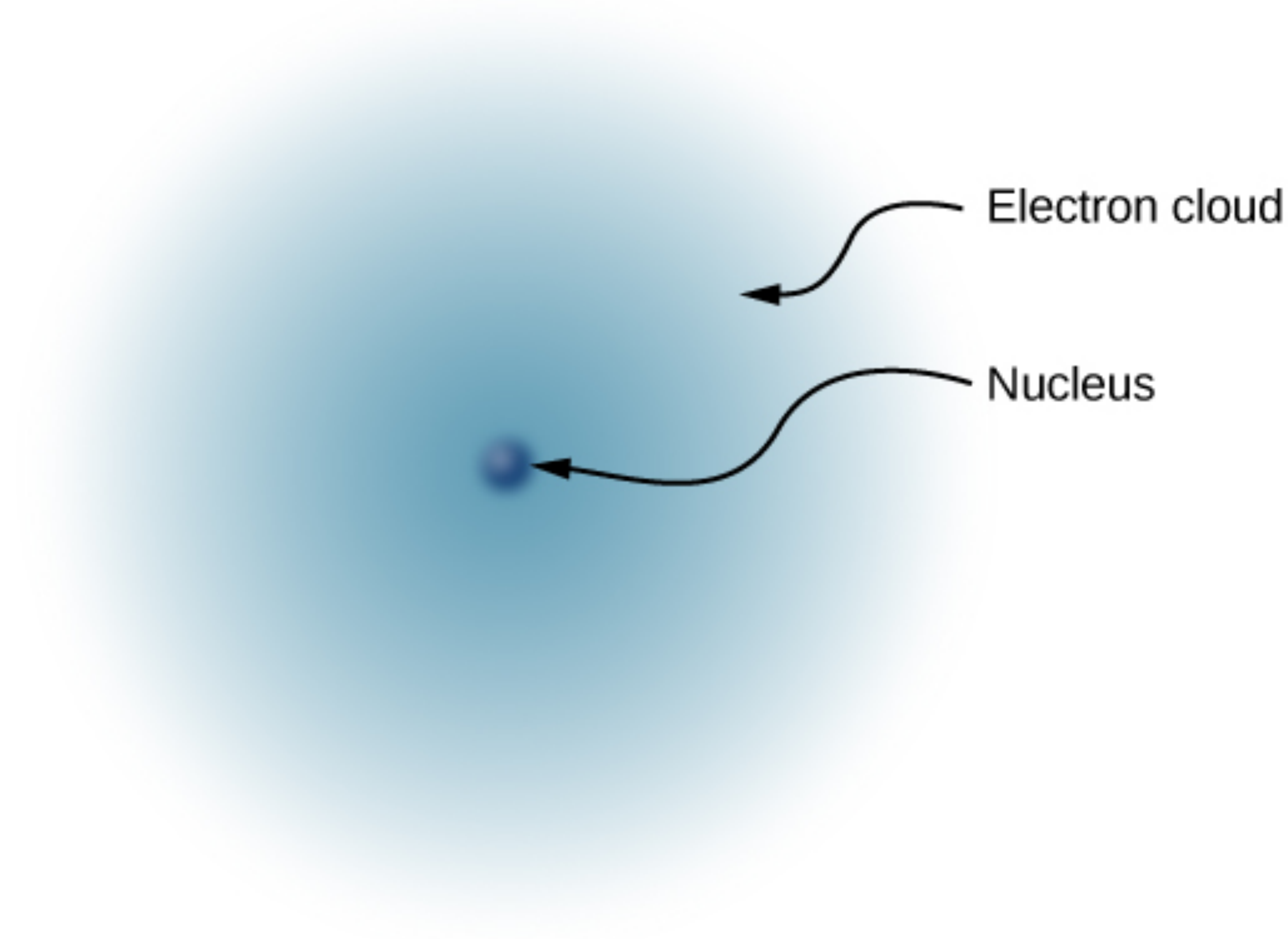
$$J_\mu(z) = J'_\mu(z - x') - J'_\mu(z - x)$$

$$J'_\mu(z) = -i \int \frac{d^d k}{(2\pi)^d} \frac{k_\mu}{k^2} e^{ik\cdot z} = -\frac{\Gamma(d/2-1)}{4\pi^{d/2}} \partial_\mu \frac{1}{z^{d-2}}$$

$$\Rightarrow \qquad G_D = \langle \bar{\phi}_{nl}(x_f) \phi_{nl}(x_i) \rangle$$

where $\phi_{nl}(x) \equiv e^{-ie \int d^D z J'_\mu(z-x) A^\mu(x)} \phi(x)$

$$\phi_{nl}(x) \equiv e^{-ie\int d^D z J'_\mu(z-x)A^\mu(x)}\phi(x)$$



Physically, $\phi_{nl}(x)$ can be interpreted as
the creation operator of a charged scalar particle (nucleus)
surrounded by a background electron cloud

This is an overall neutral object (atom)!

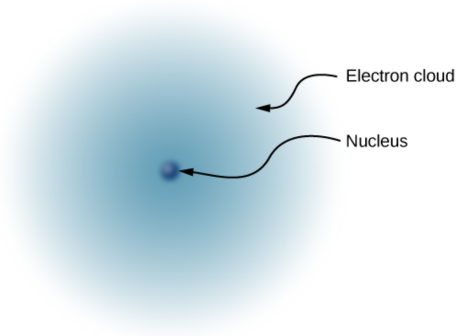
QED plasma (textbook result)

$$\mathcal{L}_{\text{QED}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}\left(i\not{D} - m\right)\psi + A_\mu J^\mu$$

$$D_\mu = \partial_\mu - i\mu\delta_{\mu 0} - igA_\mu$$

If we want finite charge density in the infinite volume limit, with a finite free energy density, neutralising background is crucial

Otherwise we have long-range electric fields giving infrared-divergent energy cost



$$\phi_{nl}(x) \equiv e^{-ie\int d^D z J'_\mu(z-x) A^\mu(x)} \phi(x)$$

$$J'_\mu(z) = -i \int \frac{d^d k}{(2\pi)^d} \frac{k_\mu}{k^2} e^{ik\cdot z} = -\frac{\Gamma(d/2-1)}{4\pi^{d/2}} \partial_\mu \frac{1}{z^{d-2}}$$

Observation:

In Landau gauge $\partial^\mu A_\mu = 0 \implies \phi_{nl}(x) = \phi(x)$

We may expect that the operator-state correspondence can be applied to access anomalous dimension of this operator

If true, we need to compute the energy of the state created by this operator

$$S = \int d^D x \left(\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + (D_\mu \phi)^\dagger D_\mu \phi + \frac{\lambda (4\pi)^2}{6} (\bar{\phi}\phi)^2 \right)$$

$$D = 4 - \epsilon$$

- Perturbative WF fixed point at 1-loop reads

$$\lambda^* = \frac{3}{20} \left(19\epsilon \pm i\sqrt{719}\epsilon \right) , \qquad a_g^* = \frac{3}{2}\epsilon \qquad a_g = \frac{e^2}{(4\pi)^2}$$

complex!

- Map to the cylinder

$$S = \int d^D x \sqrt{-g} \left(\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + (D_\mu \phi)^\dagger D^\mu \phi + m^2 \bar{\phi}\phi + \frac{\lambda (4\pi)^2}{6} (\bar{\phi}\phi)^2 \right)$$

$m^2 = (D-2)^2/4$ with radius of the cylinder R=1

- State-operator correspondence

$$G_D = \langle \bar{\phi}(x_f) \exp\left(-i\, e \int d^D x\, J^\mu(x) A_\mu(x) \right) \phi(x_i) \rangle$$

$$\langle Q | e^{-HT} | Q \rangle = \mathcal{Z}^{-1} \int_{\rho=f}^{\rho=f} \mathcal{D}\rho \mathcal{D}\chi \mathcal{D}A\, e^{-S_{\rm eff}}$$

$$\phi(x) = \frac{\rho(x)}{\sqrt{2}} e^{i\chi(x)}$$

$$S_{\rm eff} = \int_{-T/2}^{T/2} d\tau \int d\Omega_{D-1} \left(\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} (\partial\rho)^2 + \frac{1}{2} \rho^2 (\partial\chi)^2 + \frac{1}{2} m^2 \rho^2 + e\rho^2 A_\mu \partial^\mu \chi + \frac{1}{2} e^2 \rho^2 A_\mu A^\mu + \frac{\lambda (4\pi)^2}{24} \rho^4 + \frac{i\, Q}{\Omega_{D-1}} \dot{\chi} \right) + J^\mu(x) A_\mu(x)$$

- Fixing the charge of the initial and final state to Q

$$+J^{\mu}(x)A_{\mu}(x)$$

$$\partial_{\mu}F^{\mu\nu} = j^{\nu}(x) \quad \Rightarrow \quad \partial_{\mu}F^{\mu\nu} = j^{\nu}(x) + J^{\nu}(x) = 0$$

$J^{\nu}(x)$ is neutralising background

Homogeneous ground state ansatz

$$\rho(x) = f, \quad \chi(x) = -i\mu\tau, \quad A_{\mu} = 0$$

$$S = S(\phi_0) + \frac{1}{2}(\phi - \phi_0)^2 S''(\phi_0) + \ldots$$

$\downarrow$

Δ_{-1}

Homogeneous ground state

$$\rho(x) = f, \quad \chi(x) = -i\mu\tau, \quad A_\mu = 0$$

From EOM

$$\mu^3 - \mu = \frac{4}{3}\lambda Q \ , \qquad f^2 = \frac{6}{(4\pi)^2\lambda}\left(\mu^2 - m^2\right)$$

Plugging into $S_{\text{eff.}}$

$$4\,\Delta_{-1} = \frac{3^{2/3}\left(x+\sqrt{-3+x^2}\right)^{1/3}}{3^{1/3}+\left(x+\sqrt{-3+x^2}\right)^{2/3}} + \frac{3^{1/3}\left(3^{1/3}+\left(x+\sqrt{-3+x^2}\right)^{2/3}\right)}{\left(x+\sqrt{-3+x^2}\right)^{1/3}}$$

The same as in U(1) global case

$$x \equiv 6\lambda Q$$

$$S = S(\phi_0) + \frac{1}{2}(\phi - \phi_0)^2 S''(\phi_0) + \dots$$

$$\rho(x) = f + r(x)$$

$$\chi(x) = -i\mu\tau + \frac{\pi(x)}{f}$$

↓ Δ_0

Add gauge-fixing and ghost terms

$$\delta S = \frac{1}{2}\int d^d x \left(G^2 + \mathcal{L}_{\text{ghost}}\right), \qquad G = \frac{1}{\sqrt{\xi}}(\nabla_\mu A^\mu + ef\pi)$$

and expand S_{eff} to quadratic order

$$\mathcal{L}^{(2)}_{\text{eff}} = \frac{1}{2}A_\mu\left(-g^{\mu\nu}\nabla^2 + \mathcal{R}^{\mu\nu} + \left(1 - \frac{1}{\xi}\right)\nabla^\mu\nabla^\nu + (ef)^2 g^{\mu\nu}\right)A_\nu$$

$$+ \frac{1}{2}(\partial_\mu r)^2 - \frac{1}{2}2(m^2 - \mu^2)r^2 + \frac{1}{2}(\partial_\mu \pi)^2 - \frac{1}{2\xi}(ef)^2\pi^2$$

$$- 2i\mu\, r\partial_\tau \pi - 2if\mu r A^0 + ef\left(1 - \frac{1}{\xi}\right)A_\mu \partial^\mu \pi + \bar{c}[-\nabla^2 + (ef)^2]c$$

Spectrum of fluctuations

$$scalars: r, \pi, A_0, h \qquad A_i = B_i + C_i \qquad C^i = \nabla^i h$$

$$vectors: B_i \qquad \nabla_i B^i = 0$$

$$ghosts: c, \bar{c} \qquad -\nabla^2 = -\partial_\tau^2 + \left(-\nabla^2_{S^{D-1}}\right) \quad on \quad \mathbb{R} \times S^{D-1} space$$

$$B_i: \quad \int \frac{d\omega}{2\pi} \sum_\ell n_v(\ell) \det \left(-\partial_\tau^2 + J^2_{\ell(v)} + (D-2) + (ef)^2\right)^{-1/2}$$

$$c, \bar{c}: \quad \int \frac{d\omega}{2\pi} \sum_\ell n_s(\ell) \det \left[-\partial_\tau^2 + J^2_{\ell(s)} + (ef)^2\right]$$

$$scalars: \int \frac{d\omega}{2\pi} \sum_\ell n_s(\ell) \det \left[\mathcal{B}\right]^{-1/2}$$

Scalars

$$\mathcal{B} = \begin{pmatrix} -\omega^2 + J^2_{\ell(s)} + 2(\mu^2 - m^2) & -2i\mu\omega & -2ie\mu f & 0 \\ 2i\mu\omega & -\omega^2 + J^2_{\ell(s)} + \frac{1}{\xi}e^2 f^2 & -ef\left(1 - \frac{1}{\xi}\right)\omega & -ief\left(1 - \frac{1}{\xi}\right)|J_{\ell(s)}| \\ -2ie\mu f & ef\left(1 - \frac{1}{\xi}\right)\omega & -\frac{1}{\xi}\omega^2 + J^2_{\ell(s)} + (ef)^2 & i\left(1 - \frac{1}{\xi}\right)\omega|J_{\ell(s)}| \\ 0 & ief\left(1 - \frac{1}{\xi}\right)|J_{\ell(s)}| & i\left(1 - \frac{1}{\xi}\right)\omega|J_{\ell(s)}| & -\omega^2 + \frac{1}{\xi}J^2_{\ell(s)} + (ef)^2 \end{pmatrix}$$

Determinant factorizes with gauge-independent dispersion relations:

$$\xi \det \mathcal{B} = (\omega^2 + \omega_+^2)(\omega^2 + \omega_-^2)(\omega^2 + \omega_1^2)^2$$

ξ cancels out in the final result due to contribution from Z^{-1}

$$\langle Q|e^{-HT}|Q\rangle = \mathcal{Z}^{-1} \int_{\rho=f}^{\rho=f} \mathcal{D}\rho\mathcal{D}\chi\mathcal{D}A\, e^{-S_{\text{eff}}}$$

$scalars: r, \pi, A_0, h$

$vectors: B_i$

$ghosts: c, \bar{c}$

$$\Delta_0 = \frac{1}{2}\sum_{\ell=\ell_0}^{\infty} d_\ell \omega_i(\ell)$$

Field	d_ℓ	$\omega_i(\ell)$	ℓ_0
B_i	$n_v(\ell)$	$\sqrt{J^2_{\ell(v)} + (D-2) + e^2 f^2}$	1
$h(C_i)$	$n_s(\ell)$	$\sqrt{J^2_{\ell(s)} + e^2 f^2}$	1
$(c, \bar{c})$	$-2n_s(\ell)$	$\sqrt{J^2_{\ell(s)} + e^2 f^2}$	0
A_0	$n_s(\ell)$	$\sqrt{J^2_{\ell(s)} + e^2 f^2}$	0
ϕ	$n_s(\ell)$	$\sqrt{J^2_{\ell(s)} + 3\mu^2 - m^2 + \frac{1}{2}e^2 f^2 \pm \sqrt{\left(3\mu^2 - m^2 - \frac{1}{2}e^2 f^2\right)^2 + 4J^2_{\ell(s)}\mu^2}}$	0

The MSbar renormalized result reads

$$\Delta_0 = \frac{1}{16}\left(-15\mu^4 - 6\mu^2 + 8\sqrt{6\mu^2-2} + 5\right) \qquad a_g \equiv \frac{e^2}{16\pi^2}$$

$$+ \frac{1}{2}\sum_{\ell=1}\sigma(\ell) - \frac{3a_g}{8\lambda}(\mu^2-1)\left(\frac{3a_g}{\lambda}(7\mu^2+5) - 9\mu^2 + 5\right)$$

$$\sigma(\ell) = \frac{9a_g}{2\lambda\ell}\left(\mu^2-1\right)\left[\left(\frac{3a_g}{\lambda}-1\right)\left(\mu^2-1\right) - 2\ell(\ell+1)\right]$$

$$+ \frac{5}{4\ell}\left(\mu^2-1\right)^2 - 2(\ell+1)(2\ell(\ell+2)+\mu^2),$$

subtraction terms

$$+ (\ell+1)^2\left[\omega_+^*(\ell) + \omega_-^*(\ell)\right] + 2\ell(\ell+2)\omega^*(\ell)$$

$$(\omega_\pm^*)^2 = \frac{3a_g}{\lambda}\left(\mu^2-1\right) + 3\mu^2 + \ell(\ell+2) - 1$$

$$\pm\sqrt{\left(\frac{3a_g}{\lambda}\left(\mu^2-1\right) - 3\mu^2 + 1\right)^2 + 4\ell(\ell+2)\mu^2}$$

scalars

$$(\omega^*)^2 = \frac{6a_g}{\lambda}\left(\mu^2-1\right) + \ell(\ell+2) + 1$$

vectors

Put together

$$S = S(\phi_0) + \frac{1}{2}(\phi - \phi_0)^2 S''(\phi_0) + \dots$$

$S(\phi_0) \to \Delta_{-1}$, $\quad S''(\phi_0) \to \Delta_0$

$$E = E_{-1} + E_0 + \cdots = \frac{\Delta_{-1} + \Delta_0 + \cdots}{R}$$

Should correspond to $\phi_{nl}(x) \equiv e^{-ie\int d^D z J'_\mu(z-x)A^\mu(x)}\phi(x)$

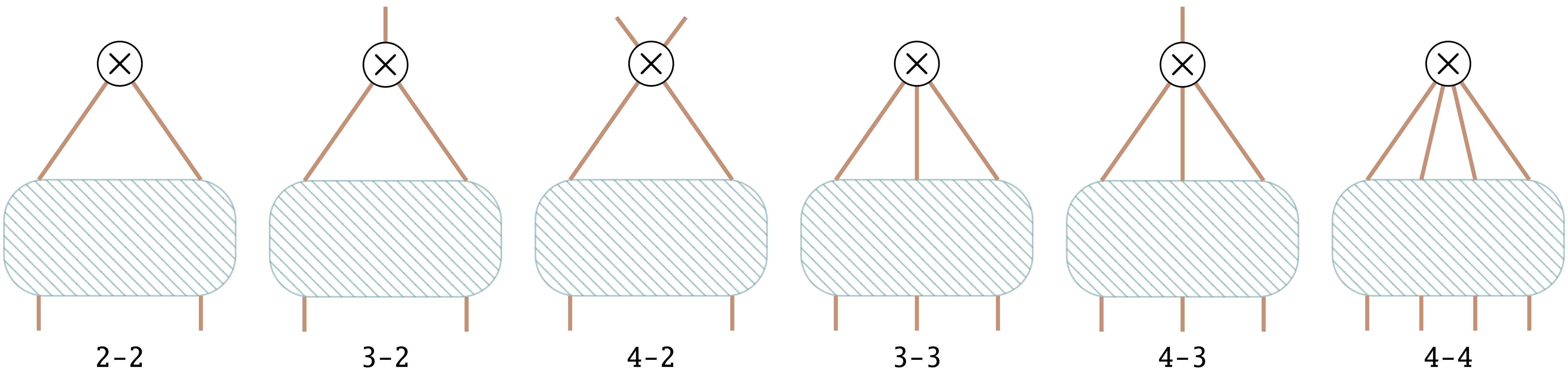
In Landau gauge $\partial^\mu A_\mu = 0 \implies \phi_{nl}(x) = \phi(x)$

To cross-check we may compute conformal dimension of $\phi(x)$ diagrammatically

The result will be gauge-dependent and we fix Landau gauge

Explicit 3-loop gauge-dependent result for ϕ^Q

We compute 3-loop AD for ϕ^Q for fixed Q=2,3,4 in $D = 4 - \epsilon$



and ``fit'' all coefficients Ckl in

$$\gamma_Q(\lambda, a_g, \xi) = \sum_{l=1}^{3} \gamma_Q^{(l-\text{loop})}(\lambda, a_g, \xi) \ , \quad \gamma_Q^{(l-\text{loop})} \equiv \sum_{k=0}^{l} C_{kl} Q^{l+1-k}$$

- $\lambda Q \ll 1$ **Comparing to ordinary perturbation theory**

	1-loop	2-loop	3-loop	
Δ_{-1}	$Q^2\lambda_0$	$Q^3\lambda_0^2$	$Q^4\lambda_0^3$	....
Δ_0	$Q\lambda_0$	$Q^2\lambda_0^2$	$Q^3\lambda_0^3$	....
Δ_1		$Q\lambda_0^2$	$Q^2\lambda_0^3$	....
Δ_2			$Q\lambda_0^3$	....
$\vdots$				

Explicit 3-loop gauge-dependent result for ϕ^Q

$$\gamma_Q^{(1)}(\lambda, a_g, \xi) = \underbrace{\frac{\lambda}{3} Q^2}_{\text{leading}} \underbrace{- Q \left(3a_g + \frac{\lambda}{3} \right)}_{\text{sub−leading}} + a_g Q^2 \xi$$

$$\gamma_Q^{(2)}(\lambda, a_g) = \underbrace{-\frac{2\lambda^2}{9} Q^3}_{\text{leading}} + \underbrace{\left(a_g^2 - \frac{4a_g\lambda}{3} + \frac{2\lambda^2}{9} \right) Q^2}_{\text{sub−leading}} + \left(\frac{7a_g^2}{3} + \frac{4a_g\lambda}{3} + \frac{\lambda^2}{9} \right) Q$$

$$\gamma_Q^{(3)}(\lambda, a_g) = \underbrace{\frac{8\lambda^3}{27} Q^4}_{\text{leading}} + \underbrace{Q^3 \left[\frac{4a_g\lambda^2}{3}(3 - 2\zeta_3) - \frac{8a_g^2\lambda}{3}(1 + 3\zeta_3) + 4a_g^3(9\zeta_3 - 1) + \frac{2\lambda^3}{27}(16\zeta_3 - 17) \right]}_{\text{sub−leading}}$$
$$+ Q^2 \left[\frac{29a_g^2\lambda}{6} + a_g^3(95 - 108\zeta_3) + \frac{\lambda^3}{18}(57 - 64\zeta_3) - \frac{4a_g\lambda^2}{9}(31 - 30\zeta_3) \right] + Q \left[\frac{13a_g^2\lambda}{6} + \frac{2a_g\lambda^2}{9}(49 - 48\zeta_3) - \frac{2\lambda^3}{27}(31 - 32\zeta_3) - a_g^3 \left(\frac{3251}{54} - 72\zeta_3 \right) \right]$$

In Landau gauge we find perfect agreement for the leading and subleading terms with large-Q results!

Towards realistic theory

To use large charge expansion technology to compute anomalous dimensions in the SU(3)xSU(2)xU(1) Standard Model with quarks, leptons and the Higgs we need, at least:

- Generalise the construction to the local symmetry ☑
- Find the way to identify the operator ☑
- Add fermions (in SM these are Yukawa interactions)

Yukawa interactions: NJLY model

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$$\mathcal{L}_{\rm NJLY} = \partial_\mu \bar{\phi} \partial^\mu \phi + \bar{\psi}_j \partial\!\!\!/ \psi^j + g \bar{\psi}_{Rj} \bar{\phi} \psi_L^j + g \bar{\psi}_{Lj} \phi \psi_R^j + \frac{\lambda}{24} \left(\bar{\phi}\phi\right)^2$$

$$\phi = f e^{i\chi}$$

$$\chi = -i\mu\tau$$

Remove phases from Yukawa term via:

$$\psi_L \to \psi_L \, e^{\mu\tau/2} \, , \qquad \psi_R \to \psi_R \, e^{-\mu\tau/2}$$

Classically: $\psi^{cl}_{L,R} = 0$ → Δ_{-1} is O(2) model result

Quadratic in fluctuations:

$$S^{(2)} = \int_{-T/2}^{T/2} d\tau \int d\Omega_{d-1} \Big[\frac{1}{2}(\partial r)^2 + \frac{1}{2}(\partial \pi)^2 - 2i\mu \, r \partial_\tau \pi + (\mu^2 - m^2) r^2$$

$$+ i\mu \, \bar{\psi}_j \gamma^0 \psi^j + \bar{\psi}^j \nabla\!\!\!\!/_{\mathcal{M}} \psi^j + g \, f \bar{\psi}_{Lj} \psi_R^j + g \, f \bar{\psi}_{Rj} \psi_L^j \Big]$$

Gaussian integral

$$\int \mathcal{D}r \mathcal{D}\pi \mathcal{D}\bar{\psi} \mathcal{D}\psi \, e^{-S^{(2)}} = \frac{\det \, F}{\det \, B}$$

Fermionic dispersions

$$\omega_{f\pm}(\ell) = \sqrt{\frac{3g^2(\mu^2 - m^2)}{8\pi^2\lambda} + \left(\frac{\mu}{2} + \lambda_{f\pm}\right)^2}$$

↑ Eigenvalues of the Laplacian on the sphere

Presence of Yukawa destroys Fermi surface!

Leading quantum correction

$$\Delta_0 = \frac{1}{2}\sum_{\ell=0}^{\infty}\left[n_\ell(\omega_+(\ell) + \omega_-(\ell)) - N_f n_{f,\ell}(\omega_{f+}(\ell) + \omega_{f-}(\ell))\right]$$

$$\Delta_0^{(f)} = Q\left(\frac{g^2}{8\pi^2} - \frac{3g^4}{32\pi^4\lambda}\right) + Q^2\left(\frac{g^2\lambda}{12\pi^2} - \frac{g^4}{32\pi^4}\right) + Q^3\left(\frac{g^6\zeta(3)}{64\pi^6} - \frac{g^2\lambda^2}{18\pi^2} + g^4\lambda\frac{1-3\zeta(3)}{48\pi^4}\right)$$

$$+ \ldots.$$

Standard model

SU(3)xSU(2)xU(1) local symmetry

- To NLO in semiclassical expansion SU(3) does not enter
- Find the way to identify the operator (without weak interactions)

$$\mathcal{O}_Q(x) = e^{-ig' Q \int d^D z J'_\mu(z-x) B^\mu(z)} H^{I_1} \ldots H^{I_Q}$$

Dressed state of Higgs bosons

- Add fermions (in SM these are Yukawa interactions)

$$\mathcal{L}_{\text{Yukawa}} = -4\pi \left(Y_u^{ij} \left(\mathcal{Q}_i^L H^c \right) u_j^R + Y_d^{ij} \left(\mathcal{Q}_i^L H \right) d_j^R + Y_l^{ij} \left(L_i^L H \right) l_j^R \right)$$

Add chemical potential for hypercharge: $D_\mu \to D_\mu - i\mu Y$

E.O.M:

$$\partial_\nu B^{\mu\nu} = 0 \ ,$$

$$-(D_\mu - i\mu\delta_{\mu 0})(D^\mu - i\mu\delta^{\mu 0})H - m^2 H - \frac{(4\pi)^2}{3}\lambda(H^\dagger H)H = 0 \ ,$$

$$\partial^\mu W^{(a)}_{\mu\nu} + 4\pi g\epsilon^{abc}A^{\mu(b)}W^{(c)}_{\mu\nu} + 4\pi i g\left[H^\dagger\frac{\tau^a}{2}\partial_\nu H - \partial_\nu H^\dagger\frac{\tau^a}{2}H\right]$$
$$+\frac{g^2(4\pi)^2}{2}W^{(a)}_\nu H^\dagger H + 8\pi g\mu\delta_{\nu 0}H^\dagger\frac{\tau^a}{2}H = 0 \ .$$

Ground state

Miransky et al 03

$$W^{(+)}_3 = (W^{(-)}_3)^* = C \neq 0 \ , \quad W^{(3)}_0 = P \neq 0 \ , \quad H = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix} \ ,$$

$$v \neq 0 \ , \quad W^{(\pm)}_{1,2} = W^{(\pm)}_0 = W^{(3)}_{1,2} = W^{(3)}_3 = 0 \ , \quad B_\mu = 0$$

Breaks rotational invariance! $SO(3)\times SU(2)_L \times U(1)_Y \to SO(2)$

Standard model: result

$$\mathcal{O}_Q(x) = e^{-ig'Q\int d^D z J'_\mu(z-x) B^\mu(z)} H^{I_1} \dots H^{I_Q}$$

$$\Delta_Q = Q + \left\{\frac{1}{3}\lambda Q^2 + \left[N\mathcal{Y}_u + N\mathcal{Y}_d + \mathcal{Y}_l - \frac{3}{4}g'^2 - \frac{\lambda}{3}\right]Q\right\} - \left\{\frac{2}{9}\lambda^2 Q^3 - \left[2N\mathcal{Y}_{uu} + 2N\mathcal{Y}_{dd} + 2\mathcal{Y}_{ll}\right.\right.$$
$$\left.\left. - \frac{2}{3}\lambda(N\mathcal{Y}_u + N\mathcal{Y}_d + \mathcal{Y}_l) - \frac{1}{3}\lambda g'^2 + \frac{g'^4}{16} + \frac{\lambda^2}{9}\right]Q^2 + C_{22}Q\right\} + \left\{\frac{8}{27}\lambda^3 Q^4 + \left[\frac{1}{16}g'^6(9\zeta(3) - 1)\right.\right.$$
$$- \frac{1}{6}g'^4\lambda(1 + 3\zeta(3)) + \frac{1}{3}g'^2\lambda^2(3 - 2\zeta(3)) + \frac{4}{27}\lambda^3(9\zeta(3) - 8) + \frac{4}{27}\left(3N\left(\lambda^2\mathcal{Y}_u - 3\lambda\mathcal{Y}_{uu}\right.\right.$$
$$+ 9\zeta(3)(\lambda\mathcal{Y}_{uu} - 2\mathcal{Y}_{uuu})\Big) + 3N\left(\lambda^2\mathcal{Y}_d - 3\lambda\mathcal{Y}_{dd} + 9\zeta(3)(\lambda\mathcal{Y}_{dd} - 2\mathcal{Y}_{ddd})\right) + 3\left(\lambda^2\mathcal{Y}_l - 3\lambda\mathcal{Y}_{ll}\right.$$
$$\left.\left.\left. + 9\zeta(3)(\lambda\mathcal{Y}_{ll} - 2\mathcal{Y}_{lll})\right)\right)\right]Q^3 + C_{23}Q^2 + C_{33}Q\Big\} + \mathcal{O}\left(\kappa_I^4 Q^5\right). \quad (7.1)$$

$$\mathcal{Y}_f = (4\pi)^2 \mathrm{Tr} Y_f Y_f^\dagger, \quad \mathcal{Y}_{ff} = (4\pi)^4 \mathrm{Tr}(Y_f Y_f^\dagger)^2, \quad \mathcal{Y}_{fff} = (4\pi)^6 \mathrm{Tr}(Y_f Y_f^\dagger)^3, \quad f = u, d, l.$$

Thank you!