String Interactions in Yang Mills Theory

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Upcoming Paper with **Siwei Zhong**

Many theories admit unbreakable string-like excitations of nonzero tension

$$l_s^{-2}$$

1. The Abrikosov string in superconductors.
2. The Nielsen-Olesen strings in the Abelian Higgs model.
3. Confining strings in Yang-Mills theory.

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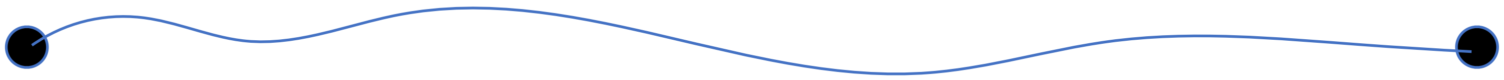
Such strings are also very useful in the description of mesons, baryons, and glueballs, especially highly excited ones.

The first step is to review the action on a string of length L. We can discuss closed strings or open strings. Let us consider open strings with Dirichlet boundary conditions for simplicity.



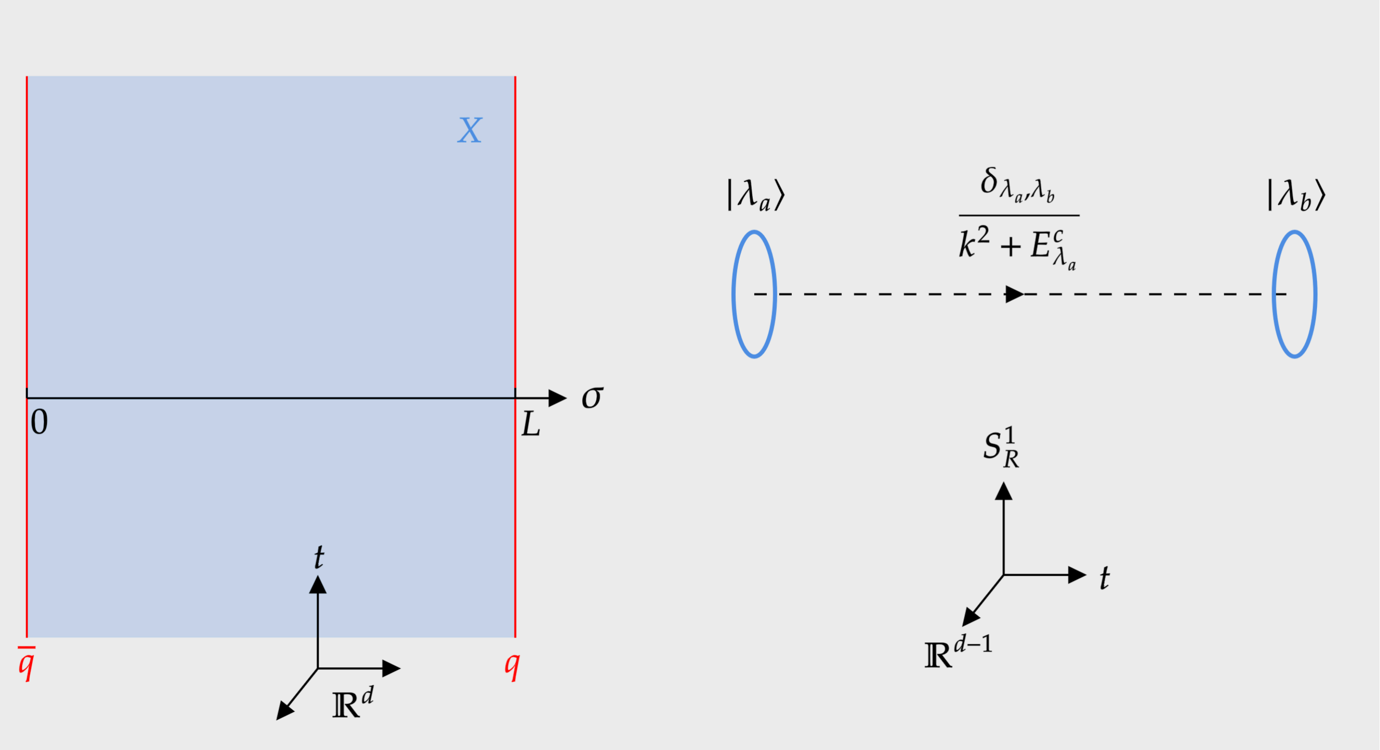
$$S_{\rm NG} = -\frac{1}{l_s^2}\int dt d\sigma \sqrt{-\det \partial_\alpha X^\mu \partial_\beta X_\mu} + \cdots$$

$$= -\frac{L}{l_s^2}\int dt + \frac{1}{2}\int dt d\sigma \left[(\partial_t X_\perp)^2 - (\partial_\sigma X_\perp)^2\right] + \cdots$$

The spectrum is simply

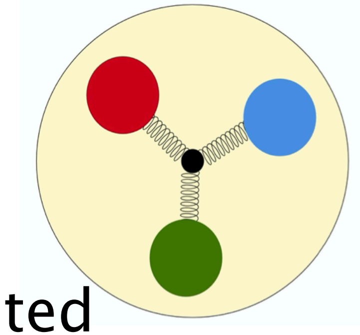
$$\omega_n = \frac{n\pi}{L} + \cdots \qquad (n > 0)$$

We build a Fock space out of these excitations, as usual. The computation of the thermal partition function has a dual interpretation as a propagation of a special state of the closed string.



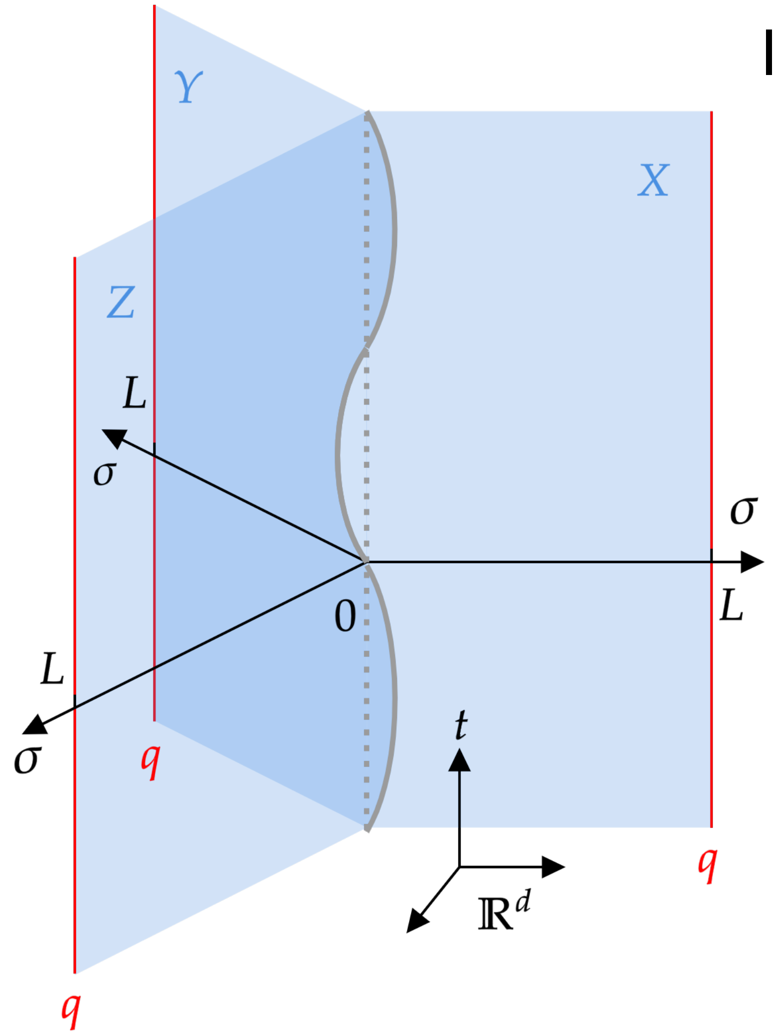
The predictions of the EFT can be contrasted with lattice simulations by inserting two Polyakov loops along the thermal circle. This has been done in exquisite detail mostly by [Athenodorou, Teper].

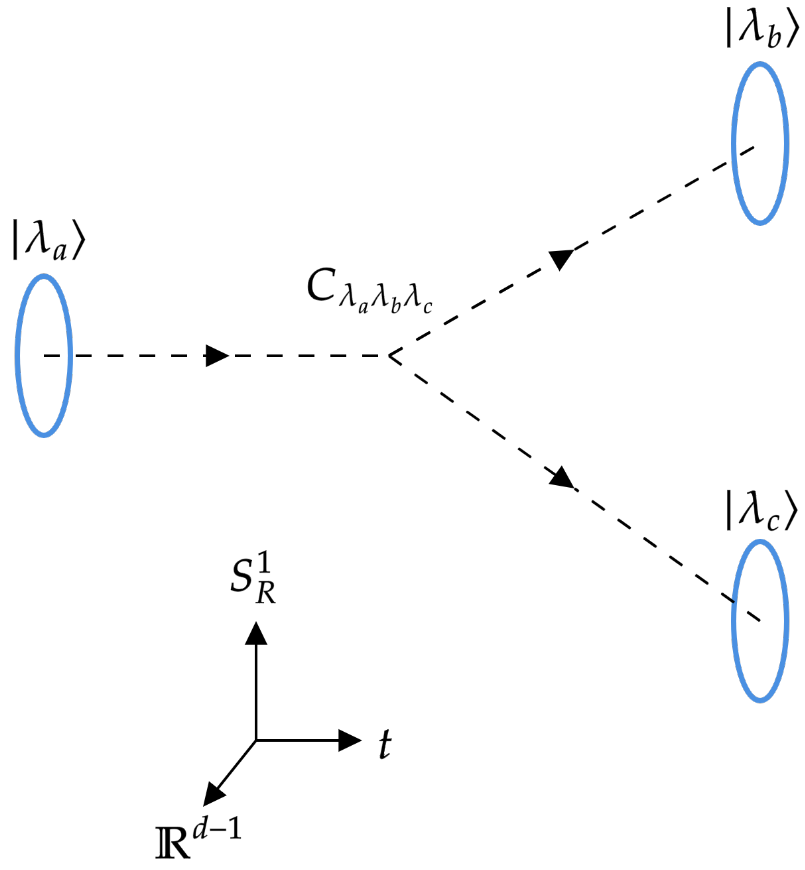
Consider the case of $SU(3)$YM theory.
Suppose we have three external quarks, qqq, rather than $q\bar{q}$. This dynamically creates a string junction. The junction is a dynamical object in the open string channel.
The fact that three strings can meet at a junction was already pointed out in 1975 and 1977 by [Artru] and [Veneziano,Rossi].



The same configuration in the closed string channel looks like a three-closed-strings interaction vertex.

The baryon junction is very easy to prepare on the lattice, by employing three Polyakov loops.

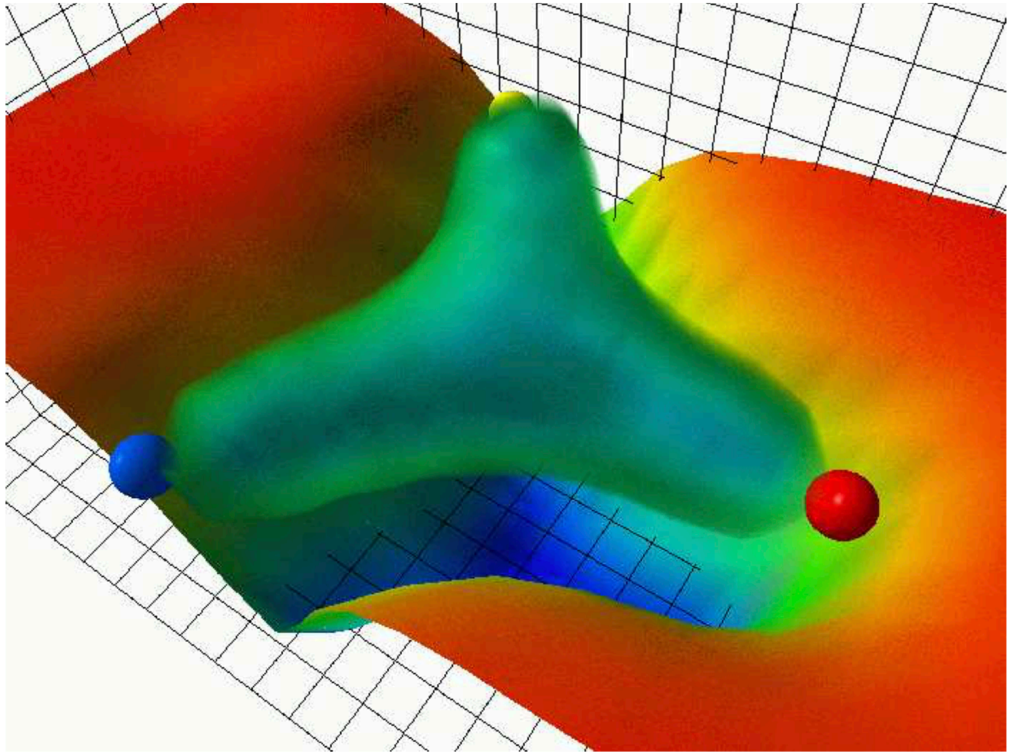




A priori nothing is known about string interactions. The interaction is due to a **"D-instanton."**

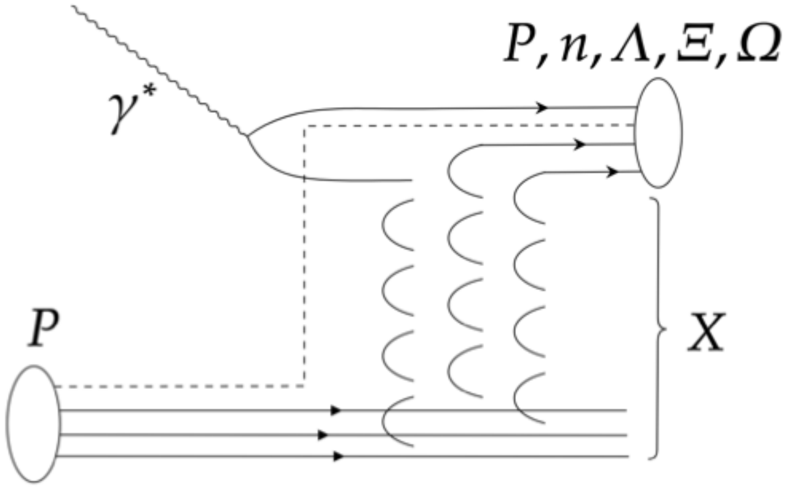
However in the open channel the problem seems more tractable. Furthermore, the dynamical junction was already observed in lattice simulations of baryons, and the junction is often called the "baryon junction".

F. Bissey et al.



The baryon junction exists in YM theory. We will use this terminology even though the word baryon is inappropriate as there are no dynamical quarks and no $U(1)_B$ symmetry.

Yet it is true that in some sense it is the baryon junction that carries the baryon number in QCD since there is pair creation changing the end points but the baryon junction remains intact. This point of view of thinking about the baryon junction as the topological object which carries baryon number leads to interesting predictions that would be testable.



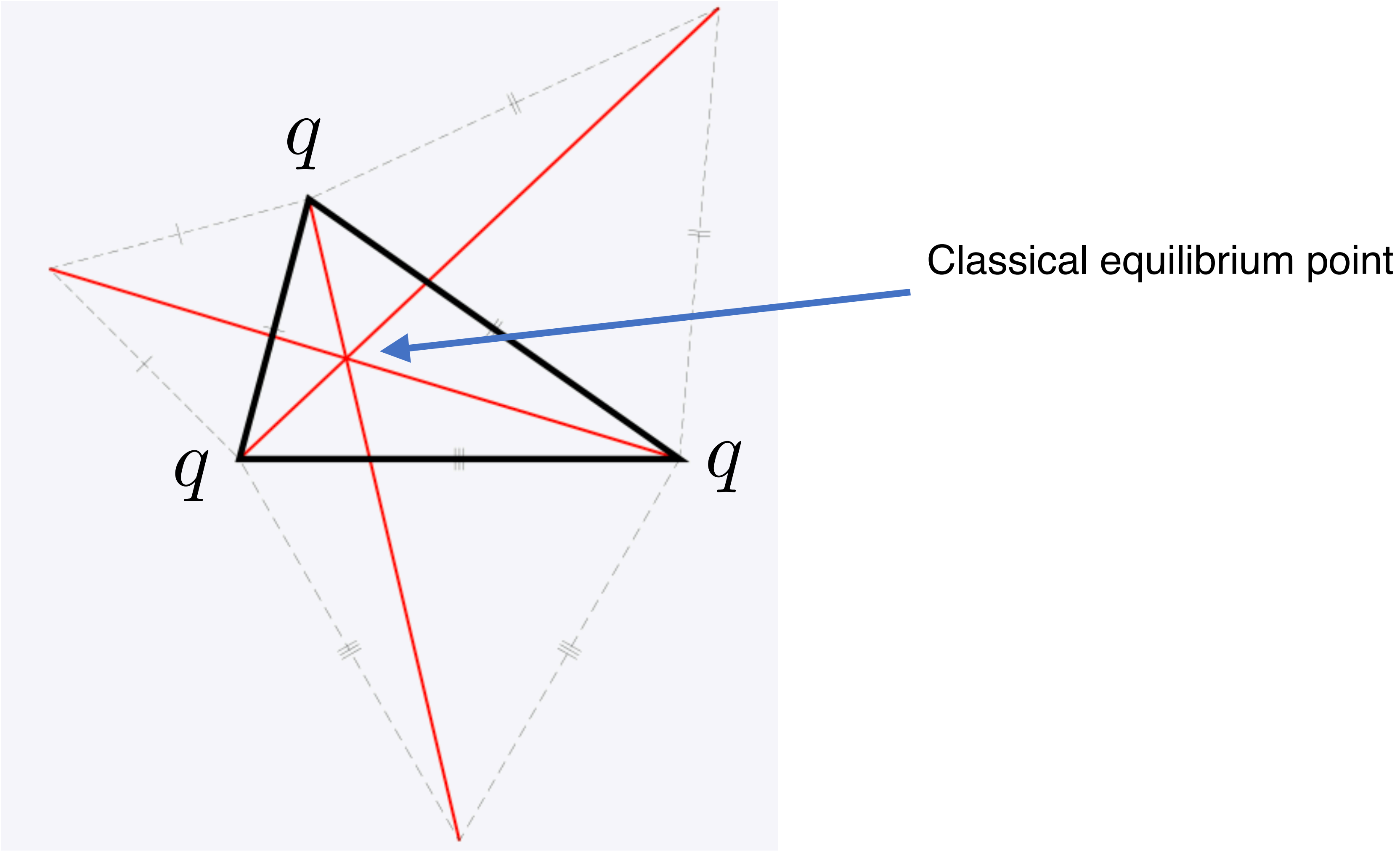
[Florio, Frenklakh, Kharzeev]

Up to and including order $1/L^3$ the theory of the junction has only one new parameter, which is the junction mass M. It is a priori not necessarily positive.

$$-M\int_{\sigma=0} dt + \frac{Ml_s^2}{2}\int_{\sigma=0} dt(\partial_t x_i)^2 + S_{NG}^{(1)} + S_{NG}^{(2)} + S_{NG}^{(3)} + O\left(1/L^3\right)$$

Other two derivative operators, especially $\int dt\partial_\sigma X_\perp^{(i)}\partial_\sigma X_\perp^{(j)}$ are all excluded!

Classically the saddle point is when the three strings meet at 120 degrees. This is the Fermat point.



The quantization then proceeds in a straightforward manner. Let us report the equilateral case for simplicity. The spectrum of normal modes is too complicated otherwise.

$$\text{Junction} \sim N^3 \otimes D^3$$

$$3\times : \quad \omega_n = \frac{n\pi}{L}$$

$$1\times : \quad \omega_n = \frac{(n+\frac{1}{2})\pi}{L}\left(1 - \frac{2M}{3TL}\right)$$

$$2\times : \quad \omega_n = \frac{(n+\frac{1}{2})\pi}{L}\left(1 - \frac{M}{3TL}\right)$$

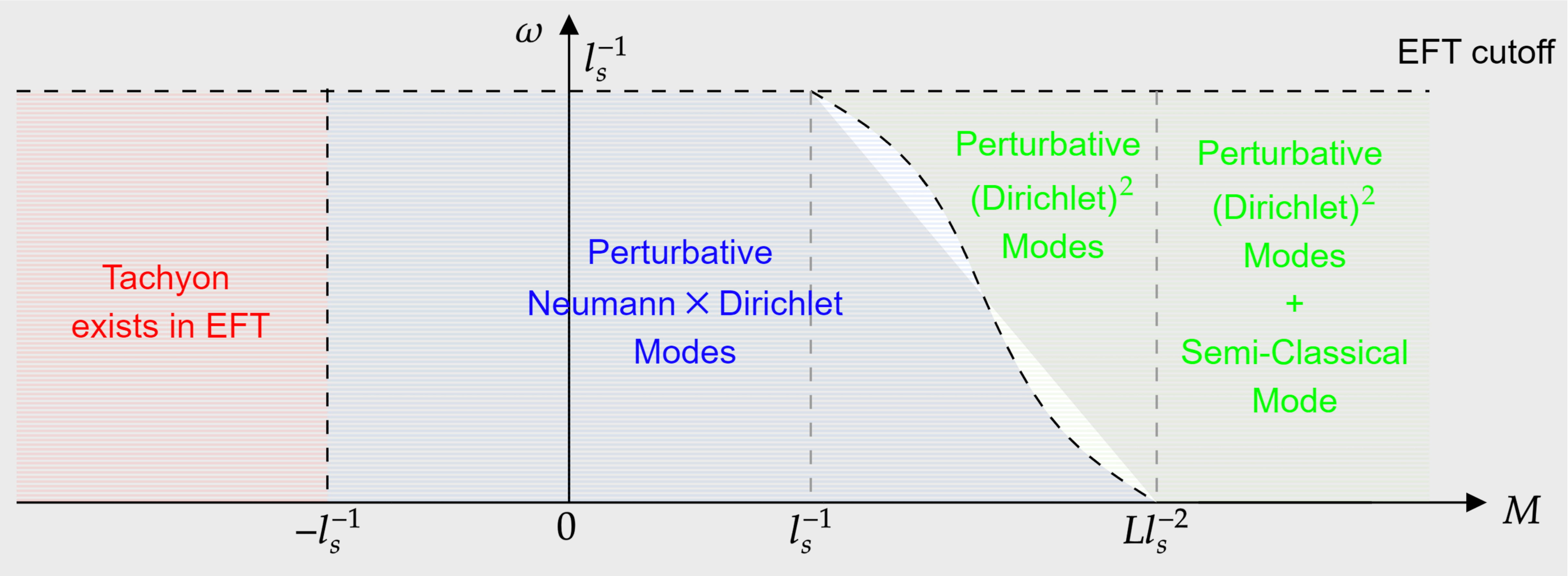
Classically the ground state energy in the equilateral case is simply

$$E_{classical} = \frac{3L}{l_s^2} + M$$

The normal modes however contribute to the ground state as usual and we get a quantum corrected ground state energy:

$$E_{quantum} = \frac{3L}{l_s^2} + M - \frac{(d-2)\pi}{16L} - \frac{(d+2)\pi M l_s^2}{144L^2} + O(\frac{1}{L^3})$$

This particular M dependence should allow a measurement of the baryon junction mass.



ω
l_s^{-1}
EFT cutoff
Tachyon
exists in EFT
Perturbative
Neumann × Dirichlet
Modes
Perturbative
(Dirichlet)^2
Modes
Perturbative
(Dirichlet)^2
Modes
+
Semi-Classical
Mode
$-l_s^{-1}$
0
l_s^{-1}
Ll_s^{-2}
M

Surprisingly, nothing is wrong in the open channel for negative junction mass, as long as it is not too large in absolute value.

To our knowledge the value of M is not known yet in YM theory. The partition function is given by

$$Z = \frac{e^{-6\pi RL/l_s^2 - 2\pi RM}}{\left[\eta(\sqrt{q})\right]^d \left[\eta(q)\right]^{d-3}} \left[1 - \frac{(d+2)Ml_s^2}{144L} \log q \left[2E_2(q) - E_2(\sqrt{q})\right]\right]$$

$$q \equiv e^{\frac{-2\pi^2 R}{L}}$$

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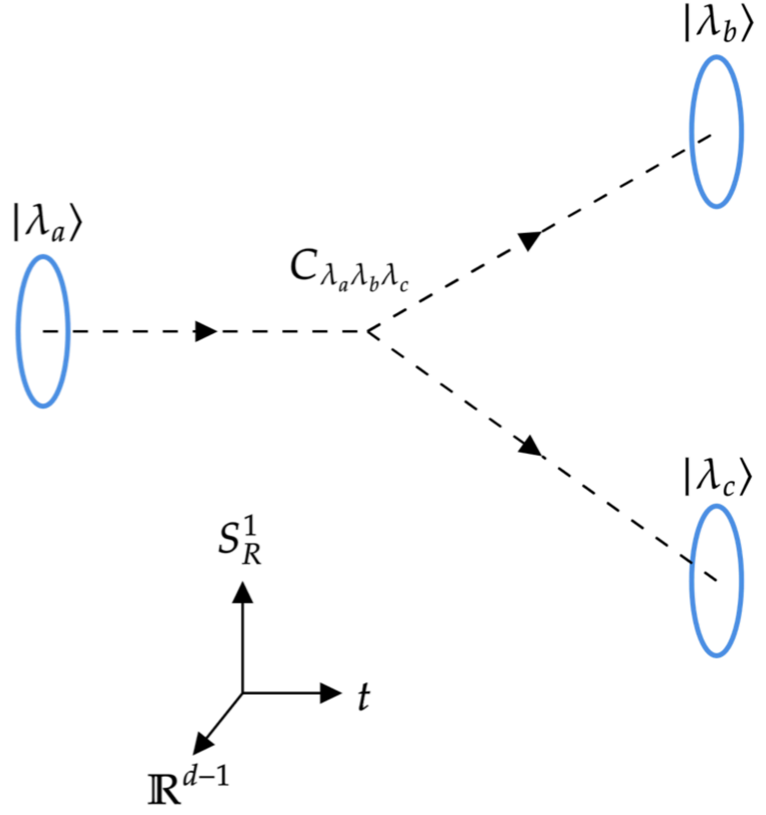
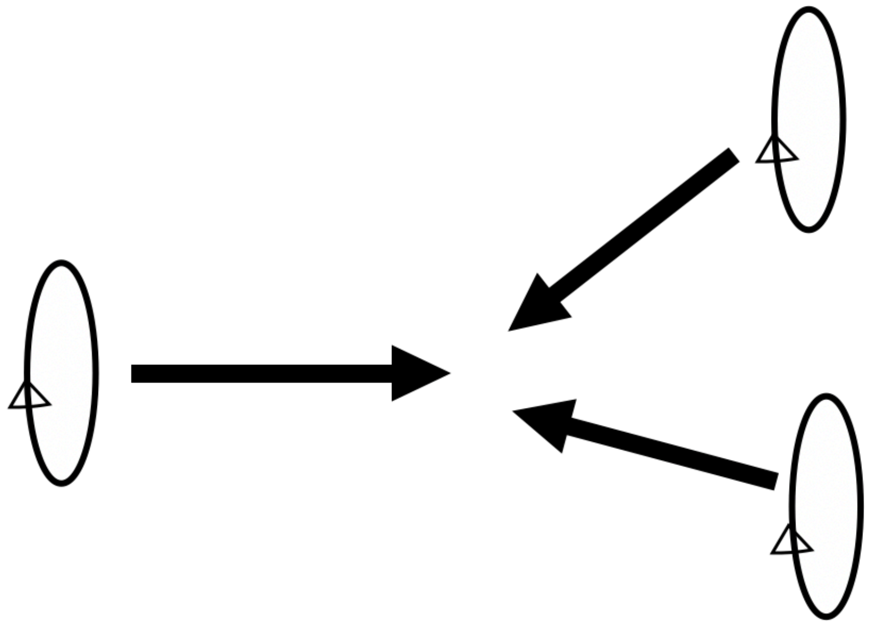
Somehow, this should also describe **local** scattering of closed string states.



The scattering has the same kinematics as scattering of ordinary particles in 2+1 dimensions. Each wrapped string in the state λ_a can be thought of as a complex string field $\Phi_{\lambda_a}(x)$ in 2+1 dimensions and our action is:

$$\sum C_{abc} \int d^3x \; \Phi_{\lambda_a} \Phi_{\lambda_b} \Phi_{\lambda_c} + \text{higher derivative} + c.c.$$

Each Polyakov loop creates a combination of string fields $\sum v_a \Phi_{\lambda_a}$ and then each of them propagates with the usual propagator in 2+1 dimensions and finally they interact through a local s-wave vertex.

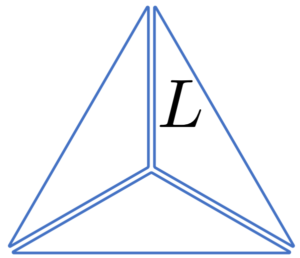


Therefore it should be true that:

$$\frac{e^{-6\pi RL/l_s^2-2\pi RM}}{\left[\eta(\sqrt{q})\right]^d\left[\eta(q)\right]^{d-3}}\left[1-\frac{(d+2)Ml_s^2}{144L}\log q\left[2E_2(q)-E_2(\sqrt{q})\right]\right]$$

$$\stackrel{?}{=}\sum v_a v_b v_c C_{abc}\int d^3x'\,\frac{e^{-M_a|x_1-x'|-M_b|x_2-x'|-M_c|x_3-x'|}}{|x_1-x'||x_2-x'||x_3-x'|}$$

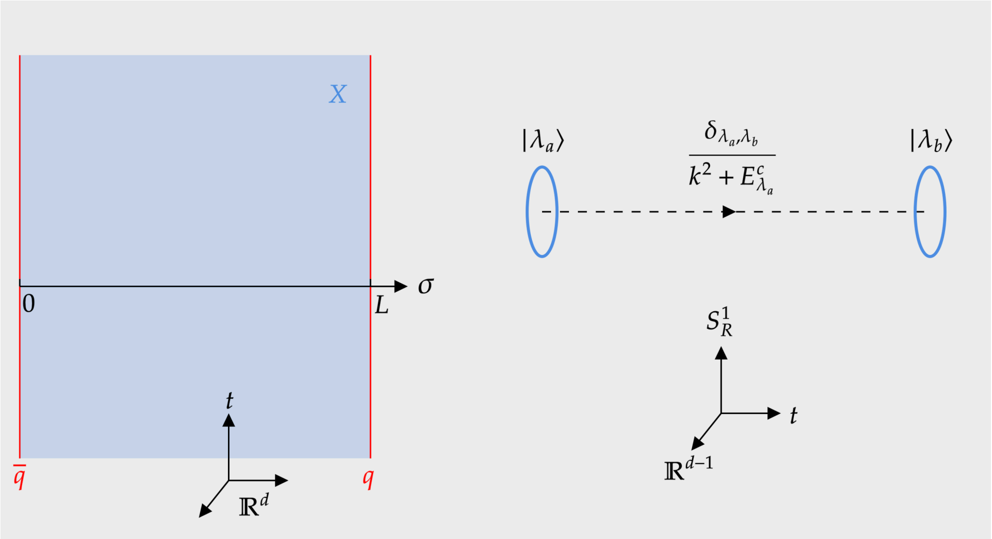
Where the locations $x_{1,2,3}$ are on an equilateral triangle



The amplitudes v_a are known from the Dirichlet-Dirichlet string which turns into a propagator in the closed channel.

For instance

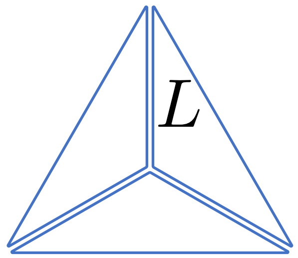
$$v_{\mathbf{0}} = 1 + \frac{(d-1)l_s^2}{48\pi R^2} + O\left(1/R^3\right)$$
$$v_{\mathbf{1}} = \sqrt{d-1}\left[1 + \frac{(d-25)l_s^2}{48\pi R^2} + O\left(1/R^3\right)\right]$$



Therefore the question is really if appropriate interaction vertices can be found.

A technical digression is to take care of the integral

$$\int d^3x' \, \frac{e^{-M_a|x_1-x'|-M_b|x_2-x'|-M_c|x_3-x'|}}{|x_1-x'||x_2-x'||x_3-x'|}$$



The masses of the closed string states are

$$M_a = \frac{2\pi R}{l_s^2} - \frac{d-1}{12R} + 2\frac{n_a}{R} + O(1/R^3) \ .$$

The integral is controlled by a saddle point where the interaction vertex is at the Fermat point (up to small deviations for generic n_a) and the variance is

$$\delta x'^2 \sim \frac{L}{R} l_s^2$$

We can therefore replace the closed string vertex ansatz as

$$\sum v_a v_b v_c C_{abc} \int d^3x' \frac{e^{-M_a|x_1-x'|-M_b|x_2-x'|-M_c|x_3-x'|}}{|x_1-x'||x_2-x'||x_3-x'|}$$

$$\sim e^{-3\frac{LR}{l_s^2}+\frac{\pi L}{R}} \sum_{abc} C_{abc} v_a v_b v_c e^{-\frac{4\pi L}{R}(n_a+n'_b+n''_c)}$$

$$= e^{-3\frac{LR}{l_s^2}+\frac{\pi L}{R}} \sum_{abc} C_{abc} v_a v_b v_c \tilde{q}^{n_a+n_b+n_c} \quad , \quad \tilde{q} \equiv e^{\frac{-4\pi L}{R}}$$

Let us test it for the leading order baryon junction partition function:

$$\frac{e^{-6\pi RL/l_s^2-2\pi RM}}{\left[\eta(\sqrt{q})\right]^d\left[\eta(q)\right]^{d-3}}\left[1-\frac{(d+2)Ml_s^2}{144L}\log q\left[2E_2(q)-E_2(\sqrt{q})\right]\right]$$

We plug d=3 and we use the usual modular properties of the eta function and Eisenstein series to switch from q to $\tilde{q}$. We find that indeed the transformed function can be interpreted as local scattering and we can read out the interaction vertices.

Since there are degeneracies in the closed string to the order we are working, we can extract for the most part only average values for the interaction vertices, but for the first few low-lying states we can extract them unambiguously:

$$C_{\mathbf{000}} = e^{-2\pi MR}\left[1 + \frac{(d+2)Ml_s^2}{36R} + O\left(1/R^2\right)\right]$$

$$C_{\mathbf{001}} =$$

$$e^{-2\pi MR}\left[\frac{d-3}{3\sqrt{d-1}} + \frac{(d+2)(d+21)Ml_s^2}{108\sqrt{d-1}R} + O\left(1/R^2\right)\right]$$

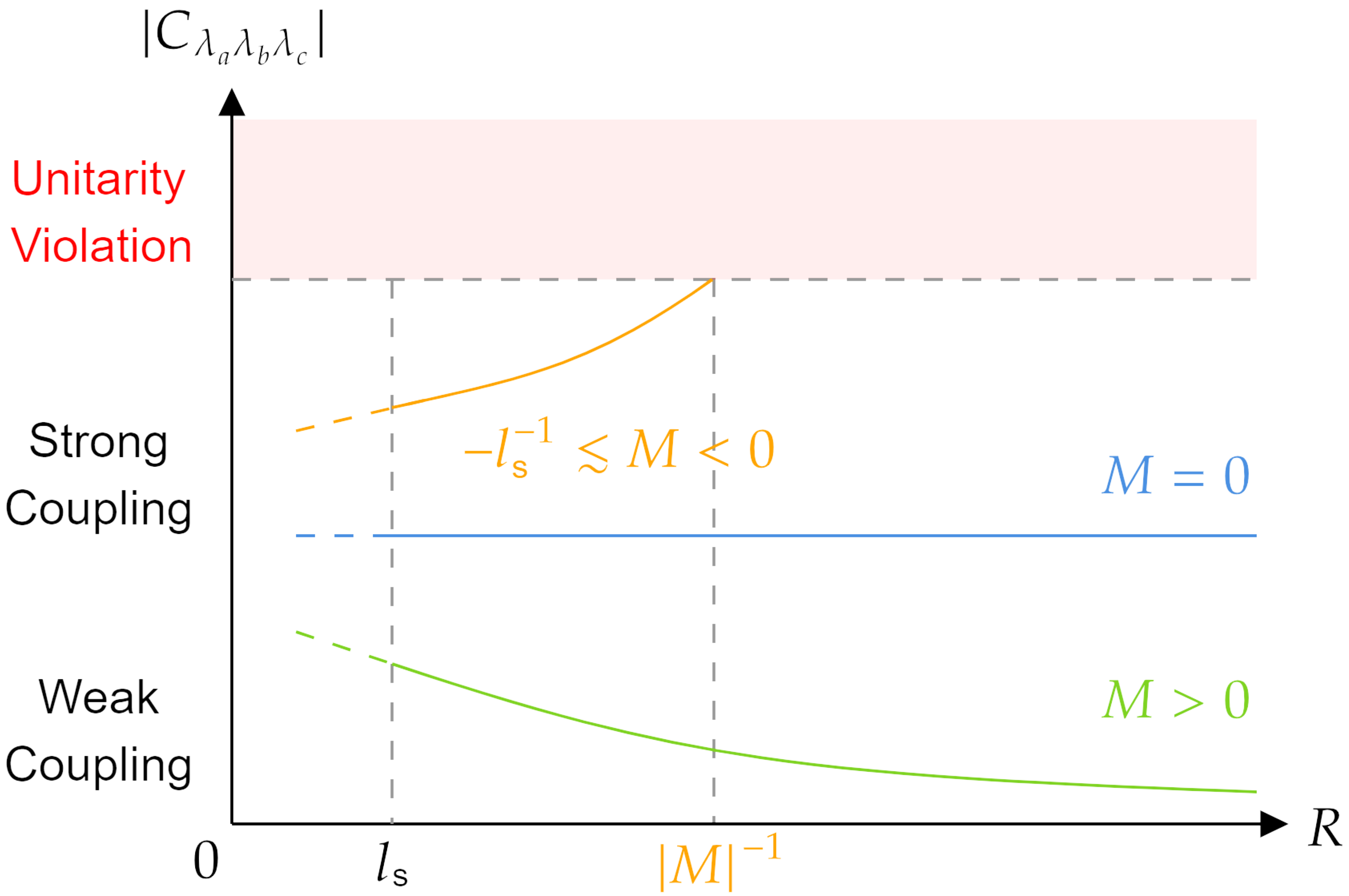
etc.

One qualitative fact which is clear from the baryon junction channel

$$\frac{e^{-6\pi RL/l_s^2 - 2\pi RM}}{\left[\eta(\sqrt{q})\right]^d \left[\eta(q)\right]^{d-3}} \left[1 - \frac{(d+2)Ml_s^2}{144L} \log q \left[2E_2(q) - E_2(\sqrt{q})\right]\right]$$

is that the interaction vertices scale like

$$C_{abc} \sim e^{-2\pi RM}$$



$|C_{\lambda_a\lambda_b\lambda_c}|$
Unitarity
Violation
Strong
Coupling
Weak
Coupling
$-l_s^{-1} \lesssim M < 0$
$M = 0$
$M > 0$
0
l_s
$|M|^{-1}$
R

In our open channel analysis we have seen that large negative M is disallowed because of a perturbative instability. Here we see that we potentially face unitarity violation for any negative M.

On the other hand in the Klebanov-Strassler model, the baryon junction does have negative mass.

[Imamura]

Table IV. The energies of baryon vertices in the KS solution. $E_{\rm junc}$ is the energy of an entire junction of branch length $L = 4$. E_0 represents the contribution of branches, computed as $E_0 = \sum_{i=1}^{3} LT(\theta_i^{\rm f})$. The energy of a vertex is defined by $E_{\rm vertex} = E_{\rm junc} - E_0$. The shaded numbers are the final results obtained by extrapolation.

$(\theta_1^{\rm f}, \theta_2^{\rm f}, \theta_3^{\rm f})$	$n_{\rm mesh}$	10	20	40	80	∞
$(\frac{4\pi}{12}, \frac{4\pi}{12}, \frac{4\pi}{12})$	$E_{\rm junc}$	131.014	131.8324	132.0261	132.0731	132.0886
$E_0 = 132.5661$	$E_{\rm vertex}$	−1.552	−0.7337	−0.5400	−0.5130	−0.4775
$(\frac{3\pi}{12}, \frac{4\pi}{12}, \frac{5\pi}{12})$	$E_{\rm junc}$	128.126	128.9356	129.1274	129.1740	129.1894
$E_0 = 129.6061$	$E_{\rm vertex}$	−1.480	−0.6705	−0.4787	−0.4321	−0.4167
$(\frac{2\pi}{12}, \frac{5\pi}{12}, \frac{5\pi}{12})$	$E_{\rm junc}$	122.473	123.3349	123.5420	123.5927	123.6095
$E_0 = 123.8908$	$E_{\rm vertex}$	−1.418	−0.5559	−0.3488	−0.2981	−0.2813

Therefore, since there is a lot of data showing that long strings exist in Yang Mills theory and they seem to be good one-particle states, we are compelled to make a prediction that $M > 0$.

The other thing is that we have in d=3 with $M = 0$ we find a strange selection rule saying that

$$C_{abc} = 0 \ \ \text{for} \ \ n_a + n_b + n_c \neq 0 \bmod 2$$

This strange selection rule is due to a T-duality like symmetry that exchanges N and D b.cs. on the baryon junction. We find that this symmetry persists even to higher order than we exhibited here.

Some obvious open questions:

1. We were able to obtain some information on string scattering pretty much out of nothing. How far can this be pushed?
2. To determine M in Yang Mills theory.
3. It is plausible that the same estimate $C_{abc} \sim e^{-2\pi RM}$ holds for long strings in mesons or glueballs. R is then related to the spin.
4. Go to higher order, and extend beyond the equilateral case. Test if local s-wave scattering still works.
5. We found a perturbative instability for large negative M and a potential problem with unitarity for any negative M. What are the end points of these instabilities? Deconfinement?
6. What happens for qqq which are a triangle with an angle above 120 degrees?

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THANK YOU!