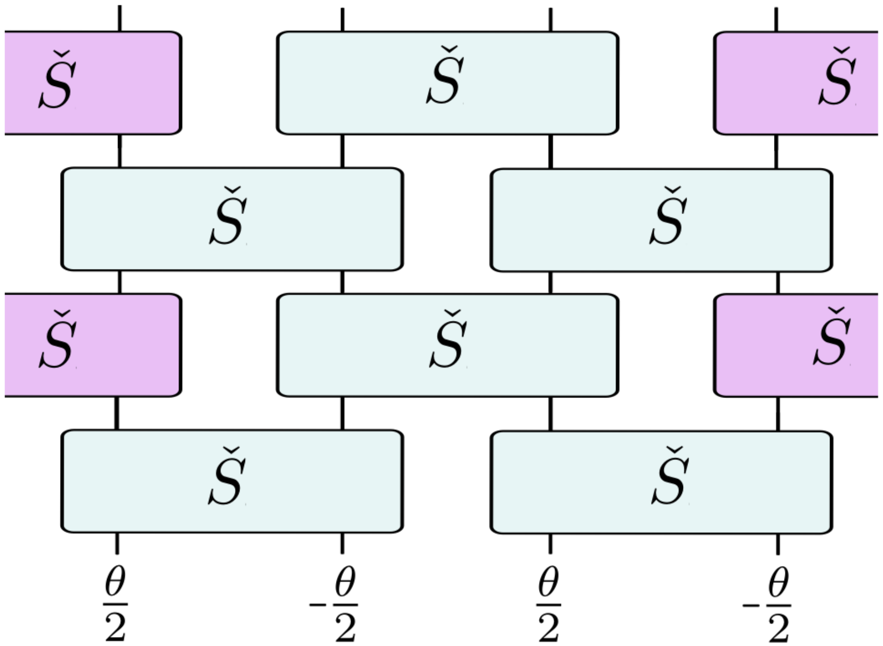
Fermionic bricks in the wall



IPMU – Tokyo– 30 Sept 2024

UNIVERSITEIT VAN AMSTERDAM

Kareljan Schoutens

Institute for Theoretical Physics

QuSoft

with

Pietro Richelli

Alberto Zorzato

QuSoft

Research Center for Quantum Software

Quantum Delta NL

Congratulation to Pietro Richelli for his QuSoft Master Certificate

Alberto Zorzato

@ TopoAnyons

Les Houches

April 2024

Bricks in the wall

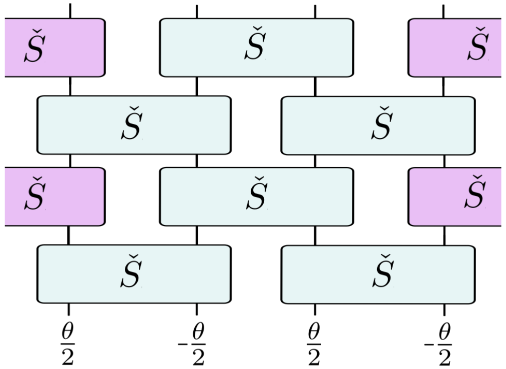
Arches NP
Utah

Brick wall quantum circuits

with a global fermionic symmetry

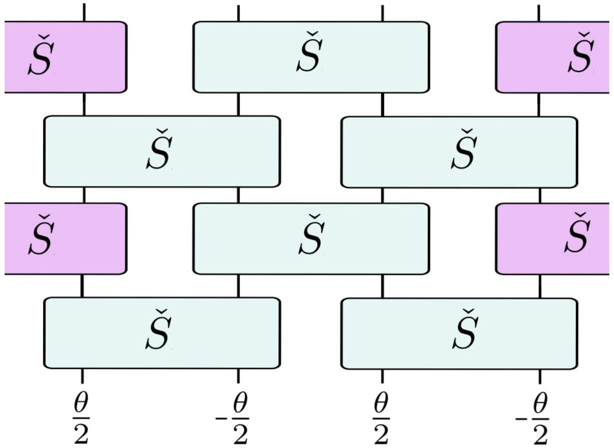
with P. Richelli, A. Zorzato

arXiv:2402.18440, SciPost Phys 17, 087 (2024)



- **Context**

  non-equilibrium quantum dynamics and digital quantum simulation



- **Origins**

  fermionic brick from factorizable particle scattering in 1+1D

- **Math**

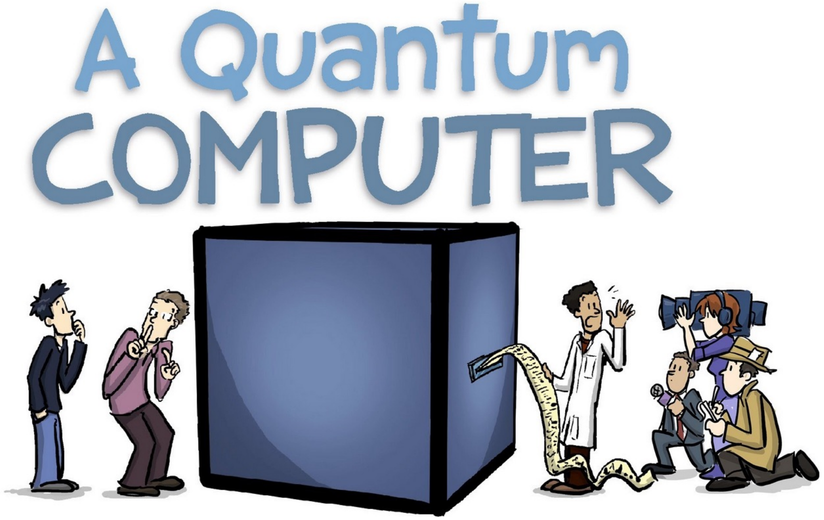
  YBE and integrability, free fermionic (matchgate) form of Š

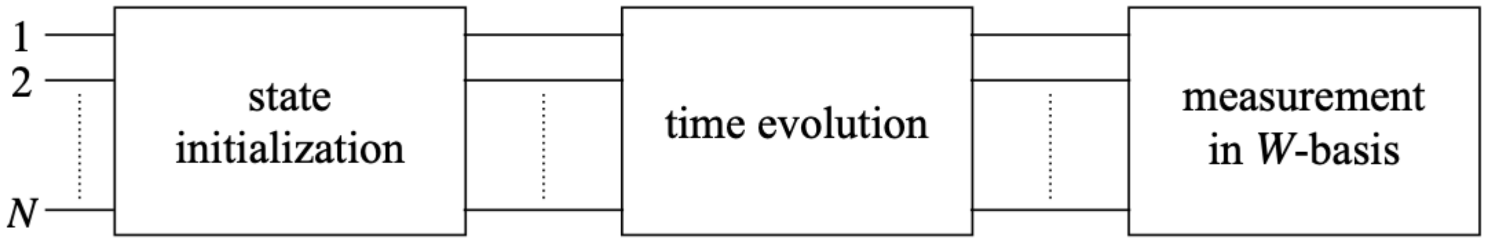
- **Phys**

  Hamiltonian limit (generalized Kitaev chains) and quantum dynamics

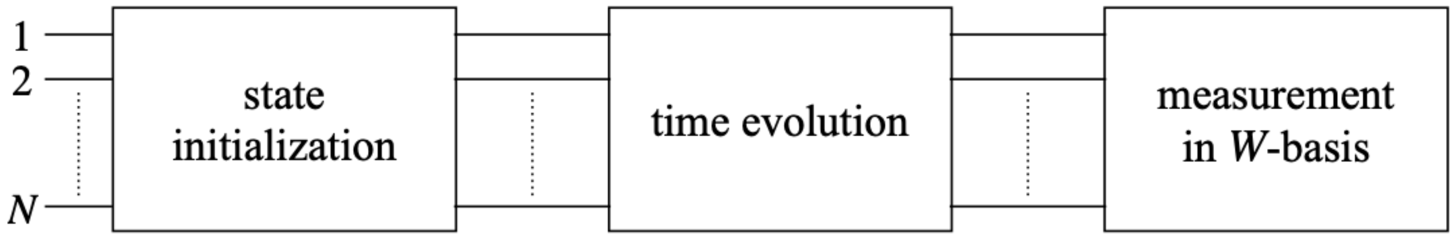
Non-equilibrium quantum dynamics and digital quantum simulation

Context

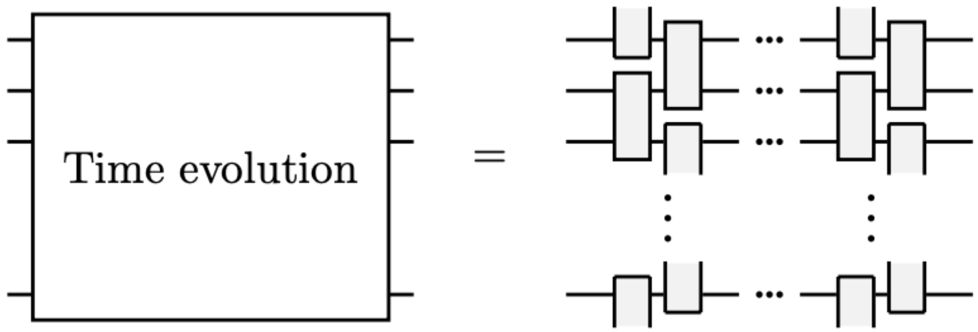




Non-equilibrium quantum dynamics and digital quantum simulation



Time evolution implemented using a **brick wall** type quantum circuit:



figures from arXiv:2208.00576

Non-equilibrium quantum dynamics and digital quantum simulation

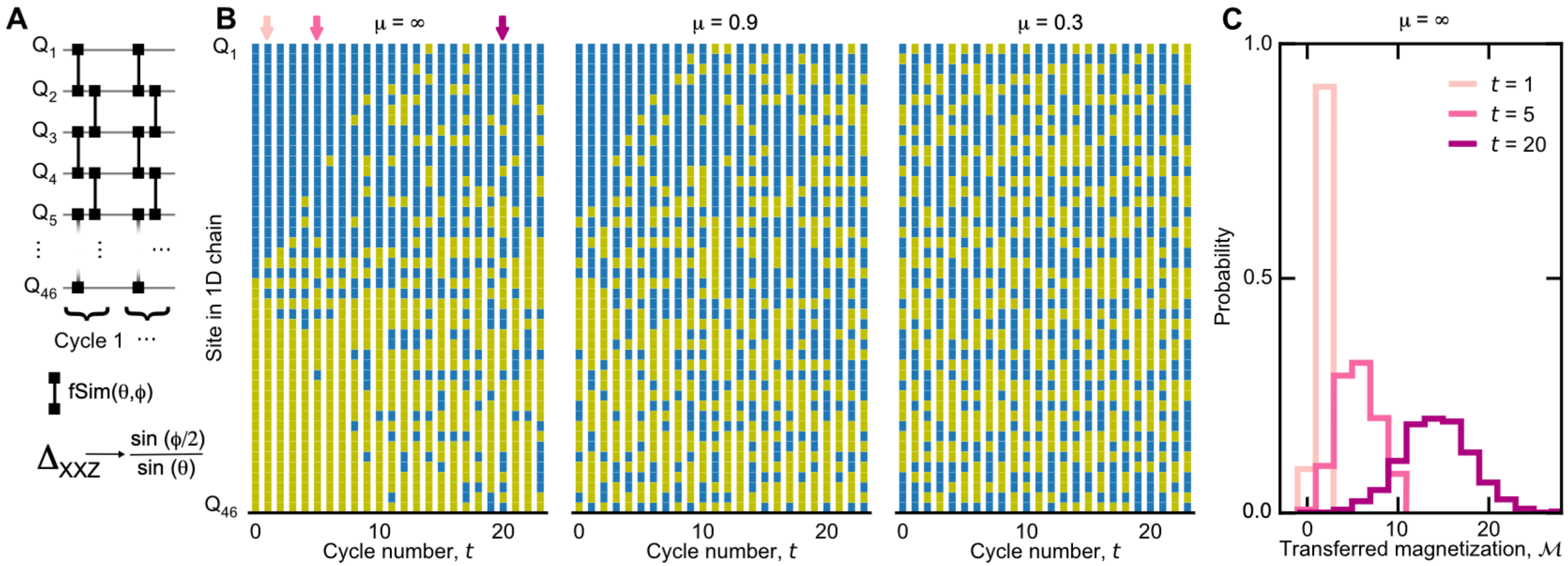


FIG. 1. **Domain wall relaxation in the Heisenberg XXZ spin chain.** (**A**) Schematic of the unitary gate sequence used in this work, where fSim gates are applied in a Floquet scheme on a 1D chain of $N_Q = 46$ qubits. (**B**) Relaxation dynamics as a function of site and cycle number for $\mu = \infty, 0.9$, and 0.3 for initially prepared domain-wall states with $2\langle S^z \rangle = \pm \tanh \mu$. (**C**) Histogram showing the probability distribution of transferred magnetization after $t = 1$, 5 and 20 cycles (arrows in **B**) for $\mu = \infty$.

Google Quantum AI and collaborators, arXiv:2306.09333

Global fermionic symmetry

Origins

Reading qubit states as

$|0> \rightarrow |\uparrow>, \quad |1> \rightarrow |\downarrow>$

natural symmetry is *SU(2)*, as in the XXX chain or *U(1)* as in an XXZ chain.

Reading qubit states as

$|0> \rightarrow |b>, \quad |1> \rightarrow |f>$

natural symmetry is fermionic.

Q: Do we have 2-qubit gates with fermionic symmetry?

Global fermionic symmetry

Clue: interpret 2-body S-matrices of 1+1D particle theories as 2-qubit gates.

If that particle theory is described by integrable QFT, these S-matrices will satisfy a Yang-Baxter Equation (YBE).

If that particle theory is supersymmetric, these S-matrices will possess a global fermionic symmetry.

If that particle theory arises as a perturbed superconformal field theory (sCFT), both these properties are possible.

Global fermionic symmetry

SUPERSYMMETRY AND FACTORIZABLE SCATTERING

Kareljan SCHOUTENS*

Institute for Theoretical Physics, State University of New York at Stony Brook, Stony Brook, NY 11794-3840, USA

Received 20 February 1990

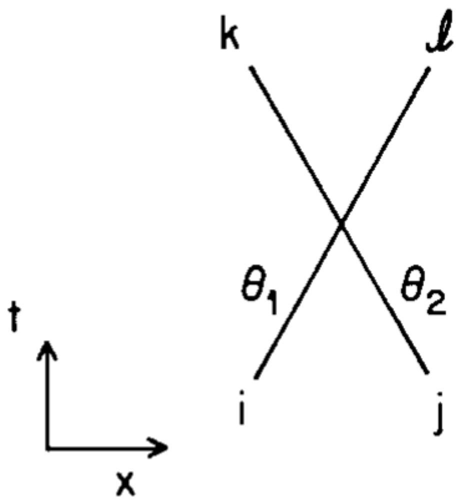
We analyze supersymmetric particle theories in 1 + 1 dimensions that exhibit factorizable scattering. We propose the general form $\hat{S} = \hat{S}_{BF}\hat{S}_{B}$ for the S-matrix, where $\hat{S}_{B}$ is a purely bosonic S-matrix and $\hat{S}_{BF}$ describes the mixing of bosonic and fermionic particles. We derive a general expression for $\hat{S}_{BF}$.

Nucl. Phys. B344, 665 (1990)

Global fermionic symmetry

Nucl. Phys. B344, 665 (1990)

S-matrix for particle scattering in 1+1D



$$|A_i(\theta_1)A_j(\theta_2)\rangle_{\text{in}} = \hat{S}_{ij}^{kl}(\theta_1-\theta_2)|A_k(\theta_2)A_l(\theta_1)\rangle_{\text{out}},$$

Global fermionic symmetry

Nucl. Phys. B344, 665 (1990)

Find 2-particle scattering matrix that solves conditions SUSY and YBE

SUSY :

$$\left[(\rho x)^{-1}Q_1 + \rho x Q_2\right]\hat{S}(\theta) = \hat{S}(\theta)\left[\rho x Q_1 + (\rho x)^{-1}Q_2\right],$$

$$\left[\rho^{-1}x\overline{Q}_1 + \rho x^{-1}\overline{Q}_2\right]\hat{S}(\theta) = \hat{S}(\theta)\left[\rho x^{-1}\overline{Q}_1 + \rho^{-1}x\overline{Q}_2\right],$$

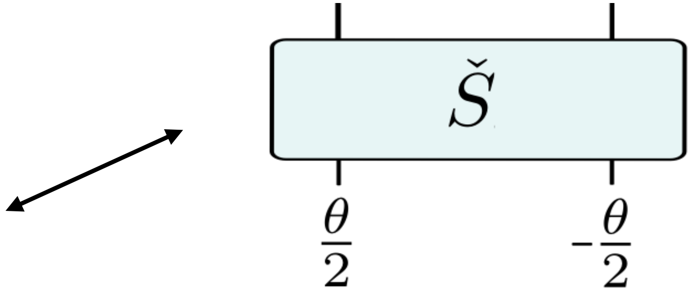
$$\theta = \theta_1 - \theta_2,\ x = \exp(\theta/4),\ \rho = (m_1/m_2)^{1/4}$$

YBE :

$$\hat{S}_{i_1 i_2}^{k_2 k_1}(\theta_{12})\hat{S}_{k_1 i_3}^{k_3 j_1}(\theta_{13})\hat{S}_{k_2 k_3}^{j_3 j_2}(\theta_{23}) = \hat{S}_{i_2 i_3}^{k_3 k_2}(\theta_{23})\hat{S}_{i_1 k_3}^{j_3 k_1}(\theta_{13})\hat{S}_{k_1 k_2}^{j_2 j_1}(\theta_{12}).$$

Global fermionic symmetry

From my 1990 paper:



$$\check{\mathbf{S}}(\alpha, \gamma, \theta)$$

→ Most general supersymmetric 2-body S-matrices satisfying (graded) YBE, with

$\boldsymbol{\alpha}$ – coupling strength

$\boldsymbol{\gamma}$ – log of particle mass ratio

$\boldsymbol{\theta}$ – difference of particle rapidities

Global fermionic symmetry

$$\check{\mathbf{S}}(\theta) = f(\theta)\begin{pmatrix} 1-t\,\tilde{t} & 0 & 0 & t+\tilde{t} \\ 0 & 1+t\,\tilde{t} & t-\tilde{t} & 0 \\ 0 & -t+\tilde{t} & 1+t\,\tilde{t} & 0 \\ -t-\tilde{t} & 0 & 0 & 1-t\,\tilde{t} \end{pmatrix} + g(\theta)\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$t = \tanh\left[\frac{\theta+\gamma}{4}\right], \qquad \tilde{t} = \tanh\left[\frac{\theta-\gamma}{4}\right], \qquad \gamma = \log\left(\frac{m_i}{m_j}\right)$$

$$f(\theta) = \frac{\alpha}{2i}\sqrt{m_i m_j}\,\frac{\cosh(\theta/2)+\cosh(\gamma/2)}{\cosh(\theta/2)\sinh(\theta/2)}\,g(\theta)$$

Global fermionic symmetry

- $\check{\mathbf{S}}(\alpha, \gamma, \theta)$ matched with specific perturbed sCFT, later confirmed by TBA analysis

M. Moriconi, KjS - 1995

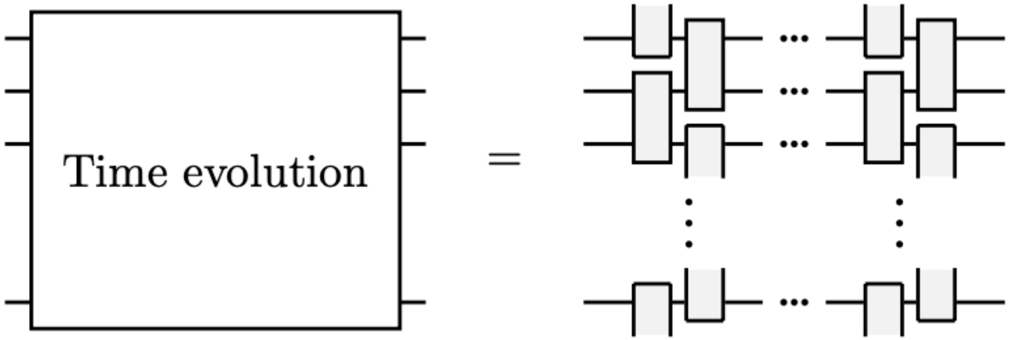
- $\check{\mathbf{S}}(\alpha, \gamma, \theta)$ satisfies `free fermion' (FF) or `matchgate' property, as a consequence we can write

$$\check{\mathbf{S}}_{i,i+1}(\alpha, \gamma, \theta) = \exp[i\mathbf{E}_{i,i+1}].$$

quadratic in fermion operators

Integrability and FF structure

Math



If the 2-qubit `brick' satisfies a Yang-Baxter Equation (YBE) the Floquet dynamics given by the quantum circuit is integrable.

Gritsev, Polkolnikov, 2017; Vanicat, Zadnik, Prosen, 2018; Miao, Gritsev, Kurlov, 2022; Maruyoshi et al, 2022; ...

Integrability and FF structure

The FF structure of $\check{\mathbf{S}}(\alpha, \gamma, \theta)$ leads to a FF form of the circuit unitary,

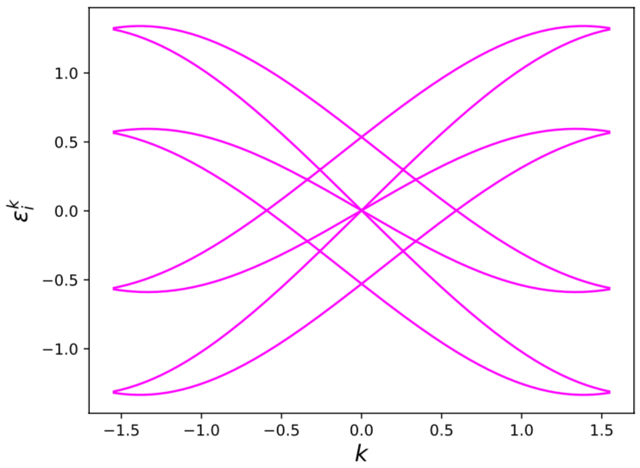
$$\mathbf{U}_{\mathbf{F}}^{\mathbf{k}}(\alpha, \gamma, \theta) = \exp\left[i \sum_i \epsilon_i^k (\eta_i^{k\,\dagger} \eta_i^k - \frac{1}{2})\right]$$

We found close-to-closed form results for the dispersions ϵ_i^k .

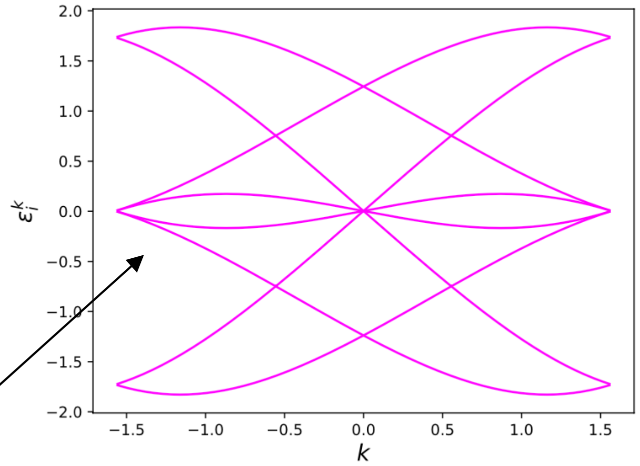
Integrability and FF structure

dispersions for

α=1, γ=1



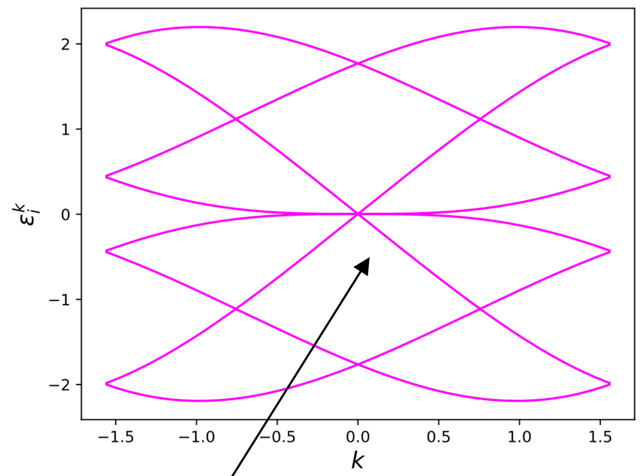
ϑ=0.3



ϑ=0.7001

gap closing for

$k= \pm\pi/2$



ϑ=1

cubic dispersion

near k=0

Hamiltonian limit

Phys

For $\vartheta \to 0$, extract Hamiltonian $\mathbf{H}_\gamma(\alpha, \gamma)$ via

$$\mathbf{U_F}(\theta) = \exp[i\mathbf{E}(\theta)] \overset{\theta \to 0}{=} \mathbf{U_F}(0) + i\mathbf{U_F}(0)\mathbf{H}_\gamma\, \theta + o(\theta^2)$$

This gives a generalized Kitaev chain Hamiltonian $\mathbf{H}_\gamma(\alpha, \gamma)$ with hopping and condensate terms through nnnn (next-next-next nearest neighbors)

Hamiltonian limit

Exploiting, again, FF structure we found 1-particle bands

$$(\epsilon_\gamma^k)_{1,2} = \frac{1}{\sqrt{2}}\sqrt{\nu_0 + \nu_1 \cos(2k) \pm \sqrt{\sum_{j=0}^{6} \mu_j \cos\left(2j\, k\right)}}$$

$$
\begin{aligned}
\nu_0 &= \frac{\operatorname{sech}^4\left(\frac{\gamma}{2}\right)}{\alpha^2}\left(1+2\cosh(\gamma)\right)+\operatorname{sech}^2\left(\frac{\gamma}{2}\right),\\
\nu_1 &= \frac{\operatorname{sech}^4\left(\frac{\gamma}{2}\right)}{\alpha^2}-\operatorname{sech}^2\left(\frac{\gamma}{2}\right),\\
\mu_0 &= \frac{\operatorname{sech}^6\left(\frac{\gamma}{2}\right)}{\alpha^4}\left(8\cosh(\gamma)\right)\\
&\quad+\frac{\tanh^2\left(\frac{\gamma}{2}\right)\operatorname{sech}^{12}\left(\frac{\gamma}{2}\right)}{32\alpha^2}\Big(300-193\cosh(\gamma)+162\cosh(2\gamma)-15\cosh(3\gamma)+2\cosh(4\gamma)\Big),\\
\mu_1 &= \frac{8\operatorname{sech}^6\left(\frac{\gamma}{2}\right)}{\alpha^4}\\
&\quad-\frac{\tanh^2\left(\frac{\gamma}{2}\right)\operatorname{sech}^{12}\left(\frac{\gamma}{2}\right)}{16\alpha^2}\Big(163-120\cosh(\gamma)+92\cosh(2\gamma)-8\cosh(3\gamma)+\cosh(4\gamma)\Big),\\
\mu_2 &= \frac{\tanh^4\left(\frac{\gamma}{2}\right)\operatorname{sech}^{10}\left(\frac{\gamma}{2}\right)}{16\alpha^2}\Big(93+4\cosh(\gamma)+31\cosh(2\gamma)\Big),\\
\mu_3 &= -\frac{\tanh^4\left(\frac{\gamma}{2}\right)\operatorname{sech}^{10}\left(\frac{\gamma}{2}\right)}{2\alpha^2}\Big(21-12\cosh(\gamma)+7\cosh(2\gamma)\Big),\\
\mu_4 &= \frac{\tanh^4\left(\frac{\gamma}{2}\right)\operatorname{sech}^{10}\left(\frac{\gamma}{2}\right)}{8\alpha^2}\Big(45-44\cosh(\gamma)+15\cosh(2\gamma)\Big),\\
\mu_5 &= -\frac{4\tanh^8\left(\frac{\gamma}{2}\right)\operatorname{sech}^{6}\left(\frac{\gamma}{2}\right)}{\alpha^2},\\
\mu_6 &= \frac{\tanh^8\left(\frac{\gamma}{2}\right)\operatorname{sech}^{6}\left(\frac{\gamma}{2}\right)}{2\alpha^2}.
\end{aligned}
$$

Hamiltonian limit

Exploiting, again, FF structure find 1-particle bands

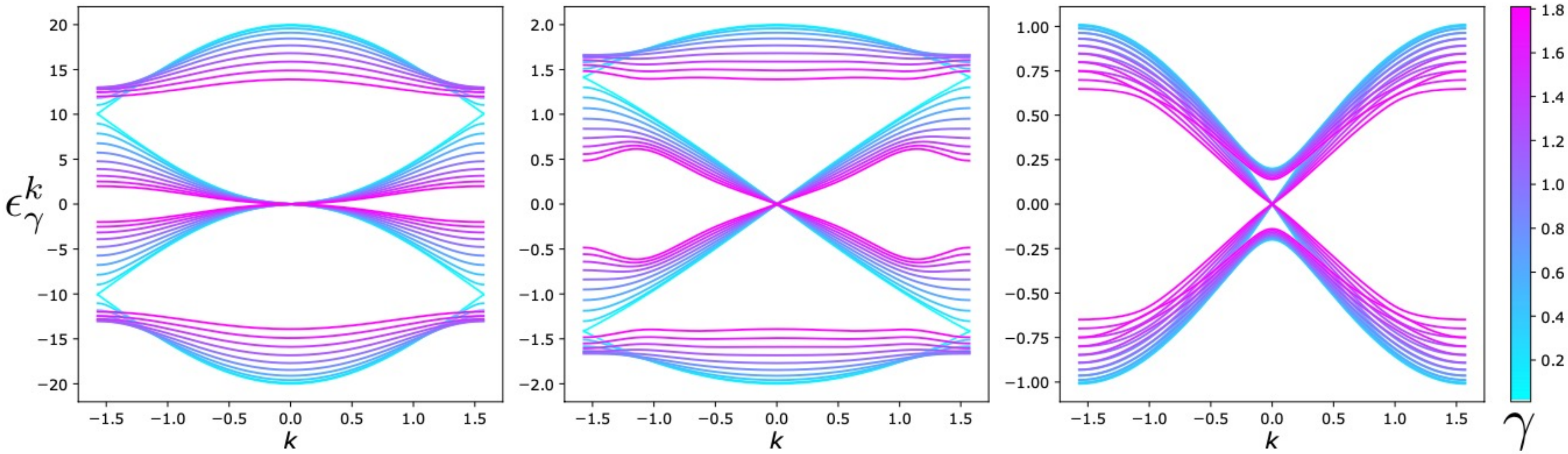
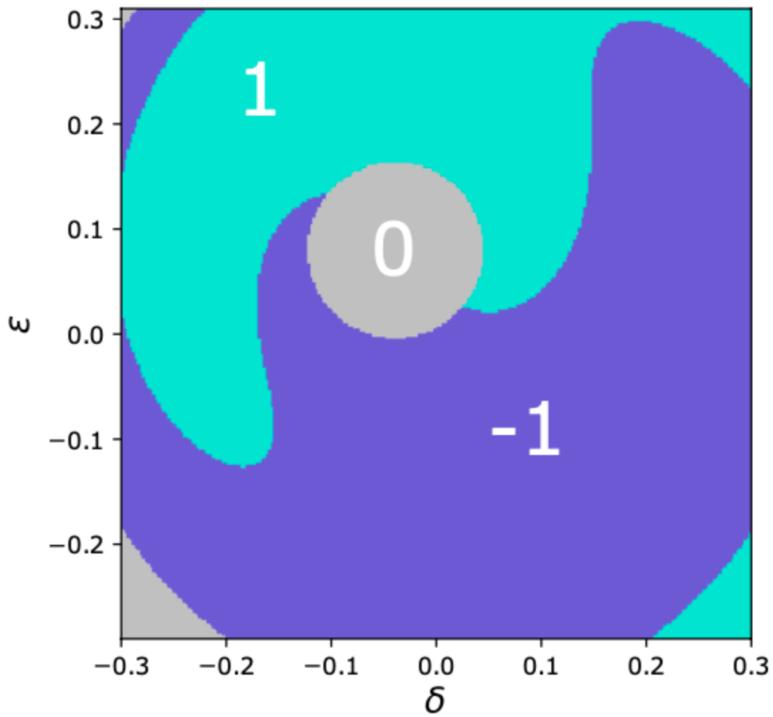


Figure 2: Dispersions $\pm\epsilon_\gamma^k$ for $\mathbf{H}_\gamma$. From the left: $\alpha = 0.1, 1, 10$.

Global fermionic symmetry guarantees that $\mathbf{H}_\gamma(\alpha, \gamma)$ is critical for all α, γ (!)

Hamiltonian limit

Perturbing $\mathbf{H}_\gamma(\alpha, \gamma)$ in general opens a gap and leads to topological phases with localized Majorana zero-modes localized at edges or defects



$\alpha = 5, \gamma = 3, \epsilon_1 = -\epsilon_2$

Quantum Delta NL

Quantum dynamics

Phys

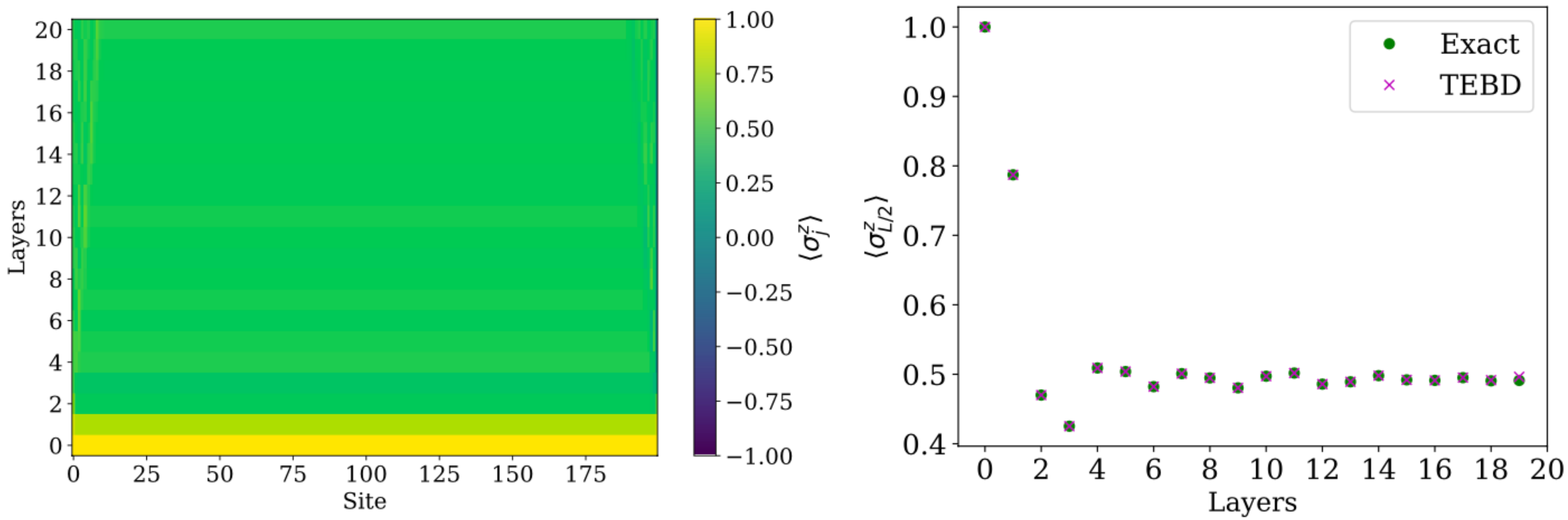
Quantum dynamics constrained by FF structure (GGE) but rich in its dependence on, in particular, the mass ratio γ.

We explored quench dynamics, focussing on observable $< \sigma^z_j > = 2\, n_j - 1$.

We compare analytical reasoning (k-space analytics and GGE), numerics (time evolving block decimation, TEBD) and preliminary implementations on IBM Q

Quantum dynamics, I

Warm-up: quench dynamics from state *bbbb*



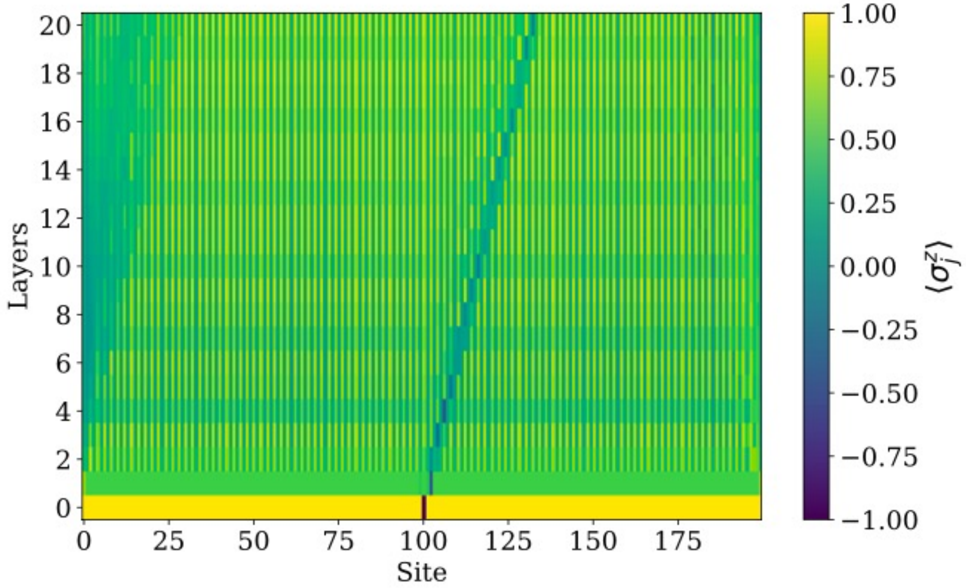
(a) (b)

Figure 9: (a) $\langle\sigma^z_j\rangle$ TEBD results for an OBC system with $L = 200$, $\alpha = 1$, $\gamma = 0$, $\theta = 0.5$ (b) Local magnetization $\langle\sigma^z_j\rangle$ with $j = L/2$ as a function of number applied $\mathbf{U_F}$ layers, exact results vs. TEBD.

→ Dephasing leads to equilibration of the $<\sigma^z{}_j>$, to a value analytically fixed by the GGE

Quantum dynamics, II

Drift: quench dynamics from state ... *bbfbb* ...



$\alpha = 1, \gamma = 3, \theta = 3.$

→ for unequal masses (non-zero γ):

- modulation of the equilibrium < σ^z_j >,
- drift velocity v_d of the seed, for $\gamma >> \theta$

$$v_d = 2\tanh(\gamma/2) = 2\frac{m_1 - m_2}{m_1 + m_2}$$

Edge modes away from criticality

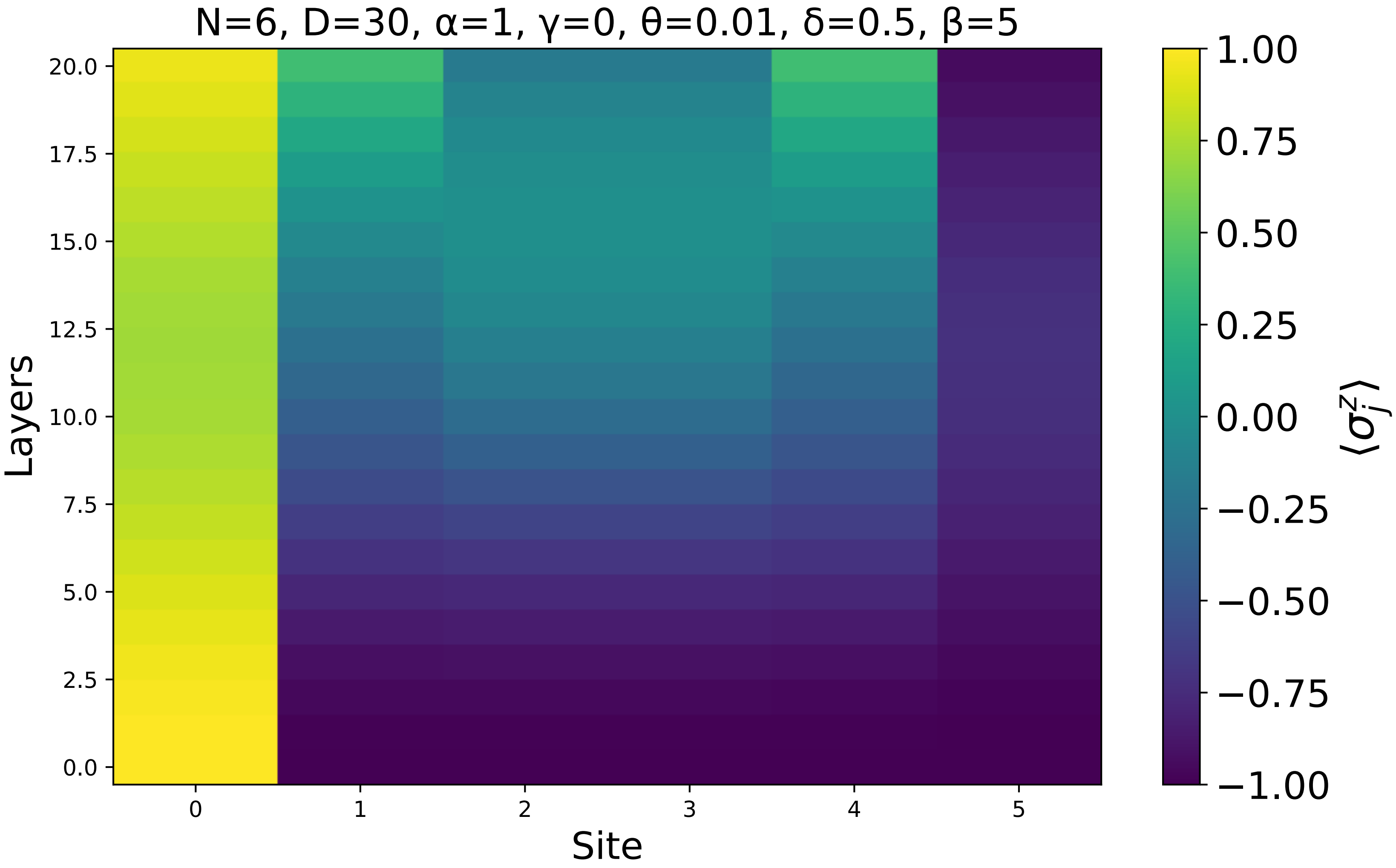
- parameter δ that moves $\check{\mathbf{S}}(\alpha, \gamma, \theta)$ off criticality
- edge chemical potential β away from susy values

- breaks both global fermionic symmetry and criticality
- opens up topological phase and stable edge modes

- study via quench from *bfff*...

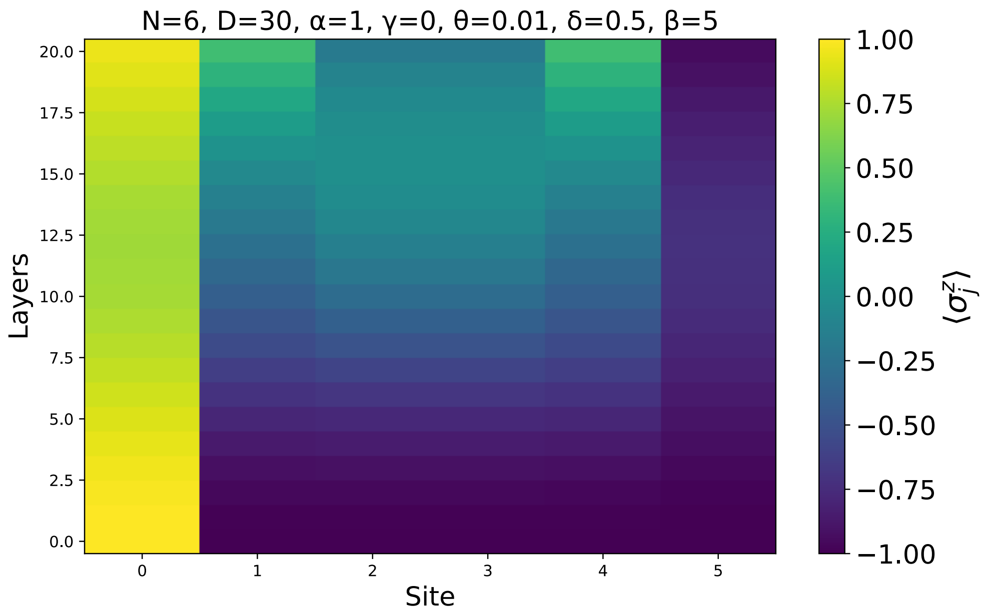
Quantum dynamics, III

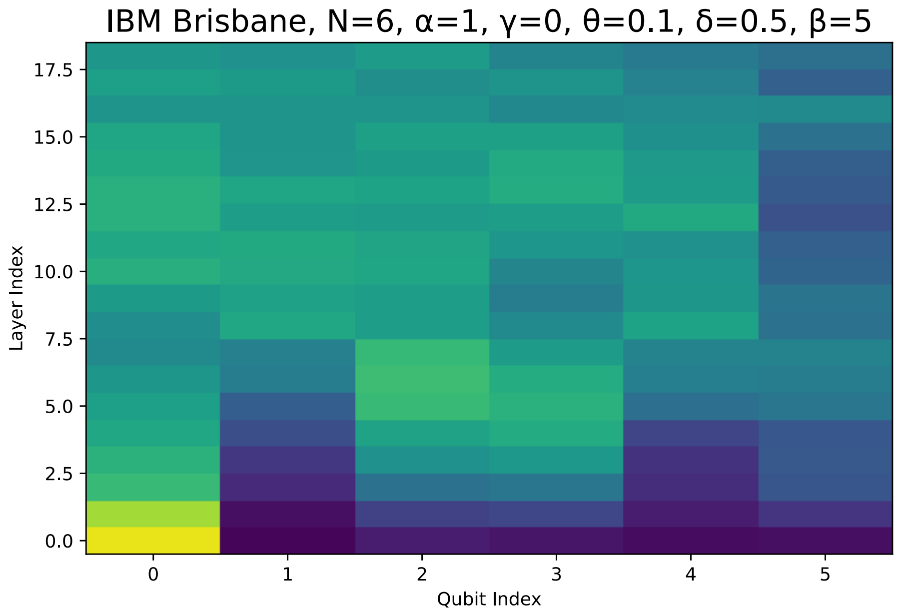
Edge modes away from criticality

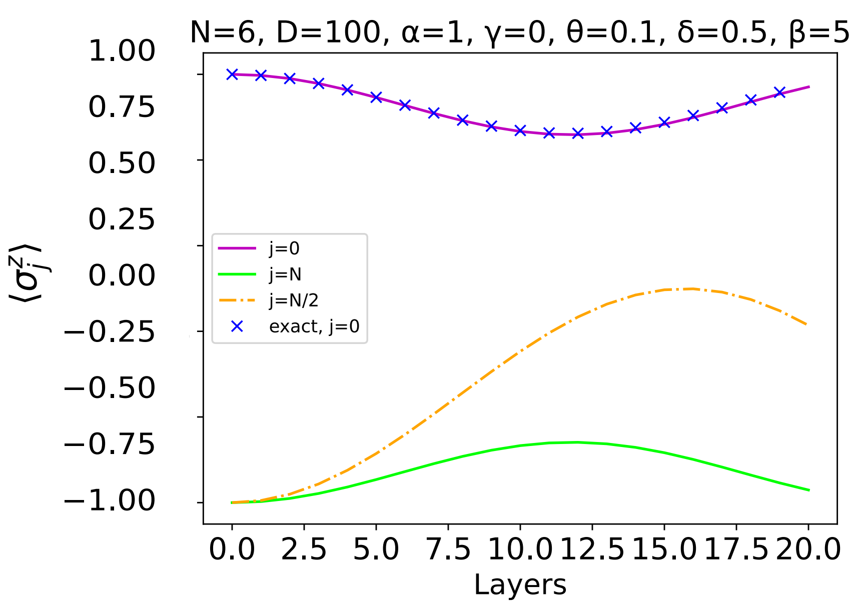


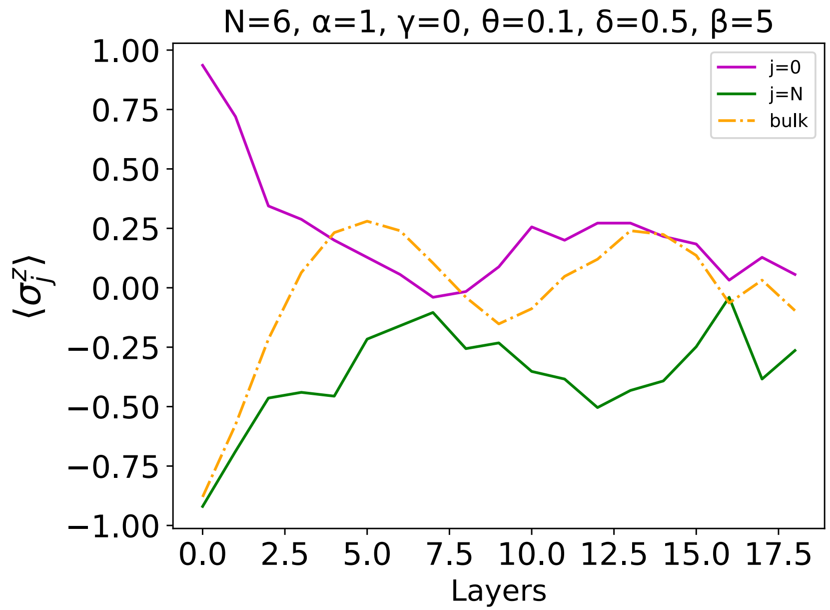
TEBD: quench from *bfff*...

TEBD vs IBM Eagle (preliminary, no Error Mitigation)









Edge modes away from criticality

- for γ=0 `standard' Kitaev chain edge modes

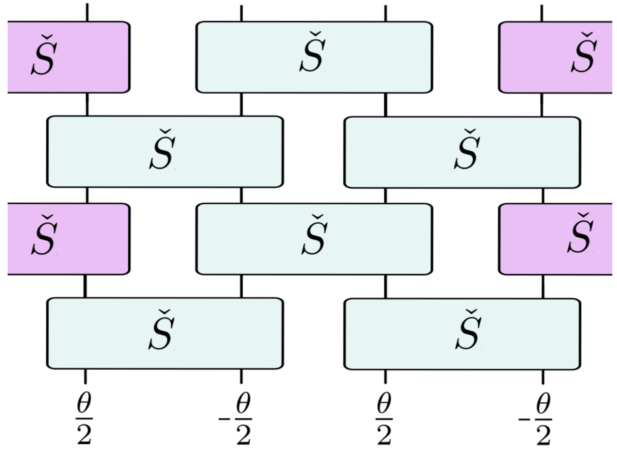
  [see also Google AI paper on noise resilient edge modes for kicked Ising model: Mi et al., Science 378, 785–790 (2022)]

- what for non-zero γ?
- QI protocols such as edge-to-edge transfer

Quantum dynamics beyond Hamiltonian limit

away from Hamiltonian limit

- probe special features of spectral structure
  (gap closings and cubic dispersion)
- topological features in quantum cellular automata



- **Conclusion**

  → this is class of circuits with high degree of analytical control yet rich and non-trivial dynamics

- **Outlook**

  → explore quantum dynamics and quantum control protocols in presence of perturbations, edges/defects, intermediate measurements, etc.