Driven (Dirac) systems and novel quantum anomalous states

Takashi Oka
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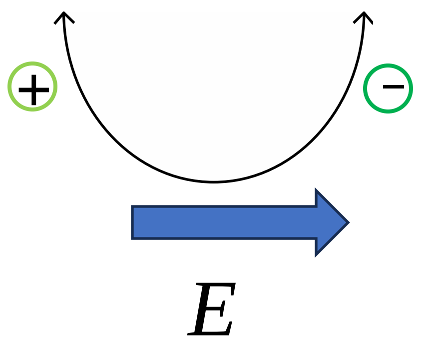
Non-Hermitian physics from dynamics of a closed system



Flat bands from periodic driving

Okazaki, Okumura, Takayoshi, TO, arXiv ’24

Kitamura, TO, arXiv ’24

Outline

1. Review of the work by Heisenberg, Euler, Schwinger

   non-adiabatic Berry phase, imaginary time method

2. Tunneling breakdown from a SPT phase

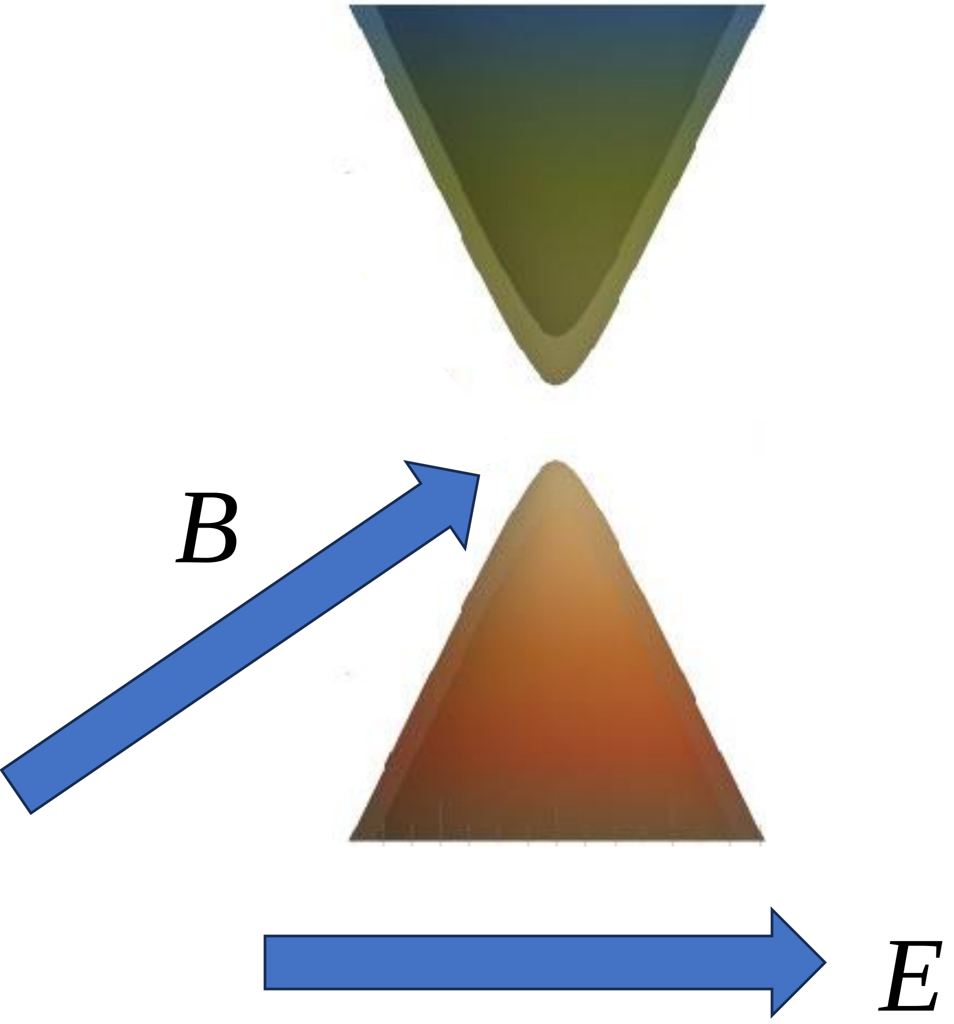
   Okazaki, Okumura, Takayoshi, TO, arXiv ’24

3. Floquet state in AC-magnetic fields

   π-Landau states, chiral anomaly induced homodyne

   Kitamura, TO, arXiv ’24

1. Review of the work by Heisenberg, Euler, Schwinger



Free Dirac fermions in static electro-magnetic fields

$$\mathcal{L} = \bar{\psi}\left[i\gamma^\mu(\partial_\mu + ieA_\mu) - m\right]\psi$$

$$\mathbf{E} = -\partial_t \mathbf{A} + \nabla A^0$$
$$\mathbf{B} = \nabla \times \mathbf{A}$$

Heisenberg-Euler's work (1936)

Dirac, sea (1928)

Heisenberg, Euler (1936)
Weisskopf (1936)

Folgerungen aus der Diracschen Theorie des Positrons.

Von **W. Heisenberg** und **H. Euler** in Leipzig.

Mit 2 Abbildungen. (Eingegangen am 22. Dezember 1935.)

Aus der Diracschen Theorie des Positrons folgt, da jedes elektromagnetische Feld zur Paarerzeugung neigt, eine Abänderung der Maxwellschen Gleichungen des Vakuums. Diese Abänderungen werden für den speziellen Fall berechnet, in dem keine wirklichen Elektronen und Positronen vorhanden sind, und in dem sich das Feld auf Strecken der Compton-Wellenlänge nur wenig ändert. Es ergibt sich für das Feld eine Lagrange-Funktion:

$$\mathfrak{L} = \frac{1}{2}(\mathfrak{E}^2-\mathfrak{B}^2) + \frac{e^2}{hc}\int_0^\infty e^{-\eta}\frac{d\eta}{\eta^3}\left\{ i\eta^2(\mathfrak{E}\mathfrak{B})\cdot\frac{\cos\left(\frac{\eta}{|\mathfrak{E}_k|}\sqrt{\mathfrak{E}^2-\mathfrak{B}^2+2i(\mathfrak{E}\mathfrak{B})}\right)+\text{konj}}{\cos\left(\frac{\eta}{|\mathfrak{E}_k|}\sqrt{\mathfrak{E}^2-\mathfrak{B}^2+2i(\mathfrak{E}\mathfrak{B})}\right)-\text{konj}} + |\mathfrak{E}_k|^2 + \frac{\eta^2}{3}(\mathfrak{B}^2-\mathfrak{E}^2)\right\}.$$

$\mathfrak{E}, \mathfrak{B}$ Kraft auf das Elektron.

$$|\mathfrak{E}_k| = \frac{m^2c^3}{e\hbar} = \frac{1}{„137“}\,\frac{e}{(e^2/mc^2)^2} = \text{„Kritische Feldstärke“}.$$

threshold (critical) field
= Schwinger limit

Ihre Entwicklungsglieder für (gegen $|\mathfrak{E}_k|$) kleine Felder beschreiben Prozesse der Streuung von Licht an Licht, deren einfachstes bereits aus einer Störungsrechnung bekannt ist. Für große Felder sind die hier abgeleiteten Feldgleichungen von den Maxwellschen sehr verschieden. Sie werden mit den von Born vorgeschlagenen verglichen.

pair creation of electron-positrons

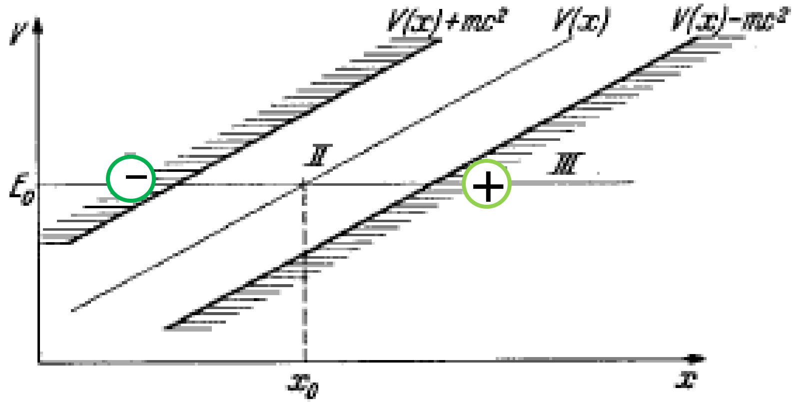


Fig. 1.

Schwinger's work (1951)

PHYSICAL REVIEW VOLUME 82, NUMBER 5 JUNE 1, 1951

On Gauge Invariance and Vacuum Polarization

JULIAN SCHWINGER
Harvard University, Cambridge, Massachusetts
(Received December 22, 1950)

This paper is based on the elementary remark that the extraction of gauge invariant results from a formally gauge invariant theory is ensured if one employs methods of solution that involve only gauge covariant quantities. We illustrate this statement in connection with the problem of vacuum polarization by a prescribed electromagnetic field. The vacuum current of a charged Dirac field, which can be expressed in terms of the Green's function of that field, implies an addition to the action integral of the electromagnetic field. Now these quantities can be related to the dynamical properties of a "particle" with space-time coordinates that depend upon a proper-time parameter. The proper-time equations of motion involve only electromagnetic field strengths, and provide a suitable gauge invariant basis for treating problems. Rigorous solutions of the equations of motion can be obtained for a constant field, and for a plane wave field. A renormalization of field strength and charge, applied to the modified lagrange function for constant fields, yields a finite, gauge invariant result which implies nonlinear properties for the electromagnetic field in the vacuum. The contribution of a zero spin charged field is also stated. After the same field strength renormalization, the modified physical quantities describing a plane wave in the vacuum reduce to just those of the maxwell field; there are no nonlinear phenomena for a single plane wave, of arbitrary strength and spectral composition. The results obtained for constant (that is, slowly varying fields), are then applied to treat the two-photon disintegration of a spin zero neutral meson arising from the polarization of the proton vacuum. We obtain approximate, gauge invariant expressions for the effective interaction between the meson and the electromagnetic field, in which the nuclear coupling may be scalar, pseudoscalar, or pseudovector in nature. The direct verification of equivalence between the pseudoscalar and pseudovector interactions only requires a proper statement of the limiting processes involved. For arbitrarily varying fields, perturbation methods can be applied to the equations of motion, as discussed in Appendix A, or one can employ an expansion in powers of the potential vector. The latter automatically yields gauge invariant results, provided only that the proper-time integration is reserved to the last. This indicates that the significant aspect of the proper-time method is its isolation of divergences in integrals with respect to the proper-time parameter, which is independent of the coordinate system and of the gauge. The connection between the proper-time method and the technique of "invariant regularization" is discussed. Incidentally, the probability of actual pair creation is obtained from the imaginary part of the electromagnetic field action integral. Finally, as an application of the Green's function for a constant field, we construct the mass operator of an electron in a weak, homogeneous external field, and derive the additional spin magnetic moment of $\alpha/2\pi$ magnetons by means of a perturbation calculation in which proper-mass plays the customary role of energy.

Schwinger's work (1951)

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One loop expression of the effective Lagrangian (s: proper time) $F = eE$

$$\Delta\mathcal{L}^{\mathrm{QED}}(F) = \frac{1}{8\pi^2}\int_0^\infty \frac{ds}{s^2}\left[F\cot(Fs) - \frac{1}{s}\right] e^{-ism_e^2}$$

Imaginary part obtained from singularity in $s \in C$

Chapter PDF

$$\Gamma(F)^{\mathrm{QED}}/L^d = \frac{\alpha F^2}{2\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\exp\left(-\frac{n\pi m_e^2}{|F|}\right)$$

Schwinger's work

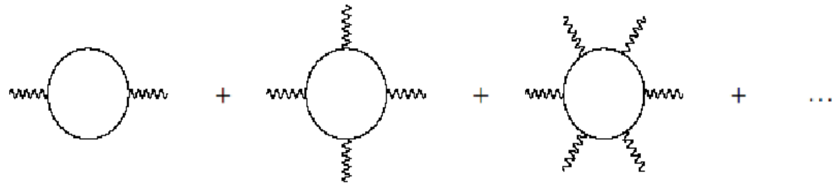
Schwinger 1951

Heisenberg-Euler Effective Lagrangian

$$\mathcal{L}_{\text{HE}}(A_{ext}) = -i \ln \int D[\psi, \bar{\psi}] e^{i \int d^D x L(A_{ext})}$$

$$= -i \ln \text{Det}\left[i\partial\!\!\!/ + ieA\!\!\!/ - m \right]$$

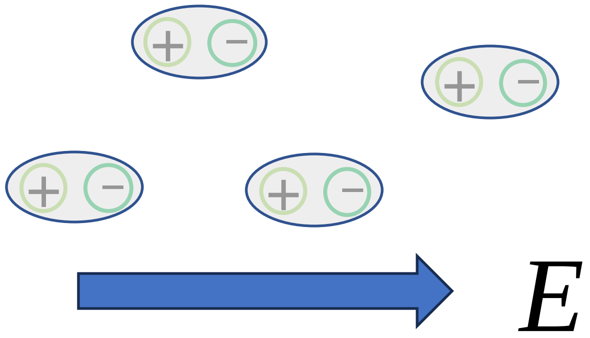
$$=$$



generating function of nonlinear responses

Polarization induced by E

$$X(E) = -\frac{\partial}{\partial E} \text{Re}\mathcal{L}_{\text{HE}}$$



$$= \tfrac{1}{2}(\mathbf{E}^2 - \mathbf{H}^2) + \frac{2\alpha^2}{45} \frac{(\hbar/mc)^3}{mc^2} \left[(\mathbf{E}^2 - \mathbf{H}^2)^2 + 7(\mathbf{E}\cdot\mathbf{H})^2 \right]$$

$$+ \frac{\alpha^2}{90} \frac{(\hbar/\mu c)^3}{\mu c^2} \left[\frac{7}{4}(\mathbf{E}^2 - \mathbf{H}^2)^2 + (\mathbf{E}\cdot\mathbf{H})^2 \right] + \cdots. \quad (3.54)$$

Berry phase theory of Polarization

Berry phase

$$P_{\parallel}^{(\lambda)} = \frac{ifq_e}{8\pi^3} \int_A d\mathbf{k}_{\perp} \sum_{n=1}^{M} \int_0^{|\mathbf{G}_{\parallel}|} dk_{\parallel} \left\langle u_{\mathbf{k}n}^{(\lambda)} \middle| \frac{\partial}{\partial k_{\parallel}} \middle| u_{\mathbf{k}n}^{(\lambda)} \right\rangle$$

King-Smith, Vanderbilt, Phys. Rev. B. **47**, 1651 (1993)

twist operator

$$\langle X \rangle = \frac{L}{2\pi} \operatorname{Im} \ln \langle \Psi_0 | e^{i\frac{2\pi}{L}\hat{X}} | \Psi_0 \rangle$$

R. Resta, Phys. Rev. Lett. **80**, 1800 (1998)

- Topological index (P=0,1/2 for lattice with inv. sym.)
- Quantum metric
- Relation to LSM theorem

Can we relate this to Schwinger's vacuum polarization?

HE-Effective Lagrangian from groundstate amplitude

Oka, Aoki 2005

DC *E*-field in the potential gauge $H = H_0 + E\hat{X}$ $\hat{X}$: position operator

$$\mathcal{L}_{HE}(E) = -\lim_{t\to\infty} \frac{i}{tV} \ln\langle 0|\hat{T}e^{-i\int_0^t E\hat{X}(s)ds}|0\rangle$$

(interaction picture)

- ~~Adiabatic~~ quench limit (fixed $tE=2\pi/L$) recovers $\langle X\rangle = \frac{L}{2\pi}\,\mathrm{Im}\ln\langle\Psi_0|e^{i\frac{2\pi}{L}\hat{X}}|\Psi_0\rangle = -\frac{\partial}{\partial E}\mathrm{Re}\mathcal{L}_{HE}$
- Generically, HE-Lagrangian is related to the groundstate amplitude (λ: parameter)

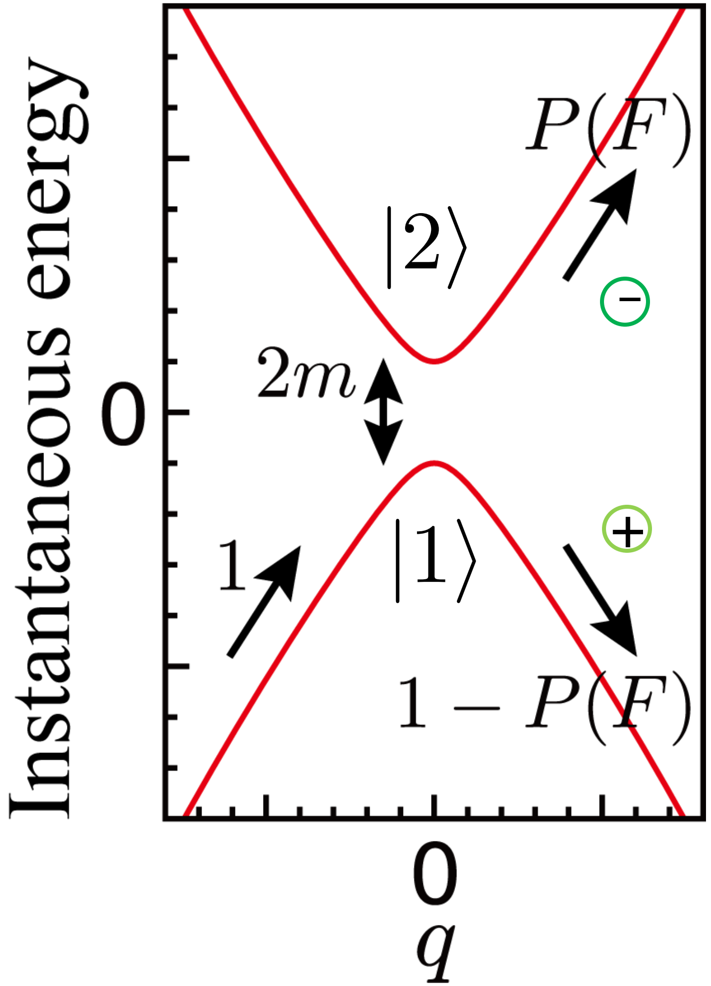
$$\Xi(t) = \langle\Psi_0(\boldsymbol{\lambda}(t))|\hat{T}e^{-i\int_0^t H(\boldsymbol{\lambda}(s))ds}|\Psi_0(\boldsymbol{\lambda}(0))\rangle e^{i\int_0^t E_0(A(s))ds}$$

adiabatic ground state time evolution opt. ground state

$$\sim e^{itV\mathcal{L}(E)}$$

Recovering Schwinger's result

$$\Xi(t) = \langle\Psi_0(\boldsymbol{\lambda}(t))|\hat{T}e^{-i\int_0^t H(\boldsymbol{\lambda}(s))ds}|\Psi_0(\boldsymbol{\lambda}(0))\rangle e^{i\int_0^t E_0(A(s))ds}$$



3+1 D Dirac model ➡ Landau Zener problem

Landau 1932
Zener 1932

$$\hat{\mathcal{H}}(q) = m\hat{\sigma}^z + vq\hat{\sigma}^x$$

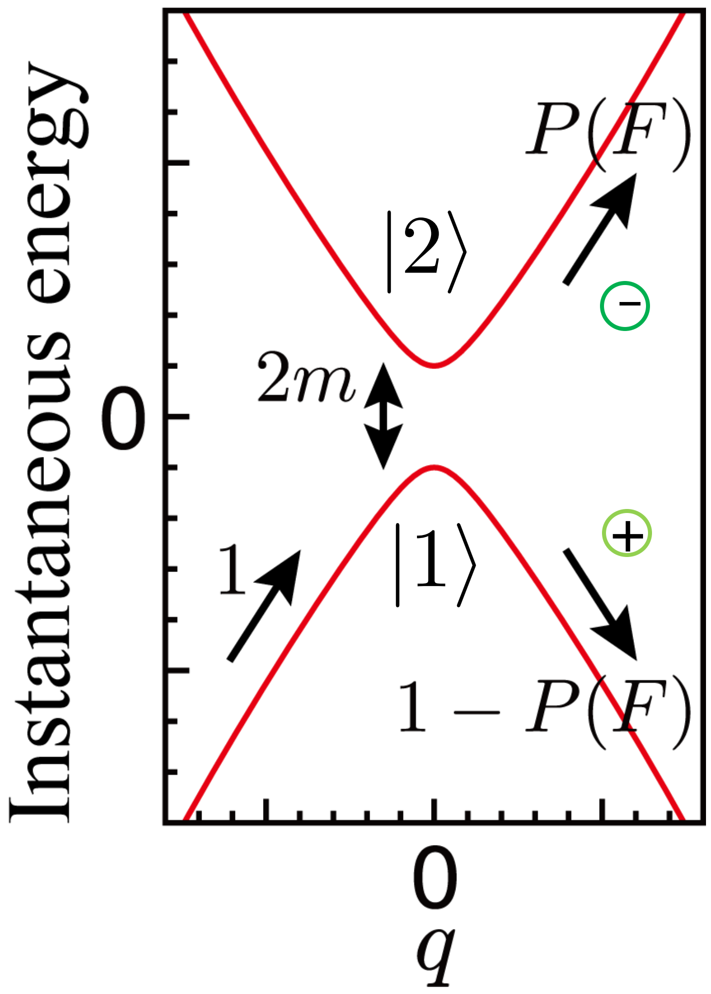
$$\lambda = q = -Ft$$

$$|\psi(t)\rangle \sim \sqrt{1-p}e^{i\gamma}|1\rangle + \sqrt{p}e^{i\beta}|2\rangle$$

Recovering Schwinger's result

Oka, Aoki 2005

$$\Xi(t) = \langle \Psi_0(\boldsymbol{\lambda}(t)) | \hat{T} e^{-i\int_0^t H(\boldsymbol{\lambda}(s))ds} | \Psi_0(\boldsymbol{\lambda}(0)) \rangle e^{i\int_0^t E_0(A(s))ds}$$



$$\mathrm{Re}\mathcal{L}(E) = -E\int_{\mathrm{BZ}} \frac{d\boldsymbol{k}}{(2\pi)^d}\frac{\tilde{\gamma}(\boldsymbol{k})}{2\pi}$$

$$\mathrm{Im}\mathcal{L}(E) = -E\int_{\mathrm{BZ}} \frac{d\boldsymbol{k}}{(2\pi)^d}\frac{1}{4\pi}\ln[1-P(\boldsymbol{k})]$$

Results of the Landau-Zener model (Landau's textbook)

$$p(\boldsymbol{k}) = \exp\left[-\pi\frac{(\Delta_{\mathrm{band}}(\boldsymbol{k})/2)^2}{vF}\right]$$

$$\gamma(\boldsymbol{k}) = \frac{1}{2}\,\mathrm{Im}\int_0^\infty ds\,\frac{e^{-i(\Delta_{\mathrm{band}}(\boldsymbol{k})/2)^2 s}}{s}\left[\cot(vFs) - \frac{1}{vFs}\right]$$

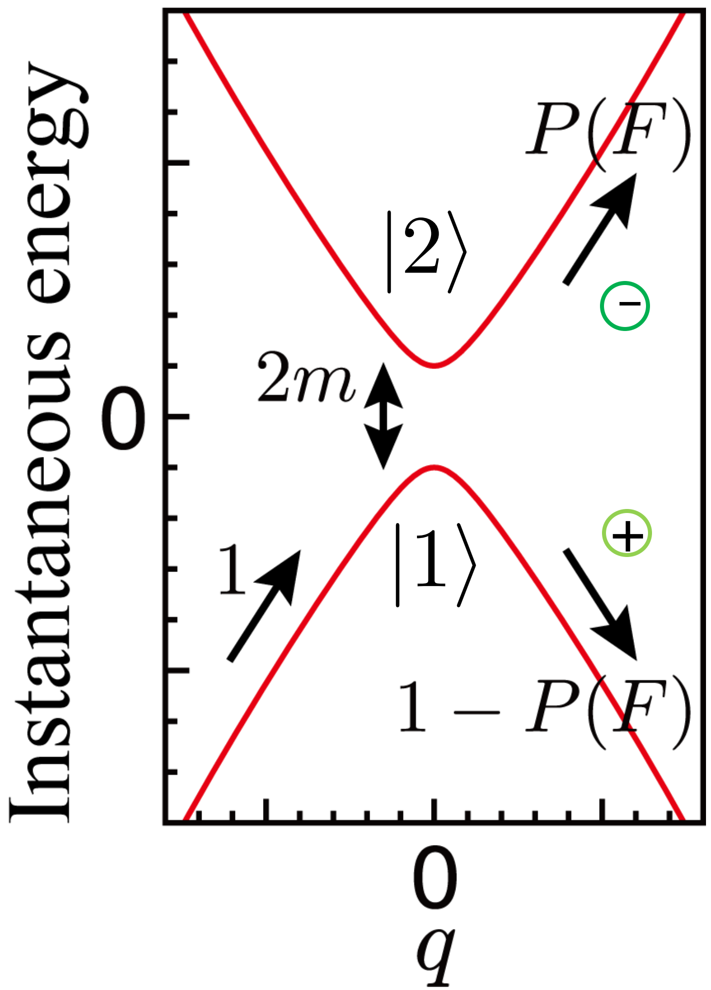
Stokes phase
(Nonadiabatic Berry phase)

Agrees with Schwinger 1951 after k-integral

Recovering Schwinger's result

Oka, Aoki 2005

$$\Xi(t) = \langle \Psi_0(\boldsymbol{\lambda}(t)) | \hat{T} e^{-i \int_0^t H(\boldsymbol{\lambda}(s)) ds} | \Psi_0(\boldsymbol{\lambda}(0)) \rangle e^{i \int_0^t E_0(A(s)) ds}$$



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Results of the Landau-Zener model (Landau's textbook)

$$p(\boldsymbol{k}) = \exp\left[-\pi \frac{(\Delta_{\mathrm{band}}(\boldsymbol{k})/2)^2}{vF}\right]$$

Can we calculate p?

$$\gamma(\boldsymbol{k}) = \frac{1}{2} \mathrm{Im} \int_0^\infty ds \frac{e^{-i(\Delta_{\mathrm{band}}(\boldsymbol{k})/2)^2 s}}{s} \left[\cot(vFs) - \frac{1}{vFs}\right]$$

Stokes phase
(Nonadiabatic Berry phase)

Agrees with Schwinger 1951 after k-integral

Imaginary time method (Landau-Dykhne theory, DDP theory)

Dykhne JETP (1962), Daviis, Pechukas, J.Chem.Phys. (1976)

Matrix version of WKB approximation for 2-level systems

$$H(t) = \begin{pmatrix} A(t) & B(t) \\ C(t) & D(t) \end{pmatrix}$$

Tunneling probability

$$p = \exp\left(-2\mathrm{Im}\int_{t_0}^{t^*} dt' [E_2(t') - E_1(t')]\right)$$

$t^* \in \mathbb{C}$: exceptional point

$$E_2(t^*) = E_1(t^*)$$

proof: saddle point approximation of the transition amplitude

$$a_+(\infty) = \int_{-\infty}^{\infty} \mathrm{d}t\, a_-(t)\,\eta(t)\,\mathrm{e}^{\mathrm{i}\Delta(t)} \qquad \Delta(t) = \mathcal{E}_+(t) - \mathcal{E}_-(t) = \int_0^t \mathrm{d}t'\,\delta E(t') \qquad \left.\frac{\partial \Delta}{\partial t}\right|_{t=t_c} = 0$$

extension Fukushima, Shimazaki ’19

Schwinger mechanism (tunneling problem) → Dirac model with complex gauge field

$$\mathcal{L} = \bar{\psi}\left[(i\partial_\mu - A_\mu)\gamma^\mu - m\right]\psi$$

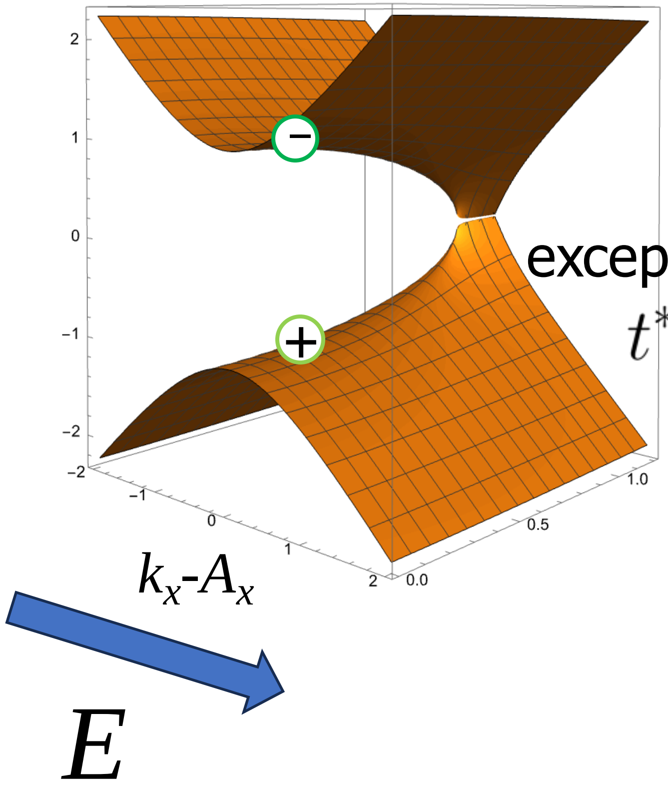
$$A_x = Et$$

Imaginary time method

Hatano-Nelson-like system

$$\mathcal{L} = \bar{\psi}\left[(i\partial_\mu - A_\mu)\gamma^\mu - m\right]\psi$$

$$A_x = iE\tau$$



exceptional point

$$t^* = \pm i\Delta_k/F$$

Two eigenstates becomes the same

$$S_{1,2} = \int_{t_0}^{t^*} dt'[E_2(\Phi(t')) - E_1(\Phi(t'))]$$

$$= i\int_{t_0}^{\Delta_k/F} dt'[\sqrt{-F^2t^2 + \Delta_k^2}]$$

$$= i\pi\frac{\Delta_k^2}{2F}$$

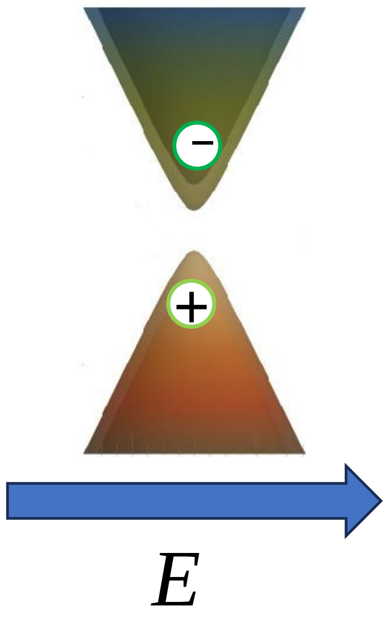
$$\Delta_k = \sqrt{m^2 + k_\perp^2}$$

$$p_k = e^{-\pi\Delta_k^2/F}$$

total $P\sim \Gamma/V = 2\mathrm{Im}\mathcal{L} \sim \frac{e^2E^2}{4\pi^3}\exp\left(-\pi\frac{m^2}{eE}\right)$

Schwinger Phys. Rev.82, (1951)

Summary of part 1



Heisenberg-Euler problem

$$\mathcal{L} = \bar{\psi}\left[(i\partial_\mu - A_\mu)\gamma^\mu - m\right]\psi$$
$$A_x = Et$$

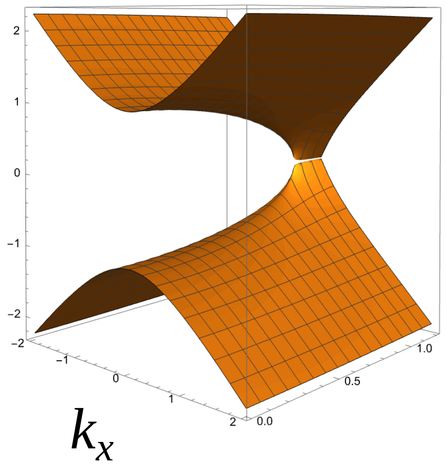
k, spin conservation →

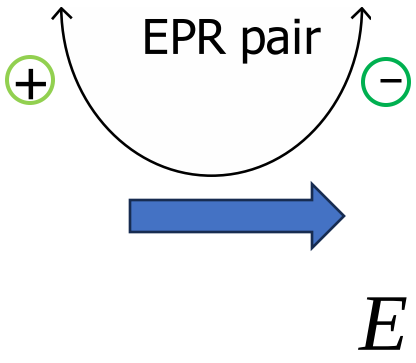
Landau-Zener model

↓ Imaginary time method (exact only for LZ model)

Dirac model with complex gauge field

$$\mathcal{L} = \bar{\psi}\left[(i\partial_\mu - A_\mu)\gamma^\mu - m\right]\psi$$
$$A_x = iE\tau$$





HE effective Lagrangian

$$\mathcal{L}_{HE}(E) = -\lim_{t\to\infty}\frac{i}{tV}\ln\langle 0|\hat{T}e^{-i\int_0^t E\hat{X}(s)ds}|0\rangle$$

=(polarization)+I (tunneling probability)

Oka, Aoki 2005

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1. Review of the work by Heisenberg, Euler, Schwinger

   non-adiabatic Berry phase, imaginary time method

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   Okazaki, Okumura, Takayoshi, TO, arXiv ’24

3. Floquet state in AC-magnetic fields

   π-Landau states, chiral anomaly induced homodyne

   TO, Bucciantini ‘16
   Kitamura, TO, arXiv ’24

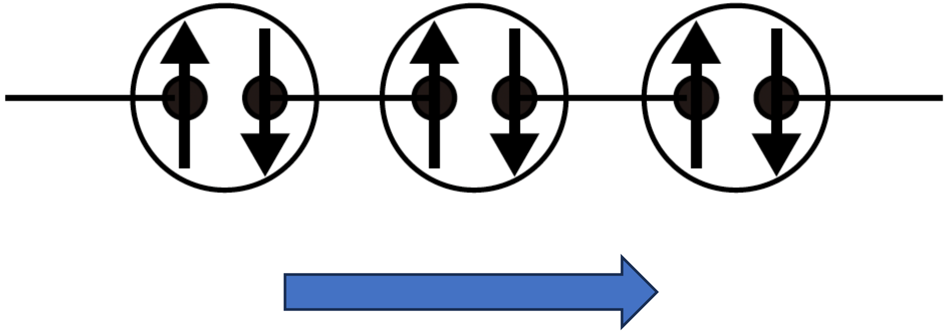
Tunneling breakdown from a SPT phase

Koichi Okazaki

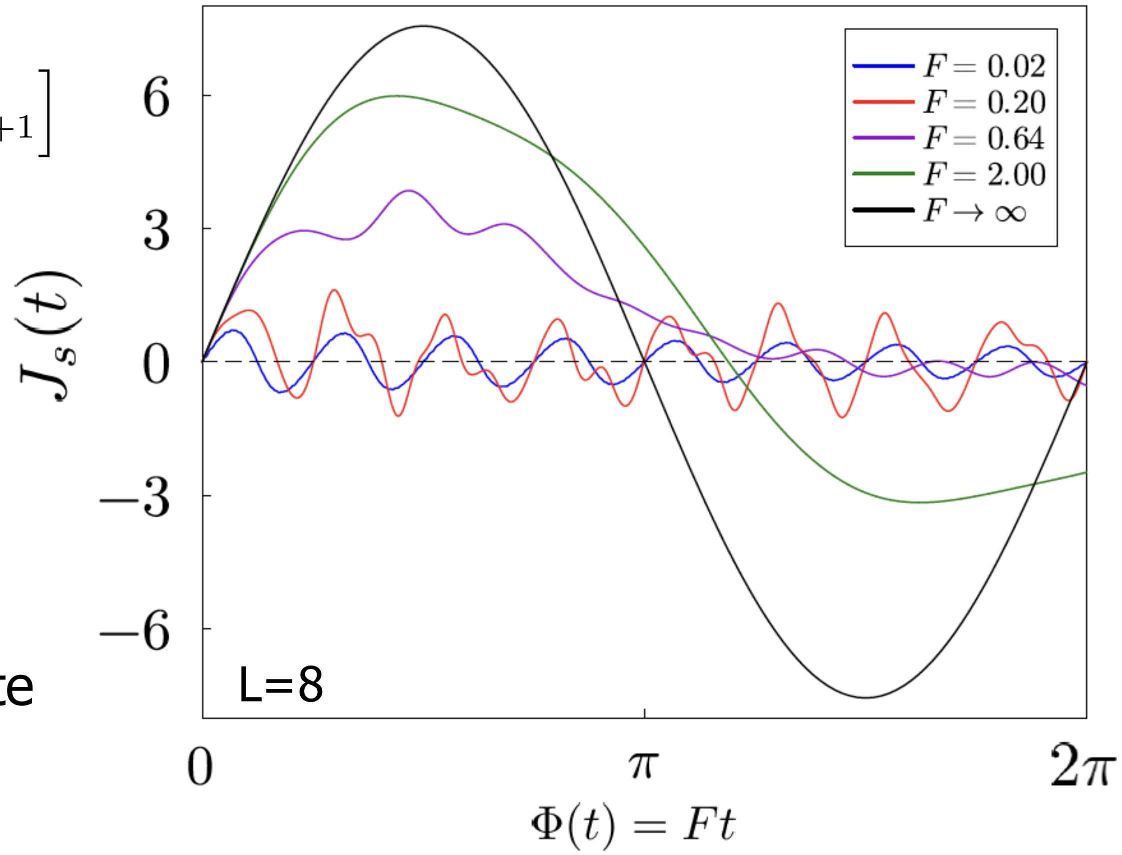
Okazaki, Okumura, Takayoshi, TO, arXiv ’24

Driven S=1 Heisenberg model

$$H(t) = \frac{J}{2}\sum_j \left[\left(e^{iFt}S_j^{\dagger}S_{j+1}^{-} + e^{-iFt}S_{j+1}^{\dagger}S_j^{-}\right) + 2S_j^zS_{j+1}^z\right]$$



Spin current



- Start from Valence Bond Solid (VBS) ground state
  (picture accurate at AKLT point)
- $F\ (= \frac{\partial B_z}{\partial x})$ is the spin electric field

Bloch oscillation
ΔΦ=2π

Tunneling breakdown from a SPT phase

Okazaki, Okumura, Takayoshi, TO, arXiv '24

Koichi Okazaki

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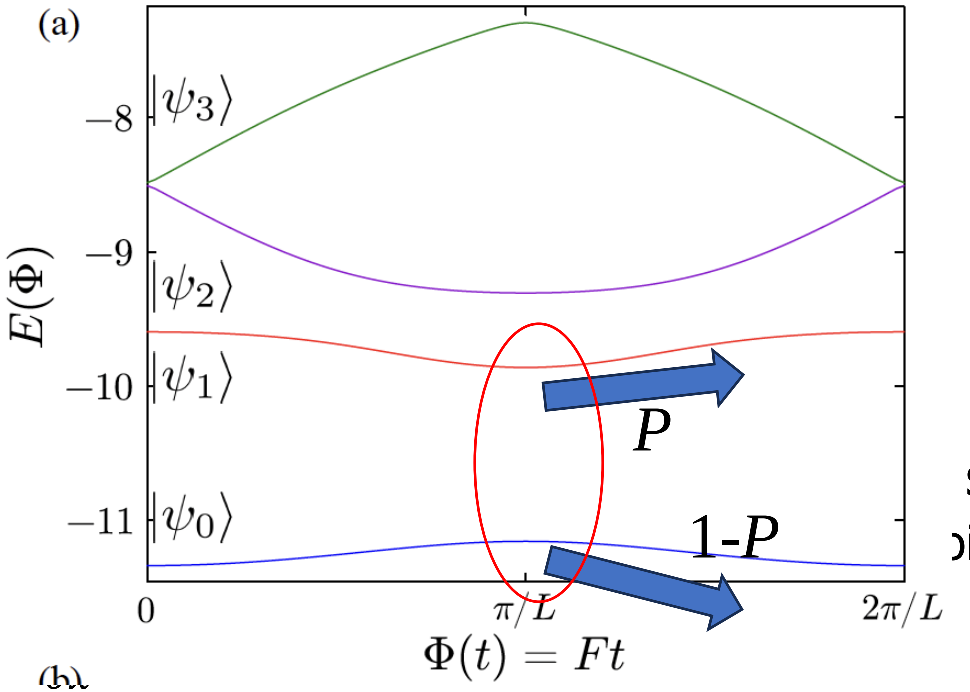
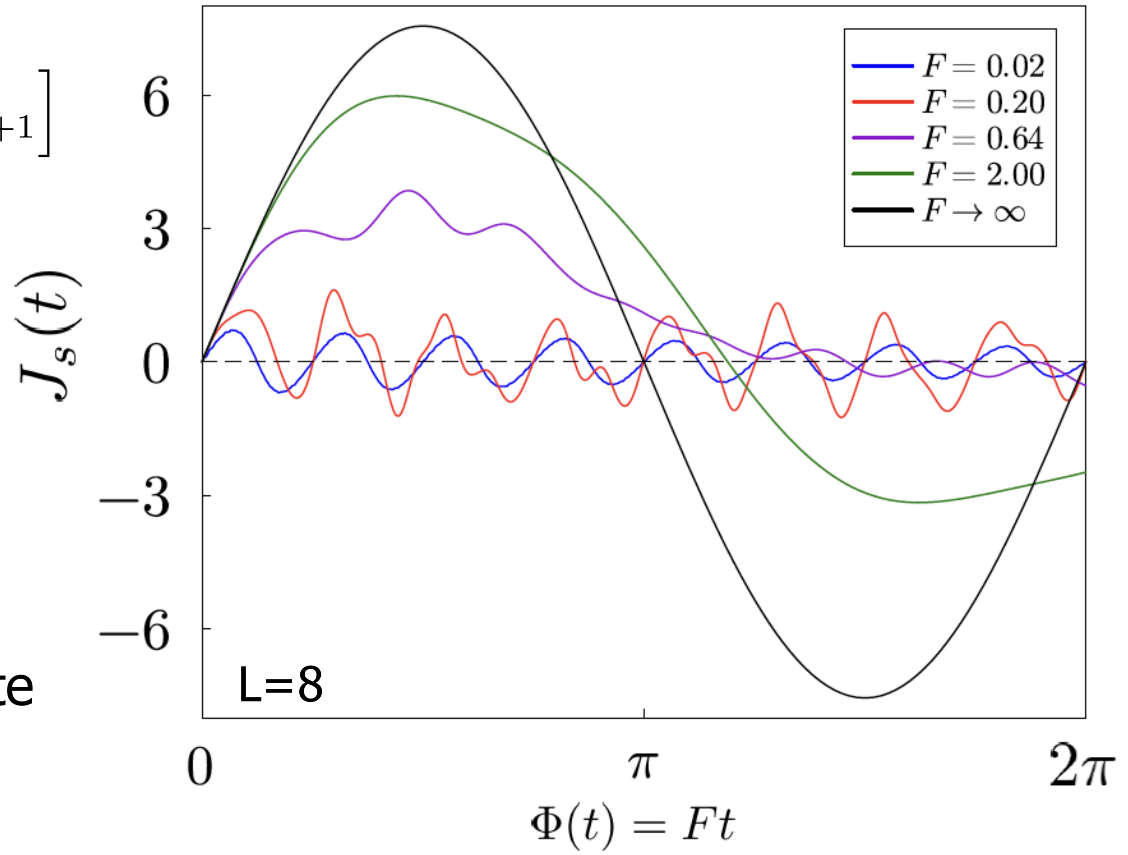


- Sta state
  oint)
- F (

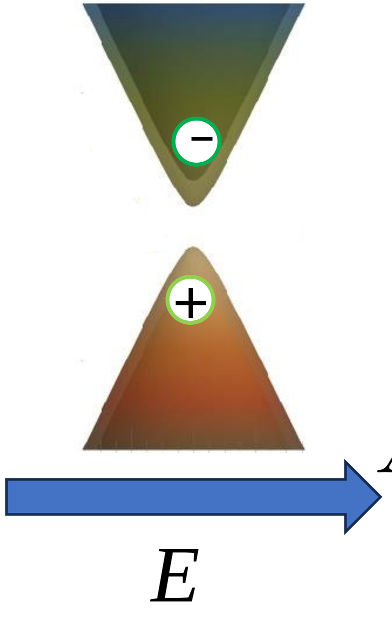
Spin current



Let's calculate P

Bloch oscillation
$\Delta\Phi = 2\pi$

Previously we had



Heisenberg-Euler problem $\xrightarrow{k,\ \text{spin}}$ Landau-Zener model

$$\mathcal{L} = \bar{\psi}\left[(i\partial_\mu - A_\mu)\gamma^\mu - m\right]\psi$$

$$A_x = Et$$

Imaginary time method
(exact only for LZ model)

Dirac model with complex gauge field

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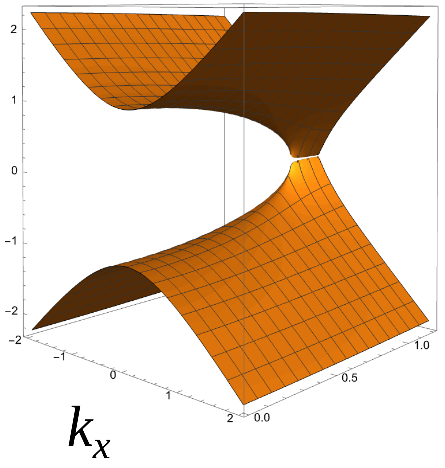
Schwinger Phys. Rev.82, (1951)



Can we do the following?

Driven S=1 Heisenberg model

$$H(t) = \frac{J}{2}\sum_{j}\left[\left(e^{iFt}S_j^{\dagger}S_{j+1}^{-} + e^{-iFt}S_{j+1}^{\dagger}S_j^{-}\right) + 2S_j^zS_{j+1}^z\right]$$

? Imaginary time method
(exact only for LZ model)

Non-Hermitian S=1 Heisenberg model

$$H(\Phi) = \frac{J}{2}\sum_{j}\left[\left(e^{\Phi}S_j^{\dagger}S_{j+1}^{-} + e^{-\Phi}S_{j+1}^{\dagger}S_j^{-}\right) + 2S_j^zS_{j+1}^z\right]$$

$$\Phi = \Phi_r + i\Phi_i$$

Let’s try

Non-Hermitian S=1 Heisenberg model

$$H(\Phi) = \frac{J}{2}\sum_j \left[\left(e^{\Phi} S_j^{\dagger} S_{j+1}^{-} + e^{-\Phi} S_{j+1}^{\dagger} S_j^{-}\right) + 2S_j^z S_{j+1}^z\right]$$

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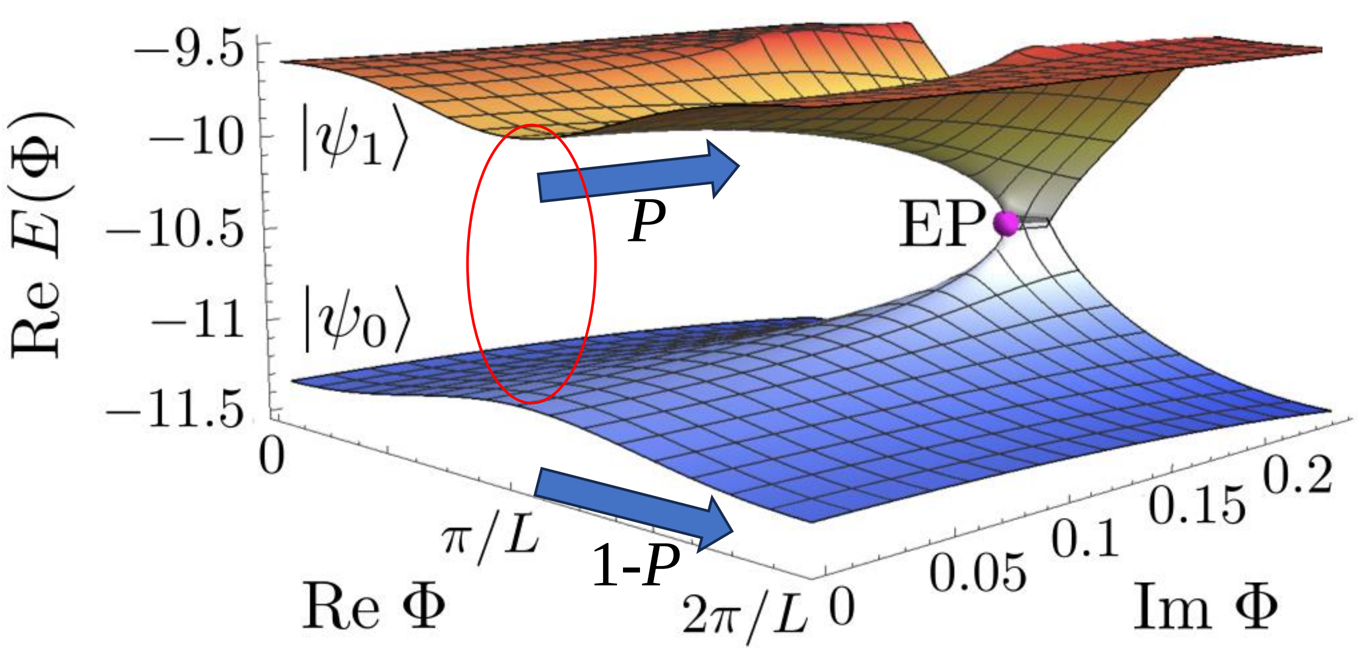
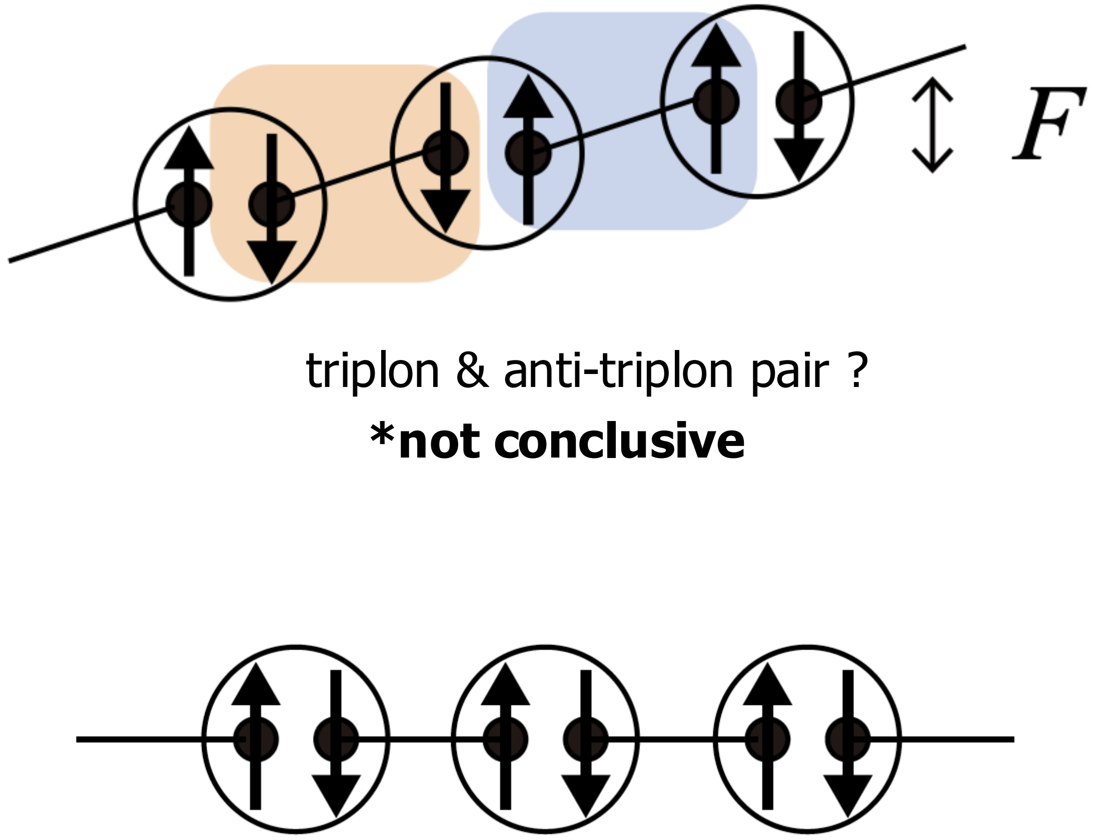




triplon & anti-triplon pair ?

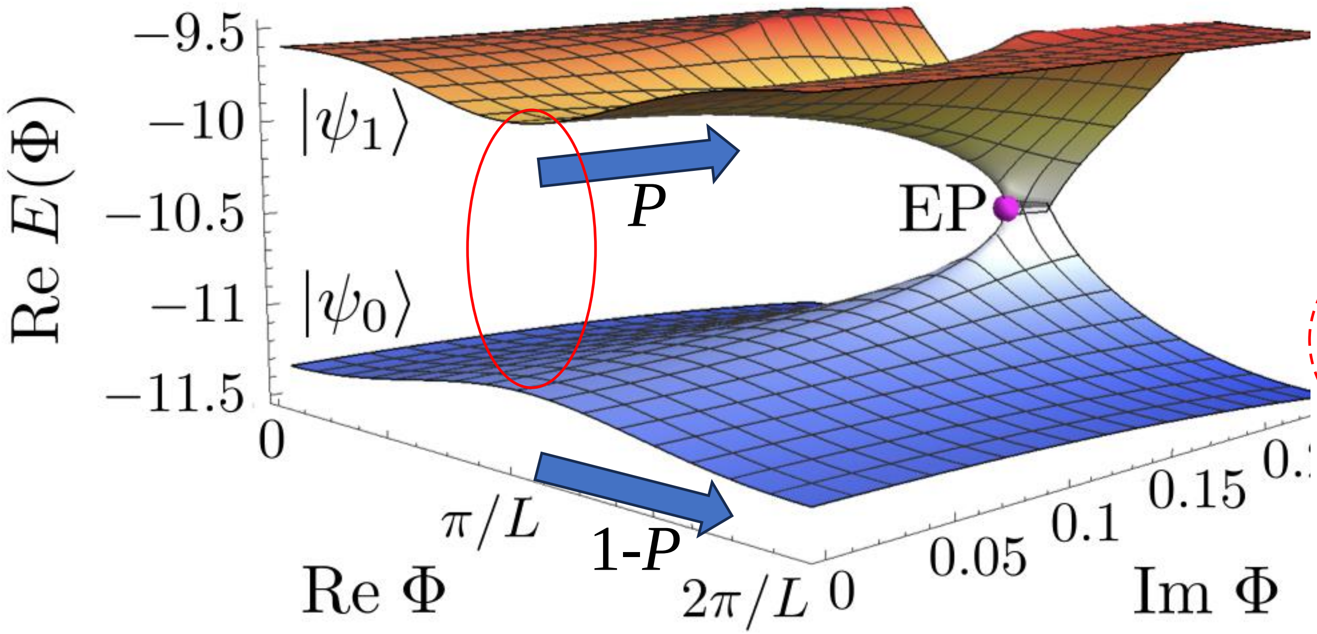
***not conclusive**

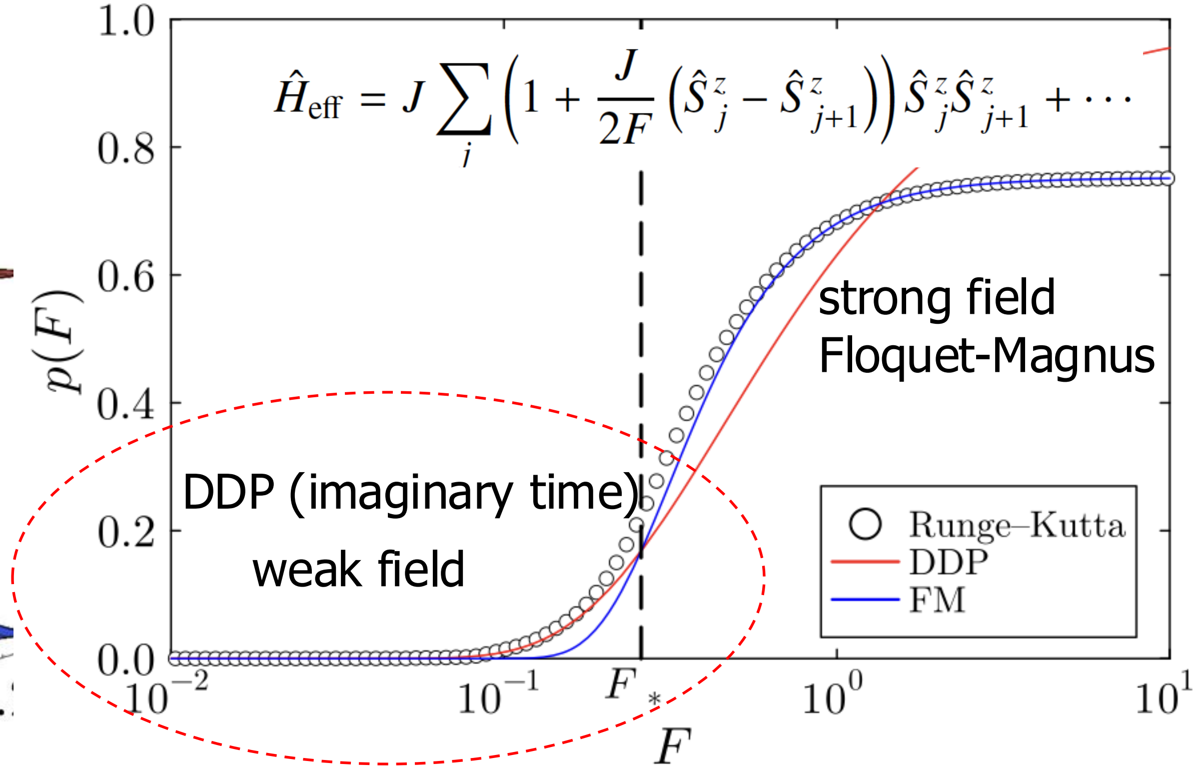
ground state

Non-Hermitian S=1 Heisenberg model

$$H(\Phi) = \frac{J}{2}\sum_j \left[\left(e^{\Phi} S_j^{\dagger} S_{j+1}^{-} + e^{-\Phi} S_{j+1}^{\dagger} S_j^{-}\right) + 2 S_j^z S_{j+1}^z\right]$$

$$\Phi = \Phi_r + i\Phi_i$$





$$p(F) = \exp(-\pi F_{\text{th}}/F)$$

Exceptional point is formed between 0 and 1 states,
and the tunneling is well-described by imaginary time method.

$$F_{\text{th}} = \frac{2}{\pi}\text{Im}\int_{\pi/L}^{\Phi_c} d\Phi[E_1(\Phi) - E_0(\Phi)]$$

Remark: 1D Hubbard model

Oka, Aoki 2010, Oka 2012

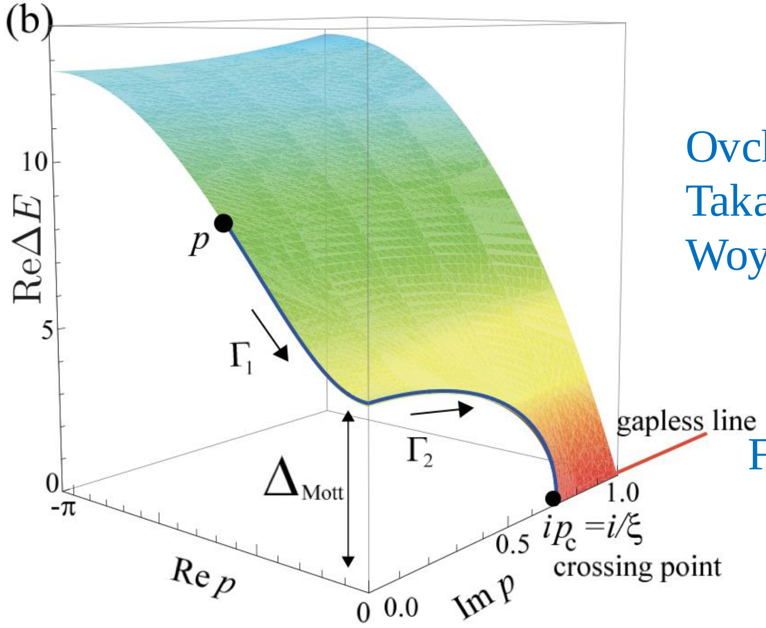
Momentum resolved dh-pair creation rate

$$\mathcal{P}_p = \exp\left(-2\mathrm{Im}\int_\Gamma \Delta E(p-\Phi)\frac{-1}{F(\Phi)}d\Phi\right)$$

ΔE: d-h energy, Γ: complex path, F: Jacobian (E-field)

Imaginary time method + Bethe ansatz

Can study effects of DC, AC, or pulse fields



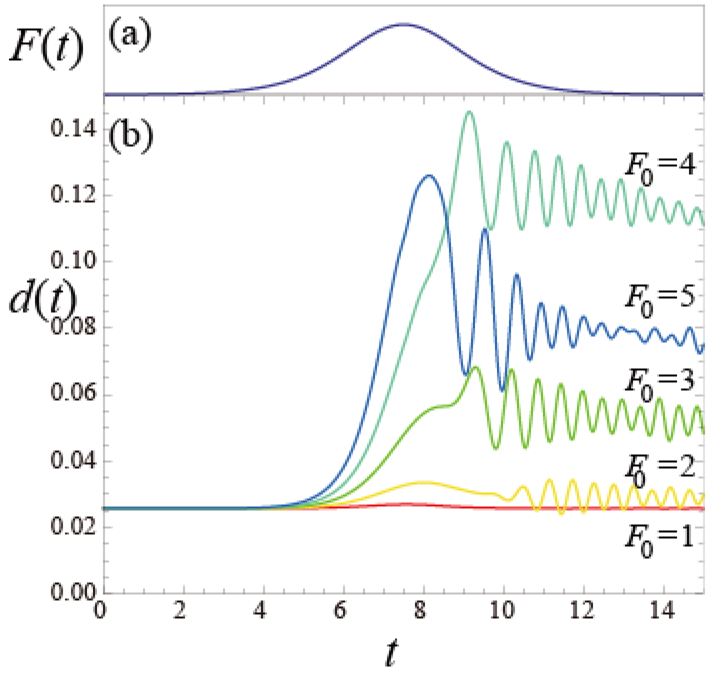
String states

Ovchinikov JETP, 30 (1970), Coll, PRB 9 (1974),
Takahashi, Prog. Theor. Phys. 47 (1972)
Woynarovich, J.Phys.C, (1982)

Non-Hermitian extension

Fukui-Kawakami (1998), Nakamura-Hatano (2006)

$$\varepsilon(k) = 2u + 2\cos(k) + 2\int_0^\infty \frac{e^{-u\omega}}{\omega\cosh u\omega}J_1(\omega)\cos(\omega\sin k)d\omega.$$



Experiments Yamakawa et al. (Okamoto gr.) 2017, Li et al. (Hsieh gr.) 2022

Outline

1. Review of the work by Heisenberg, Euler, Schwinger

   non-adiabatic Berry phase, imaginary time method

2. Tunneling from a SPT phase

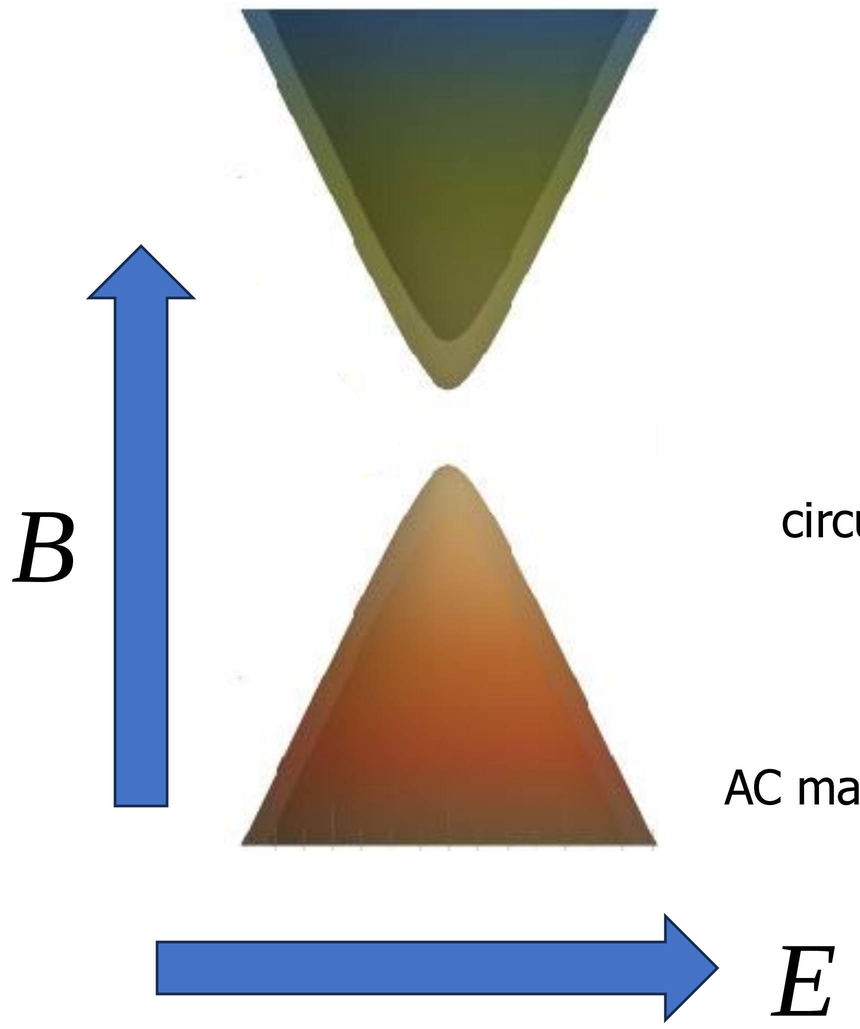
   Okazaki, Okumura, Takayoshi, TO, arXiv '24

3. Floquet state in AC-magnetic fields

   π-Landau states, chiral anomaly induced homodyne

   Kitamura, TO, arXiv '24

Part 3: Floquet state in AC-magnetic fields



Free Dirac fermions in electro-magnetic fields

$$\mathcal{L} = \bar{\psi}\left[i\gamma^{\mu}(\partial_{\mu} + ieA_{\mu}) - m\right]\psi$$

static $\boldsymbol{E} = \boldsymbol{E}_0$ Heisenberg-Euler 1936

circularly pol. $\boldsymbol{E} = E_0(\cos\Omega t, \sin\Omega t, 0)$ TO-Aoki 2009

Floquet topological phases (FTI, FATI,..)

AC magnetic field $\boldsymbol{B} = B_0(0, 0, \cos\Omega t)$ Kitamura-TO 2024

π-Landau levels
chiral anomaly induced heterodyne effect

* non-relativistic TO-Bucciantini 2016

Q. Can we get Landau levels from oscillating magnetic fields?

1. Nonrelativistic particle

2. 2D Dirac particle

$$B_z(t) = B\cos\Omega t$$

Q. Can we get Landau levels from oscillating magnetic fields?

1. Nonrelativistic particle

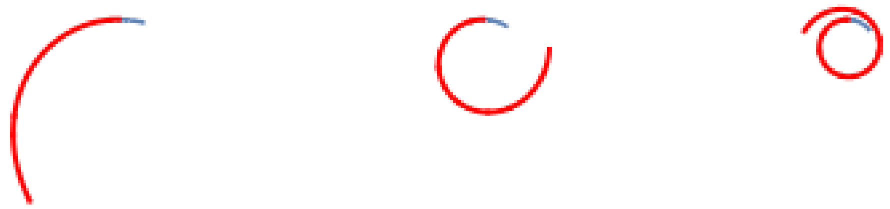
2. 2D Dirac particle

$$B_z(t) = B\cos\Omega t$$

right → left → right → left

↔

Period T



TO-Bucciantini 2016

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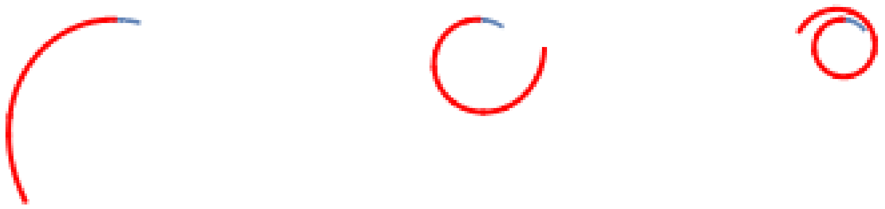


TO-Bucciantini 2016

~~electron → hole → electron → hole~~

Period T

upper → lower → upper → lower

Kitamura, TO, arXiv ’24

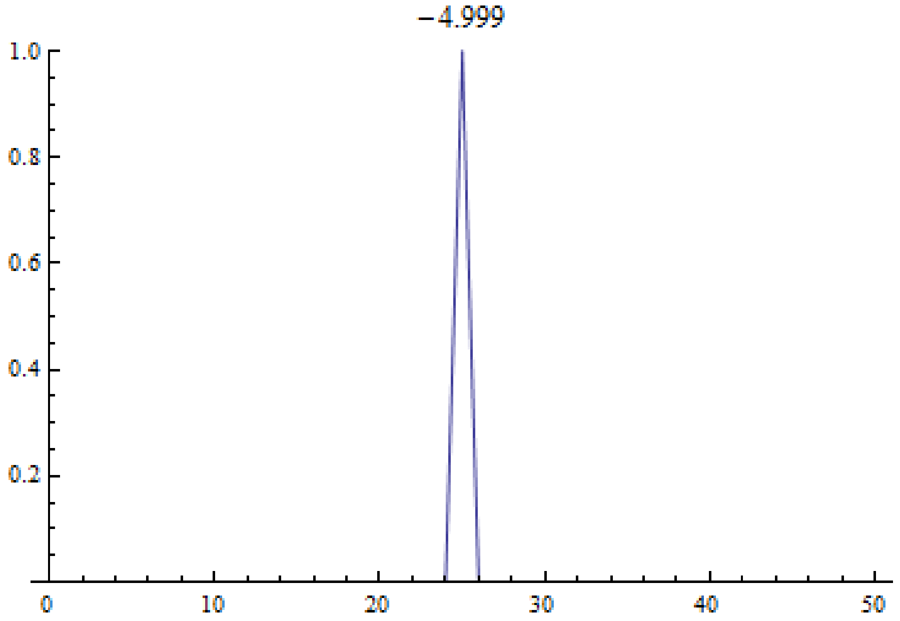
Classical Floquet state

Dunlap and Kenkre '86

Quantum Floquet state

"Dynamic localization"

$$H(t) = -J\sum_i \left(e^{-i\theta(t)A\cos\Omega t}c_{i+1}^\dagger c_i + \text{h.c.}\right)$$



momentum space $H(t) = \sum_k (-2J)\cos(k - A\cos\Omega t)c_k^\dagger c_k$

"time average" $H_F = J_0(A)(-2J)\cos(k)$ = Effective Floquet Hamiltonian

Floquet theory

$$H(t+T) = H(t)$$

"weird helicopter" (youtube)

slow

Stroboscopic motion

$$t = 0, T, 2T, \ldots$$

- Almost like a static system
- We can define (quasi)-energy and state

Effective Floquet Hamiltonian

H_F = time average + effective terms

fast

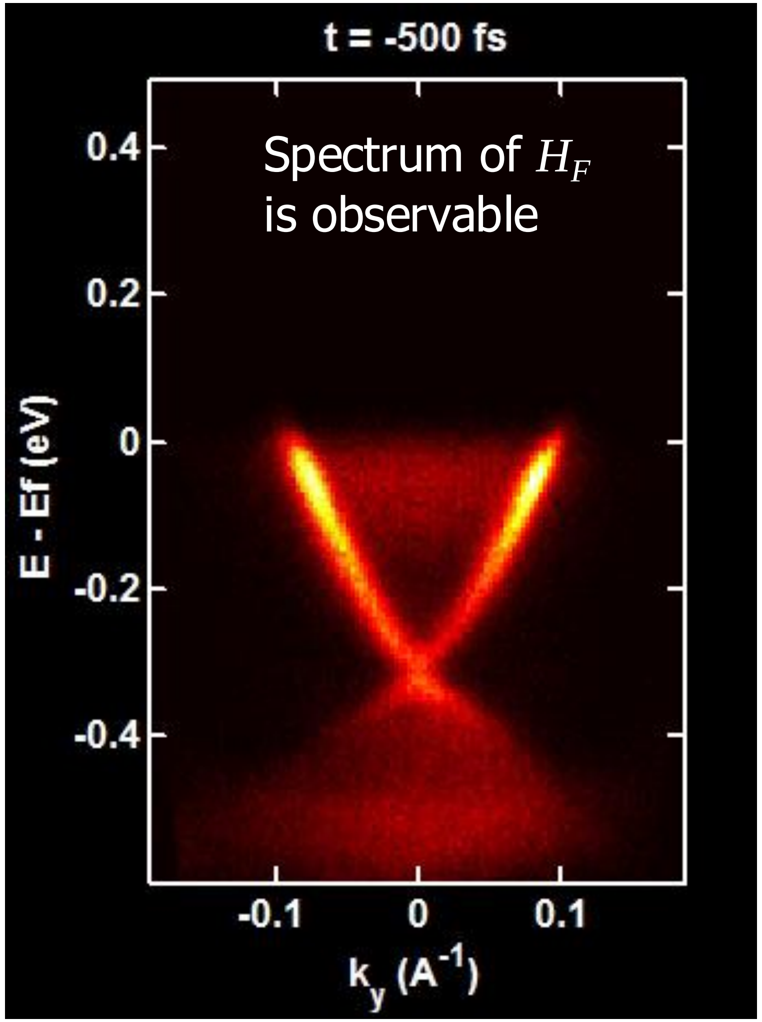
Micromotion

$$t: 0 \rightarrow T$$

- Floquet state

2D Dirac + circularly pol. laser

$$H = \boldsymbol{\sigma} \cdot (\boldsymbol{p} - \boldsymbol{A}(t))$$



Gedik @MIT, Science '13

Floquet theory

$$H(t+T) = H(t)$$

slow

Stroboscopic motion

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- Almost like a static system
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Effective Floquet Hamiltonian

H_F = time average + effective terms

fast

Micromotion

t: $0 \rightarrow T$

- Floquet state

Theory of Floquet states

Time periodic systems $H(t+T) = H(t)$

Floquet theorem

time evolution

fast slow

$$U(t,0) \equiv \hat{T} e^{-i\int_0^t H(s)ds} = V(t) e^{-iH_F t}$$

time ordering

proof:

step 1.

Define H_F from 1step evolution

$$e^{-iH_F T} \equiv U(T,0)$$

step 2.

Define Kick operator $V(t)$

$$V(t) \equiv U(t,0) e^{iH_F t}$$

$$V(t+T) = V(t)$$

time periodic

How can we calculate Floquet states?

1. Solve the Schrodinger equation and obtain U

2. Sambe's space-time picture (Move to Fourier space)

$$H(t) = \sum_{m=\infty}^{\infty} e^{-im\Omega t} H_m$$

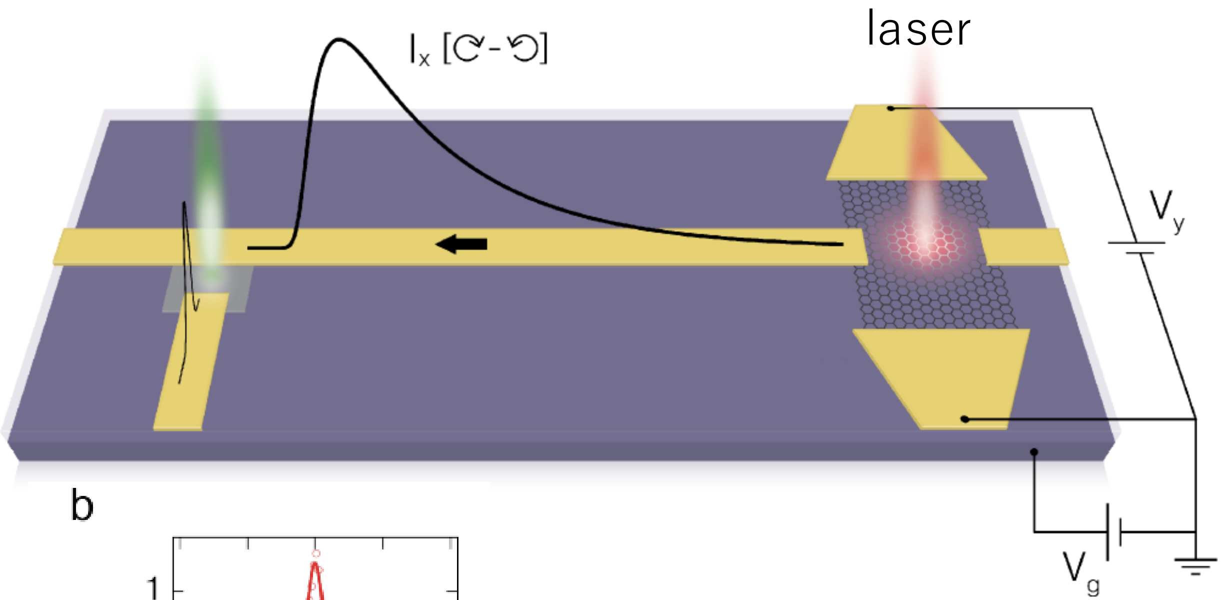
3. High frequency expansion
   (1/Ω expansion, Floquet-Magnus expansion)

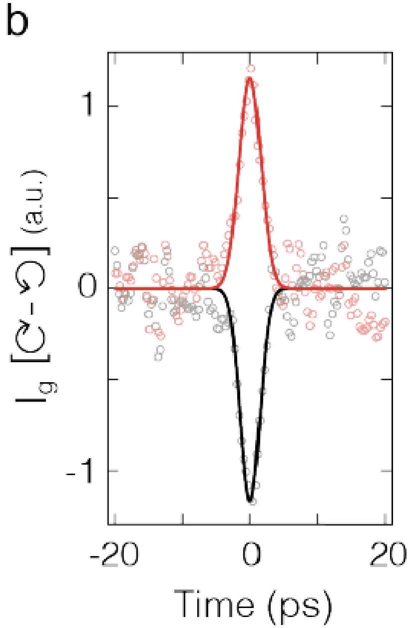
$$H_F = H_0 + \frac{1}{\Omega}\sum_{m>0}[H_{-m}, H_m]$$
$$+\frac{1}{\Omega^2}\left(\sum_{m\neq 0}\frac{1}{2m^2}[[H_{-m}, H_0], H_m] + \sum_{m(\neq 0), n(\neq 0, m)}\frac{1}{3mn}[[H_{-m}, H_{m-n}], H_n]\right) + \mathcal{O}(\Omega^{-3})$$

Eckardt, Rev. Mod. Phys. **89** 2017
TO, Kitamura Ann. Rev. CMP 2019
Rudner, Lindner, Nat. Rev. Phys. 2 2020

Application to Quantum Electronics

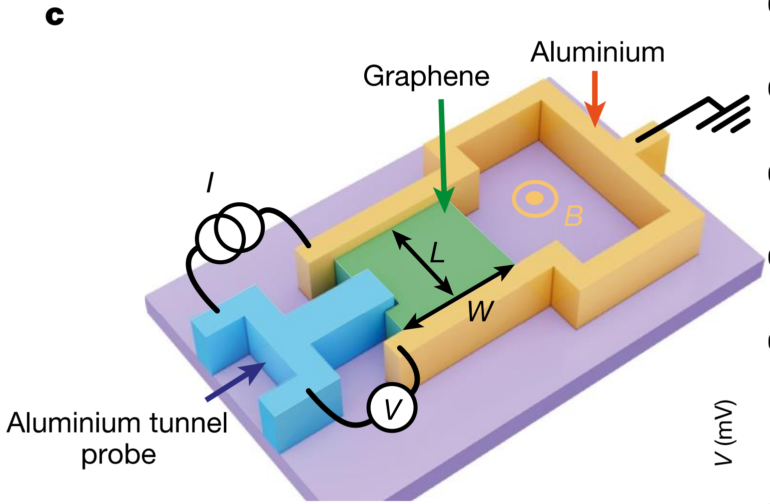
On-chip THz current



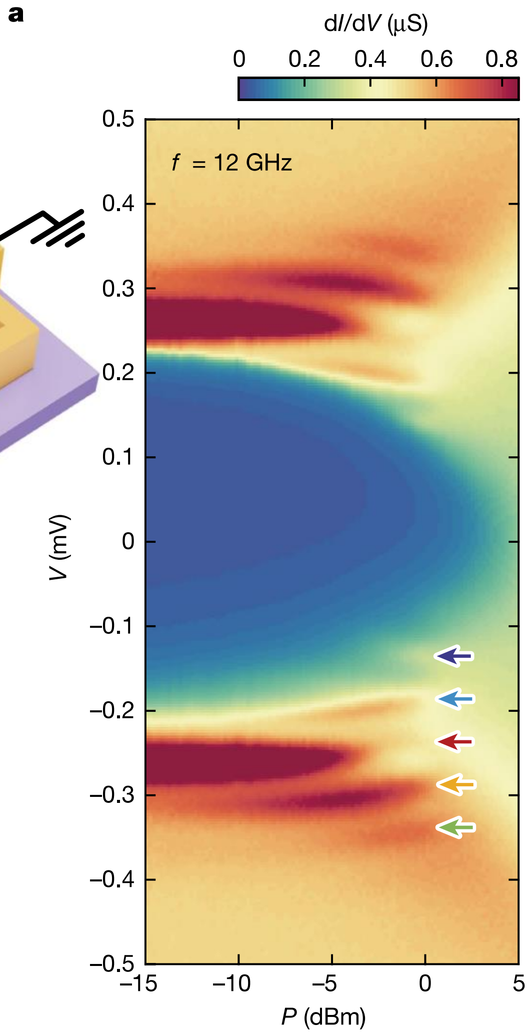


Laser induced Hall current

Floquet-Andreev bound state



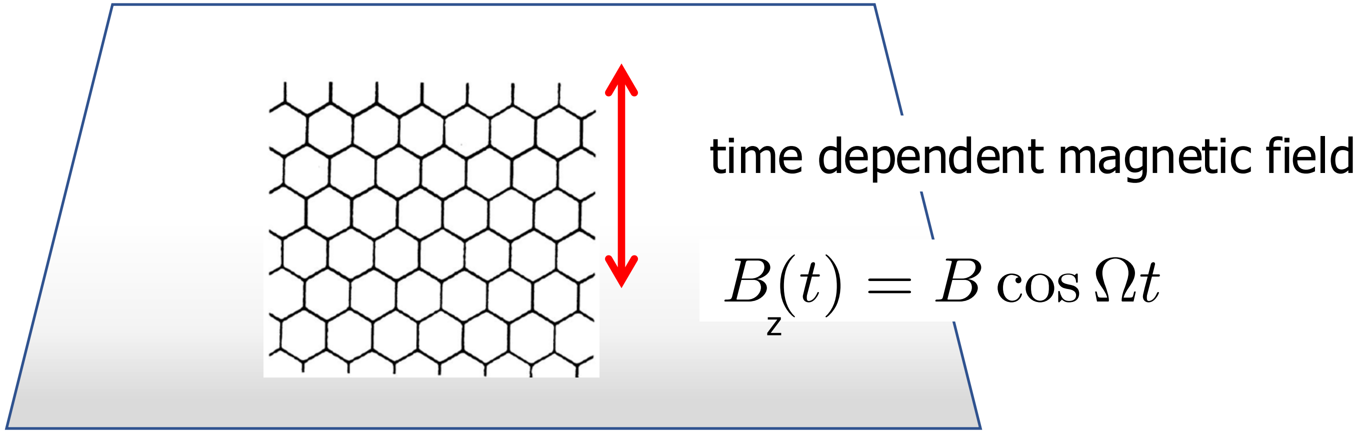
Microwave emitted from an antenna in the cryostat



Mciver, Cavalleri et al. Nat. Phys. '19

Sein Park,.., Gil Young Cho, Gil-Ho Lee, Nature '22

2D Dirac electron in oscillating B field

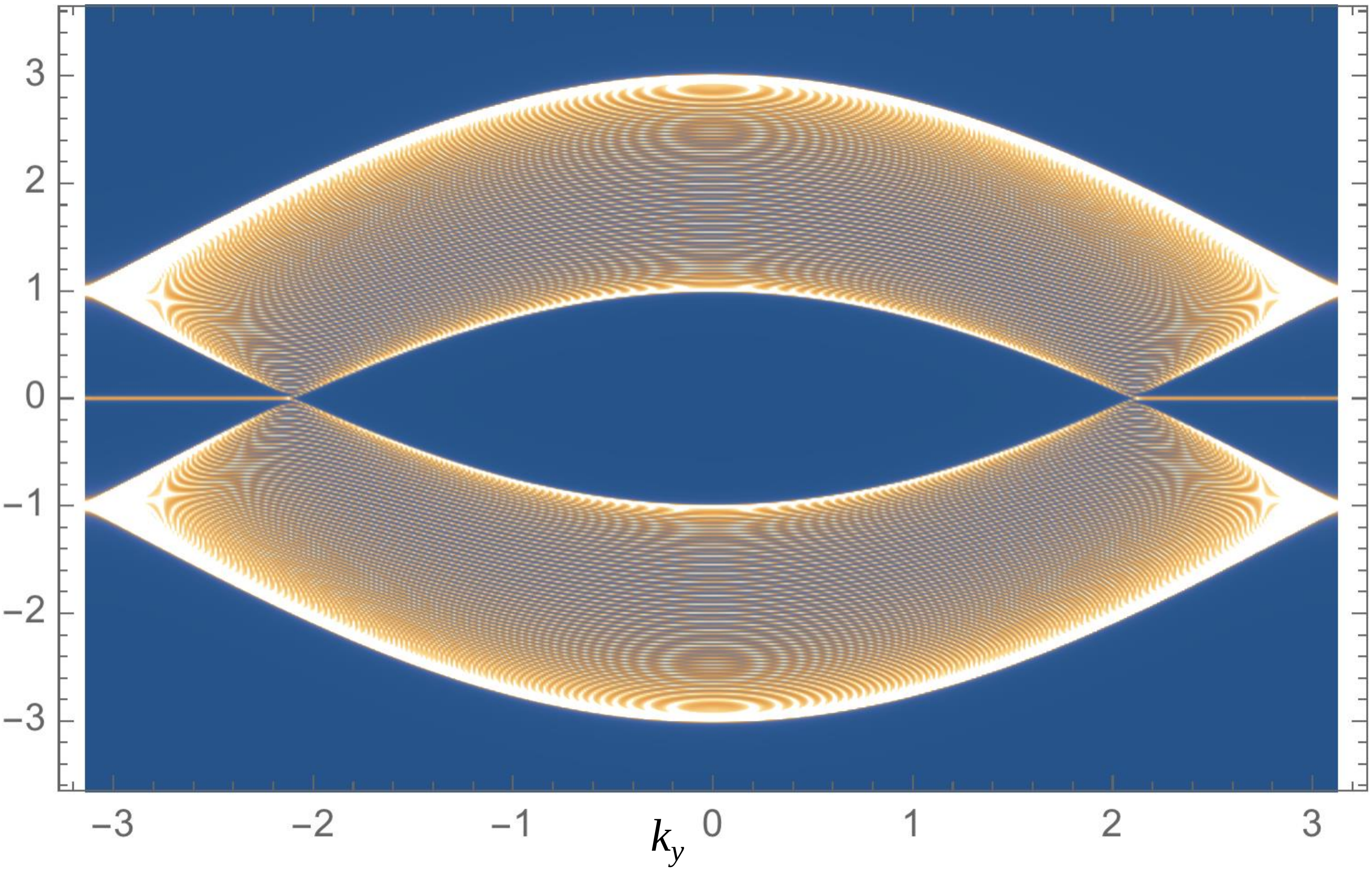


graphene, surface of 3D TI, …

Dirac equation

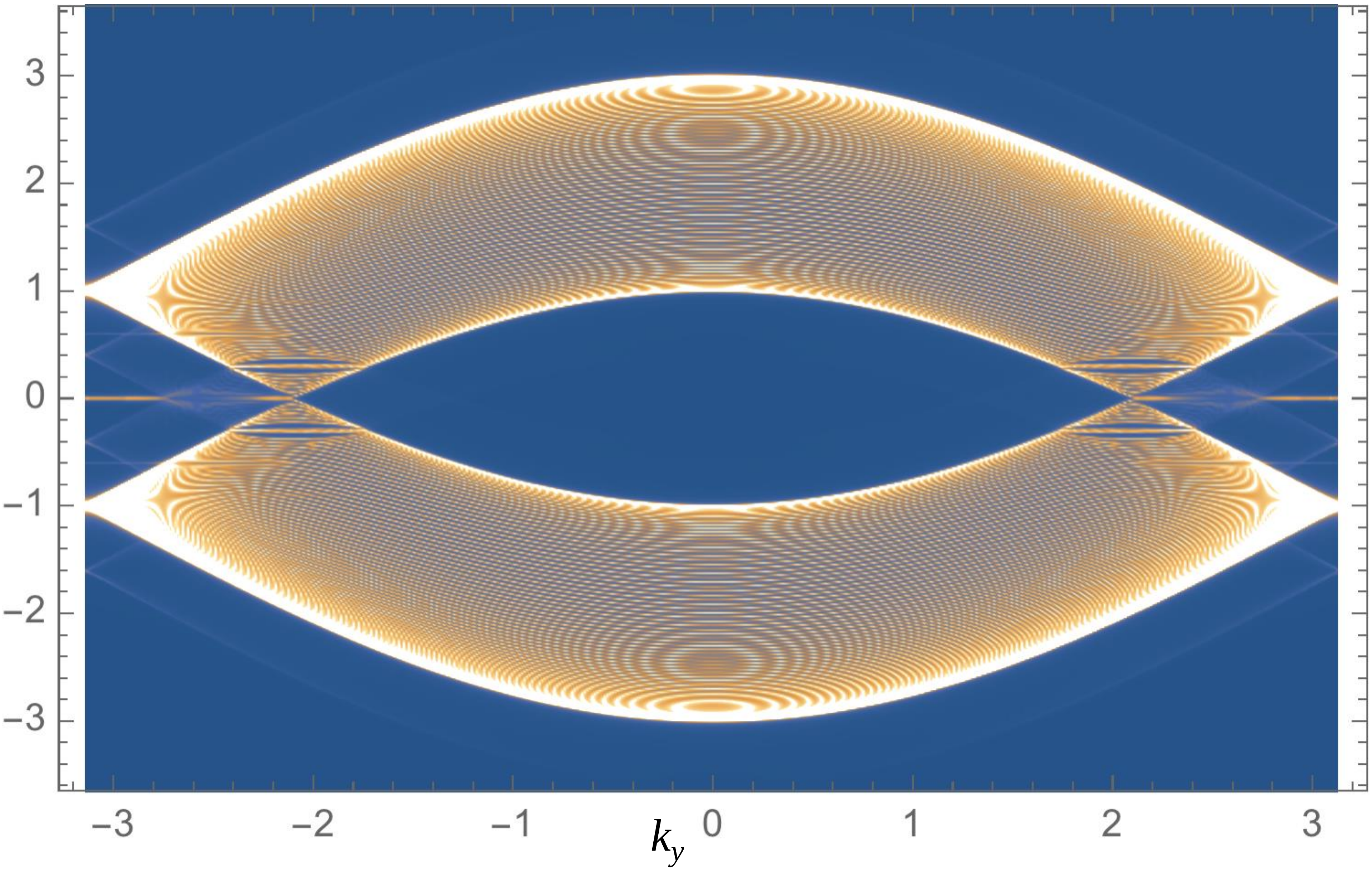
$$H_{\mathrm{Dirac}} = \sigma_x \hat{p}_x + \sigma_y (p_y - \underbrace{B\cos\Omega t x}_{= A_y})$$

Spectrum of H_F



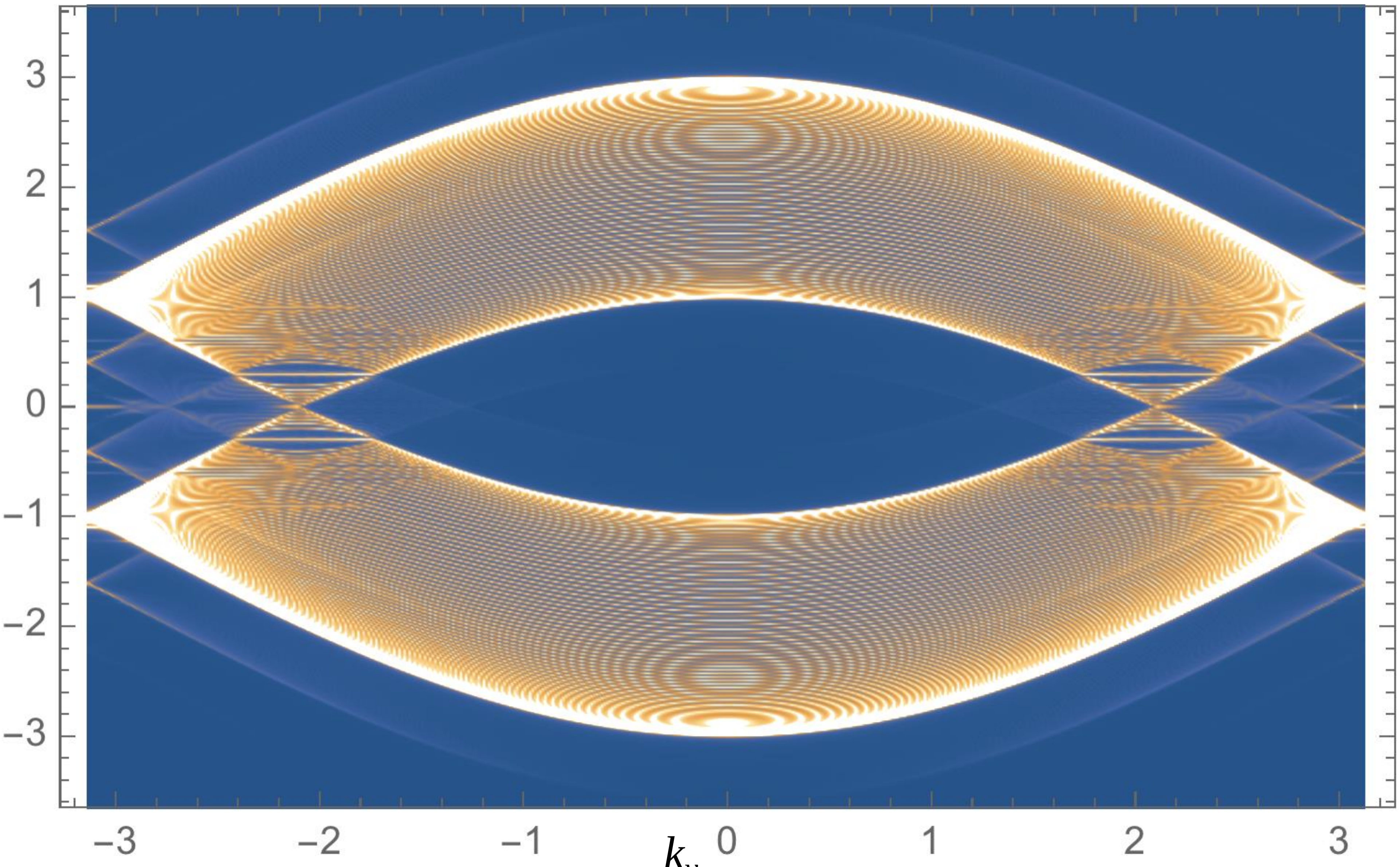
Ω=*0.6*, **B/a=0.000**, E_x=0.0 honeycomb, zigzag edge

Spectrum of H_F



Ω=*0.6*, **B/a=0.0010**, E_x=0.0   honeycomb, zigzag edge

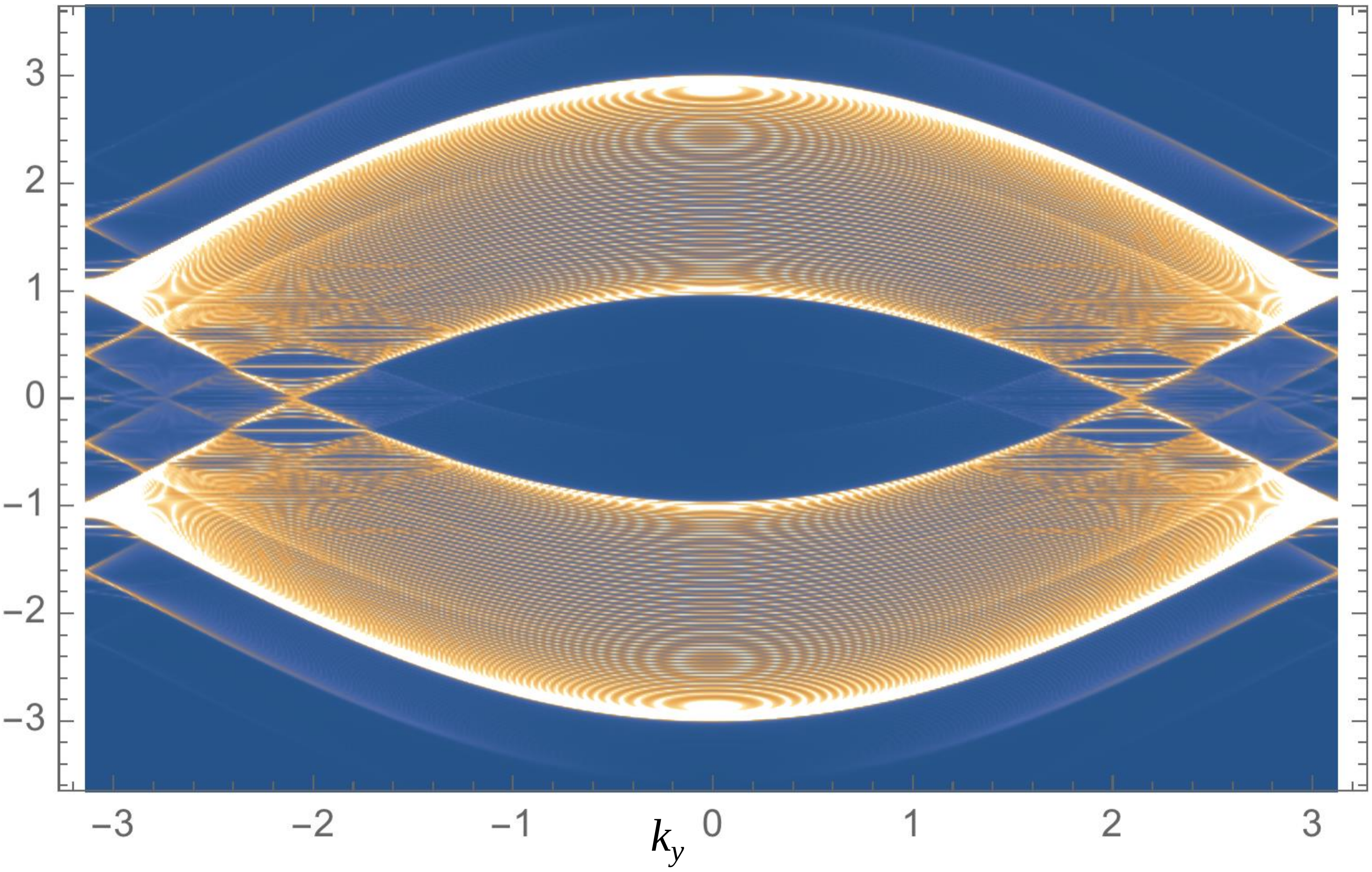
Spectrum of H_F



k_y

Ω=*0.6*, **B/a=0.002**0, E_x=0.0 honeycomb, zigzag edge

Spectrum of H_F



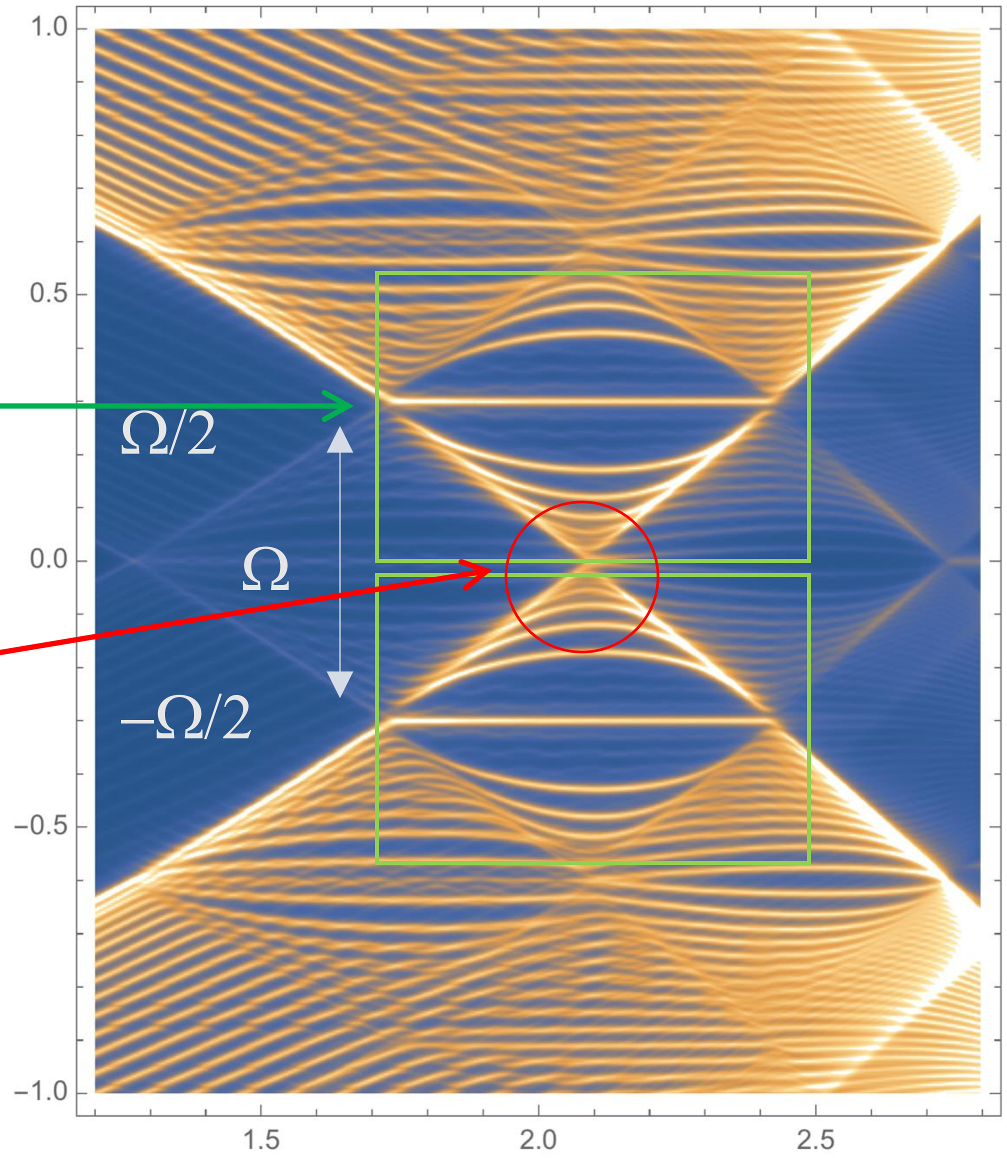
Ω=0.6, **B/a=0.0030**, E_x=0.0 honeycomb, zigzag edge

Spectrum of H_F

“π-Landau levels”

Flat bands at $\varepsilon = \pm\Omega/2$

Dirac node at $\varepsilon = 0$



$\Omega = 0.6$, $B/a = 0.0030$, $E_x = 0.0$

Effective Hamiltonian for the “π-Landau levels”

$$H_{\mathrm{Dirac}} = \sigma_x \hat{p}_x + \sigma_y (p_y - B\cos\Omega t x)$$

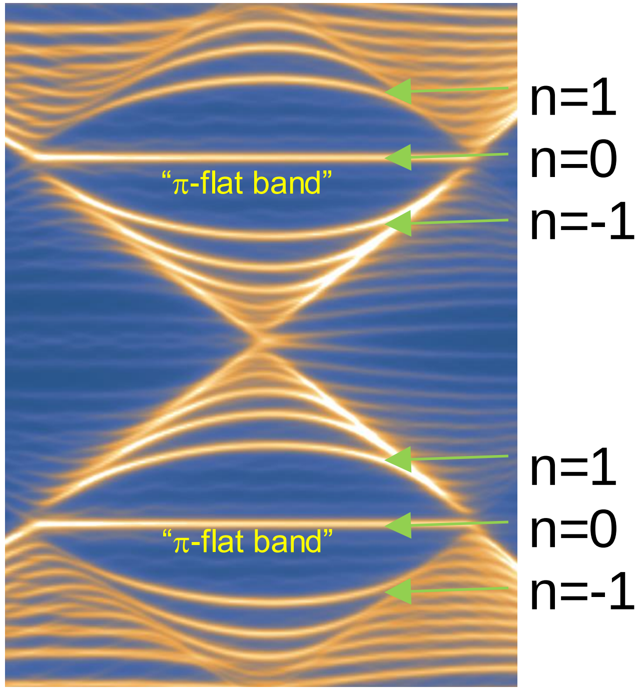
↓ rotating frame transformation
+ time average

Slow motion

$$H_{\mathrm{eff}} = \cos\theta \begin{pmatrix} 0 & -i\partial_x + i\frac{B}{2}x \\ -i\partial_x - i\frac{B}{2}x & 0 \end{pmatrix}$$

Landau levels of 2D Dirac system

$$\varepsilon_n = \sqrt{\Omega^2 - p_y^2}\sqrt{Bn} \pm \Omega/2$$

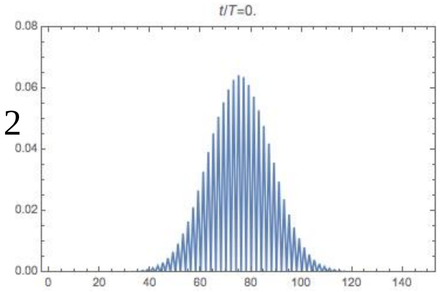


two n=0 states

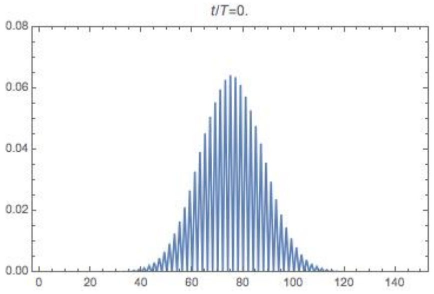
two n=1 states

Fast motion

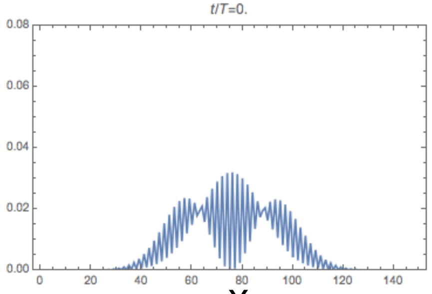
$|\psi|^2$



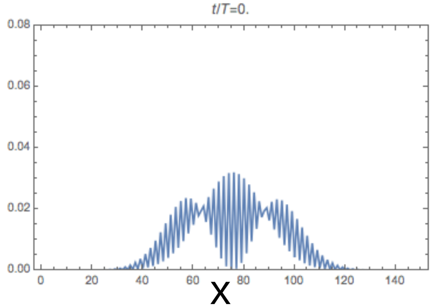
x



x



x



x

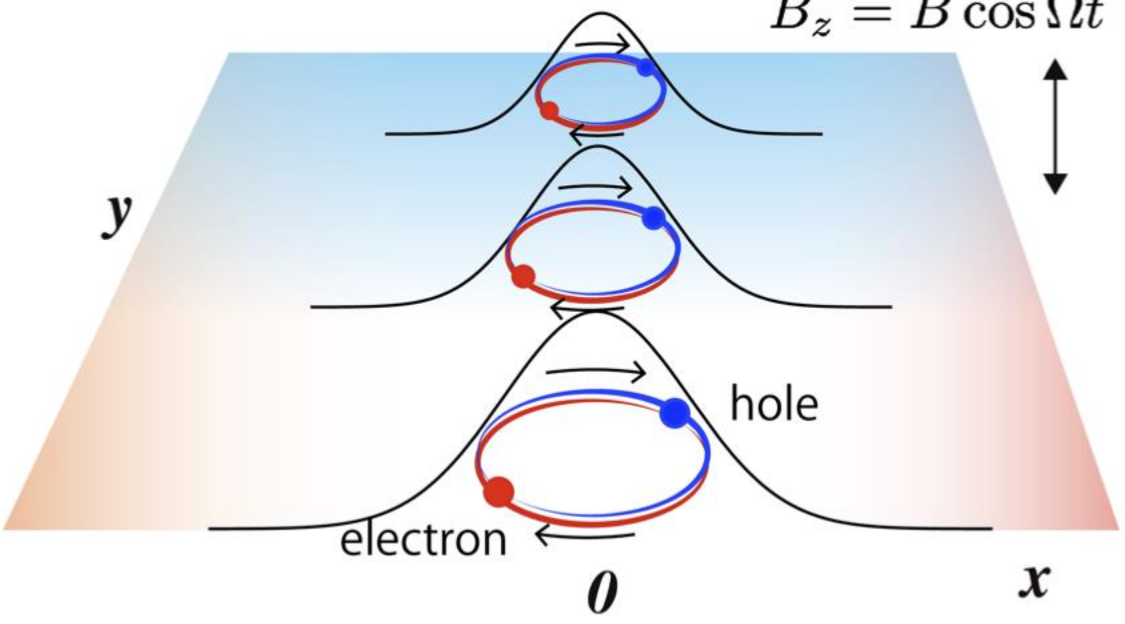
Degenerate flat bands protected by time-glide symmetry (Morimoto-Po-Vishwanath’17)

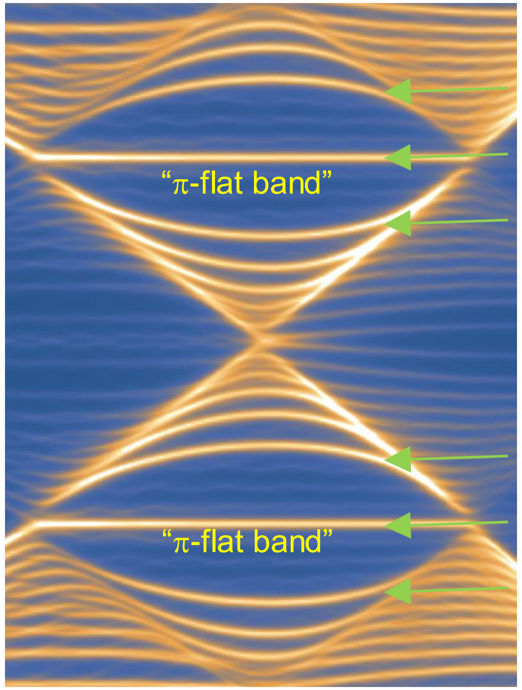
Effective Hamiltonian for the “π-Landau levels”

Rabi-oscillation between electron and hole 0th LL

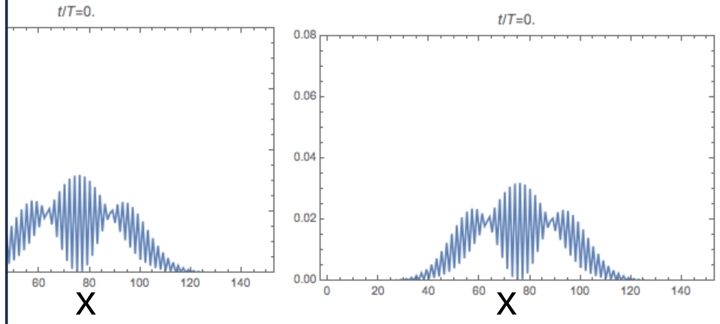
$$|\phi(t)\rangle \sim \cos\left(\frac{\Omega}{2}t\right)|\psi_+\rangle + \sin\left(\frac{\Omega}{2}t\right)|\psi_-\rangle$$

0LL of +B   0LL of -B



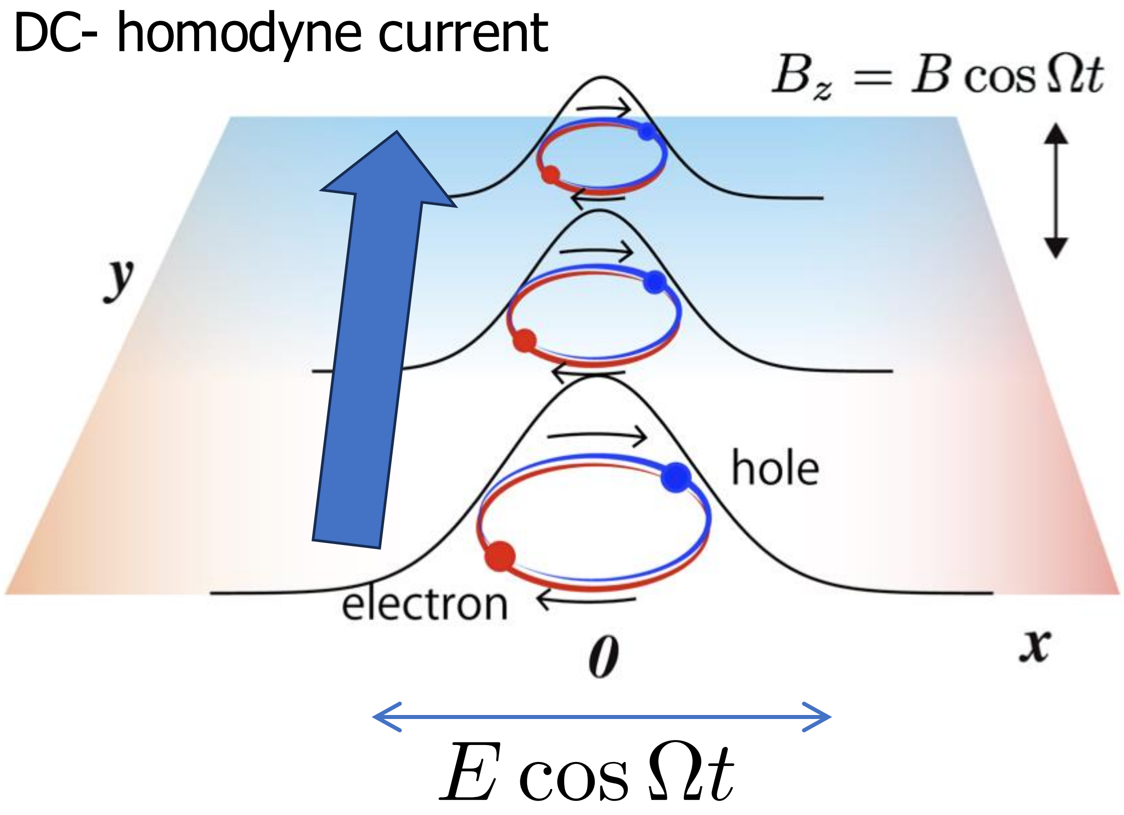


two n=1 states



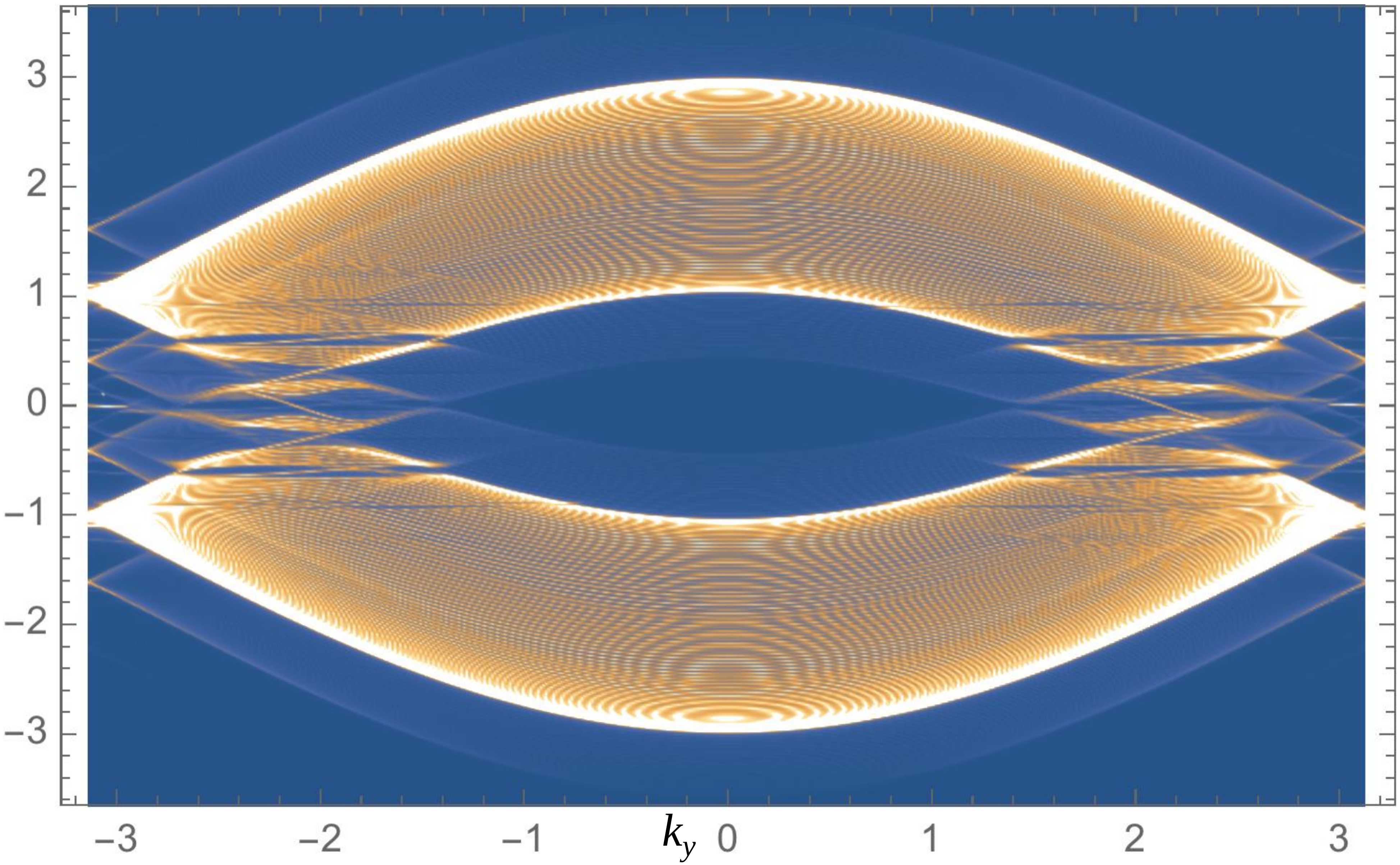
The flat band is protected by time-glide symmetry (Morimoto-Po-Vishwanath’17)

2D Dirac electron in oscillating B and E fields



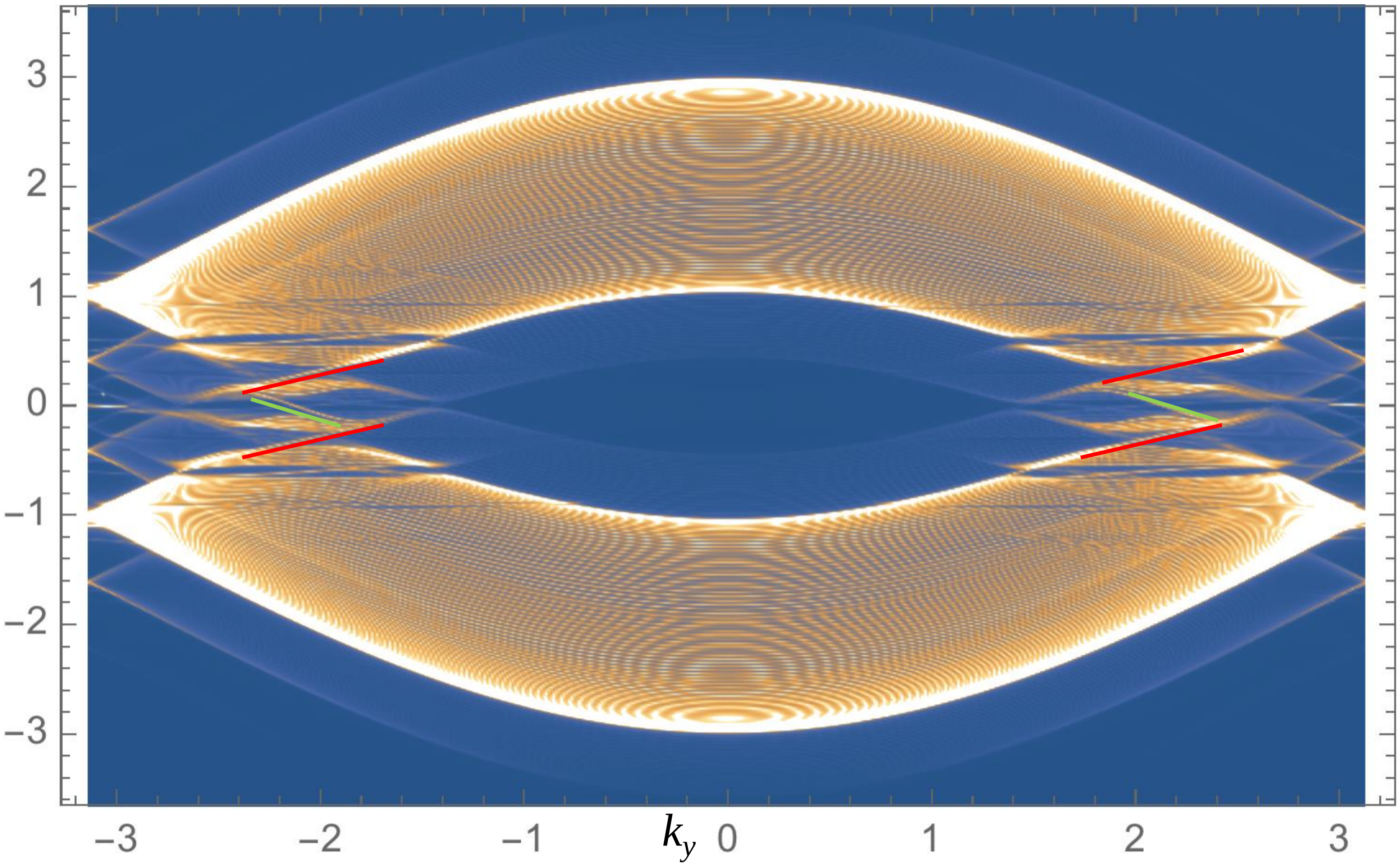
$$H_{\rm 2DD} = \sigma_x\left(p_x + \frac{E_x}{\Omega}\sin(\Omega t)\right) + \sigma_y\left(p_y - B_z\cos(\Omega t)x\right)$$

Heterodyne Hall effect (add B and E)



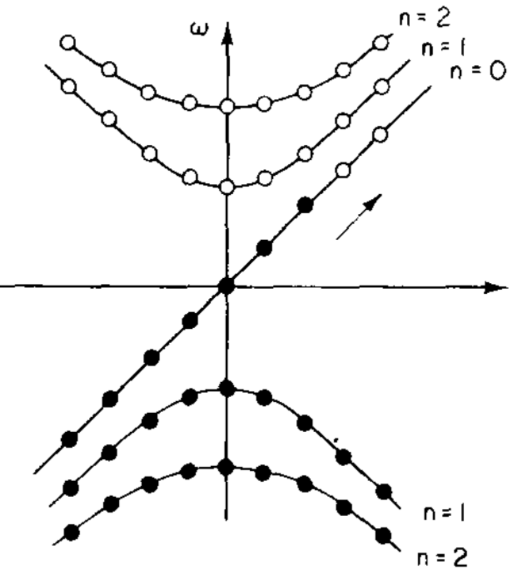
Ω=*0.6, B/a*=0.0020, E_x=0.20 honeycomb, zigzag edge

Heterodyne Hall effect (add B and E)



Ω=*0.6, B/a*=0.0020, E_x=0.20 honeycomb, zigzag edge

Many chiral bands



Nielsen-Ninomiya, Phys. Lett.'83

"π-Landau levels"

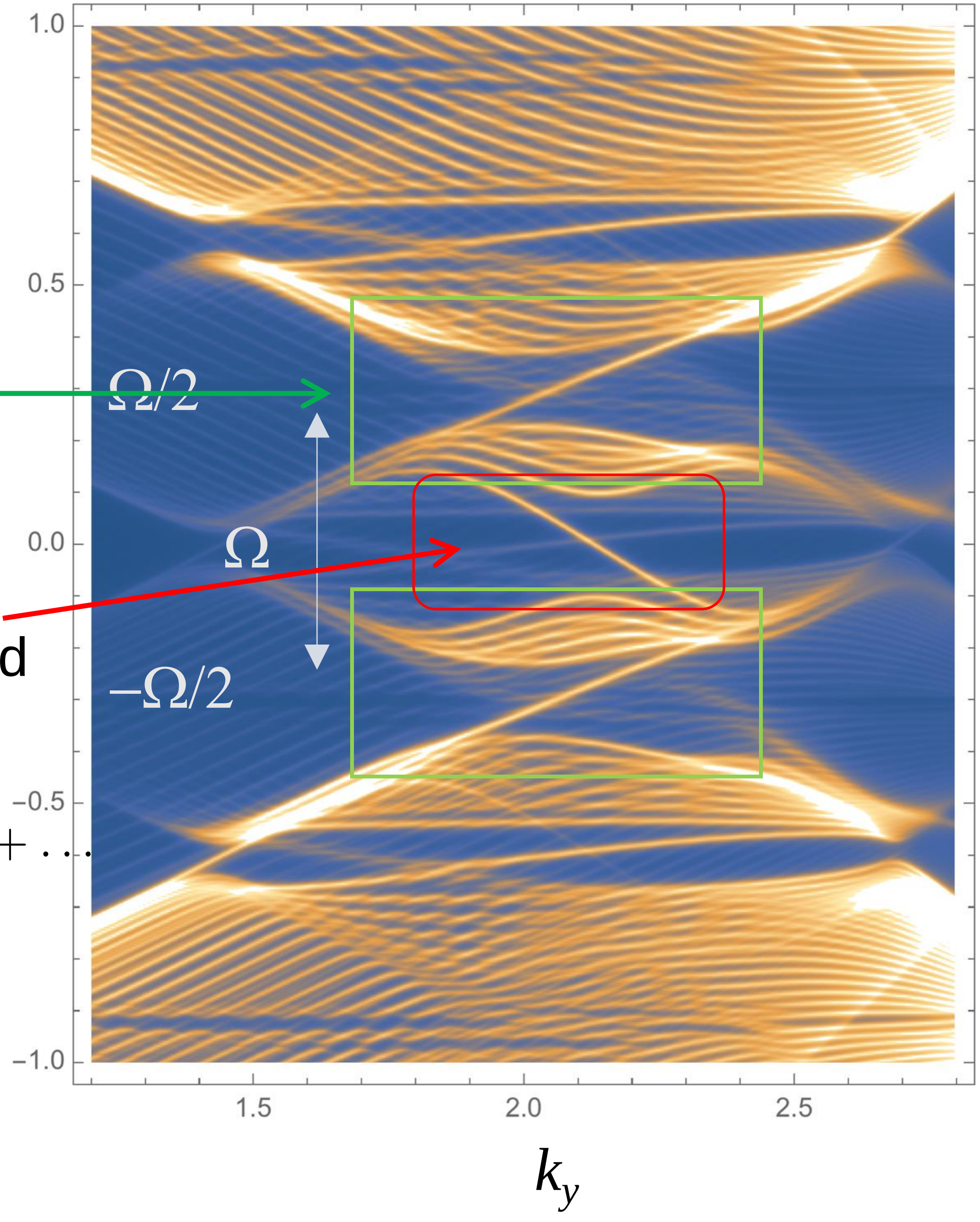
chiral bands at ε=±Ω/2

Dirac node→chiral band

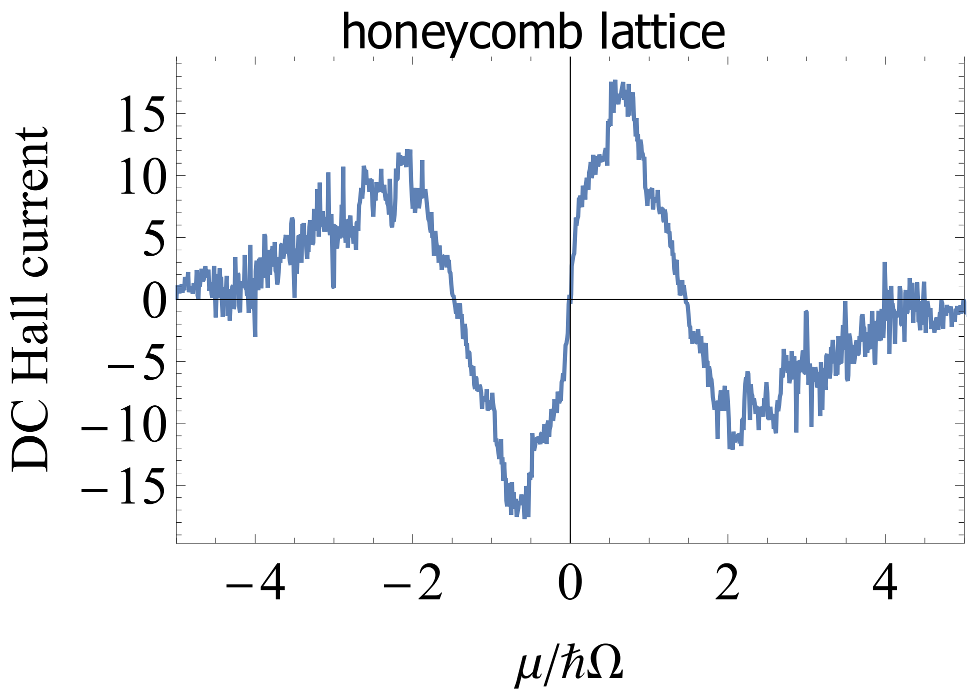
Floquet effective Ham.

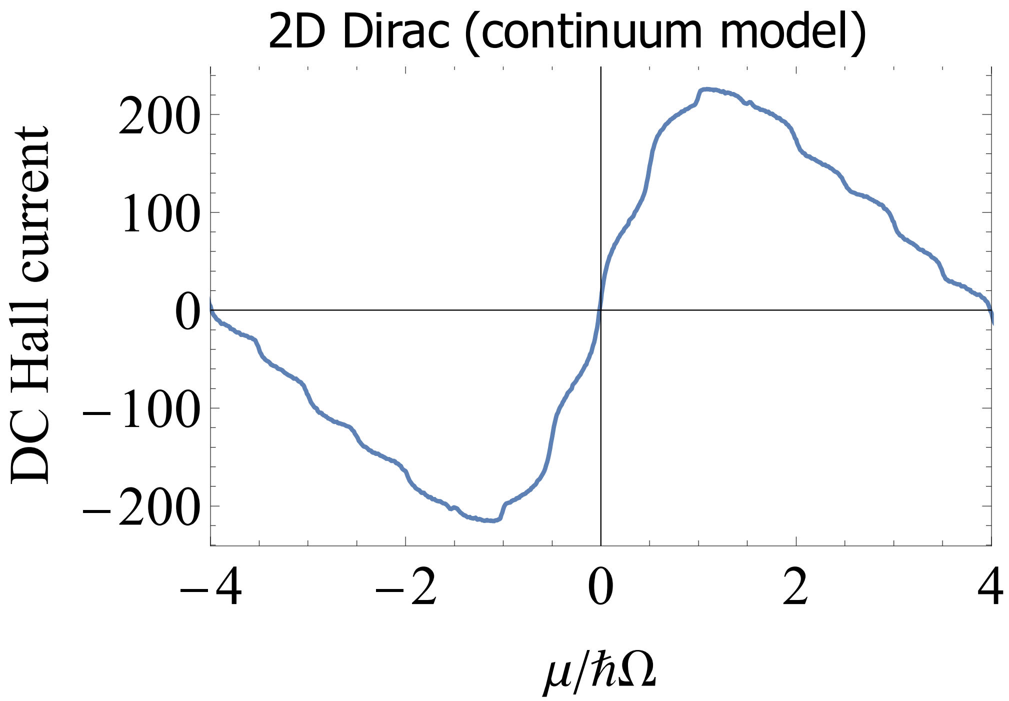
$$H_{\rm F} = \sigma_x \hat{p}_x + \sigma_y p_y + \sigma_z \frac{E_x B}{\Omega^2} x + \ldots$$

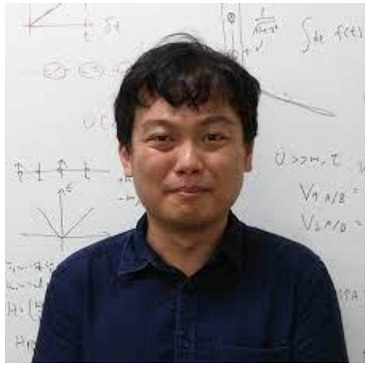
~ 3D Weyl in "$B_{\rm eff}$"



Chiral anomaly induced homodyne effect







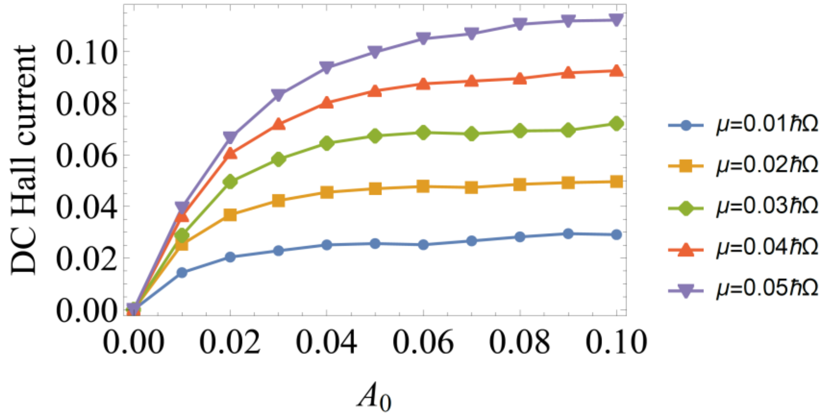
Sota Kitamura
(U-Tokyo)

The Floquet-CME behavior

$$J_y = 2\frac{e^2}{h}\mu$$

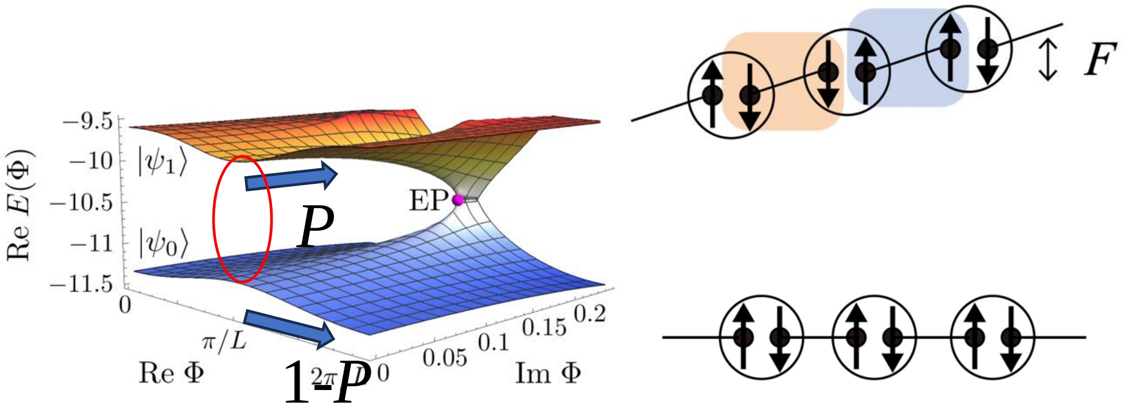
similar to

Fukushima-Kharzeev-Warringa '08



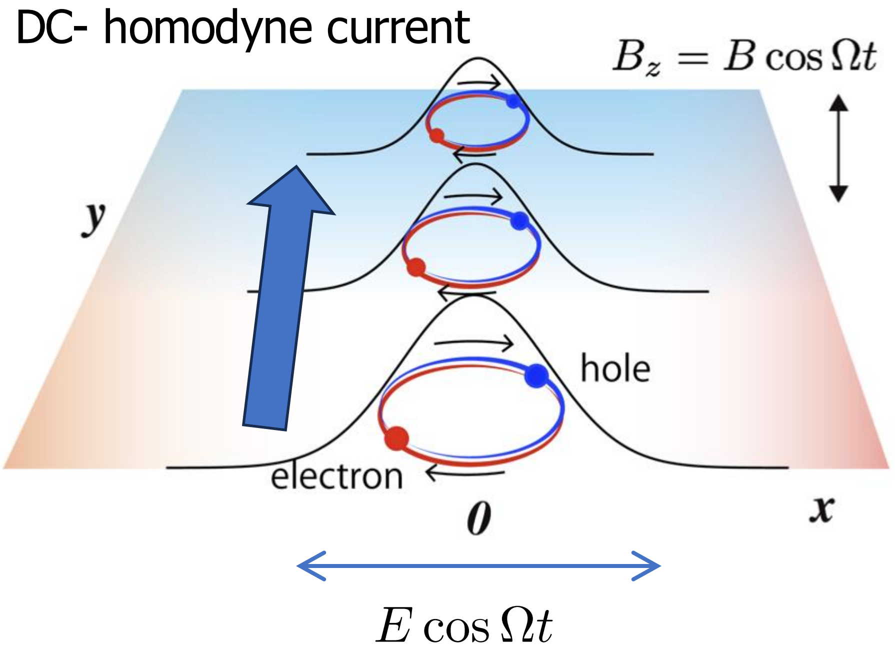
Summary of part 2, 3

2. Tunneling breakdown from a SPT phase



- Information of tunneling
  in the non-Hermitian model
- Non-Hermitian MPS state
- Future: General topologically ordered state

3. Floquet state in AC-magnetic fields



- Landau-levels and chiral bands are formed
- Future: Disorder, interaction, fractional filling