An operator growth "hypothesis" in open quantum systems

Pratik Nandy

Yukawa Institute for Theoretical Physics (YITP), Kyoto U
&
iTHEMS, RIKEN

Kavli IPMU, Tokyo

Based on
1) arXiv: 2212.06180 (JHEP) with B. Bhattacharjee (IBS), X. Cao (ENS) and T. Pathak (YITP)
2) arXiv: 2311.00753 (JHEP) with B. Bhattacharjee (IBS) and T. Pathak (YITP)

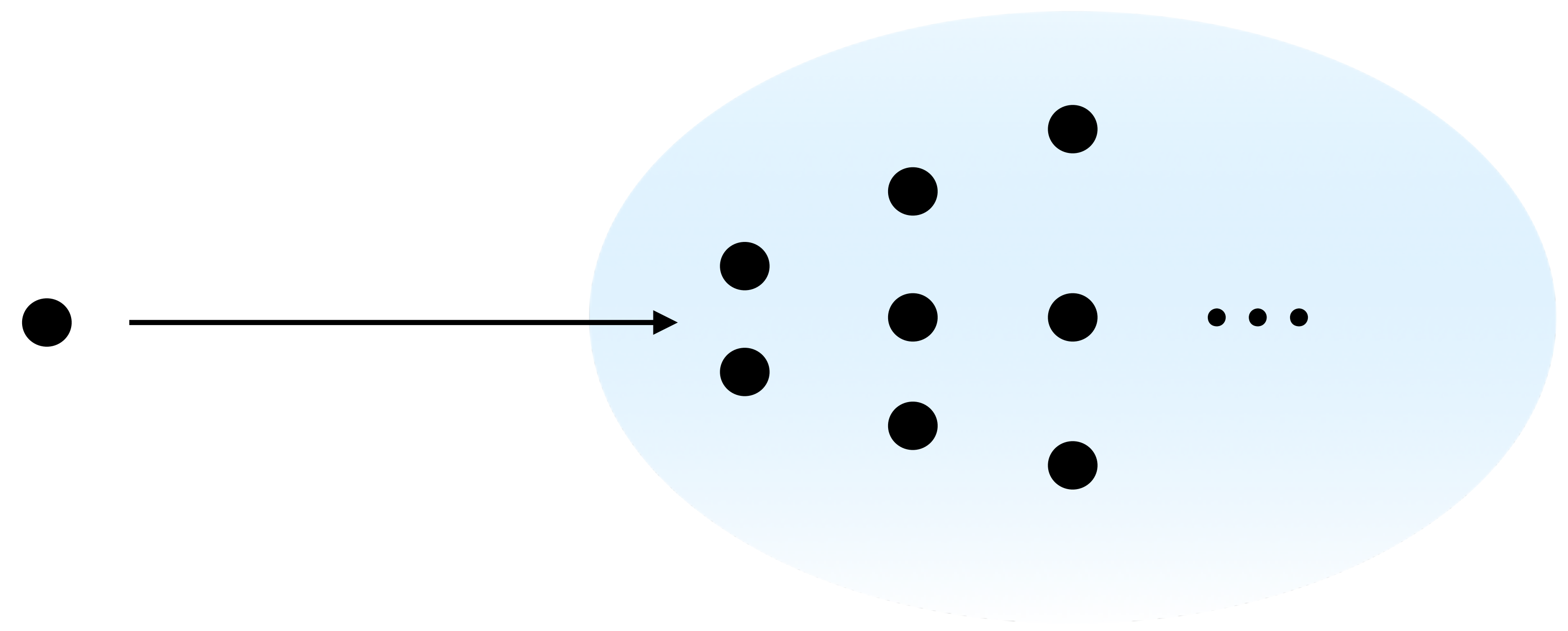
Review (Phys. Rept.): arXiv: 2405.09628 - with A.M. Roubeas (Lux), P. M. Azcona (Lux), A. Dymarsky (Kentucky) and A. del Campo (Lux)

YUKAWA INSTITUTE FOR THEORETICAL PHYSICS

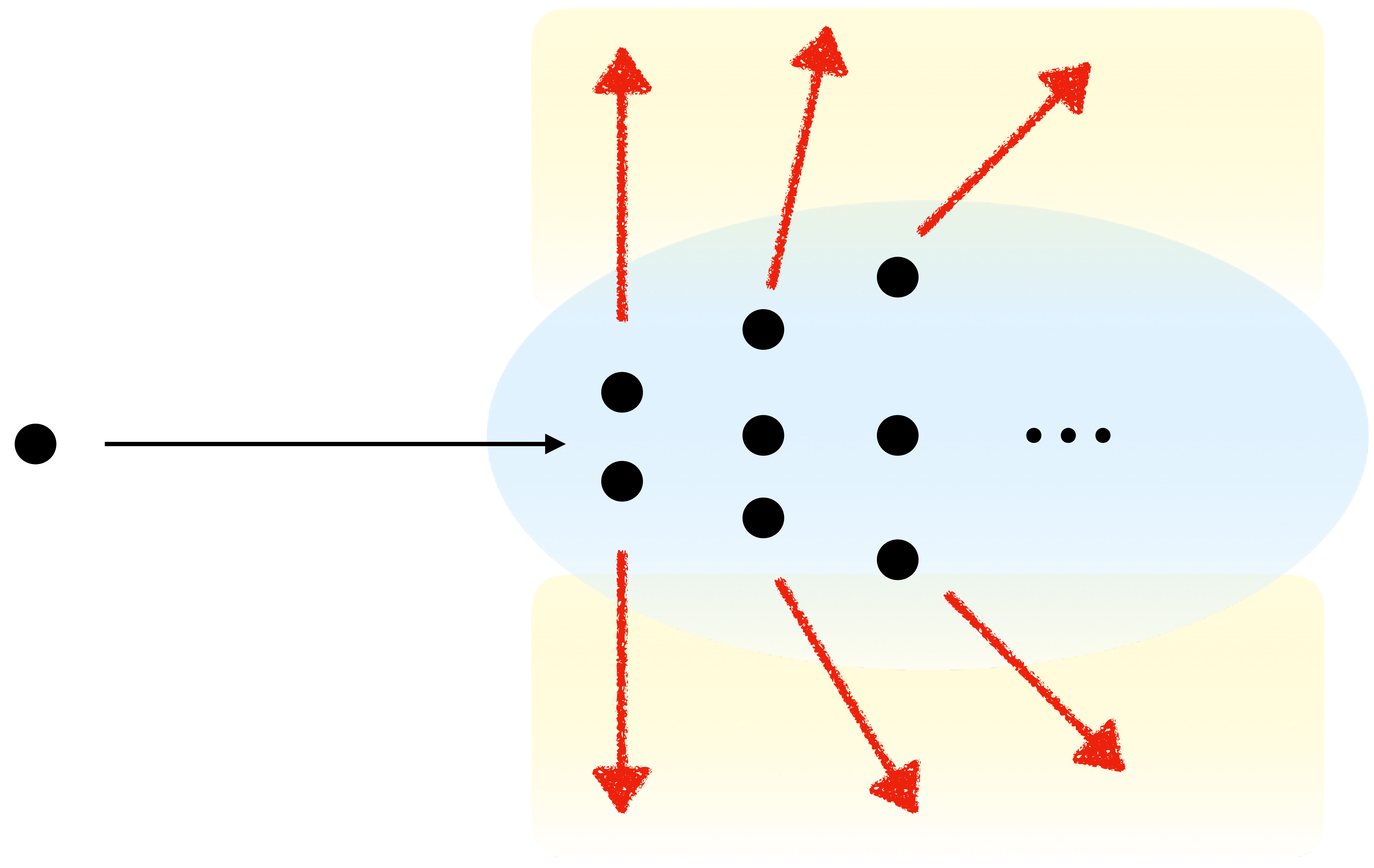
Grant-in-Aid for Transformative Research Areas A
Extreme Universe

RIKEN iTHEMS
RIKEN interdisciplinary Theoretical & Mathematical Sciences

Information flow across a system



$t_* \sim \log N$



$t_* \sim \log N \qquad t_d \sim ?? \qquad X \sim ??$

Questions:

1) How does the information flow across a system which is connected with a dissipative environment?

2) Can we quantify some universal quantity for such "dissipative" information flow?

We put the information to a system in terms of an operator.
The information flow is measured how the operator evolves in time.

1. Operator growth: what is it?

2. Operator growth hypothesis: Lanczos algorithm and Krylov complexity

Example: dissipative SYK

3. Operator growth in dissipative systems : Introducing bi-Lanczos algorithm

4. Information flow in open quantum systems: Motivate “dissipative quantum chaos”

5. Conclusion and summary

Other probes of quantum chaos

1. Level statistics: Chaotic systems is supposed to follow Wigner-Dyson statistics

Wigner (1958), Dyson (1962), Bohigas-Giannoni-Schmit (1984)

Generalized to open quantum systems

Sa-Ribeiro-Prosen (2019), Kawabata-Xiao-Ohtsuki-Shindou (2022)

2. Spectral form factor

CGHPS^{4}T (2016)

Generalized to open quantum systems

Xu-Chenu-Prosen-del Campo (2020), **PN**-Pathak-Tezuka (2021)

Interesting from the semiclassical gravity and holographic side

Saad-Shenker-Stanford (2019)

3. Operator size distribution

Roberts-Stanford-Susskind (2014), Roberts-Stanford-Streicher (2018)

Schuster-Yao (2022), Zhang-Wu (2023)

4. OTOC

Maldacena-Shenker-Stanford (2015)

Defined on closed systems and can be generalized to open quantum systems

Syzranov-Gorshkov-Galitski (2018)

Operator growth

Consider any Hamiltonian H. Start with a initial simple operator, say Z_1 . Under the time evolution, the simple operator becomes a complicated operator $Z_1(t) = e^{iHt} Z_1(0)\, e^{-iHt}$.

$$Z_1(t) = Z_1 - it[H, Z_1] - \frac{t^2}{2!}[H,[H,Z_1]] + \frac{it^3}{3!}[H,[H,[H,Z_1]]] + \cdots = Z_1 - it\mathscr{L}\, Z_1 - \frac{t^2}{2!}\mathscr{L}^2 Z_1 + \frac{it^3}{3!}\mathscr{L}^3 Z_1 \cdots = e^{i\mathscr{L}t} Z_1 \,.$$

Example: $$H = -\sum_{i=1}^{N} Z_i Z_{i+1} - g\sum_{i=1}^{N} X_i - h\sum_{i=1}^{N} Z_i$$

Evaluate the commutators

$$\mathscr{L}\, Z_1 = [H, Z_1] \sim Y_1$$

Liouvillian $\mathscr{L} \cdot = [H, \cdot\,]$

$$\mathscr{L}^2 Z_1 = [H,[H,Z_1]] \sim Y_1 + X_1 Z_2$$

Increasing support of many operators

$$\mathscr{L}^3 Z_1 = [H,[H,[H,Z_1]]] \sim Y_1 + X_1Y_2 + Y_1Z_2$$

$$\mathscr{L}^4 Z_1 = [H,[H,[H,[H,Z_1]]]] \sim X_1 + Y_1 + Z_1 + X_1X_2 + Y_1Y_2 + Z_1Z_2 + X_1Z_2 + Y_1Z_3 + Y_1Z_2Y_2 + Z_1X_2X_1 + X_2Z_3X_1$$

Given an initial operator $\mathcal{O}_0$, the time evolution is expanded on a basis of nested commutators.

$$\tilde{\mathcal{O}}_n = \mathcal{L}^n \mathcal{O}_0 \quad n = 0,1,2, \cdots$$

We basis states may not be orthonormal. So we use a Gram-Schmidt (GS) orthonormalisation procedure to produce **orthonormal basis (Krylov basis).**

Inner product

$$\tilde{\mathcal{O}}_n \xrightarrow{\text{GS}} \mathcal{O}_n \qquad \langle \mathcal{O}_m | \mathcal{O}_n \rangle = \delta_{mn} \qquad \langle A | B \rangle = \frac{1}{D}\mathrm{Tr}(A^\dagger B)$$

Lanczos algorithm: The procedure to obtain such **orthonormal Krylov basis.**

The time evolution is in Krylov basis: $\quad | \mathcal{O}(t) \rangle = \sum_n i^n \varphi_n(t) | \mathcal{O}_n \rangle$

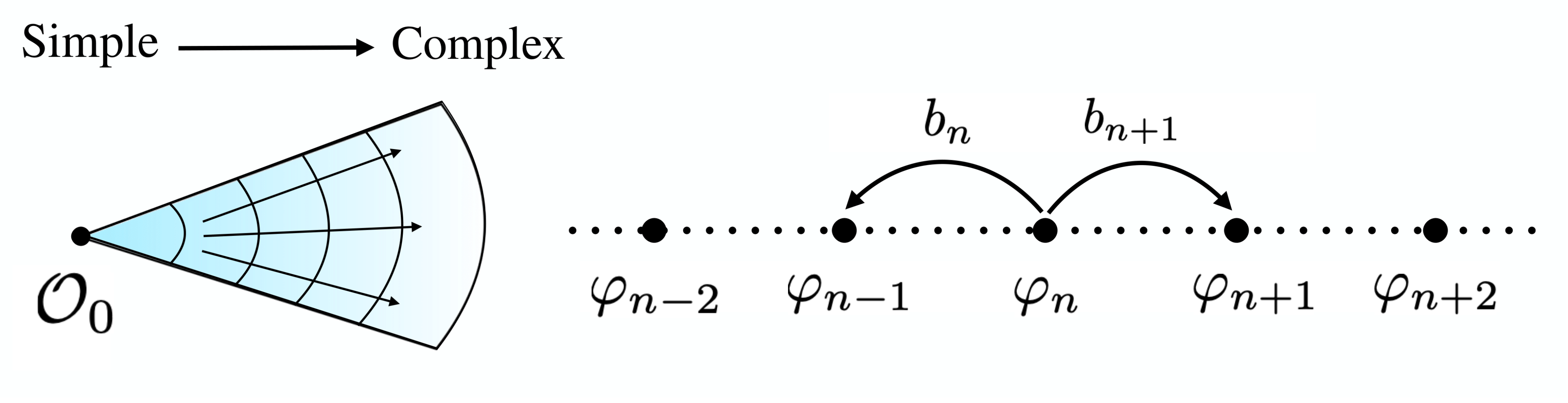
Autocorrelation function $\quad C(t) \equiv \varphi_0 = \langle \mathcal{O}(t) | \mathcal{O}_0 \rangle \quad$ contains the full information of the growth

Viswanath-Muller (1994)

Parker-Cao-Avdoshkin-Scaffidi-Altman (2018)

The φ_n's satisfy the following equation which is simple

$$\dot{\varphi}_n(t) = b_n \varphi_{n-1}(t) - b_{n+1} \varphi_{n+1}(t)$$



Inputs:		Outputs:
Hamiltonian H and the initial operator $\mathcal{O}_0$	Lanczos algorithm	Lanczos coefficients $\{b_n\}$ and the Krylov basis $\{\mathcal{O}_n\}$

Liouvillian becomes tridiagonal in Krylov basis

$$\mathcal{L} | \mathcal{O}_n \rangle = b_{n+1} | \mathcal{O}_{n+1} \rangle + b_n | \mathcal{O}_{n-1} \rangle$$

$$\mathcal{L} = \begin{pmatrix} 0 & b_1 & 0 & \cdots & 0 \\ b_1 & 0 & b_2 & \cdots & 0 \\ 0 & b_2 & 0 & b_3 & \cdots \\ \cdots & \cdots & b_3 & \cdots & \cdots \\ 0 & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & \cdots & \cdots \end{pmatrix} .$$

Information about b_n $\equiv$ information about $\varphi_n(t)$

Unitarity: $\sum_n |\varphi_n(t)|^2 = 1$

K-complexity: average position of the particle in Krylov chain

Parker-Cao-Avdoshkin-Scaffidi-Altman (2018)

$$K(t) := \frac{\sum_n n\,|\varphi_n(t)|^2}{\sum_n |\varphi_n(t)|^2} = \sum_n n\,|\varphi_n(t)|^2$$

Define Krylov operator: $\hat{K}\,|\mathcal{O}_n\rangle = n\,|\mathcal{O}_n\rangle$ (Number operator in Krylov space)

K-complexity: expectation value of the Krylov operator in the time-evolved state $K(t) = \langle \mathcal{O}(t)\,|\,\hat{K}\,|\,\mathcal{O}(t)\rangle$

Cumulant generating functional: $\log\langle e^{\lambda\hat{K}}\rangle = \log(\mathcal{O}(t)\,|\,e^{\lambda\hat{K}}\,|\,\mathcal{O}(t)) = \log\left(\sum_n e^{\lambda n}\,|\varphi_n(t)|^2\right).$

n-th cumulant: $k_n = \partial_\lambda^k \log\langle e^{\lambda\hat{K}}\rangle\,|_{\lambda=0}$ first cumulant = K-complexity, second cumulant = K-variance etc.

Universal operator growth hypothesis

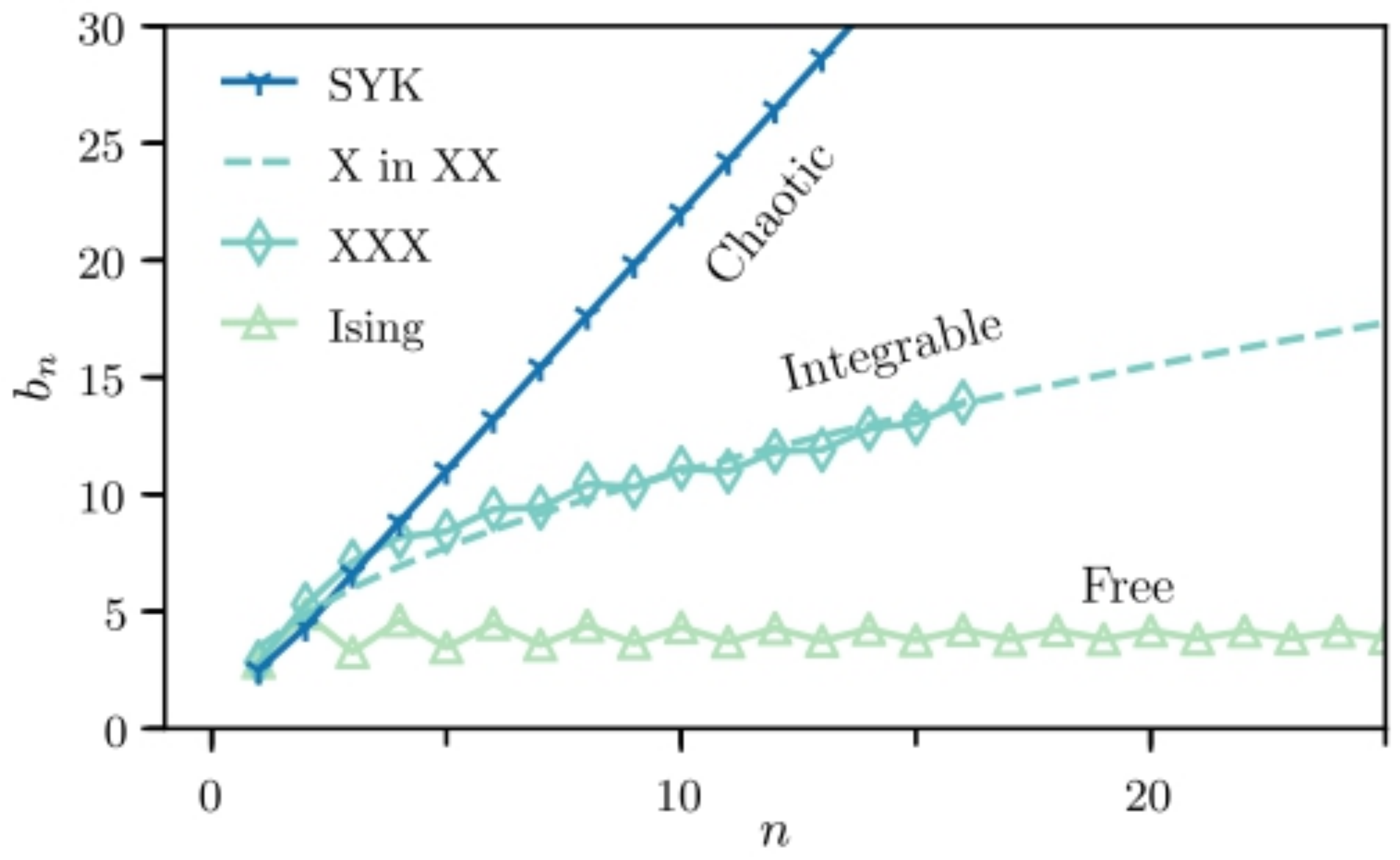
Parker-Cao-Avdoshkin-Scaffidi-Altman (2018)

"For chaotic systems, the Lanczos coefficients grow linearly, and this is the maximum growth possible" $b_n \sim \alpha\, n\,.$

For chaotic systems, K-complexity grows exponentially: $K(t) \sim e^{2\alpha t}$

Integrable systems **usually** show sublinear growth: $b_n \sim \alpha n^{\delta}, \quad 0 < \delta < 1$

K-complexity shows power-law like growth: $K(t) \sim (\alpha t)^{\frac{1}{1-\delta}}$



Growth of Lanczos coefficients
[Parker et al. (2018)]

The reverse statement is not always true

"The linear growth of Lanczos coefficients does not necessary imply chaos".

Dymarsky-Smolkin (2021)

Bhattacharjee-Cao-**PN**-Pathak (2022)

Relaxing the smoothness condition, the above hypothesis may require modifications.

Avdoshin-Dymarsky-Smolkin (2022), Camargo et al. (2022)

Spectral function: $$\Phi(\omega) = \int_{-\infty}^{\infty} C(t)\, e^{-i\omega t}\, dt$$

Moments: $$m_{2n} = \frac{1}{2\pi}\int_0^{\infty} \mathrm{d}\omega\, \omega^{2n}\, \Phi(\omega) = \frac{1}{i^n} \lim_{t\to 0} \frac{d^n C(t)}{dt^n}\,,$$ (after contour integration)

The autocorrelation function: $$C(t) := \sum_{n=0}^{\infty} m_n \frac{(it)^n}{n!}$$

Iteratively find the Lanczos coefficients as

$$M_k^{(0)} = (-1)^k m_k\,, \quad L_k^{(0)} = (-1)^{k+1} m_{k+1}\,,$$

$$M_k^{(n)} = L_k^{(n-1)} - L_{n-1}^{(n-1)} \frac{M_k^{(n-1)}}{M_{n-1}^{(n-1)}}\,, \qquad L_k^{(n)} = \frac{M_{k+1}^{(n)}}{M_n^{(n)}} - \frac{M_k^{(n-1)}}{M_{n-1}^{(n-1)}}\,, \qquad k \geq n\,,$$

$$b_n = \sqrt{M_n^{(n)}}, \qquad a_n = -\,L_n^{(n)}\,.$$

For unitary evolution, $m_{2n+1} = 0$ and thus $a_n = 0$

Auto-correlation function

$\downarrow$

Moments

$\downarrow$

Lanczos coefficients

$\downarrow$

Krylov wave functions

$\downarrow$

Krylov complexity, or other cumulants

Example: Sachdev-Ye-Kitaev (SYK) model

Sachdev-Ye (1993), Kitaev (2015)

Maldacena-Stanford (2016)

Hamiltonian $H = i^{q/2} \sum_{1 \le i_1 < i_2 < \cdots < i_q \le N} j_{i_1 \cdots i_q} \psi_{i_1} \psi_{i_2} \cdots \psi_{i_q}$

Mean: $\langle j_{i_1 \cdots i_q} \rangle = 0$

Variance: $\langle j^2_{i_1 \cdots i_q} \rangle = 2^{q-1} \dfrac{(q-1)!\mathcal{J}^2}{qN^{q-1}}$

We expand the auto-correlation function for the initial operator $\mathcal{O}_0 = \sqrt{2}\,\psi_1$

$$C(t) = 1 + \frac{g(t)}{q} + \cancel{\frac{h(t)}{q^2}} + \cdots$$

SD Eq: $\partial_t^2 g(t) = -2\mathcal{J}^2 e^{g(t)}$

BC: $g(0) = 0, \quad g'(0) = 0$

Solution:

$$g(t) = 2\ln(\operatorname{sech} \mathcal{J} t)$$

Moments: $m_{2n} = \dfrac{1}{q} \mathcal{J}^{2n} T_{n-1} + O(1/q^2)\,, \qquad n \ge 1\,.$

Tangent numbers: $\{T_{n-1}\}_{n=1}^{\infty} = \{1, 2, 16, 272, 7936, \cdots\}$

Parker-Cao-Avdoshkin-Scaffidi-Altman (2018)

Lanczos coefficients $b_n = \begin{cases} \mathcal{J}\sqrt{2/q} + O(1/q)\,, & n = 1 \\ \mathcal{J}\sqrt{n(n-1)} + O(1/q)\,, & n > 1 \end{cases}$

K-Complexity: $K(t) = \dfrac{2}{q} \sinh^2(\mathcal{J} t) + O(1/q^2)$

Open quantum systems

System + environment (bath) undergoes a unitary evolution

$$i\frac{d|\psi_{SE}(t)\rangle}{dt} = H_{SE}|\psi_{SE}(t)\rangle \qquad |\psi_{SE}(t)\rangle = U_{SE}(t,t_0)|\psi_{SE}(t_0)\rangle$$

Evolution of density matrices $\rho_{SE}(t) = U_{SE}(t,t_0)\,\rho_{SE}(t_0)\,U^{\dagger}_{SE}(t,t_0)$

We are interested in the evolution of system density matrices $\rho_S(t) = \mathrm{Tr}_E\,\rho_{SE}(t)$

We are mostly ignorant about the specific details of the environment.

The evolution of system density matrices (in generic cases)

$$\rho_S(t) = \sum_k E_k\,\rho_S(t_0)\,E_k^{\dagger}$$

$E_k \rightarrow$ Kraus operators

The Kraus operators satisfy the constraint

$$\sum_k E_k^{\dagger}E_k = I$$

We are interested in the Markovian dynamics, where $\rho_S(t+dt) = \rho_S(t) + O(dt)$ is completely determined by $\rho_S(t)$.

The evolution of system density matrix is governed by the Lindbladian (we omit the suffix S for system)

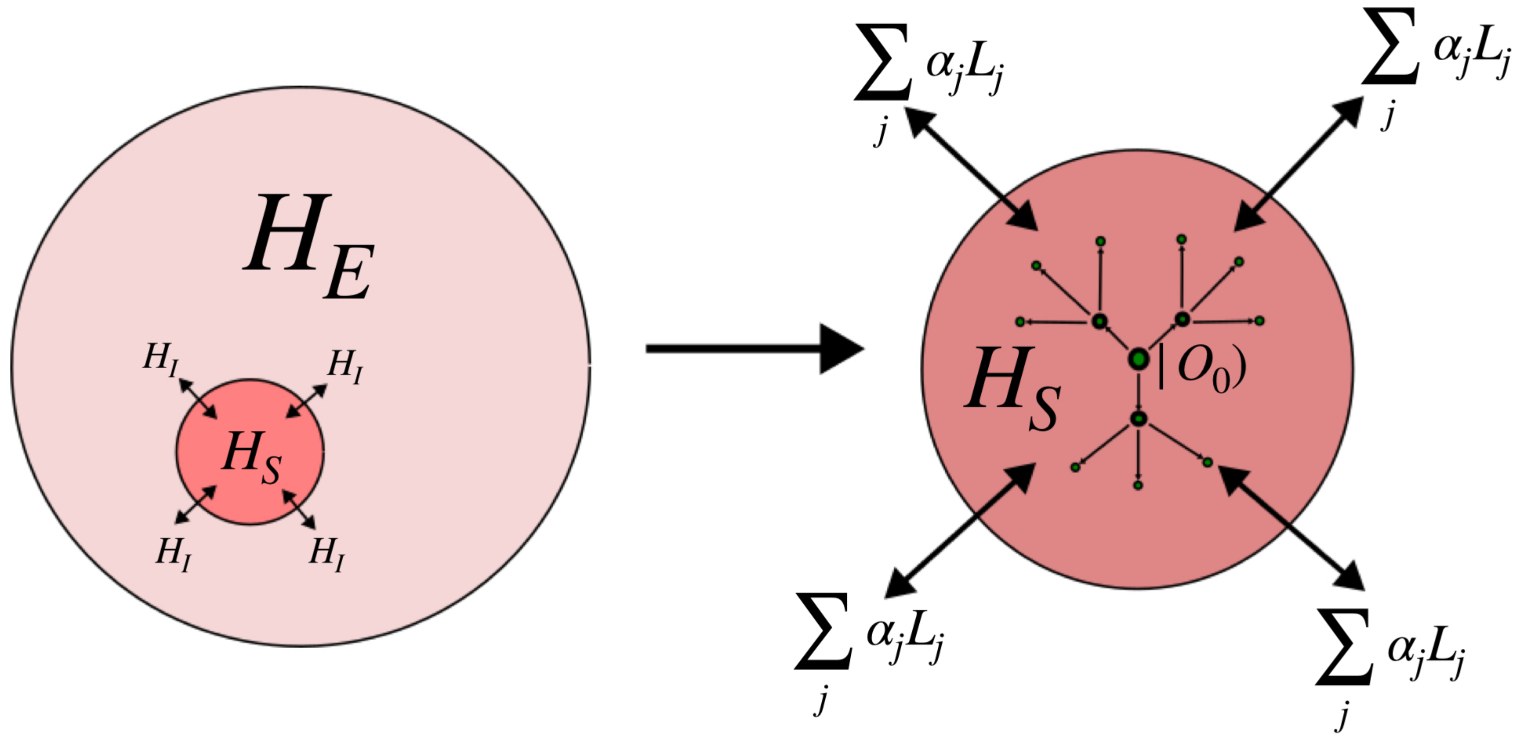
Lindblad (1976)

Gorini-Kossakowski-Sudarshan (1976)

$$\dot{\rho} = -i[H,\rho] + \sum_k \left[L_k \rho L_k^\dagger - \frac{1}{2}\{L_k^\dagger L_k, \rho\}\right]$$

The evolution of any operator is governed by the adjoint of Lindbladian

$$\mathcal{O}(t) = e^{i\mathcal{L}_o^\dagger t}\mathcal{O}_0\,, \qquad \mathcal{L}_o^\dagger \mathcal{O} = [H,\mathcal{O}] - i\sum_k \left[\pm L_k^\dagger \mathcal{O} L_k - \frac{1}{2}\left\{L_k^\dagger L_k, \mathcal{O}\right\}\right].$$



The operators L_k are known as jump (Lindblad) operators and they encode the information between the system and the interaction.

In double Hilbert space, its "vectorization" form

$$\mathcal{L}_o^\dagger \equiv (H\otimes I - I \otimes H^T) - i\sum_k \left[\pm L_k^\dagger \otimes L_k^T - \frac{1}{2}\left(L_k^\dagger L_k \otimes I + I \otimes L_k^T L_k^*\right)\right],$$

Vectorization rule

$$A\,\mathcal{O}\,B \to (A^T \otimes B)(\mathrm{vec}\,\mathcal{O})\,.$$

In the unitary evolution, the Liouvillian is Hermitian. Since the evolution is not unitary in the presence of dissipation, Lanczos algorithm cannot be applied.

We apply a more generic algorithm known as bi-Lanczos algorithm. Other algorithms such as Arnoldi iteration can also be applied.

Bhattacharya-**PN**-Nath-Sahu (2022, 2023)

Create two bi-orthonormal spaces $\langle q_m | p_n \rangle = \delta_{mn}$

The Lindbladian and its adjoint act differently

$$c_j |p_{j+1}\rangle = \mathscr{L}_o^\dagger |p_j\rangle - a_j |p_j\rangle - b_{j-1} |p_{j-1}\rangle$$

$$b_j^* |q_{j+1}\rangle = \mathscr{L}_o |q_j\rangle - a_j^* |q_j\rangle - c_{j-1}^* |q_{j-1}\rangle ,$$

In other words, construct to separate Krylov spaces

$$\mathbb{K}^j(\mathscr{L}_o^\dagger, |p_1\rangle) = \{ |p_1\rangle, \mathscr{L}_o^\dagger |p_1\rangle, (\mathscr{L}_o^\dagger)^2 |p_1\rangle, \dots \} ,$$

$$\mathbb{K}^j(\mathscr{L}_o, |q_1\rangle) = \{ |q_1\rangle, \mathscr{L}_o |q_1\rangle, \mathscr{L}_o^2 |q_1\rangle, \dots \} .$$

In this bi-orthonormal basis, the Lindbladian takes an “ideal” tridiagonal form

$$\mathscr{L}_o^\dagger = \begin{pmatrix} a_1 & b_1 & 0 & \cdots & 0 \\ c_1 & a_2 & b_2 & \cdots & 0 \\ 0 & c_2 & a_3 & b_3 & \cdots \\ \cdots & \cdots & c_3 & \cdots & \cdots \\ 0 & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & \cdots & \cdots \end{pmatrix}.$$

$$\xrightarrow{\text{We numerically observe } b_n = c_n \text{ and } a_n = i|a_n|\text{. Analytic arguments can be given for the imaginary } a_n}$$

$$\mathscr{L}_o^\dagger = \begin{pmatrix} i|a_1| & b_1 & 0 & \cdots & 0 \\ b_1 & i|a_2| & b_2 & \cdots & 0 \\ 0 & b_2 & i|a_3| & b_3 & \cdots \\ \cdots & \cdots & b_3 & \cdots & \cdots \\ 0 & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & \cdots & \cdots \end{pmatrix}.$$

Note: An alternate form of the diagonal coefficient = $\sqrt{b_n c_n}$ has also been suggested. Strivatsa-von Keyserlingk (2023)

Motivation 1: to understand the growth of a_n and b_n in dissipative SYK (analytically and numerically) and to conjecture/motivate the growth of a_n in any generic chaotic open systems.

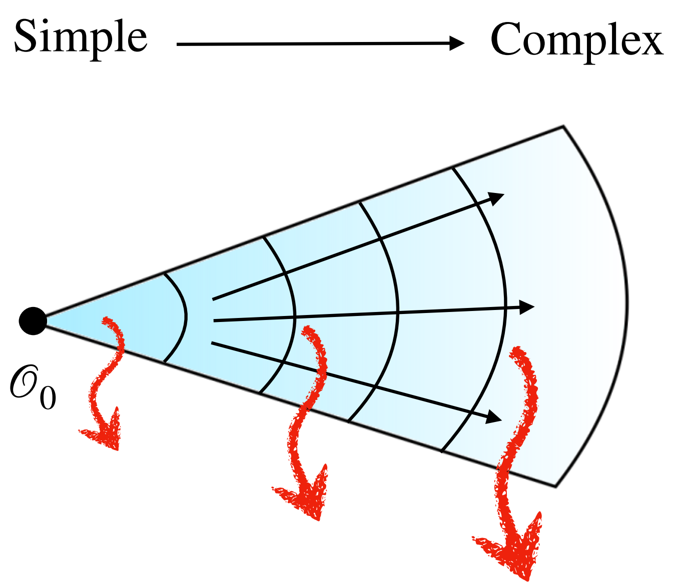
The wave functions satisfy

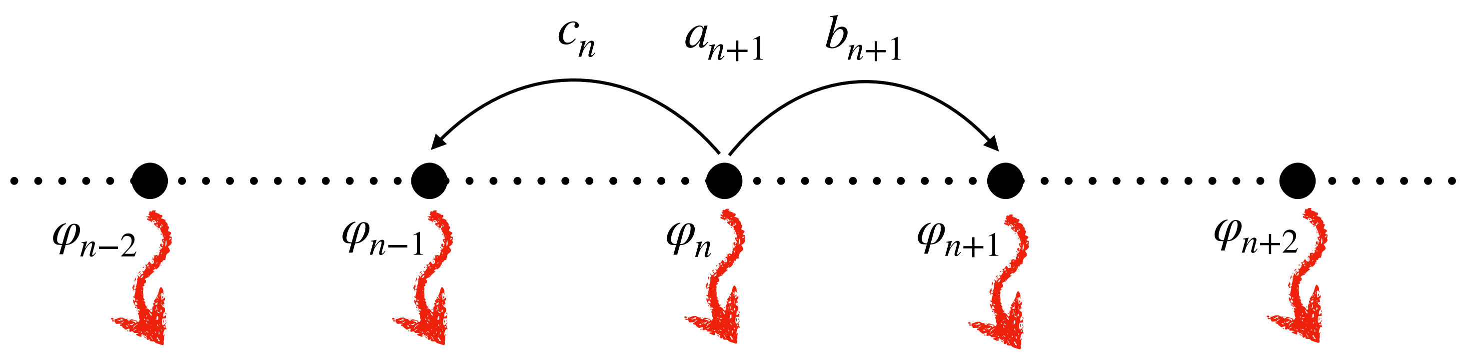
$$\partial_t \varphi_{n-1} = b_{n-1}\varphi_{n-2} + i a_n \varphi_{n-1} - b_n \varphi_n \,, \qquad n \geq 1 \,.$$

Can be understood in terms of non-Hermitian tight-binding model.

Probability is not conserved $\sum_n |\varphi_n|^2 \neq 1$

Krylov complexity
$$K(t) = \frac{\sum_n n\,|\varphi_n(t)|^2}{\sum_n |\varphi_n(t)|^2} \,.$$





Motivation 2: to understand the growth of Krylov complexity in dissipative SYK and to motivate the notion of dissipative quantum chaos any generic chaotic open systems.

Open SYK: Lindbladian dynamics

Kulkarni-Numasawa-Ryu (2021)

Hamiltonian

$$H = i^{q/2} \sum_{1 \le i_1 < i_2 < \cdots < i_q \le N} j_{i_1 \cdots i_q} \psi_{i_1} \psi_{i_2} \cdots \psi_{i_q}$$

Mean: $\langle j_{i_1 \cdots i_q} \rangle = 0$

Variance: $\langle j^2_{i_1 \cdots i_q} \rangle = 2^{q-1} \dfrac{(q-1)!\,\mathcal{J}^2}{qN^{q-1}}$

Lindblad operators:

$$L_i = \sqrt{\lambda}\, \psi_i\,, \qquad i = 1,2,\cdots,N\,.$$

We expand the auto-correlation function

$$C(\tilde{\lambda}, t) = 1 + \frac{g(\tilde{\lambda}, t)}{q} + \cdots$$

Initial operator: $\propto \psi_1$

$$g(\tilde{\lambda}, t) = \log \left(\frac{\alpha^2}{\mathcal{J}^2 \cosh^2(\alpha t + \aleph)} \right)$$

$$\tilde{\lambda} = \lambda q\,, \qquad \alpha = \mathcal{J} \sqrt{\left(\frac{\tilde{\lambda}}{2\mathcal{J}} \right)^2 + 1}\,, \qquad \aleph = \sinh^{-1} \left(\frac{\tilde{\lambda}}{2\mathcal{J}} \right).$$

We expand the auto-correlation function and computing moments are straightforward.

We are interested in computing Lanczos coefficients

$$a_n = i\tilde{\lambda} n + O(1/q) \qquad n \geq 0\,, \qquad \tilde{\lambda} = \lambda q\,.$$

$$b_n = \mathcal{J}\sqrt{\frac{2}{q}}\,, \qquad n = 1$$

$$= \mathcal{J}\sqrt{n(n-1)} + O(1/q)\,, \qquad n > 1\,.$$

Bhattacharjee-Cao-**PN**-Pathak (2022)

Comparison to closed system SYK

$$b_n = \mathcal{J}\sqrt{\frac{2}{q}}\,, \qquad n = 1$$

$$= \mathcal{J}\sqrt{n(n-1)} + O(1/q)\,, \qquad n > 1\,.$$

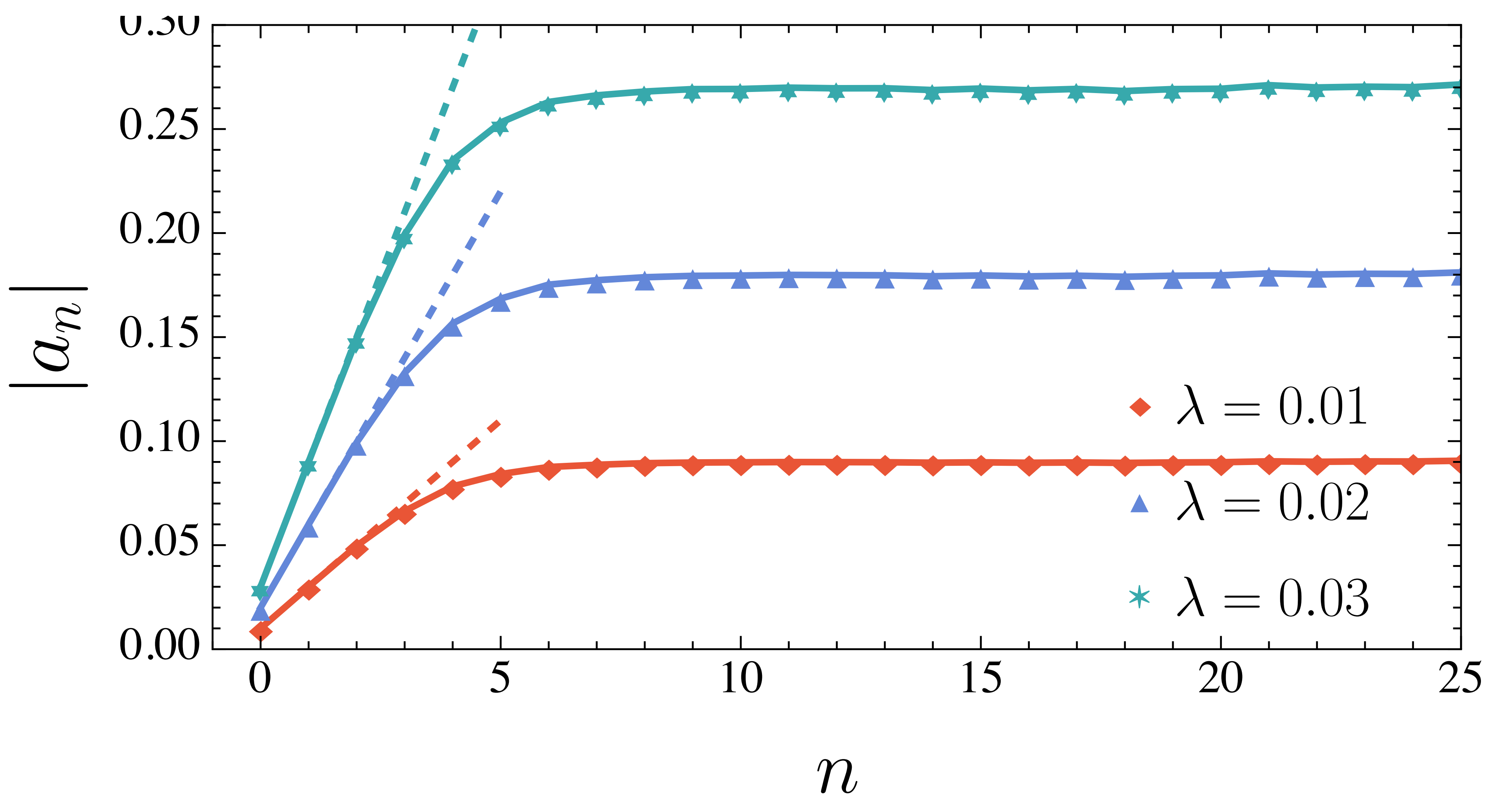
Parker-Cao-Avdoshkin-Scaffidi-Altman (2018)

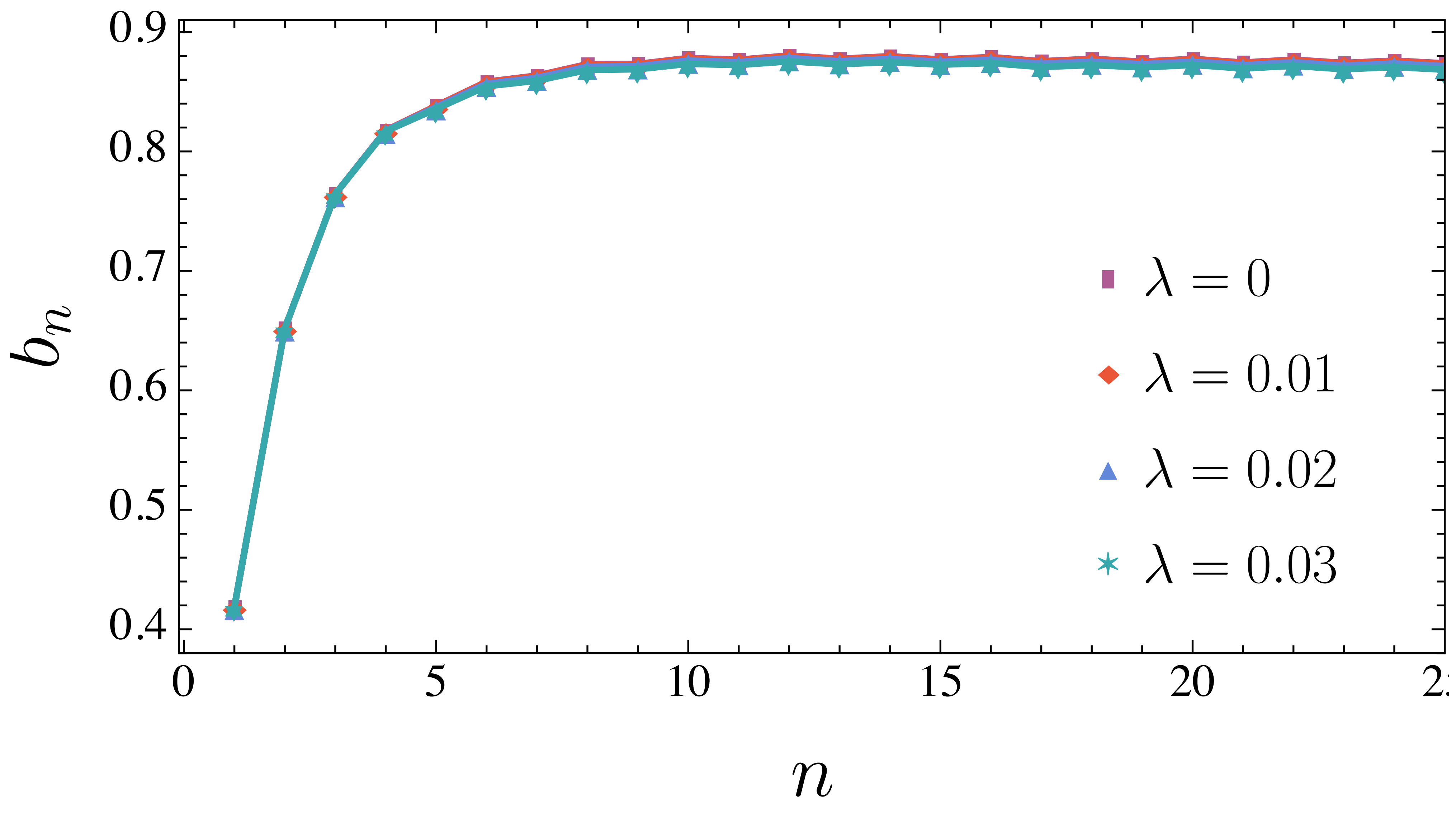
Observations for large q SYK

1. $a_n's$ linearly depend on the dissipative factor while the $b_n's$ are independent of it.
2. $a_n's$ are purely imaginary while $b_n's$ are real.
3. For large-n, both a_n and b_n are linear in n.

Applying the bi-Lanczos algorithm

Bhattacharjee-**PN**-Pathak (2023)





The set b_n exactly equals to the close system counterparts and does not depend on the dissipation.

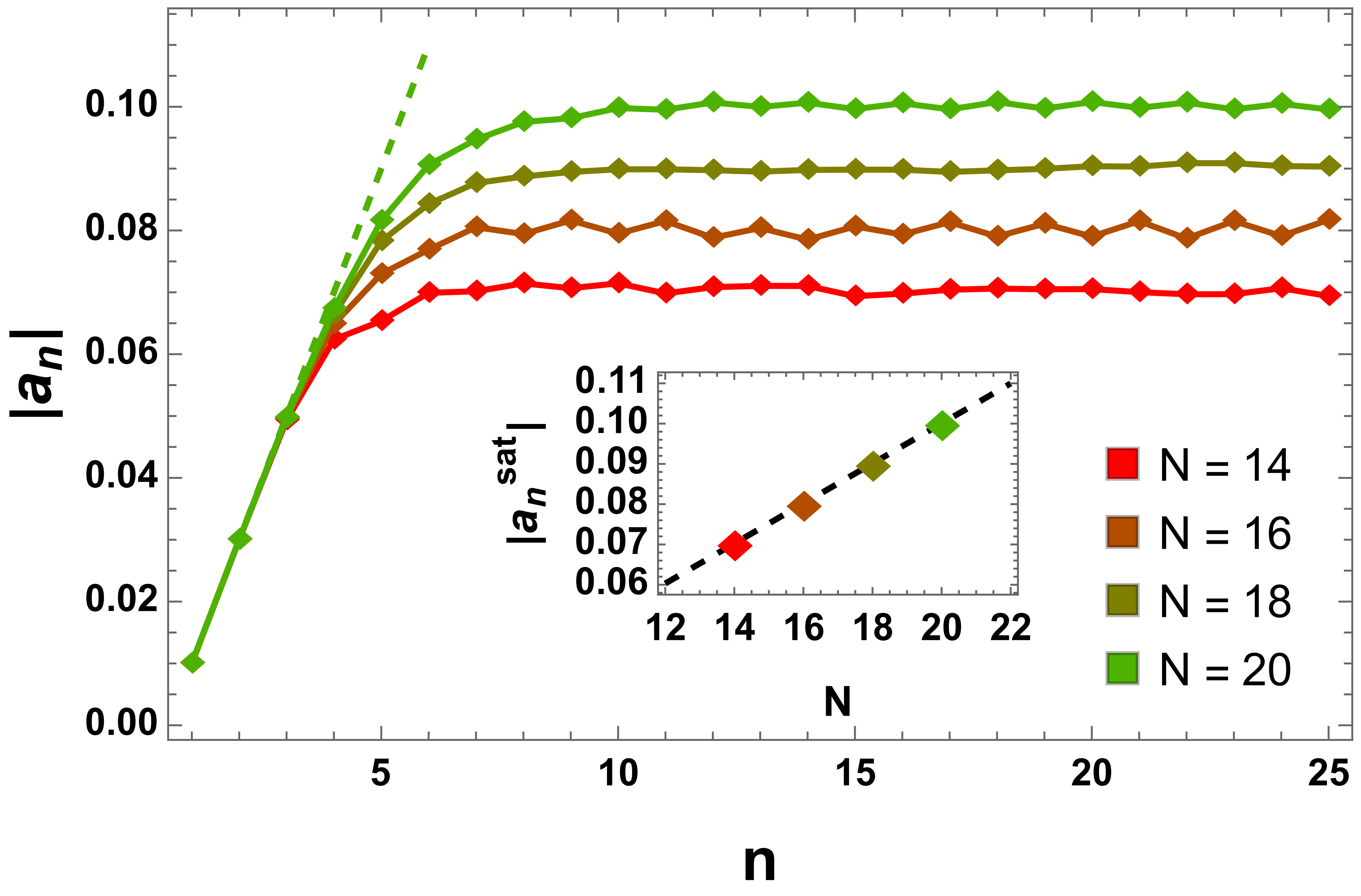
Initial operator $\propto \psi_1$

system size $N = 18$

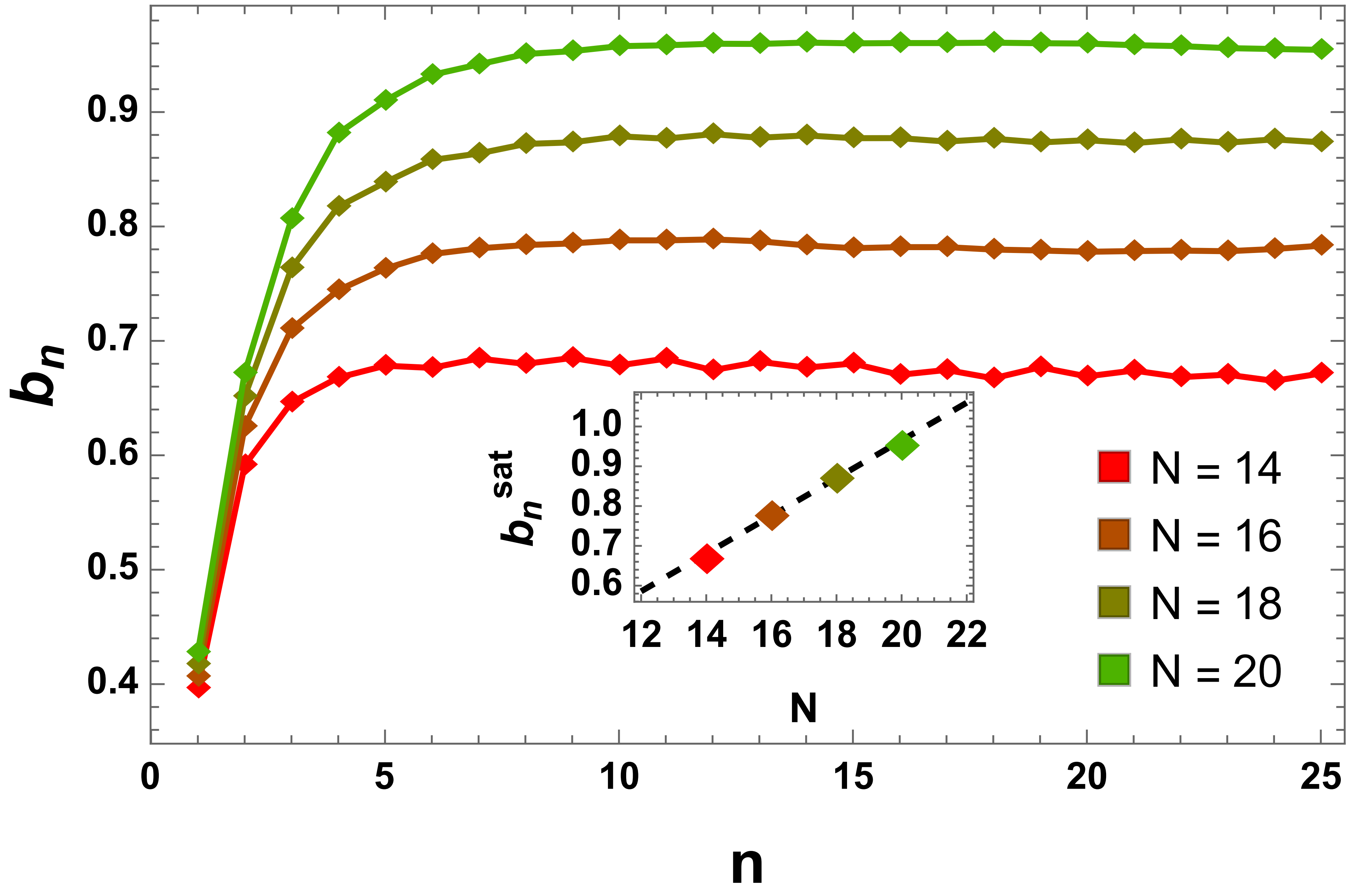
The slope of a_n is linear $\quad |a_n| = \lambda\,(2n+1)\,,$

The saturation is due to the finite size of the system

$$|a_n^{\text{sat}}| \propto N, \quad b_n^{\text{sat}} \propto N,$$



Fixed dissipation
$\lambda = 0.01$



In the thermodynamic limit $N \to \infty$, we will only be concerned about the growth

We obtain the asymptotic growth of the Lanczos coefficients

$$a_n \sim i\chi\mu n \qquad\qquad b_n \sim \alpha n$$

The most general version of "operator growth hypothesis"

Given the asymptotic growth, we can compute the Krylov complexity by recursively solving the equation

$$\partial_t \varphi_n(t) = i a_n \varphi_n(t) - b_{n+1}\varphi_{n+1}(t) + b_n \varphi_{n-1}(t)\,.$$

We take the coefficients of the form

$$b_n^2 = (1-u^2)n(n-1+\eta)\,, \quad a_n = iu(2n+\eta)\,.$$

Reduces to the asymptotic growth for

$$\alpha = 1-u^2\,, \quad \chi\mu = 2u\,.$$

The Krylov wave functions are

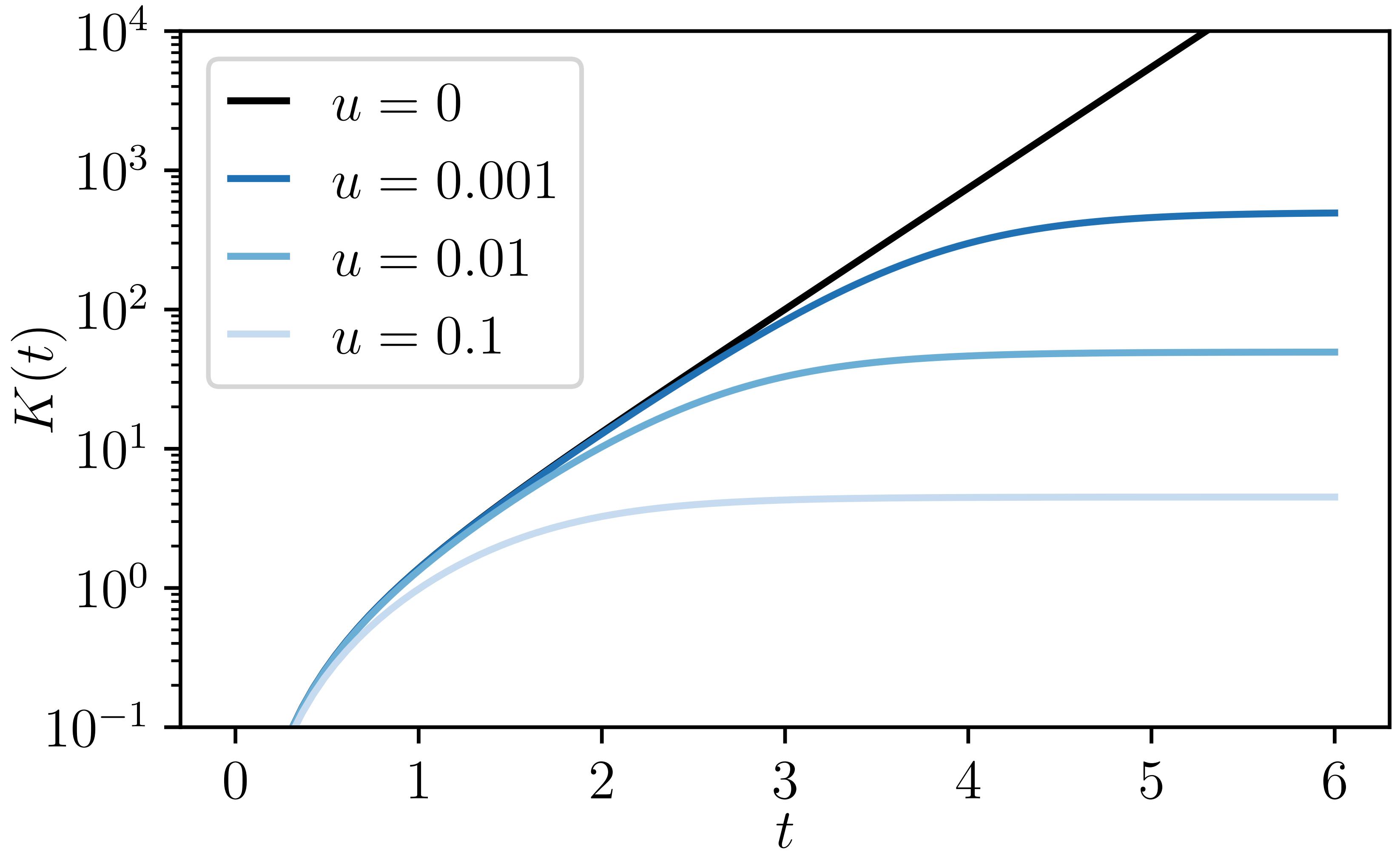
$$\varphi_n(t) = \frac{\mathrm{sech}(t)^\eta}{(1+u\tanh(t))^\eta} \times (1-u^2)^{\frac{n}{2}}\sqrt{\frac{(\eta)_n}{n!}}\left(\frac{\tanh(t)}{1+u\tanh(t)}\right)^n\,.$$

Krylov complexity

$$K(t) = \frac{1}{\mathcal{Z}} \sum_n n\,|\varphi_n(t)|^2 = \frac{\eta\left(1-u^2\right)\tanh^2(t)}{1+2u\tanh(t)-\left(1-2u^2\right)\tanh^2(t)}\,.$$

Weak dissipation limit

$$K(t) = \eta\left[\sinh^2(t) - 2u\sinh^3(t)\cosh(t) + O(u^2)\right]\,,$$



A systematic asymptotic analysis gives

$K(t) \sim 1/u \qquad t_* \sim \ln(1/u)$

Similar growth has been obtained in OTOC and operator size distribution in RUC and dissipative SYK.

Schuster-Yao (2022), Bhattacharjee-Cao-**PN**-Pathak (2022), Liu-Meyer-Xian (2024)

Krylov complexity is inversely depends on the dissipation and the dissipative time scale is logarithmic to the dissipation

$$K(t) \sim 1/u \qquad\qquad t_* \sim \ln(1/u)$$

How general/universal is this conclusion? Does it depend on the choice of the specific model or the specific choice of Lindblad operators?

Random quadratic jump operators

$$H = i^{q/2} \sum_{1 \le i_1 < i_2 < \cdots < i_q \le N} j_{i_1 \cdots i_q} \psi_{i_1} \psi_{i_2} \cdots \psi_{i_q}$$

$$L^a = \sum_{1 \le i \le j \le N} V^a_{ij} \psi_i \psi_j , \qquad a = 1,2,\cdots, M .$$

$$\langle V^a_{ij} \rangle = 0 \qquad\qquad \langle |V^a_{ij}|^2 \rangle = \frac{2V^2}{N^2} \quad \forall i, j, a$$

Kulkarni-Numasawa-Ryu (2021)
Sa-Ribeiro-Prosen (2021)

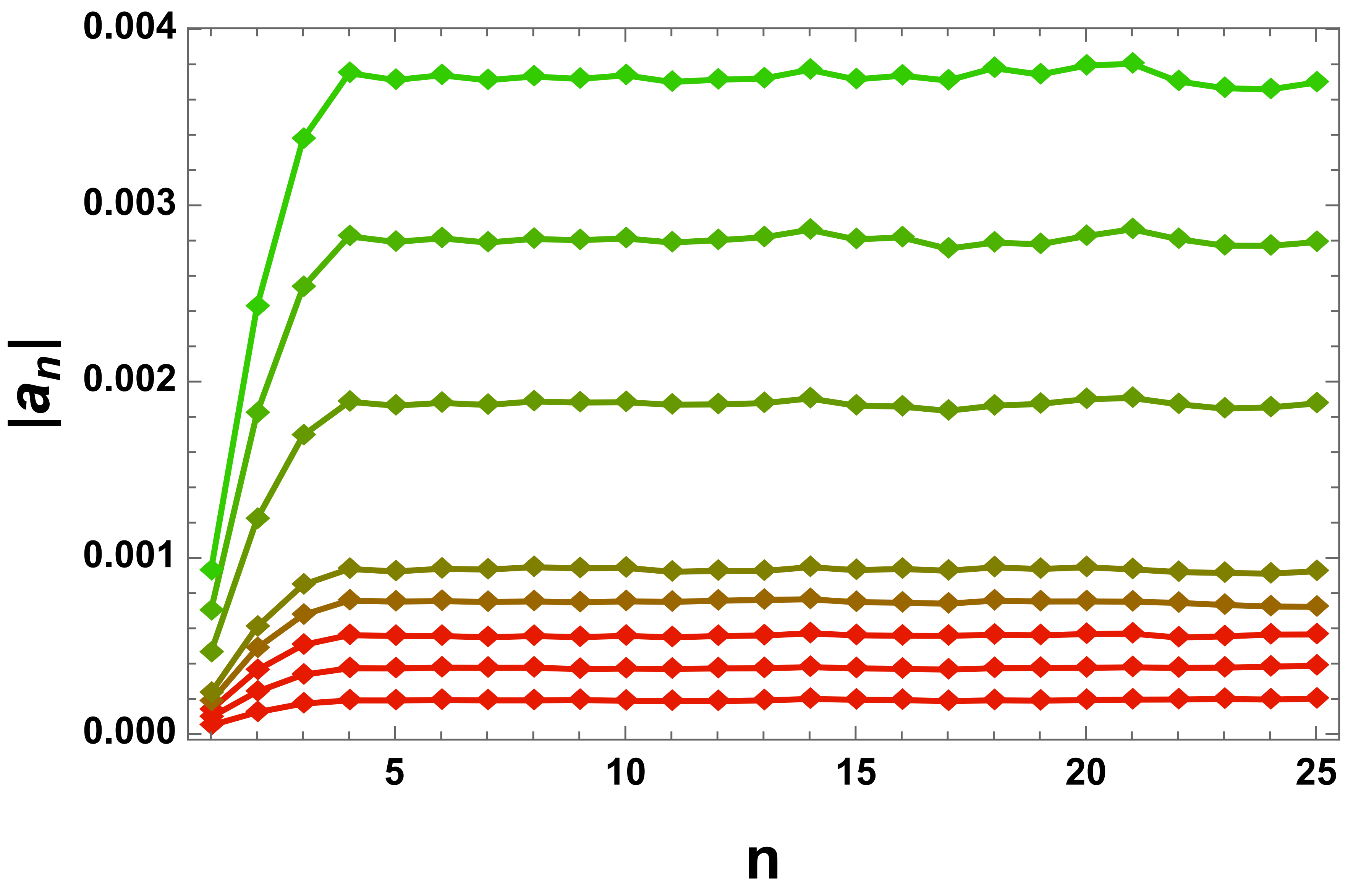
Analytically solvable in $N, M \to \infty$ limit keeping $R = M/N$ finite (a special "double scaling limit").

It is possible to prove

Bhattacharjee-**PN**-Pathak (2023)

$$\mathscr{L}_D^\dagger(\psi_{i_1}\psi_{i_2}\cdots\psi_{i_s}) \propto iV^2 R\, n\, (\psi_{i_1}\psi_{i_2}\cdots\psi_{i_s}), \qquad \Rightarrow \qquad a_n \propto iV^2 R\, n$$

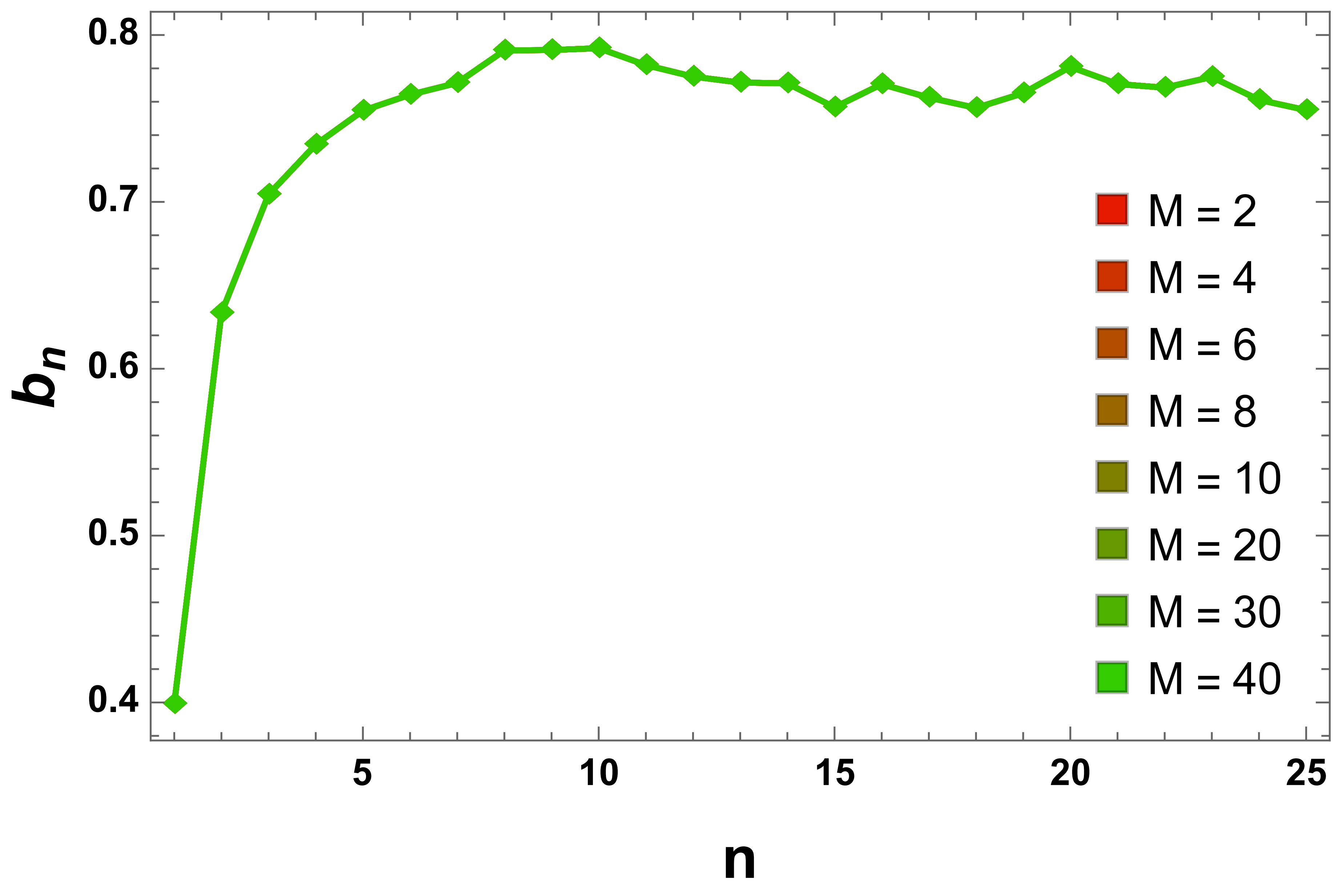
We apply the bi-Lanczos algorithm with quadratic dissipator



Initial operator $\propto \psi_1$

system size $N = 16$

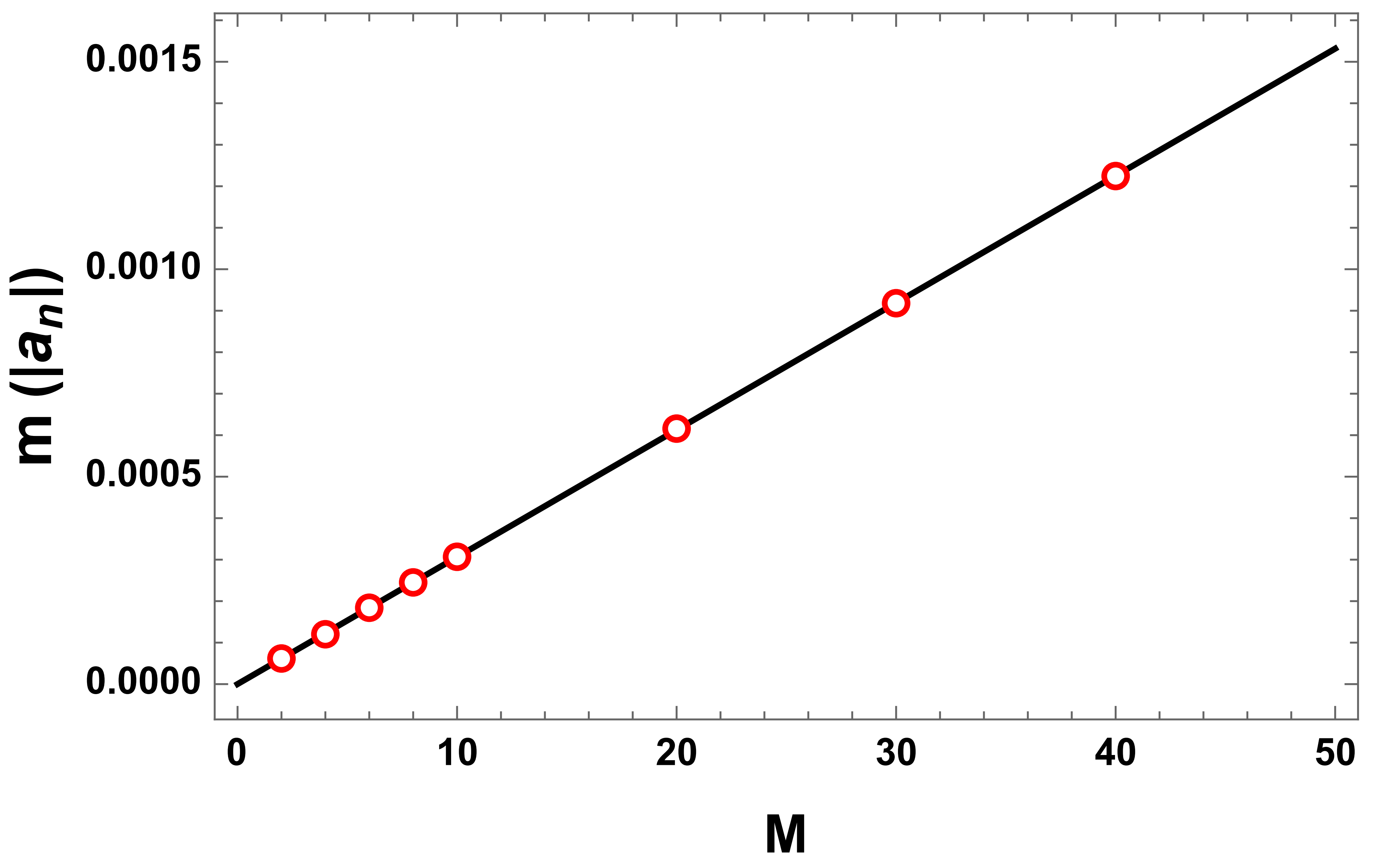
Fixed dissipation
$V = 0.02$

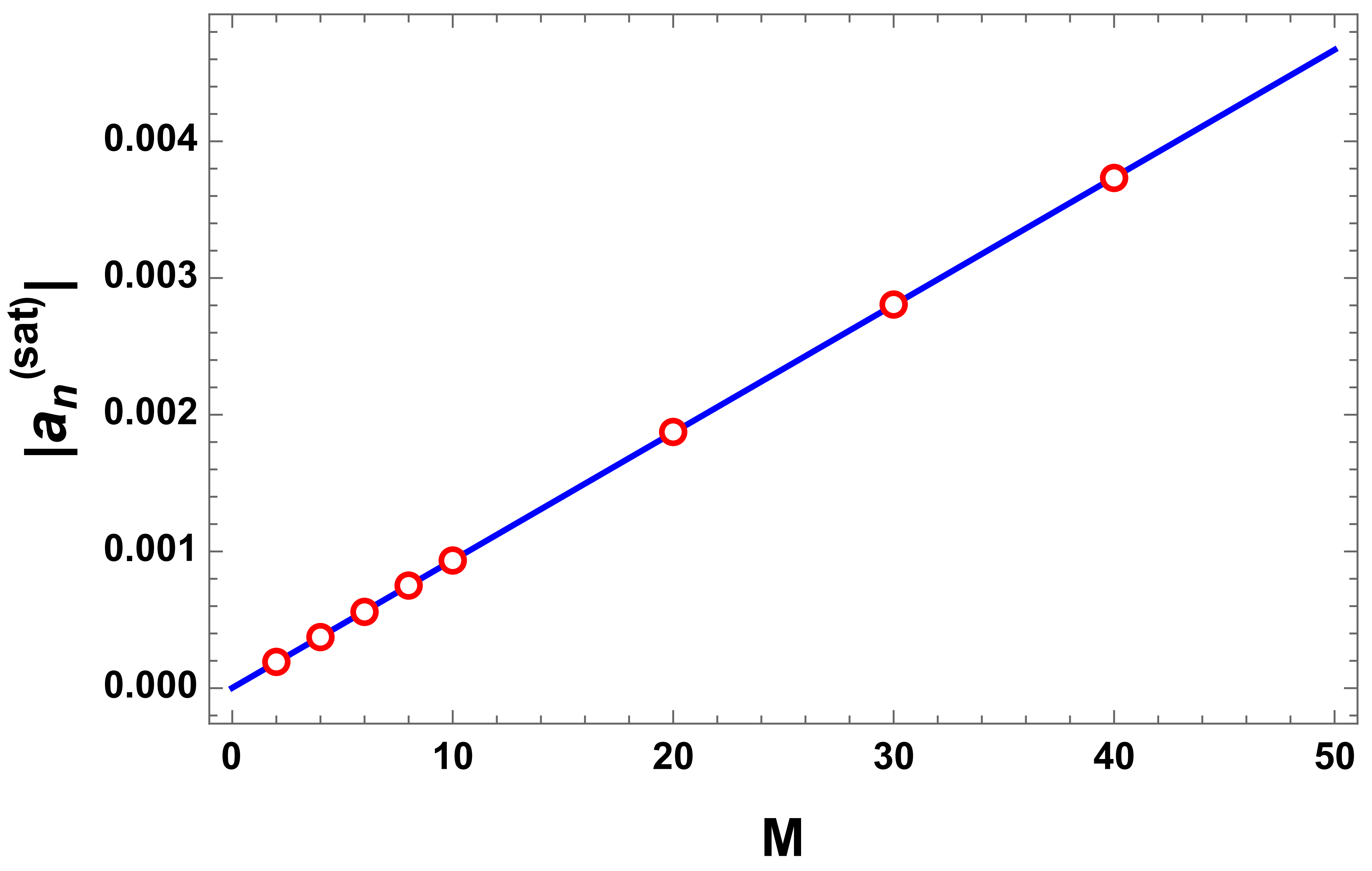


Dissipation only effects a_n but not b_n

Both the slopes and the saturation values increases linearly with M

$$m(|a_n|) \propto M\,, \quad \text{and} \quad |a_n^{(\mathrm{sat})}| \propto M\,, \quad \text{fixed}\, N\,.$$

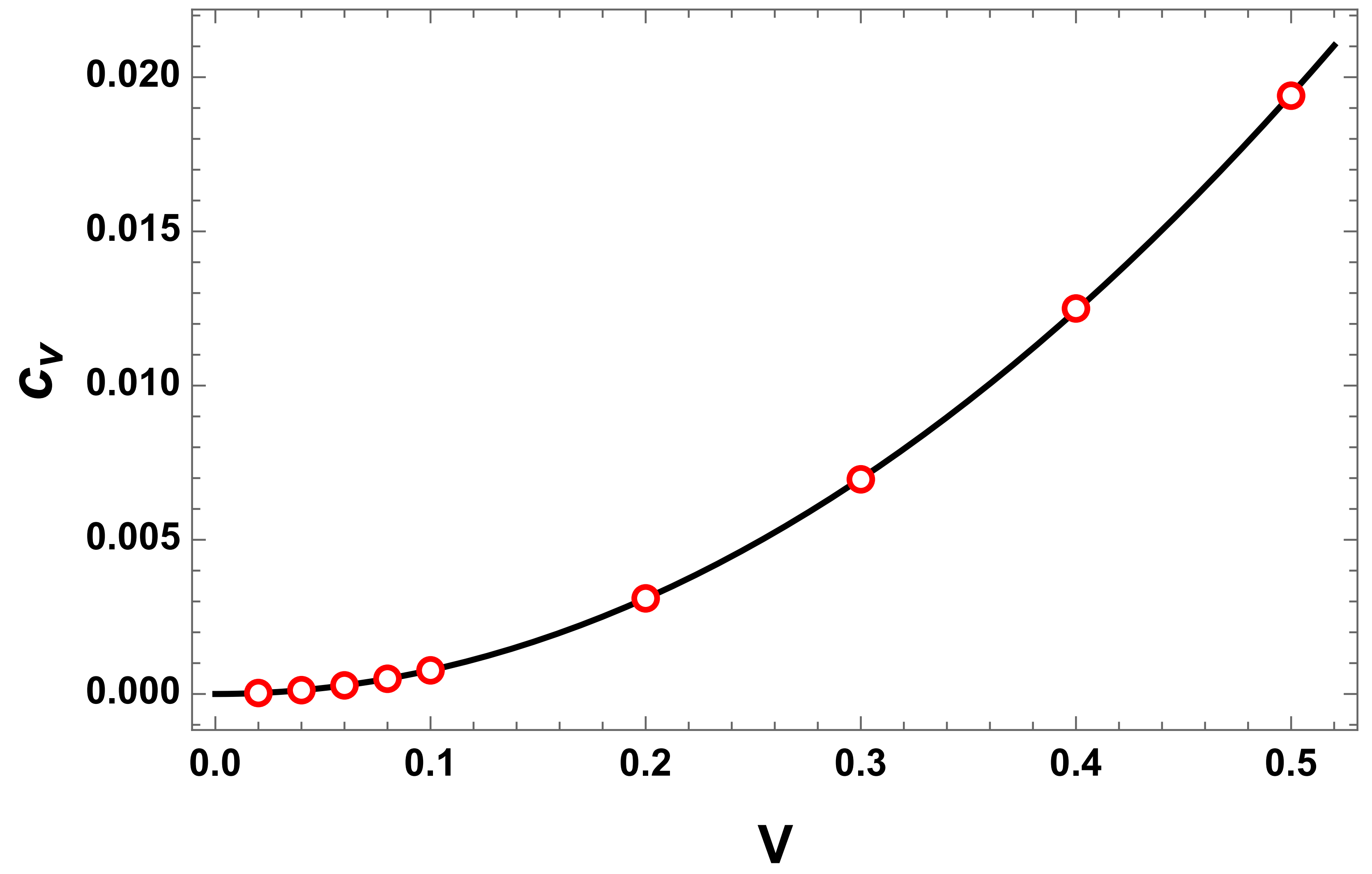




We will be concerned about the slope only $\quad m(|a_n|) := \dfrac{d|a_n|}{dn} \propto M\,,$ Implies $\quad a_n \sim i c_V M n = i c_V R N n\,.$

Assumption: The dependence of c_V in V is of the form

$$c_V = \xi\, V^{\beta}\,,$$



Data fit gives $\beta \approx 2$

The dependence of c_V in V is quadratic

Hence we obtain $$a_n \propto iV^2 R\, n\,.$$

The dissipative time scale and Krylov complexity $$t_d \sim \ln\left(\frac{1}{V}\right)\,, \qquad K_{\text{sat}} \sim \frac{1}{V^2}\,.$$

This can be proved by generic p body dissipative operator in combinatorial approach in large q and large N limit.

Questions:

1) How does the information flow across a system which is connected with a dissipative environment?

2) Can we quantify some universal quantity for such “dissipative” information flow?

Answers:

1) It’s complicated!

2) Yes, we can, at least a large class of chaotic systems! We can define a quantity - Krylov complexity which saturates inversely to the dissipation strength $K(t) \sim 1/u$ at a timescale which is logarithmically dependent on it, i.e., $t_d \sim \ln(\alpha/u)$.

Outlook:

1. We motivate to understand "dissipative quantum chaos". The results can be interpreted from the perspective of quantum measurement.

2. We believe that the dissipative timescale and and the saturation is generic and robust for any all-to-all dissipative chaotic systems. A valid question is to understand how this dissipative time scale is related to the scrambling lime.

3. What happens for non-Markovian evolution?

4. Connections with holography? Bulk picture of dissipative chaos? Keldysh wormhole. Garcia-Garcia et al. (2023)

Particle falling inside the black hole.

Kawabata et al. (2023)
(Wip) Erdmenger-**PN-**Pathak-Xian

5. Alternate formulation in terms of singular value decomposition (SVD).

6. Other interesting questions than you can think about…

Thank you for your attention!

ご清聴ありがとうございました!