Misalignment production of vector boson dark matter from axion-SU(2) inflation

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Based on a collaboration with Tomohiro Fujita (Waseda Univ.), Kazunori Nakayama (Tohoku Univ.), and Wen Yin (Tohoku Univ.)

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FY2023 “What is dark matter?” symposium
March 7th, 2024

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Collaboration across 3 groups!

Kai Murai Work in A01 and B06
Tohoku University

B01
Based on a collaboration with Tomohiro Fujita (Waseda Univ.),
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A01

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Vector dark matter

In this talk, I focus on **vector dark matter** (VDM).

There are various known scenarios to produce dark photon particles:

- Gravitational production [Graham, Mardon, Rajendran (2015), Kolb & Long (2020), …]
- Through axion-like couplings [Agrawal et al. (2018), Co et al. (2018), Bastero-Gil et al. (2018), …]
- Thermal production with the bose enhancement [Yin (2023)]
- Decay of cosmic strings [Long & Wang (2019), Kitajima & Nakayama (2022)]

⋮

These scenarios produce dark photon **particles**.

How about **coherent oscillation** of VDM?

Vector dark matter

- Misalignment mechanism for vector

Massive vector field: $\mathscr{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{m^2}{2}A_\mu A^\mu \supset -\frac{m^2}{2a^2}A_i A_i$

It is convenient to define a "physical" field:

$$Q_i \equiv \frac{A_i}{a}$$

Then, the energy density and equation of motion become

$$\rho_A = \frac{1}{2}\left[\dot{Q}_i^2 + (m^2 + H^2)Q_i^2 + 2H\dot{Q}_i Q_i\right]$$

$$\ddot{Q} + 3H\dot{Q} + (m^2 + \dot{H} + 2H^2)Q = 0$$

ρ_A is exponentially damped during inflation.

Vector dark matter

Avoiding the damping

Coupling to the inflaton [Nakayama (2019), Kitajima & Nakayama (2023)]

$$-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} \rightarrow -\frac{f^2(\phi)}{4}F_{\mu\nu}F^{\mu\nu}$$

$A_i(t)$ is generated during inflation

By introducing a curvaton field, we can avoid

- Statistical anisotropy of adiabatic perturbations
- Isocurvature perturbations

Kitajima-san's talk

In this talk, I introduce another possible model
using the dynamics of an axion and SU(2) gauge field.

Axion-SU(2) inflation

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g f^{abc} A^b_\mu A^c_\nu,$$
$$\tilde{F}^{a\mu\nu} = \frac{\epsilon^{\mu\nu\rho\sigma}}{2\sqrt{-\det[g_{\mu\nu}]}} F^a_{\rho\sigma},$$

- Chromo-natural inflation (CNI)

[Adshead & Wyman (2012)]

$$\mathscr{L} = \underbrace{\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)}_{\text{axion/inflaton}} \underbrace{- \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu}}_{\text{SU(2) gauge fields}} + \underbrace{\frac{\phi}{4f}F^a_{\mu\nu}\tilde{F}^{a\mu\nu}}_{\text{Axion-gauge fields coupling}}$$

Source gauge fields

Slow-rolling axion

Homogeneous isotropic SU(2) gauge fields

$$A^a_i \propto \delta^a_i$$

Effective potential

This gauge field configuration is isotropic:

$$A^a_i \propto \delta^a_i \;\Rightarrow\; {}^\forall \underbrace{R}_{\text{spatial rotation}}, {}^\exists \underbrace{G}_{\text{gauge transf.}} : R_{ij}A^a_j = G^{ab}A^b_i \,.$$

→ **Free from statistical anisotropy.**

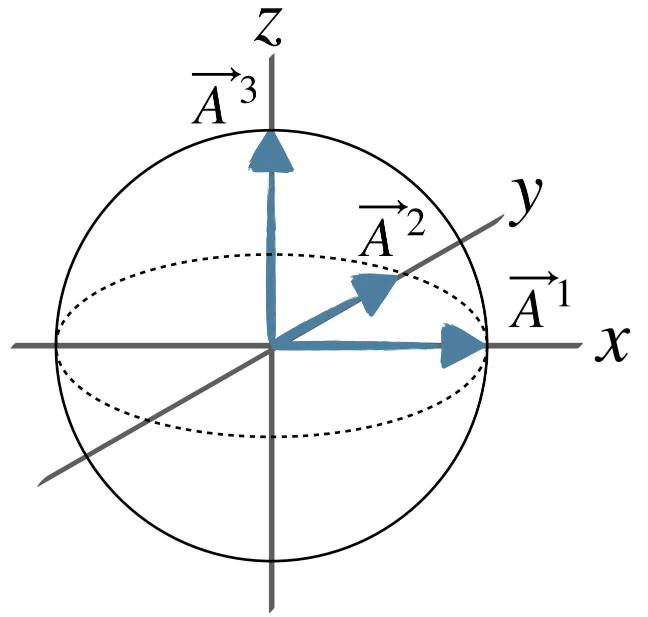
Equations of motion

Using the temporal gauge $A_0^a = 0$ and an ansatz $A_i^a(t) = \delta_i^a a(t) \underbrace{Q(t)}_{\text{gauge field amplitude}}$,

we obtain the background EoMs:

$$\ddot{\phi} + 3H\dot{\phi} + \partial_\phi V(\phi) = \underline{-\frac{3g}{f} Q^2 \left(\dot{Q} + HQ\right)},$$

$$\ddot{Q} + 3H\dot{Q} + \left(\dot{H} + 2H^2\right) Q + 2g^2 Q^3 = \underline{\frac{g}{f} Q^2 \dot{\phi}}\,.$$



In the slow-roll limit, we have a solution of

$$m_Q \equiv \frac{gQ}{H} \simeq \left(\frac{-g^2 f \partial_\phi V}{3H^4}\right)^{1/3}, \qquad \xi \equiv \frac{\dot{\phi}}{2fH} \simeq m_Q + m_Q^{-1}\,.$$

Axion-SU(2) inflation

- End of inflation

We consider that Q becomes dark matter in the later universe.

The end of inflation is determined by

$$\epsilon_H \equiv -\frac{\dot{H}}{H^2} = \epsilon_\phi + \epsilon_E + \epsilon_B = 1$$

$$\text{with} \quad \epsilon_\phi \equiv \frac{\dot{\phi}^2}{2M_{\rm Pl}^2 H^2}, \quad \epsilon_E \equiv \frac{\left(\dot{Q} + HQ\right)^2}{M_{\rm Pl}^2 H^2}, \quad \epsilon_B \equiv \frac{g^2 Q^4}{M_{\rm Pl}^2 H^2}$$

In our setup, ϵ_B makes the dominant contribution.

At the end of inflation,

$$\epsilon_B \equiv \frac{g^2 Q^4}{M_{\rm Pl}^2 H^2} \simeq 1 \rightarrow m_{Q,\rm end} = \frac{g Q_{\rm end}}{H_{\rm end}} \simeq \sqrt{\frac{g M_{\rm Pl}}{H_{\rm end}}}$$

Evolution of vector fields

- Spontaneous breaking of SU(2)

As dark matter, Q should be massive.

We consider the SSB of SU(2) by introducing an SU(2) doublet Φ.

$$\mathscr{L}_{\mathrm{SSB}} = D_\mu \Phi^\dagger D^\mu \Phi - V_\Phi(\Phi)$$

$$D_\mu \Phi = \partial_\mu \Phi - ig A_\mu^a \frac{\sigma^a}{2} \Phi, \;\; V_\Phi(\Phi) = \frac{\lambda}{4} \left(\Phi^\dagger \Phi - v^2 \right)^2$$

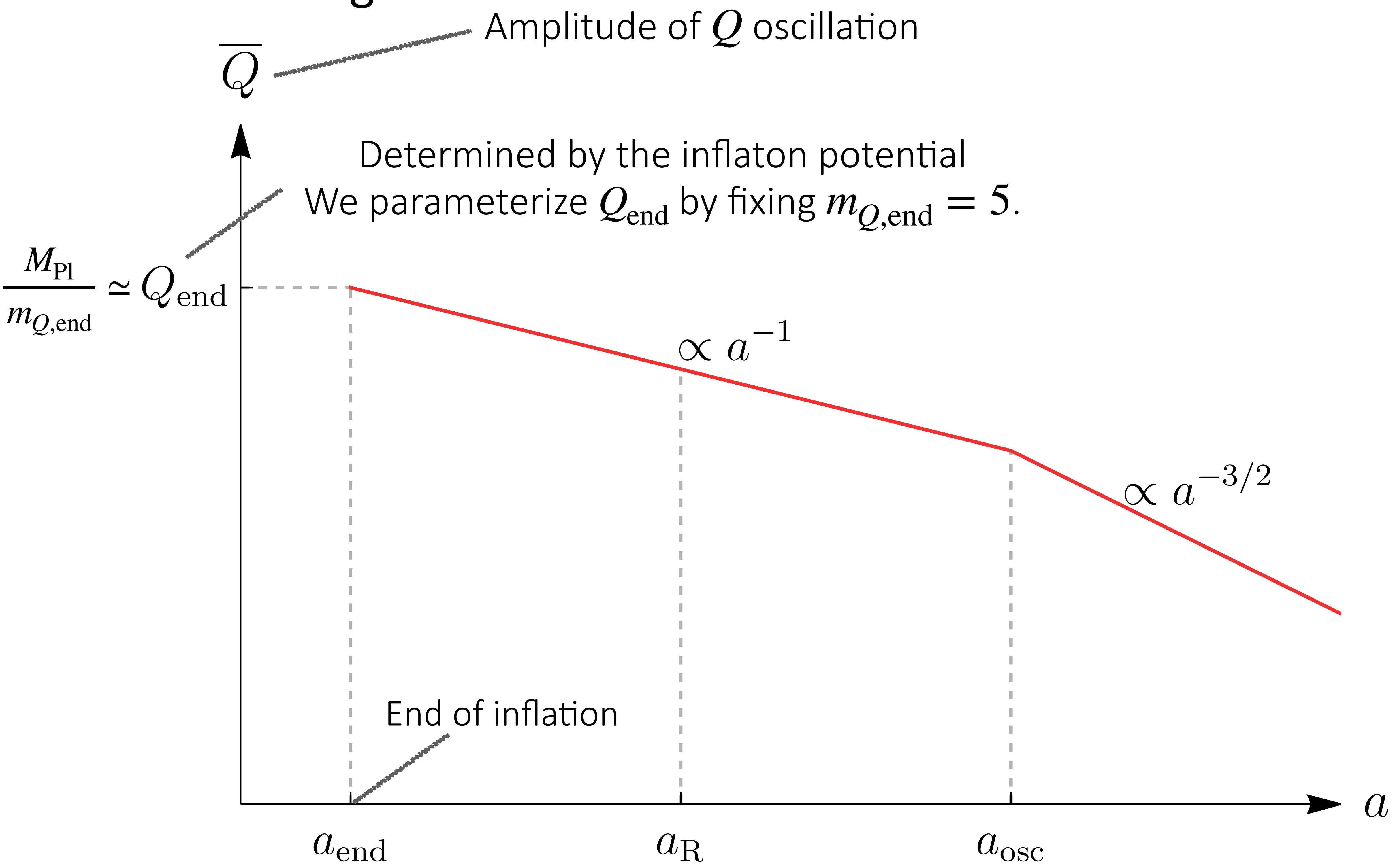
When Φ acquires a VEV of $\Phi^\dagger \Phi = v^2$,

$$\mathscr{L} \supset \frac{m^2}{2} A_\mu^a A^{a\mu}, \qquad m = gv/\sqrt{2}$$

If $m \ll H_{\mathrm{inf}}$, the inflationary dynamics is unchanged.

Evolution of vector fields

- After reheating

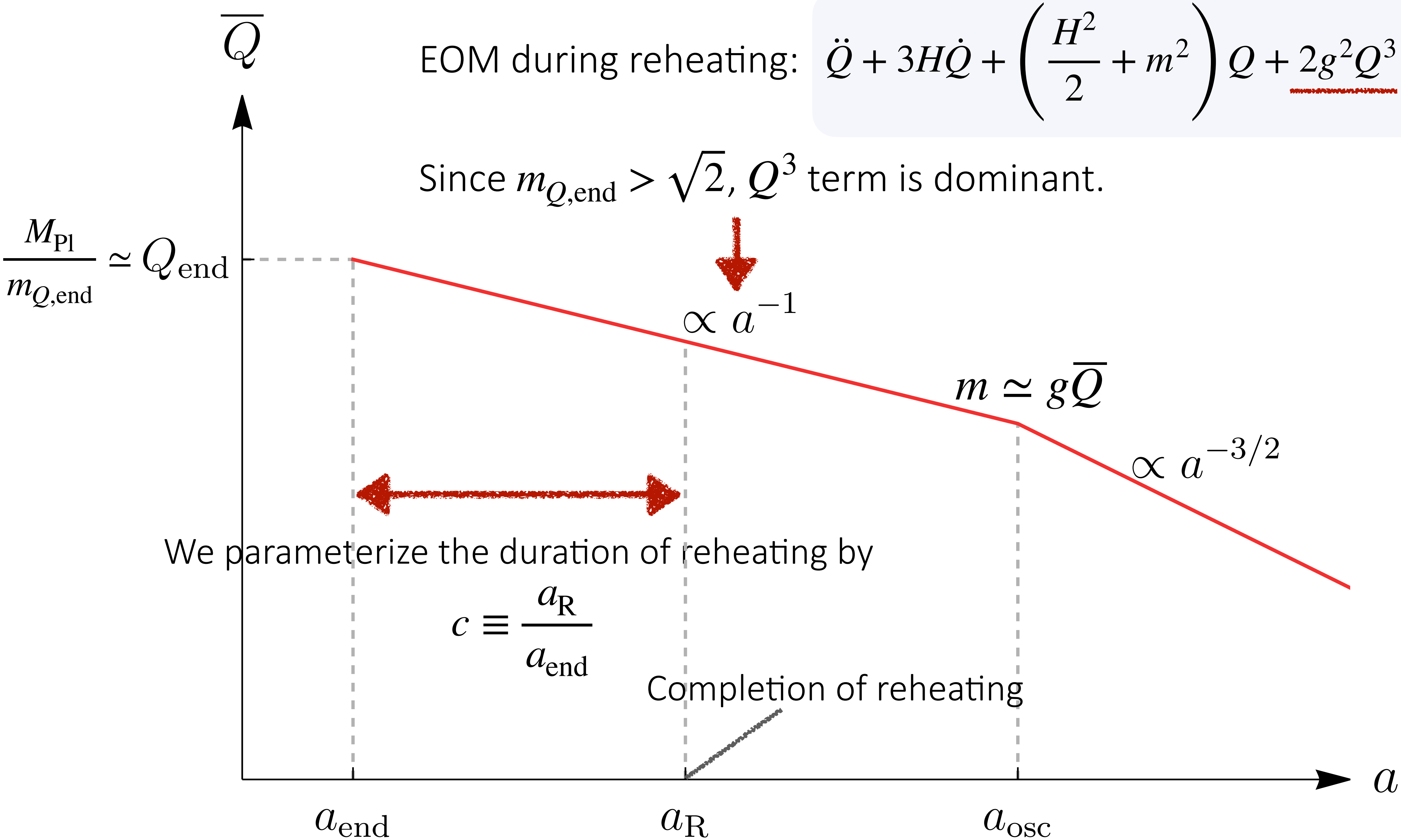


Evolution of vector fields

- After reheating

EOM during reheating: $\ddot{Q} + 3H\dot{Q} + \left(\frac{H^2}{2} + m^2\right)Q + 2g^2Q^3 = 0$

Since $m_{Q,\text{end}} > \sqrt{2}$, Q^3 term is dominant.



We parameterize the duration of reheating by

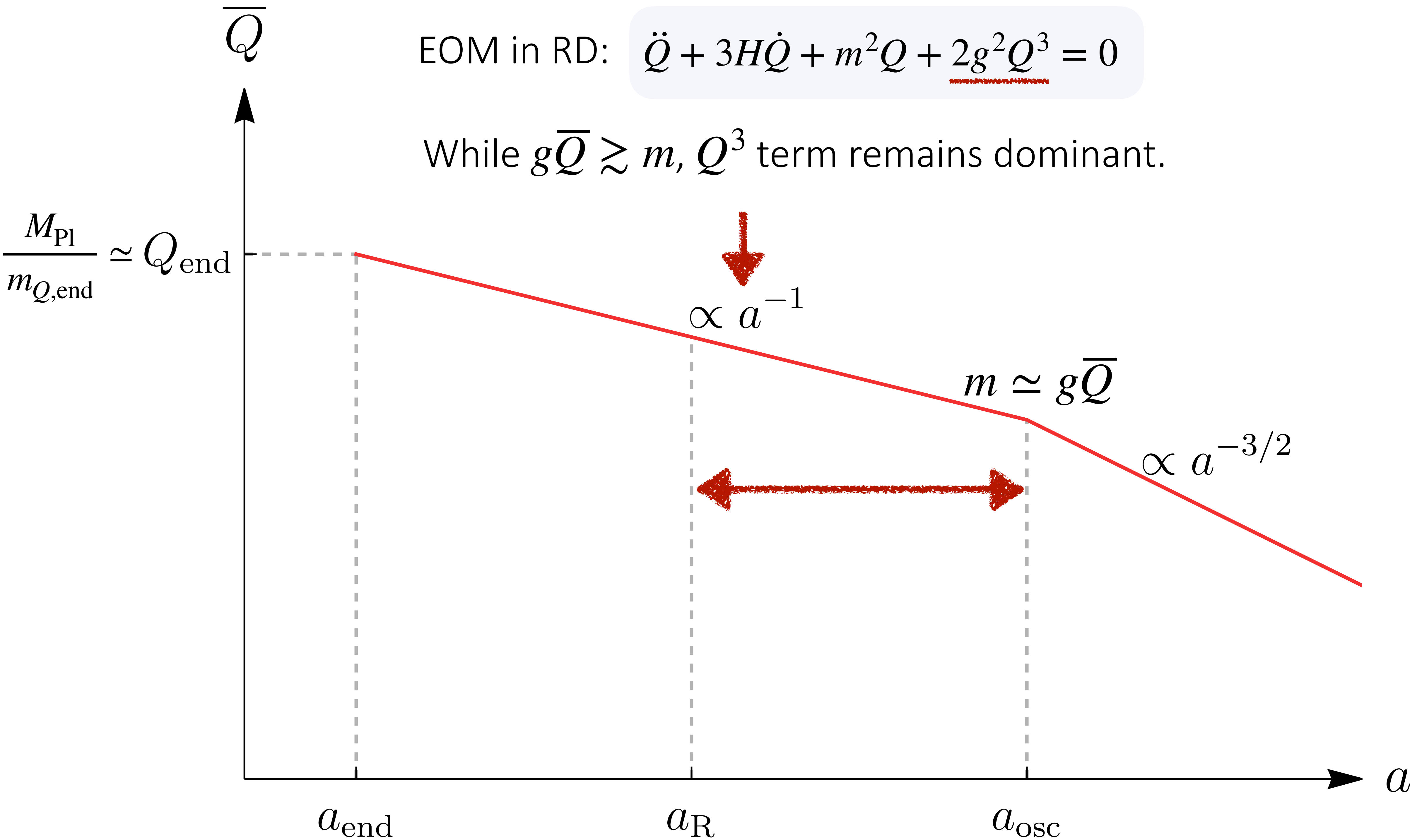
$$c \equiv \frac{a_{\text{R}}}{a_{\text{end}}}$$

Evolution of vector fields

- After reheating

EOM in RD: $\ddot{Q} + 3H\dot{Q} + m^2 Q + \underline{2g^2 Q^3} = 0$

While $g\overline{Q} \gtrsim m$, Q^3 term remains dominant.

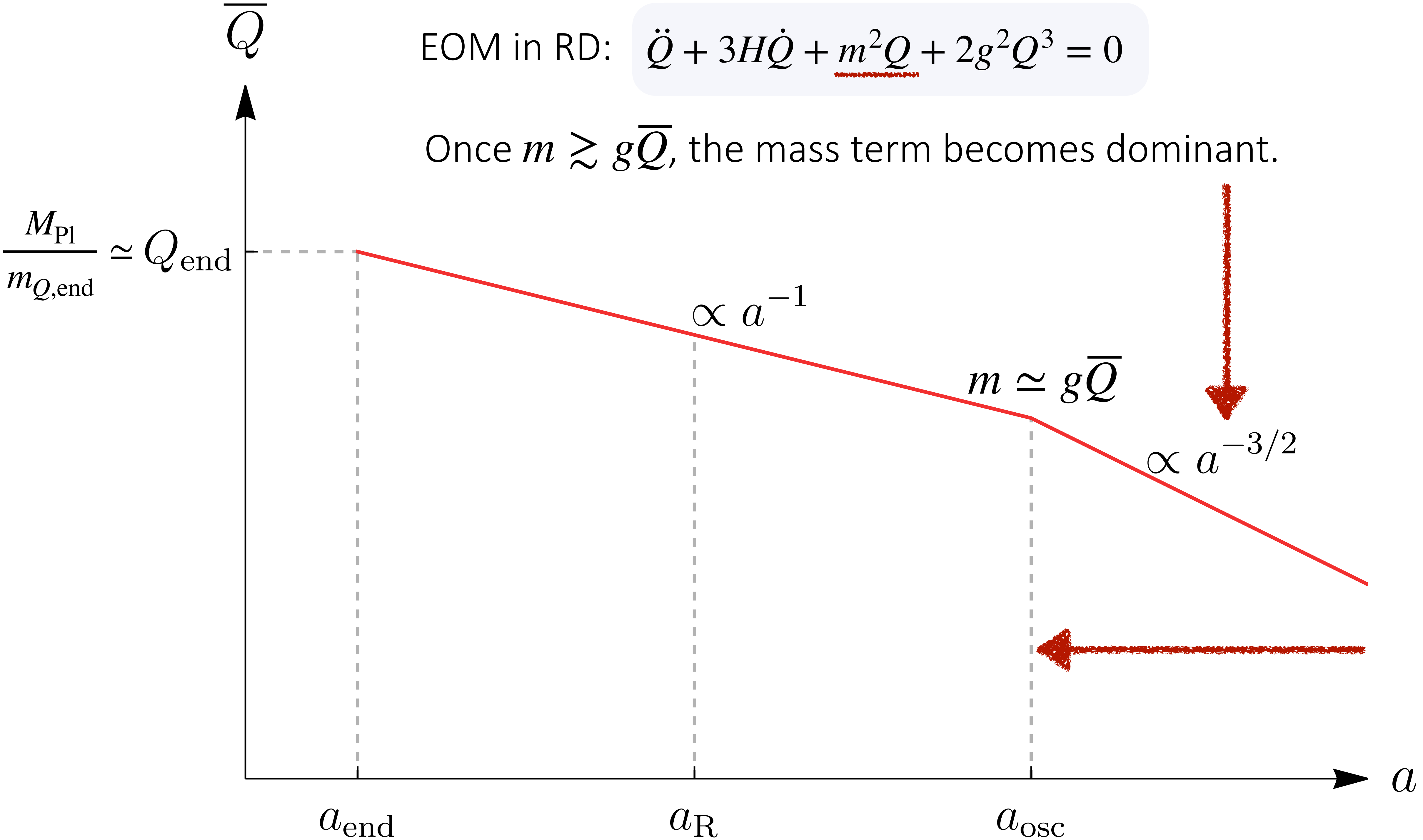


Evolution of vector fields

- After reheating

EOM in RD: $\ddot{Q} + 3H\dot{Q} + m^2 Q + 2g^2 Q^3 = 0$

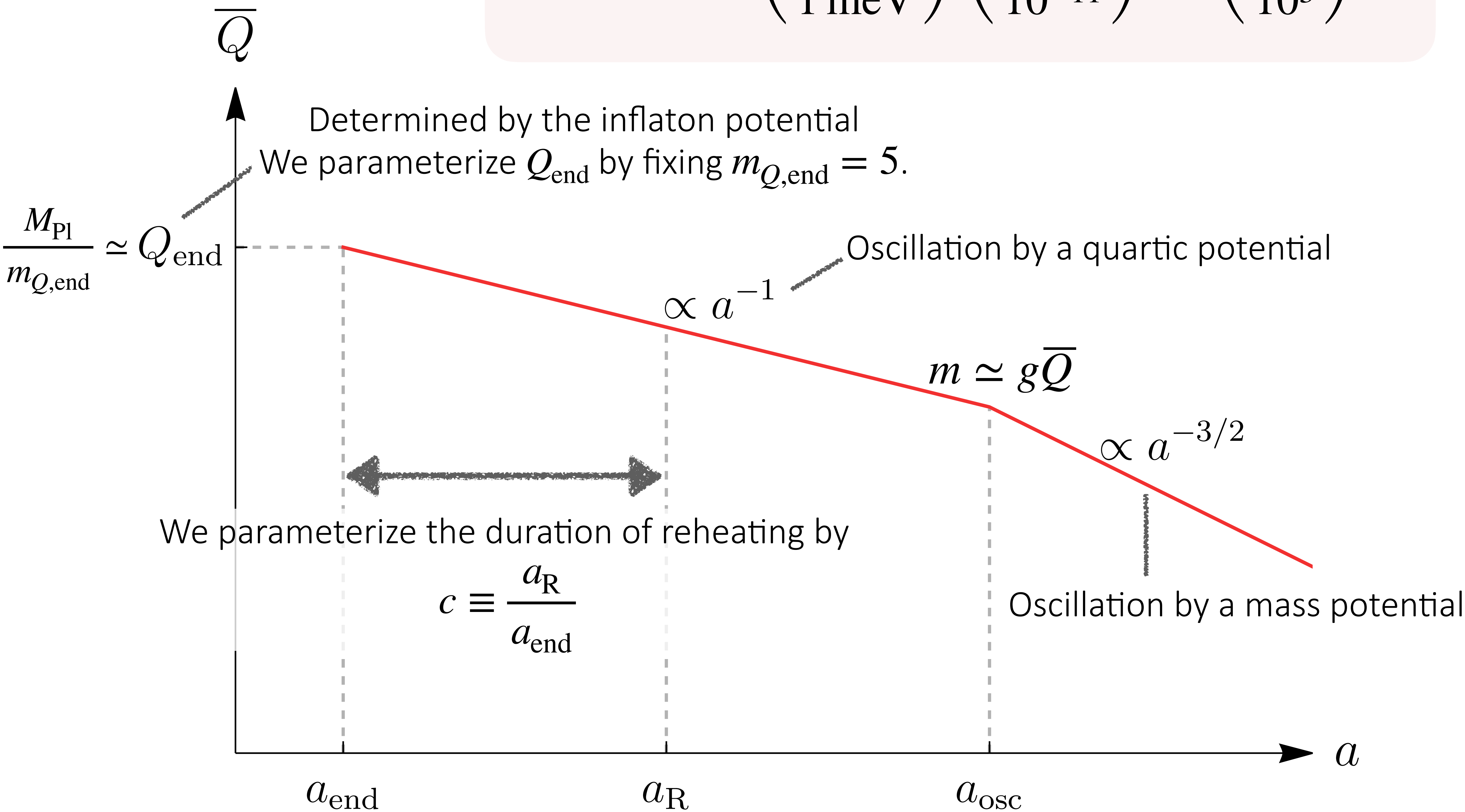
Once $m \gtrsim g\overline{Q}$, the mass term becomes dominant.



Evolution of vector fields

- DM abundance

$$\Omega_{Q,0}h^2 \simeq 0.1 \left(\frac{m}{1\,\mathrm{meV}}\right)\left(\frac{g}{10^{-11}}\right)^{-1/2}\left(\frac{c}{10^3}\right)^{-3/4}$$



VDM from axion-SU(2) inflation

- Parameter space

Correct DM abund. for
$10^{-13}\,\mathrm{eV} \lesssim m \lesssim 1\,\mathrm{eV}$

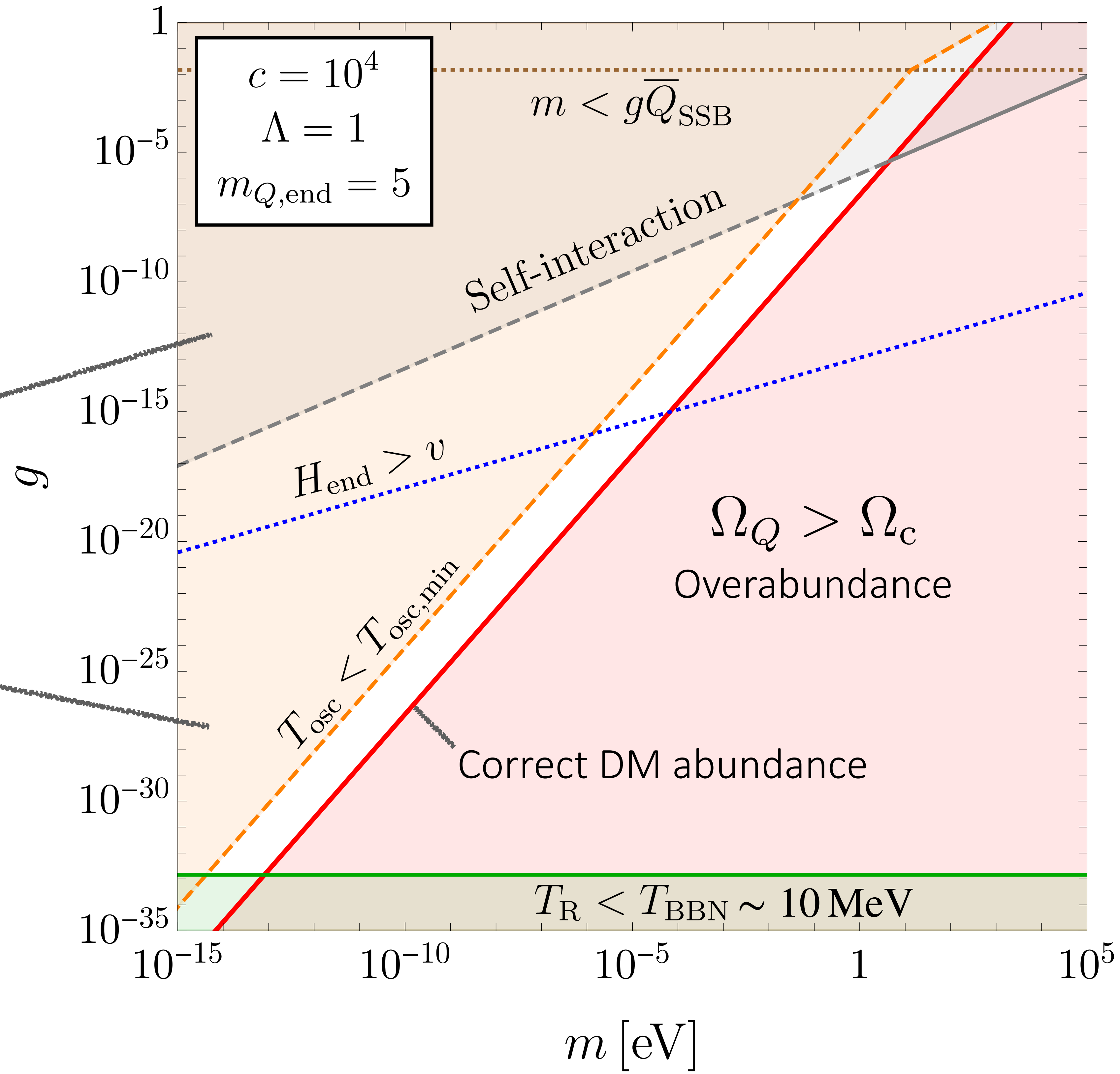
Self-int. of SU(2) gauge field

$$\frac{\sigma}{m} \sim \frac{g^4}{m^3} \lesssim \mathcal{O}(1)\,\mathrm{cm}^2/\mathrm{g}$$

Small-scale structure

Redshift when Q starts to oscillate by its mass:

$$z_{\rm osc} > 5.5 \times 10^6$$



VDM from axion-SU(2) inflation

- Kinetic mixing

In general, the SU(2) gauge field can couple with the SM photons as

SM photon

$$\mathscr{L} \supset \frac{\kappa}{2f^2}\Phi^\dagger F^a_{\mu\nu}\sigma^a F^\gamma_{\mu\nu}\Phi$$

dark SU(2)

When the SSB occurs, $\Phi = (v, 0)^{\mathrm{T}}$, it induces the kinetic mixing:

$$\mathscr{L} \supset \frac{\kappa m^2}{g^2 f^2} F^\gamma_{\mu\nu} F^{\mu\nu 3}$$

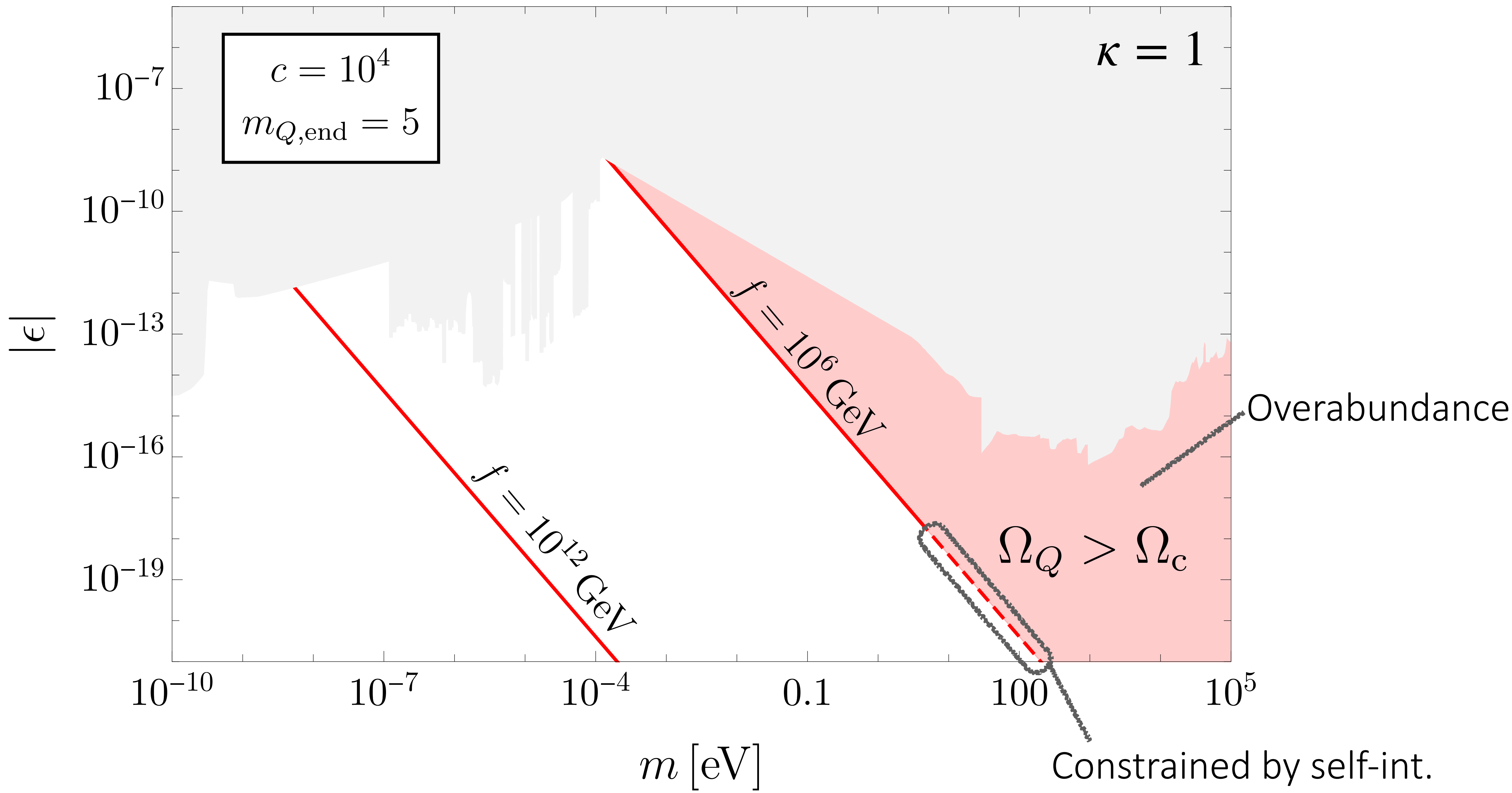
between the SM photon and A_i^3. ➔ 1/3 of DM has a kinetic mixing.

$$\mathscr{L} \supset -\frac{\epsilon}{2} F^\gamma_{\mu\nu} F^{\mu\nu 3}, \quad \epsilon \equiv -\frac{2\kappa m^2}{g^2 f^2}$$

VDM from axion-SU(2) inflation

$$\mathscr{L} \supset \frac{\kappa}{2f^2}\Phi^\dagger F^a_{\mu\nu}\sigma^a F^\gamma_{\mu\nu}\Phi$$

- Kinetic mixing



Summary

We present a new mechanism to generate a coherently oscillating VDM using axion-SU(2) gauge field dynamics during inflation.

- Vector field is not damped during inflation
- Free from statistical anisotropies of the adiabatic perturbations.
- Free from isocurvature perturbations

Our scenario can be probed through

- kinetic mixing of 1/3 of VDM
- $\Delta N_{\rm eff}$
- Self-interactions

Our scenario can be extended into

- SU(N) models
- Other SSB models (e.g., SU(2) triplets)