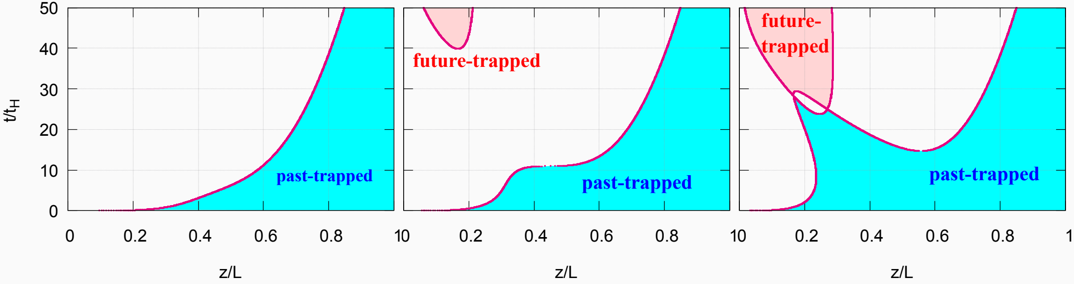
Numerical Simulation of Type II PBH Formation



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arxiv: 2401.06329

Outline

Introduction: matter side

Separate closed universe: geometry side

PBH formation from Type II fluctuation

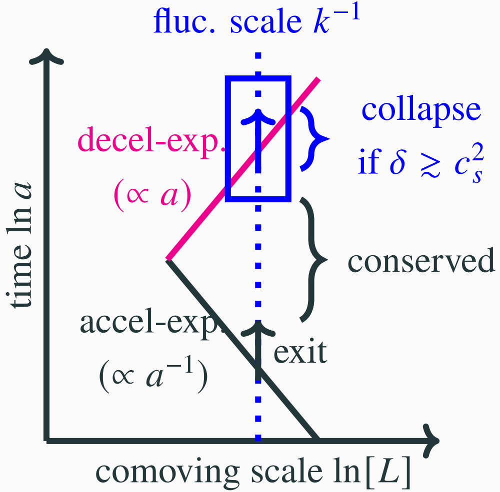
Conclusion

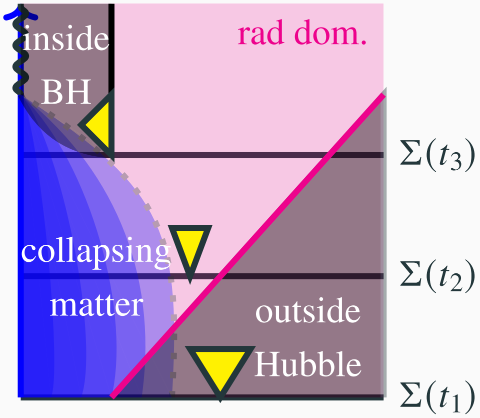
Primordial Black Hole formation

Why PBHs? Remnants and probes of non-linear primordial inhomogeneity which can be detected as BHs by GW interferometers

How do they form? Gravitational collapse of high-density regions during the Radiation-Dominated era (conventional scenario)

Collapse of Density fluctuation $\delta := \delta\rho/\rho_{\mathbf{bg}}$



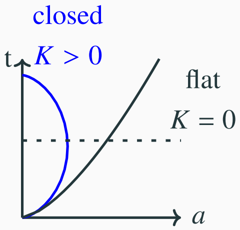


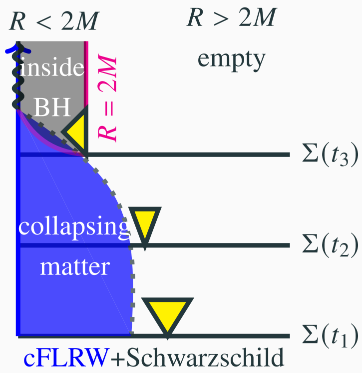
BH formation: The Simplest Example

Oppenheimer and Snyder 1939

collapsing matter: Homogeneous dust

= dust closed Friedmann solution





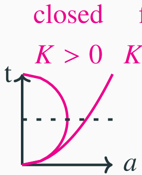
outside: empty = Schwarzschild sol.

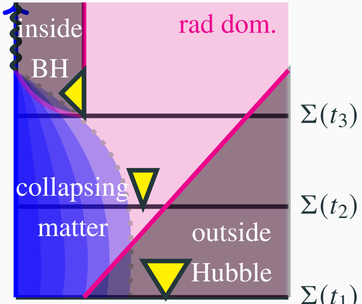
BH formation in the Early universe

Zel'dovich and Novikov 1967
Hawking 1971
Carr and Hawking 1974

collapsing matter: Inhom. rad. fluid

$= \text{rad. closed Friedmann} + \mathcal{O}(\epsilon^2)$





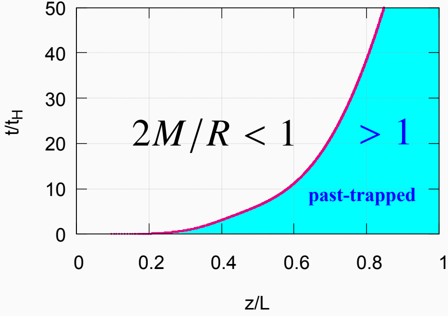
outside: expanding rad. dom. universe

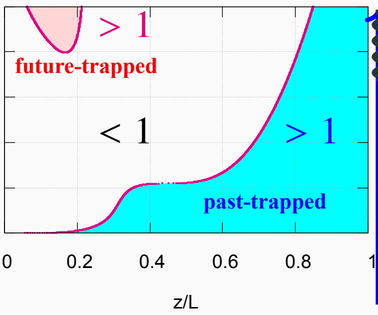
= asymptotically rad. flat Friedmann b.g.

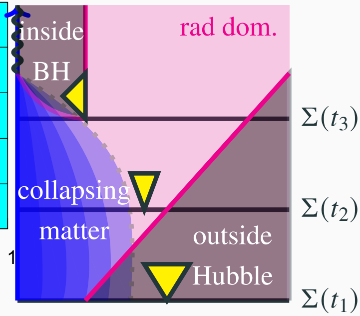
PBH formation: typical result of GR-simulation

Numerically solving 1+1 dim. full geometry and fluid

loci of horizon $2M/R(t,r) = 1$







$\mu = 0.5$ $\mu = 1.2$

Outline

“Upper limits on the size of a PBH”

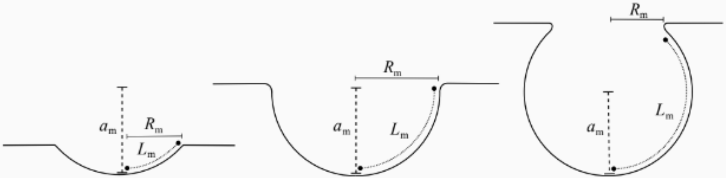
Carr and Hawking 1974
Harada and Carr 2005
Kopp, Hofmann, and Weller 2011

Initial data: perturbative sol. of curvature ζ of a timeslice

density flucs. $\delta = \delta(\zeta)$ at maximum expansion with

$c_s^2 \lesssim \delta$ “ $\lesssim \delta_{\mathrm{SU}}$ ” collapses to BH

no-SU condition: avoiding separate closed universe



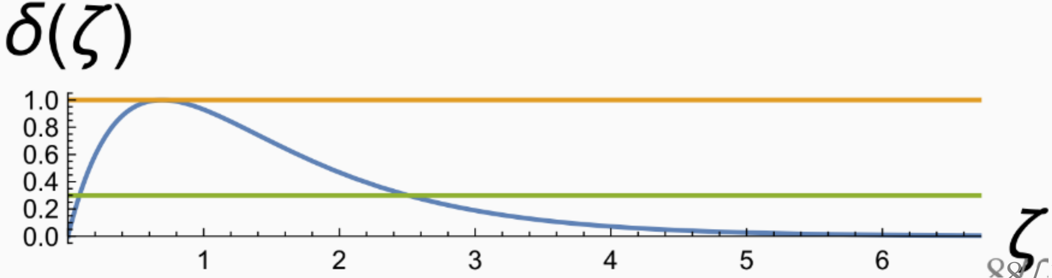
“ SUs do not constrain PBHs ”

Kopp, Hofmann, and Weller 2011
Carr and Harada 2015

the no-SU condition $\delta(\zeta) \lesssim \delta_{\mathrm{SU}}$ corresponds to $\zeta < \infty$

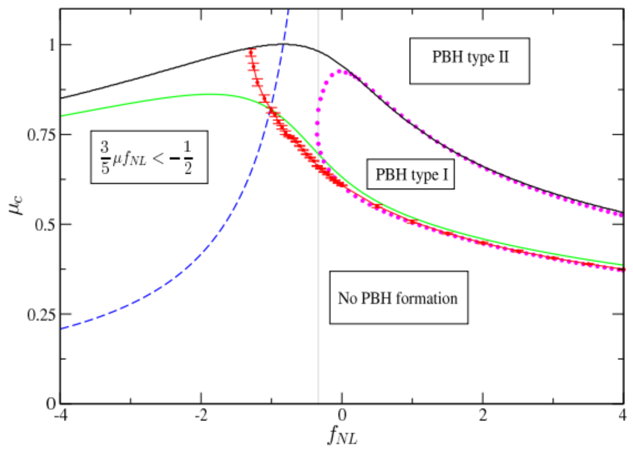
several large amplitudes of ζ give rise to twin δ_I & δ_{II}

$\Rightarrow$ type I / II fluctuation



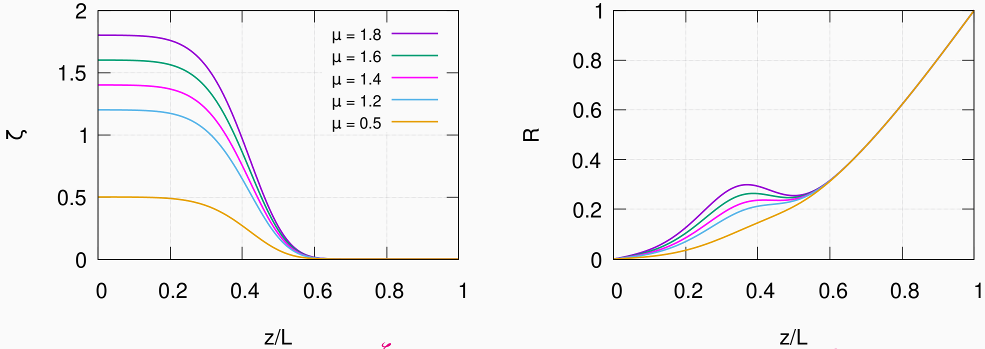
Outline

PBH from large ζ & non-Gaussianity



Escrivà, Tada, Yokoyama, and Yoo, 2022

Initial Data: Profile of fluctuaiton $\zeta(r)$

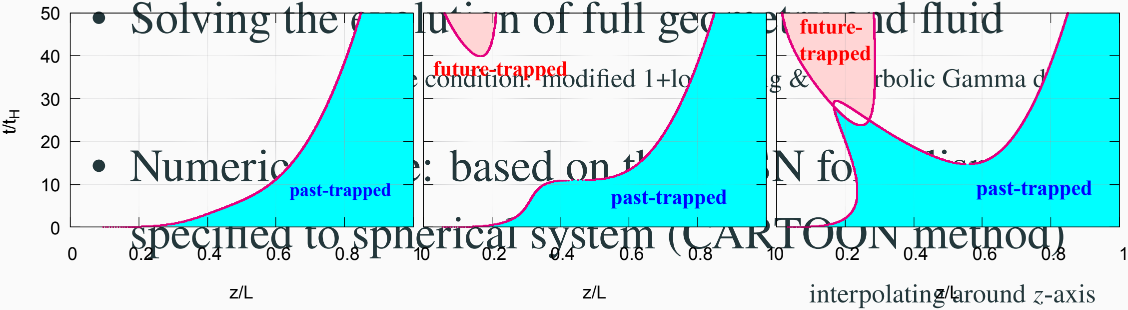


Areal radius $R := ae^{\zeta}r$ get non-monotonic $\exists\, \partial_r R = 0$

with large amplitude parameter $\mu \Rightarrow$ Type II fluctuation

COSMOS-S: BSSN based spherical code

Yoo, Okawa, and Nakao, 2013, Yoo, Harada, Hirano, Okawa, and Sasaki, 2022

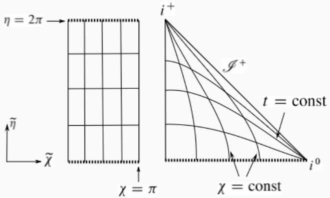


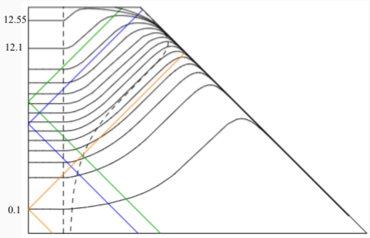
- Solving the e… lu… n of full geo… nd fluid
  - … condition: modified 1+lo… g & … rbolic Gamma d…
- Numeric… : based on t… N fo…
  specified to spherical system (CARTOON method)
  - interpolating around z-axis
- The numerial difficulty of 0/0 term does not arise
- Evolution of the type II fluctuation can be solved !!

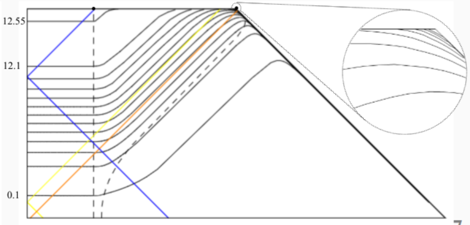
Dust case

Kopp, Hofmann and Weller, 2011

- Kopp et al. studied the type II fluctuation and PBH formation for dust fluid case.
- maching conformal diagrams by using an interpolating function from LTB system analytically.







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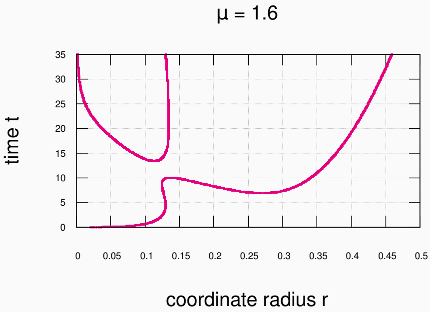
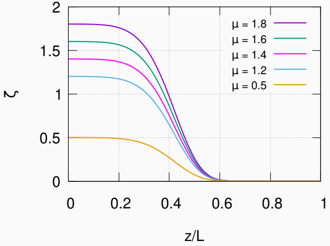
Critical case: fluctuation-II collapse to PBH-I

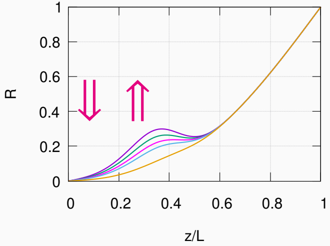


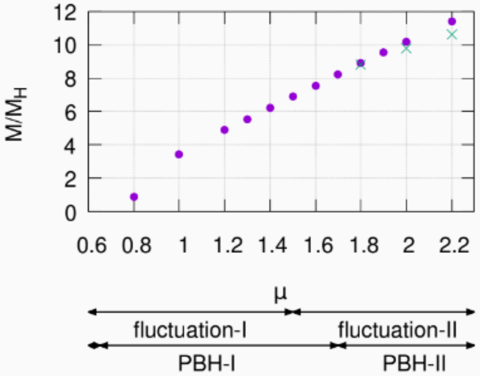
fluc.-II (having neck $\partial_r R(r) = 0$) results PBH-I (no bifurcating

The classifications of initial fluc.-type and of PBH formation-type are not always common

Result: Mass M from BH horizon $2M/R = 1$







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Conclusion & Future work

- We succeeded numerical simulation of PBH-II formation for RD case
- Fluc-II (neck of $R(r)$) does not always forms PBH-II (bifurcating horizon) due to the pressure
- PBH mass from each initial amp. of ζ can be obtained (but profile dependent)

Future work: Analysis of the profile/ $f_{\rm NL}$/ EoS-deps.

Initial Data: Cosmological long-wavelength sol.

Shibata and Sasaki 1999, Harada, Yoo, Nakama, and Koga 2015

Cosmological conformal 1+1 system (spherical sym.)

$$ds^2 = -\alpha^2 dt^2 + a^2(t) e^{2\zeta(t,r)} \left(\widetilde{\gamma}_{rr} (dr + \beta dt)^2 + r^2 d\Omega^2 \right),$$

$$= -dt^2 + a^2(t) e^{2\zeta(r)} \left(dr^2 + r^2 d\Omega^2 \right) + \mathcal{O}\left(\epsilon^2\right).$$

A known numerical difficulty in simulation

Misner and Sharp, 1964, Lasky and Lun, 2006, Kopp, Hofmann, and Weller, 2011

$$ds^2 = -\alpha^2 dt^2 + \frac{(\partial_r R(t,r))^2}{1+E(t,r)} dr^2 + R(t,r)^2 d\Omega^2,$$

$$(\partial_t R)^2 = \alpha^2 \left(\frac{2M}{R} + E \right),$$

$$\partial_t E = -2 \frac{1+E}{\rho + p} \frac{\partial_r p}{\partial_r R} \partial_t R,$$

$$\partial_t M = -4\pi p R^2 \partial_t R,$$

1

$\xrightarrow[\partial_r R = 0 \text{ exists}]{\text{Type II fluc.}}$ difficult to simulate

J. R. Oppenheimer and H. Snyder. On continued gravitational contraction. *Phys. Rev.*, 56:455–459, Sep 1939. doi: 10.1103/PhysRev.56.455. URL `https://link.aps.org/doi/10.1103/PhysRev.56.455`.

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Charles W. Misner and David H. Sharp. Relativistic equations for adiabatic, spherically symmetric gravitational collapse. *Phys. Rev.*, 136:B571–B576, Oct 1964. doi: 10.1103/PhysRev.136.B571. URL `https://link.aps.org/doi/10.1103/PhysRev.136.B571`.

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