Report from Group A03

Initial Spins of Primordial Black Holes Formed with a Soft Equation of State

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Based on Saito, Harada, YK, and Yoo, *JCAP 07 (2023) 030.*

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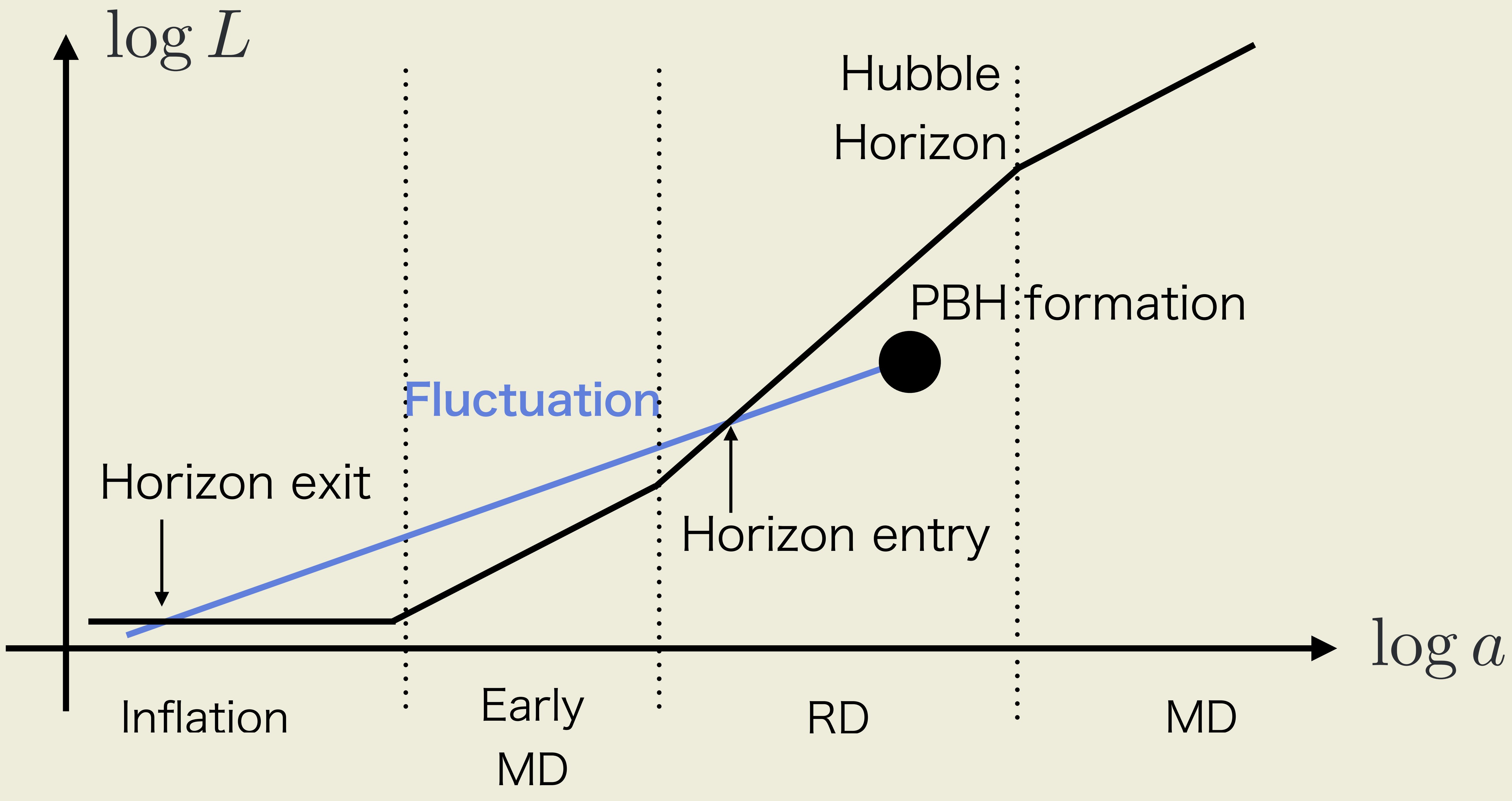
Introduction: What is PBH? / Why PBH?

PBH: BH formed in the early universe

- Candidate for a macroscopic DM
- Source of GW (BH binary merger)

- PBHs can be formed from:
  - density fluctuations,
  - vacuum bubbles,
  - cosmic strings,…



Introduction: Formation of PBH

$\delta = \frac{\rho - \rho_b}{\rho_b}$: density fluctuation

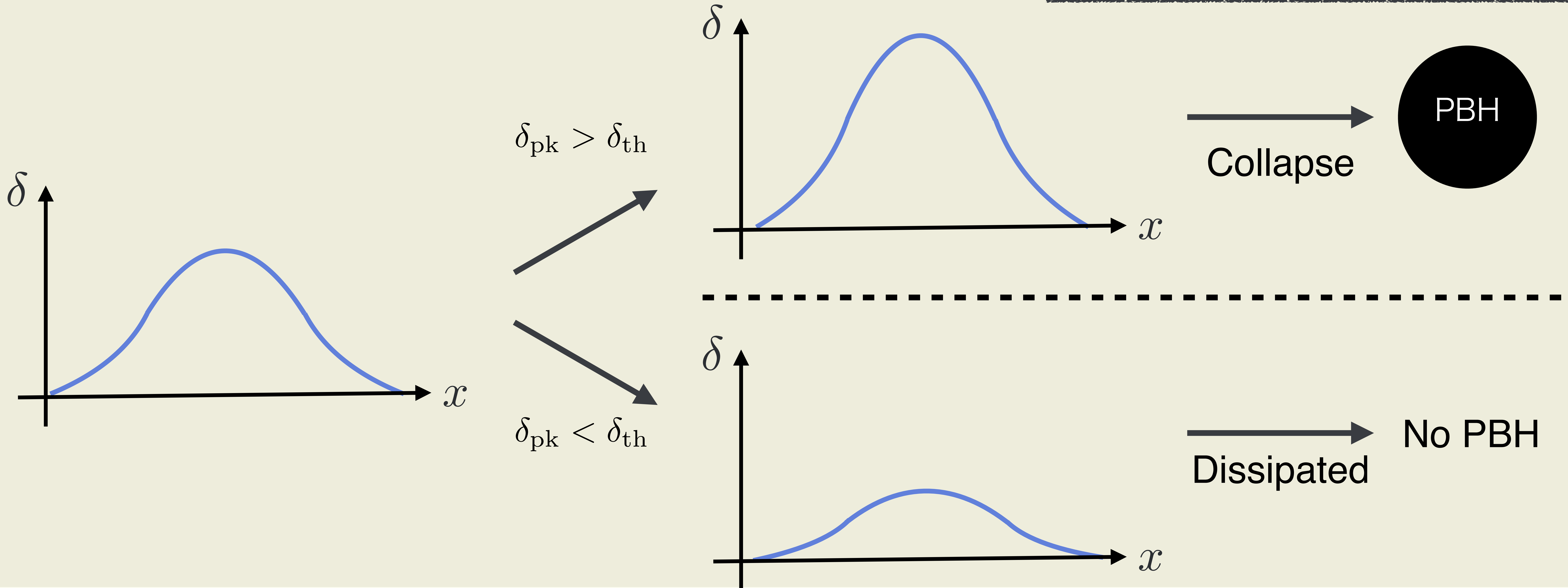
$\delta_{\rm pk}$: peak of δ

$\delta_{\rm th}$: threshold value

PBH formation due to density fluctuation

PBH is formed if $\delta_{\rm pk} > \delta_{\rm th}$

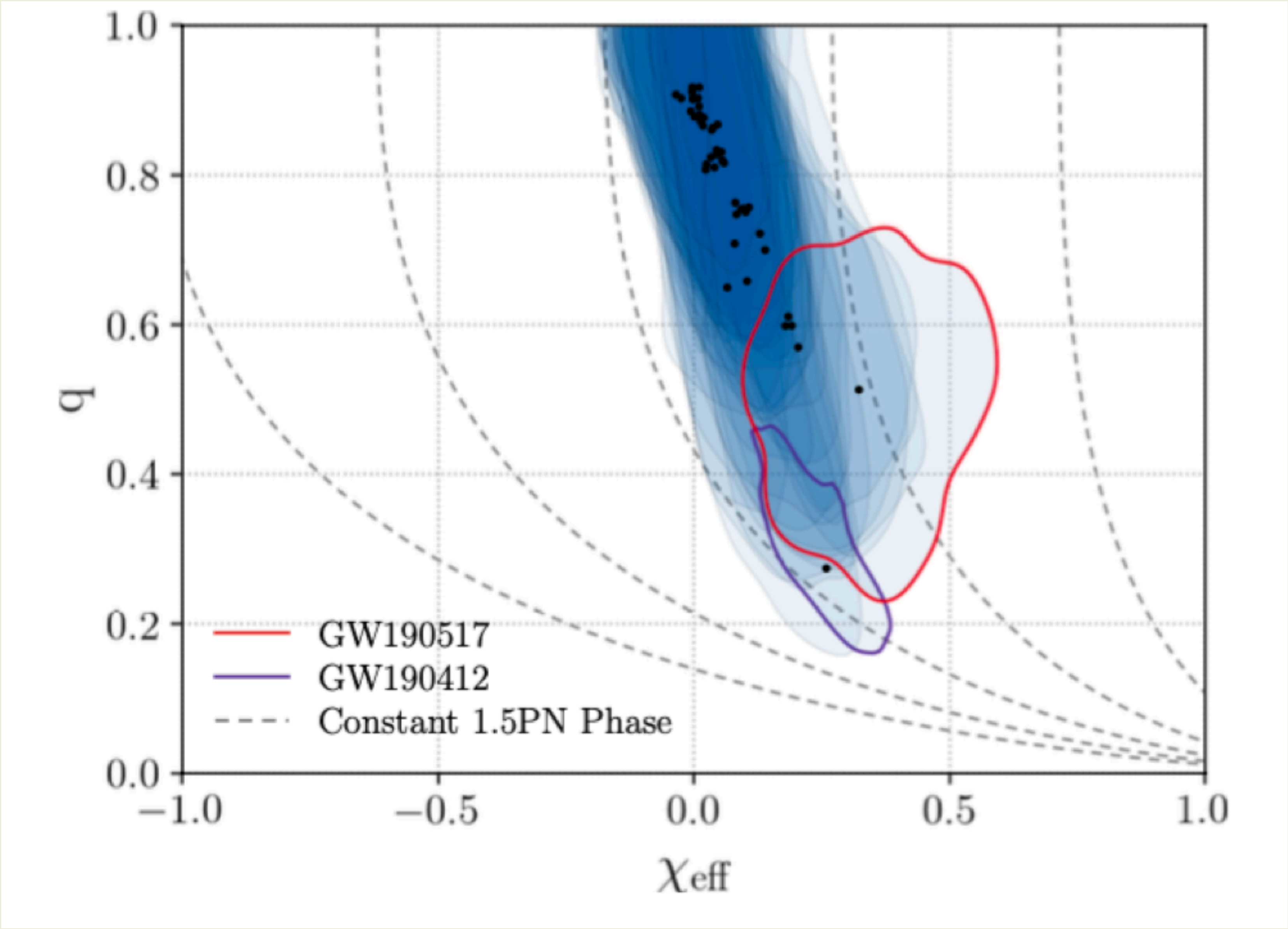
(if gravity overcomes pressure)



Introduction: Motivation of the work

Mass & spin of BHs are investigated in GW observation

⟶ Estimate the expected value of the PBH spin



[Callister+ (2021)]

Previous works on PBH spins: perfect fluid with $p = w\rho$

- RD era $(w = 1/3)$ (Harada+ 2021) $\sqrt{\langle a_*^2 \rangle} \sim O(10^{-3})$
- MD era $(w = 0)$ (Harada+ 2017) $a_* \simeq 1$

a_*: non-dimensional Kerr parameter

This work: RMS of PBH spin for $0 < w \leq 1/3$

- Application: QCD phase transition $(0.23 < w < 1/3)$

Introduction: Goal and assumptions

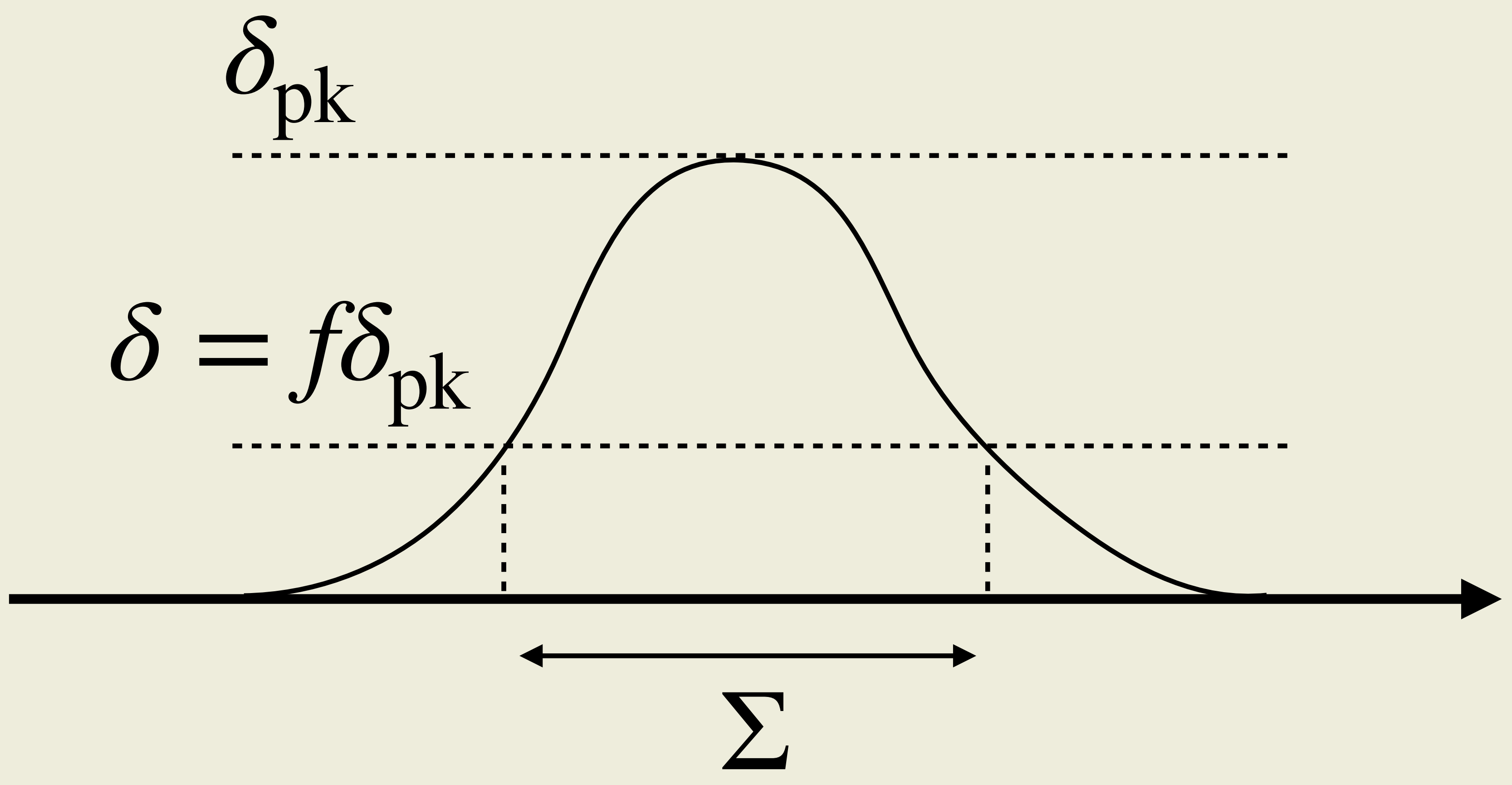
Goal: Evaluate initial spin of a PBH with the soft EOS.

Assumptions:

- Background: flat FLRW $ds^2 = a^2(-d\eta^2 + dx^2 + dy^2 + dz^2)$
- Matter: perfect fluid with $p = w\rho$ $0 < w \leq 1/3$
- Spacetime: $ds^2 = -\alpha a^2 d\eta^2 + a^2 e^{-2\zeta} \left(dx^2 + dy^2 + dz^2\right)$
- Focus on linear order effects of cosmological perturbation
- Curvature perturbation $\zeta(\eta)$: Random Gaussian field
- Power spectrum $P_\zeta(k)$: almost monochromatic

Spin of a closed region

(De Luca+ 2019, Harada+ 2021)



Assume the region Σ will collapse into a BH:

$$\Sigma = \{\vec{x} | \delta(\vec{x}) > f\delta_{\rm pk}\} \sim \text{ellipsoid} \quad (0 < f < 1)$$

Total angular momentum of Σ (Komar integral):

$$S_i(\Sigma) := -\int_\Sigma T^{ab} n_a (\phi_i)_b d\Sigma$$

$$\simeq -(1+w)a^4 \rho_b \epsilon_{ijk} v^k{}_l \int_\Sigma x^j x^k d^3x \quad (\vec{x}_{\rm pk} = \vec{0})$$

$$\propto ([a_2^2 - a_3^2]v_{23}, [a_3^2 - a_1^2]v_{13}, [a_1^2 - a_2^2]v_{12})$$

n_a : unit normal to the time slice

$(\phi_i)_b$: rotational Killing vector

$v^k{}_l = \dfrac{\partial v^k}{\partial x^l}$: velocity shear

a_i : principal axes of $\sum$

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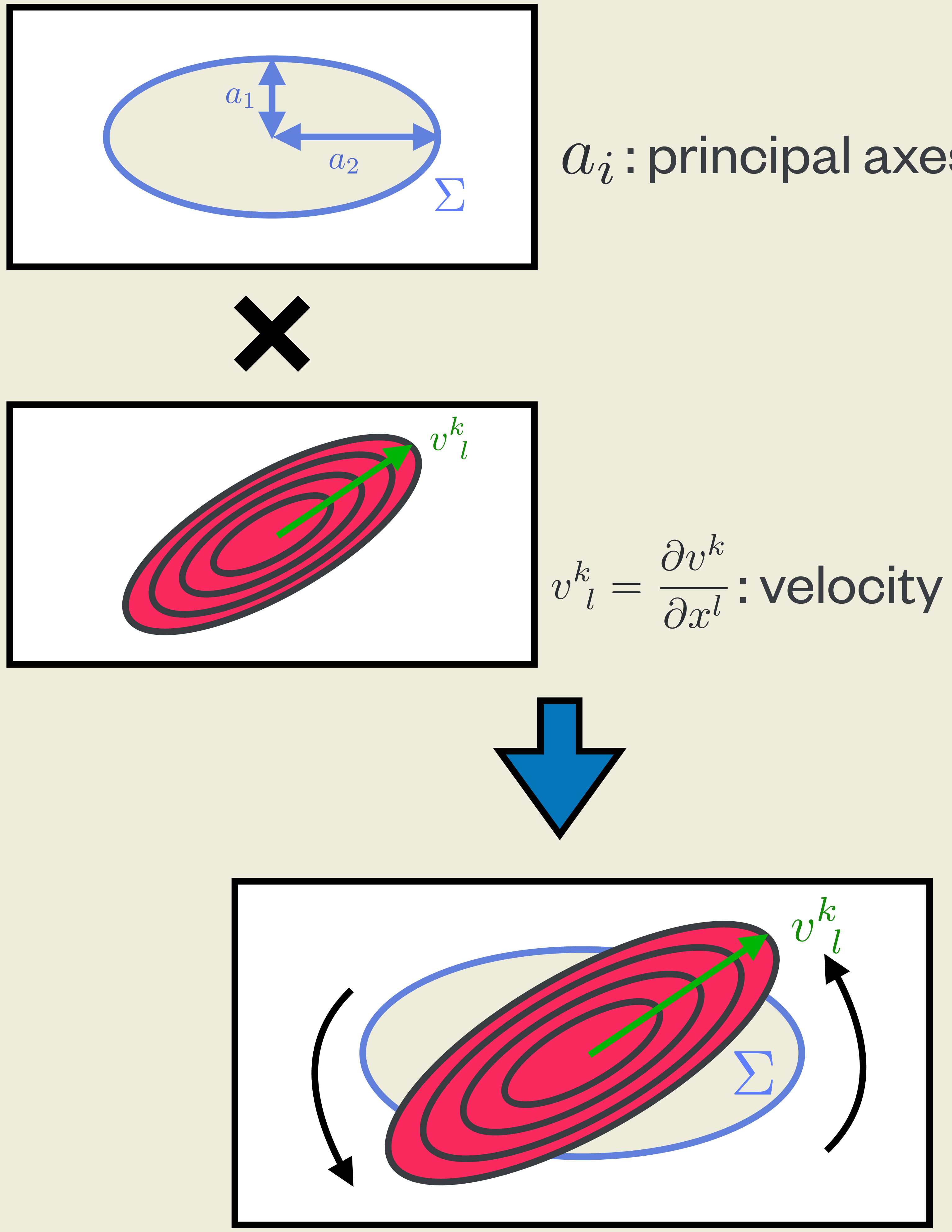


a_i : principal axes of Σ

$v^k_{\ l} = \frac{\partial v^k}{\partial x^l}$: velocity shear

If the principal axes & the shear are misaligned, $S_i \neq 0$. ***(tidal torque)***

Statistics of perturbation: Initial condition

At the initial time $\eta_{\mathrm{init}} < \eta_{\mathrm{H}}$:

- Curvature perturbation ζ: a random Gaussian field (assumption).
- Density contrast δ & velocity gradient ${v^i}_j$: given by the long wave-length solution (Harada+ 2015),

$$\delta \simeq \frac{2}{3a^2 H_b^2} \Delta\zeta,$$

$$v^i_j = \int \frac{d^3\vec{k}}{(2\pi)^3} \frac{k^i k_j}{k} v_{\vec{k}}(\eta) e^{i\vec{k}\cdot\vec{x}}, \quad v_{\vec{k}}(\eta) \propto T_v(k,\eta)\zeta$$

(CMC gauge) T_v : transfer function

in the leading order of gradient expansion, $\epsilon := k/aH_b \ll 1$.

➡ δ & ${v^i}_j$ are also random Gaussian fields.

Peak theory and analytic formula

Peak theory (BBKS 1984): statistics of Gaussian variables at the peak

Gaussian distribution:

$$f(V_i)d^{16}V_i = \frac{1}{(2\pi)^8\sqrt{\det M}} \exp\left[-\frac{1}{2}V_i(M^{-1})^{ij}V_j\right]$$

$V_i = \delta, \delta_i, \delta_{ij}, {v^i}_j$ $\quad \delta_i = \partial_i\delta \quad \delta_{ij} = \partial_i\partial_j\delta$

$M_{ij} = \langle V_i V_j \rangle$: correlation matrix

PDF of the peak values:

$$P(\nu, \lambda_i, w_i) = P_{\nu\lambda}(\nu, \lambda_i)P_w(w_i)$$

$\nu = \frac{\delta_{\rm pk}}{\sigma_0}$, λ_i: eigenvalues of $\frac{\delta_{ij}}{\sigma_2}$, w_i: non-diag comp of $-\frac{v_{ij}}{\sigma_0}$, $\sigma_j^2 = \frac{4}{9}\int \frac{dk}{k}\left(\frac{1}{aH_b}\right)^4 k^{2j+4}P_\zeta(k)$.

Assuming a high peak ($\nu \gg 1$), RMS of the spin parameter is

$$\sqrt{\langle a_*^2 \rangle} = \frac{\sqrt{\langle S^2(\eta)\rangle}}{M^2} \propto \frac{(1+w)^2}{5+3w}\left(\frac{a(\eta)}{a(\eta_H)}\right)^{3w+1} |T_v|\sqrt{1-\gamma^2}\left(\frac{\nu}{8}\right)^{-1}(1-f)^{-1/2}$$

M : mass of Σ, $\gamma = \frac{\sigma_1^2}{\sigma_0\sigma_2}$: width of $P_\zeta(k)$

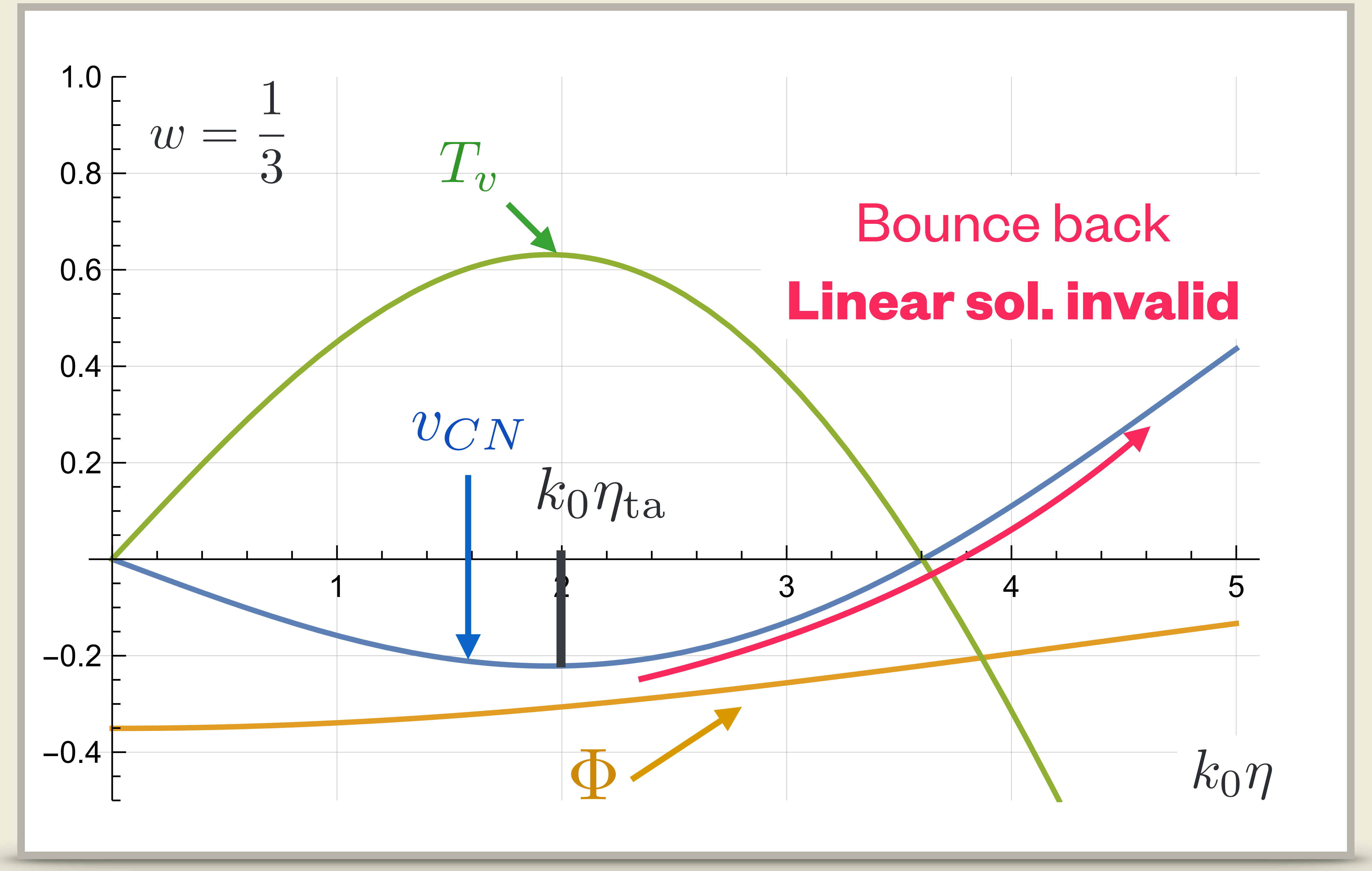
Turn-around time

Assumption on turn-around:

The evolution of Σ is decoupled from the cosmic expansion at the turn-around time $\eta_{\rm ta}$ defined by

$$v'_{\rm CN}(\eta_{\rm ta}) = 0,$$

and the spin is conserved for $\eta > \eta_{\rm ta}$ until the PBH is formed.



Result: turn-around time

Turn around time η_{ta}: $v'_{\rm CN}(\eta_{\rm ta}) = 0$

Horizon entry of $1/k_0$($\sim$ wavelength)

Hubble horizon entry η_{HH}

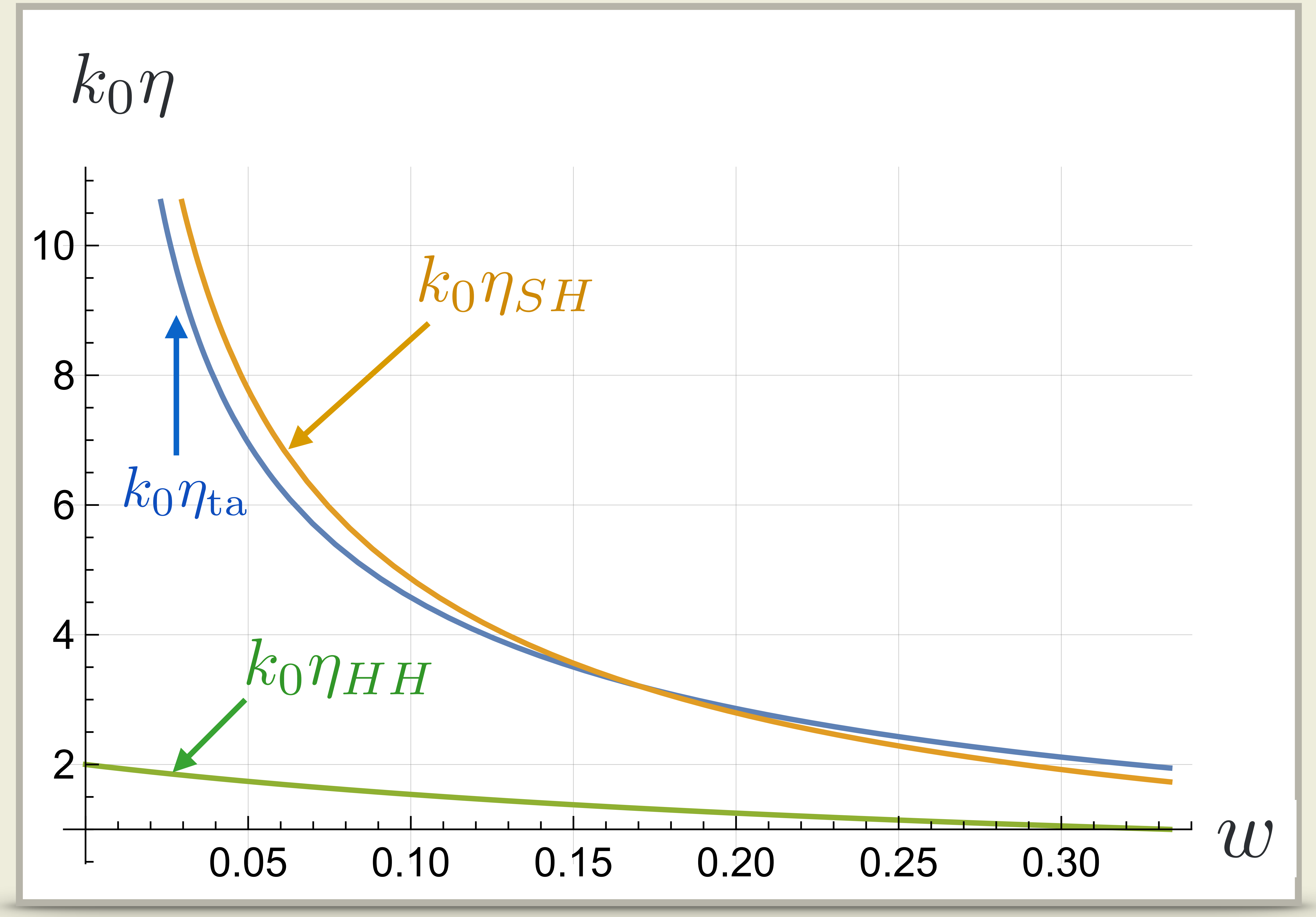
$$\frac{a(\eta_{HH})}{k_0} = \frac{1}{H(\eta_{HH})}$$

Sonic horizon entry η_{SH}

$$\frac{a(\eta_{SH})}{k_0} = \frac{\sqrt{w}}{H(\eta_{SH})}$$

Sonic horizon

Turn around $\sim$ sonic horizon entry of $1/k_0$



Result: RMS of spin

$\nu = 8 \quad \gamma = 0.85 \quad M \sim M_{\rm H}$ Amp. of fluctuation: threshold

Spin decreases with w

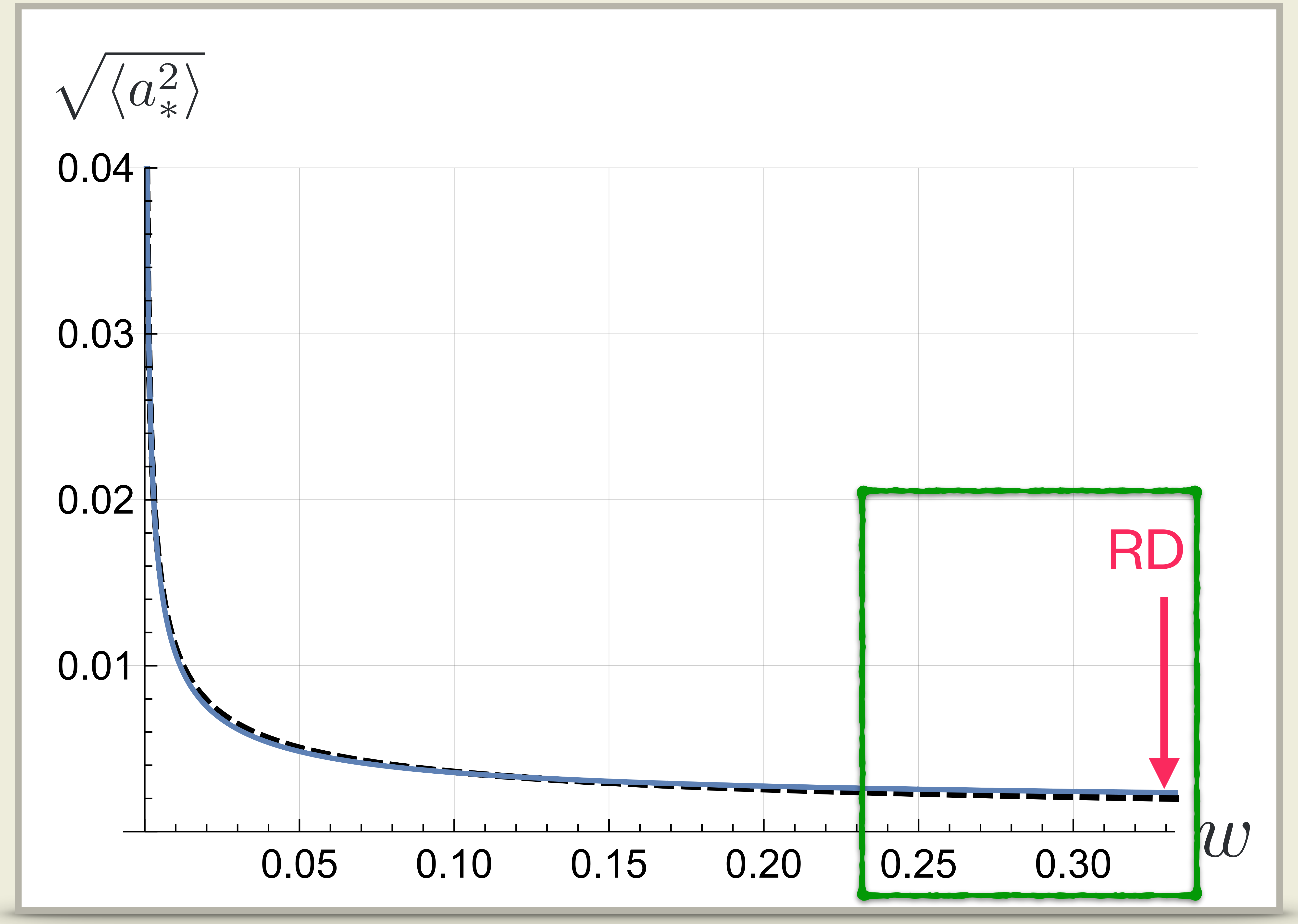
$\sqrt{\langle a_*^2 \rangle} = 2.3 \times 10^{-3}$ for RD

$\sqrt{\langle a_*^2 \rangle} \geq O(10^{-2})$ for $w \leq 10^{-2}$

$\sqrt{\langle a_*^2 \rangle} = 2.6 \times 10^{-3}$ for $w = 0.23$

QCD phase transition: $0.23 < w < 1/3$
Not so significantly changed

Fitted by $\sqrt{\langle a_*^2 \rangle} \propto w^{-0.49}$ (dashed line)



Discussion

$$\sqrt{\langle a_*^2\rangle} \propto \frac{(1+w)^2}{5+3w}\left(\frac{a(\eta_{ta})}{a(\eta_{HH})}\right)^{2w+1}|T_v(k_0,\eta_{ta})|$$

growth of scale factor

Hubble horizon entry → turn-around

$\sim \eta_{SH}$

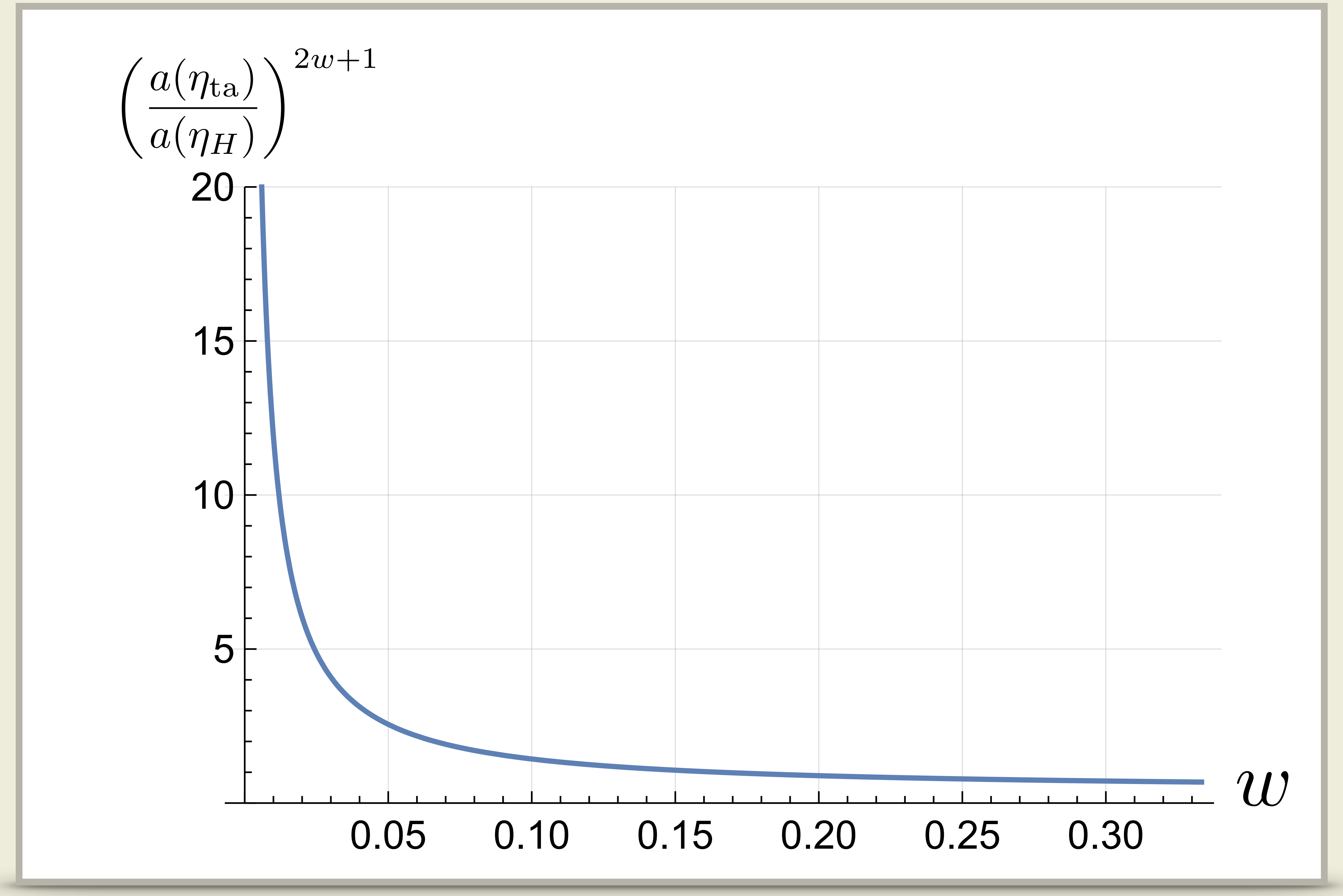
$$\frac{a(\eta_{ta})}{a(\eta_{HH})} \simeq \left(\frac{\eta_{SH}}{\sqrt{6}\eta_{HH}}\right)^{\frac{2}{1+3w}} = \left(\frac{1}{6w}\right)^{\frac{1}{1+3w}}$$

Smaller w

→ smaller sonic horizon

→ larger growth until turn-around

→ larger value of spin



Summary

- We have estimated the PBH spin due to matter with $0 < w \leq 1/3$
- The spin decreases with w

  $\sqrt{\langle a_*^2 \rangle} = 2.3 \times 10^{-3}$ for RD

  $\sqrt{\langle a_*^2 \rangle} = O(10^{-2})$ for $w = O(0.01)$ $\sqrt{\langle a_*^2 \rangle} \propto w^{-0.49}$

  No significant effect during QCD phase transition
- Spacetime expansion ($a(\eta_{HH}) \to a(\eta_{SH})$) contributes
- Future work

  -profile dependence

  -consider highly non-spherical (low peak) situations

  Crucial for MD era (on-going)

Thank you for your attention!