

SIDM halo structure and evolution: Semi-analytical and effective models

Shunichi Horigome @ Kavli IPMU

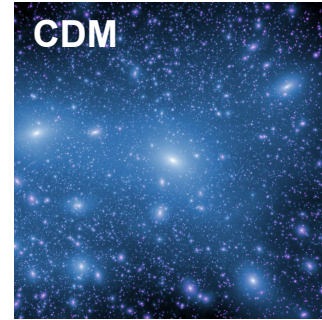
based on

S. Ando, S. Horigome and M. Shirasaki [in prep.]

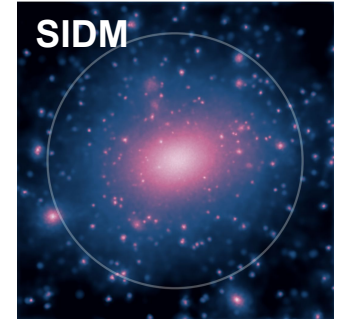
S. Ando, S. Horigome, E. O. Nadler, D. Yang, H.-B. Yu [in prep.]

CDM vs. SIDM

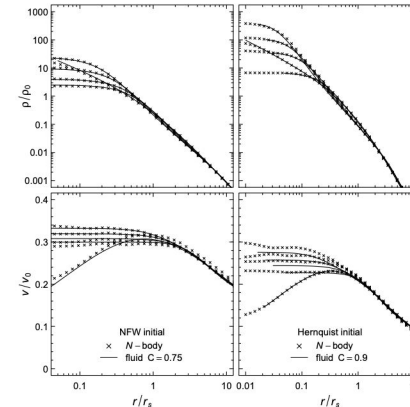
- CDM
 - Great explanation of LSS
 - Small scale...?
 - Cored-cuspy problem
 - Diversity problem
 - Too-big-to-fail problem
 - (baryonic feedback?)
- SIDM
 - Alternative model which can explain the small scale structure
 - **Core formation due to the self-interaction**
 - **Gravothermal core-collapse of subhalos**



[Millennium-II simulation](#)



[Brinckmann+\(2017\)](#)



[\[Koda+\(2011\)\]](#)

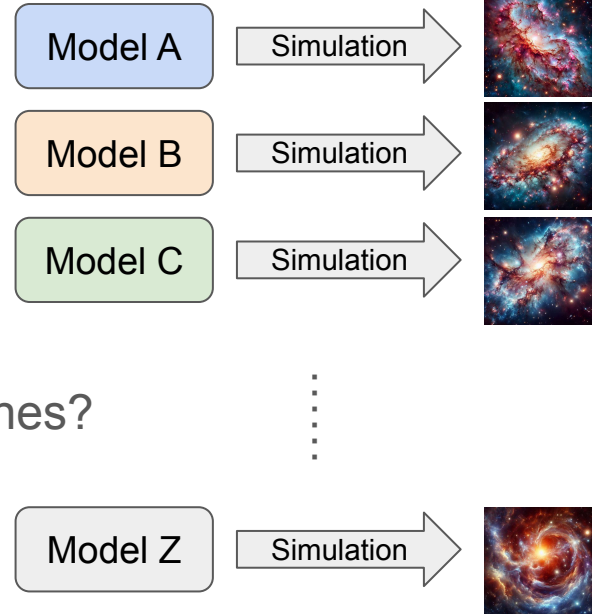
SIDM

SIDM is interesting but challenging...

Problems of N-body simulations

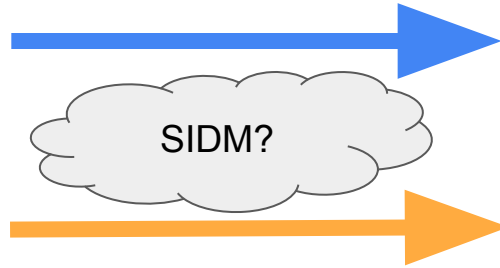
- Resolution
- Computational costs

What can we do for quick (particle) SIDM searches?



SIDM

Two directions of SIDM searches



Semi-analytical modeling of SIDM halos

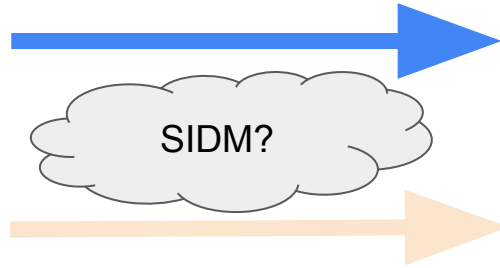
- Beyond the limitation of N-body simulation
 - Resolution
 - Quickness

Effective model of SIDM evolution

- Gravothermal fluid model
- Better understanding of physics behind SIDM halos

SIDM

Two directions of SIDM searches



Semi-analytical modeling of SIDM halos

- Beyond the limitation of N-body simulation
 - Resolution
 - Quickness

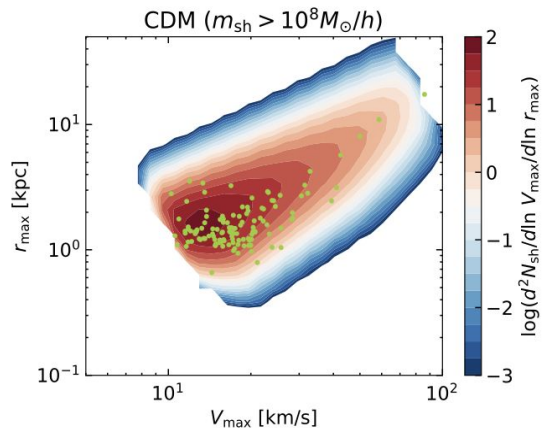
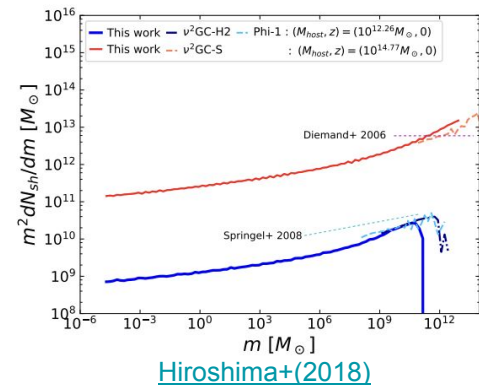
Effective model of SIDM evolution

- Gravo-thermal fluid model
- Better understanding of physics behind SIDM halos

SASHIMI

Semi-Analytical SubHalo Inference Modelling (SASHIMI)

- Simulate subhalo properties using
 - Extended-Press-Schechter model
 - Tidal stripping model calibrated by N-body simulation
- Outcomes
 - Continuous subhalo catalogue
 - Subhalo mass function
 - Probability distribution of
 - V_{\max} (maximum circular velocity)
 - r_{\max} ($V(r_{\max}) = V_{\max}$)
 - Subhalo size
 - Subhalo density
 - ...



Various SASHIMI

~ Menu ~

SASHIMI-C

for CDM

SASHIMI-W

for WDM

SASHIMI-SIDM

for SIDM

Today's Special



SASHIMI-SIDM

SASHIMI with....

- Parametric model of SIDM halos: [\[Yang+\(2023\)\]](#)

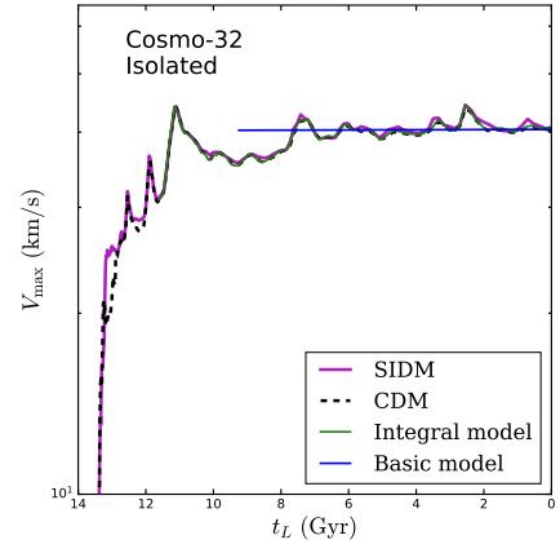
$$\frac{V_{\max}}{V_{\max,0}} = 1 + 0.1777\tau - 4.399\tau^3 + 16.66\tau^4 - 18.87\tau^5 + 9.077\tau^7 - 2.436\tau^9$$

$$\frac{R_{\max}}{R_{\max,0}} = 1 + 0.007623\tau - 0.7200\tau^2 + 0.3376\tau^3 - 0.1375\tau^4,$$

$$\tau = t/t_c$$

Core-collapse time scale:

$$t_c = \frac{150}{C} \frac{1}{(\sigma_{\text{eff}}/m)\rho_{\text{eff}}r_{\text{eff}}} \frac{1}{\sqrt{4\pi G\rho_{\text{eff}}}},$$

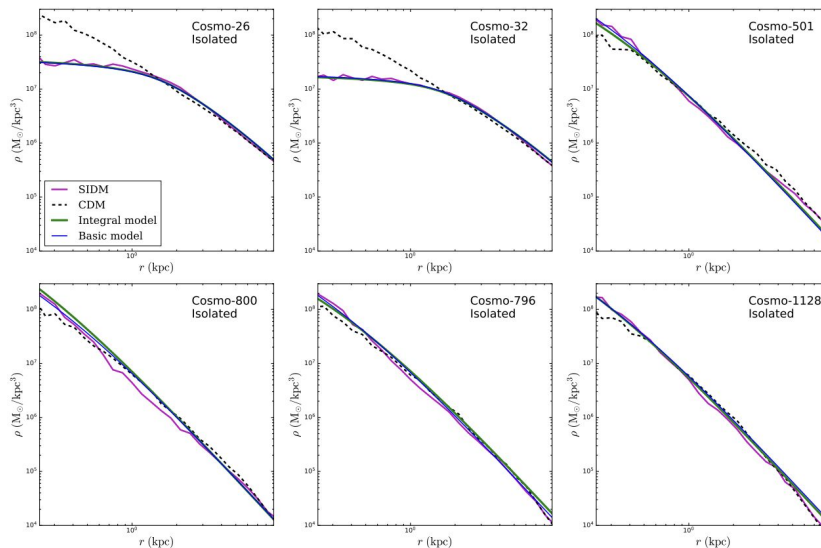


[Yang+\(2023\)](#)

SASHIMI-SIDM

Parametric model of SIDM halos: [\[Yang+\(2023\)\]](#)

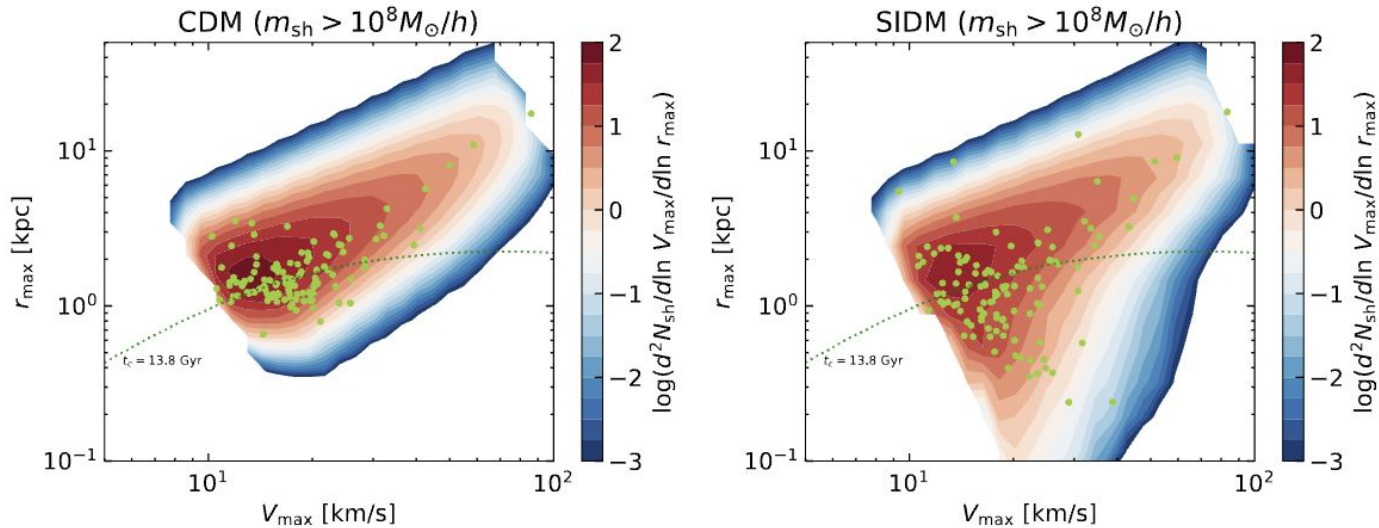
Nice fitting of SIDM profiles and their evolution



[Yang+\(2023\)](#)

Testing and Tasting SASHIMI-SIDM

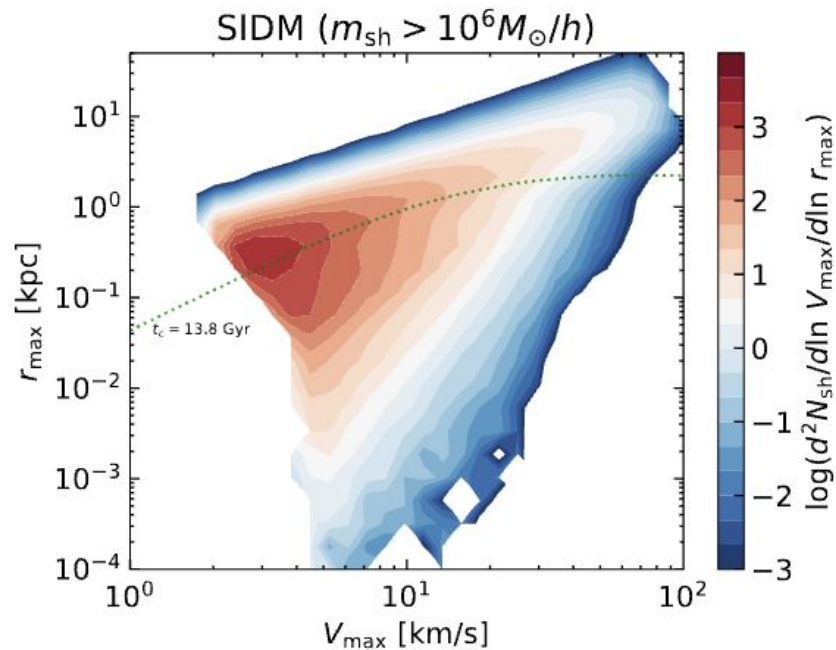
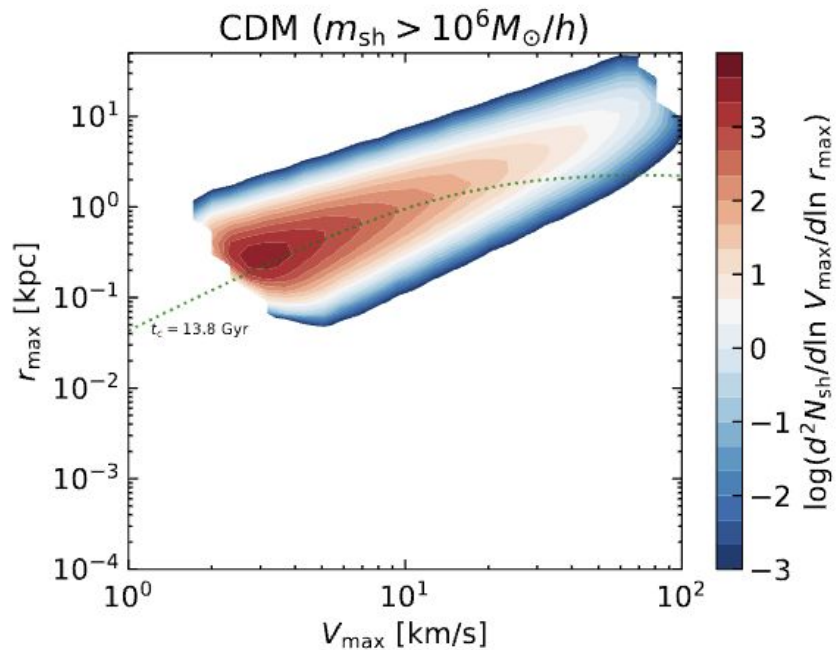
SASHIMI-SIDM can reproduce N-body results



SASHIMI-SIDM allows us to play with SIDM easily in a few minutes

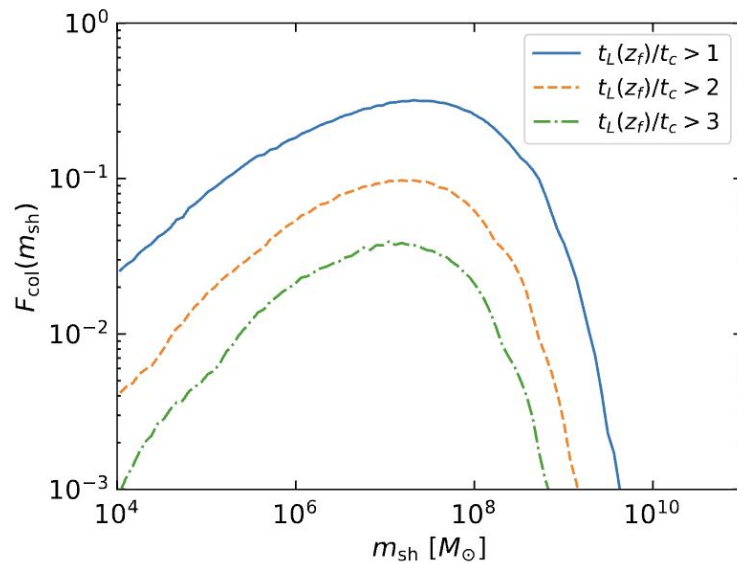
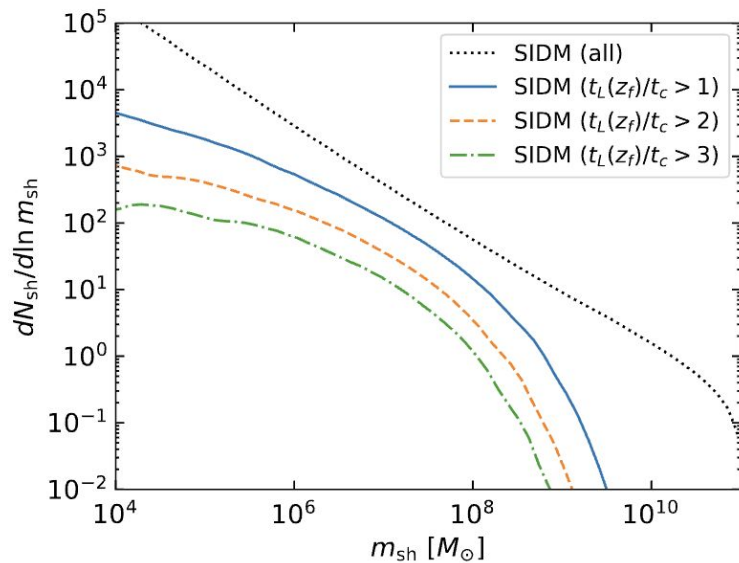
SASHIMI-SIDM

Use Case: Property for lighter subhalos



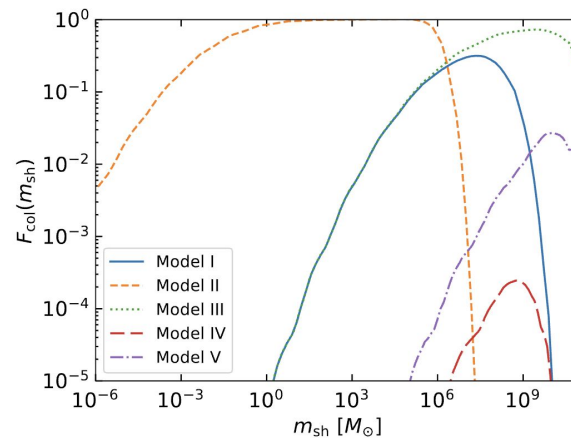
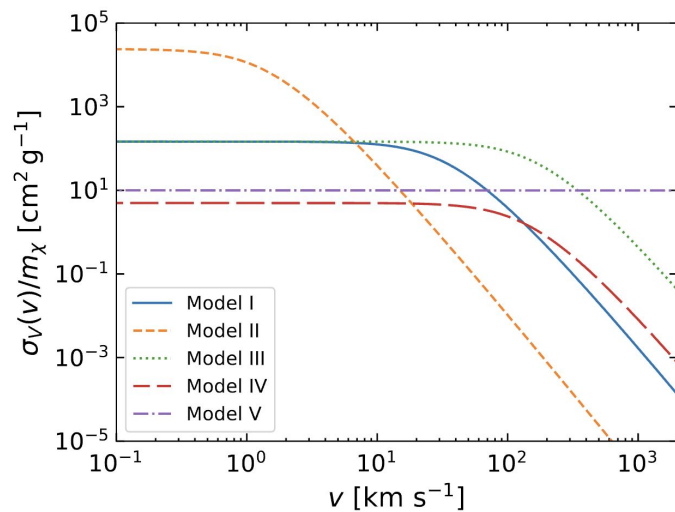
SASHIMI-SIDM

Use case: Subhalo mass function



SASHIMI-SIDM

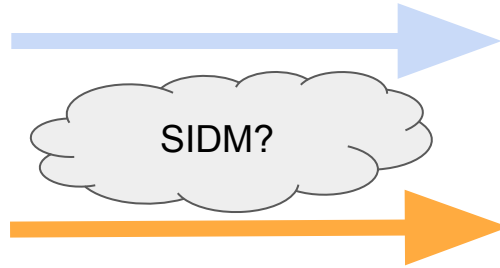
Subhalo mass function for various SIDM models



It works for lighter subhalo masses

SIDM

Two directions of SIDM searches



Semi-analytical modeling of SIDM halos

- Beyond the limitation of N-body simulation
 - Resolution
 - Quickness

Effective model of SIDM evolution

- Gravothermal fluid model
- Better understanding of physics behind SIDM halos

Effective model of SIDM evolution

- Gravo-thermal fluid model [[Balberg+\(2002\)](#)]
 - DM is modelled as a thermally conducting fluid in quasistatic virial equilibrium:

- $\frac{\partial M}{\partial r} = 4\pi r^2 \rho$ Mass conservation

- $\frac{\partial(\rho\sigma_v^2)}{\partial r} = -G\frac{M\rho}{r}$ Hydrostatic equilibrium

- $\frac{L}{4\pi r^2} = -\kappa\frac{\partial T}{\partial r} = -\kappa\rho\frac{\partial}{\partial r}\left(\frac{3\sigma_v^2}{2}\right)$ Heat conduction (the coefficient κ has been calibrated with simulations of SIDM isolated halos in Koda & Shapiro 2011)

- $\frac{\partial L}{\partial r} = -4\pi r^2 \rho \left[\left(\frac{\partial}{\partial t}\right)_M \frac{3\sigma_v^2}{2} + p \left(\frac{\partial}{\partial t}\right)_M \frac{1}{\rho} \right] = -4\pi r^2 \rho v^2 \left(\frac{\partial}{\partial t}\right)_M \log\left(\frac{\sigma_v^3}{\rho}\right)$ 1st law of thermodynamics

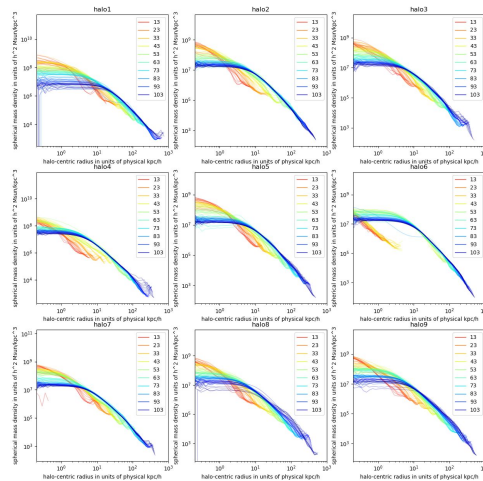
Note that $p = \rho\sigma_v^2$

- Nice fitting with isolated N-body simulation [[Koda+\(2011\)](#)]
 - → How about Cosmological N-body simulation?

Effective model of SIDM evolution

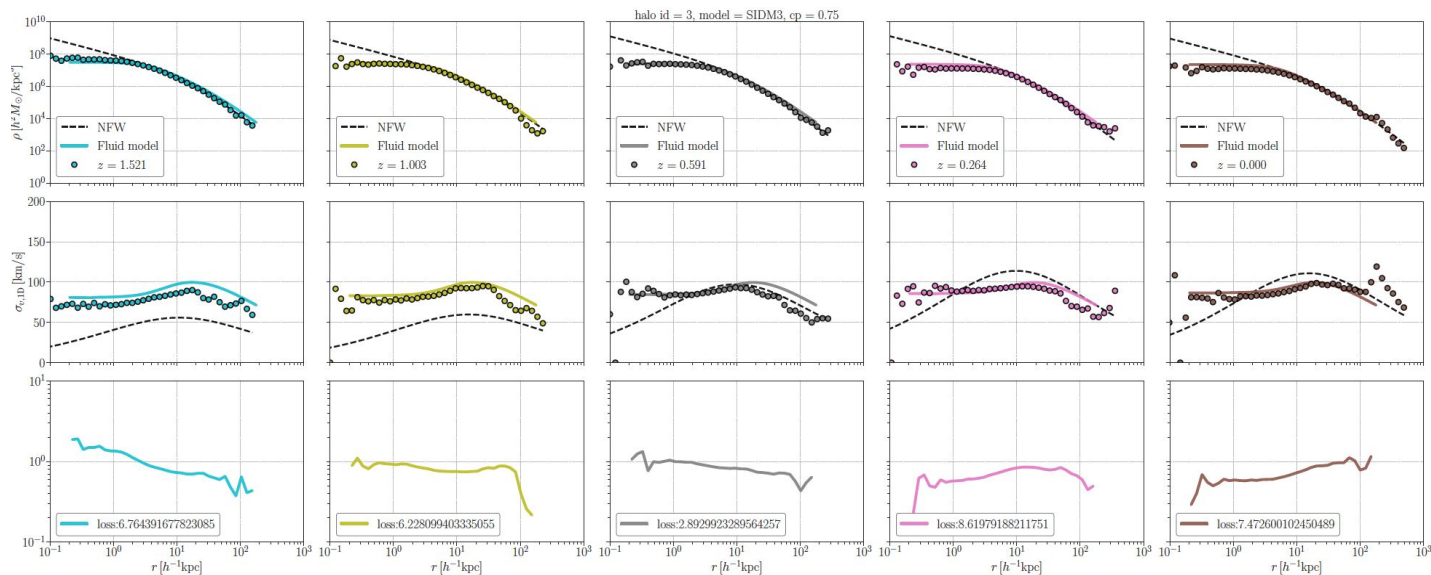
Dataset: SIDM N-body simulation [Ebisu, Ishiyama, and Hayashi (2022)]

- 1024^3 particles / $(8 \text{ Mpc/h})^3$ (comoving)
- CDM / SIDM_1 / SIDM_3 ($\sigma/m = 1$ or $3 \text{ cm}^2/\text{g}$)
- 9 MW-sized halos



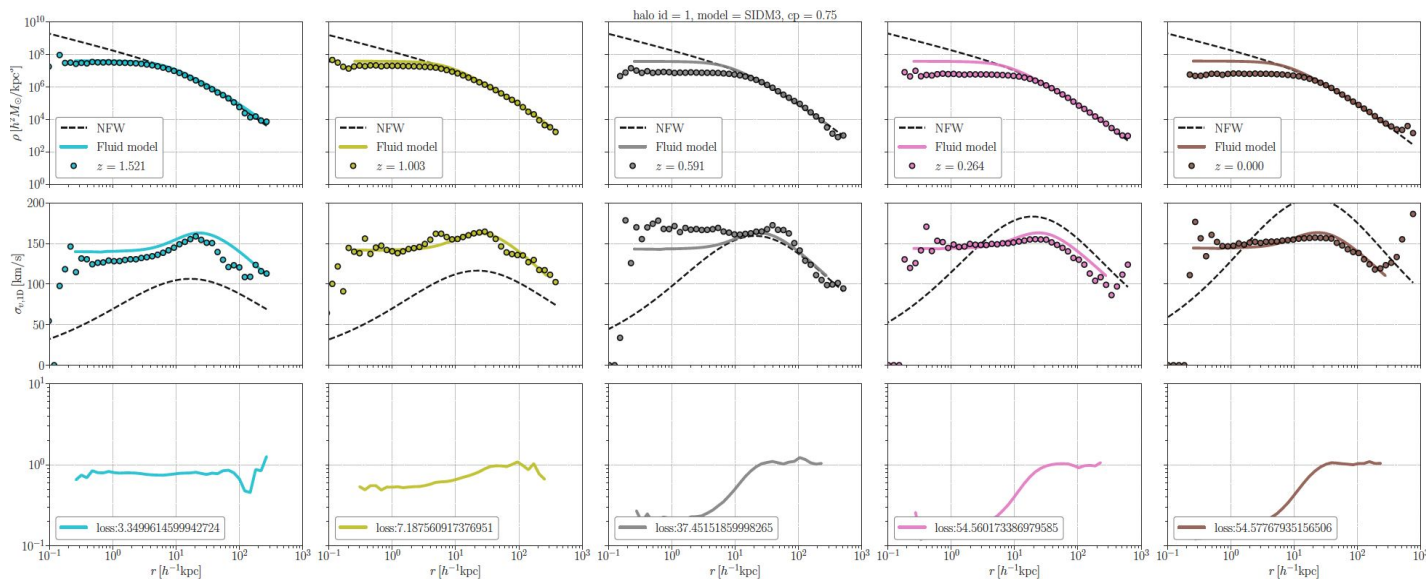
Effective model of SIDM evolution

Results (preliminary): e.g. halo 3... consistent



Effective model of SIDM evolution

Results (preliminary): e.g. halo 1... deviation

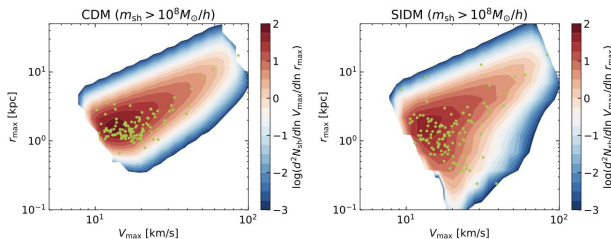


Future work: analysing the origin of the deviation

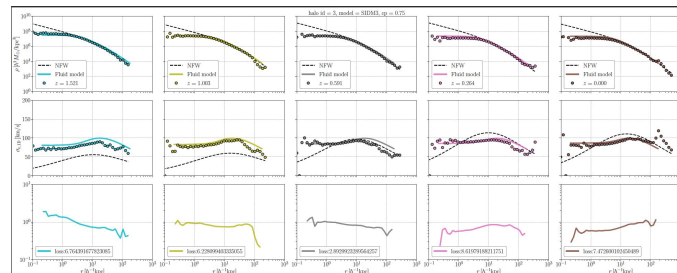
SIDM halo structure and evolution: Semi-analytical and effective models

- SIDM is an interesting candidate to solve small scale problems in CDM
- Difficulties of N-body SIDM simulations: Resolution, computation cost
- Two options:

- Semi-analytical modelling of SIDM halo
 - Quick SIDM simulator based on semi-analytical models



- Effective modelling of SIDM halo
 - Understanding the physics behind the halo structure



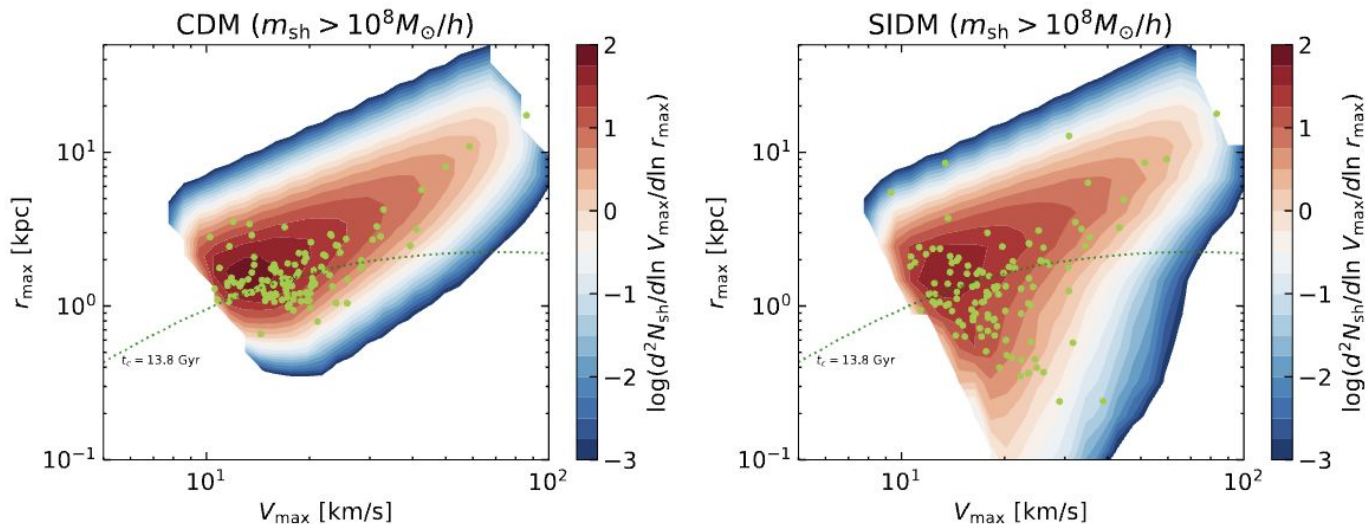
Backup

SIDM halo structure and evolution: Semi-analytical and effective models

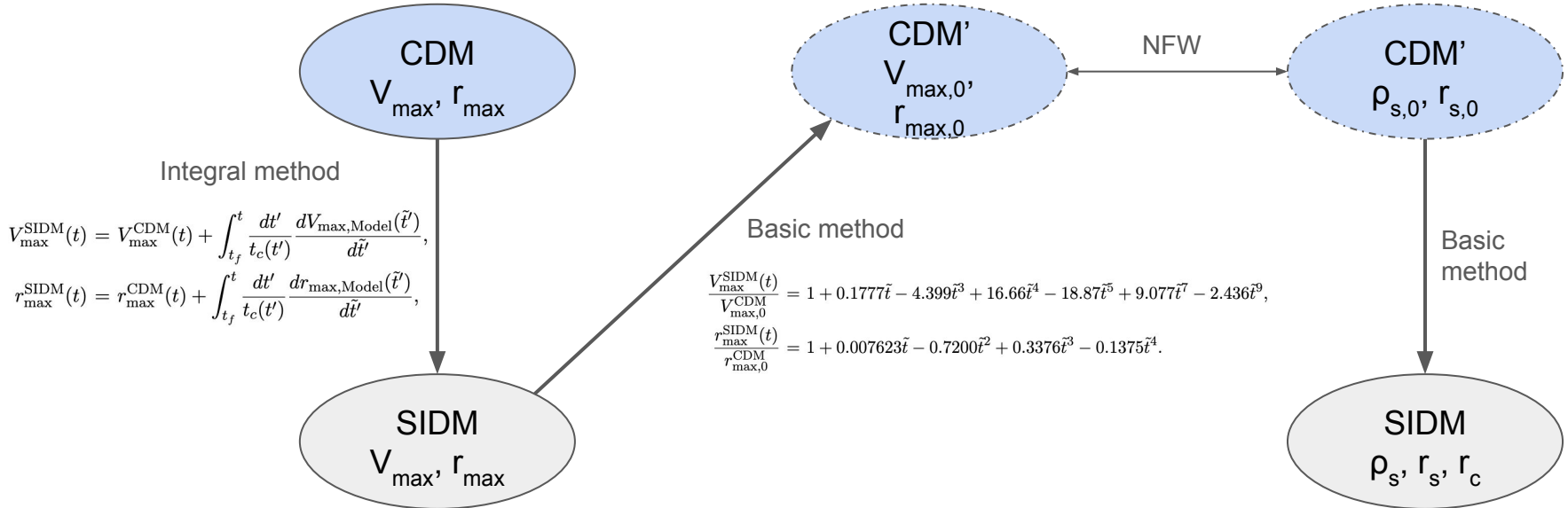
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D. Yang, H.-B. Yu [in prep.]

SASHIMI-SIDM: Semi-analytical approach to simulate SIDM models

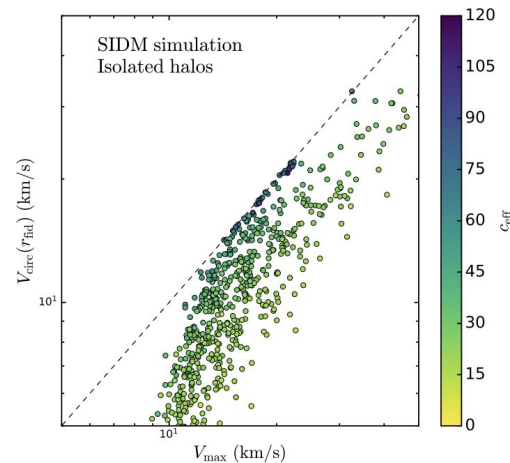
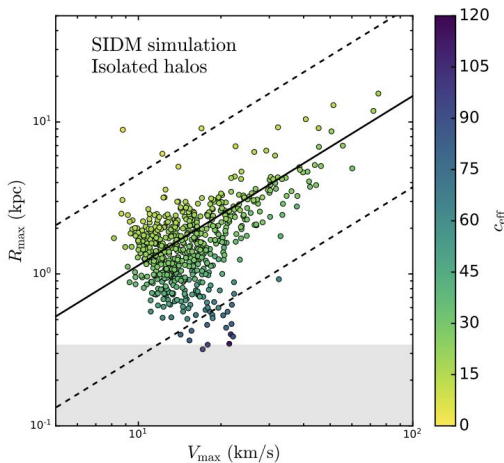
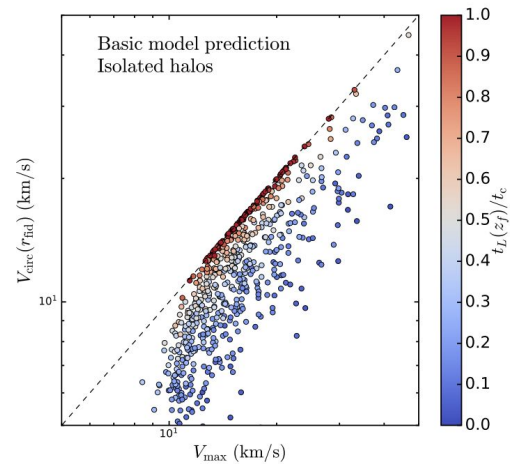
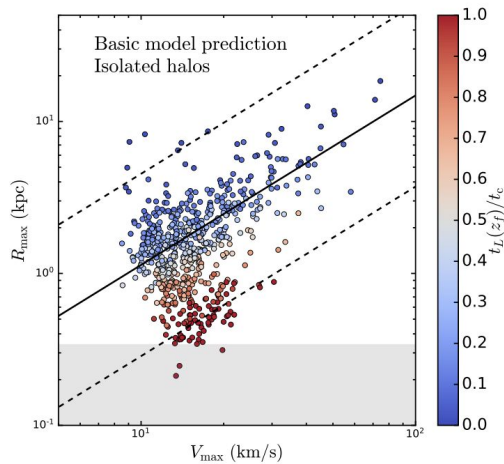
- Quick calculation of SIDM halo properties and subhalo mass functions



Parametric Model



Parametric Model



[Yang+\(2023\)](#)

$$r_{\text{fid}} = 2V_{\text{max}}/(70 \text{ km/s}) \text{ kpc}$$

Effective model of SIDM evolution

Parameters:

- Heat conductance

$$\kappa = b_* \sigma_v \left[\left(\frac{1}{\lambda} \right) + \left(\frac{b_* \sigma_v t_r}{C_* H_g} \right) \right]^{-1} \text{ with } b_* = 0.25, C_* = 0.75$$

- Initial redshift
 - $M_{z,\text{init}} = f * M_0$, $f = 0.04$
- mass, scale radius
 - CDM counterpart @ $z=0$

where $H_g \equiv \sqrt{\sigma_v^2 / (4\pi G \rho)}$ is the gravitational scale height of the system, $\lambda = (\rho \sigma / m)^{-1}$ is the collisional scale for the mean free path, $t_r \equiv \lambda / (a \sigma_v)$ is the relaxation time with a coefficient of order of unity being a , and we adopt $a = \sqrt{16/\pi}$ for hard-sphere scattering of particles with a Maxwell-Boltzmann velocity distribution (Reif 1965).

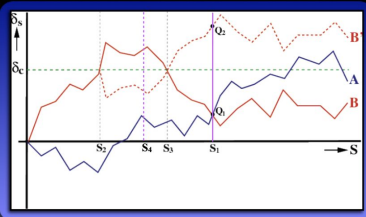
In the limit of $\lambda \ll H_g$, the thermal conductivity is given by $\kappa \simeq (3/2)(k_B/m)b_*\rho\lambda^2/(at_r)$ and b_* can be regarded as an effective impact parameter among particle collisions. In the limit of $\lambda \gg H_g$, one finds $\kappa \simeq (3/2)(k_B/m)C_*\rho H_g^2/t_r$, reproducing an empirical formula of gravothermal collapse of globular clusters (Lynden-Bell & Eggleton 1980).

EPS formalism

[Press & Schechter(1974)]

Ref: http://www.astro.yale.edu/vdbosch/astro610_lecture9.pdf

The Excursion Set Formalism



Three trajectories corresponding to three different mass elements in a Gaussian random field. Note that B' is obtained mirroring trajectory B in the line $\delta_S = \delta_c$ for $S > S_2$. Since the trajectories are Markovian and B and B' are equally likely!

The problem with the PS ansatz is that it fails to account for trajectories such as B when counting mass elements in haloes with mass $M > M_1$.

Correcting for this is easy though, by realizing that each trajectory B has a mirror version, B', that is equally likely (as a result of the Markovian nature of the trajectories).

Double-counting trajectories with $\delta_S > \delta_c$ at S_1 corrects for 'missed trajectories'....

A natural explanation for the fudge-factor two in PS formalism!

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The Excursion Set Formalism

In the excursion set formalism, also called the Extended Press-Schechter (EPS) formalism, one uses the (statistics of) Markovian random walks (the trajectories of mass elements in (S, δ_S) -space) to infer the halo mass function (and more).

PS ansatz: fraction of mass elements with $\delta_S > \delta_c(t)$ is equal to the mass fraction that at time t resides in haloes with masses $> M$, where S and M are related according to $S = \sigma^2(M)$

EPS ansatz: fraction of trajectories with a first upcrossing (FU) of the barrier $\delta_S = \delta_c(t)$ at $S > S_1 = \sigma^2(M_1)$ is equal to the mass fraction that at time t resides in haloes with masses $M < M_1$

Since, each trajectory is guaranteed to upcross the barrier $\delta_S = \delta_c(t)$ at some (arbitrarily large) S , the EPS ansatz predicts that every mass element is in a halo of some (arbitrarily low) mass

$F(< M_1) = 1 - F(> M_1)$

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The EPS Mass Function

Based on the EPS ansatz, we can write the EPS mass function as:

$$n(M, t) dM = \frac{\bar{\rho}}{M} \frac{\partial F(> M)}{\partial M} dM = -\frac{\bar{\rho}}{M} \frac{\partial F(< M)}{\partial M} dM$$

$$= -\frac{\bar{\rho}}{M} \frac{\partial F_{FU}(> S)}{\partial S} \frac{dS}{dM} dM = \frac{\bar{\rho}}{M} f_{FU}(S, \delta_c) \left| \frac{dS}{dM} \right| dM$$

Here $f_{FU}(S, \delta_c) dS$ is the fraction of trajectories that have their first upcrossing of barrier $\delta_c(t)$ between S and $S + dS$.

Without proof: $f_{FU}(\nu) = \frac{1}{\sqrt{2\pi}} \frac{\delta_c}{S^{3/2}} \exp\left[-\frac{\delta_c^2}{2S}\right] = \frac{1}{2S} f_{PS}(\nu)$ (see MBW §7.2.2 for derivation)

where, as before, we defined $\nu = \delta_c(t)/\sigma(M) = \delta_c/\sqrt{S}$ and we expressed the result in terms of the PS multiplicity function $f_{PS}(\nu) = \sqrt{2/\pi} \nu \exp(-\nu^2/2)$

It is straightforward to show that this yields exactly the same halo mass function as before, but this time there has been no need for a fudge factor....

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SASHIMI

Tidal stripping: [[Hiroshima+\(2018\)](#)]

$$\dot{m}(z) = -A \frac{m(z)}{\tau_{\text{dyn}}(z)} \left[\frac{m(z)}{M(z)} \right]^{\zeta},$$

$$\log A = \left[-0.0003 \log \left(\frac{M_{\text{host}}}{M_{\odot}} \right) + 0.02 \right] z$$
$$+ 0.011 \log \left(\frac{M_{\text{host}}}{M_{\odot}} \right) - 0.354,$$

$$\zeta = \left[0.00012 \log \left(\frac{M_{\text{host}}}{M_{\odot}} \right) - 0.0033 \right] z$$
$$- 0.0011 \log \left(\frac{M_{\text{host}}}{M_{\odot}} \right) + 0.026.$$

