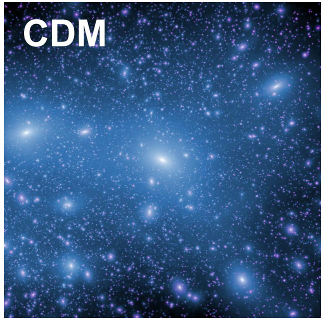
SIDM halo structure and evolution: Semi-analytical and effective models

Shunichi Horigome @ Kavli IPMU

based on
S. Ando, S. Horigome and M. Shirasaki [in prep.]
S. Ando, S. Horigome, E. O. Nadler, D. Yang, H.-B. Yu [in prep.]

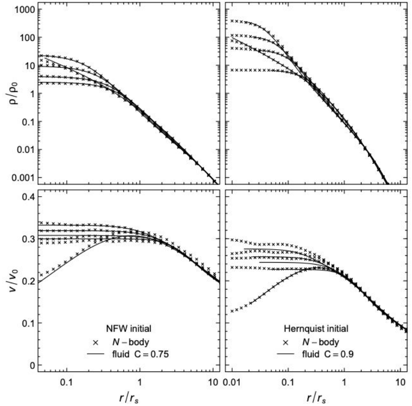
CDM vs. SIDM

- CDM
  - Great explanation of LSS
  - Small scale…?
    - Cored-cuspy problem
    - Diversity problem
    - Too-big-to-fail problem
      - (baryonic feedback?)
- **SIDM**
  - Alternative model which can explain the small scale structure
    - **Core formation due to the self-interaction**
    - **Gravothermal core-collapse of subhalos**



Millennium-II simulation

Brinckmann+(2017)



[Koda+(2011)]

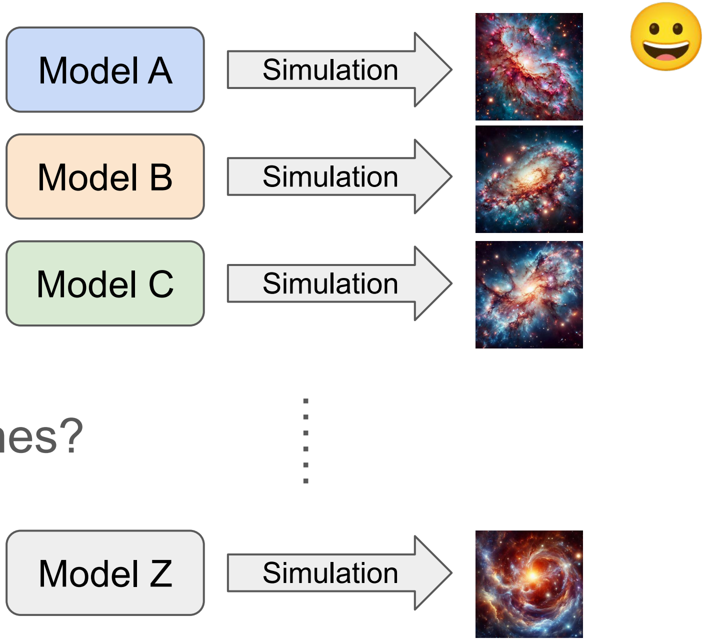
SIDM

SIDM is interesting but challenging…

Problems of N-body simulations

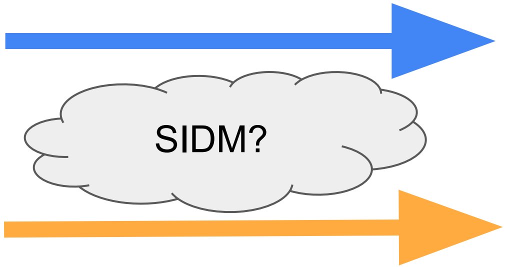
- Resolution
- Computational costs

What can we do for quick (particle) SIDM searches?



SIDM

Two directions of SIDM searches



Semi-analytical modeling of SIDM halos

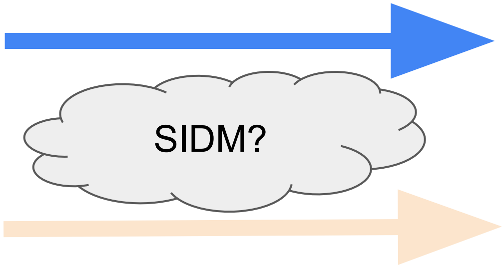
- Beyond the limitation of N-body simulation
  - Resolution
  - Quickness

Effective model of SIDM evolution

- Gravothermal fluid model
- Better understanding of physics behind SIDM halos

SIDM

Two directions of SIDM searches



Semi-analytical modeling of SIDM halos

- Beyond the limitation of N-body simulation
  - Resolution
  - Quickness

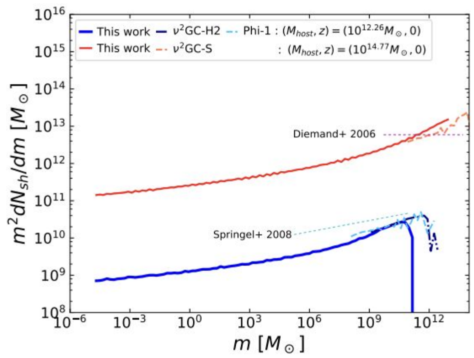
Effective model of SIDM evolution

- Gravothermal fluid model
- Better understanding of physics behind SIDM halos

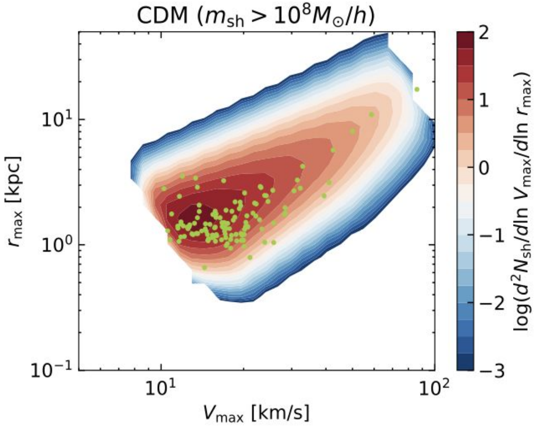
SASHIMI

Semi-Analytical SubHalo Inference ModelIng (SASHIMI)

- Simulate subhalo properties using
  - Extended-Press-Schechter model
  - Tidal stripping model calibrated by N-body simulation
- Outcomes
  - Continuous subhalo catalogue
    - Subhalo mass function
    - Probability distribution of
      - V_{max} (maximum circular velocity)
      - r_{max} ($V(r_{max}) = V_{max}$)
      - Subhalo size
      - Subhalo density
      - …



Hiroshima+(2018)



Various SASHIMI

~ *Menu* ~

SASHIMI-C

for CDM

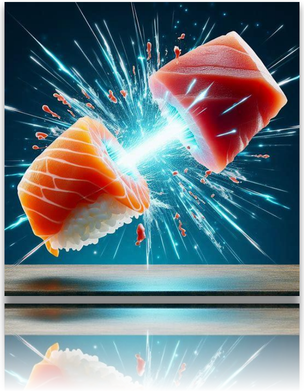
SASHIMI-W

for WDM

Today's Special

SASHIMI-SIDM

for SIDM



SASHIMI-SIDM

SASHIMI with….

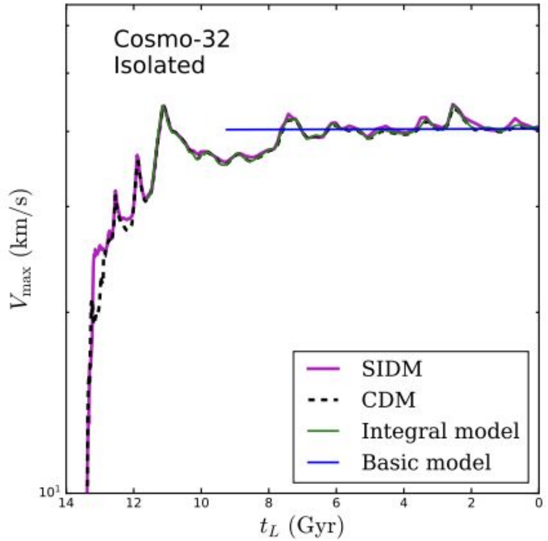
- Parametric model of SIDM halos: [Yang+(2023)]

$$\frac{V_{\max}}{V_{\max,0}} = 1 + 0.1777\tau - 4.399\tau^3 + 16.66\tau^4 - 18.87\tau^5 + 9.077\tau^7 - 2.436\tau^9$$

$$\frac{R_{\max}}{R_{\max,0}} = 1 + 0.007623\tau - 0.7200\tau^2 + 0.3376\tau^3 - 0.1375\tau^4,$$

$$\tau = t/t_c$$

Core-collapse time scale: $$t_c = \frac{150}{C}\frac{1}{(\sigma_{\rm eff}/m)\rho_{\rm eff} r_{\rm eff}}\frac{1}{\sqrt{4\pi G\rho_{\rm eff}}},$$

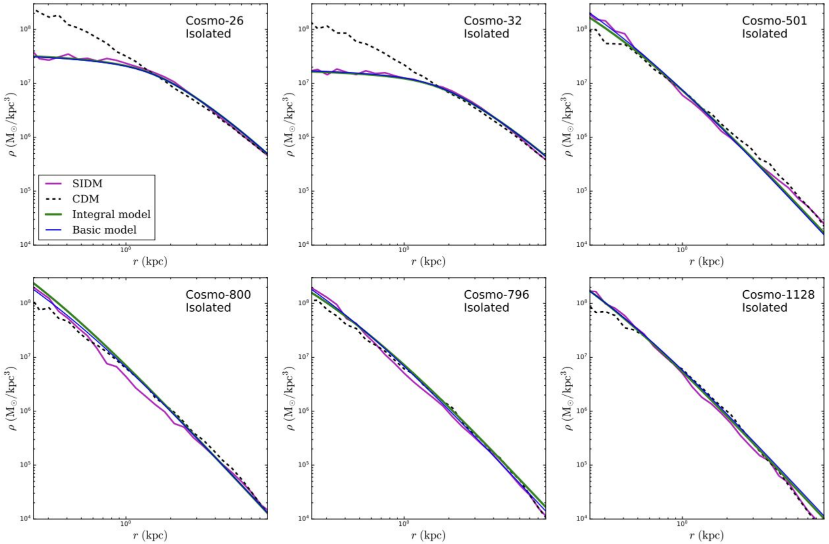


Yang+(2023)

SASHIMI-SIDM

Parametric model of SIDM halos: [Yang+(2023)]

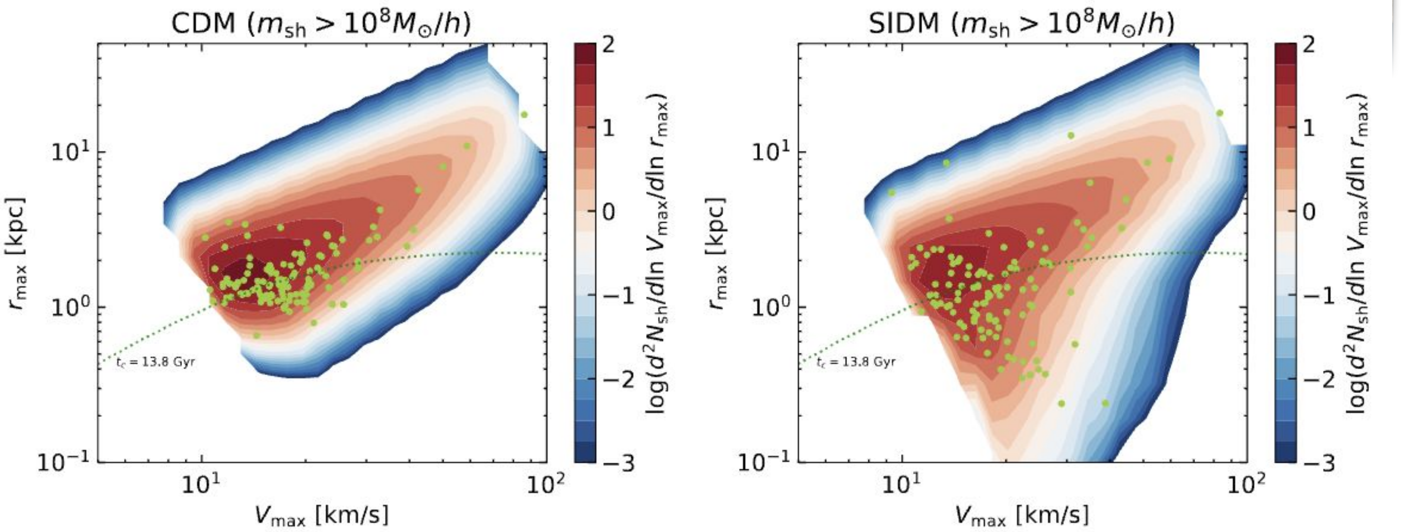
- Nice fitting of SIDM profiles and their evolution



Yang+(2023)

Testing and Tasting SASHIMI-SIDM

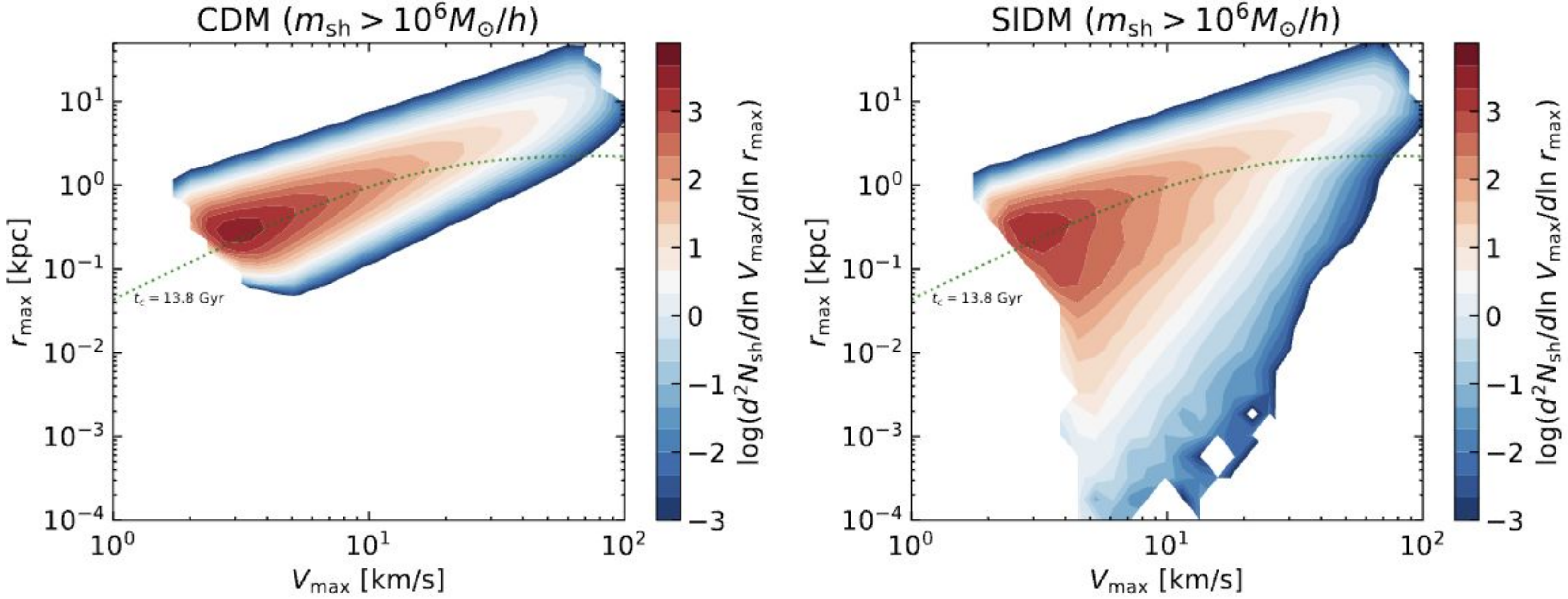
SASHIMI-SIDM can reproduce N-body results



SASHIMI-SIDM allows us to play with SIDM easily in a few minutes

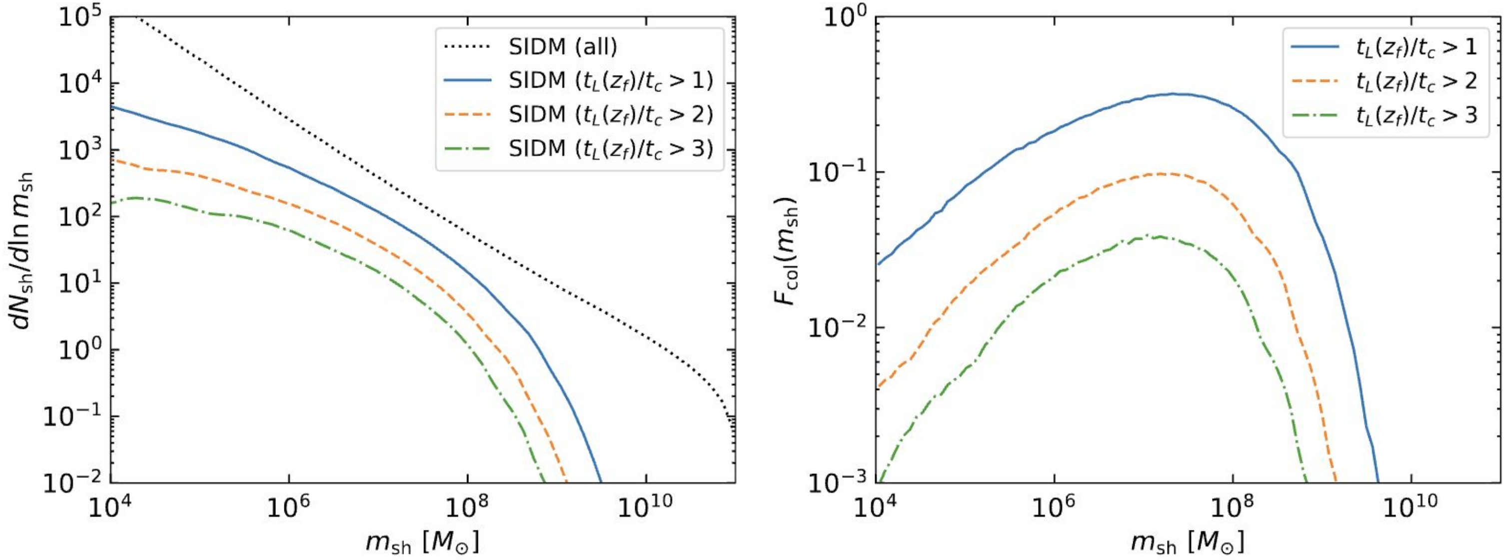
SASHIMI-SIDM

Use Case: Property for lighter subhalos



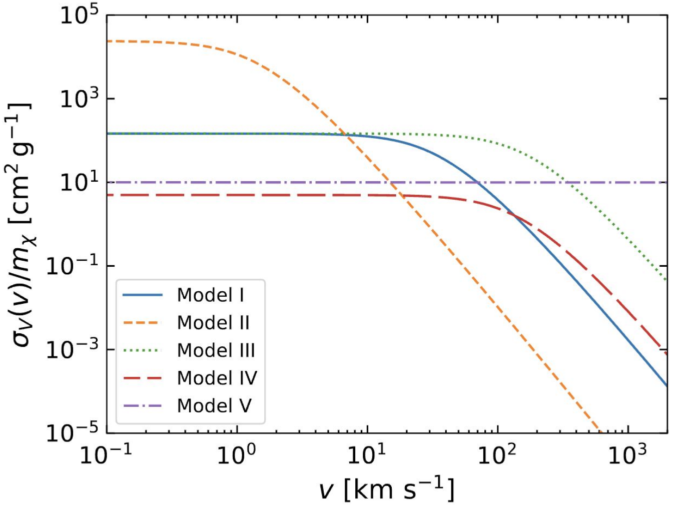
SASHIMI-SIDM

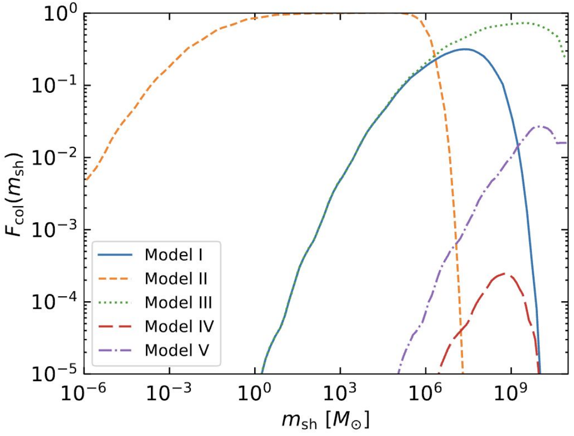
Use case: Subhalo mass function



SASHIMI-SIDM

Subhalo mass function for various SIDM models

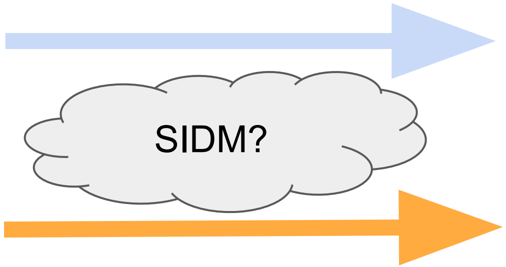




It works for lighter subhalo masses

SIDM

Two directions of SIDM searches



Semi-analytical modeling of SIDM halos

- Beyond the limitation of N-body simulation
  - Resolution
  - Quickness

Effective model of SIDM evolution

- Gravothermal fluid model
- Better understanding of physics behind SIDM halos

Effective model of SIDM evolution

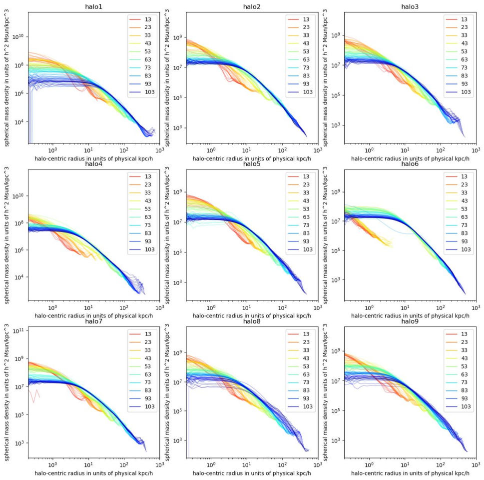
- Gravothermal fluid model [Balberg+(2002)]
  - DM is modelled as a thermally conducting fluid in quasistatic virial equilibrium:
    - $\frac{\partial M}{\partial r} = 4\pi r^2 \rho$ Mass conservation
    - $\frac{\partial(\rho\sigma_v^2)}{\partial r} = -G\frac{M\rho}{r}$ Hydrostatic equilibrium
    - $\frac{L}{4\pi r^2} = -\kappa\frac{\partial T}{\partial r} = -\kappa\rho\frac{\partial}{\partial r}\left(\frac{3\sigma_v^2}{2}\right)$ Heat conduction (the coefficient κ has been calibrated with simulations of SIDM isolated halos in Koda & Shapiro 2011)
    - $\frac{\partial L}{\partial r} = -4\pi r^2\rho\left[\left(\frac{\partial}{\partial t}\right)_M \frac{3\sigma_v^2}{2} + p\left(\frac{\partial}{\partial t}\right)_M \frac{1}{\rho}\right] = -4\pi r^2 \rho v^2 \left(\frac{\partial}{\partial t}\right)_M \log\left(\frac{\sigma_v^3}{\rho}\right)$ 1st law of thermodynamics

    Note that $p = \rho\sigma_v^2$
  - Nice fitting with isolated N-body simulation [Koda+(2011)]
    - → How about Cosmological N-body simulation?

Effective model of SIDM evolution

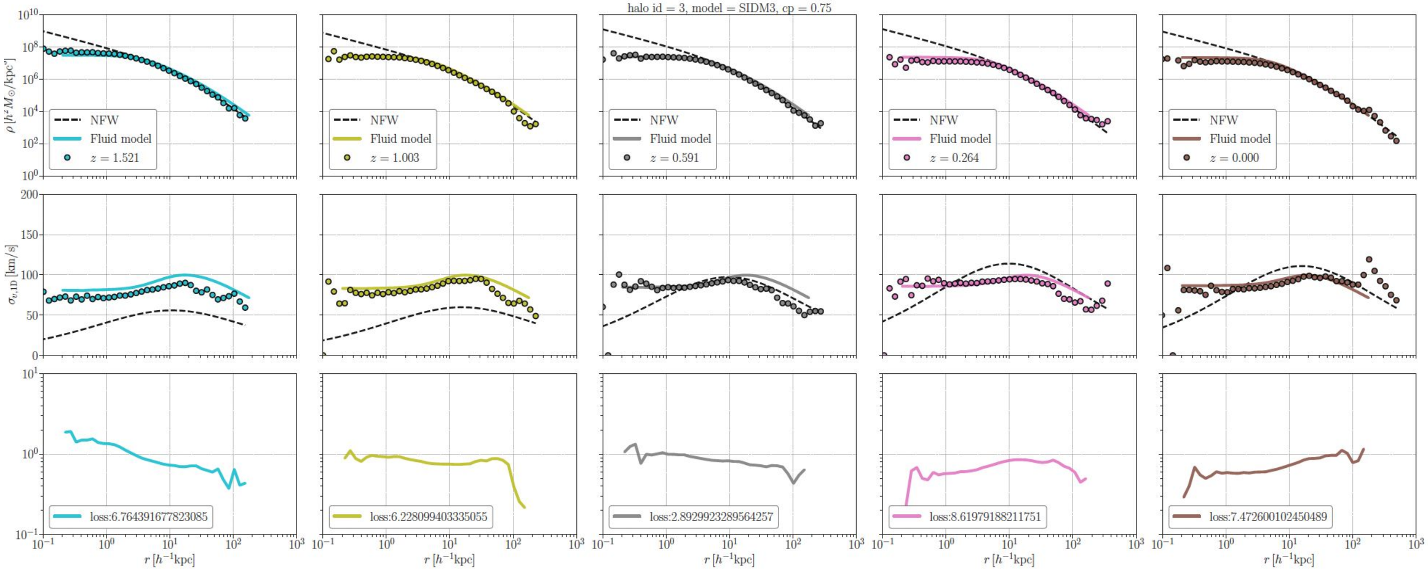
Dataset: SIDM N-body simulation [Ebisu, Ishiyama, and Hayashi (2022)]

- 1024^3 particles / (8 Mpc/h)^3 (comoving)
- CDM / SIDM$_1$ / SIDM$_3$ (σ/m = 1 or 3 cm^2/g)
- 9 MW-sized halos



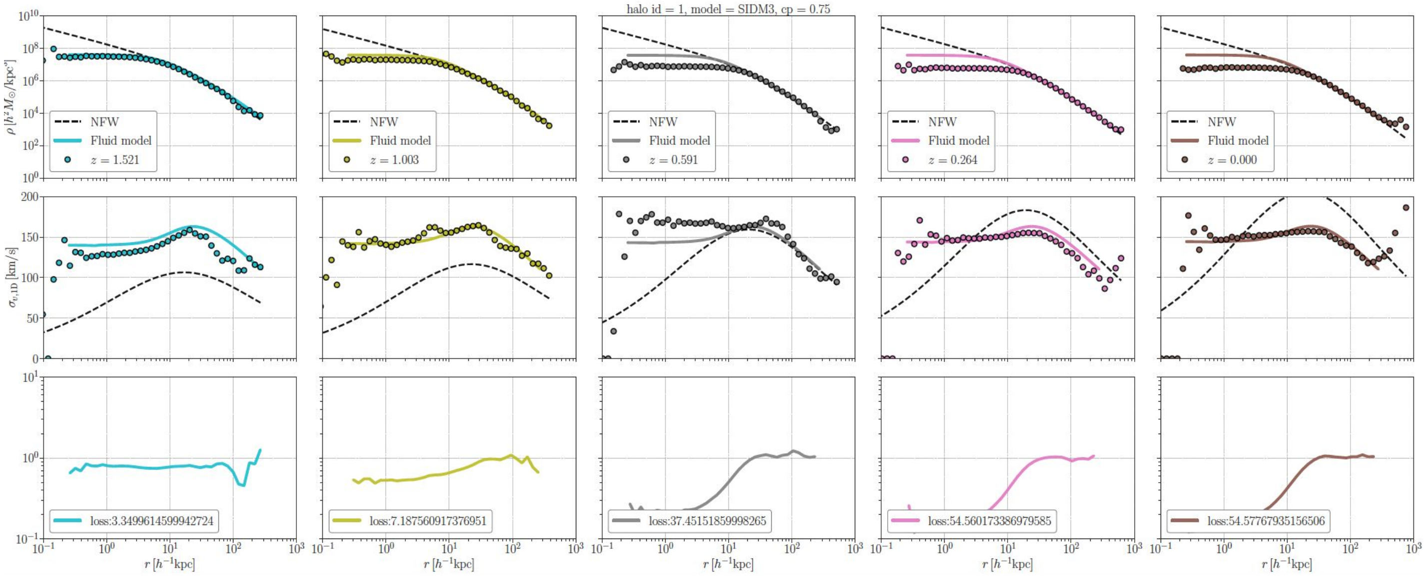
Effective model of SIDM evolution

Results (preliminary): e.g. halo 3… consistent



Effective model of SIDM evolution

Results (preliminary): e.g. halo 1… deviation

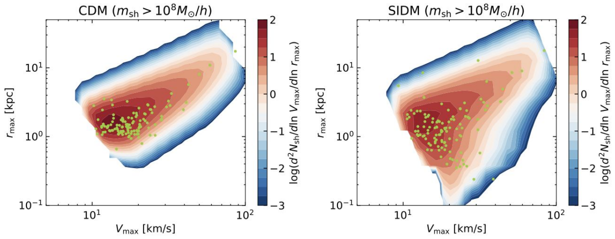


Future work: analysing the origin of the deviation

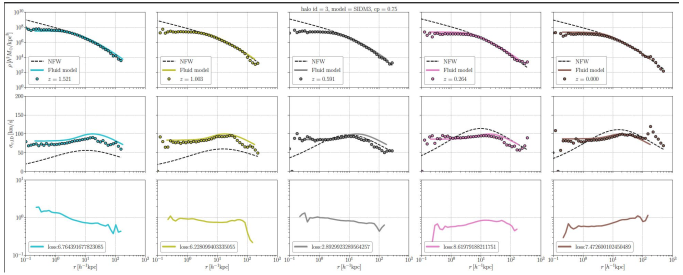
SIDM halo structure and evolution: Semi-analytical and effective models

- SIDM is an interesting candidate to solve small scale problems in CDM
- Difficulties of N-body SIDM simulations: Resolution, computation cost
- Two options:

- Semi-analitical modelling of SIDM halo
  - Quick SIDM simulator based on semi-analytical models



- Effective modelling of SIDM halo
  - Understanding the physics behind the halo structure



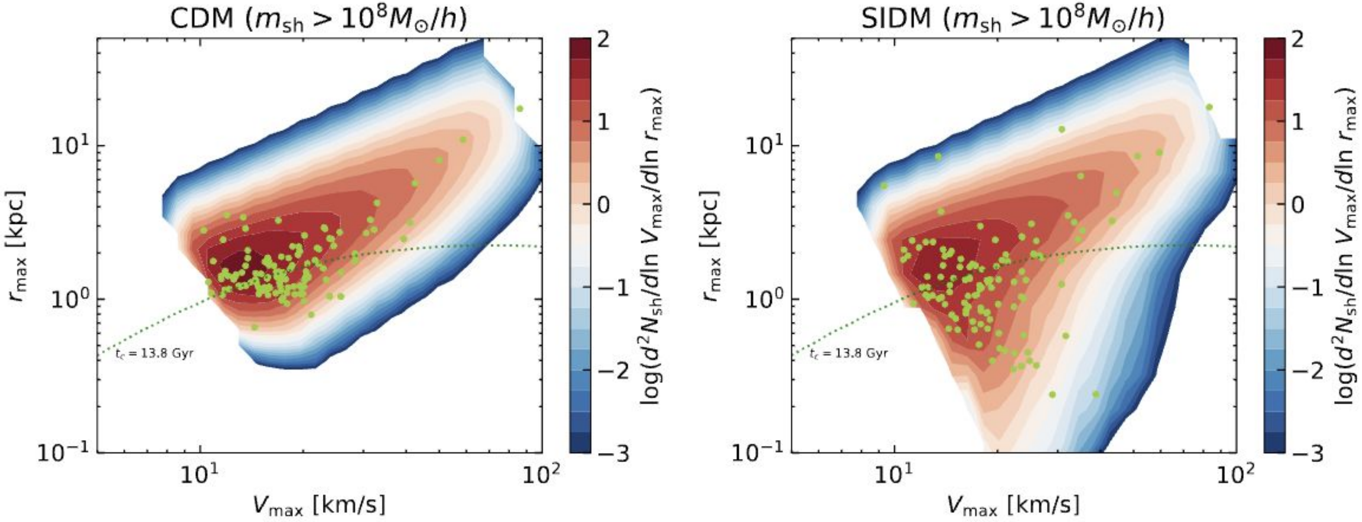
Backup

SIDM halo structure and evolution: Semi-analytical and effective models

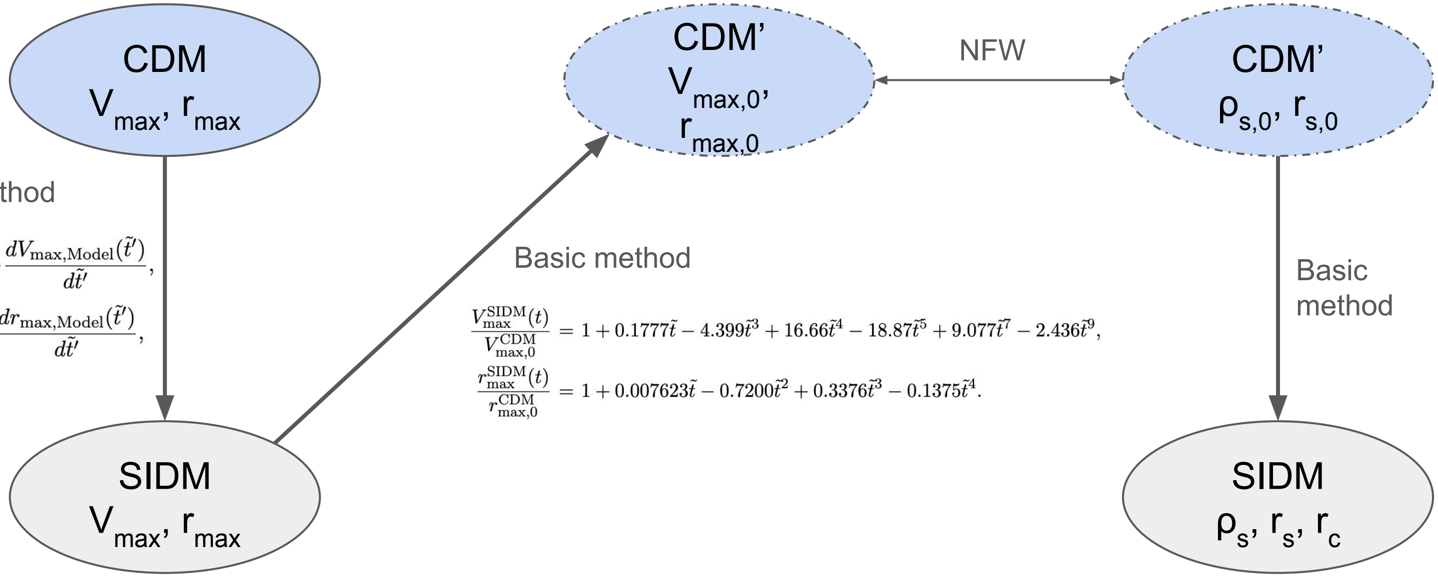
S. Ando, S. Horigome, E. O. Nadler,
D. Yang, H.-B. Yu [in prep.]

SASHIMI-SIDM: Semi-analytical approach to simulate SIDM models

- Quick calculation of SIDM halo properties and subhalo mass functions



Parametric Model



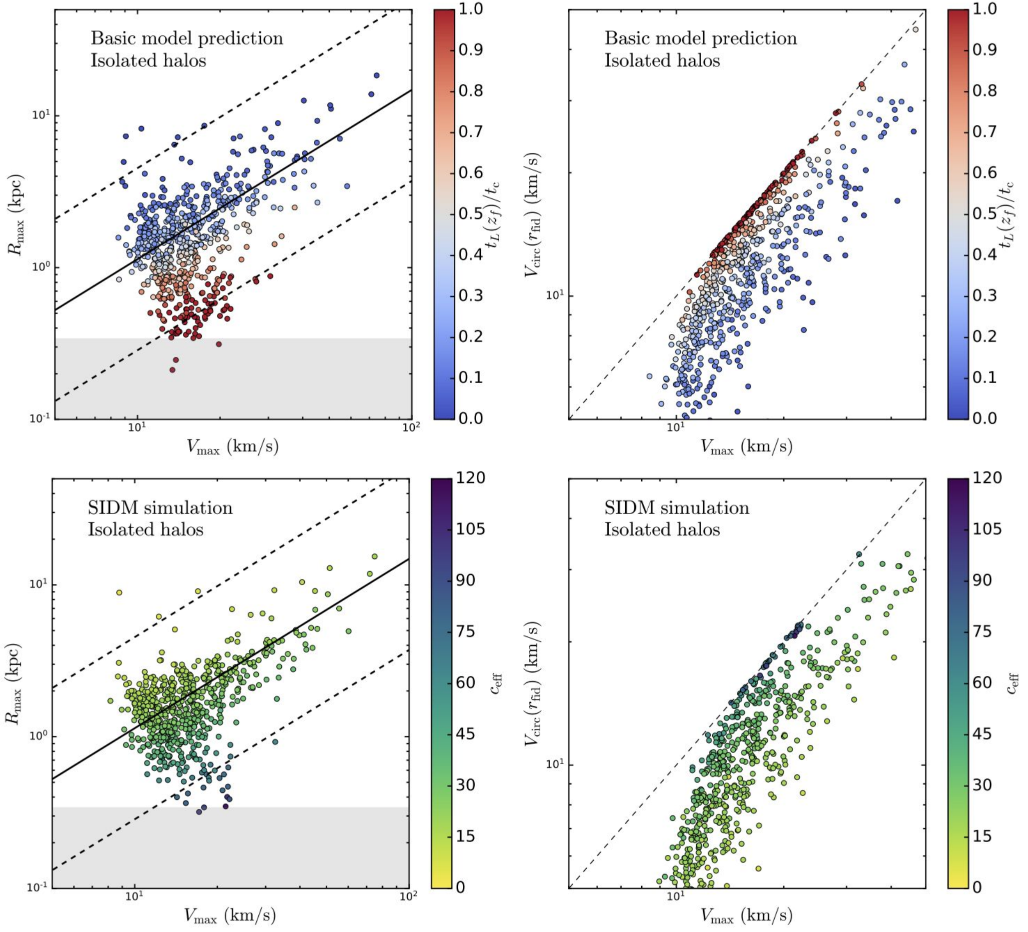
$$V_{\max}^{\mathrm{SIDM}}(t) = V_{\max}^{\mathrm{CDM}}(t) + \int_{t_f}^{t} \frac{dt'}{t_c(t')} \frac{dV_{\max,\mathrm{Model}}(\tilde{t}')}{d\tilde{t}'},$$

$$r_{\max}^{\mathrm{SIDM}}(t) = r_{\max}^{\mathrm{CDM}}(t) + \int_{t_f}^{t} \frac{dt'}{t_c(t')} \frac{dr_{\max,\mathrm{Model}}(\tilde{t}')}{d\tilde{t}'},$$

$$\frac{V_{\max}^{\mathrm{SIDM}}(t)}{V_{\max,0}^{\mathrm{CDM}}} = 1 + 0.1777\tilde{t} - 4.399\tilde{t}^3 + 16.66\tilde{t}^4 - 18.87\tilde{t}^5 + 9.077\tilde{t}^7 - 2.436\tilde{t}^9,$$

$$\frac{r_{\max}^{\mathrm{SIDM}}(t)}{r_{\max,0}^{\mathrm{CDM}}} = 1 + 0.007623\tilde{t} - 0.7200\tilde{t}^2 + 0.3376\tilde{t}^3 - 0.1375\tilde{t}^4.$$

Parametric Model



Yang+(2023)

$$r_{\rm fid} = 2V_{\rm max}/(70\ {\rm km/s})\ {\rm kpc}$$

Effective model of SIDM evolution

Parameters:

- Heat conductance

$$\kappa = b_* \sigma_v \left[\left(\frac{1}{\lambda} \right) + \left(\frac{b_* \sigma_v t_r}{C_* H_g} \right) \right]^{-1} \text{ with } b_* = 0.25, C_* = 0.75$$

- Initial redshift
  - $M_{z,init} = f * M_0$, $f = 0.04$
- mass, scale radius
  - CDM counterpart @ z=0

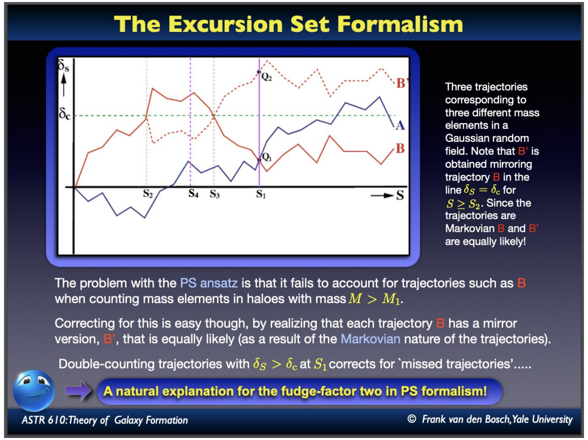
where $H_g \equiv \sqrt{\sigma_v^2/(4\pi G\rho)}$ is the gravitational scale height of the system, $\lambda = (\rho\sigma/m)^{-1}$ is the collisional scale for the mean free path, $t_r \equiv \lambda/(a\sigma_v)$ is the relaxation time with a coefficient of order of unity being a, and we adopt $a = \sqrt{16/\pi}$ for hard-sphere scattering of particles with a Maxwell-Boltzmann velocity distribution (Reif 1965).

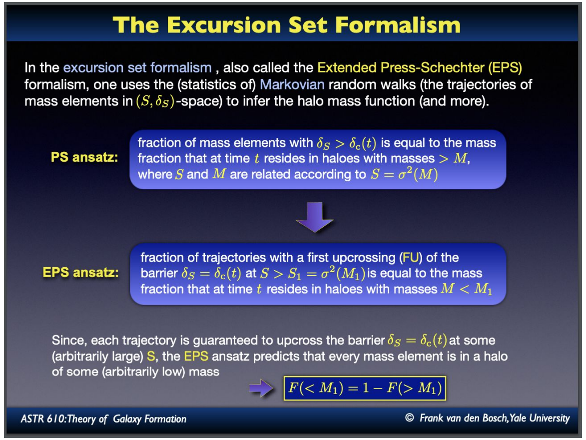
In the limit of $\lambda \ll H_g$, the thermal conductivity is given by $\kappa \simeq (3/2)(k_B/m)b_*\rho\lambda^2/(at_r)$ and b_* can be regarded as an effective impact parameter among particle collisions. In the limit of $\lambda \gg H_g$, one finds $\kappa \simeq (3/2)(k_B/m)C_*\rho H_g^2/t_r$, reproducing an empirical formula of gravothermal collapse of globular clusters (Lynden-Bell & Eggleton 1980).

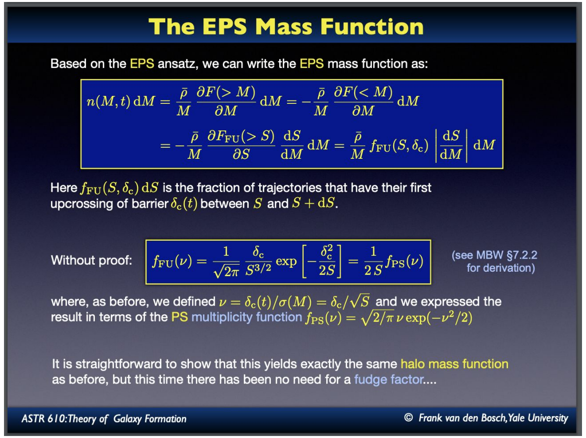
EPS formalism

[Press & Schechter(1974)]

Ref: http://www.astro.yale.edu/vdbosch/astro610_lecture9.pdf







SASHIMI

Tidal stripping: [Hiroshima+(2018)]

$$\dot{m}(z) = -A\frac{m(z)}{\tau_{\rm dyn}(z)}\left[\frac{m(z)}{M(z)}\right]^{\zeta},$$

$$\begin{aligned}
\log A &= \left[-0.0003\log\left(\frac{M_{\rm host}}{M_\odot}\right)+0.02\right]z \\
&\quad +0.011\log\left(\frac{M_{\rm host}}{M_\odot}\right)-0.354, \\
\zeta &= \left[0.00012\log\left(\frac{M_{\rm host}}{M_\odot}\right)-0.0033\right]z \\
&\quad -0.0011\log\left(\frac{M_{\rm host}}{M_\odot}\right)+0.026.
\end{aligned}$$