Vlasov-Poisson Simulation for Warm Dark Matter Halos

Confirmation with N-body simulation

Satoshi Tanaka

(YITP, Kyoto Univ.)

collaborator

Atsushi Taruya, Yohsuke Enomoto (Kyoto Univ., C02), Takahiro Nishimichi (Kyoto Sangyo Univ. , C02),

Kohji Yoshikawa (Univ. of Tsukuba) and Naoki Yoshida (Univ. of Tokyo/IPMU)

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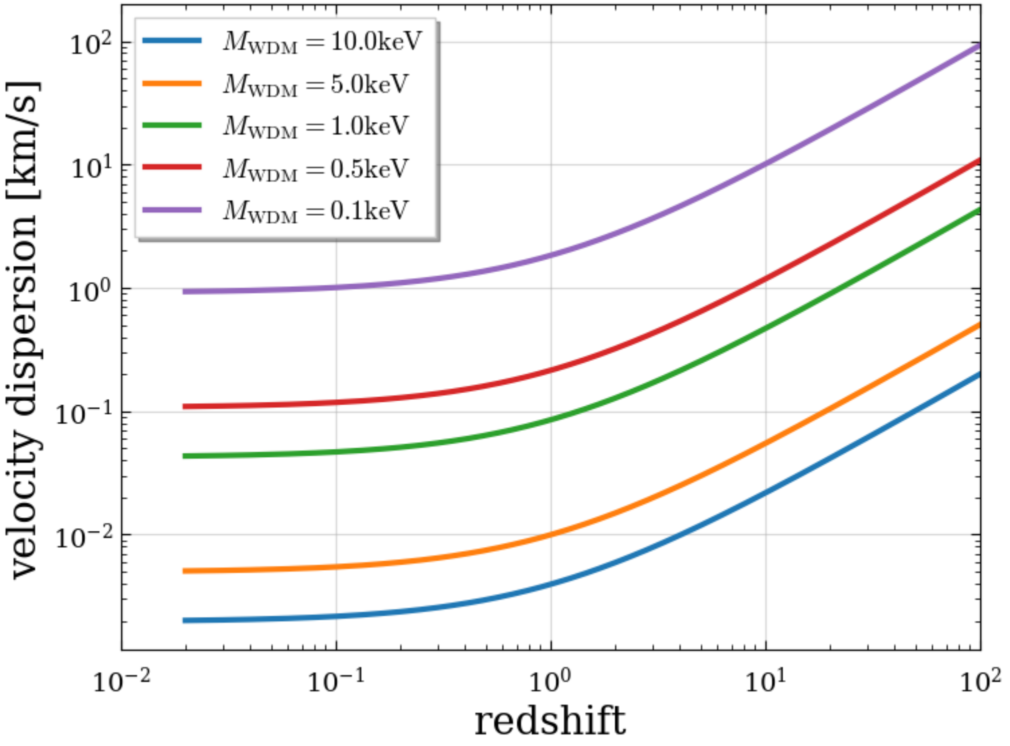
WDM velocity dispersion

- WDM has velocity dispersion for CDM (i.e. $\sigma_{\mathrm{CDM}} \sim 0$, $\sigma_{\mathrm{WDM}} < 1\ \mathrm{km/s}$, $\sigma_{\mathrm{neutrino}} > 1000\ \mathrm{km/s}$)
- Density fluctuations are suppressed by velocity dispersion below the free stream scale

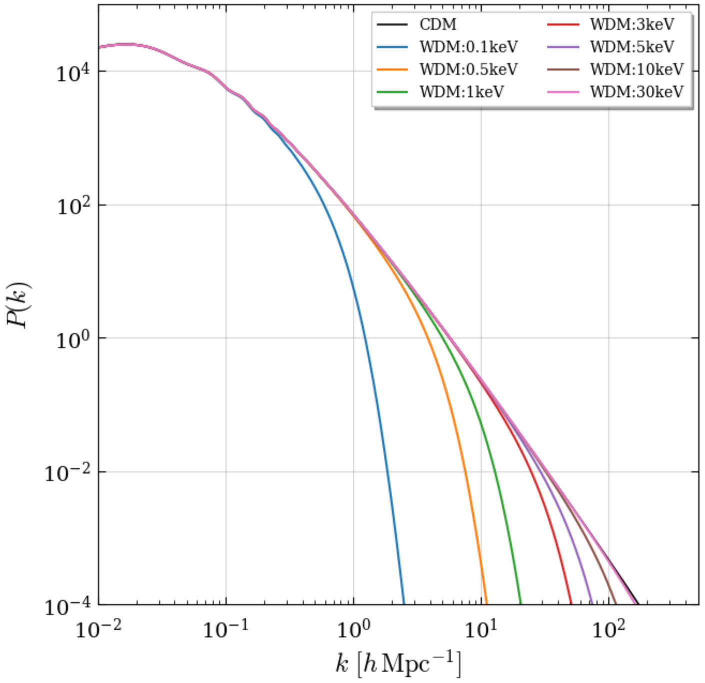
For thermal velocity (Leo et al. 2017)

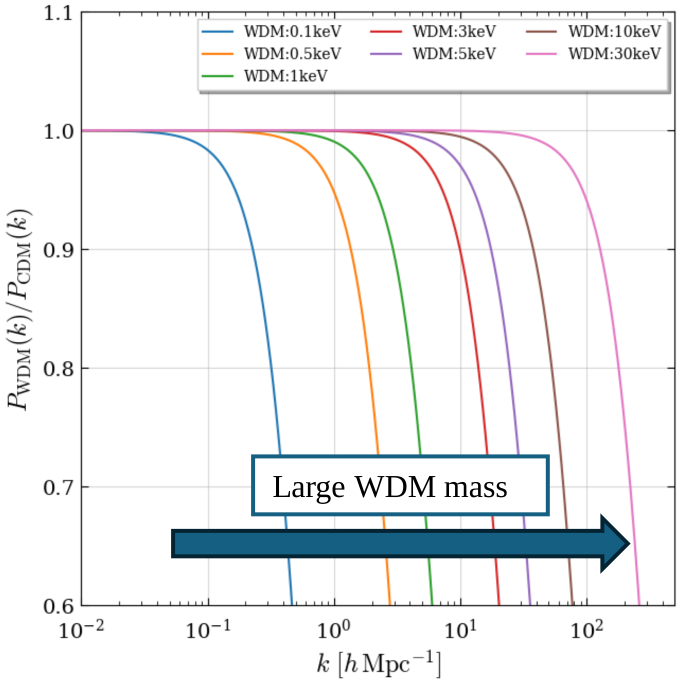
$$\sigma_{\mathrm{therm}} \equiv \sqrt{\langle v_{\mathrm{therm}}^2 \rangle} = 0.0429 \left(\frac{\Omega_{\mathrm{WDM}}^0}{0.3}\right)^{1/3} \left(\frac{h}{0.7}\right)^{2/3} \left(\frac{\mathrm{keV}}{m_{\mathrm{WDM}}}\right)^{4/3} (1+z)\ \ \mathrm{km/s}.$$



σ_{WDM} is less than 1 km/s at z<10

- ☐ WDM lower mass limit (Banik+2021)
- $\boldsymbol{M_{\mathrm{WDM}} > 3.6\ \mathrm{keV}}$ (from the tidal streams)
- $\boldsymbol{M_{\mathrm{WDM}} > 6.2\ \mathrm{keV}}$ (+ dwarf satellite counts)





WDM *N*-body simulation

- *N*-body simulations are commonly used to build WDM models that incorporate free-streaming effects.
- In previous studies, a cutoff corresponding to the σ_{WDM} of the WDM is put into the transition function for the initial conditions.
- N-body simulation is the same as for CDM case.
- The *N*-body simulation itself does not properly reflect the free-streaming effect of σ_{WDM}.

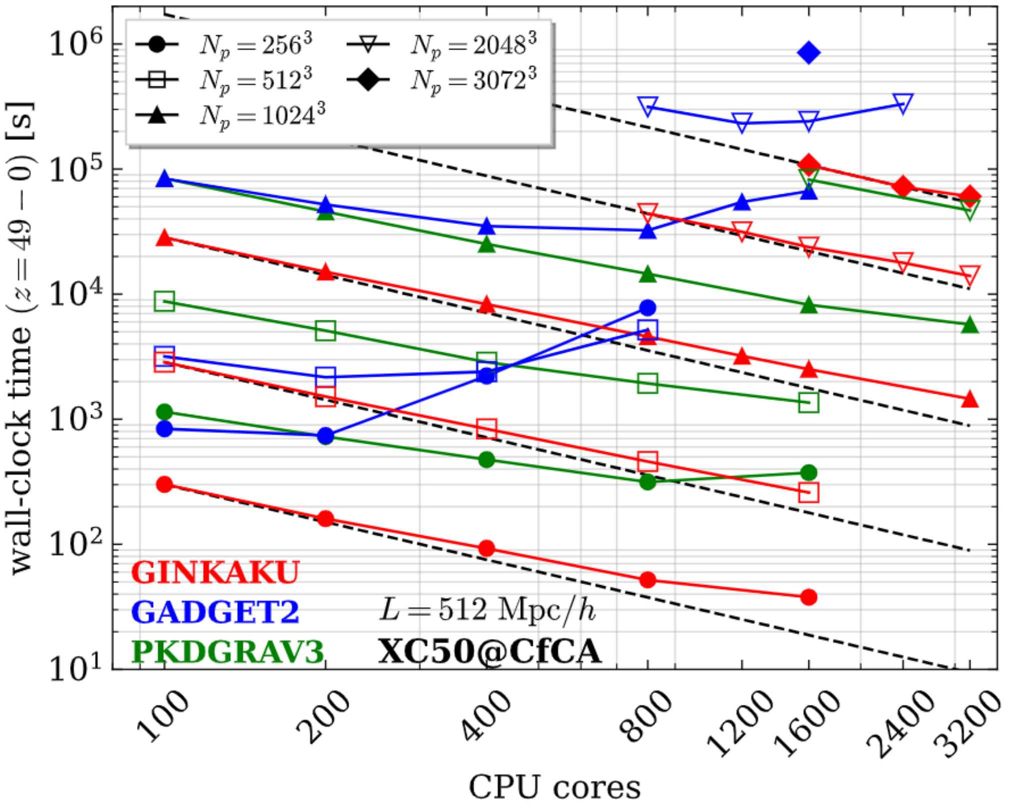
First, the behavior of WDM is confirmed by *N*-body simulation.

N-body simulation setting

- $N = 1024^3$
- $L = 10\ h^{-1}\mathrm{Mpc}/h$
- $m_\mathrm{p} = 8.16 \times 10^4\ M_\odot/h$
- **0.5, 1, 5 keV WDM** and CDM

➢ **Ginkaku** (Nishimichi, ST+submitted soon); TreePM N-body simulation code based on FDPS

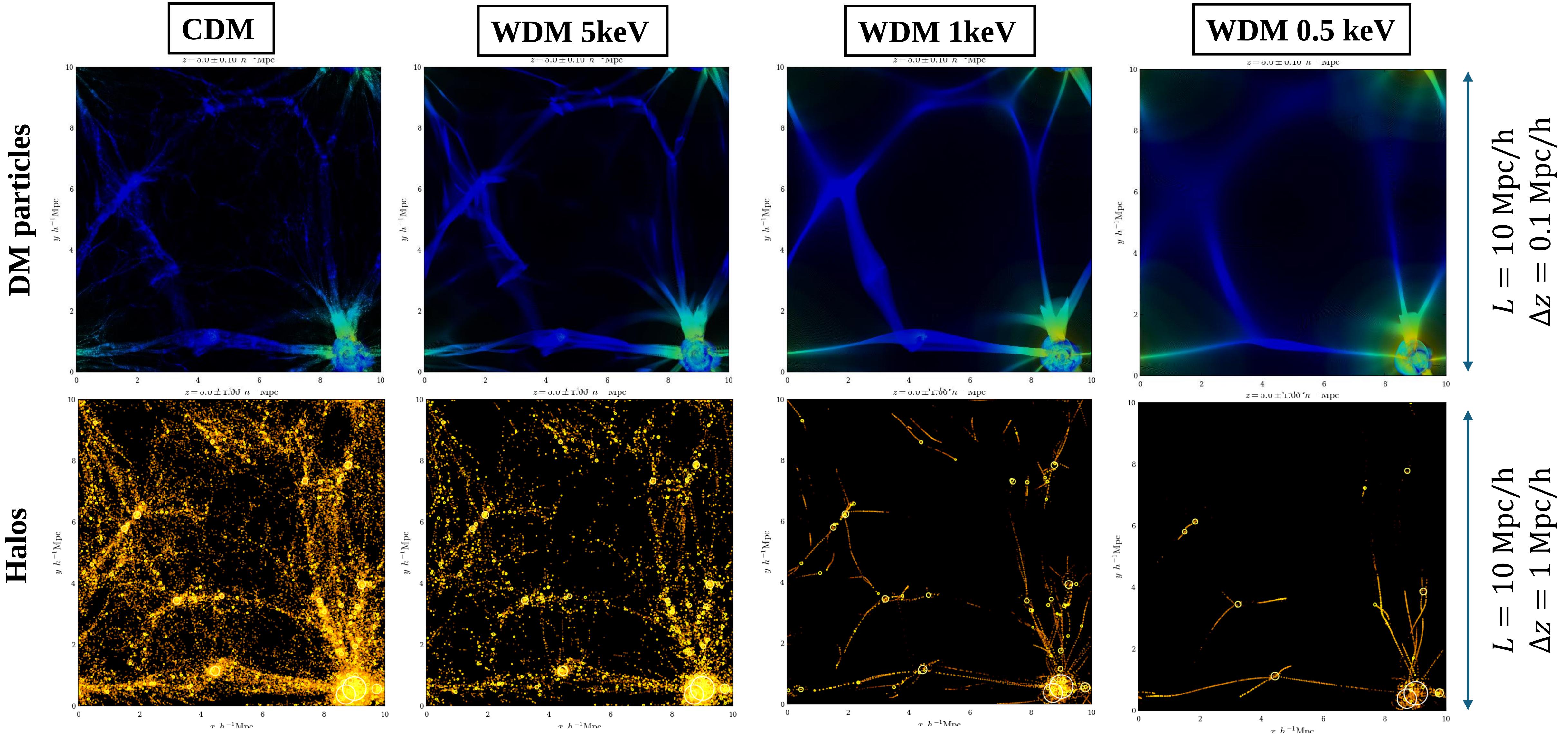
➢ Rockstar halo finder (Behroozi+2013)



✓ Ginkaku code is about 10 times faster than Gadget-2 code in a modern supercomputer.

✓ The parallelization efficiency of strong scaling is 90%.

WDM *N*-body simulation in Large-scale



Spurious artificial halos appear in small WDM masses

WDM *N*-body simulation

- **On a cosmological scale,** the spurious artificial halos are formed for light WDM mass.
  - Small halo formation suppressed due to cutoff of initial conditions

- **On a specific halo,** we do not know if the σ_{WDM} is reflected due to shot noise.
  - There are a few N-body simulations (Colin+2008) of WDM particles with σ_{WDM}, but I don't think they correctly resolve the WDM velocity dispersion.

- It is difficult to simultaneously resolve the thermal σ_{WDM} (< 1 km/s) and the bulk velocity (~100 km/s) of each halo on a cosmological scale.

- Focus on a halo formation that considers the effects of σ_{WDM} for a specific halo.

Therefore, investigate whether WDM halo formation is able be simulated by **Vlasov simulation** that correctly treat velocity dispersion.

Vlasov-Poisson Simulation

Vlasov (collisionless Boltzmann) equation

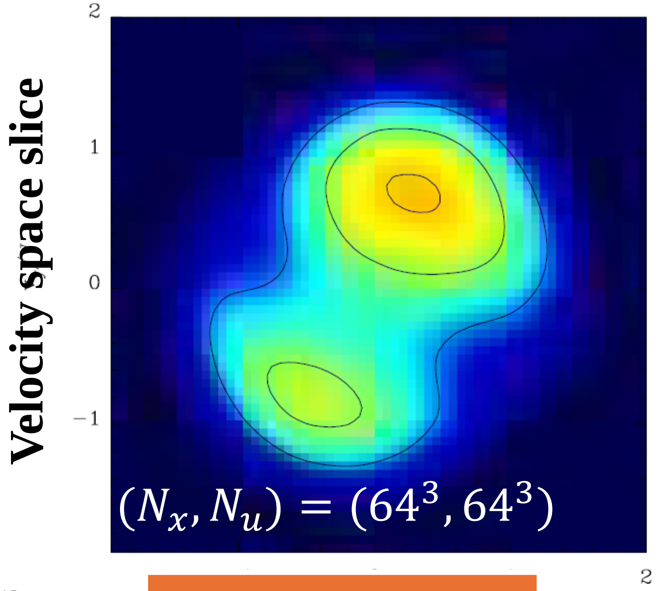
$$\frac{\partial f}{\partial t} + \boldsymbol{v} \cdot \frac{\partial f}{\partial \boldsymbol{x}} - \nabla \phi \cdot \frac{\partial f}{\partial \boldsymbol{v}} = 0$$

$$\nabla^2 \phi = 4\pi G \rho = 4\pi G \int f d^3 \boldsymbol{v}$$

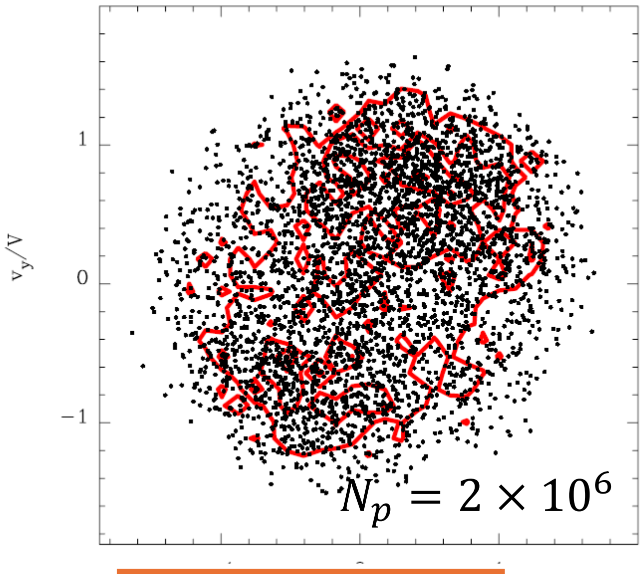
In principle, for a given initial condition, the evolution of $f(x, y, z, u, v, w)$ should be followed in 6D phase-space, coupled with Poisson's equation.

Computational cost and memory require are both $O(N^6)$ since the calculation is performed on 3D configuration + 3D velocity space.

Demonstration of the collision of two King spheres (not WDM)



Vlasov simulation

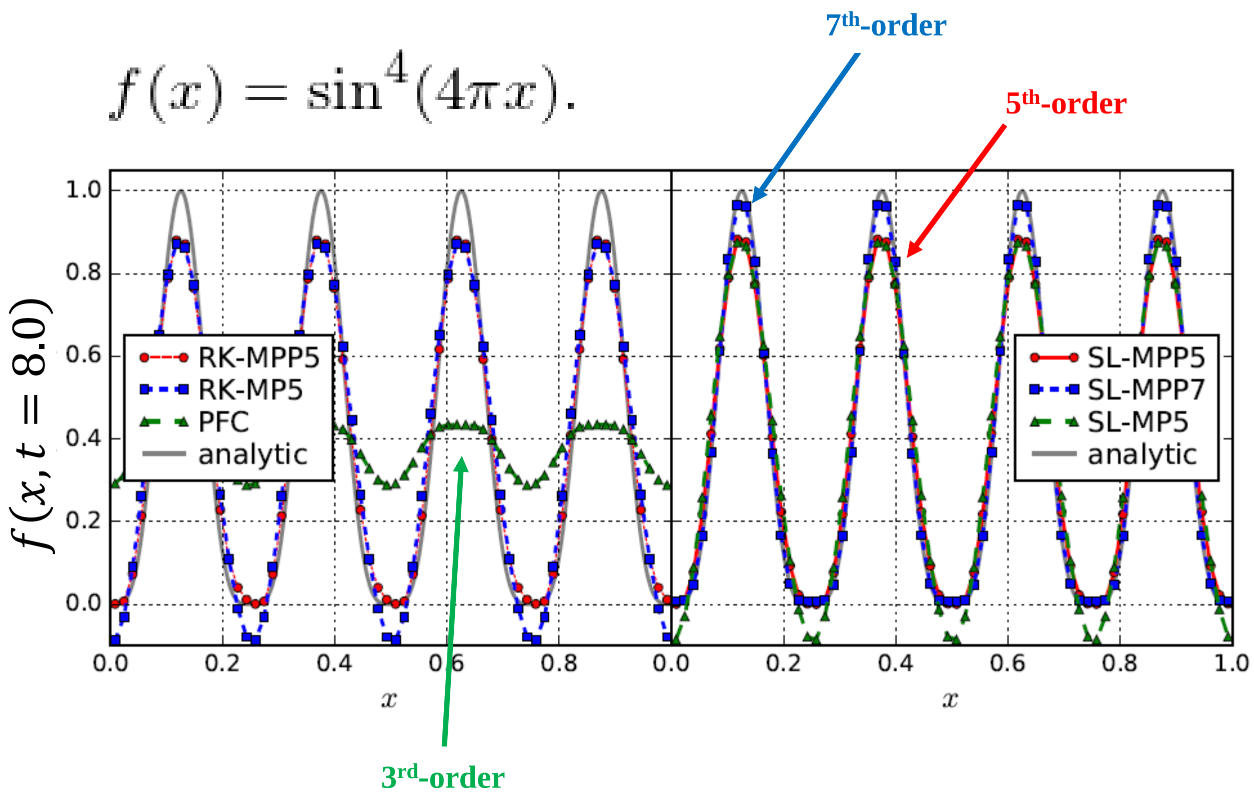


***N*-body simulation**

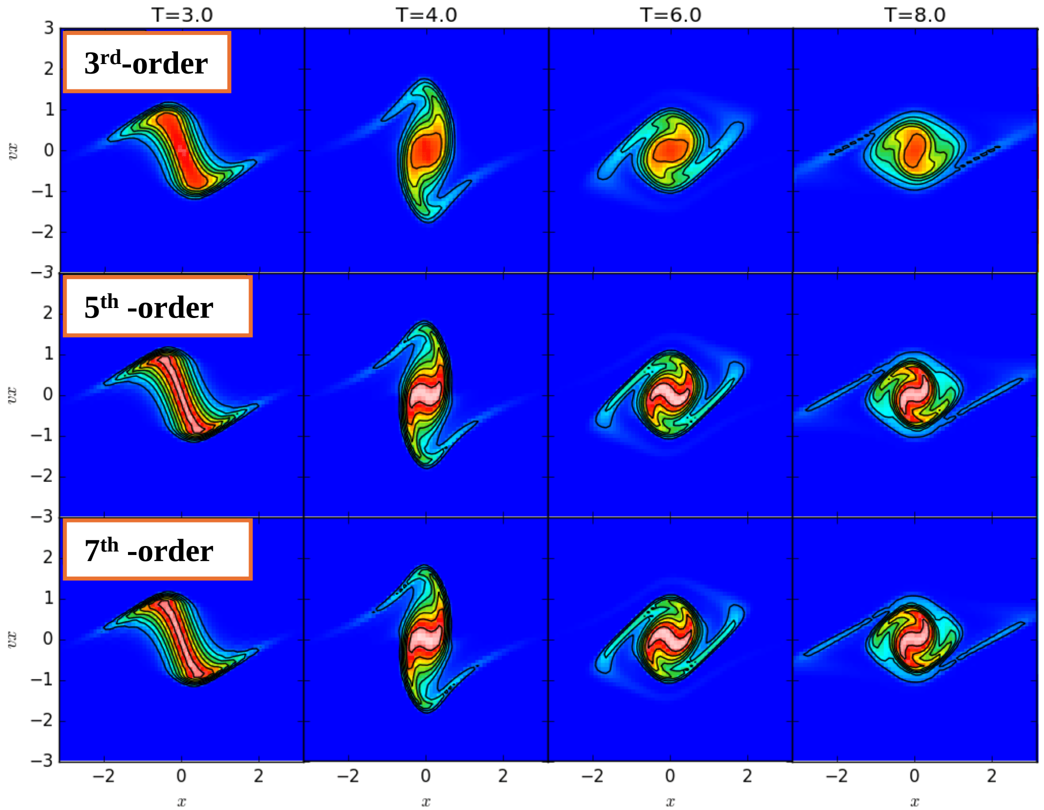
- ✓ N-body simulations cannot eliminate shot noise no matter how much the number of particles is increased in principle
- ✓ Vlasov simulation is very smooth on phase space.

Yoshikawa, Yoshida and Umemura, 2013

Phase-Space Self-Gravitating System



Spherical Collapse of a uniform density sphere



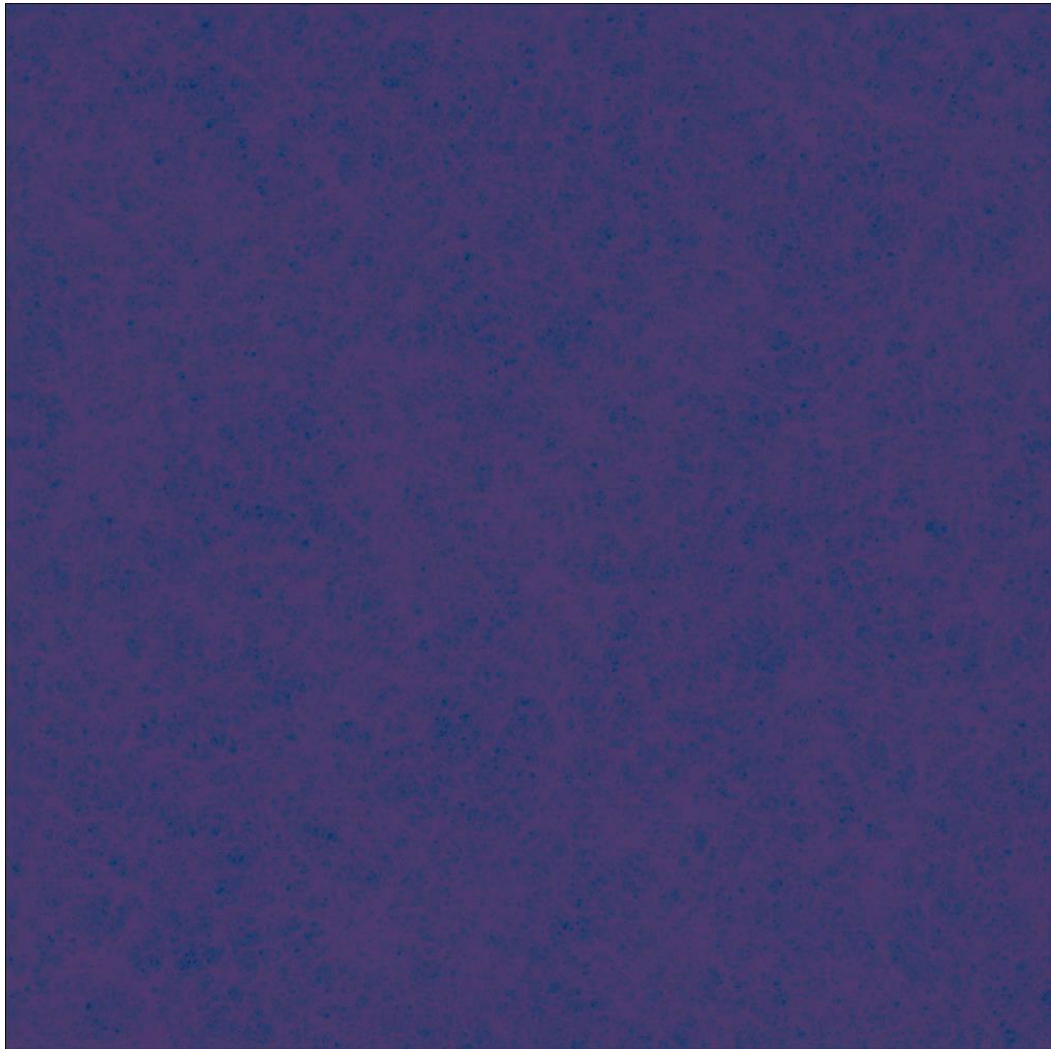
- Development of higher-order (5 or 7 and more) accuracy schemes to achieve high accuracy with fewer meshes. (ST+2017)
- **Successfully applied to neutrinos (hot component)** (Yoshikawa, ST+2020)

$$(N_x, N_v) = (64^3, 64^3)$$

A 400 trillion-grid Vlasov simulation + 330 billion CDM particles

Demonstration with hot components

Performed on Fugaku full system (147,456 nodes (7,077,888 CPU cores))

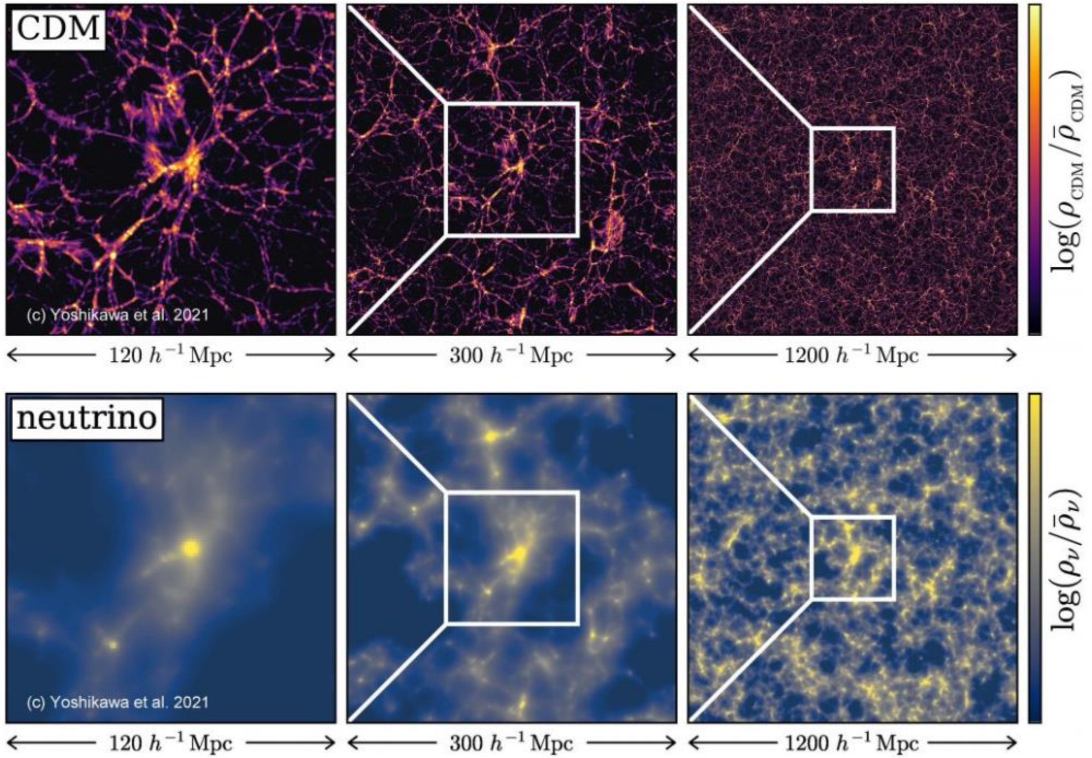


480 h^{-1}Mpc

Neutrinos Vlasov mesh : $(N_x,\ N_v) = (1152^3, 64^3)$

CDM particles : $N_p = 6912^3$

Nonlinear clustering and neutrino "halos"



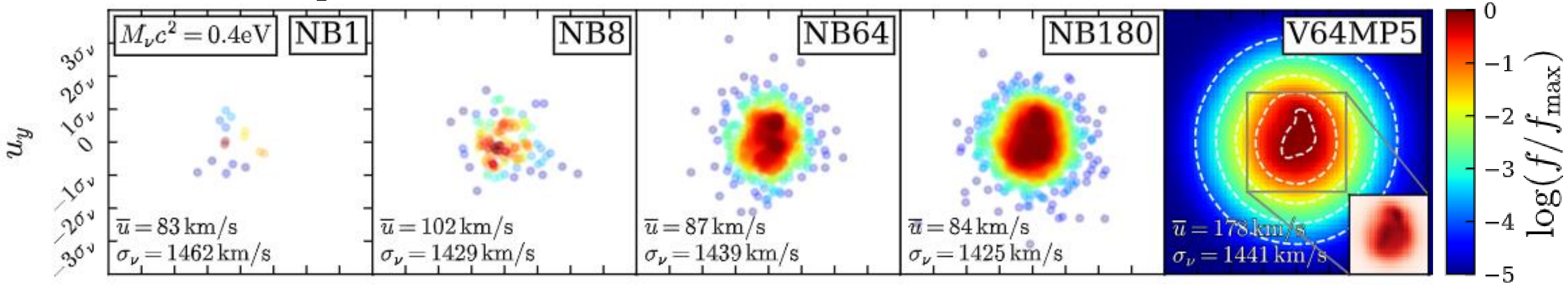
Yoshikawa, ST and Yoshida 2021

2021 Gordon Bell Finalist

This year's award is Quantum Supremacy in China group.

Distribution function of Hot component

Demonstration with hot components



$\frac{N_\nu}{N_{\mathrm{CDM}}} = 1$ $\frac{N_\nu}{N_{\mathrm{CDM}}} = 8$ $\frac{N_\nu}{N_{\mathrm{CDM}}} = 64$ $\frac{N_\nu}{N_{\mathrm{CDM}}} = 180$ $(N_x, N_v) = (192^3, 64^3)$

typical setting for N-body neutrino sim.

Same amont of memory usage

- Vlasov simulations can also correctly evaluate velocity dispersion, especially for hot components as massive neutrino.
- The high-velocity component is the poor sampling even if the larger number of particles in the N-body simulation.

Not easy in the case of WDM because the velocity dispersion is much smaller

Vlasov simulation for WDM

Thermal velocity dispersion (σ) [km/s]

		z=20	z=5	z = 0
Hot	0.1 eV /3	59565	32490	5415
	0.4 eV/3	14890	8122	1354
Warm	0.5 keV	1.11	0.61	0.1
	1 keV	0.44	0.24	0.04
	5 keV	0.052	0.028	0.0047
	10 keV	0.020	0.011	0.0019

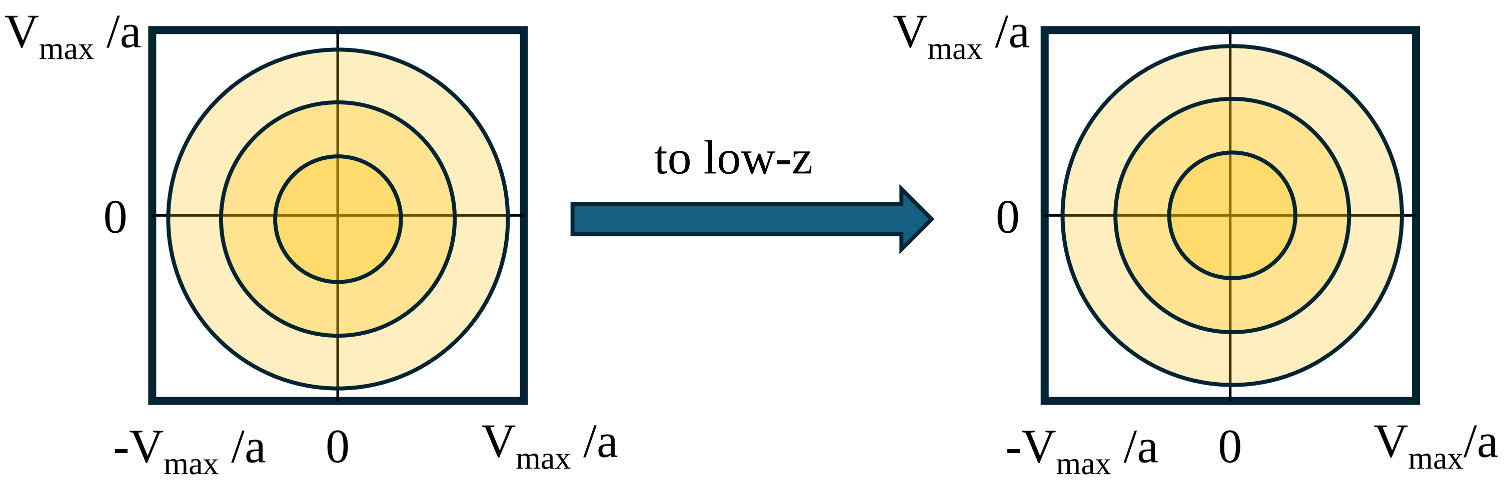
- In WDM, the velocity dispersion is very small compared to neutrinos, so it is impossible to apply as same.
- If σ is smaller than the velocity space grid ΔV, it cannot be evaluated.

$$\sigma_{\mathrm{v}} = 180(1+z)\left(\frac{m_\nu c^2}{1\ \mathrm{eV}}\right)^{-1}\ \mathrm{km\ s^{-1}}.$$

$$\sigma_{\mathrm{therm}} \equiv \sqrt{\langle v_{\mathrm{therm}}^2\rangle} = 0.0429\left(\frac{\Omega_{\mathrm{WDM}}^0}{0.3}\right)^{1/3}\left(\frac{h}{0.7}\right)^{2/3}\left(\frac{\mathrm{keV}}{m_{\mathrm{WDM}}}\right)^{4/3}(1+z)\ \mathrm{km/s}.$$

Vlasov simulation for WDM

- **Hot component case**



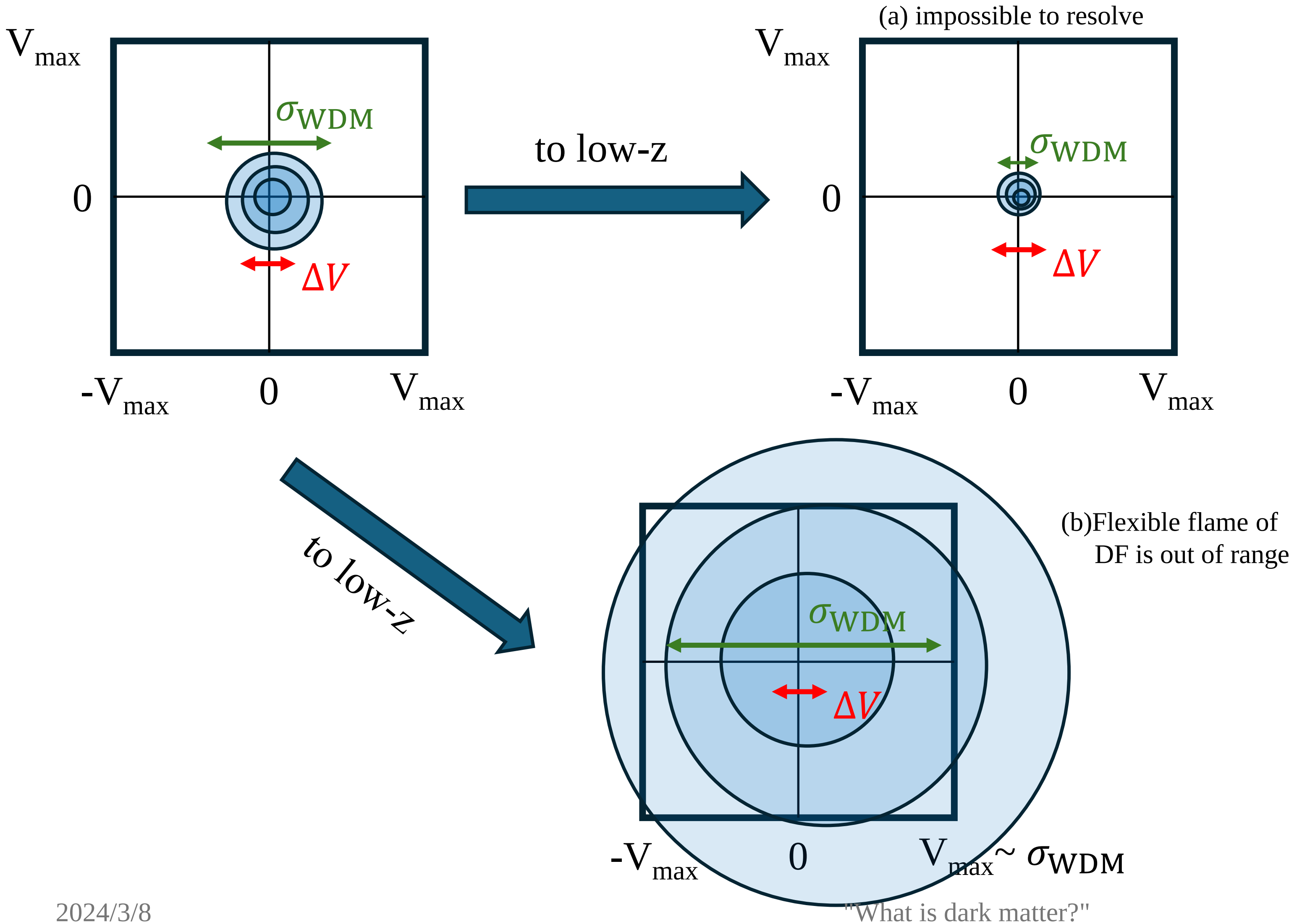
Vlasov simulation in comoving flame

$$\sigma_\nu(z) \gg V_{\text{bulk}},$$
$$V_{\max}(z) \sim 4\sigma_\nu(z)$$
and
$$\Delta V = 2V_{\max}(z)/N_\nu,$$
$$\Delta V \ll \sigma_\nu(z)$$

- ✓ The distribution function always fits in the velocity space region

Vlasov simulation for WDM

- **Warm component**

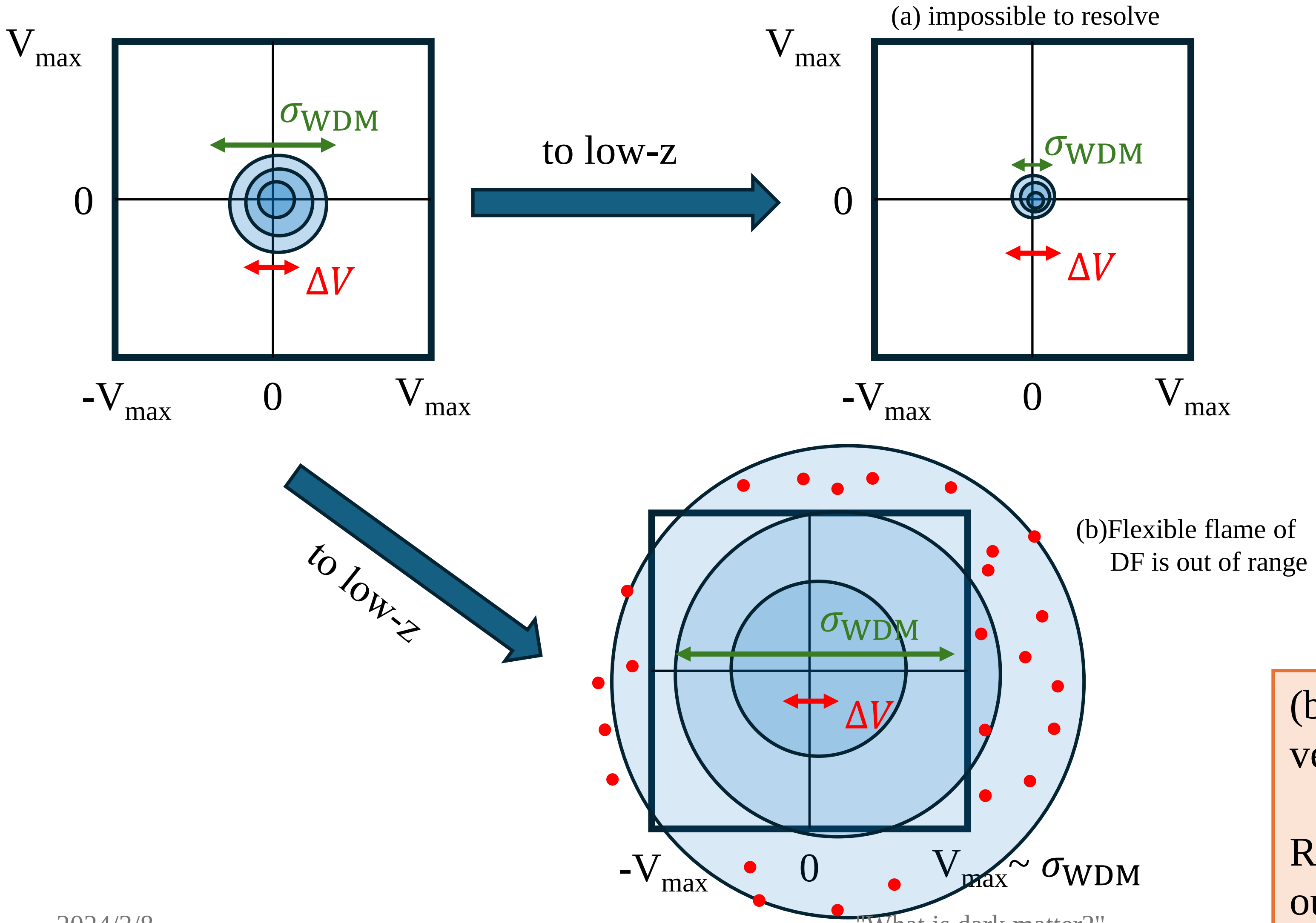


- V_{bulk} can always set to zero in the halo rest frame in velocity space.
  - ✓ Cosmological simulation with multiple halos cannot be a halo rest frame.

- Even in a halo rest frame,
  - ✓ (a) If V_{max} is fixed, at some point it will be impossible to resolve the σ_{WDM} per velc mesh.
  - ✓ (b) If V_{max} is flexible to the resolution limit of the σ_{WDM}, the constituent high velocity particles in the halo will out of the region.

Vlasov simulation for WDM

- **Warm component**



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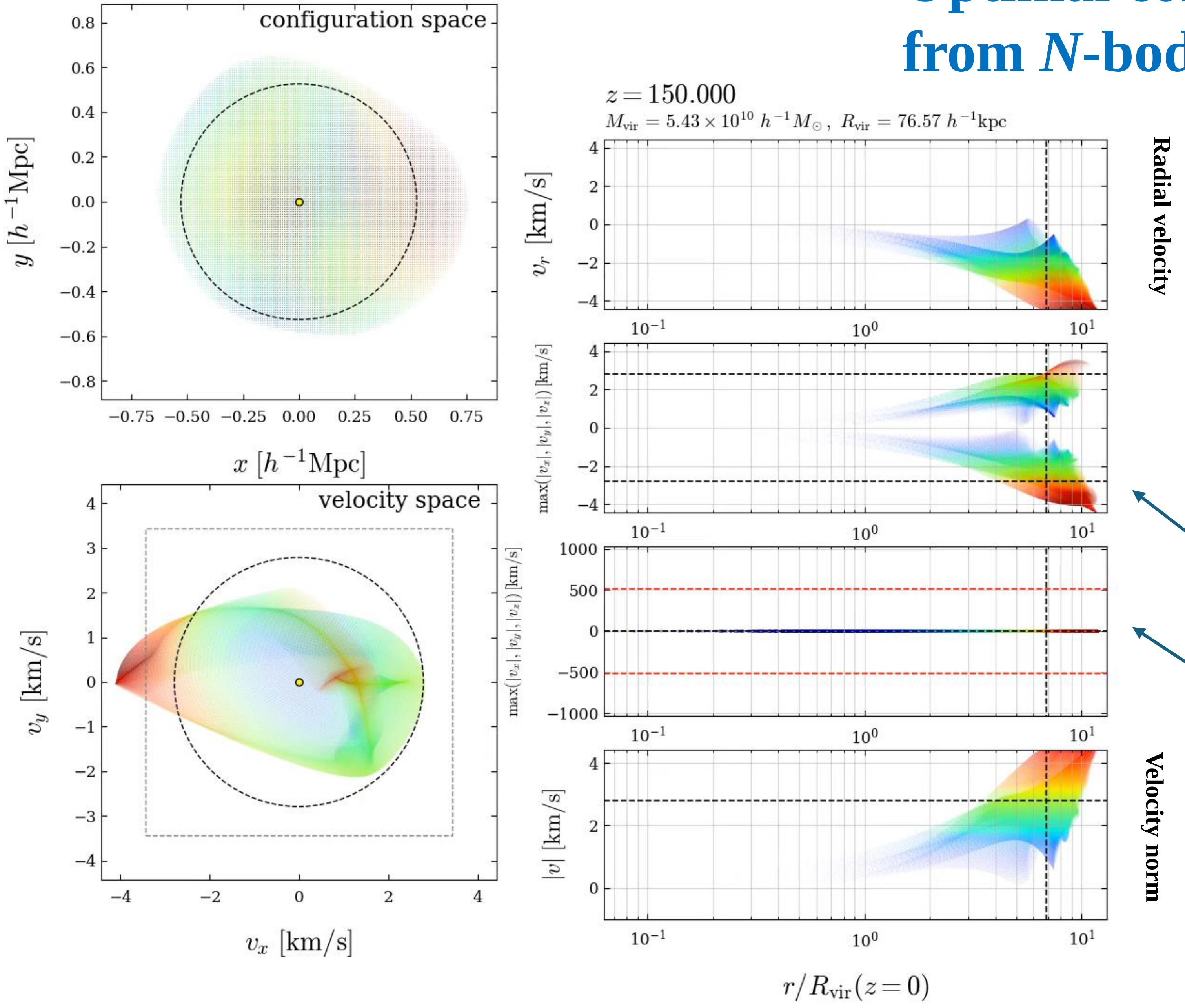
  - ✓ (b) If V_{max} is flexible to the resolution limit of the σ_{WDM}, the constituent high velocity particles in the halo will out of the region.

(b) can consider the σ_{WDM} near the low velocity component.

Replacing with N-body particles for the outside domain DF.

Optimal conditions for Vlasov simulation from *N*-body simulation



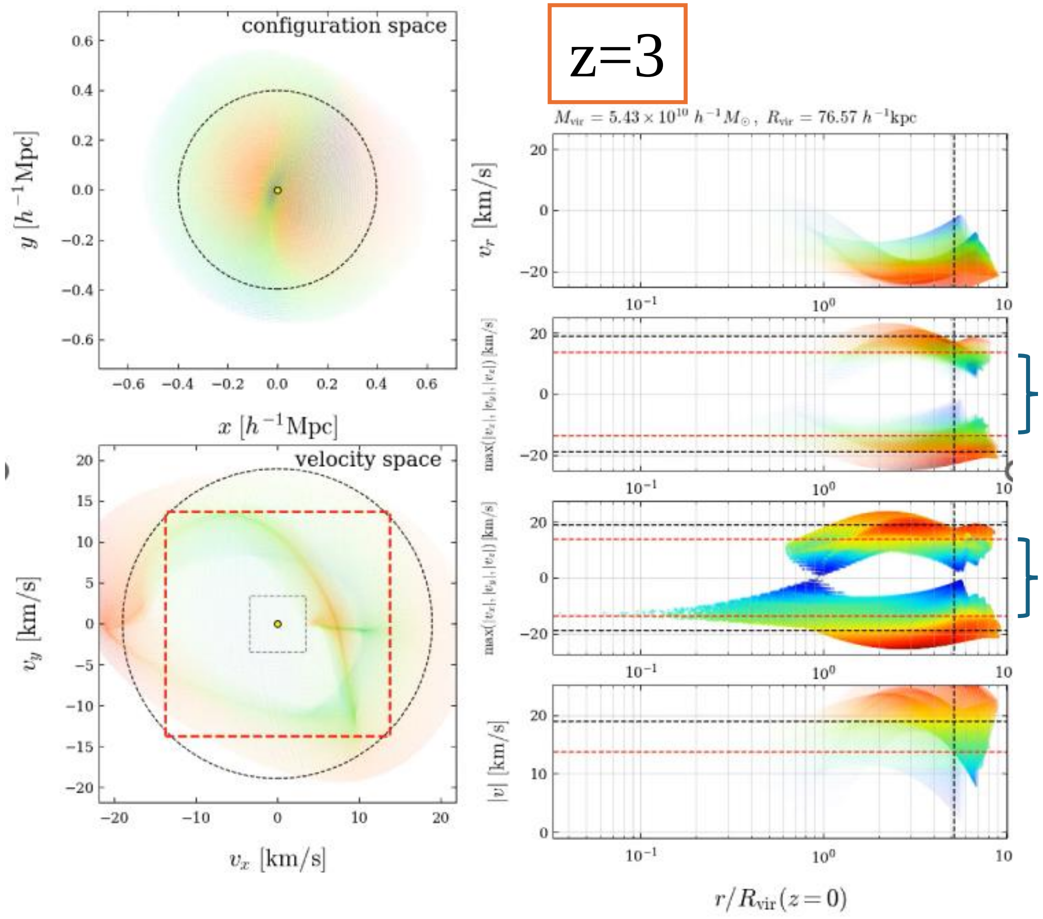
- $M_{\rm WDM} = 0.5$ keV
- $M_{\rm vir} = 5.43 \times 10^{10}\ M_\odot/h$
- $R_{\rm vir} = 76.6\ \ \mathrm{kpc}/h$

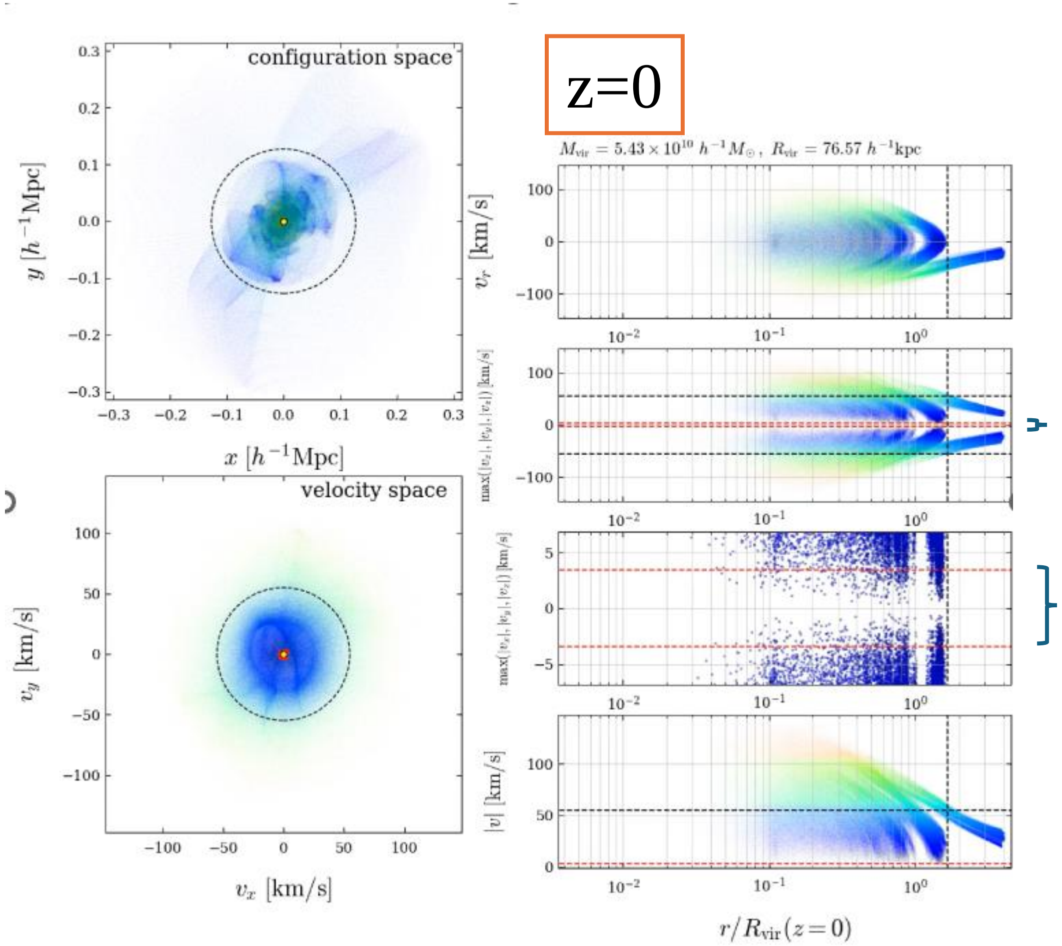
➢ WDM particle traces for particles in 4Rvir at z=0. (Enomoto+2023)

Maximum velocity of particles

Red lines are the limit of velocity dispersion that can be resolved by Vlasov simulation ($N_v = 64^3$).
(And the red frame in the lower left panel)

Optimal conditions for Vlasov simulation from *N*-body simulation





- The velocity dispersion can be handled correctly until the halo prompt cusp is formed.
- Conversely, low-velc particles are not sampled in the *N*-body simulation.

- There is almost no matter that can resolve the velocity dispersion in Vlasov simulation at low-z.

Summary

- Known problems with WDM simulation in N-body method
  - ✓ Spurious halos in cosmological simulation
  - ✓ Effect of shot noise in a halo formation. Resolve of tiny velocity dispersion of WDM.
- Vlasov simulations can evaluate velocity dispersion in phase space.
  - ✓ Cosmological simulations such as neutrinos are currently difficult in WDM models.
  - ✓ Evaluation of σ_{WDM} for a specific halo formation is possible in a halo rest flame.
  - ✓ However, it can only be simulated to up to the prompt cusp formation.
- The next step is to model the WDM halo with the velocity dispersion from the Vlasov WDM simulation.
  - ✓ Target WDM masses smaller than the lower limit of observation for comparison with *N*-body.
- To WDM simulate in full, the 6D phase calculation needs to AMR, which is very difficult.

Summary

- In Vlasov simulations, thermal velocity resolution of the high velocity component is not possible due to the size of the velocity space mesh number, as we know from before
- In Vlasov, thermal velocity resolution of low velocity components is possible by zooming in.
- N-body cannot represent very low velocity components very well.
- However, the mass ratio of the extremely low velocity component at z=0 is almost negligible
- Vlasov simulation is worthwhile in that it allows us to correctly follow the speeds up to about z~5, where the halo cores are formed.
- If the low velocity component is solved correctly, the halo center profile can be properly evaluated.

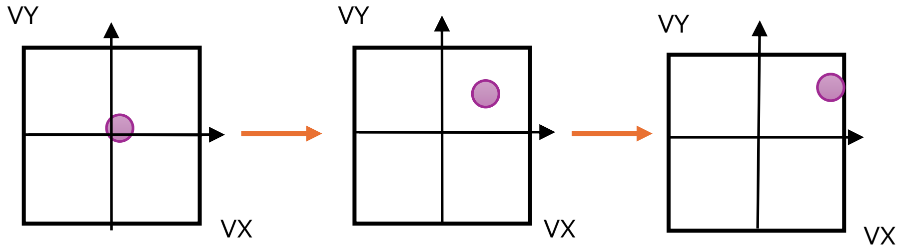
Cosmological Vlasov-Poisson Equation

◆Vlasov simulation in comoving flame

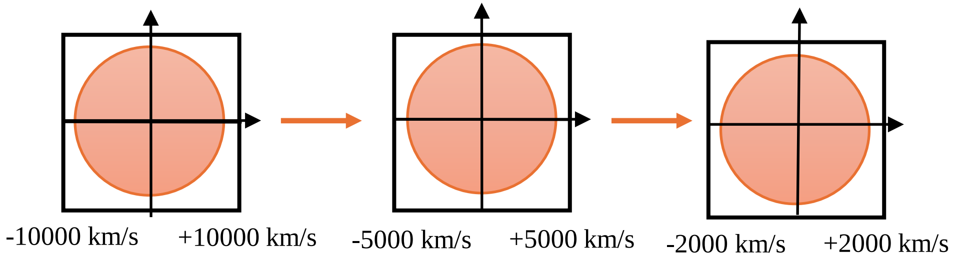
$$\frac{\partial f}{\partial t} + \frac{\boldsymbol{p}}{a^2}\cdot\frac{\partial f}{\partial \boldsymbol{x}} - \nabla\phi\cdot\frac{\partial f}{\partial \boldsymbol{p}} = 0, \qquad \boldsymbol{p} = a^2\dot{\boldsymbol{x}}$$

$$\nabla^2\phi = 4\pi G a^2\bar{\rho}\delta = 4\pi G a^2\bar{\rho}\int f d^3\boldsymbol{p} - 1.$$

- 宇宙膨張によって a^2 で運動量が増加
- あらかじめ設定した速度空間の範囲をすぐにはみ出す



- ニュートリノの場合は平均速度<<速度分散かつ、膨張によって速度分散も小さくなるので速度分散の範囲を狭めることが可能



◆Vlasov equation with peculiar velocity $(v = a\dot{x})$

$$\frac{\partial f}{\partial t} + \frac{\boldsymbol{v}}{a}\cdot\frac{\partial f}{\partial \boldsymbol{x}} - \left[H\boldsymbol{v} + \frac{\nabla\phi}{a}\right]\cdot\frac{\partial f}{\partial \boldsymbol{v}} = 0.$$

Advection equation in velocity space depends on velocity itself.

$$\nabla^2\phi = 4\pi G a^2\bar{\rho}\delta = 4\pi G a^2\bar{\rho}\int f d^3\boldsymbol{p} - 1.$$

- a で運動量が増加するが上記よりかは穏やか
- 速度分散に対して平均速度が大きくなるCDMなどではこちらが必要