SUSY GUT and the Doublet-Triplet Splitting Problem

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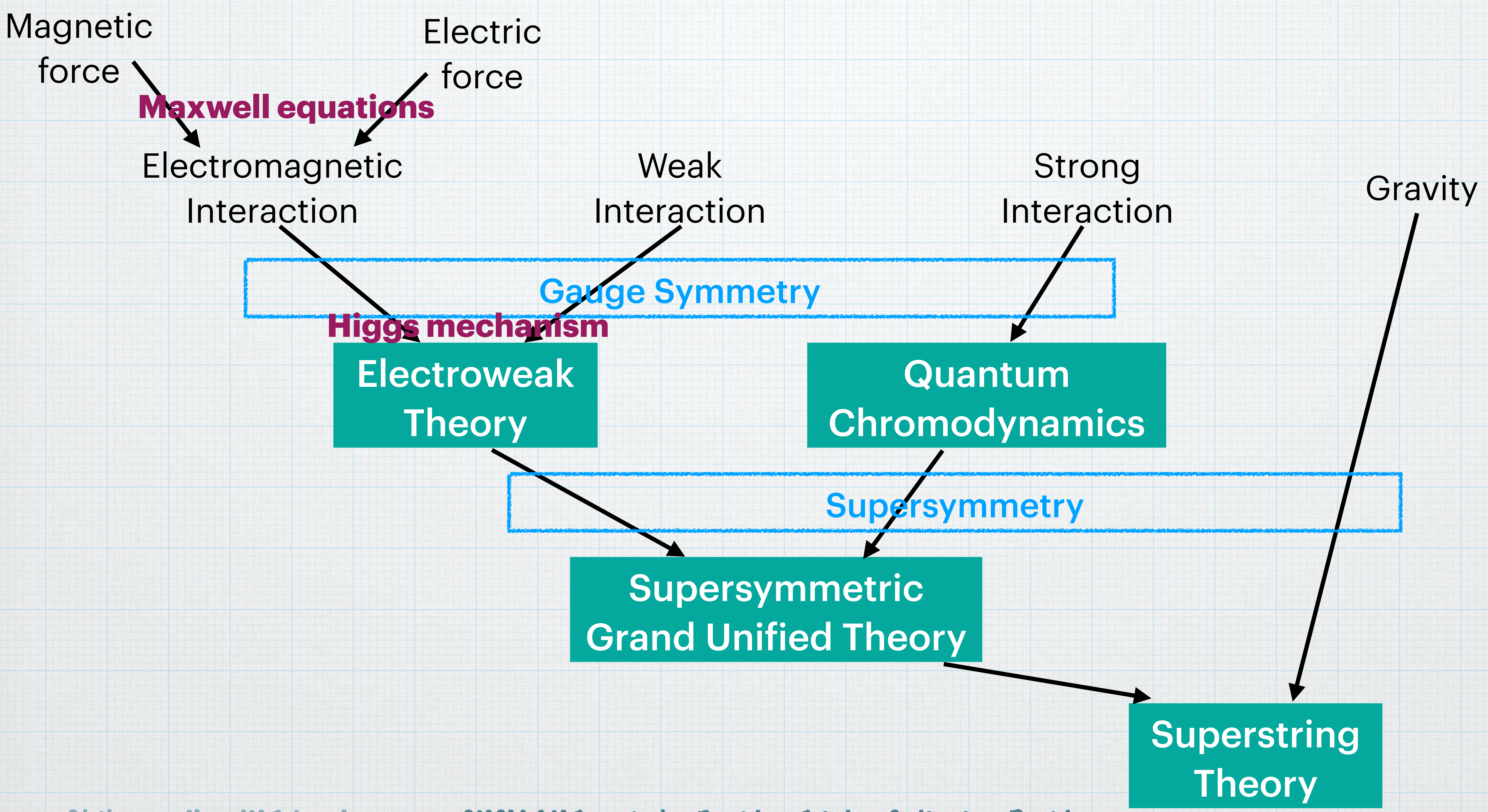
Outline

* 1. Grand Unified Theories and the Doublet-Triplet Splitting
* 2. Missing Partner Mechanism
* 3. Example: R-symmetric Flipped SU(5)
* 4. Nambu-Goldstone Higgs
* 5. Example: NG Higgs SUSY SU(5)
* 6. Summary

1. Grand Unified Theories and the Doublet-Triplet Splitting

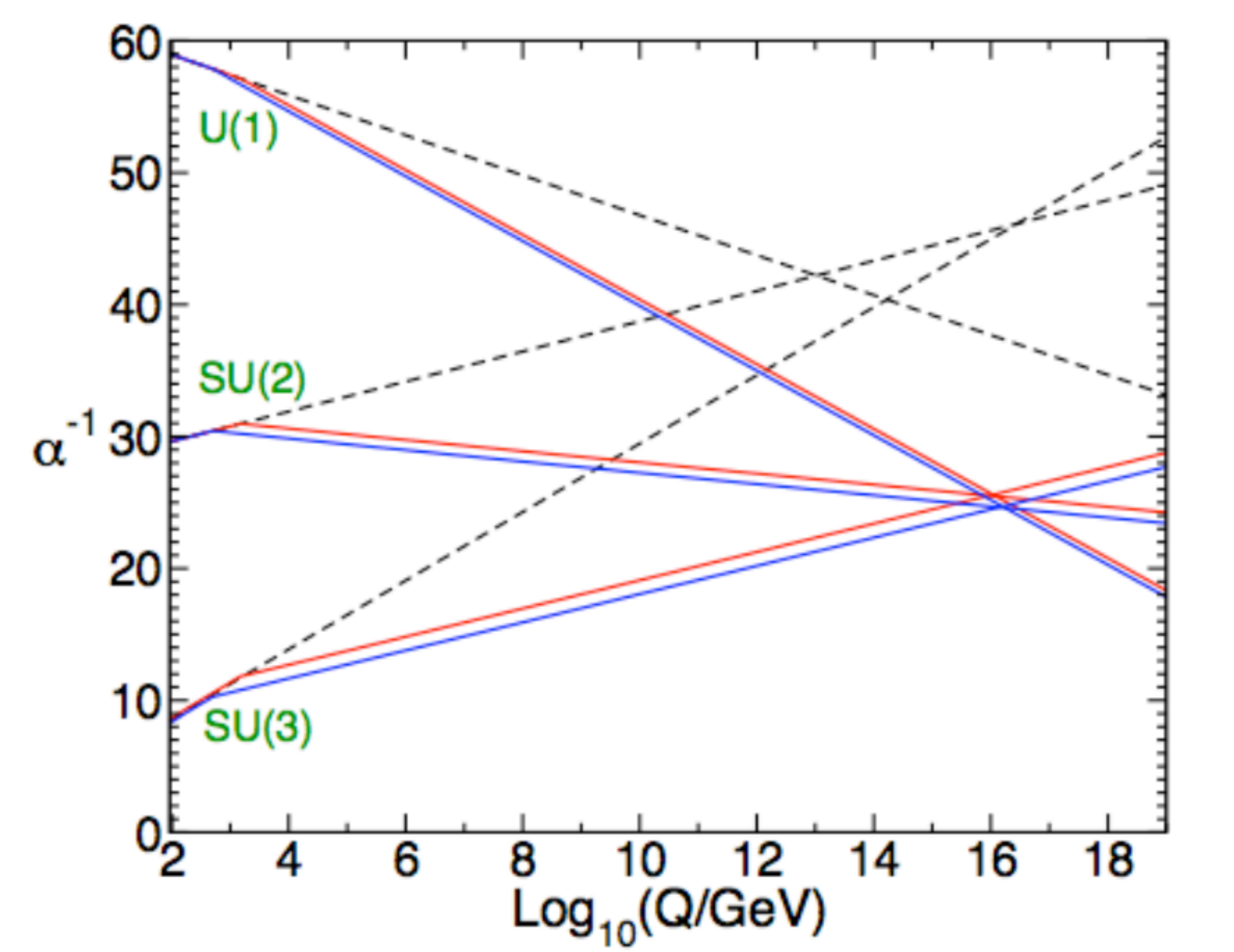


Magnetic force
Electric force
Maxwell equations
Electromagnetic Interaction
Weak Interaction
Strong Interaction
Gravity
Gauge Symmetry
Higgs mechanism
Electroweak Theory
Quantum Chromodynamics
Supersymmetry
Supersymmetric Grand Unified Theory
Superstring Theory

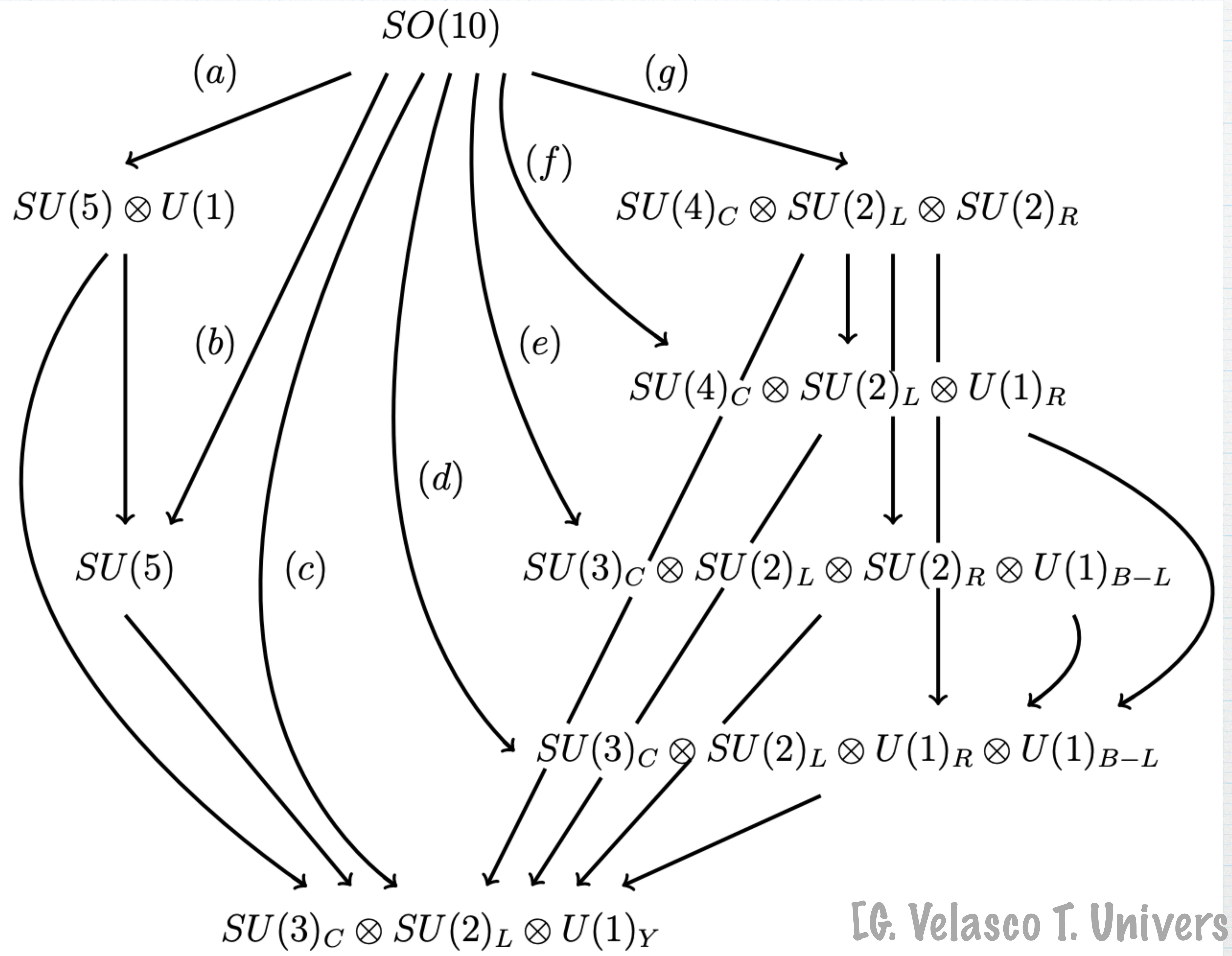
Grand Unification

* Coupling unification (SUSY GUT)
    * $g_1(M_{\mathrm{GUT}}) = g_2(M_{\mathrm{GUT}}) = g_3(M_{\mathrm{GUT}})$
    * SUSY GUT
* SUSY GUT: EW scale stabilization
* Unification of quarks and leptons
* Charge quantization
* Proton decay

[P. Langacker, Phys. Rept. 72 (1981) 185]



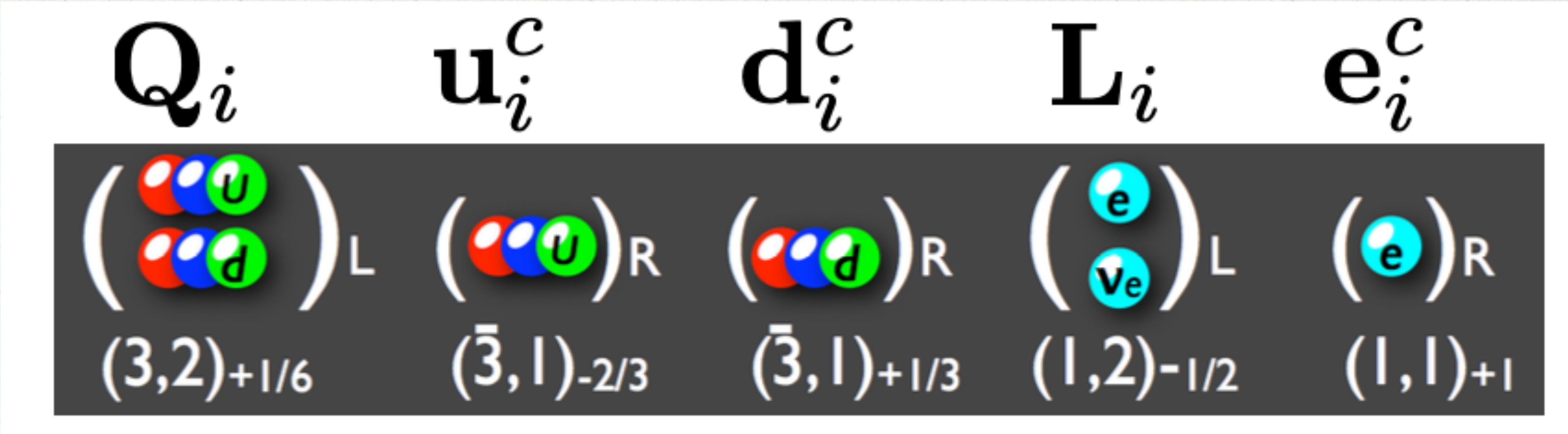
Grand Unification



[G. Velasco T. University Coll. London (2015)]

SU(5) GUT

- SU(5) gauge theory
- $$F_i = \mathbf{10} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & u^c_{i3} & -u^c_{i2} & u^1_i & d^1_i \\ -u^c_{i3} & 0 & u^c_{i1} & u^2_i & d^2_i \\ u^c_{i2} & -u^c_{i1} & 0 & u^3_i & d^3_i \\ -u^1_i & -u^2_i & -u^3_i & 0 & e^c_i \\ -d^1_i & -d^2_i & -d^3_i & -e^c_i & 0 \end{pmatrix}, \qquad \bar{f}_i = \bar{\mathbf{5}} = \begin{pmatrix} d^c_{i1} \\ d^c_{i2} \\ d^c_{i3} \\ e_i \\ -\nu_i \end{pmatrix}, \qquad l^c_i = \mathbf{1} = \left(\nu^c_i\right)$$
- $SU(5) \rightarrow SU(3) \times SU(2) \times U(1)$ ⇨ Hypercharge assignments
  - Broken by $\Phi = 24 \rightarrow (8,1)_0 + (1,3)_0 + (1,1)_0 + (3,2)_{+5/6} + (\bar{3},2)_{-5/6}$
  - $10 \rightarrow (\bar{3},1)_{-2/3} + (3,2)_{+1/6} + (1,1)_{+1}$
  - $\bar{5} \rightarrow (\bar{3},1)_{+1/3} + (1,2)_{-1/2}$



Doublet-Triplet Splitting Problem

* MSSM Higgs: embedded in 5 and $\bar{5}$ of SU(5)
    * $H = \begin{pmatrix} \zeta_u \\ H_u \end{pmatrix}, \quad \bar{H} = \begin{pmatrix} \bar{\zeta}_d \\ H_d \end{pmatrix}$

$\zeta_u, \bar{\zeta}_d$: colored Higgs —> need to be heavy
H_u, H_d : MSSM Higgs —> need to be light

⇩

Doublet-triplet splitting problem

* Superpotential $W \supset \lambda(H\Phi\bar{H} + MH\bar{H})$
    * $< \Phi > = \mathrm{diag}(b, b, b, -\frac{3}{2}b, -\frac{3}{2}b)$
    * $W_{\mathrm{eff}} = \lambda(b+M)\zeta_u\bar{\zeta}_d + \lambda(-\frac{3}{2}b + M)H_uH_d$
    * $\frac{3}{2}b = M$ —> fine-tuning problem

Doublet-Triplet Splitting Problem

* Solutions:
    * Sliding singlet mechanism
    * Missing partner mechanism
    * Dimopoulos-Wilczek mechanism
    * Nambu-Goldstone bosons

2. Missing Partner Mechanism

Missing Partner Mechanism in SU(5)

* Mixing with additional triplets
* $c_1 50 \cdot 75 \cdot 5 + c_2 \overline{50} \cdot 75 \cdot \bar{5} + c_3 50 \cdot \overline{50}$
* Symmetry breaking: $SU(5) \to SU(3) \times SU(2) \times U(1)$
  * $75 \to (\bar{3},1)_{-5/3} + (\bar{6},2)_{-5/6} + (3,2)_{-5/6} + (8,3)_0 + (1,1)_0$
    $+(8,1)_0 + (\bar{3},2)_{+5/6} + (6,2)_{+5/6} + (3,1)_{+5/3}$
  * $50 \to (\bar{6},1)_{-4/3} + (8,2)_{-1/2} + (6,3)_{+1/3} + (\bar{3},1)_{+1/3} + (3,2)_{+7/6} + (1,1)_{+2}$
  * $5 \to (3,1)_{-1/3} + (1,2)_{+1/2}$

Missing Partner Mechanism in SU(5)

* Large representation
  * $24 \to 75 + 50 + \bar{50}$
* Color Higgs becomes heavy without fine-tuning.
* $\mu 5\bar{5}$ term is dropped by hand in original missing partner model.
* Missing partner model with the minimal matter content does not work well.

Flipped SU(5) GUT

* Flipped SU(5) GUT: SU(5)xU(1) gauge theory
* $$F_i = \mathbf{10} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & d_{i3}^c & -d_{i2}^c & u_i^1 & d_i^1 \\ -d_{i3}^c & 0 & d_{i1}^c & u_i^2 & d_i^2 \\ d_{i2}^c & -d_{i1}^c & 0 & u_i^3 & d_i^3 \\ -u_i^1 & -u_i^2 & -u_i^3 & 0 & \nu_i^c \\ -d_i^1 & -d_i^2 & -d_i^3 & -\nu_i^c & 0 \end{pmatrix}, \quad \bar{f}_i = \bar{\mathbf{5}} = \begin{pmatrix} u_{i1}^c \\ u_{i2}^c \\ u_{i3}^c \\ e_i \\ -\nu_i \end{pmatrix}, \quad l_i^c = \mathbf{1} = \left(e_i^c \right)$$
* $$Y = \frac{1}{\sqrt{15}} T_{24} + \sqrt{\frac{8}{5}} Q_X$$
* The doublet-triplet splitting problem
* Missing partner mechanism

Missing Partner Mechanism in Flipped SU(5)

* **Doublet-triplet splitting problem** is solved by the **missing partner mechanism**.
* $W_{\rm DT} = \frac{1}{4}\lambda_4\epsilon_{\alpha\beta\gamma\delta\epsilon}H^{\alpha\beta}H^{\gamma\delta}h^{\epsilon} + \frac{1}{4}\lambda_5\epsilon^{\alpha\beta\gamma\delta\epsilon}\bar{H}_{\alpha\beta}\bar{H}_{\gamma\delta}\bar{h}_{\epsilon}\ ,$
* $10_{+1} \rightarrow (\bar{3},1)_{+1/3} + (3,2)_{+1/6} + (1,1)_0$
* $5_{-2} \rightarrow (3,1)_{-1/3} + (1,2)_{-1/2}$
* Triplets obtain masses while doublets remain massless

3. Example: R-Symmetric Flipped SU(5)

R-Symmetric Flipped SU(5)

* Standard flipped SU(5) GUT
  * MSSM Higgs reside in h and $\bar{h}$
  * H and $\bar{H}$ break the SU(5)xU(1) down to the SM gauge group
* Newly introduced in our model
  * Global U(1)R symmetry
  * Additional singlet field S

Fields	Components	SU(5)	U(1)	$U(1)_R$
F_i	d_i^c, Q_i, ν_i^c	$\mathbf{10}$	$+1$	$17/36$
$\bar{f}_i$	u_i^c, L_i	$\bar{\mathbf{5}}$	-3	$17/36$
ℓ_i^c	e_i^c	$\mathbf{1}$	$+5$	$17/36$
H	d_H^c, Q_H, ν_H^c	$\mathbf{10}$	$+1$	$1/36$
$\bar{H}$	$d_{\bar{H}}^c, Q_{\bar{H}}, \nu_{\bar{H}}^c$	$\overline{\mathbf{10}}$	-1	$17/36$
h	D, H_d	$\mathbf{5}$	-2	$19/18$
$\bar{h}$	$\bar{D}, H_u$	$\bar{\mathbf{5}}$	$+2$	$19/18$
S	S	$\mathbf{1}$	0	$1/9$

Superpotential

- Superpotential
  - $W_{\text{Yukawa}} = -\frac{1}{4}\lambda_1^{ij}\epsilon_{\alpha\beta\gamma\delta\epsilon}F_i^{\alpha\beta}F_j^{\gamma\delta}h^\epsilon + \sqrt{2}\lambda_2^{ij}F_i^{\alpha\beta}\bar{f}_{j\alpha}\bar{h}_\beta + \lambda_3^{ij}\bar{f}_{i\alpha}\ell_j^c h^\alpha$,
  - $W_{\text{DT}} = \frac{\lambda_4}{4\Lambda_{\text{DT}}^8}\epsilon_{\alpha\beta\gamma\delta\epsilon}S^8 H^{\alpha\beta}H^{\gamma\delta}h^\epsilon + \frac{1}{4}\lambda_5\epsilon^{\alpha\beta\gamma\delta\epsilon}\bar{H}_{\alpha\beta}\bar{H}_{\gamma\delta}\bar{h}_\epsilon$,
  - $W_{\text{neutrino}} = \frac{c_{ij}}{2\Lambda_N^2}S(F_i^{\alpha\beta}\bar{H}_{\alpha\beta})(F_j^{\gamma\delta}\bar{H}_{\gamma\delta})$,
  - $W_{HS} = \frac{\lambda_H}{4\Lambda_{HS}^5}(H^{\alpha\beta}\bar{H}_{\alpha\beta})^4 + \frac{\lambda_{HS}}{18\Lambda_{HS}^{10}}(H^{\alpha\beta}\bar{H}_{\alpha\beta})^2 S^9 + \frac{\lambda_S}{18\Lambda_{HS}^{15}}S^{18}$,
- Unwanted bilinear terms, $H\bar{H}$ and $h\bar{h}$, are both forbidden by the U(1)R symmetry.
- U(1)R symmetry highly restricts the possible forms of the operators in W_{HS}

Doublet-Triplet Splitting

* Doublet-triplet splitting problem is solved by the missing partner mechanism.

* $$W_{\rm DT} = \frac{\lambda_4}{4\Lambda_{\rm DT}^8}\epsilon_{\alpha\beta\gamma\delta\epsilon}S^8 H^{\alpha\beta}H^{\gamma\delta}h^{\epsilon} + \frac{1}{4}\lambda_5\epsilon^{\alpha\beta\gamma\delta\epsilon}\bar{H}_{\alpha\beta}\bar{H}_{\gamma\delta}\bar{h}_{\epsilon} ,$$

* Triplets obtain masses while doublets remain massless

  * $$M_{H_C} = \lambda_4 |\langle\Phi\rangle| \left(\frac{\langle S\rangle}{\Lambda_{\rm DT}}\right)^8$$
    $$\simeq \lambda_4 \times 7\times 10^{11} \times \left(\frac{|\langle\Phi\rangle|}{10^{16}\ \mathrm{GeV}}\right)\left(\frac{\langle S\rangle}{3\times 10^{17}\ \mathrm{GeV}}\right)^8\left(\frac{\Lambda_{\rm DT}}{10^{18}\ \mathrm{GeV}}\right)^{-8}\ \mathrm{GeV} ,$$

    Light color-triplet Higgs

  * $$M_{\bar{H}_C} = \lambda_5 |\langle\Phi\rangle| .$$

Proton Decay

* Dimension-6 proton decay

* $$\Gamma(p \to \pi^0 e^+) = \frac{m_p}{32\pi}\left(1 - \frac{m_\pi^2}{m_p^2}\right)^2 \left|V_{ud}\right|^2 \left|\left(U_\ell\right)_{11}\right|^2 \left(\langle \pi^0 | (ud)_R u_L | p \rangle_e\right)^2 \times \left[\frac{(A_1^H)^2 (A_1^R)^2}{|\langle \Phi \rangle|^4} + \frac{(A_2^R)^2}{M_{H_C}^4}\left\{y_d(M_{H_C}) y_e(M_{H_C})\right\}^2\right],$$

* $$\Gamma(p \to \pi^0 \mu^+) = \frac{m_p}{32\pi}\left(1 - \frac{m_\pi^2}{m_p^2}\right)^2 \left|V_{ud}\right|^2 \left|\left(U_\ell\right)_{12}\right|^2 \left(\langle \pi^0 | (ud)_R u_L | p \rangle_\mu\right)^2 \times \left[\frac{(A_1^H)^2 (A_1^R)^2}{|\langle \Phi \rangle|^4} + \frac{(A_2^R)^2}{M_{H_C}^4}\left\{y_d(M_{H_C}) y_\mu(M_{H_C})\right\}^2\right],$$

* $$\Gamma(p \to K^0 e^+) = \frac{m_p}{32\pi}\left(1 - \frac{m_K^2}{m_p^2}\right)^2 \left|V_{us}\right|^2 \left|\left(U_\ell\right)_{11}\right|^2 \left(\langle K^0 | (us)_R u_L | p \rangle_e\right)^2 \times \left[\frac{(A_1^H)^2 (A_1^R)^2}{|\langle \Phi \rangle|^4} + \frac{(A_2^R)^2}{M_{H_C}^4}\left\{y_s(M_{H_C}) y_e(M_{H_C})\right\}^2\right],$$

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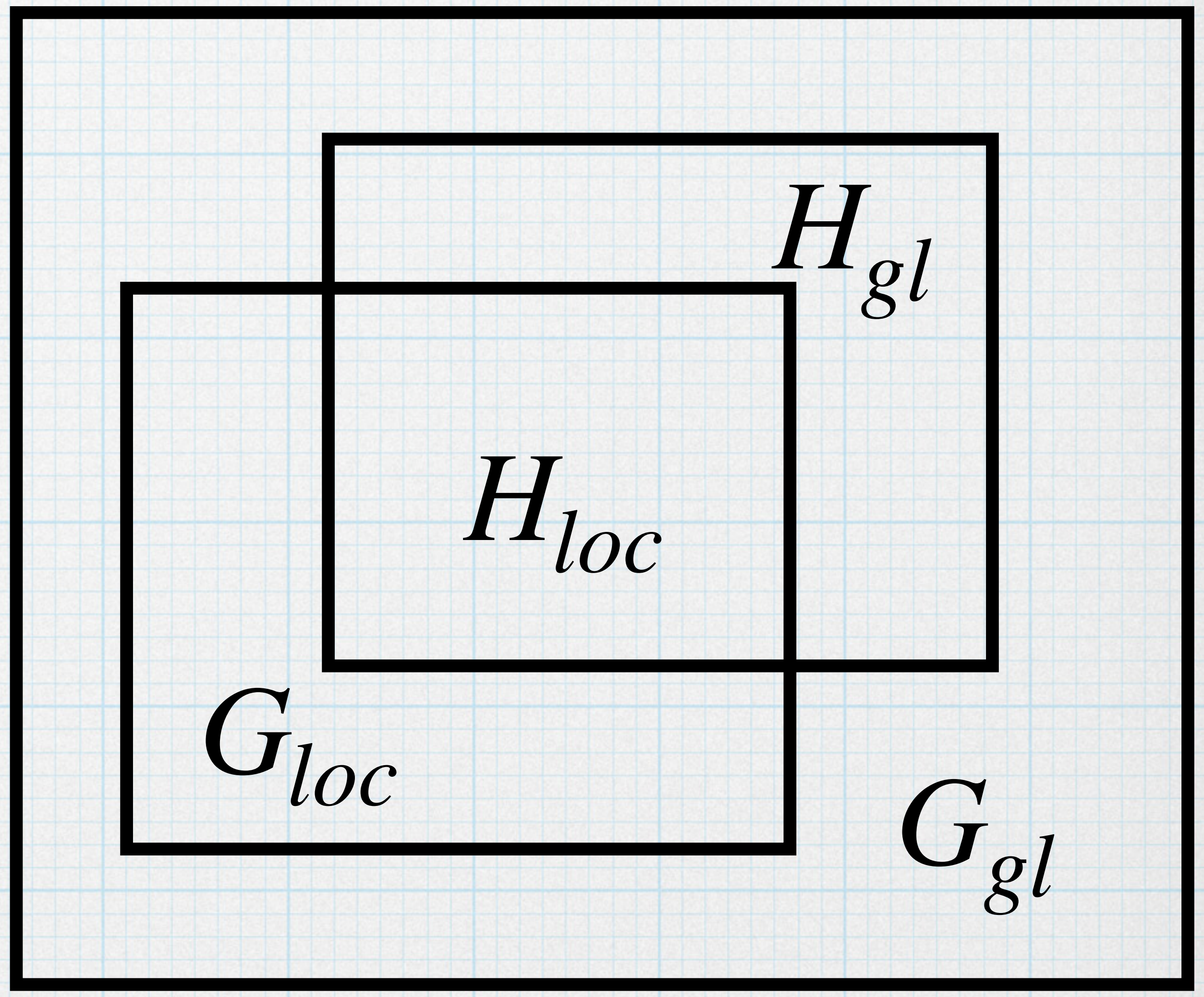
* The color-triplet Higgs exchange process: induced by the Yukawa interactions —> the decay modes that contain the second generations

4. Nambu-Goldstone Higgs

Global and Local Symmetry Breaking

* Quasi-goldstone bosons from the SSB of an approximate global sim.
* $W = W_{\text{Higgs}}(\hat{\Sigma}) + W(\hat{\Sigma}, \hat{f})$
* $W_{\text{Higgs}}(\hat{\Sigma})$: invariant under a global sym. group G_{gl}
* $H_{loc} = H_{gl} \cap G_{loc}$
* Goldstone $\hat{P} \sim \left[\text{adj}(G_{gl}) - \text{adj}(H_{gl})\right] - \left[\text{adj}(G_{loc}) - \text{adj}(H_{loc})\right]$
* $H_{loc} = SU(3) \times SU(2) \times U(1)$



Global and Local Symmetry Breaking

TABLE 1
Content of the $\hat{P}$ supermultiplet under $SU(3) \otimes SU(2) \otimes U(1)$

	G_{loc}	G_{gl}	H_{gl}	$\hat{P}$
A	SU(5)	SU(6)	$SU(4) \otimes SU(2) \otimes U(1)$	$\hat{D}$
B	SU(5)	SU(6)	$SU(3) \otimes SU(2) \otimes U(1)$	$\hat{D} \oplus \hat{T} \oplus \hat{S}$
C	SO(10)	SO(11)	$SU(3) \otimes SU(2) \otimes U(1)$	$\hat{D} \oplus \hat{T}$
D	SO(10)	SO(12)	$SU(4) \otimes SU(2) \otimes U(1)$	$2\hat{D} \oplus \hat{T}$
E	SO(10)	SO(12)	$SU(3) \otimes SU(2) \otimes U(1)$	$2\hat{D} \oplus 2\hat{T} \oplus \hat{S}$
F	SO(10)	E_6	$SU(3) \otimes SU(2) \otimes U(1)$	$2\hat{D} \oplus 2\hat{T} \oplus \hat{S} \oplus \hat{Q}$

$\hat{D} = (1, 2)_{1/2} \oplus (1, 2)_{-1/2}$, $\hat{T} = (3, 1)_{-1/3} \oplus (\bar{3}, 1)_{1/3}$, $\hat{S} = (1, 1)_0$, $\hat{Q} = (3, 2)_{1/6} \oplus (\bar{3}, 2)_{-1/6}$.

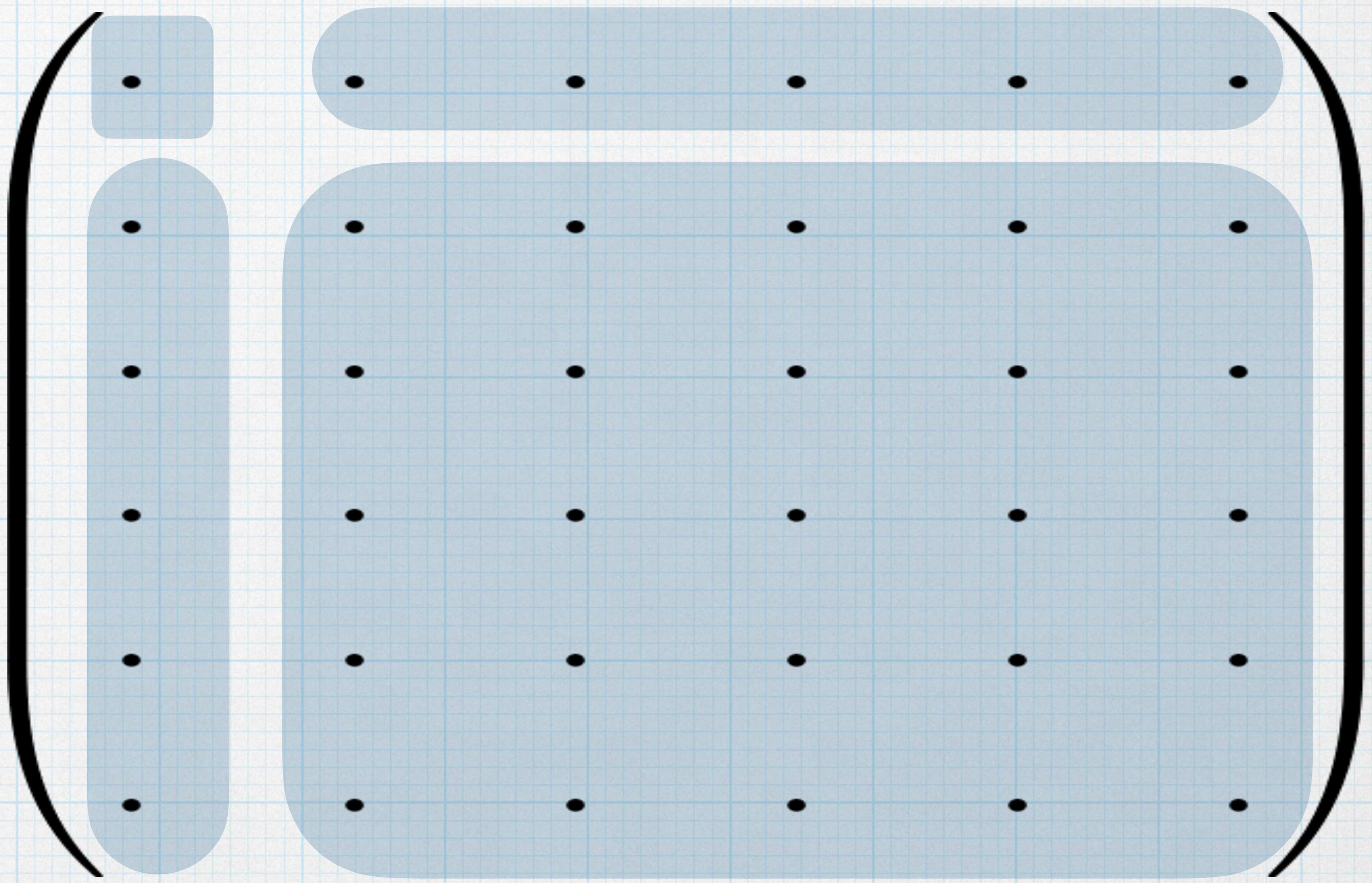
[R. Barbieri, G. Dvali, A. Strumia, Nuclear Physics B391 (1993) 487-500.]

5. Example: NG Higgs SUSY SU(5)

NG Higgs SUSY SU(5) Model

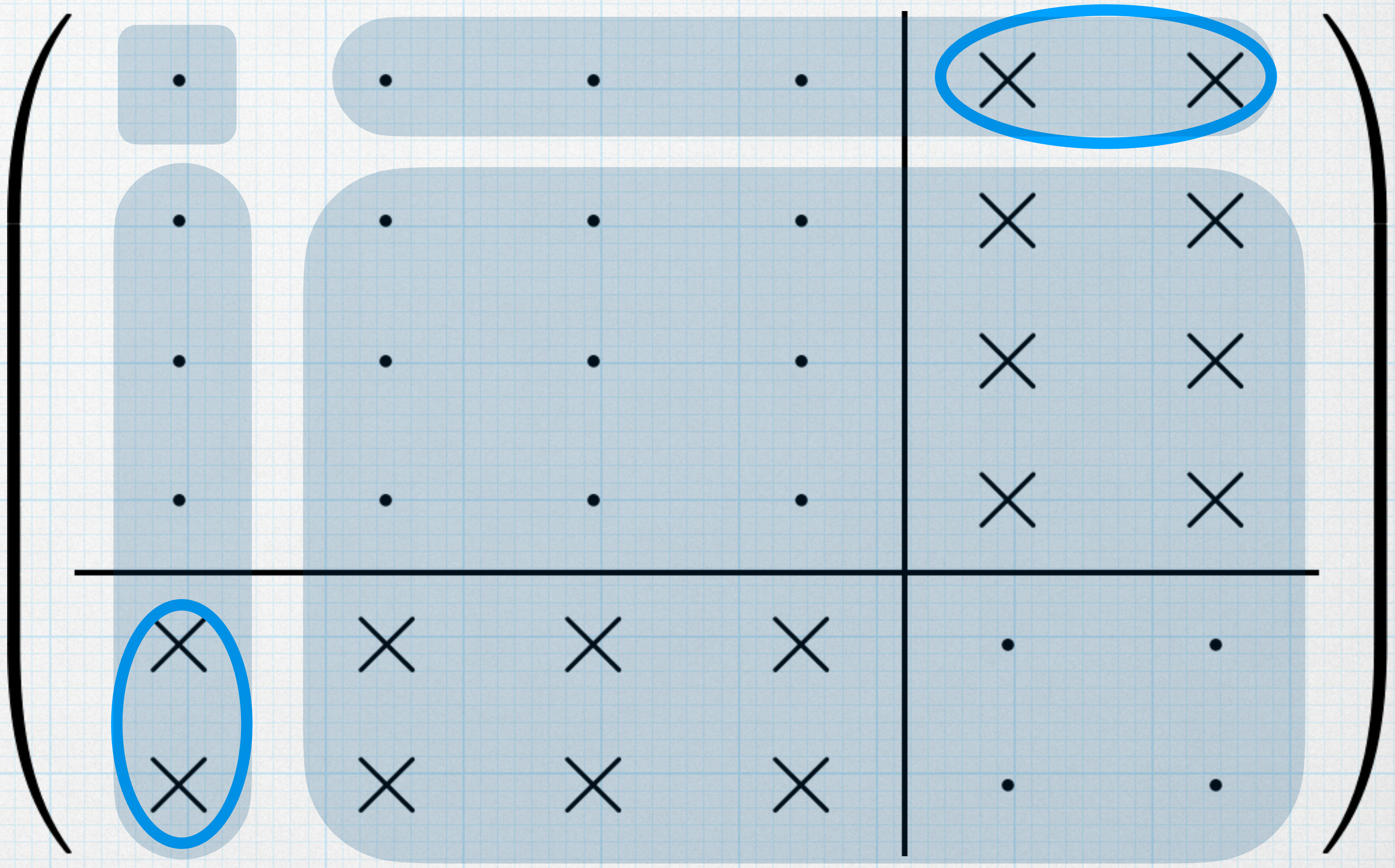
[Frist proposed by K. Inoue, A. Kakuto and H. Tkano, Próg. Theor. Phys. 75 (1986) 664.]

* SUSY SU(5) grand unified theory
* Higgs sector: a global SU(6) symmetry
  * MSSM Higgs: in the adjoint rep. of SU(6)
  * $$W_{\text{Higgs}}(\hat{\Sigma}) = \frac{1}{3}\lambda \text{Tr}\hat{\Sigma}^3 + \frac{1}{2}M\text{Tr}\hat{\Sigma}^2$$
* Relations among the couplings (compared to the usual SU(5) GUT)



NG Higgs SUSY SU(5) Model

- Symmetry breaking
  - Global SU(6) —> SU(4)xSU(2)xU(1)
  - Gauged SU(5) —> SU(3)xSU(2)xU(1)
  - $\langle \hat{\Sigma} \rangle = \hat{V} \cdot \mathrm{diag}(1,1,1,1,-2,-2)\ , \qquad \hat{V} = M/\lambda$
  - Goldstone modes: $(3,2) + (\bar{3},2) + (1,2) + (1,2)$
  - MSSM Higgs
    - $M_H = 0$ massless to all order (non-renormalization theorem)
    - Radiative corrections after soft SUSY breaking



Coupling Constants

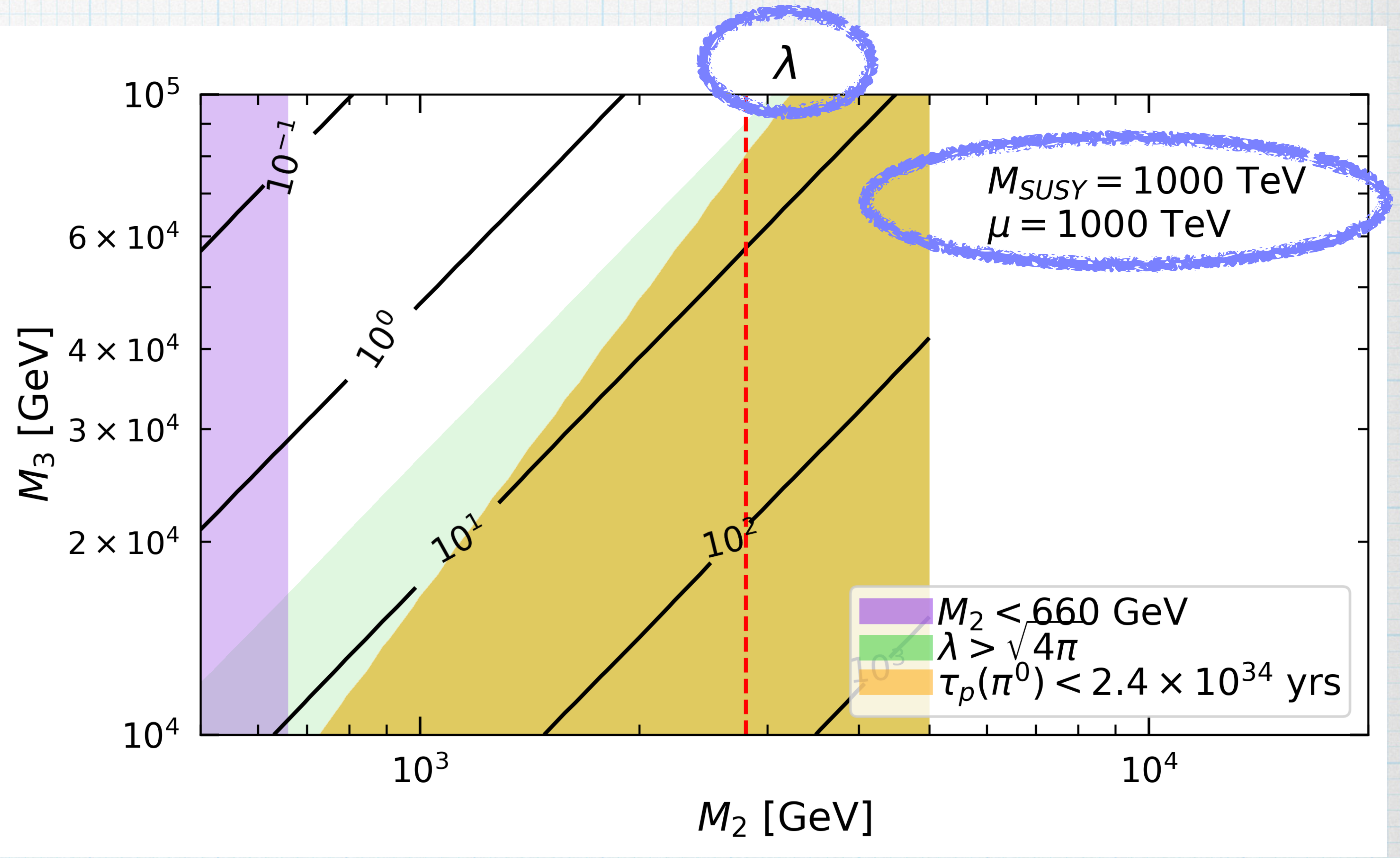
- SU(6) global sym. —> Relations among the couplings (compared to the usual SU(5) GUT)
- The dependence on the low-energy SUSY spectrum
- Approximate relation (one-loop):

$$\frac{1}{\alpha_2(Q_G)} - \frac{1}{\alpha_3(Q_G)} = -\frac{5}{7}\frac{1}{\alpha_1(m_Z)} + \frac{12}{7}\frac{1}{\alpha_2(m_Z)} - \frac{1}{\alpha_3(m_Z)} + \frac{1}{28\pi}\ln\left(\frac{m_{\widetilde{W}}^{32}\cdot m_{\widetilde{H}}^{12}\cdot m_A^{3}}{m_Z^{19}\cdot m_{\widetilde{g}}^{28}}\right) + \frac{1}{28\pi}\sum_i \ln\left[\left(\frac{m_{\widetilde{Q}_i}^{7}}{m_{\widetilde{\bar{u}}_i}^{5}\cdot m_{\widetilde{\bar{e}}_i}^{2}}\right)\left(\frac{m_{\widetilde{L}_i}^{3}}{m_{\widetilde{\bar{d}}_i}^{3}}\right)\right].$$

- The perturbativity condition
  - Upper limit on $m_{\widetilde{W}}$, $m_{\widetilde{H}}$ and m_A
  - Lower limit on $m_{\widetilde{g}}$
- We use two-loop RGEs and one-loop threshold corrections in numerical computation

Results: High-scale SUSY

- Minimal SUSY SU(5) with low-scale SUSY-breaking : rapid proton-decay problem
- High-scale SUSY: a SUSY-breaking scale of $\mathcal{O}(1)$ PeV
- Results:
- Purple region: LHC limit
- Red line: $\Omega_{\widetilde{W}} h^2 = \Omega_{DM} h^2$
- $M_{H_C} \lesssim M_X$
- $M_{H_C} \lesssim 10^{16}$ GeV
- A small wino mass M_2 and a large gluino mass M_3
- $M_{SUSY} \sim 1000\ TeV$



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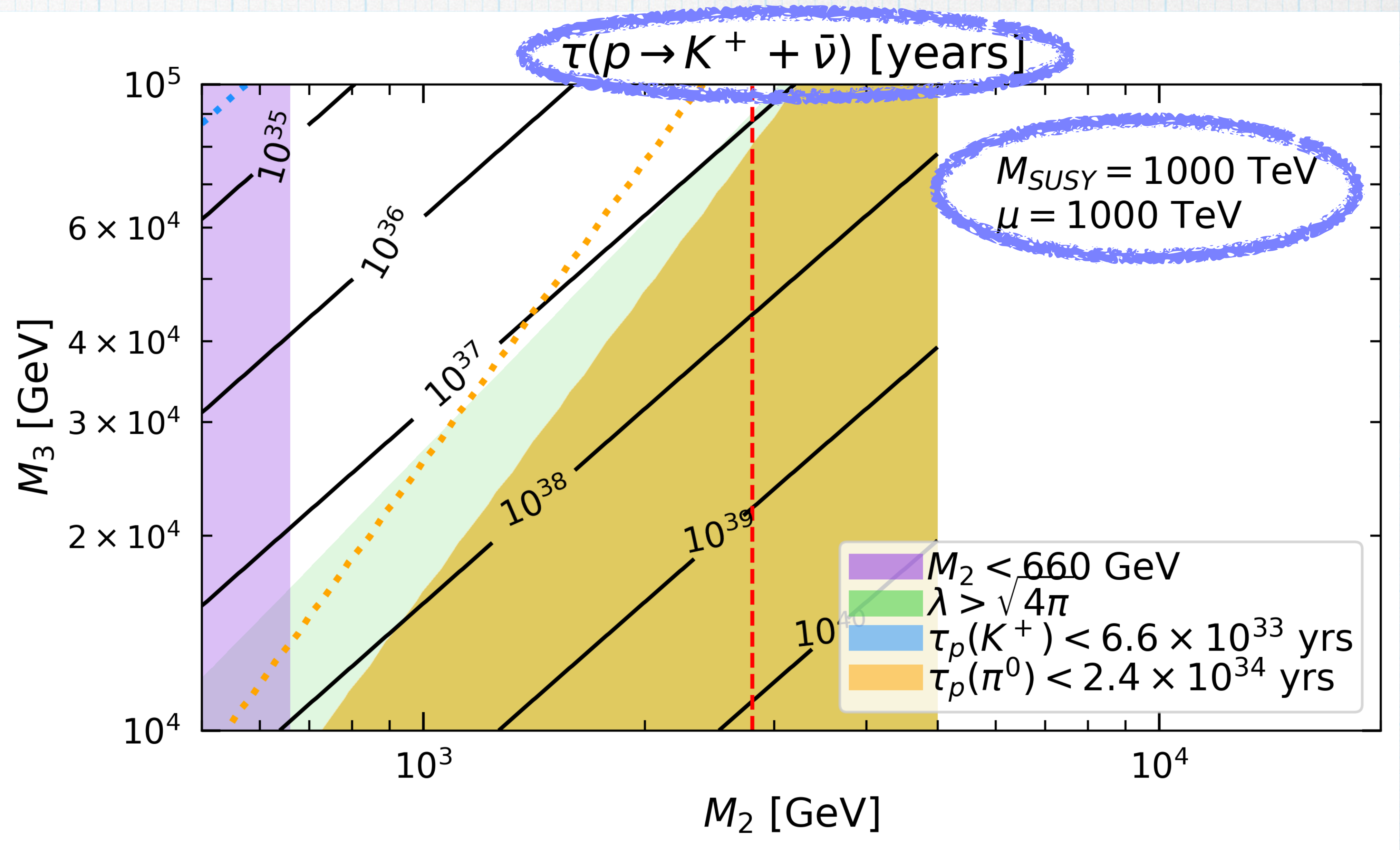

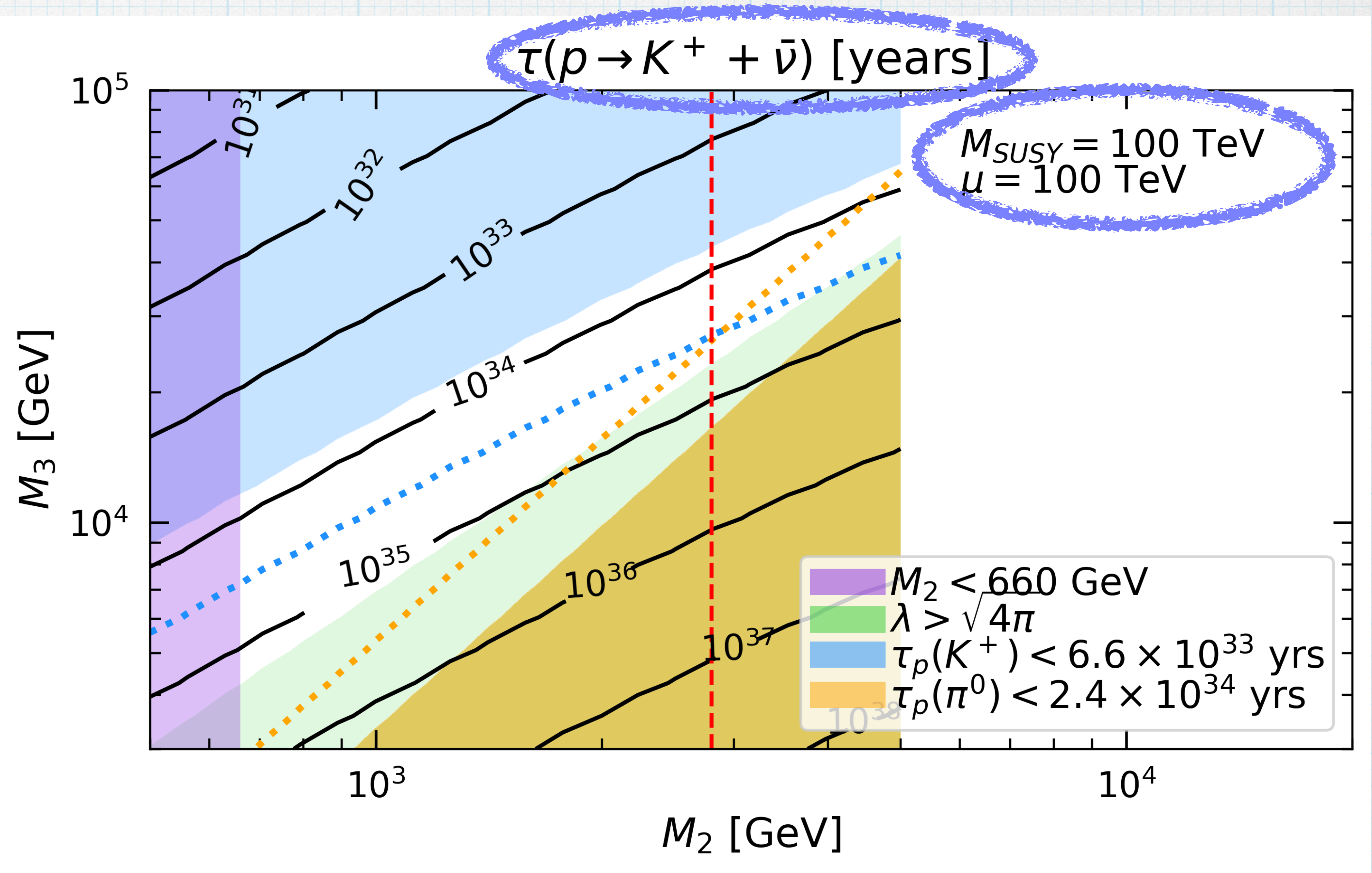

6. Summary

Summary

* SUSY GUT: the doublet-triplet splitting problem (light Higgs doublets)
* Missing partner mechanism —> R-symmetric flipped SU(5)
  * A flipped SU(5) SUSY GUT model with an additional global U(1)R symmetry
  * The μ-terms of the Higgs field are forbidden by the U(1)R sym.
  * A light color-triplet Higgs $M_{H_C} \sim \mathcal{O}(10^{12})$ GeV
  * Proton decay $p \to \pi^0 \mu^+$ and $p \to K^0 \mu^+$
* NG Higgs —> NG Higgs SUSY SU(5)
  * The global symmetry —> relations among the GUT parameters, and restricts the SUSY particle masses through the RGEs
  * $M_{H_C} \lesssim M_X$
  * A small wino mass ($< \mathcal{O}(1)$ TeV) and a relatively large gluino mass ($> \mathcal{O}(1) - \mathcal{O}(100)$ TeV)
  * High-scale SUSY scenario with $M_{\rm SUSY} \approx \mathcal{O}(1)$ PeV is favored.