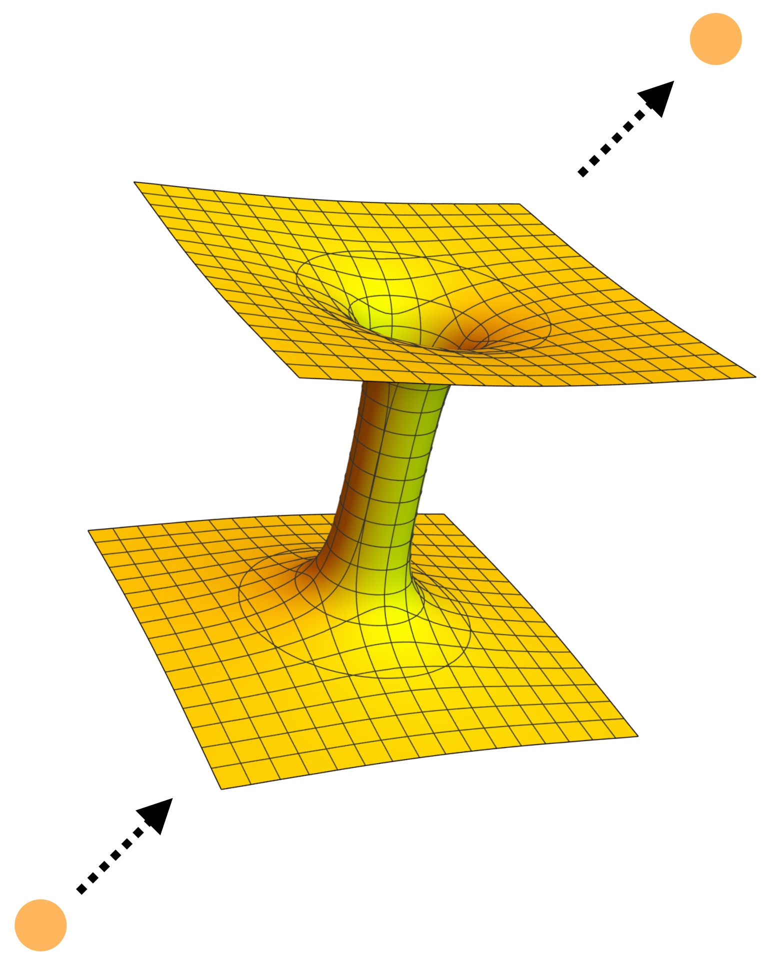
Phenomenological impacts of axionic wormholes



Yoshiki Kanazawa (University of Tokyo)

東京大学 THE UNIVERSITY OF TOKYO

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Axionic wormhole

- Gravitational instanton solution in axion theories. $G_{\mu\nu} = \frac{1}{M_P^2} T_{\mu\nu}$

- Characterized by the Peccei-Quinn charge. $Q = \int d^3x \; J^0_{\mathrm{PQ}}$

- Induce effective PQ-violating operators below a typical scale of wormholes

$$c_{m,n} \frac{|\Phi|^{2m} \Phi^n}{M_P^{2m+n-4}} + \mathrm{h.c.} \qquad \Phi(x) = \frac{f(x)}{\sqrt{2}} e^{i\theta(x)} \qquad \theta(x): \text{Axion}$$

⟶ **Axion quality problem**

The magnitude of PQ-violation by wormholes highly depends on models

Previous works

1. Giddings-Strominger model — No quality problem

$f(x) = f_a$

2. Model with dynamical radial field — **Quality problem**

$\Phi(x) = \frac{f(x)}{\sqrt{2}} e^{i\theta(x)}$ θ : Axion

My works

3. Model with non-minimal coupling $\xi |\Phi|^2 R$

(An extension of Model 2)

For large ξ, we can avoid the quality problem

- Metric formulation
- Palatini formulation

Contents of this talk

- Axion quality problem
- Axionic wormholes
- Axionic wormholes in model with non-minimal coupling

Axion quality problem

QCD axion

Strong CP problem

$$\theta \equiv \theta_{\rm QCD} + \sum_{\rm quark} \arg M_q \qquad \mathscr{L}_{\rm QCD} \supset \frac{\theta_{\rm QCD}}{32\pi^2} G\tilde{G}$$

- A combination of QCD theta angle and quark mass phases is a physical parameter that characterizes CP-violation.
- Measurements of the neutron EDM indicates that the theta is unnaturally tiny.

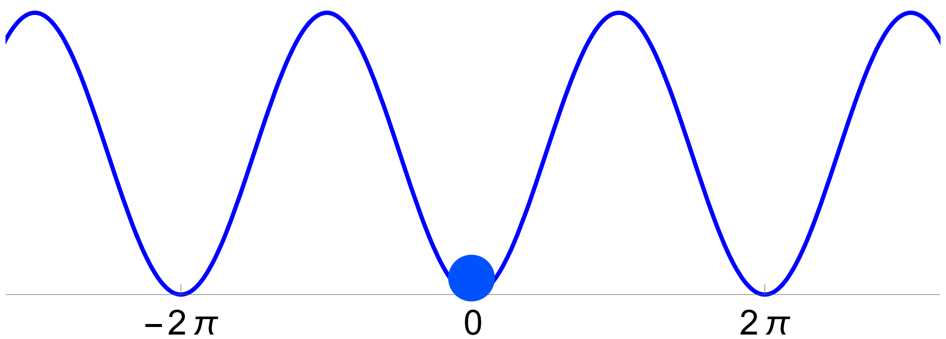
$$|d_n| < 1.8 \times 10^{-26}\ e \cdot {\rm cm}\ (90\,\%\ {\rm C.L.}) \longrightarrow |\theta| \lesssim 10^{-10}$$

C. Abel et al., Phys. Rev. Lett. 124, 081803 (2020).

Peccei-Quinn mechanism

- Introduce a global U(1) PQ symmetry and associated NG boson, **"axion"**, whose VEV plays the role of effective theta angle.
- The axion acquires a periodic potential due to QCD instanton.

$$V_{\rm QCD}(\theta) = -\Lambda_{\rm QCD}^4 \cos\theta \longrightarrow \theta_{\rm eff} = 0 \quad \text{exactly}$$



R. D. Peccei and H. R. Quinn, Phys. Rev. Lett 38, 1440 (1977);
R. D. Peccei and H. R. Quinn, Phys. Rev. D 16, 1791 (1977).

PQ mechanism in the presence of gravity

- It is expected that global symmetries are explicitly violated by gravity.

T. Banks and N. Seiberg, 2011; E. Witten, 2018; D. Harlow and H. Ooguri, 2019.

- Axion potential acquires gravitational corrections.

$$c_{m,n}\frac{|\Phi|^{2m}\Phi^n}{M_P^{2m+n-4}} + \mathrm{h.c.}$$

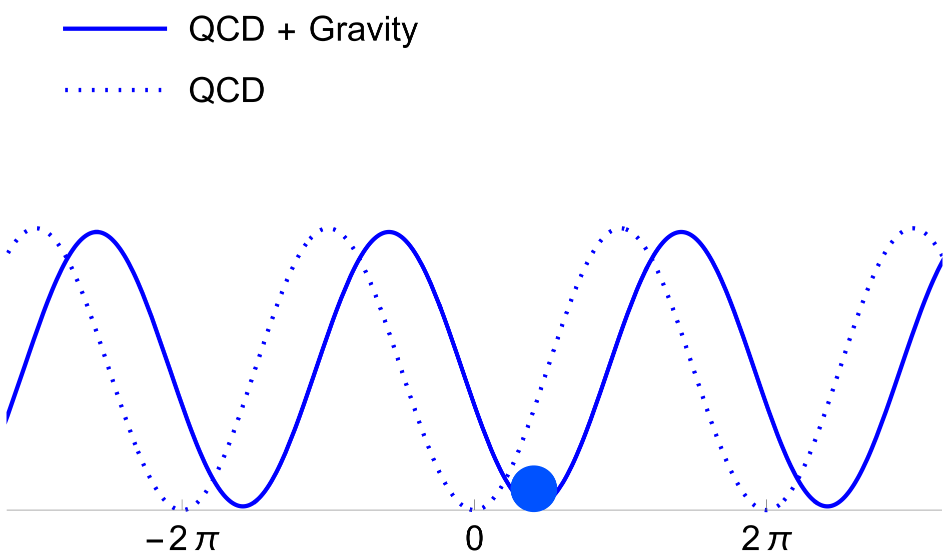
dim = $2m+n$, PQ charge = n

$c_{m,n}$: a complex constant

$$\Phi = \frac{f}{\sqrt{2}}e^{i\theta} \qquad \langle|\Phi|\rangle = \frac{f_a}{\sqrt{2}}$$

$$\longrightarrow \quad V_{\mathrm{grav.}}(\theta) = 2|c_{m,n}|M_P^4\left(\frac{f_a}{\sqrt{2}M_P}\right)^{2m+n}\cos(n\theta+\delta_{m,n}) \qquad \delta_{m,n} \equiv \arg\, c_{m,n}$$

- The minimum deviates from CP-conserving points due to the relative phase $\delta_{m,n}$.



PQ mechanism in the presence of gravity

Effective theta angle $\theta_{\rm eff} \simeq 2n\,|c_{m,n}|\sin\delta_{m,n}\left(\frac{M_P}{\Lambda_{\rm QCD}}\right)^4\left(\frac{f_a}{\sqrt{2}M_P}\right)^{2m+n}$

$$\sim 10^{76}$$

For the case of dim 5, PQ charge 1, $c_{2,1}\frac{|\Phi|^4\Phi}{M_P} + {\rm h.c.} \qquad (m=2,\ n=1)$

Strong CP problem

$|\theta_{\rm eff}| \lesssim 10^{-10}$ $\longrightarrow$

Axion quality problem

$|c_{2,1}|\sin\delta_{2,1} < 10^{-54}$ for $f_a = 10^{12}$ GeV

Extremely small

cf. Axion window $10^8\ {\rm GeV} \lesssim f_a \lesssim 10^{12}\ {\rm GeV}$

M. Kamionkowski and J. March-Russell, Phys. Lett. B 282, 137 (1992);
R. Holman, S. D. H. Hsu, T. W. Kephart, E. W. Kolb, R. Watkins, and L. M. Widrow, Phys. Lett. B 282, 132 (1992).

Sources of U(1) PQ violation

Non-perturbative effects

Wormhole ⋯ Emerge within the Einstein gravity

Worldsheet instanton
Brane instanton ⋯ Derived from string theories

P. Svrcek and E. Witten, JHEP 06, 051 (2006).

⋮

- PQ-violating operators are exponentially suppressed by the instanton action.

$$|c| \sim e^{-S} \qquad S: \text{instanton action}$$

- Instanton corrections include operators with a positive power of the Planck scale, such as

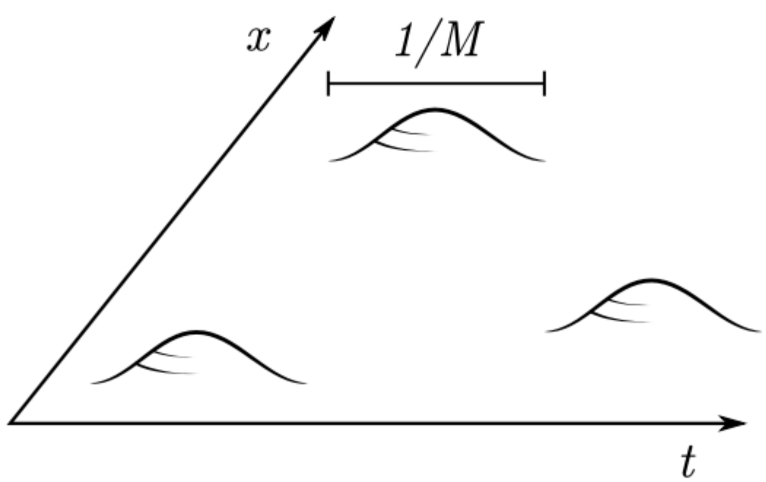
$$c_{0,1} M_P^3 \Phi + \mathrm{h.c.}$$

Axionic wormholes

What is wormhole?

Comparison between QCD instanton and wormhole

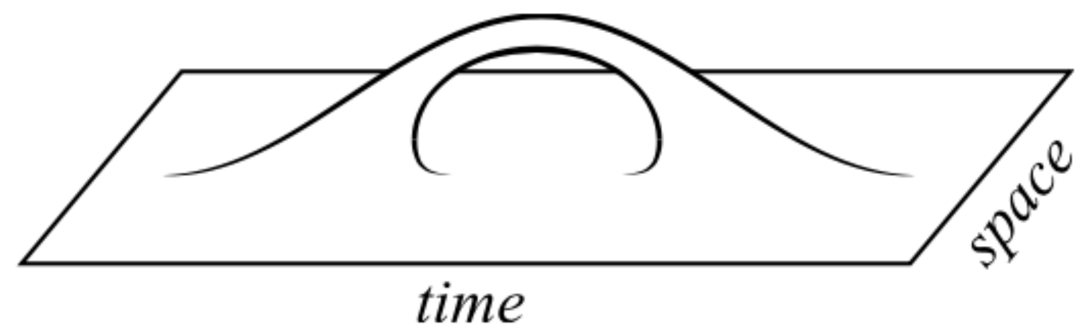
<u>QCD instanton</u>



- Localized at one point
- Characterized by the topological charge

$$Q = \frac{1}{32\pi^2}\int d^4x\ G\tilde{G} \in \mathbb{Z}$$

<u>Wormhole</u>



- Localized at two points
- Characterized by the PQ charge

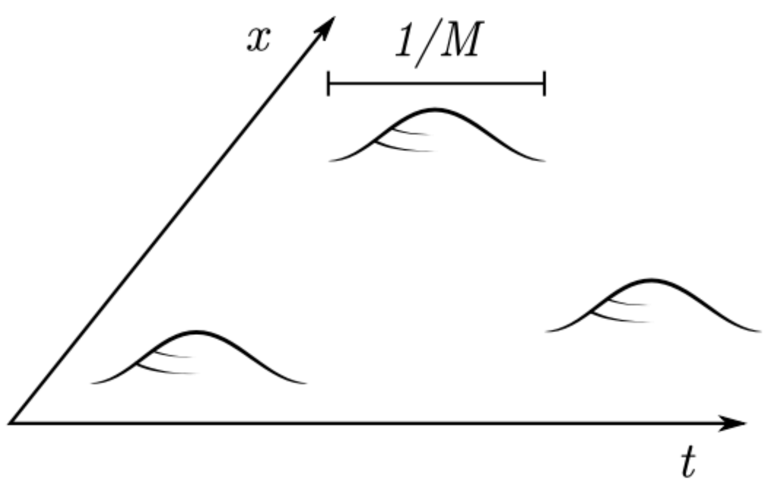
$$Q = \int d^3x\ J^0_{\mathrm{PQ}} \in \mathbb{Z}$$

A. Hebecker, T. Mikhail and P. Soler, Front. Astron. Space Sci. 5, 35 (2018).

What is wormhole?

Comparison between QCD instanton and wormhole

QCD instanton



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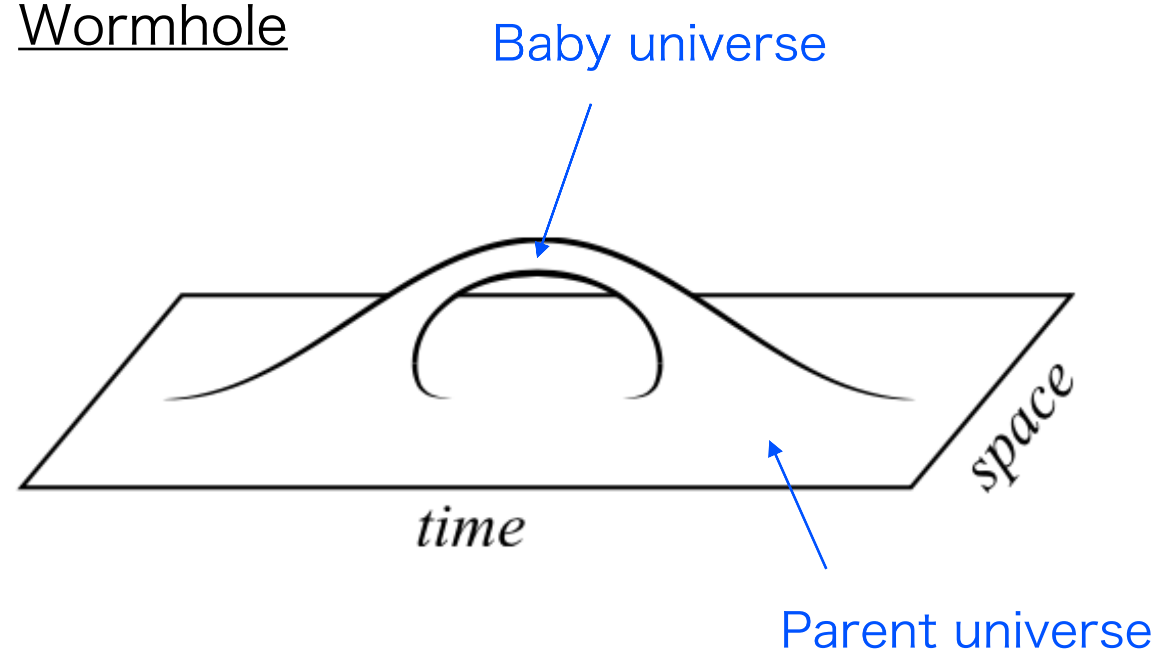

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Giddings-Strominger wormhole

Setup

- O(4)-symmetric Euclidean geometry

$$ds^2 = dr^2 + a(r)^2 d^2\Omega_3$$

r : Euclidean time

$a(r)$: Radius of baby universe

- Action

$$S = \int d^4x \sqrt{g} \left[-\frac{M_P^2}{2} R + \frac{f_a^2}{2} (\partial_\mu \theta)^2 \right]$$

$$\theta \sim \theta + 2\pi$$

- Global U(1) PQ symmetry

$$\theta(r) \to \theta(r) + \varepsilon$$

Current conservation

$$\partial_\mu J^\mu_{\rm PQ} = 0 \qquad J^\mu_{\rm PQ} \equiv \sqrt{g} f_a^2 g^{\mu\nu} \partial_\nu \theta$$

Conserved charge

$$Q = \int d^3x \; J^0_{\rm PQ} = 2\pi^2 a^3 f_a^2 \partial_r \theta = n \in \mathbb{Z}$$

Quantized by an integer

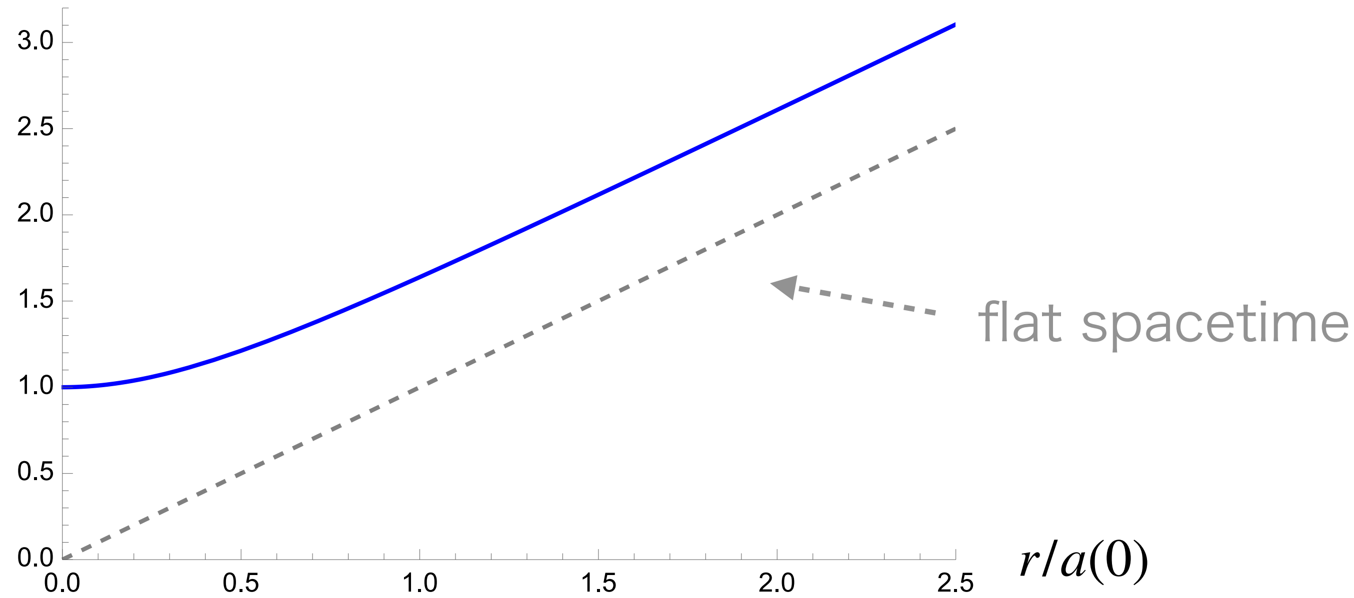
S. B. Giddings and A. Strominger, Nucl. Phys. B 306, 890 (1988).

Giddings-Strominger wormhole

<u>Solution</u>

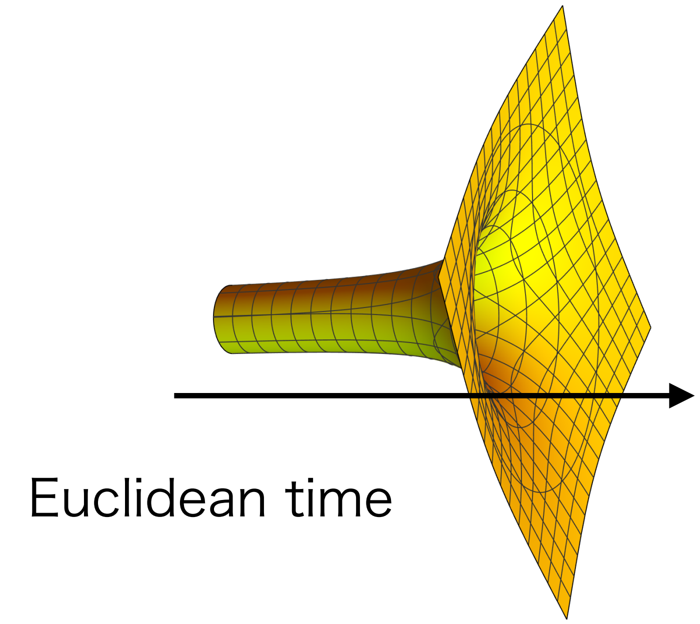
- EoMs $a'^{\,2} - 1 = -\frac{n^2}{24\pi^4 f_a^2 M_P^2}\frac{1}{a^4}$ $2aa'' + a'^{\,2} - 1 = \frac{n^2}{8\pi^4 M_P^2 f_a^2}\frac{1}{a^4}$
- Boundary condition $a'(0) = 0$ ⟶ $a(0) = \left(\frac{n^2}{24\pi^4 f_a^2 M_P^2}\right)^{\frac{1}{4}}$ Size of wormhole throat

$a(r)/a(0)$ $a(r) \to r \quad (r \to \infty)$



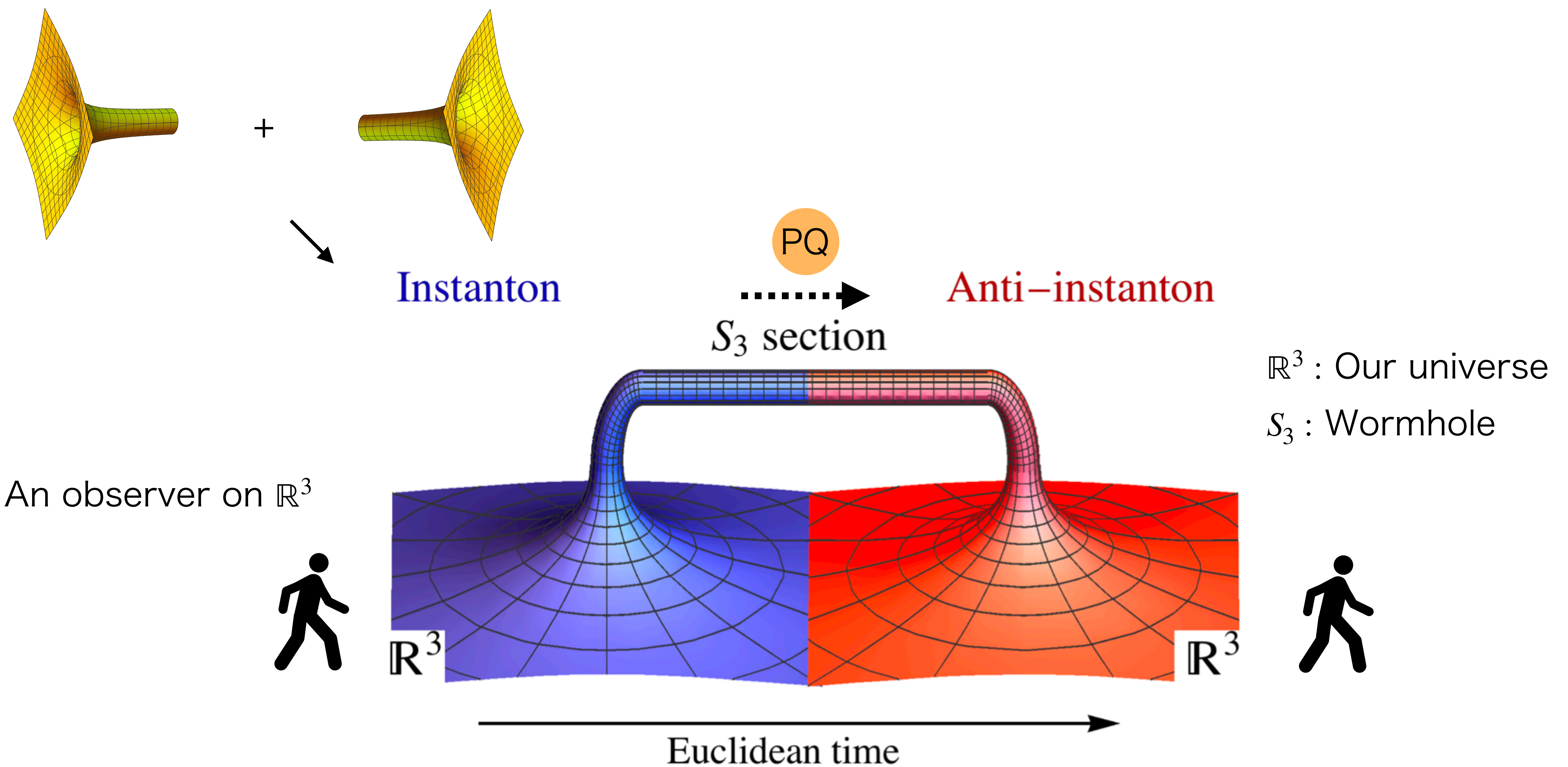
finite size at $r = 0$

<u>"Semi-"wormhole</u>



S. B. Giddings and A. Strominger, Nucl. Phys. B 306, 890 (1988).

U(1) PQ violation by wormholes

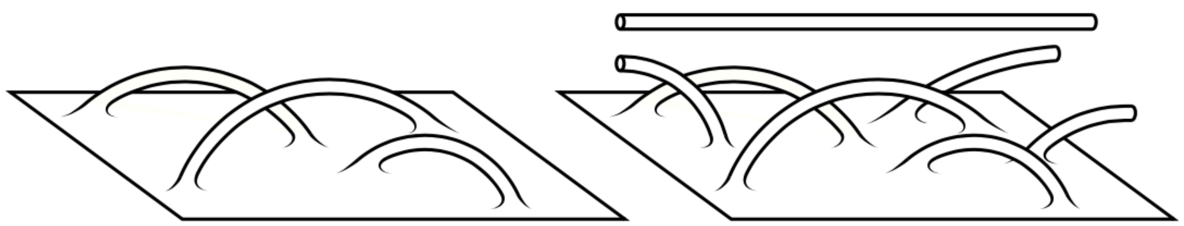


- An observer on $\mathbb{R}^3$ feels that PQ charge passes through a wormhole.
- The PQ symmetry is explicitly **violated** on $\mathbb{R}^3$, while **conserved** on $\mathbb{R}^3 \oplus S_3$.

R. Alonso and A. Urbano, JHEP 02, 136 (2019).

U(1) PQ violation by wormholes

- Sum over all possible configurations in the path integral under dilute gas approximation



All of
full-wormhole,
semi-wormhole,
disconnected wormhole

- Effective action

$$I = \sum_{n \in \mathbb{N}} \sqrt{K_n} e^{-S_n} \int d^4x \sqrt{g(x)} (\mathcal{O}_n(x)\alpha_n + \mathcal{O}_n^*(x)\alpha_n^*)$$

PQ-violating

$\mathcal{O}_n(x)$: A set of local operators with PQ charge n

α_n : A parameter that characterizes our vacuum

S_n : Wormhole action with PQ charge n

- **The wormhole action S_n significantly depends on models.**

S. R. Coleman, Nucl. Phys. B307, 867-882 (1988);
A. Hebecker, T. Mikhail and P. Soler, Front. Astron. Space Sci. 5, 35 (2018).

U(1) PQ violation by wormholes

1. Giddings-Strominger model $S = \int d^4x \sqrt{g} \left[-\frac{M_P^2}{2} R + \frac{f_a^2}{2} (\partial_\mu \theta)^2 \right]$ $ds^2 = dr^2 + a(r)^2 d^2\Omega_3$

- Wormhole corrections to the axion potential

$$V_{\text{grav.}} = 2|\alpha_1| e^{-S_1} L^{-4} \cos(\theta + \delta_1) \qquad \delta_1 \equiv \arg \alpha_1$$

Wormhole action $S_1 = \sqrt{\frac{3\pi^2}{8} \frac{M_P}{f_a}}$ Size of the wormhole throat $L \equiv a(0) \sim (M_P f_a)^{-\frac{1}{2}}$

- Effective theta angle $|\theta_{\text{eff}}| \simeq \left(\frac{L^{-1}}{\Lambda_{\text{QCD}}} \right)^4 e^{-S_1} \ll 10^{-10}$ for $f_a \lesssim 10^{16}$ GeV

assuming $|\alpha_1| \sin \delta_1 \sim \mathcal{O}(1)$

No quality problem

U(1) PQ violation by wormholes

2. Model with dynamical radial field $\Phi = \frac{f}{\sqrt{2}} e^{i\theta} \quad \langle |\Phi| \rangle = \frac{f_a}{\sqrt{2}}$

$$S = \int d^4x \sqrt{g} \left[-\frac{M_P^2}{2} R + |\partial_\mu \Phi|^2 + \lambda (|\Phi|^2 - f_a^2/2)^2 \right] = \int d^4x \sqrt{g} \left[-\frac{M_P^2}{2} R + \frac{f^2}{2} (\partial_\mu \theta)^2 + \frac{1}{2} (\partial_\mu f)^2 + \frac{\lambda}{4} (f^2 - f_a^2)^2 \right]$$

- EoMs

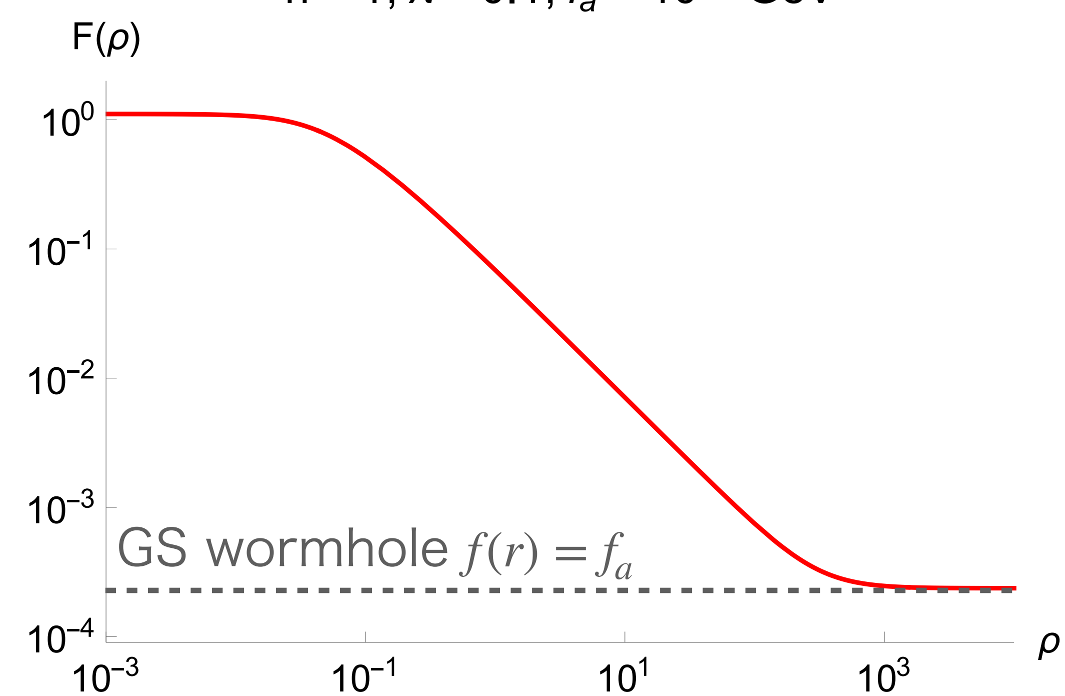
$$a'^2 - 1 = -\frac{a^2}{3M_P^2} \left[-\frac{1}{2} f'^2 + \frac{\lambda}{4} (f^2 - f_a^2)^2 + \frac{n^2}{8\pi^4 f^2 a^6} \right]$$

$$2aa'' + a'^2 - 1 = -\frac{a^2}{M_P^2} \left[\frac{1}{2} f'^2 + \frac{\lambda}{4} (f^2 - f_a^2)^2 - \frac{n^2}{8\pi^4 f^2 a^6} \right]$$

$$f'' + 3\frac{a'}{a} f' = \lambda f (f^2 - f_a^2) - \frac{n^2}{4\pi^4 f^3 a^6}$$

$$F \equiv \frac{f}{\sqrt{3} M_P}$$

n = 1, λ = 0.1, f_a = 10^{15} GeV



$$\rho \equiv \sqrt{3\lambda} M_P r$$

- Boundary conditions

$$a'(0) = 0, \quad f'(0) = 0, \quad f(\infty) = f_a$$

U(1) PQ violation by wormholes

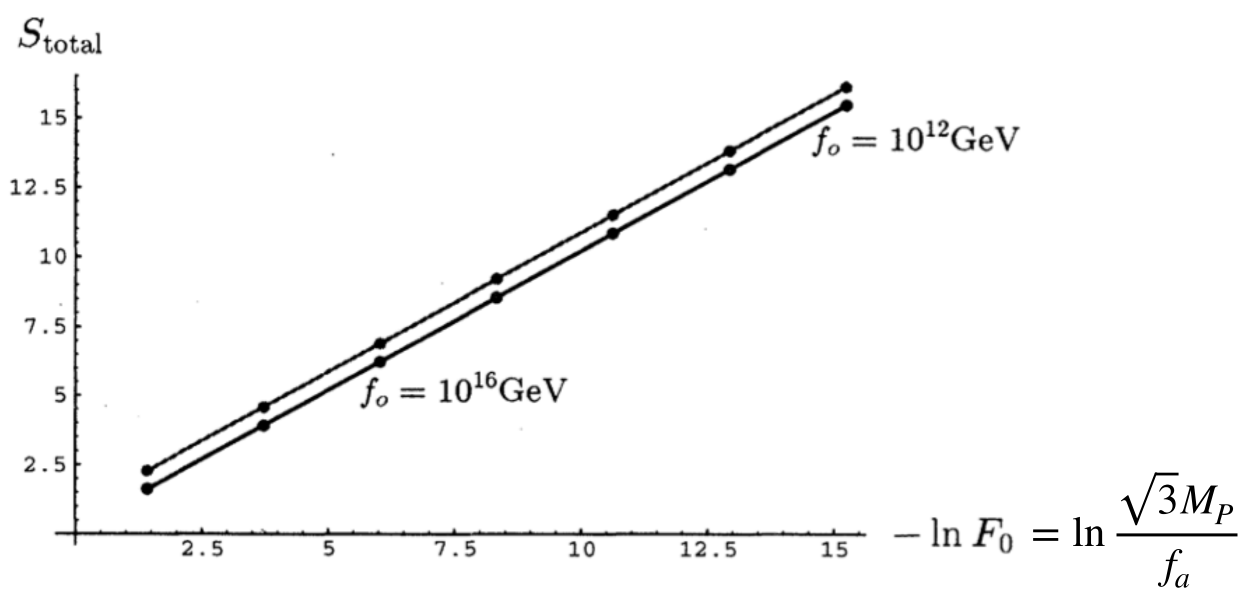
2. Model with dynamical radial field $\Phi = \frac{f}{\sqrt{2}} e^{i\theta} \quad \langle |\Phi| \rangle = \frac{f_a}{\sqrt{2}}$

- Wormhole corrections to the axion potential

$$V_{\rm grav.} = \alpha_1 e^{-S_1} L^{-3} \Phi + {\rm h.c.} = \sqrt{2} |\alpha_1| e^{-S_1} L^{-3} f_a \cos(\theta + \delta_1)$$

Wormhole action $S_1 \simeq \log \frac{M_P}{f_a}$

Size of the wormhole throat $L \sim M_P^{-1}$



R. Kallosh, A. D. Linde, D. A. Linde and L. Susskind, Phys. Rev. D 52, 912 (1995).

U(1) PQ violation by wormholes

2. Model with dynamical radial field $\quad \Phi = \frac{f}{\sqrt{2}} e^{i\theta} \quad \langle |\Phi| \rangle = \frac{f_a}{\sqrt{2}}$

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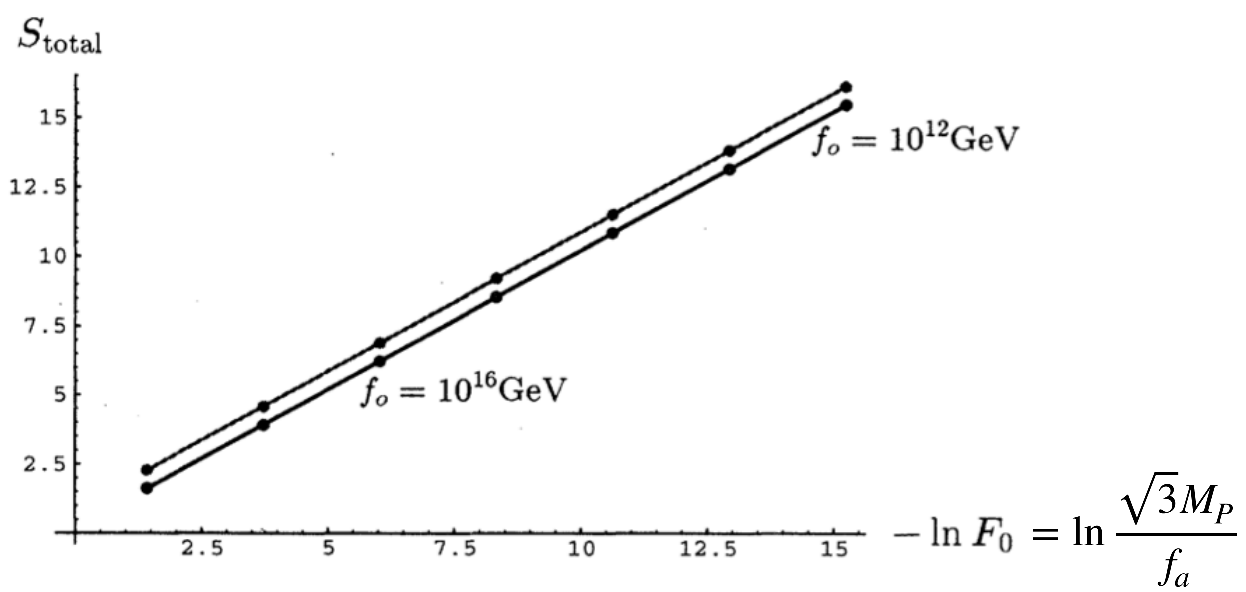
Wormhole action $\quad S_1 \simeq \log \frac{M_P}{f_a} \ll 190$

Size of the wormhole throat $\quad L \sim M_P^{-1}$



Strong CP problem **Axion quality problem**

$|\theta_{\text{eff}}| \lesssim 10^{-10}$ ⟶ $S_1 \gtrsim 190$ for $f_a = 10^{12}$ GeV assuming $|\alpha_1| \sin\delta_1 \sim \mathcal{O}(1)$

R. Kallosh, A. D. Linde, D. A. Linde and L. Susskind, Phys. Rev. D 52, 912 (1995).

Axionic wormhole in model with non-minimal coupling

Our model

3. Model with non-minimal gravitational coupling of Φ

$$S = \int d^4x \sqrt{g}\left[-\frac{M_P^2}{2}R - \xi(|\Phi|^2 - f_a^2/2)R + |\partial_\mu \Phi|^2 + \lambda(|\Phi|^2 - f_a^2/2)^2\right]$$

$$= \int d^4x \sqrt{g}\left[-\frac{M_P^2}{2}\Omega^2(f)R + \frac{f^2}{2}(\partial_\mu \theta)^2 + \frac{1}{2}(\partial_\mu f)^2 + \frac{\lambda}{4}(f^2 - f_a^2)^2\right]$$

$$\Omega^2(f) \equiv 1 + \frac{\xi(f^2 - f_a^2)}{M_P^2}$$

Our model

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$$= \int d^4x \sqrt{g} \left[-\frac{M_P^2}{2} \Omega^2(f) R + \frac{f^2}{2} (\partial_\mu \theta)^2 + \frac{1}{2} (\partial_\mu f)^2 + \frac{\lambda}{4} (f^2 - f_a^2)^2 \right]$$

$$\Omega^2(f) \equiv 1 + \frac{\xi(f^2 - f_a^2)}{M_P^2}$$

The value of the wormhole action depends on the formulation of gravity.

- Metric formulation

$$\Gamma^\lambda_{\mu\nu} = \frac{1}{2} g^{\lambda\alpha} (g_{\mu\alpha,\nu} + g_{\alpha\nu,\mu} - g_{\mu\nu,\alpha})$$

Levi-Civita connection

- Palatini formulation

$\Gamma^\lambda_{\mu\nu}$ and $g_{\mu\nu}$ are independently introduced in the action.

EoM →

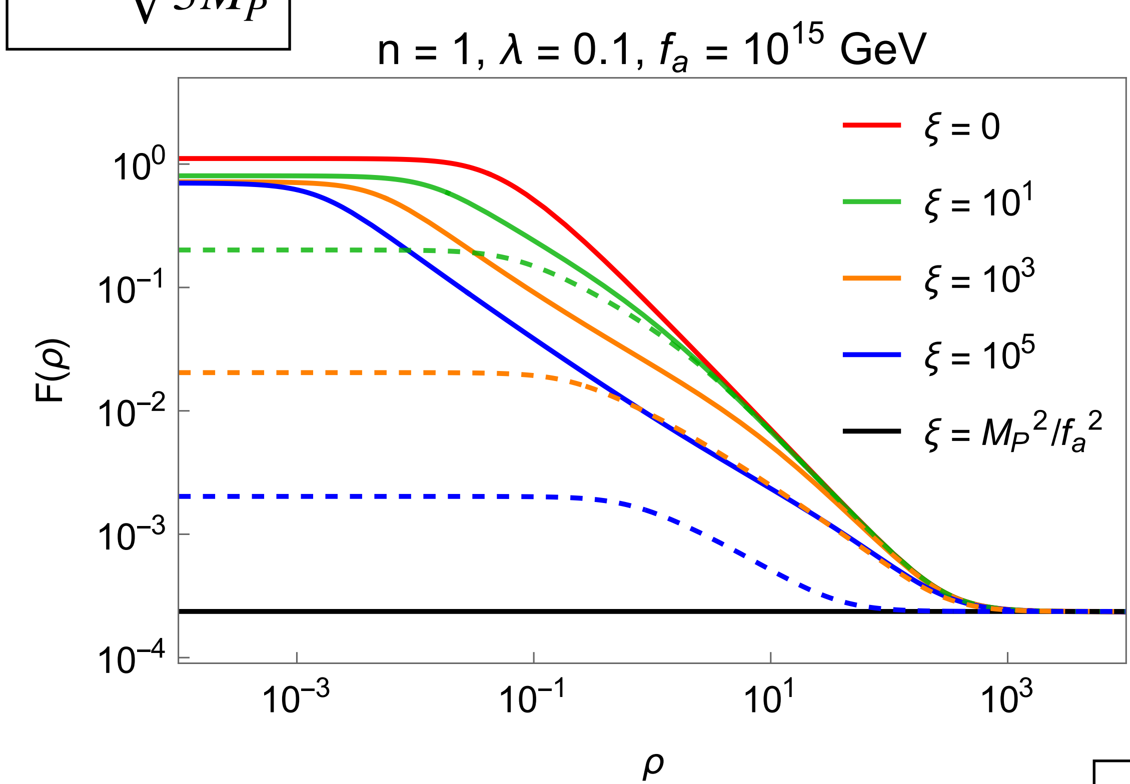
$$\Gamma^\lambda_{\mu\nu} = \frac{1}{2} g^{\lambda\alpha} (g_{\mu\alpha,\nu} + g_{\alpha\nu,\mu} - g_{\mu\nu,\alpha}) + \underbrace{\delta^\lambda_\mu \partial_\nu \omega + \delta^\lambda_\nu \partial_\mu \omega - g_{\mu\nu} \partial^\lambda \omega}_{\text{additional terms}}$$

$$\omega(f) \equiv \log \sqrt{\Omega^2(f)}$$

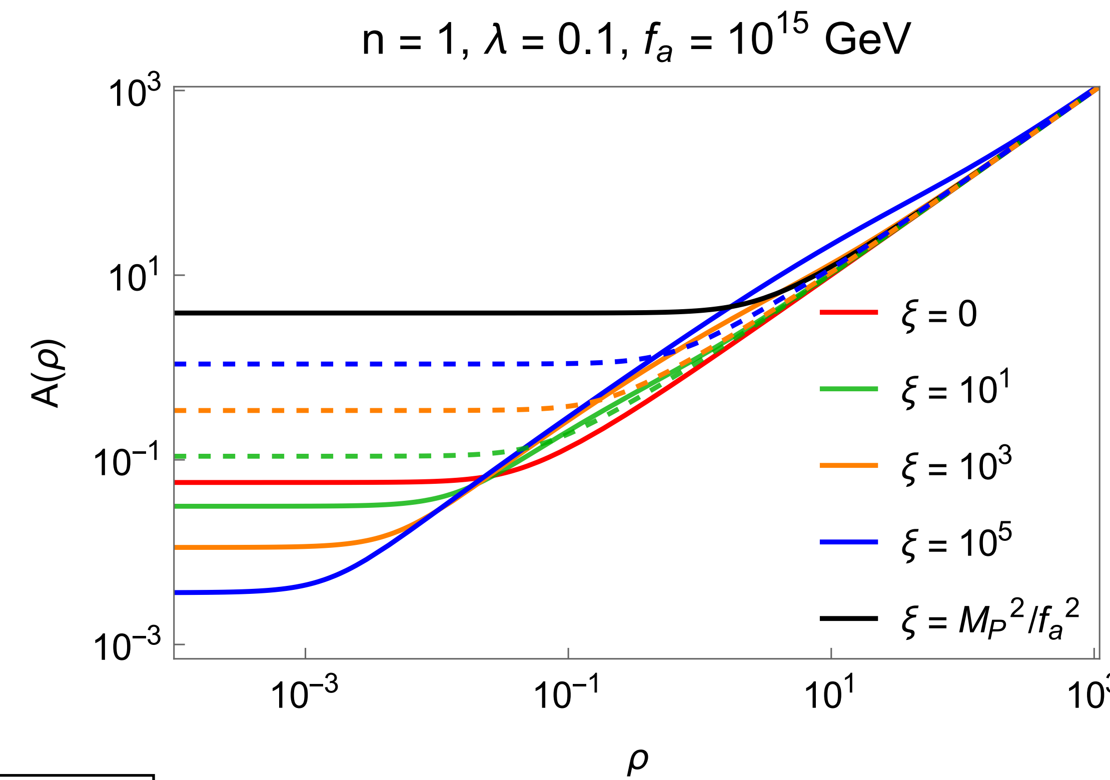
Wormhole solution

Palatini (solid)
Metric (dashed)

$$F \equiv \frac{f}{\sqrt{3}M_P}$$



$$A \equiv \sqrt{3\lambda}M_P a$$

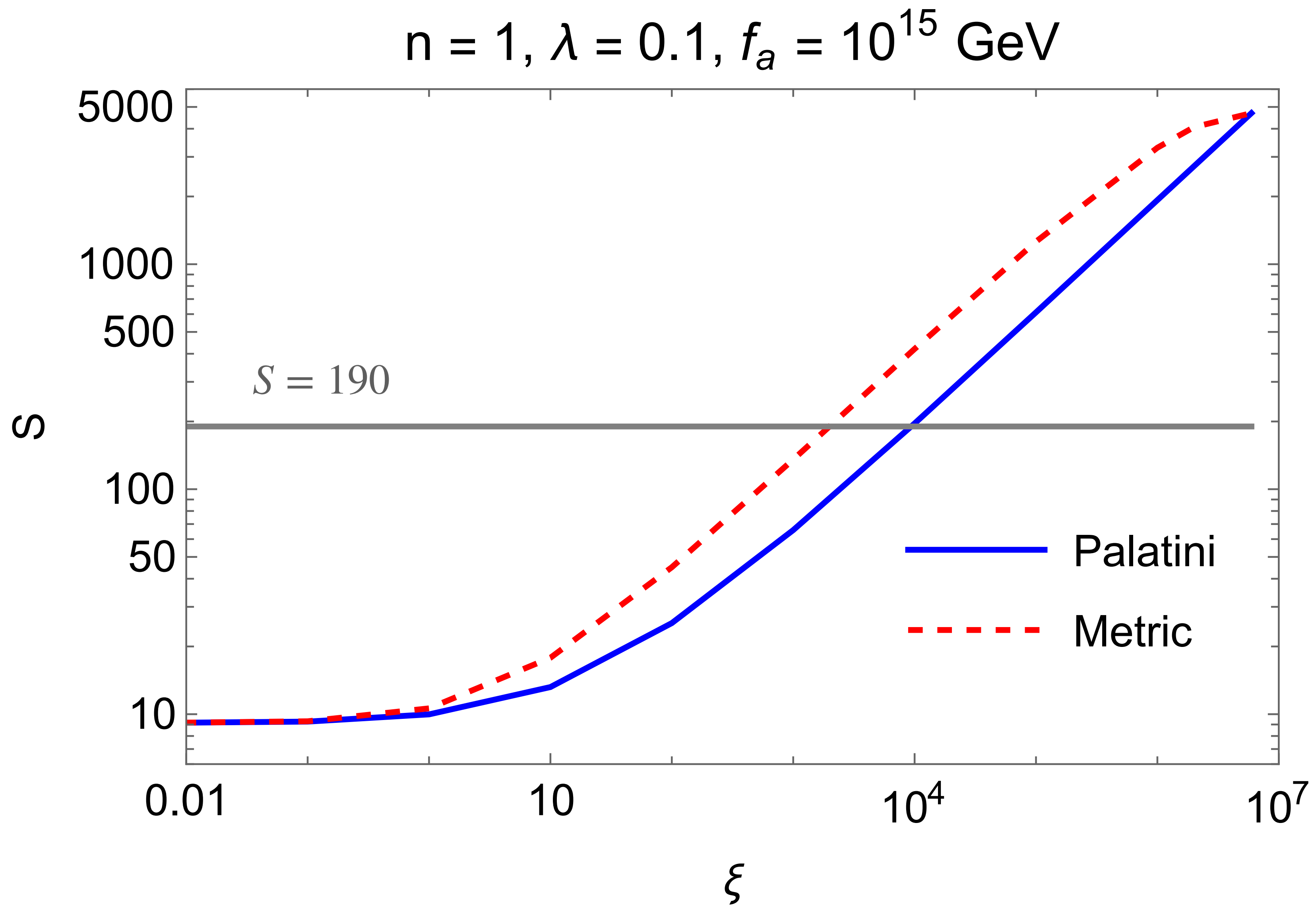


$$\rho \equiv \sqrt{3\lambda}M_P r$$

- The case of $\xi = M_P^2/f_a^2$ (black) is identical to the Giddings-Strominger solution.
- For intermediate values of ξ, the solutions are quite different between the Palatini and metric formulations.

K. Hamaguchi, **YK** and N. Nagata, Phys. Rev. D 105, 076008 (2022), arXiv:2108.13245;
D. Y. Cheong, K. Hamaguchi, **YK**, S. M. Lee, N. Nagata and S. C. Park, arXiv:2210:11330.

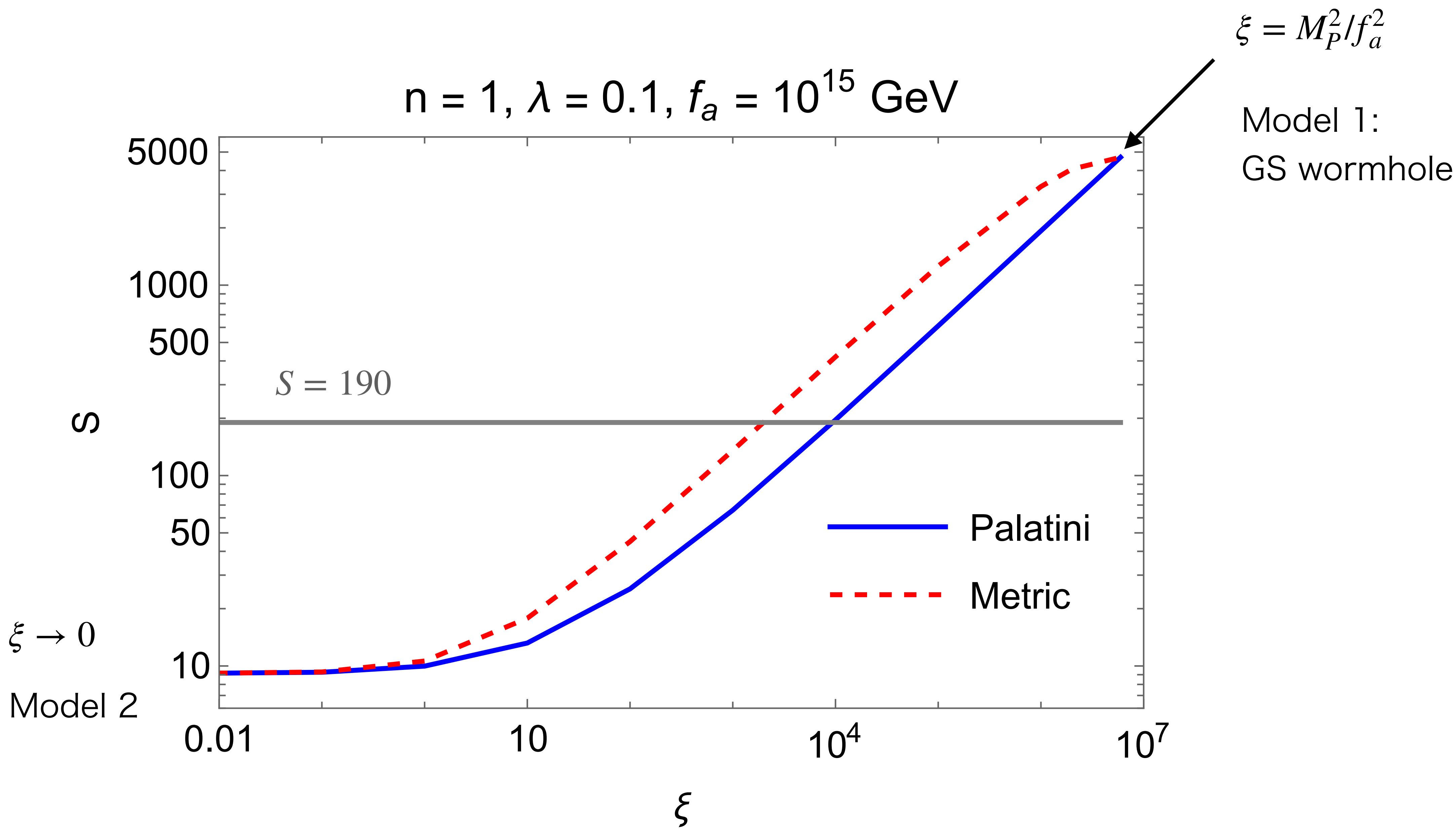
Wormhole action



- To avoid the quality problem

→ Metric: $\xi \gtrsim 2 \times 10^3$, Palatini: $\xi \gtrsim 1 \times 10^4$

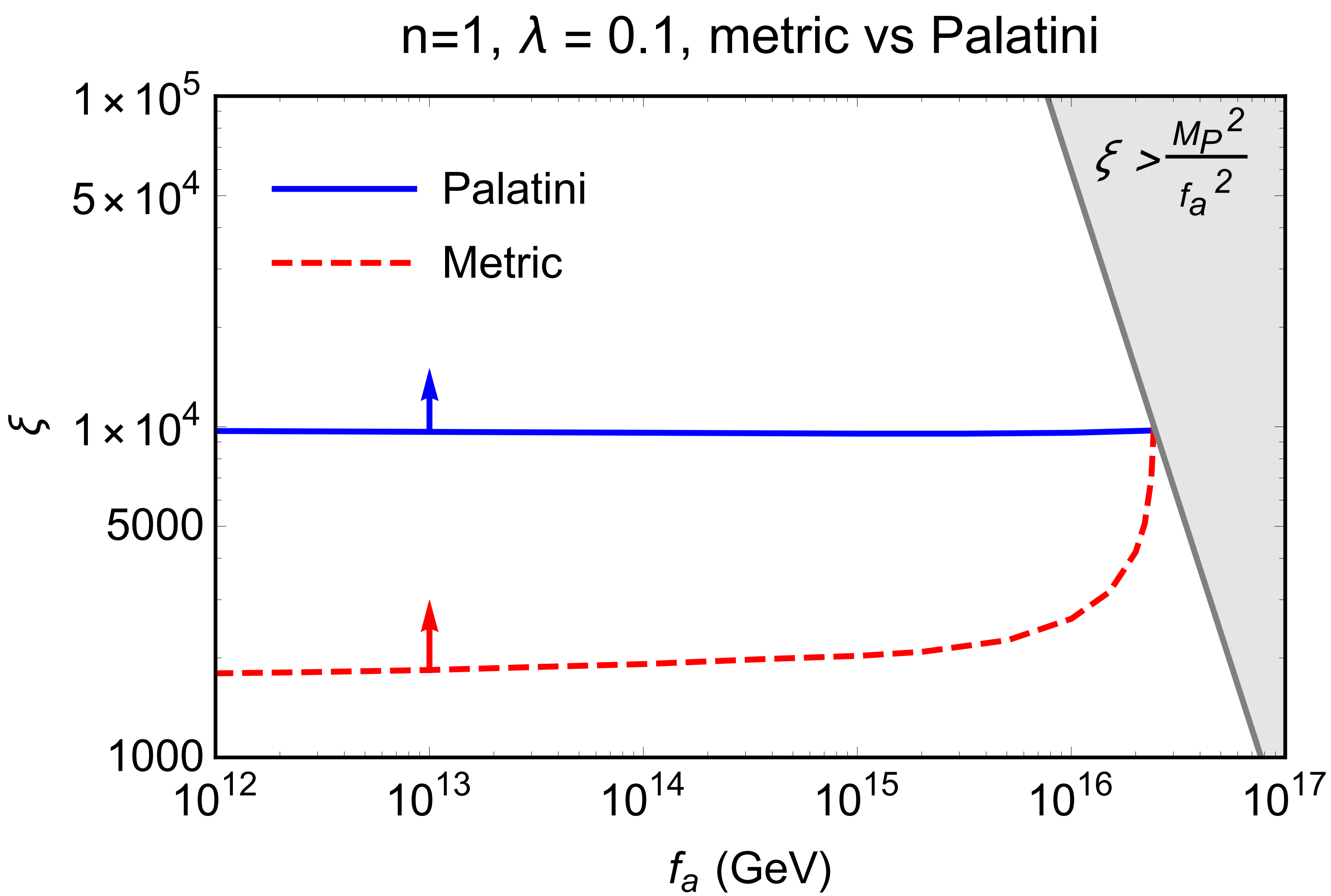
Wormhole action



- To avoid the quality problem

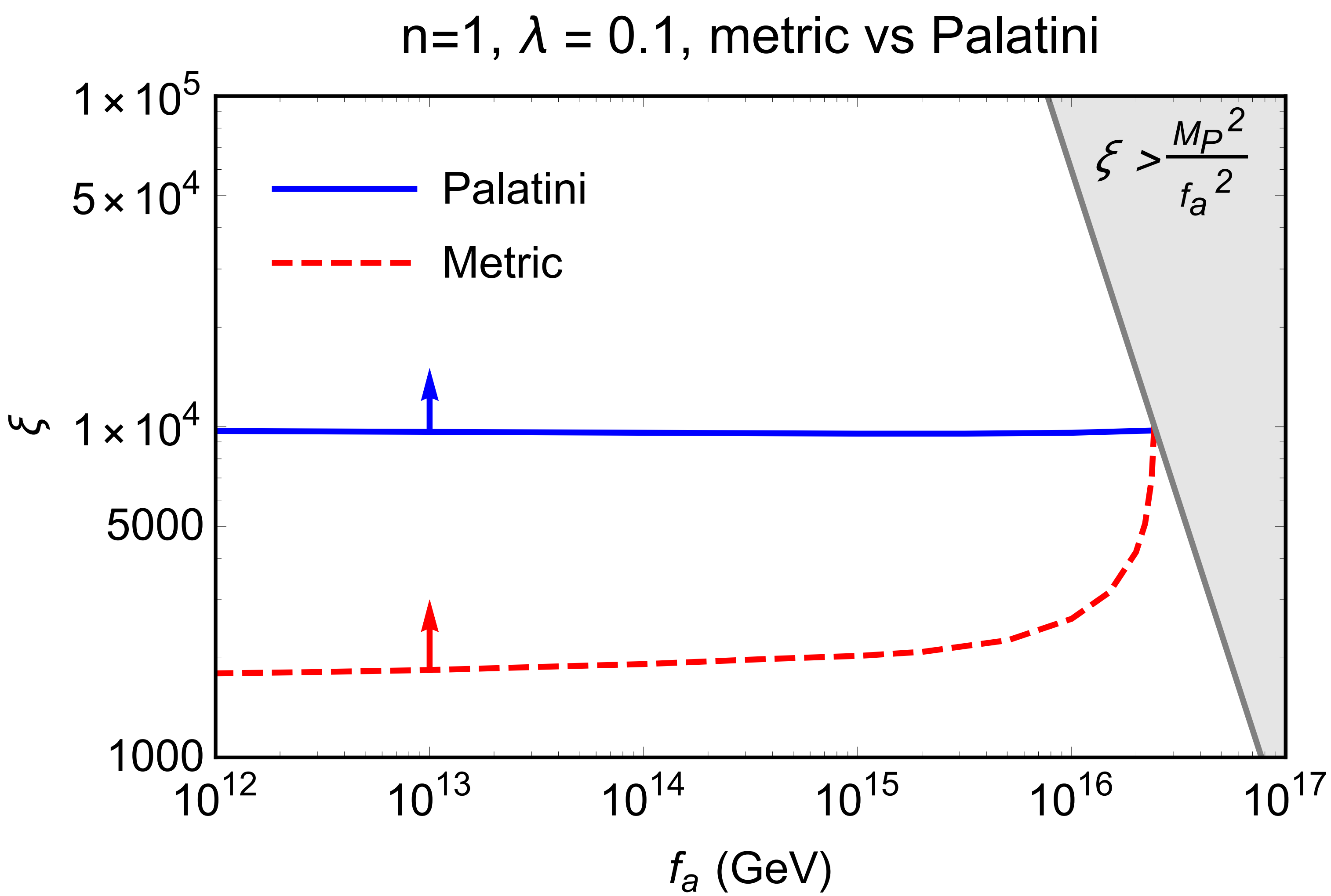
⟶ Metric: $\xi \gtrsim 2 \times 10^3$, Palatini: $\xi \gtrsim 1 \times 10^4$

Parameter region to avoid the quality problem



- The quality problem can be avoided for $f_a \lesssim 2.5 \times 10^{16}$ GeV.

Parameter region to avoid the quality problem



- The quality problem can be avoided for $f_a \lesssim 2.5 \times 10^{16}$ GeV.

- Our model can realize **inflation** where the radial field f plays the role of inflaton.

CMB constraint $\log(10^{10} A_s) = 3.044 \pm 0.014$ Planck 2018

$\rightarrow$

$\xi = 4.9 \times 10^4 \sqrt{\lambda}$ (metric)

$\xi = 1.4 \times 10^{10} \lambda$ (Palatini)

for 60 e-folding

Summary

- Axionic wormholes can spoil the PQ solution to the strong CP problem.
- The value of the wormhole action highly depends on models.
- The PQ violation by wormholes is sufficiently suppressed by a large value of the non-minimal gravitational coupling for both the metric and the Palatini formulations.

Summary

- Axionic wormholes can spoil the PQ solution to the strong CP problem.
- The value of the wormhole action highly depends on models.
- The PQ violation by wormholes is sufficiently suppressed by a large value of the non-minimal gravitational coupling for both the metric and the Palatini formulations.

Thank you for your attention!

Backup

Procedures to get correct EoMs

1. Search saddle point solutions in pure imaginary direction of axion, $\theta \to i\theta$

$$S = \int d^4x \sqrt{g} \left[-\frac{M_P^2}{2} R - \frac{f_a^2}{2} (\partial_\mu \theta)^2 \right]$$

We need to add a boundary term σ when computing the transition amplitude.

$$S + \sigma = \int d^4x \sqrt{g} \left[-\frac{M_P^2}{2} R + \frac{f_a^2}{2} (\partial_\mu \theta)^2 \right] \qquad \sigma = \int d^3x \ J^0 \theta = \int d^4x \ J^\mu \partial_\mu \theta$$

J^μ : conserved current

2. Introduce J^μ as a variable of path integral, and θ as a Lagrange multiplier

$$S = \int d^4x \sqrt{g} \left[-\frac{M_P^2}{2} R + \frac{1}{2 g f_a^2} J^\mu J_\mu + \frac{1}{\sqrt{g}} \theta (\partial_\mu J^\mu) \right]$$

Variation in terms of θ $\longrightarrow$ $\partial_\mu J^\mu = 0$

Variation in terms of J^μ $\longrightarrow$ $J^\mu = \sqrt{g} g^{\mu\nu} f_a^2 \partial_\nu \theta$

S. R. Coleman and K. Lee, Nucl. Phys. B 329 (1990) 387.

Palatini formulation

- $\Gamma^{\lambda}_{\mu\nu}$ and $g_{\mu\nu}$ are independently introduced in the action.

- The Ricci tensor $R_{\mu\nu}$ is given by an explicit function of the affine connection $\Gamma^{\lambda}_{\mu\nu}$.

$$R_{\mu\nu}(\Gamma) = \partial_\mu \Gamma^{\lambda}_{\lambda\nu} - \partial_\lambda \Gamma^{\lambda}_{\mu\nu} + \Gamma^{\lambda}_{\mu\sigma}\Gamma^{\sigma}_{\lambda\nu} - \Gamma^{\sigma}_{\mu\nu}\Gamma^{\lambda}_{\lambda\sigma}$$

- $\Gamma^{\lambda}_{\mu\nu}$ is generally different from one in the metric formulation beyond the minimal gravity.

Example

$$S = \int d^4x \sqrt{g} \left[-\frac{M_P^2}{2} \Omega^2 g^{\mu\nu} R_{\mu\nu}(\Gamma) + \cdots \right]$$

Ω^2 is a function of matter fields.

$\xrightarrow{\text{EoM}}$ $\nabla_\lambda [M_P^2 \Omega^2 \sqrt{g} g^{\mu\nu}] = 0$ or $\Gamma^{\lambda}_{\mu\nu} = \frac{1}{2} g^{\lambda\alpha}(g_{\mu\alpha,\nu} + g_{\alpha\nu,\mu} - g_{\mu\nu,\alpha}) + \delta^{\lambda}_{\mu}\partial_\nu \omega + \delta^{\lambda}_{\nu}\partial_\mu \omega - g_{\mu\nu}\partial^{\lambda}\omega$ $\quad \omega \equiv \log\sqrt{\Omega^2}$

Action in Jordan and Einstein frames

- Jordan frame action

$$S = \int d^4x \sqrt{g} \left[-\frac{M_P^2}{2}\Omega^2(f)R + \frac{f^2}{2}(\partial_\mu\theta)^2 + \frac{1}{2}(\partial_\mu f)^2 + V \right] \qquad \Omega^2(f) \equiv 1 + \frac{\xi(f^2 - f_a^2)}{M_P^2}$$

↓ Weyl transformation $\quad g_{\mu\nu} \to \Omega^{-2} g_{\mu\nu}$

- Einstein frame action

$$S_E = \int d^4x\sqrt{g} \left[-\frac{M_P^2}{2}\left(R - \zeta \frac{3(\partial_\mu \Omega^2)^2}{2\Omega^4} \right) + \frac{1}{2\Omega^2}(\partial_\mu f)^2 + \frac{f^2}{2\Omega^2}(\partial_\mu\theta)^2 + \frac{V}{\Omega^4} \right]$$

$$= \int d^4x\sqrt{g} \left[-\frac{M_P^2}{2}R + \frac{1}{2}(\partial_\mu\phi)^2 + \frac{f(\phi)^2}{2\Omega^2(f(\phi))}(\partial_\mu\theta)^2 + \frac{V(f(\phi))}{\Omega^4(f(\phi))} \right]$$

$\zeta = 0$ Palatini

$\zeta = 1$ metric

$$\frac{d\phi}{df} = \sqrt{\frac{1}{\Omega^2} + \zeta\frac{6\xi^2 f^2}{M_P^2\Omega^4}}$$

ϕ : canonically normalized field in Einstein frame

- $\xi = M_P^2/f_a^2 \qquad \frac{f^2}{2\Omega^2}(\partial_\mu\theta)^2 = \frac{f_a^2}{2}(\partial_\mu\theta)^2$

Axion θ decouples to the radial field f. ⟶ **Giddings-Strominger wormhole**

Equations for Palatini formulation

$$\Omega^2(f) \equiv 1 + \frac{\xi(f^2 - f_a^2)}{M_P^2}, \quad \omega(f) \equiv \log\sqrt{\Omega^2(f)}$$

- Variation with respect to $g_{\mu\nu}$

$$\Omega^2(a^{'\,2} - 1 + 2aa'\omega' + a^2\omega^{'\,2}) = -\frac{a^2}{3M_P^2}\left[-\frac{1}{2}f^{'\,2} + V(f) + \frac{n^2}{8\pi^4 f^2 a^6}\right]$$

$$\Omega^2(2aa'' + a^{'\,2} - 1 + 4aa'\omega' + a^2\omega^{'\,2} + 2a^2\omega'') = -\frac{a^2}{M_P^2}\left[\frac{1}{2}f^{'\,2} + V(f) - \frac{n^2}{8\pi^4 f^2 a^6}\right]$$

- Variation with respect to f

$$f'' + 3\frac{a'}{a}f' = \frac{dV}{df} - \frac{n^2}{4\pi^4 f^3 a^6} + 6\xi f\left[\frac{a'' + a^{'\,2} - 1}{a^2} + \omega^{'\,2} + \omega'' + 3\frac{a'}{a}\omega'\right]$$

Equations for metric formulation

$$\Omega^2(f) \equiv 1 + \frac{\xi(f^2 - f_a^2)}{M_P^2}$$

- Variation with respect to $g_{\mu\nu}$

$$\Omega^2(a'^2 - 1) + \frac{2\xi}{M_P^2} a a' f f' = -\frac{a^2}{3M_P^2}\left[-\frac{1}{2}f'^2 + V(f) + \frac{n^2}{8\pi^4 f^2 a^6}\right]$$

$$\Omega^2(2aa'' + a'^2 - 1) + \frac{2\xi a^2}{M_P^2}\left[ff'' + f'^2 + 2\frac{a'}{a}ff'\right] = -\frac{a^2}{M_P^2}\left[\frac{1}{2}f'^2 + V(f) - \frac{n^2}{8\pi^4 f^2 a^6}\right]$$

- Variation with respect to f

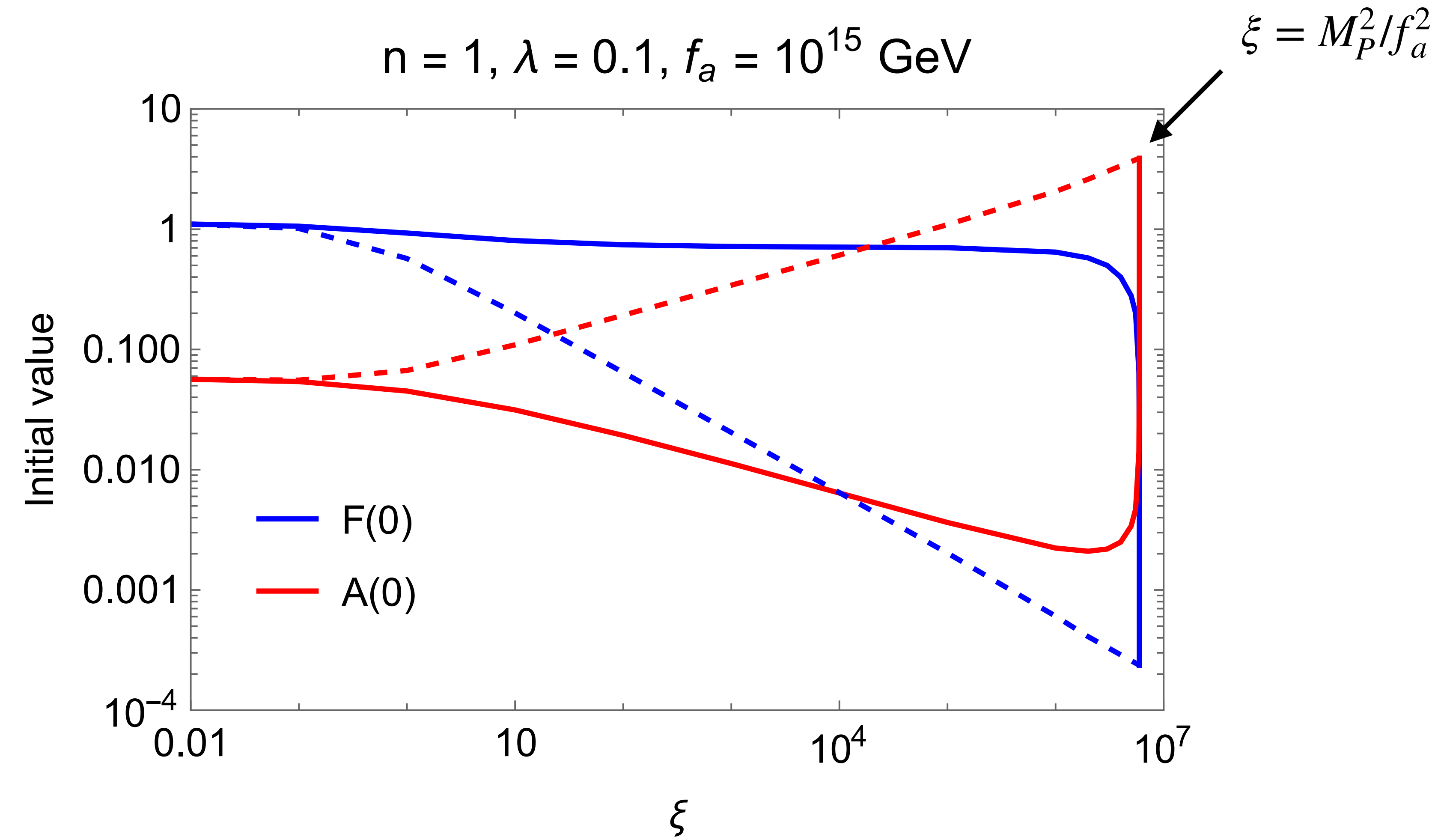
$$f'' + 3\frac{a'}{a}f' = \frac{dV}{df} - \frac{n^2}{4\pi^4 f^3 a^6} + 6\xi f\left[\frac{a'' + a'^2 - 1}{a^2}\right]$$

Wormhole solution

$$F(0) = \frac{f(0)}{\sqrt{3}M_P}$$

$$A(0) = \sqrt{3\lambda}M_P a(0)$$

Palatini (solid)
Metric (dashed)

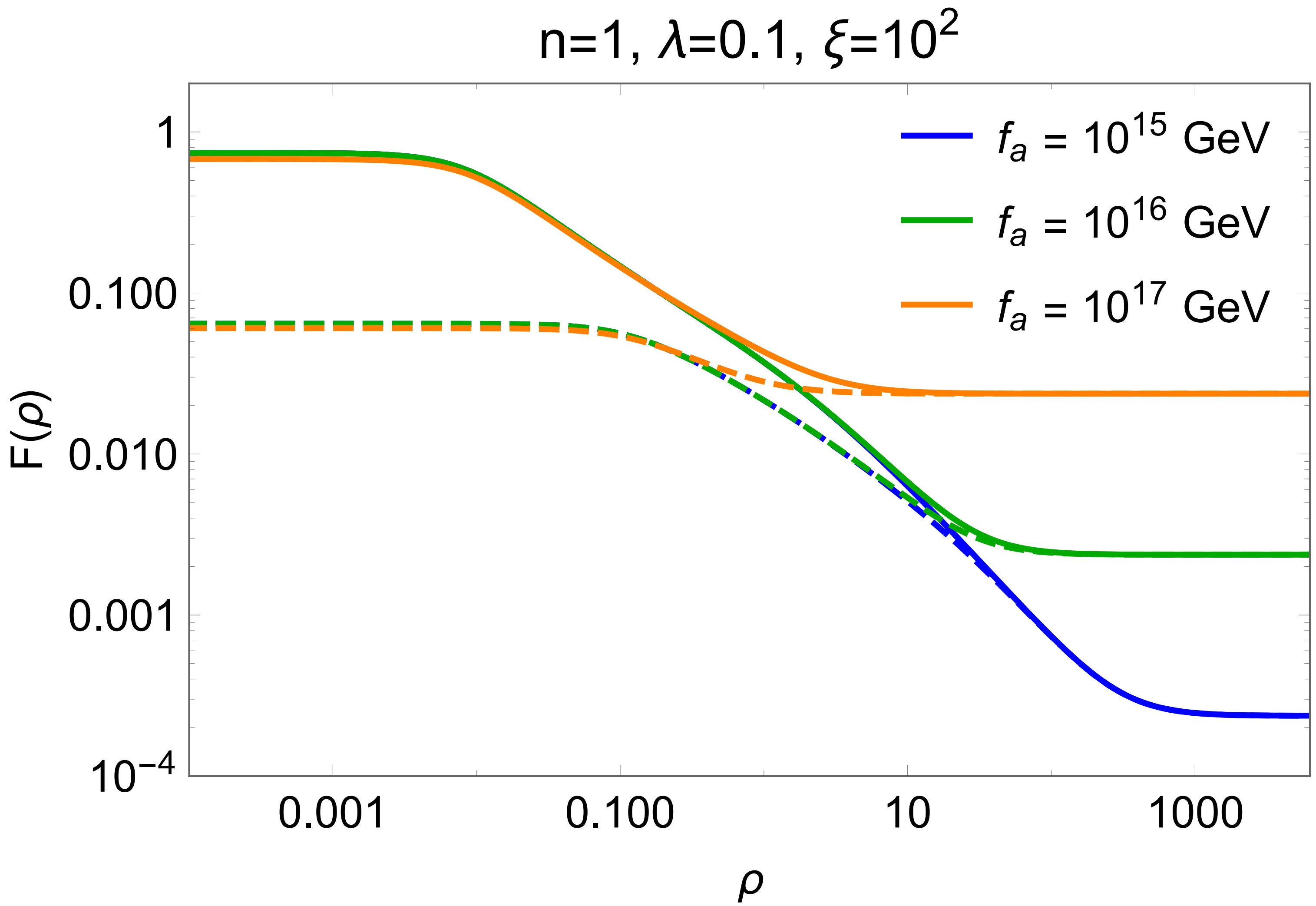


- For the Palatini case (solid), the initial values sharply vary close to $\xi = M_P^2/f_a^2$.

Wormhole solution

$$F \equiv \frac{f}{\sqrt{3}M_P}$$

Palatini (solid)
Metric (dashed)



$$\rho \equiv \sqrt{3\lambda}M_P r$$

- The solution hardly depends on f_a close to the wormhole throat.

Values for $\xi \gg 1$

Initial values

Metric $f(0) \sim \xi^{-\frac{1}{2}} M_P$

$a(0) \sim n^{\frac{1}{2}} \xi^{\frac{1}{4}} M_P^{-1}$

Palatini $f(0) \sim M_P$

$a(0) \sim n^{\frac{1}{2}} \xi^{-\frac{1}{4}} M_P^{-1}$

Wormhole action

$$S = 2\pi^2 \int dr \left[a^3 \frac{f^2}{2} \theta'^{\,2} + \cdots \right] = 2\pi^2 \int dr \left[\frac{n^2}{8\pi^4 a^3 f^2} + \cdots \right] \sim \frac{n^2}{a(0)^2 f(0)^2} \sim n \xi^{\frac{1}{2}}$$

Perturbative unitarity cut-off in the Jordan frame

$$\Lambda_J \left(f \ll \frac{M_P}{\sqrt{\xi}} \right) \simeq \begin{cases} M_P/\xi & \text{(metric)} \\ M_P/\sqrt{\xi} & \text{(Palatini)} \end{cases}$$

$$\Lambda_J \left(f \gg \frac{M_P}{\sqrt{\xi}} \right) \simeq \begin{cases} \xi f^2/M_P & \text{(metric)} \\ \sqrt{\xi} f^2/M_P & \text{(Palatini).} \end{cases}$$

$a(0)^{-1} < \Lambda_J$ is satisfied for both formulations.

F. Bezrukov, A. Magnin, M. Shaposhnikov, and S. Sibiryakov, JHEP 01, 016 (2011);
I. Antoniadis, A. Guillen, and K. Tamvakis, JHEP 08, 018 (2021), [Addendum: JHEP 05, 074 (2022)].

Gibbons-Hawking-York boundary term

$$S_{\rm GHY} = -M_P^2 \int_{\partial V} d^3x \sqrt{\tilde{g}}\,\Omega^2(f)(K - K_0)$$

$$= -M_P^2 \int_{\partial V} d\Omega_3\ \Omega^2(f) a^3 \frac{3(a' - 1)}{a}$$

$$= -6\pi^2 M_P^2 a(0)^2 \Omega^2(f(0))$$

$\tilde{g}$: Metric on 3d sphere

K : Extrinsic curvature

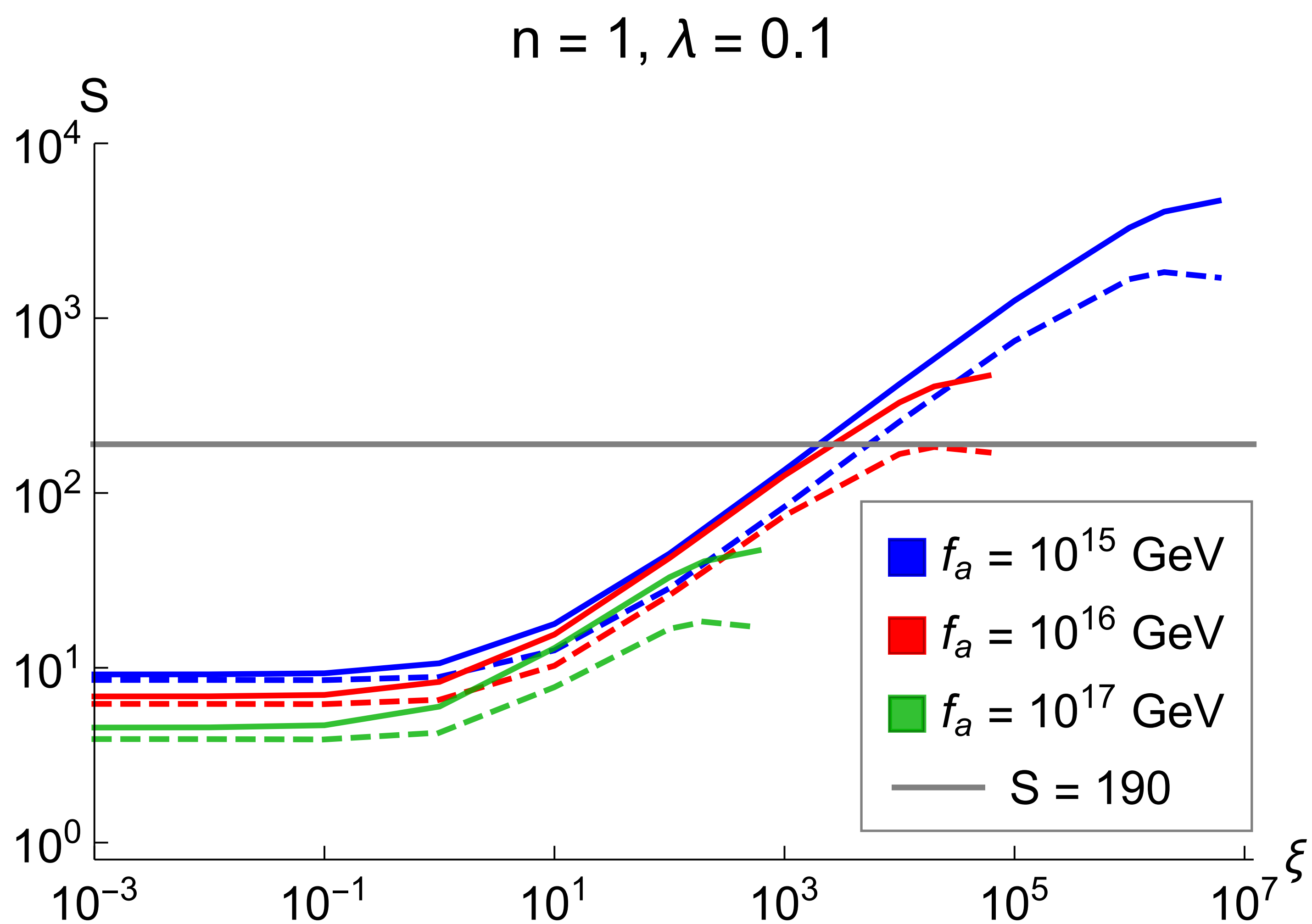
K_0 : Remove divergence of K at $r \to \infty$

- Remove the 2nd-order derivative in the Ricci scalar to be consistent with the uncertainty principle.

- The value of the GHY term is negative.

J. W. York, Jr., Phys. Rev. Lett. 28, 1082 (1972);
G. W. Gibbons and S. W. Hawking, Phys. Rev. D 15, 2752 (1977).

Wormhole action (metric)



Without GHY (solid)
With GHY (dashed)

- The value of the action is smaller for the case with the Gibbons-Hawking-York term.

- $f_a \lesssim 9.0 \times 10^{15}$ GeV and $\xi \gtrsim 5 \times 10^3$ is needed to solve the quality problem.