Multiphonon Processes in Spin-Dependent Dark-Matter Scattering

BETHANY SUTER, UC BERKELEY

BERKELEY WEEK

IN COLLABORATION WITH P. MUNBODH, S. KNAPEN, S. GORI, & T. LIN

Outline

1. The experimental outlook
2. Why phonons + DM
3. Derivation of Multiphonon scattering
4. Results!

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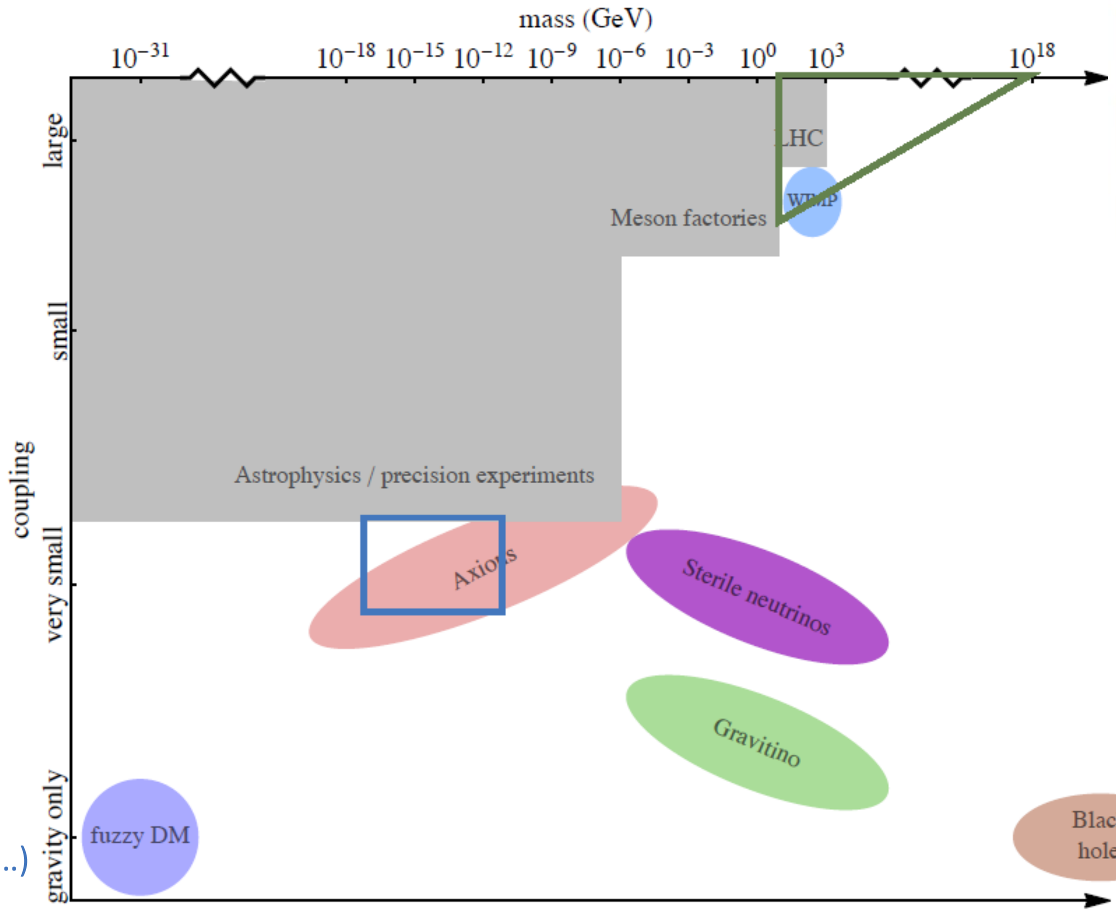
The Dark Matter Landscape

Resonant cavities (ADMX, MADMAX, DMradio…)



Nuclear recoils (LZ, XENON1T, …)

Figure by S. Knapen

The Dark Matter Landscape

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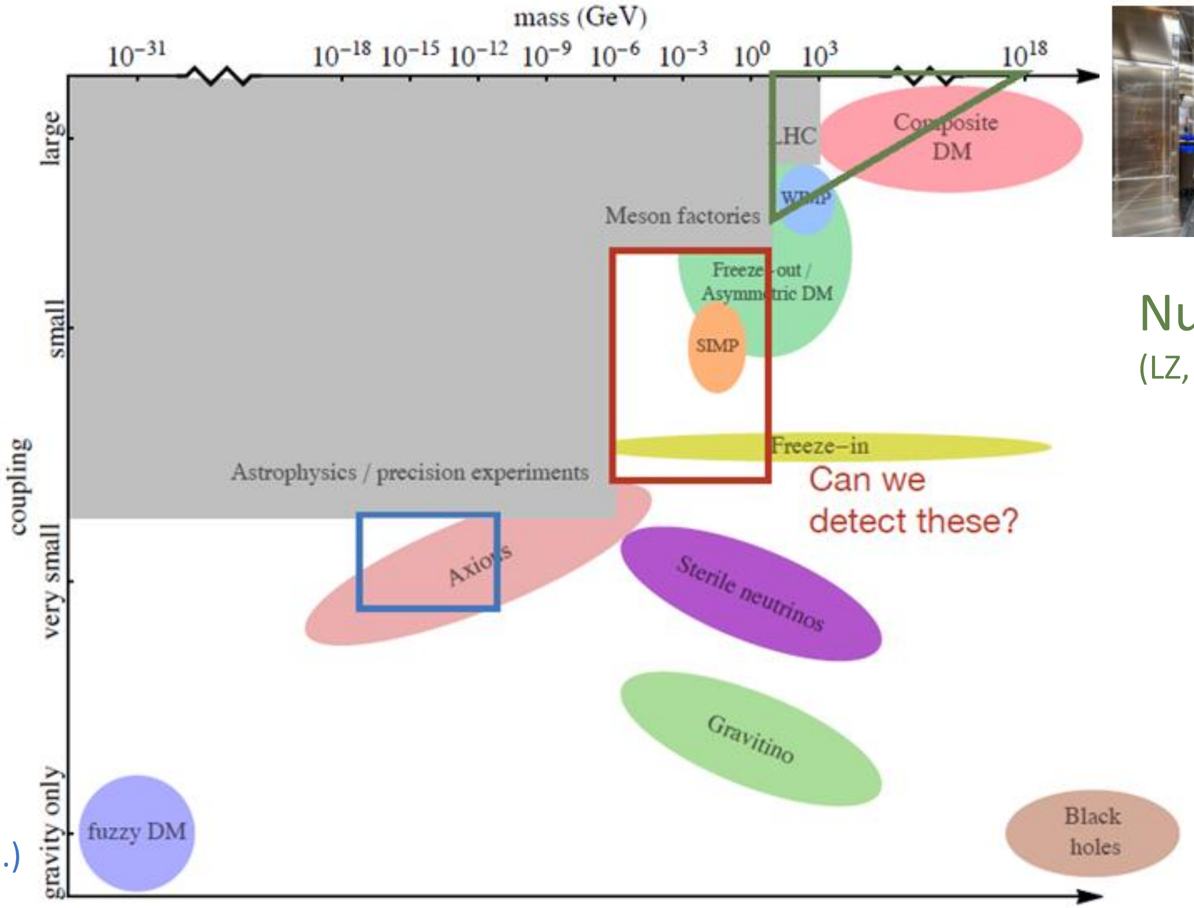


Can we detect these?

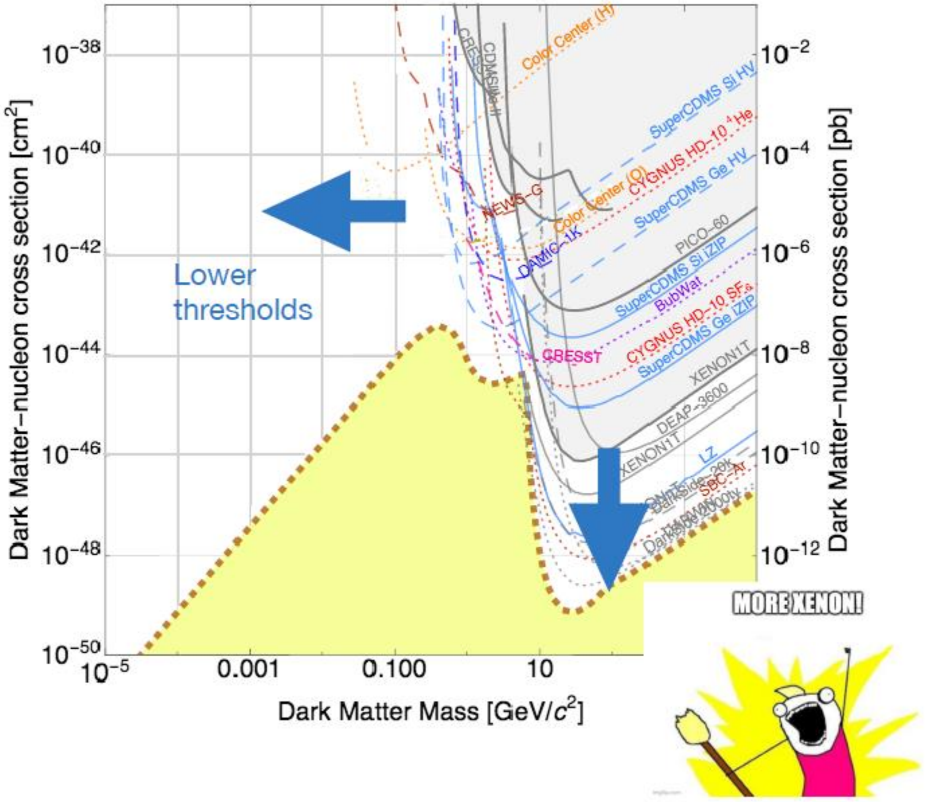
Nuclear recoils

(LZ, XENON1T, …)

Figure by S. Knapen

Light Dark Matter Direct Detection

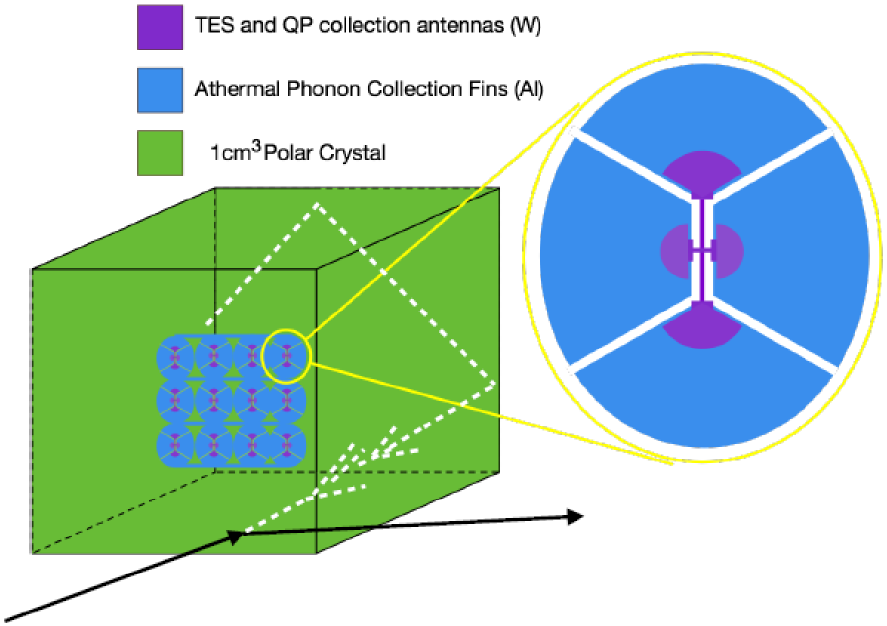
What do we need?



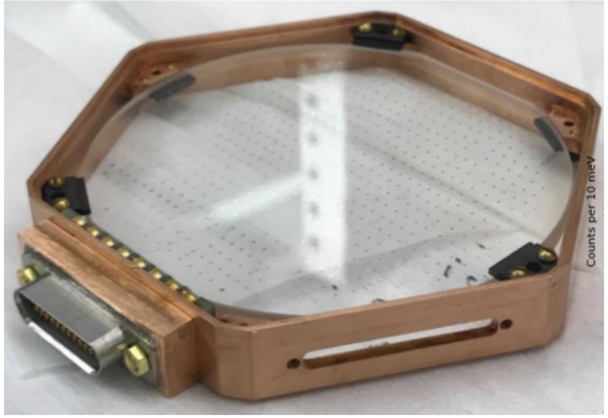
M. Battaglieri: 1707.04591
Meme credit: S. Knapen

Phonon Detector: SPICE

SPICE / HeRALD
TESSERACT



3’’ sapphire detector



- Polar Materials: GaAs or Sapphire
- Scintillation & phonons
  - Background discrimination!
- Low energy TES

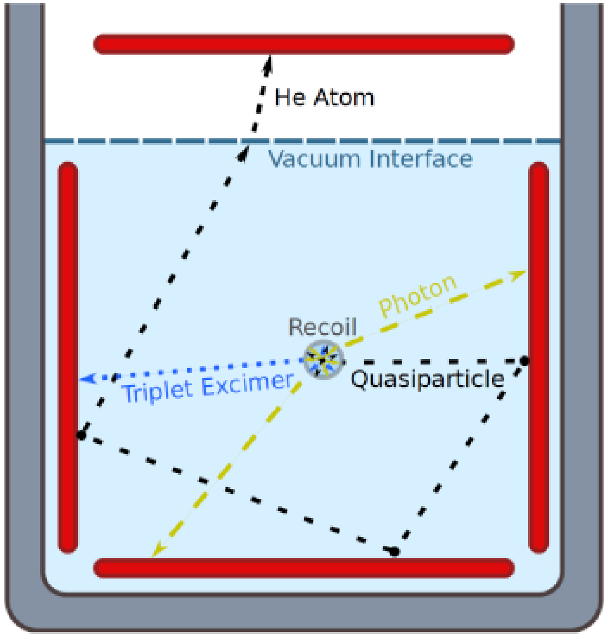
~ 10 meV threshhold

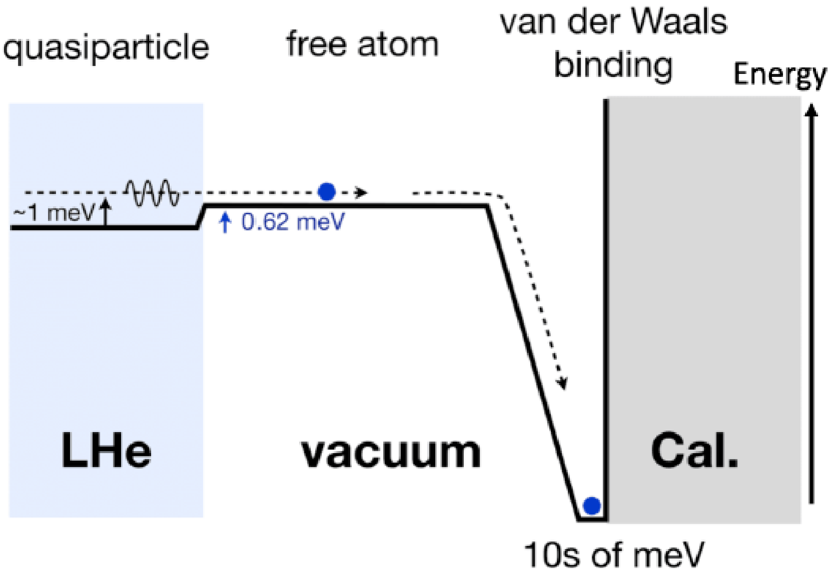
Figure from M. Pyle

Picture from TESSERACT Website

Phonon Detector: HeRALD

SPICE / HeRALD
TESSERACT





- Calorimeters with TES readout
- Quantum evaporation of He atoms

S. Hertel, A. Biekert, J.Lin, V.Velan, & D. McKinsey arXiv:1810.06283
Figures from J. Lin slides

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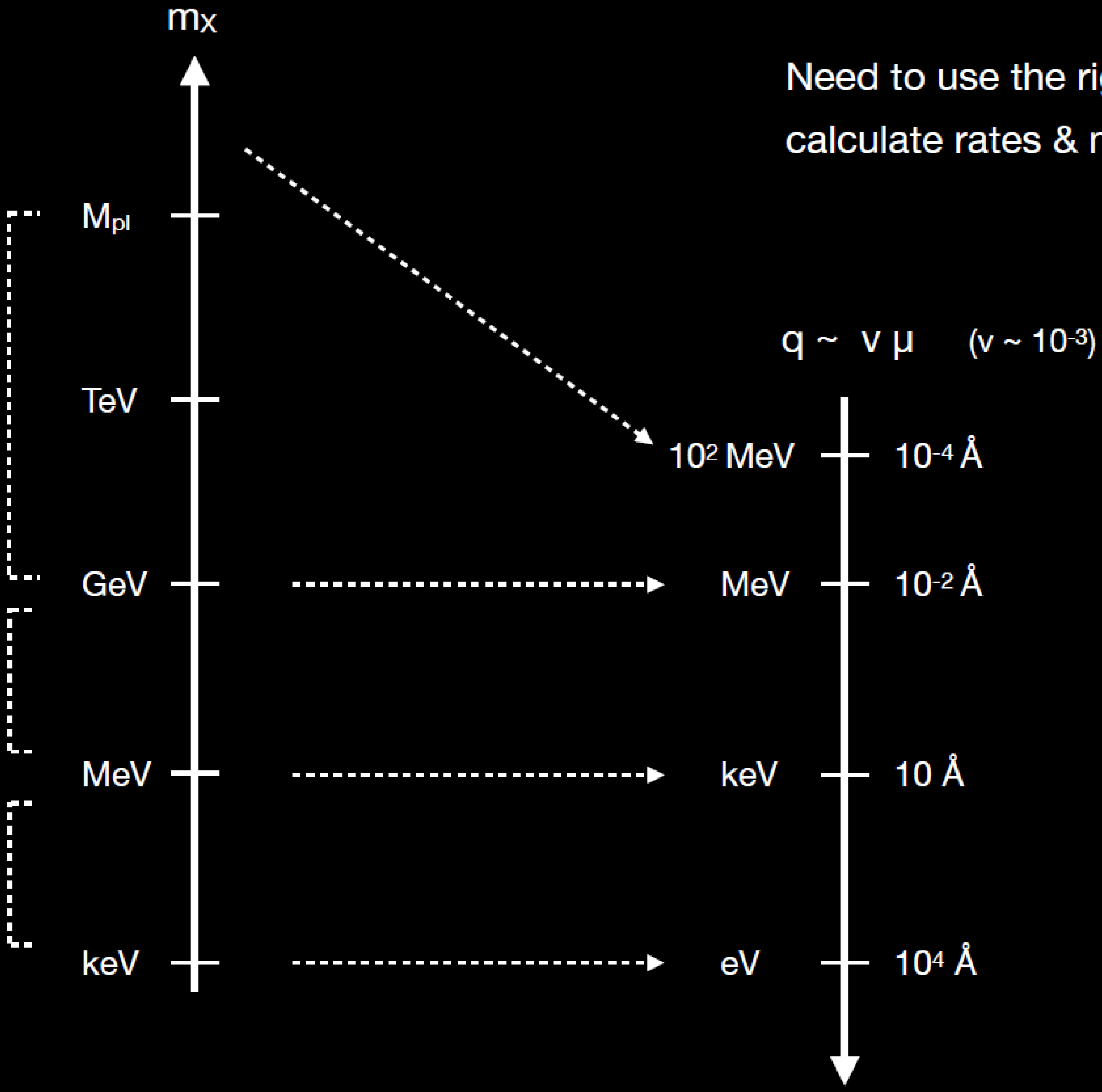
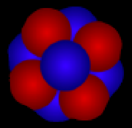
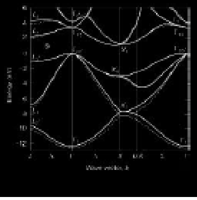
The need for theory

Need to use the right effective theory to calculate rates & match it to the UV theory

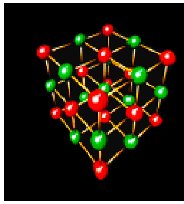




Nuclear recoil



Electronic structure



Phonon excitations

Slide by S. Knapen

Phonons

Very complicated system of coupled harmonic oscillators

Acoustic Phonons

- Coherent motion of the lattice atoms
- Wavelength → 0 corresponds to displacement of the crystal

Optical Phonons

- Out of phase motion of the lattice atoms
- If material is polar, couples to EM field

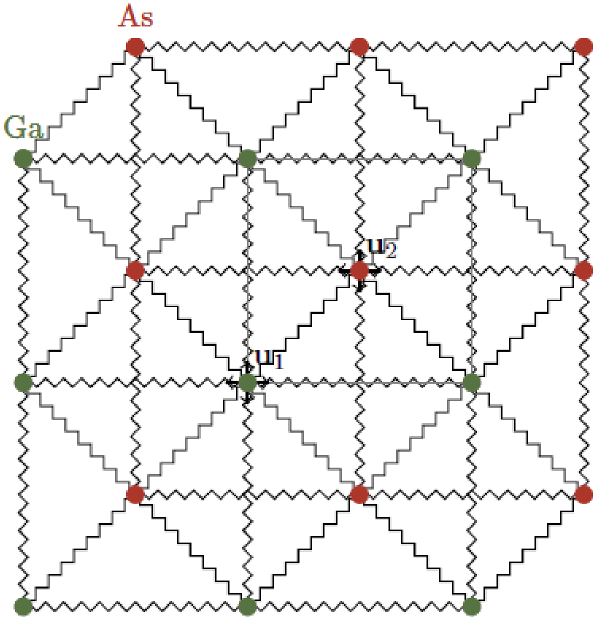


Figure from S. Knapen

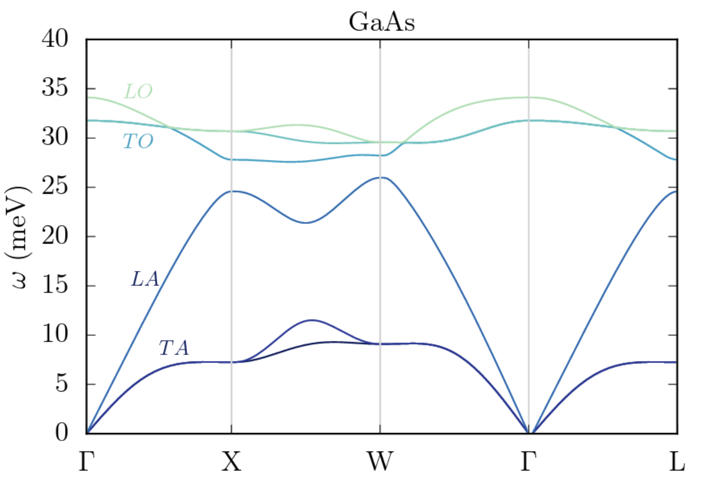
Acoustic vs Optical

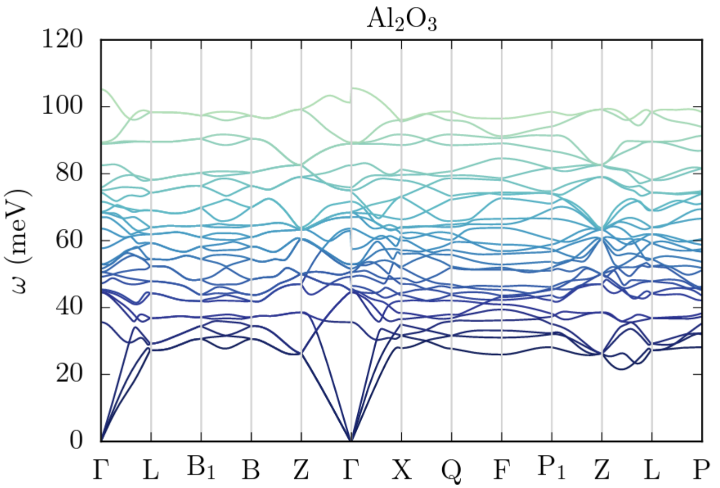
- Low momentum transfer = only excite acoustic phonon in 1st Brillouin zone

$$\omega = c_s|q| \approx 2c_s v m_X \sim 7\ meV \times \frac{m_X}{100keV}$$

- Best threshold is in 10 – 100 meV range, so difficult or impossible to detect
- Optical modes don’t have this scaling:

$$\omega \sim 30\ meV \text{ as } |q| \to 0$$



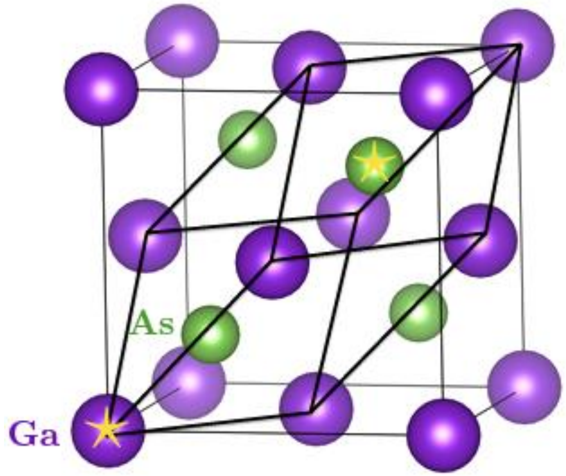


Figures from arXiv:1807.10291

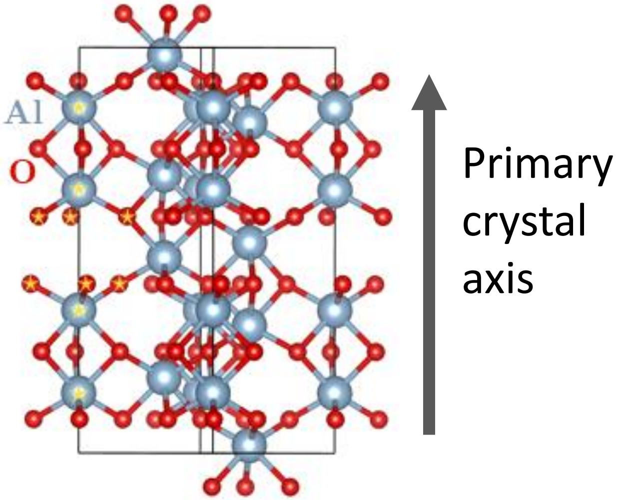
Polar Materials

- At least two different atoms with **different** effective charges
- Each unit cell forms an electric dipole
- E field or dark photon causes vibrations
  - → Optical phonons
- GaAs
  - 2 atoms in unit cell
  - 3 acoustic phonons, 3 optical phonons
- Sapphire
  - 10 atoms in unit cell
  - 3 acoustic phonons, 27 optical phonons

GaAs



Al_2O_3 (Sapphire)



Figures from arXiv:1807.10291

Benefits of Polar Materials

- Gapped dispersion of optical phonons
  - Single or multiphonon
- Anisotropic crystal structures
  - Daily modulation in rate
- Low screening
  - Required: few free electrons, high polarizability
  - Gap for electronic excitations $\sim O(1 - 10\ eV)$
  - Kinetic mixing with dark photon couples to dipole moment
- Easy to fabricate

S. Griffin, S. Knapen, T. Lin, M. Pyle, K. Zurek: 1807.10291

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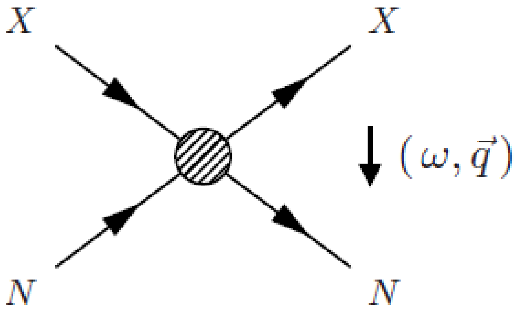
DM - Phonon EFT

Nuclear Recoil:

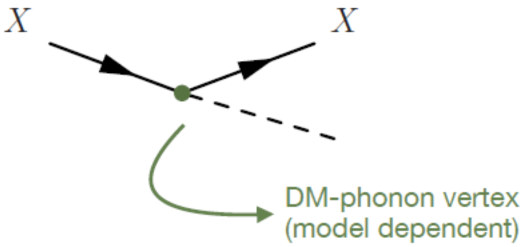
$$\omega = \frac{q^2}{2m_N}$$



Phonon Regime:

$$q \ll \sqrt{2m_N\omega}$$

- Momentum is a good expansion parameter



DM-phonon vertex (model dependent)

$\mathcal{O}(q)$, $\mathcal{O}(q^2)$ or $\mathcal{O}(q^4)$

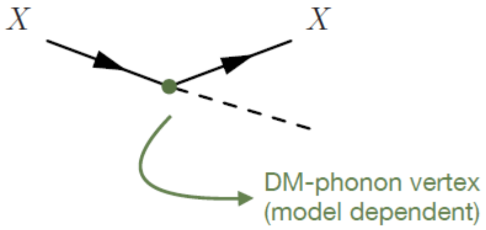
(Depends on DM model & phonon branch)

S. Knapen, T. Lin, M. Pyle, K. Zurek: 1712.06598
S. Griffin, S. Knapen, T. Lin, M. Pyle, K. Zurek: 1807.10291

Figures from S. Knapen

DM-Multiphonon Expansion

LO



DM-phonon vertex
(model dependent)

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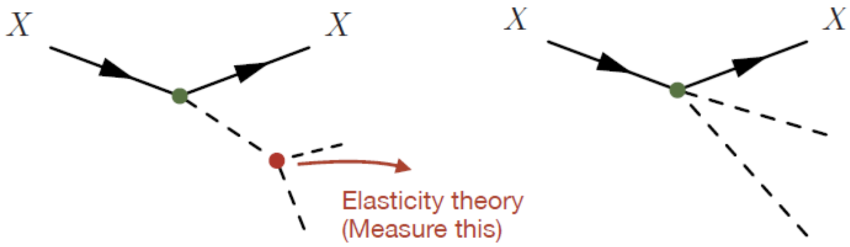
NLO

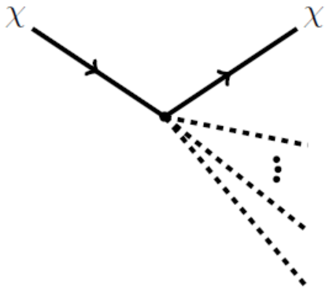


Elasticity theory
(Measure this)

$\mathcal{O}(q^4)$

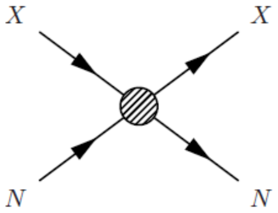
B. Campbell-Deem, P. Cox, S. Knapen, T. Lin, T. Melia: 1911.0348

N^{n}LO



$\mathcal{O}(q^{2n})$

N$^\infty$LO = nuclear recoil



$$\sim \delta\left(\omega - \frac{q^2}{2m_N}\right)$$

B. Campbell-Deem, S. Knapen, T. Lin, E. Villarama: 2205.02250

Figures from S. Knapen

Multiphonon Rate (SI)

Begin with Fermi's golden rule:

$$\frac{d\sigma}{d^3q\,d\omega} \sim \sum_{i,f} \left| \sum_{\ell}^{N} \langle \lambda_f | e^{iq\cdot r_\ell} | \lambda_i \rangle \right|^2 \delta(E_f - \omega)$$

Let's do SI first (easier)

Initial and final crystal configurations:
Spin & phonon excitations

Replace delta function:

$$\frac{d\sigma}{d^3q\,d\omega} \sim \int_{-\infty}^{\infty} dt \sum_{i,f} \sum_{\ell,\ell'}^{N} \langle \lambda_i | e^{iq\cdot r_\ell} | \lambda_f \rangle \langle \lambda_f | e^{-iq\cdot r_{\ell'}} | \lambda_i \rangle \; e^{i(E_f-\omega)t}$$

Use completeness of states:

$$\frac{d\sigma}{d^3q\,d\omega} \sim \int_{-\infty}^{\infty} dt \sum_{i} \sum_{\ell,\ell'}^{N} \langle \lambda_i | e^{iq\cdot r_\ell(0)} e^{-iq\cdot r_{\ell'}(t)} | \lambda_i \rangle \, e^{i\omega t}$$

Multiphonon Rate (SI)

How can we calculate this for an arbitrary crystal?

$$\frac{d\sigma}{d^3q\,d\omega} \sim \int_{-\infty}^{\infty} dt \sum_i \sum_{\ell,\ell'}^{N} \langle \lambda_i \underbrace{| e^{iq\cdot r_\ell(0)} e^{-iq\cdot r_{\ell'}(t)} | \lambda_i \rangle}_{?} \, e^{i\omega t}$$

Incoherent Approximation:

- Set $\ell = \ell'$ → no interference between atoms
- Good when $q \gg 2\pi/a$

Harmonic Approximation:

- Decompose into a sum of harmonic oscillators weighted by the phonon density of states
- Good for crystals with few anharmonicities

$$\langle 0 | e^{iq\cdot r_\ell(0)} e^{-iq\cdot r_\ell(t)} | 0 \rangle \sim e^{\langle 0 | q\cdot r_\ell(0)\, q\cdot r_\ell(t) | 0 \rangle} + \cdots$$

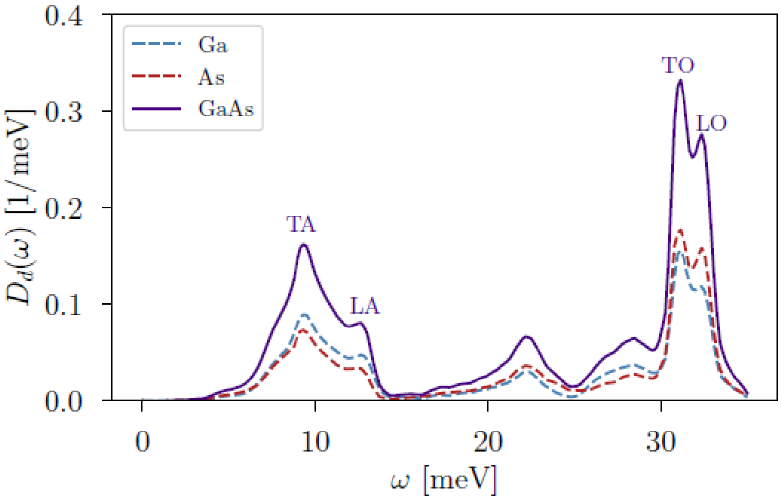
Multiphonon Rate (SI)

$$\frac{d\sigma}{d^3q\,d\omega} \sim \sum_d^N f_d^2 e^{-2W_d(q)} \sum \left(\frac{q^2}{2m_d}\right)^n \frac{1}{n!} \left(\prod_{i=1}^n \int d\omega_i \frac{D_d(\omega_i)}{\omega_i}\right) \delta\left(\sum_j \omega_j - \omega\right)$$

$$\Big\downarrow \quad q \gg \sqrt{2\omega m_d}$$

Impulse Approximation

Partial density of States



$$\frac{d\sigma}{d^3q\,d\omega} \sim \sum_d^N f_d^2 \sqrt{\frac{2\pi}{\Delta_d^2}} \exp\left(-\frac{\left(\omega - \frac{q^2}{2m_d}\right)^2}{2\Delta_d^2}\right)$$

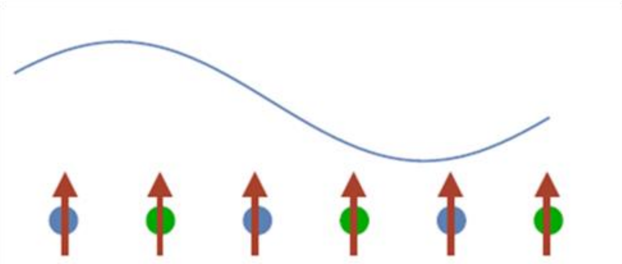
$$\Big\downarrow \quad q \ggg \sqrt{2\omega m_d}$$

$$\frac{d\sigma}{d^3q\,d\omega} \sim \sum_d^N f_d^2 \times \delta\left(\omega - \frac{q^2}{2m_d}\right)$$

Free nuclear recoil limit!

B. Campbell-Deem, S. Knapen, T. Lin, E. Villarama: 2205.02250

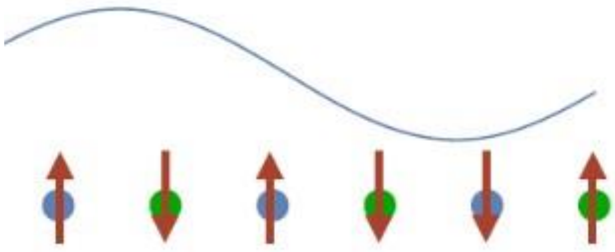
Spin Dependent Derivation



Nuclear Spin Polarized

Similar to spin independent case, with a form factor correction

(see T. Trickle et. al. 2009.13534)



Nuclear Spin Unpolarized

Cross section doesn't average away:

$\ell = \ell'$ (incoherent) terms contribute $\sim \langle S_N^2 \rangle$

Previous calculation holds!

Figures by S. Knapen

Spin Dependent Derivation

- The crystal dynamics remain the same:

$$\frac{d\sigma}{d^3q\,d\omega} \sim q^2 \langle S_\chi^2 \rangle \sum_{\ell,d}^{N} \left(\frac{\overline{f_d^2 \langle S_d^2 \rangle}}{m_d^2} \right) \boxed{\int_{-\infty}^{\infty} dt \, \langle \lambda_0 | e^{iq\cdot r_\ell(0)} e^{-iq\cdot r_{\ell'}(t)} | \lambda_0 \rangle \, e^{i\omega t}}$$

  - The bar denotes an average over isotope abundances
  - $\langle S_d^2 \rangle$ is the averaged squared spin of the nucleus

Exactly the same as SI case

- Approximations:
  - ~~Incoherent~~
  - Harmonic
  - Isotropic crystal
  - Spherically symmetric spins

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Model Dependence: EFT operators

Pseudoscalar mediator & fermion DM

$$(O_4^f) : \mathcal{L} \supset g_\chi a \bar{\chi} \gamma^5 \chi + g_p a \, \bar{N} \gamma^5 N$$

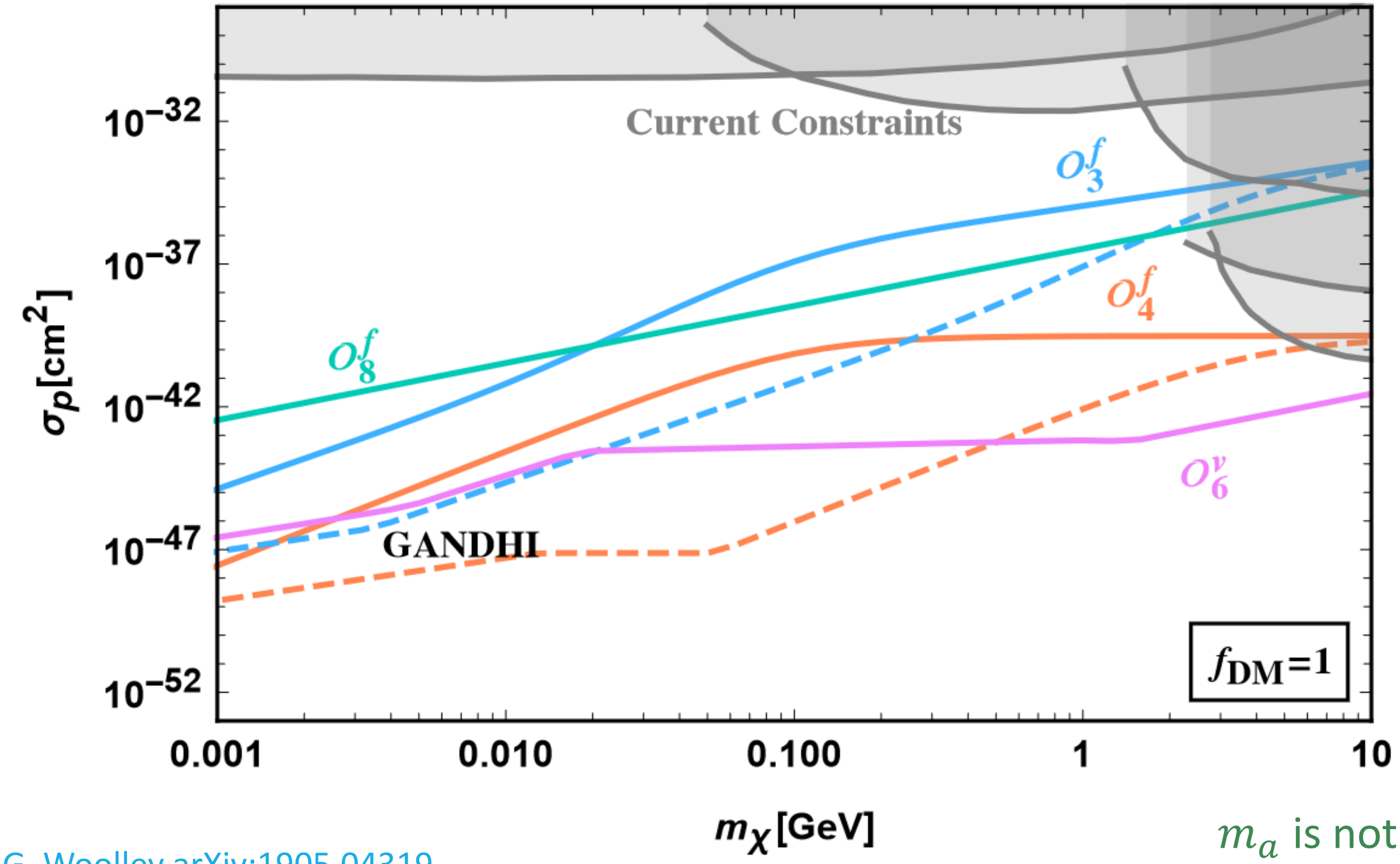
$$\mathcal{L}_{NR} \sim (q \cdot S_N)(q \cdot S_\chi)$$

$$(O_3^f) : \mathcal{L} \supset g_\chi a \bar{\chi} \chi + g_p a \, \bar{N} \gamma^5 N$$

$$\mathcal{L}_{NR} \sim (S_N \cdot S_\chi)$$

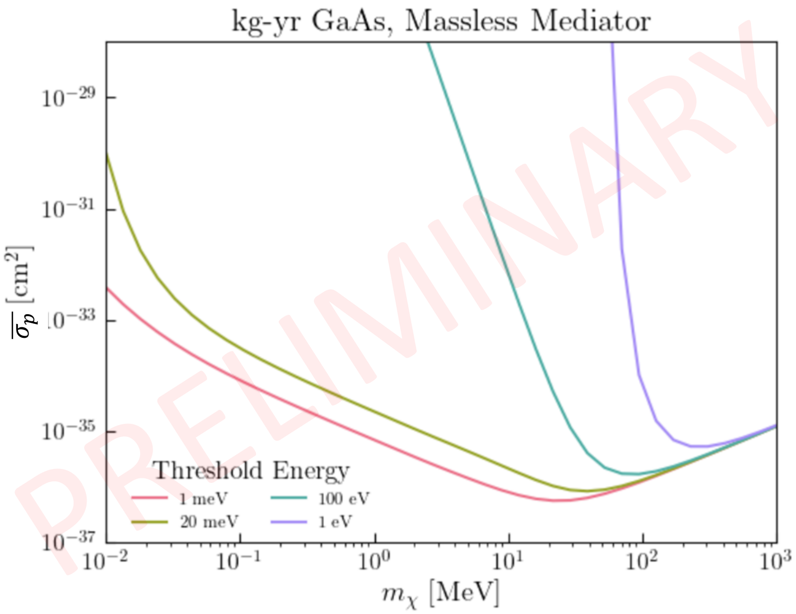
A. Fitzpatrick, W. Haxton, E. Katz, N. Lubbers, Y. Xu arXiv:1203.3542

Spin Dependent Constraints

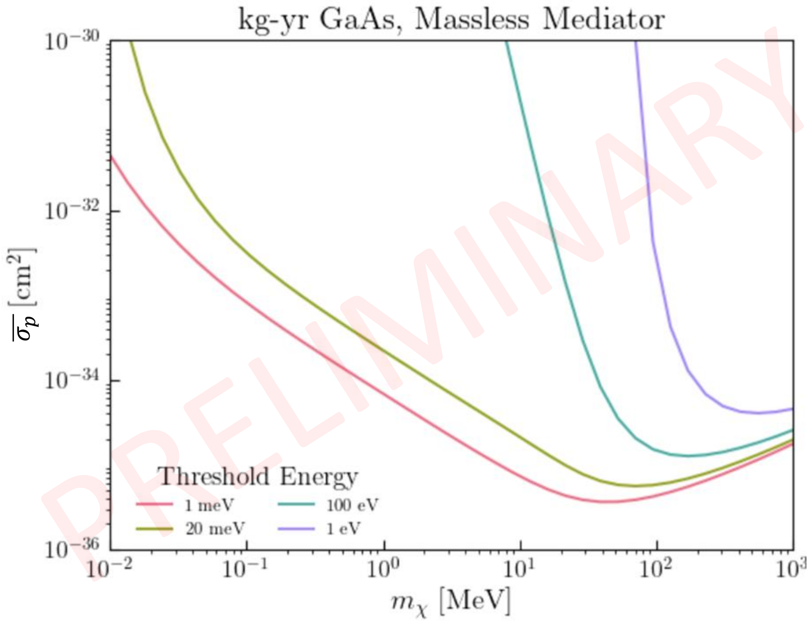


m_a is not fixed

H. Ramani, G. Woolley arXiv:1905.04319



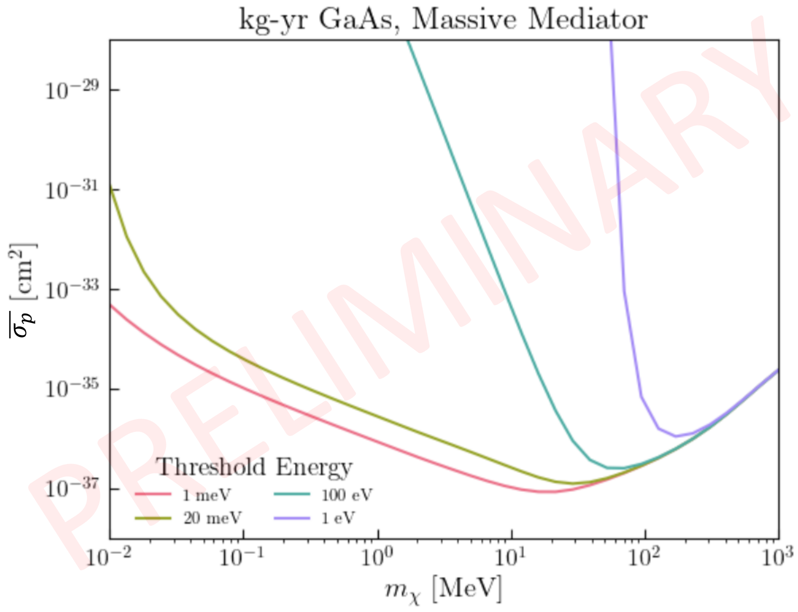
Pseudoscalar DM, massless mediator



Scalar DM, massless mediator

Results

Cross sections needed for 3 events/kg-year rate for GaAs using several threshold energies



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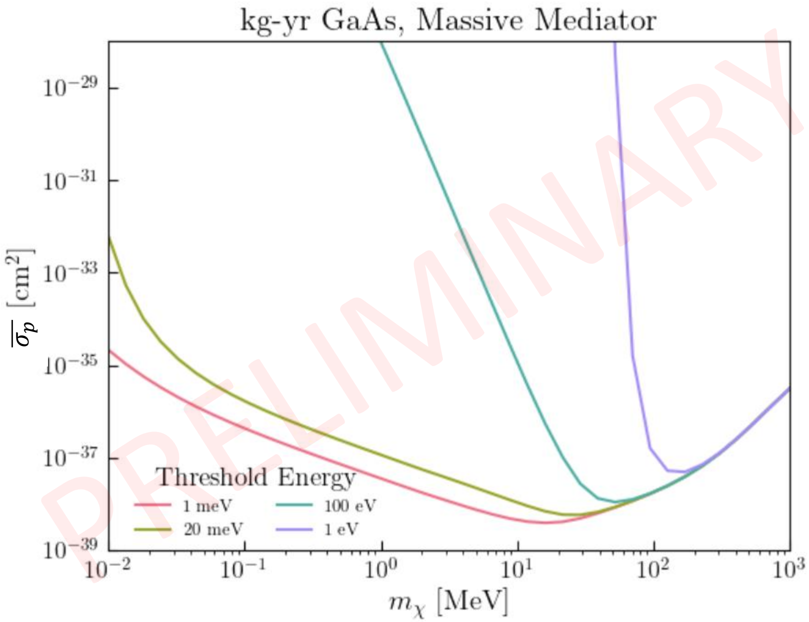


Scalar DM, massive mediator

Results

Cross sections needed for 3 events/kg-year rate for GaAs using several threshold energies

Summary

Lots of new proposed experiments are looking for dark matter-phonon interactions

Multiphonon – DM interactions cover intermediate DM mass ranges between nuclear recoil and single phonon detection

Spin dependent multiphonon scattering utilizes similar methods as spin independent when crystal spins are randomly distributed

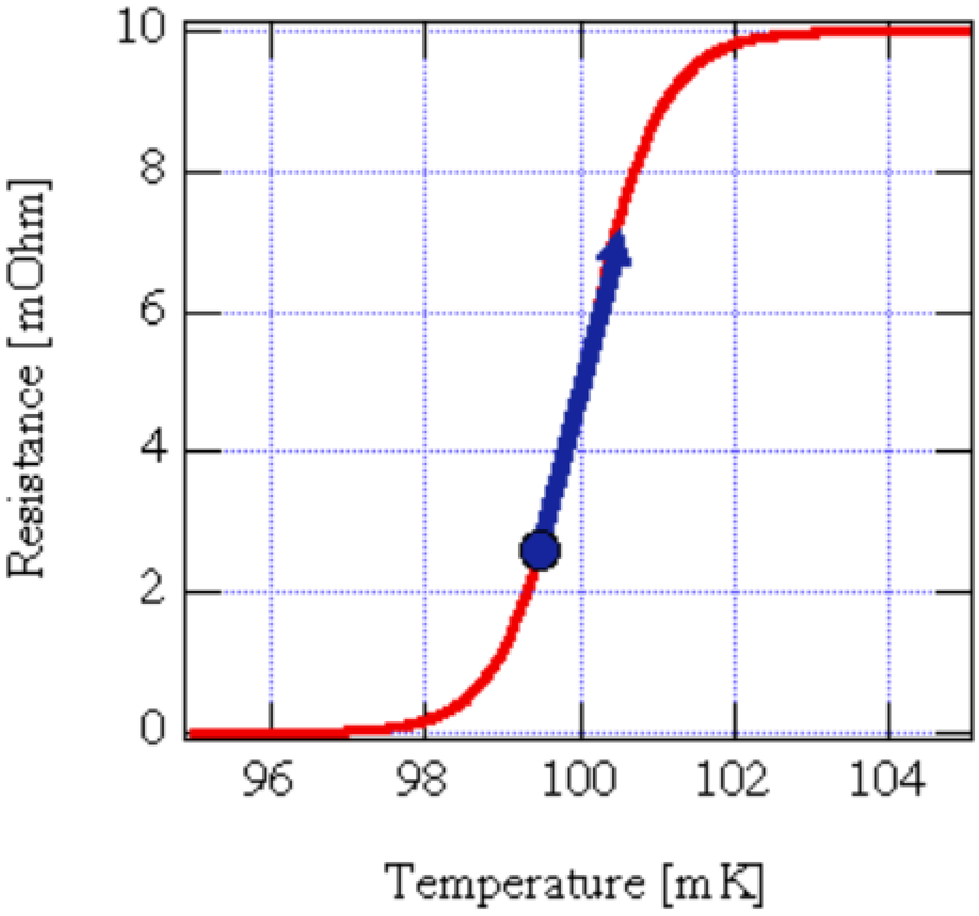
The cross sections for spin dependent cases are within the range of future experiments – spin dependent multiphonon experiments should be created to probe intermediate DM masses

Questions?

Backup Slides

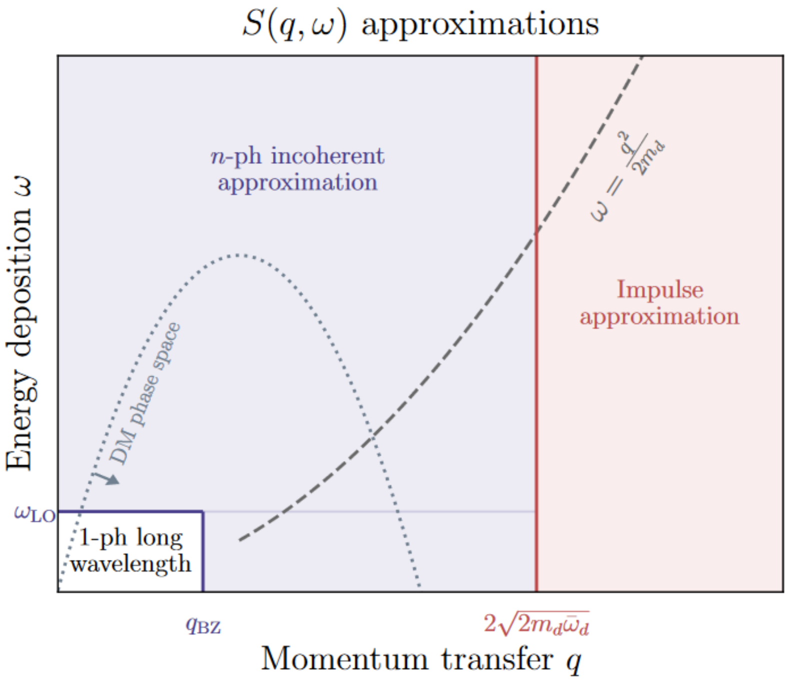
Transition Edge Sensors (TES)

- Superconducting film acting at the phase "transition edge"
- Large change in resistance with tiny shifts in temperature
- Smaller band widths → lower threshold energies

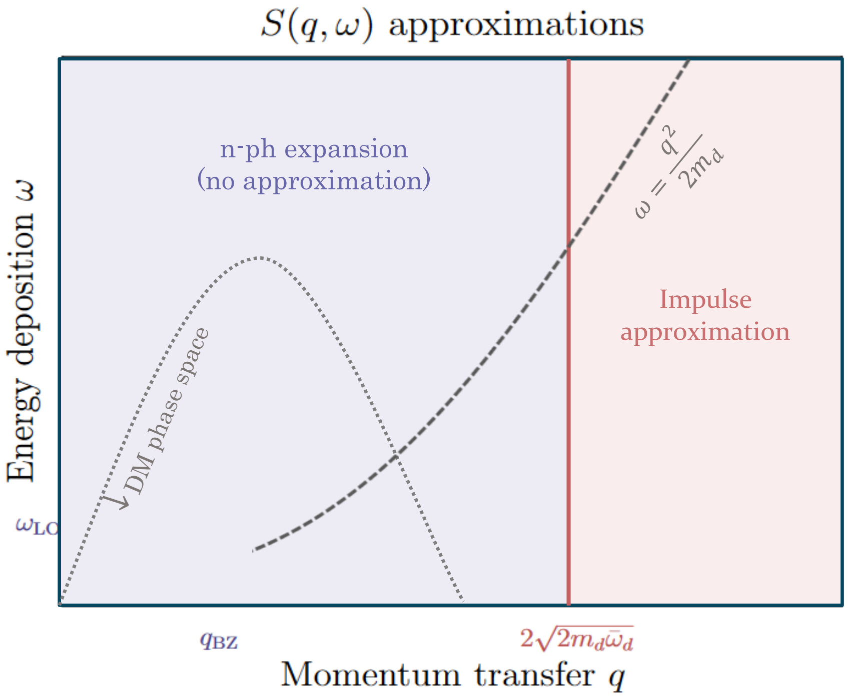


Approximations

Spin Independent



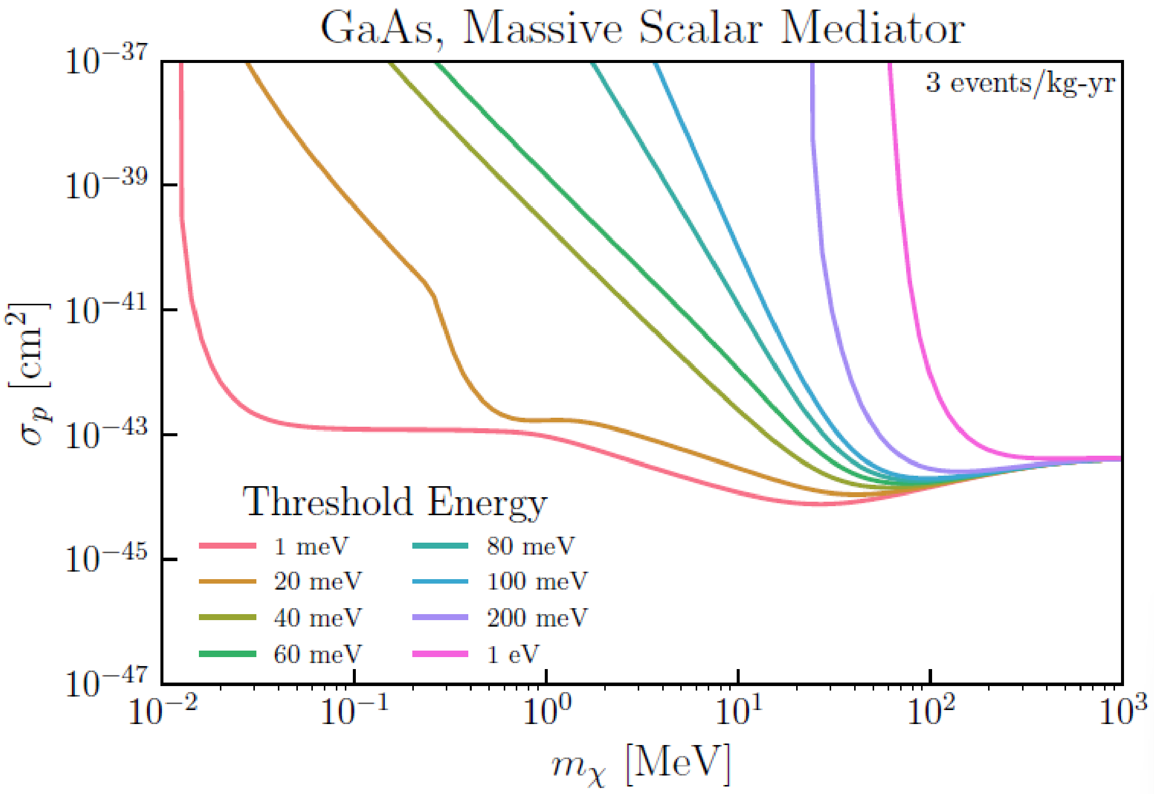
Spin Dependent



B. Campbell-Deem, S. Knapen, T. Lin, E. Villarama: 2205.02250

Spin Independent Results

- Assumes a coupling $\sim A_d$
- Isotropic approximation
- Anharmonic corrections around $m_\chi \sim 1 - 10\ MeV$: 2309.10839



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