Axion Mass from Small Instantons?

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Introduction

What is **axion**?

- Axion appears in Peccei-Quinn (PQ) mechanism, which solves the strong CP problem. Axion is also a dark matter candidate.

Energy scale of **Peccei-Quinn mechanism**

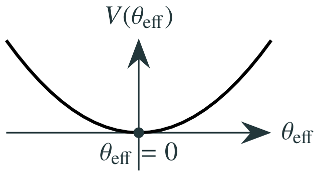
- Observational constrainton decay constant: $f_{\rm PQ} \gtrsim 10^9 {\rm GeV}$
- Even larger $f_{\rm PQ}$ is preferred to exlain all of the dark matter

"Axion Quality Problem"

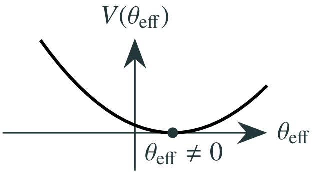
Known global symmetries are accidentally realized to preserve gauge symmetries (e.g. Baryon number).

If PQ symmetry $\mathrm{U}(1)_{\mathrm{PQ}}$ is accidental,

- Higher dimensional $\mathrm{U}(1)_{\mathrm{PQ}}$ (explicitly-)breaking operators can contribute to the effective θ-angle and θ can easily exceed the experimental upper bound.



Axion potential from anomaly.

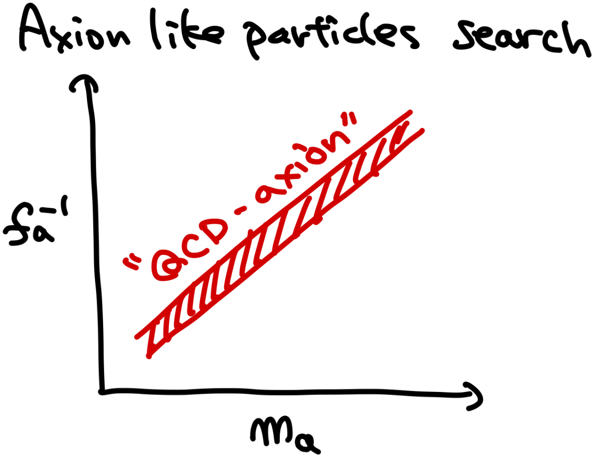


Additional PQ-breaking$\rightarrow$ $\theta \neq 0$.

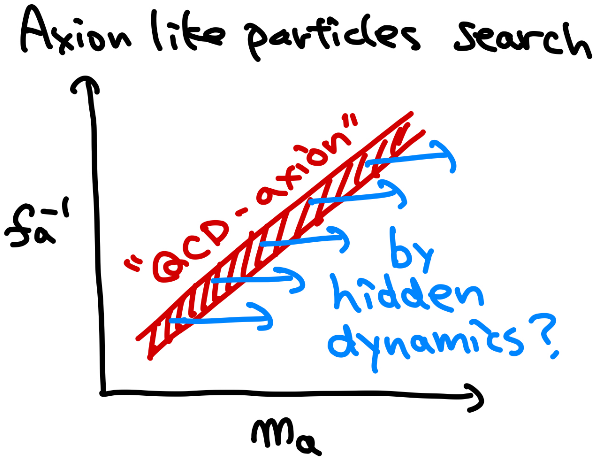
Introduction

Axion quality problem $\rightarrow$ Need for hidden dynamics to solve the strong CP problem?

Axion quality problem → Need for hidden dynamics to solve the strong CP problem?



Axion quality problem → Need for hidden dynamics to solve the strong CP problem?



Introduction

1. Suppresions for $\mathrm{U(1)_{PQ}}$-breaking operators can be achieved with some gauge symmetries in UV.

2. In such models, instantons not in QCD ("**small instantons**") might affect the axion potential. [Agrawl & Howe (2018), Csáki et al (2020)].

3. **Axion mass from small instantons?**
   $\rightarrow$ It might affect search strategies of QCD axion.

Composite Axion Models

- ”Massless quark”: θ_{QCD} is absorbed by a chiral rotation → No strong CP problem.

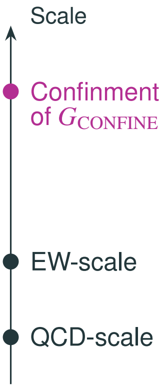
Composite axion models:

- In the similar way, can the strong CP problem be solved by massless fermions?

1. Massless fermions must not remain in low energy.
   → Confinement of hidden gauge dynamics in high-energy.
2. An axion appears as a “pion” of hidden gauge dynamics.
   → **Composite axion**.

- Higher-dimensional PQ-breaking operators are suppressed enough in some models. → Not suffering from the axion quality problem.

Composite Axion Models



Symmetry breaking by G_{CONFINE}:

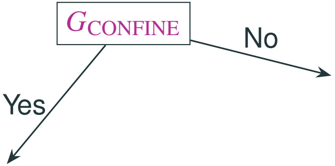
- GAUGE: $G \supset SU(3)_{\text{QCD}}$
- GLOBAL: PQ ($\longrightarrow$ Axion)

G depends on models.

Dynamical scales: $\Lambda_{\text{CONFINE}} \gg \Lambda_G$

Composite Axion Models

Q. Is PQ symmetry **anomalous under**. . . ?



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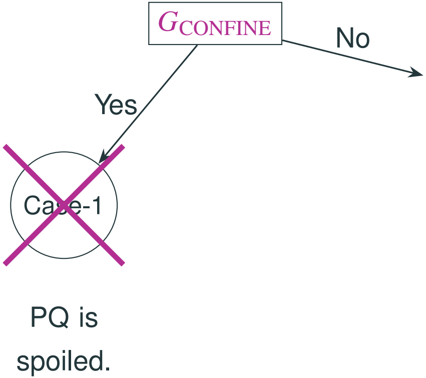

Composite Axion Models

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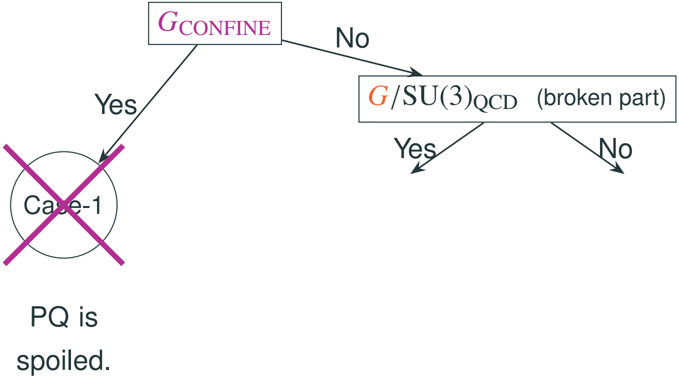

Composite Axion Models

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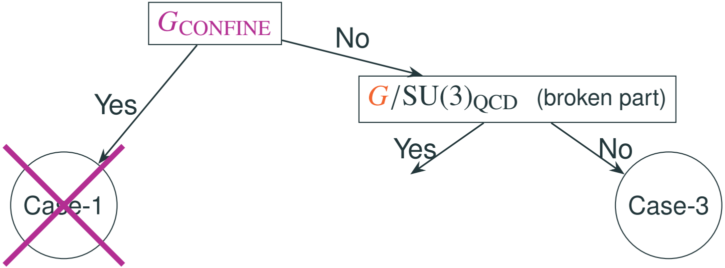
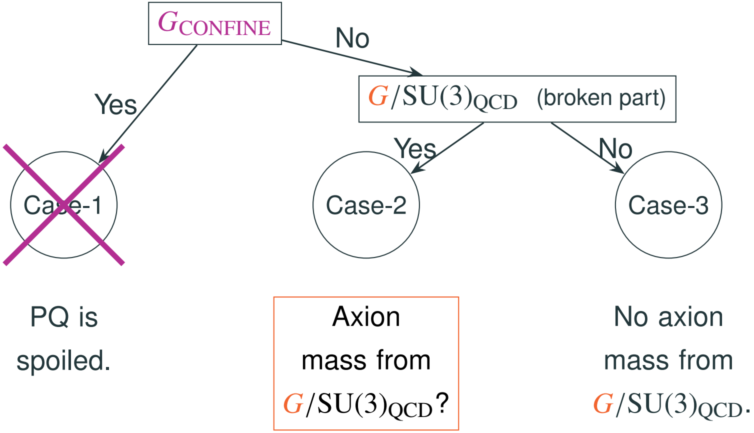


PQ is spoiled.

No axion mass from $G/\mathrm{SU}(3)_{\rm QCD}$.

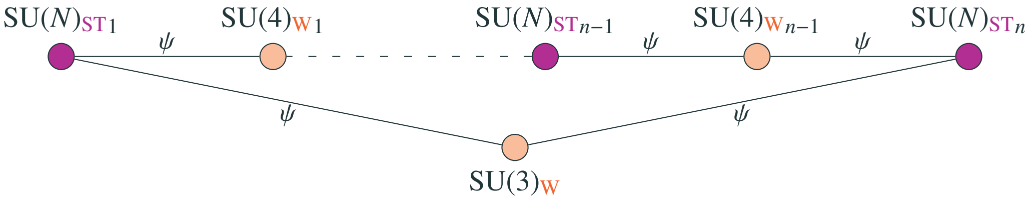
Q. Is PQ symmetry **anomalous under**. . . ?



Model

An example of "Case-2": **Composite accidental axion model** [Redi & Sato (2016)].

$$G_{\text{CONFINE}} = [SU(N)_{\text{ST}}]^n$$
$$G = SU(3)_{\text{W}} \times [SU(4)_{\text{W}}]^{n-1}$$



$[SU(N)_{\text{ST}}]^n$ confine at a scale $\Lambda\,(\sim f_a) \gg \Lambda_{\text{QCD}}$.

(In the following, n=2 as an example.)

Model

Scale

Confinment of $[SU(N)_{ST}]^2$

GAUGE: $SU(3)_W \times SU(4)_W \to SU(3)_{QCD}$

GLOBAL: Accidental PQ is broken($\to$ Axion)

QCD-scale

Model

Properties of PQ symmetry in this model:

- **High-quality**: PQ-breaking operators are restricted by gauge symmetries.
- **Anomalous under $SU(N)_{ST1} \times SU(N)_{ST2}$?: NO**

  $\rightarrow$ No axion potential from $SU(N)_{ST1} \times SU(N)_{ST2}$

- **Anomalous under $SU(3)_W \times SU(4)_W / SU(3)_{QCD}$?: YES**

  $\rightarrow$ Non-QCD instanton effect on axion mass?

Small Instantons

- A "purely-$\mathrm{SU}(3)_{\mathrm{W}}$" instanton exist with a size $\lesssim \Lambda^{-1}$ (Λ :confinement scale).
  Non-QCD small instanton.

- Naive estimate for instanton effects:

  1. Matching of couplings: $\dfrac{1}{g^2_{\mathrm{QCD}}} = \dfrac{1}{g^2_{\mathrm{SU(3)_W}}} + \dfrac{1}{g^2_{\mathrm{SU(4)_W}}}$ (at the breaking scale Λ.)
  2. Suppressions coming from the action for instantons:

  $$\exp\left(-\frac{8\pi^2}{g^2_{\mathrm{QCD}}}\right) \ll \exp\left(-\frac{8\pi^2}{g^2_{\mathrm{SU(3)_W}}}\right) \quad \text{is possible.}$$

$\rightarrow$ Naively, $\mathrm{SU}(3)_{\mathrm{W}}$ small instantons might enhance the axion mass.

Fermion Zero Modes

Instanton contribution to the axion mass may vanish, if **fermion zero modes** exist.

An example of vanishing physical quantity

$$\mathcal{L} = \xi^{+} i\sigma^{\mu} D_{\mu} \xi + \eta^{\dagger} i\sigma^{\mu} D_{\mu} \eta$$

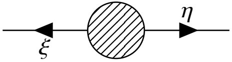
Situation: $\sigma^{\mu} D_{\mu}$ has a zero modes ψ_0.

Decomposing ξ and η as

$$\xi(x) = \xi_0 \cdot \psi_0(x) + \text{(non-zero modes)}, \quad \eta(x) = \eta_0 \cdot \psi_0(x) + \text{(non-zero modes)},$$

Path integral is

$$\int \mathrm{d}\xi^{\dagger} \mathrm{d}\xi \mathrm{d}\eta^{\dagger} \mathrm{d}\eta \ \exp[-S]\mathcal{O} \ \propto \int d\xi_0 d\eta_0 \ \exp[\text{const.}]\mathcal{O}.$$



This is vanishing, if $\mathcal{O}$ does not include ξ_0 or η_0.

Fermion Zero Modes

We also need to look carefully at **interactions** of fermions.

An example of non-vanishing physical quantity

$$L = \xi^{+} i\sigma^{\mu} D_{\mu}\xi + \eta^{\dagger} i\sigma^{\mu} D_{\mu}\eta + m(\eta\xi + \xi^{\dagger}\eta^{\dagger})$$

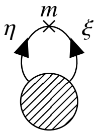
Situation: $\sigma^{\mu} D_{\mu}$ has a zero modes ψ_0.

Decomposing ξ and η as

$$\xi(x) = \xi_0 \cdot \psi_0(x) + \text{(non-zero modes)}, \quad \eta(x) = \eta_0 \cdot \psi_0(x) + \text{(non-zero modes)},$$

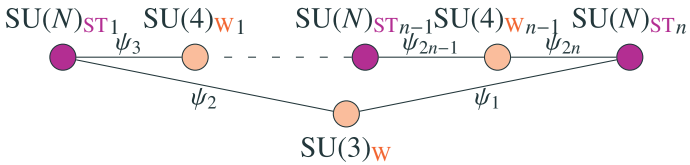
Path integral is

$$\int \mathrm{d}\xi^{\dagger}\mathrm{d}\xi\mathrm{d}\eta^{\dagger}\mathrm{d}\eta\ \exp[-S]\mathcal{O} \propto \int d\xi_0 d\eta_0\ \exp\left[-\int \mathrm{d}x^4 m\psi_0(x)\psi_0(x)\eta_0\xi_0\right]\mathcal{O} \neq 0.$$



This is non-vanishing, due to the interactions.

Fermion Zero Modes



- Lines (ψs) are fermions.
- Charges are as in the table. (n=2 model)

	$SU(N)_{ST2}$	$SU(3)_W$	$SU(N)_{ST1}$	$SU(4)_W$
ψ_1	**N**	$\bar{\mathbf{3}}$		
ψ'_1	**N**	**1**		
ψ_2		**3**	$\bar{\mathbf{N}}$	
ψ'_2		**1**	$\bar{\mathbf{N}}$	
ψ_3			**N**	$\bar{\mathbf{4}}$
ψ_4	$\bar{\mathbf{N}}$			**4**

- Fermions obtain zero modes around $SU(3)_W$ or $SU(4)_W$ instantons.

Fermion Zero Modes

Can fermion zero modes interact in a way **generating the axion potential**?

What we found: **NO**, in the model by Redi & Sato (2016)

Example: $SU(3)_W$ or $SU(4)_W$ **single** instanton.

$\rightarrow$ Fermions are not closed and $\Delta V(a)$ **= 0**

Exception: $SU(3)_W$ and $SU(4)_W$ **paired** instantons.

$\rightarrow$ Fermions are closed and $\Delta m_a \neq 0$

This is an (effectively-)QCD instanton with a size $\sim \Lambda^{-1}$ ($\rightarrow$ No mass enhancement.)

Summary

1. With Hidden gauge dynamics, massless fermions in UV can solve the strong CP problem.
   $\rightarrow$ **small instantons** might enhance the axion mass.

2. However, close look at **fermion zero modes** is needed.
   We found that **axion mass is not enhanced** in a model by Redi and Sato (2016).

3. **Future prospect**: A generalization of our result, to be applied to other models.

BACKUP

Global Symmetry

	$SU(N)_{ST2}$	$SU(3)_W$	$SU(N)_{ST1}$	$SU(4)_W$	$U(1)_{PQ}^{(SSB)}$	$U(1)_1$	$U(1)_2$	$U(1)_3$
ψ_1	$\mathbf{N}$	$\overline{\mathbf{3}}$			1	1	1	1
ψ'_1	$\mathbf{N}$	$\mathbf{1}$			−3	1	−3	1
ψ_2		$\mathbf{3}$	$\overline{\mathbf{N}}$		1	1	−1	−1
ψ'_2		$\mathbf{1}$	$\overline{\mathbf{N}}$		−3	1	3	−1
ψ_3			$\mathbf{N}$	$\overline{\mathbf{4}}$	0	−1	0	1
ψ_4	$\overline{\mathbf{N}}$			$\mathbf{4}$	0	−1	0	−1

- Global symmetries which are non-anomalous under $SU(N)_{ST1} \times SU(N)_{ST2}$ are shown.
- $U(1)$ symmetries in green colored columns are anomalous under $SU(3)_W$ or $SU(4)_W$
- Condensations: $\langle \psi_1^{(\prime)} \psi_4 \rangle \sim \Lambda^3$, $\langle \psi_2^{(\prime)} \psi_3 \rangle \sim \Lambda^3$ $\rightarrow$ Spontaneous breaking of $U(1)_{PQ}$.

  ($SU(3)_W$, $SU(4)_W$, and $[SU(3)_{ST}]^2$ indices are omitted.)

Larger Model (n=3)

	$SU(N)_{ST3}$	$SU(3)_{W}$	$SU(N)_{ST1}$	$SU(4)_{W1}$	$SU(N)_{ST2}$	$SU(4)_{W2}$
ψ_1	$\mathbf{N}$	$\overline{\mathbf{3}}$				
ψ_1'	$\mathbf{N}$	$\mathbf{1}$				
ψ_2		$\mathbf{3}$	$\overline{\mathbf{N}}$			
ψ_2'		$\mathbf{1}$	$\overline{\mathbf{N}}$			
ψ_3			$\mathbf{N}$	$\overline{\mathbf{4}}$		
ψ_4				$\mathbf{4}$	$\overline{\mathbf{N}}$	
ψ_5					$\mathbf{N}$	$\overline{\mathbf{4}}$
ψ_6	$\overline{\mathbf{N}}$					$\mathbf{4}$

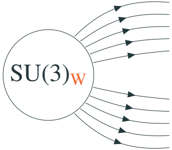
	$U(1)_{PQ}^{(SSB)}$	$U(1)_1$	$U(1)_2$	$U(1)_3$	$U(1)_4$
ψ_1	1	1	1	1	0
ψ_1'	−3	1	−3	1	0
ψ_2	1	1	−1	−1	0
ψ_2'	−3	1	3	−1	0
ψ_3	0	−1	0	1	0
ψ_4	0	0	0	−1	−1
ψ_5	0	0	0	1	1
ψ_6	0	−1	0	−1	0

- Only $U(1)_{PQ}$ is spontaneously broken, also for larger n.
- Additional $n-2$ anomalous (and unbroken) $U(1)$s, cancelling the additional θ angles.

Absence of Axion Mass Enhancement: in Detail

Small instanton effect on the axion mass is prohibited by global symmetries.

Example: A **single** $\mathrm{SU}(3)_{\mathrm{W}}$ instanton background.



$\psi_1\psi_1\psi_1\psi_1\psi_1$

$\psi_2\psi_2\psi_2\psi_2\psi_2$

$$\propto \Big(\overbrace{\psi_1\psi_1\cdots\psi_1}^{\text{baryon of } \mathrm{SU}(N)_{\mathrm{ST}2}} \Big)\cdot\Big(\overbrace{\psi_2\psi_2\cdots\psi_2}^{\text{baryon of } \mathrm{SU}(N)_{\mathrm{ST}1}} \Big)$$

This operator ('t Hooft vertex) includes the axion as its phase.

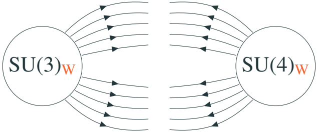
However, the anomalous rotation by $\mathrm{U}(1)_1$ can shift the phase.

Shift symmetry is realized for the axion, porhibiting the axion potential.

Absence of Axion Mass Enhancement: in Detail

[!] Axion shift symmetry is not realized in paticular instanton backgrounds.

Example: A **paired** $\mathrm{SU}(3)_{\mathrm{W}}$ and $\mathrm{SU}(4)_{\mathrm{W}}$ instantons background.



$$\propto \big(\underbrace{\psi_1\psi_1\cdots\psi_1}_{\text{baryon of } \mathrm{SU}(N)_{\mathrm{ST}\,2}}\big)\cdot\big(\underbrace{\psi_2\psi_2\cdots\psi_2}_{\text{baryon of } \mathrm{SU}(N)_{\mathrm{ST}\,1}}\big)\big(\underbrace{\psi_1^\dagger\psi_1^\dagger\cdots\psi_1^\dagger}_{\text{baryon of } \mathrm{SU}(N)_{\mathrm{ST}\,2}}\big)\cdot\big(\underbrace{\psi_2^\dagger\psi_2^\dagger\cdots\psi_2^\dagger}_{\text{baryon of } \mathrm{SU}(N)_{\mathrm{ST}\,1}}\big)$$

$\mathrm{U}(1)_1$ cannot shift the axion in this background. → Non-zero effect on axion mass.

This is just an effectively-QCD instanton.