A Novel Non-Fourier Framework to Address Light cone Effects in 21cm Observations

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What is the lightcone effect?

Compared with the cosmic microwave background, which is a snapshot of the first lights of the Universe, the observation of very large 3-dimensional volumes enables us to observe the evolution of the Universe along the depth of the volume. However, the observed volume is so large that for the Epoch of Reionization, the far side of the observed region would be characterized by a totally neutral universe, with the emergence of the first bubbles of ionized hydrogen. By contrast, the near end of the volume would present a fully ionized universe.

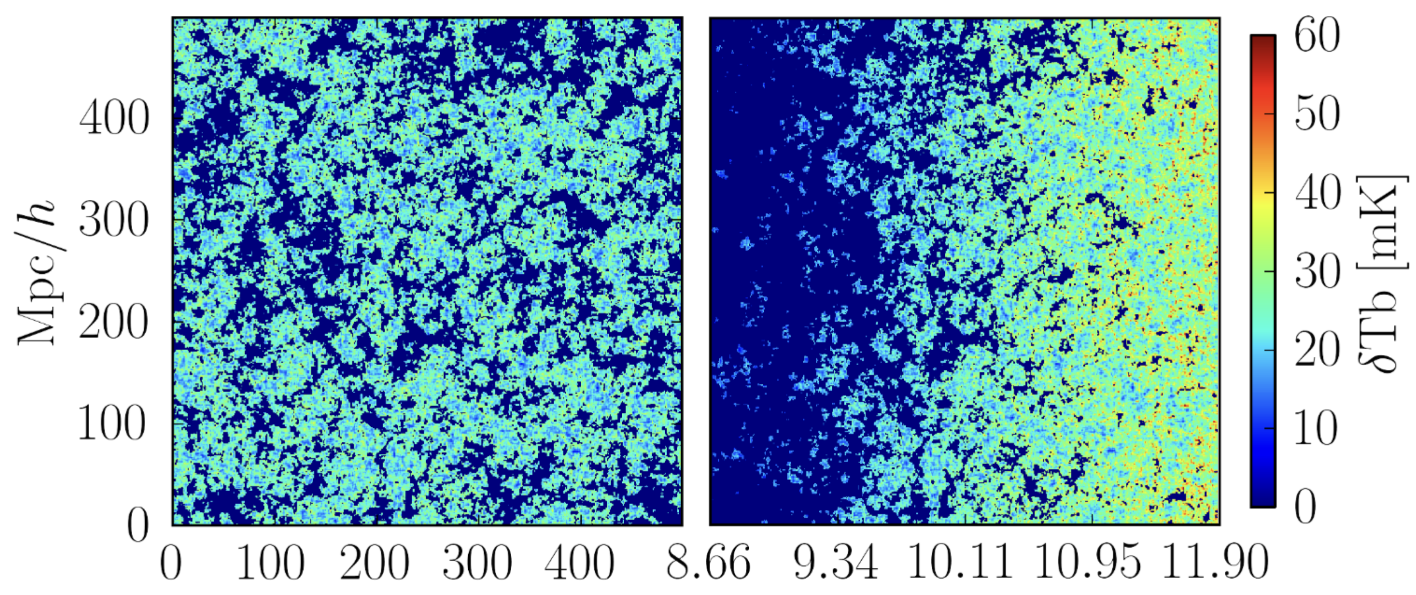

Figure 1. A graphical representation of the evolution in 21 cm brightness from numerical simulation. Left: A coeval sub-box with a side length of 500 Mpc/h at redshift 9. Right: The corresponding light cone cube, depicting the evolution of the ionization field. The x-axis on the right represents redshift rather than comoving distance. Figure taken from [1].

As a result, the properties of the universe, such as the density of free electrons, the size and morphology of ionized hydrogen bubbles, are dependent on distance from the observer. This effect of cosmic evolution along the redshift axis is known as the lightcone effect, which **introduces non-stationarity of statistics with respect to the observer**.

Consequences of the lightcone effect

More formally, if translational invariance is broken along the redshift axis, two important consequences must be considered:

- This means that the covariance matrix or the two-point correlation function is no longer diagonal in Fourier space along the redshift axis. Therefore, the power spectrum alone will not capture all the correlations of the field.

$$\xi(\mathrm{k}_1, \mathrm{k}_2) = \langle \widetilde{T}(\mathrm{k}_1)\widetilde{T}^*(\mathrm{k}_2)\rangle_V \neq (2\pi)^3\delta^D(\mathrm{k}_1 - \mathrm{k}_2)P(\mathrm{k}_1) \quad (1)$$

  with $\delta^D(..)$ the Dirac delta.
  All correlations and information from non-diagonal terms $\langle \widetilde{T}(\mathrm{k}_1)\widetilde{T}^*(\mathrm{k}_2)\rangle_V$ with $\mathrm{k}_1 \neq \mathrm{k}_2$ are not captured by the power spectrum.
- The light cone effect can also complicate the interpretation of data and upper limits on the 21 cm power spectrum. It can lead to a reduction in the amplitude of the 21 cm power spectrum, particularly for certain scales.

What is the basis that diagonalizes the correlation function with the light cone effect?

By enforcing the diagonalization of the correlation function along the redshift (Equation 1), we obtain the following basis. It consists of sinusoidal functions, but with amplitudes modulated along the redshift, encapsulated within an envelope.

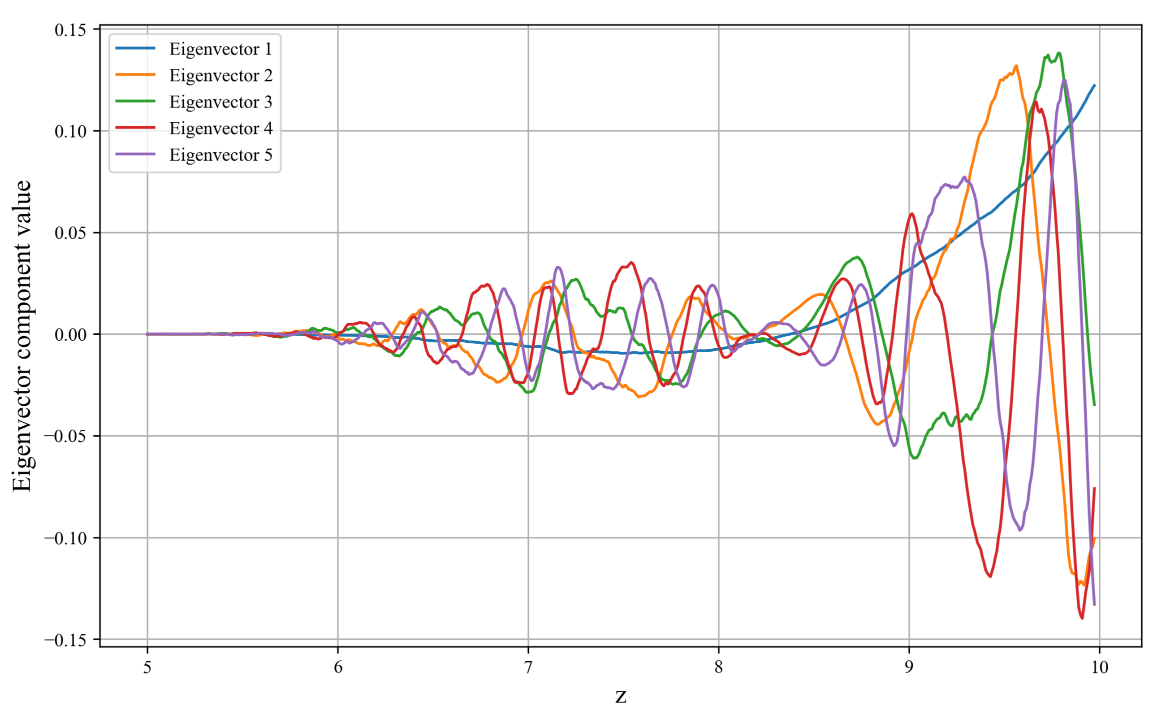

Figure 2. The first 5 basis vectors obtained by diagonalizing the covariance matrix along the redshift extracted from a 21cmFAST simulation from redshift 5 to 10 [2].

Without cosmic evolution, the Fourier modes are well recovered while they become modulated in the presence of the lightcone effect.

A new lightcone spectrum

The goal is to use this new basis along the redshift axis:

$$\delta T_b(x, y, z) \to \delta T_b(k_x, k_y, z) \to \delta T_b(k_x, k_y, z) = \sum_i \alpha_i(k_x, k_y) v_i(z)$$

with $\alpha_i(k_x, k_y)$ being the eigenvalue corresponding to the eigenvector $v_i(z)$.

Mathematical definition

$$P_{LC}(i, k_\perp) = |\alpha_i(k_\perp)|^2 \quad (2)$$

Advantages of this approach

By using the newly obtained basis, we gain access to all correlations between the different Fourier modes in addition to those captured by the power spectrum $P(k_\perp, k_\parallel)$. Obtaining the coefficients $\alpha_i(k_\perp)$ is also much faster in terms of computation time than a two-point correlation function.

Improvement of Constraints on the Epoch of Reionization Parameters

We aim to place constraints on the following key parameters: the stellar mass to halo mass ratio $f_{*,10}$ with the corresponding index α_*, the ionizing UV photon escape fraction $f_{\mathrm{esc},10}$ with the corresponding index α_{esc}, and M_{turn} is the characteristic halo mass below which galaxies become inefficient at forming stars.

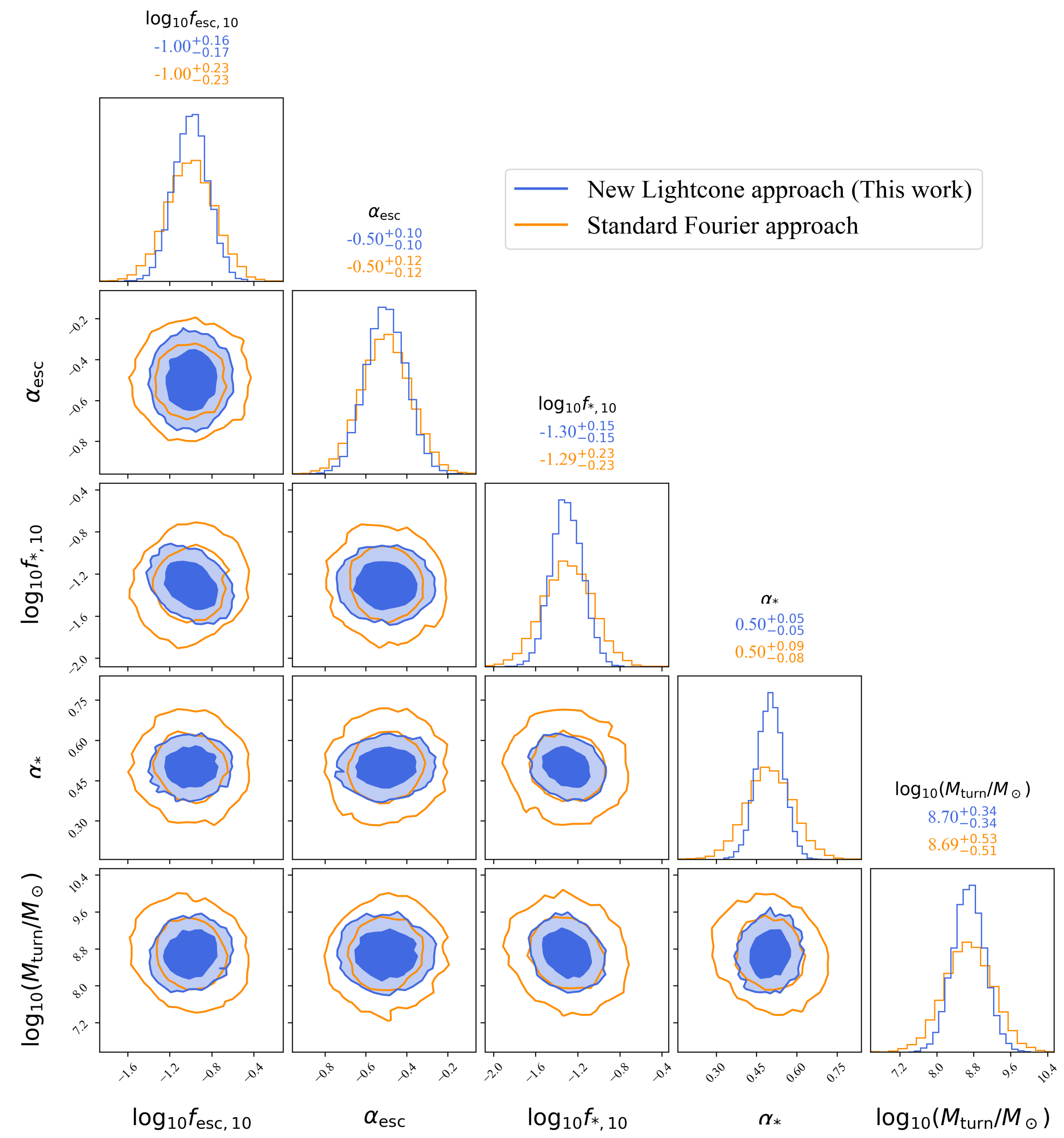

Figure 3. Parameter constraints obtained from the standard $P(k_\perp, k_\parallel)$ Fourier approach (blue) compared to the new lightcone approach $|\alpha_i(k_\perp)|^2$ (orange). The top right plots show 1D marginalized posteriors and the lower triangle plots show the 2D marginalized posteriors. The contours show 1σ, 2σ confidence intervals for the posteriors.

We compare the performance of this new summary statistic with the standard approach $P(k_\perp, k_\parallel)$. To do this, a Fisher Forecast was conducted on the constraints obtained on the parameters of the Epoch of Reionization. We assume that the 21cmFAST model incorporates error bars arising from thermal noise and cosmic variance, using 21cmSENSE, based on the HERA experiment within a volume of 250^2 Mpc^2 $\times 1690$ Mpc from $z = 5$ to $z = 10$ [3,4].

[1] Park J., Mesinger A., Greig B., Gillet N. *2019, MNRAS, MNRAS, 484, 933*
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[3] Plante P. L., Battaglia N., Natarajan A., Peterson J. B., Trac H., Cen R., Loeb A. *2014, ApJ, 789, 31*
[4] Murray S., Pober J., Kolopanis M. *2024, J. Open Source Softw., 9, 6501*

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