Matter bounce scenario in $f(R,T)$ gravity

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Payel Sarkar and **Prasanta Kumar Das**

BITS Pilani, K K Birla Goa Campus

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Abstract

We investigate the cosmic bounce in $f(R,T)(=R+2\lambda T)$ gravity theory. Cosmic bounce requires the scale factor $a(t)\neq 0$ at time $t=0$ (the point of bounce) which eliminate the initial singularity problem. We show how the Hubble parameter H, the deceleration parameter q varies with time in $f(R,T)$ gravity for different scale factor $a(t)$. The energy conditions in these bouncing models are found to be *violated*. The models are highly unstable at the point of bounce, but become stable at later time.

Introduction

- Plank2015[1] findings: $r<0.1 \rightarrow$ eliminates conventional inflationary models[2].
- Response to Planck2015 $\rightarrow$ *Cosmic bounce*[3] in which the initial singularity

$$t=0, a(t)=0 \rightarrow R(t)\sim 1/a(t)\rightarrow\infty$$

is replaced by a big non-singular bounce.

Bouncing cosmology :

- Starting from an initial contracting phase ($\dot{a}<0$), a bouncing universe goes to non-vanishing minimum radius ($a(t=0)\neq 0$) and then evolves to an expanding phase($a>0$).
- At the bouncing point($t=0$), $\dot{a}=0$ i.e. $H=0$ and $\ddot{a}>0$.
- A successful bounce for a flat universe ($p=\omega\rho$) requires,

$$H=-12(p+\rho),\ \ =-12(1+\omega)p>0\rightarrow\omega<-1$$

- A cosmic bounce requires a transition of the EOS from $\omega<-1$ to $\omega>-1\rightarrow$ a transition from $\rho+p<0$ to $\rho+p>0$.
- Energy condition(NEC)[4]: $\rho+3p\geq 0, \rho+p\geq 0$ are *violated* - a necessary prerequisite for any scale factor $a(t)$ that may exhibit bouncing.

Field Equations in Modified Gravity

The Einstein-Hilbert action in $f(R,T)$[5] gravity is

$$\mathcal{S}=\frac{1}{16\pi G}\int d^4x\sqrt{-g}\,f(R,T)+\int d^4x\sqrt{-g}\,\mathcal{L}_m$$

Here $f(R,T)=R+2\lambda T$. The modified Einstein's equations:

$$f_R(R,T)R_{\mu\nu}-\frac{1}{2}f(R,T)g_{\mu\nu}+(g_{\mu\nu}\Box-\nabla_\mu\nabla_\nu)f_R(R,T)=T_{\mu\nu}-f_T(R,T)T_{\mu\nu}-f_T(R,T)\theta_{\mu\nu}$$

Here $f_R(R,T)=\frac{\partial f(R,T)}{\partial R}$, $f_T(R,T)=\frac{\partial f(R,T)}{\partial T}$ and $\theta_{\mu\nu}=g^{\alpha\beta}\frac{\partial T_{\alpha\beta}}{\partial g^{\mu\nu}}$.
After simplification, the modified Einstein's equation become

$$R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R=T_{\mu\nu}+\lambda(2T_{\mu\nu}+2pg_{\mu\nu}+Tg_{\mu\nu})$$

where $T_{\mu\nu}=(\rho+p)u_\mu u_\nu-pg_{\mu\nu}$. The Friedmann equations are

$$3H^2=\rho(1+3\lambda)-\lambda p,\ \ 2\dot{H}+3H^2=-p(1+3\lambda)+\lambda\rho$$

The density(ρ) and pressure(p) are calculated as

$$\rho=\frac{3H^2(1+2\lambda)-2\lambda\dot{H}}{(1+3\lambda)^2-\lambda^2},\ \ p=-\frac{3H^2(1+2\lambda)+2\dot{H}(1+3\lambda)}{(1+3\lambda)^2-\lambda^2}$$

The EoS parameter $\omega(=p/\rho)$ is

$$\omega=-\frac{3H^2(1+2\lambda)+2\dot{H}(1+3\lambda)}{3H^2(1+2\lambda)-2\lambda\dot{H}}$$

Analyzing bouncing with $a(t)=ln(t^2+\beta)$

- The scale factor $a(t)=ln(t^2+\beta)$, β is a parameter.
- The Hubble parameter H, $\dot{H}$ are defined as [6]

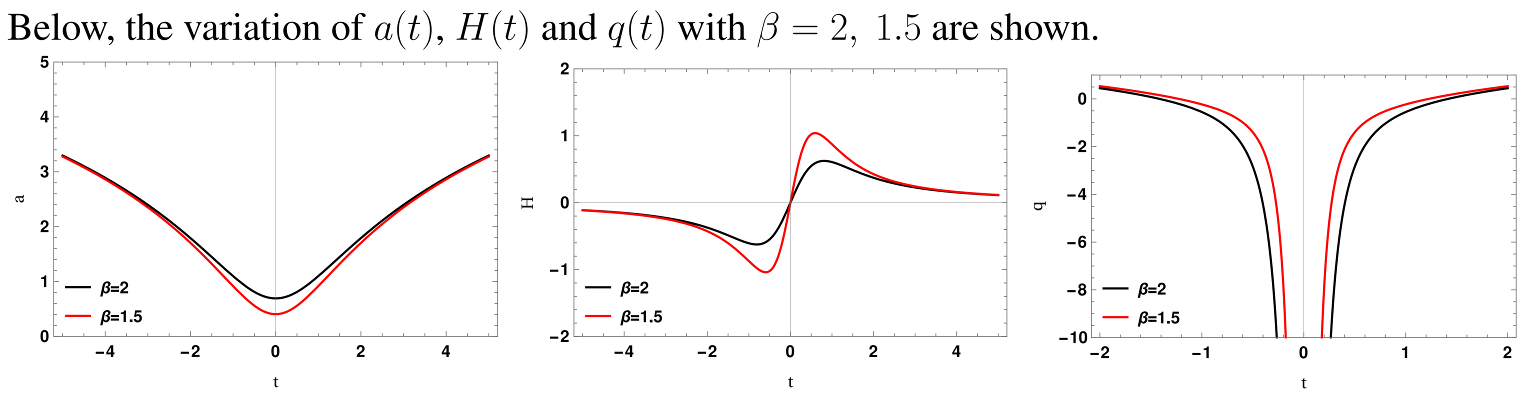
$$H=\frac{2t}{(t^2+\beta)\ln(t^2+\beta)}\rightarrow 0 \text{ at } t=t_b(=0)$$

$$\dot{H}=\frac{-4t^2-2(t^2-\beta)ln(t^2+\beta)}{(t^2+\beta)^2\ln(t^2+\beta)^2}\rightarrow\frac{2}{\beta ln(\beta)}>0 \text{ for } \beta>1 \text{ at } t=t_b(=0)$$

- The deceleration parameter q is defined as

$$q=-\frac{a\ddot{a}}{\dot{a}^2}=\frac{(t^2-\beta)\ln(t^2+\beta)}{2t^2}$$

Below, the variation of $a(t)$, $H(t)$ and $q(t)$ with $\beta=2,\ 1.5$ are shown.



Variation of $a(t)$, $H(t)$ and $q(t)$ with t with $\beta=2, 1.5$.

- At the bounce ($t=0$), we see $a\neq 0$ and $H=0$.
- At the bounce, we also see $q<0$ i.e. the Universe is accelerating.
- The Universe is passing from a contracting phase(H < 0) to an expanding phase(H > 0) through a bounce (H = 0) (Figure 1b).

Analysis ..continue

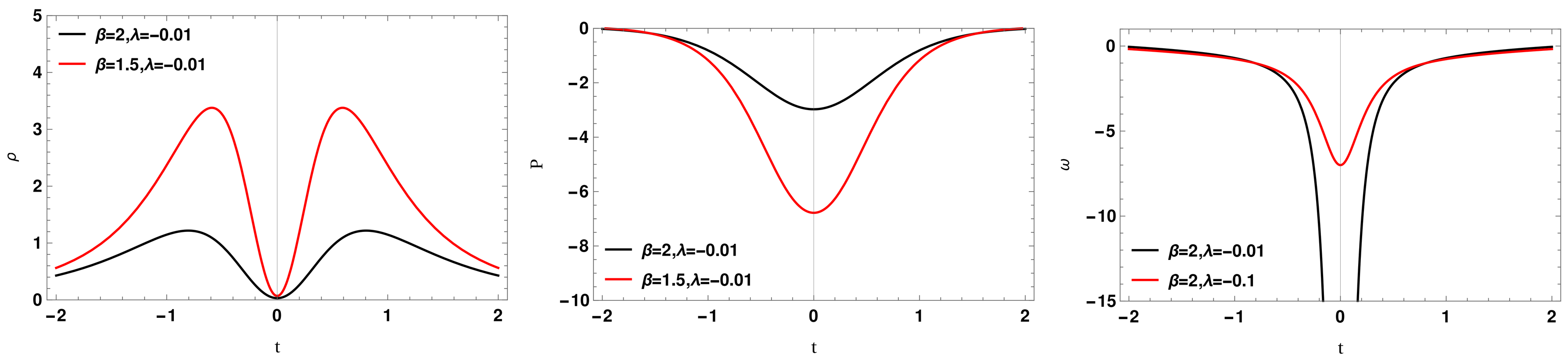
- The density(ρ), pressure(p) and EoS(ω) are defined as

$$\rho=\frac{4t^2(3+8\lambda)+4(t^2-\beta)\lambda ln(t^2+\beta)}{(t^2+\beta)^2(1+6\lambda+8\lambda^2)ln(t^2+\beta)^2},$$

$$p=\frac{-4t^2+4(t^2-\beta)(1+3\lambda)ln(t^2+\beta))}{(t^2+\beta)^2(1+6\lambda+8\lambda^2)ln(t^2+\beta)^2}$$

and $\omega=p/\rho$.

- Below $\rho(t)$, $p(t)$ and $\omega(t)$ are plotted for $\beta=2,\ 1.5$ with $\lambda=0.01$.



- At the bounce($t=0$), $\rho=0\ \rightarrow\ H=0$.
- Also at $t=0$, we see $p<0\ \rightarrow\ \dot{H}(=-(\lambda+1/2)(p+\rho))$. This means $\ddot{a}>0$ iff $\lambda>-1/2$: a necessary condition for bounce!

Stability analysis under the linear perturbation

- Stability test of $a(t)$ against the linear perturbation.
- Linear perturbation: $H(t)=H_n(t)(1+\delta(t)),\ \rho(t)=\rho_n(t)(1+\delta_n(t))$.
- The Friedmann equation in $f(R,T)$ can be written as

$$H^2=\frac{1}{3}(\rho+2\lambda(\rho+p)+f(T))$$

- The Eqn. of motion for linear perturbation gives

$$\dot{\delta_n}+\frac{T(1+3\lambda)}{2H_n}\delta_n(t)=0$$

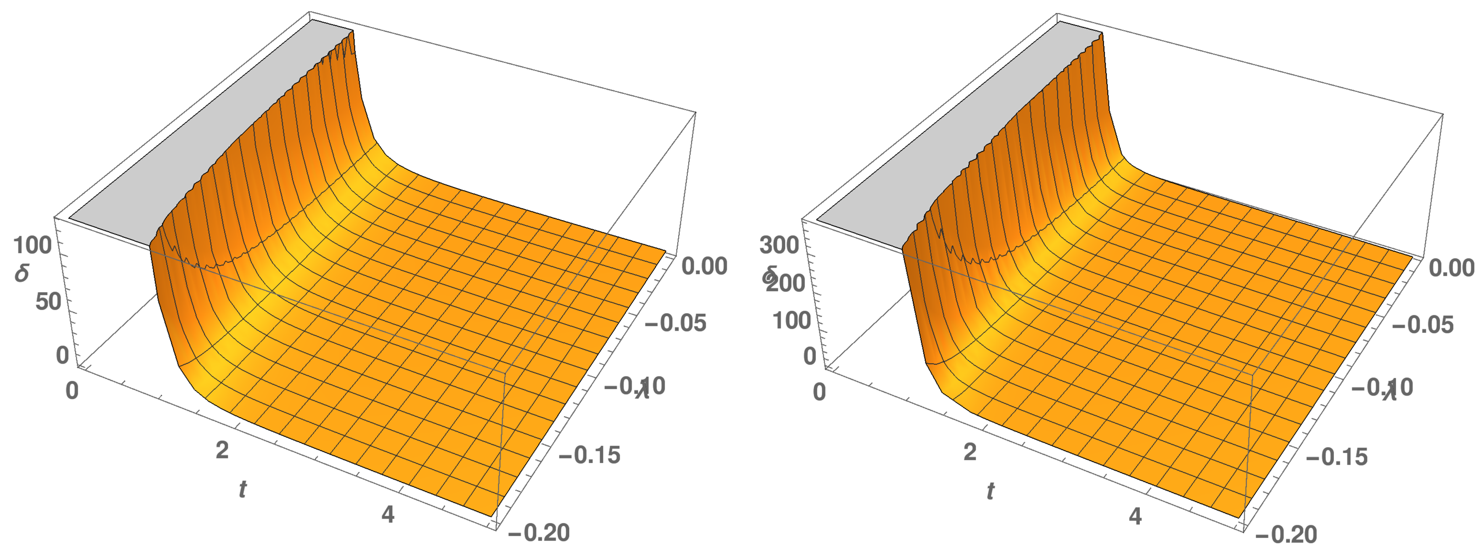
- Integrating, we get

$$\delta_n(t)=C\,Exp\left(-\frac{1+3\lambda}{2}\int\frac{T}{H_h}dt\right)$$

- Substituting δ_n into the eqn. of H^2, we get

$$\delta(t)=\frac{T(1+3\lambda)}{6H_h^2}\,CExp\left(-\frac{1+3\lambda}{2}\int\frac{T}{H_h}dt\right)$$

- Plot showing the evolution of $\delta_n(t)$(left one) and $\delta(t)$ as a function of time(Gyr) for fixed $\beta=1.5$.



Conclusion

- The bouncing cosmology with the scale factor $a(t)=ln(t^2+\beta)$ in $f(R,T)=R+\lambda T$ is studied. [1]
- The universe pass from a contracting phase ($H<0,\omega<-1$) to an expanding phase ($H>0,\omega>-1$) through a bounce.
- Highly unstable during bounce, but the perturbation decays very rapidly away from bounce making the universe stable at late time.

References

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[1] The analysis with $a(t)=1+(t^2+\beta)+(t^2+\beta)^2$ along with $a(t)=ln(t^2+\beta)$ is reported in [6]