Theory of gravity with nonminimal matter-nonmetricity coupling and the de-Sitter swampland conjectures

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Abstract

In this study, we investigate swampland conjectures within the setup of matter and non-metricity nonminimal coupling theories of gravity. We examine how the inflationary solution produced by a single scalar field can be resolved with the swampland criteria in string theory regarding the formation of de Sitter solutions. The new important findings are that the inflationary scenario in our study differs from the one in general relativity because of the presence of a nonminimal coupling term, and that difference gives the correction to general relativity. In addition, we observe that the slow-roll conditions and the swampland conjectures are incompatible with each other for a single scalar field within the framework of nonminimally coupled alternative gravity theories. We predict that these results will hold for a wide range of inflationary scenarios in the context of nonminimal coupling gravitational theories.

I. Introduction

Before going to work on the inflationary profiles within the framework of the nonminimal coupled theory of gravity, let us briefly discuss the relations between the slow-roll parameters (i.e., ϵ and η, which play major roles in the inflationary scenario) and the swampland conjectures. To be more specific, the swampland conjectures restrict a few limits on the scalar fields arising at low energy, namely ϕ, given by,

$$\frac{\Delta\phi}{M_p} < c_1, \quad M_p\frac{|\partial_\phi V|}{V} > c_2, \tag{1}$$

where $\Delta\phi$ is the variation range of the scalar field, $M_p = M_{pl}/\sqrt{8\pi}$ is the reduced planck mass, $\partial_\phi V = \frac{\partial V}{\partial \phi}$, $V(\phi)$ is the scalar field potential, and c_1, c_2 are contacts with order one. In further argument, one should consider a more redefined condition

$$M_p^2\frac{|\partial^2_{\phi\phi} V|}{V} < -c_3, \tag{2}$$

here, $\partial^2_{\phi\phi}V = \frac{\partial^2 V}{\partial \phi^2}$ and c_3 is a constant of order one.

Now, the above conditions can be easily compared with the slow-roll parameters in the single-field inflation for the scalar field (ϕ),

$$\epsilon = \frac{M_p}{2}\left(\frac{|\partial_\phi V|}{V}\right)^2, \quad \eta = M_p^2\frac{\partial^2_{\phi\phi}V}{V}. \tag{3}$$

It is well known that the slow-roll parameter satisfies $\epsilon \ll 1$ and $|\eta| \ll 1$ during the inflation period, whereas $\epsilon \sim 1$ and $|\eta| \sim 1$ at the end of inflation. The latest constraints on the slow-roll parameters are presented by using the CMB data

$$\epsilon < 0.0044, \quad \eta = -0.015 \pm 0.006. \tag{4}$$

From these results, one can clearly see that the swampland conjecture parameters do not match the requirements on c_2 and c_3. The incompatibility applies to any number of scalar fields driving inflation as long as their kinetic energy terms are canonical. Swampland theories, on the other hand, can be reconciled with data with warm inflationary model's setup for one or more scalar fields.

II. Non-minimal matter-nonmetricity coupling theory of gravity

Let us start by considering the action for non-minimal matter-nonmetricity coupling theory of gravity proposed by Harko et al.,

$$S = \int d^4x\sqrt{-g}\left[\frac{M_p^2}{2}f_1(Q) + f_2(Q)\mathcal{L}_m\right], \tag{5}$$

here, $f_1(Q)$ & $f_2(Q)$ are the arbitrary Lagrangian functions of the non-metricity scalar Q, g is the determinant of metric, and $\mathcal{L}_m$ is the matter Lagrangian.

Here, the nonmetricity scalar Q, which is the trace of non-metricity tensor has the following form, given by

$$Q = -Q_{\gamma\mu\nu}P^{\gamma\mu\nu}. \tag{6}$$

To simplify the formulation, let us introduce the following notations

$$f = M_p^2 f_1(Q) + 2f_2(Q)\mathcal{L}_m, \quad F = M_p^2 f_1'(Q) + 2f_2'(Q)\mathcal{L}_m, \tag{7}$$

where primes (') represent the derivatives of functions $f_1(Q)$ & $f_2(Q)$ with respect to Q.

Taking the variation of action (5) with respect to metric tensor, one can find the gravitational field equations given by

$$\frac{2}{\sqrt{-g}}\nabla_\gamma\left(\sqrt{-g}FP^\gamma{}_{\mu\nu}\right) + \frac{M_p^2}{2}g_{\mu\nu}f_1 + F\left(P_{\mu\gamma i}Q_\nu{}^{\gamma i} - 2Q_{\gamma i\mu}P^{\gamma i}{}_\nu\right) = -f_2T_{\mu\nu}. \tag{8}$$

With the above setup, we can investigate various cosmological scenarios in this modified alternative gravitational theory of nonminimal matter-nonmetricity coupling.

III. The FLRW cosmology

To investigate the cosmology in this nonminimal gravitational theory, we consider a spatially flat metric with homogeneous and isotropic in nature, given by

$$ds^2 = -N^2(t)dt^2 + a^2(t)\delta_{ij}dx^i dx^j, \tag{9}$$

where $i,\ j$ ranges over $1, 2, 3$ and represents the space coordinates, $N(t)$ represents lapse function, and we can take $N = 1$ at any time for the usual time reparametrization freedom. δ_{ij} is the Kronecker delta. For the FLRW metric, the non-metricity scalar Q reads as $Q = 6(H/N)^2$. The generalization of two Friedman equations can be written as:

$$f_2\rho = \frac{M_p^2 f_1}{2} - 6F\frac{H^2}{N^2}, \tag{10}$$

$$-f_2 p = \frac{M_p^2 f_1}{2} - \frac{2}{N^2}[(\dot{F} - FT)H + F(\dot{H} + 3H^2)], \tag{11}$$

respectively. One can obtain the general relativistic limit by considering the Lagrangian functions as $f_1(Q) = -Q$ and $f_2(Q) = 1$.

IV. Inflation in nonminimal coupled theory

To proceed further, we presume the Lagrangian function $f_1,\ f_2$ as follows

$$f_1(Q) = -Q, \;\; f_2(Q) = 1 + \alpha\left(\frac{Q}{6M_p^2}\right)^n, \tag{12}$$

for simplicity, we consider $\gamma = \frac{\alpha}{(6M_p^2)^n}$. The above-considered Lagrangian functions $f_1(Q)$ and $f_2(Q)$ have the standard general relativistic limit, which can be achieved by inserting $\alpha = 1$ and $n = 0$. In our study, we have explored by restricting $n \neq 0$ so that the modified gravitational effect can be measured.

We have also presumed that Lagrangian density $\mathcal{L}_m = p$ with a homogeneous scalar field ϕ, for which the energy density and pressure given by

$$\rho = \frac{\dot\phi^2}{2} + V(\phi), \;\; p = \frac{\dot\phi^2}{2} - V(\phi). \tag{13}$$

Furthermore, we assume quasi-exponential inflation i.e., $V \gg \dot\phi^2$, which implying $\rho \simeq -p \simeq V$. Now, we can find the following solutions from field equations

$$H^2 \simeq \frac{M_p^2\tilde{V}}{3}\left(1 - \frac{5}{27}\alpha\tilde{V}^3\right), \;\; \dot{H} \simeq -\frac{\dot\phi^2}{2M_p^2}\left(1 - \frac{2\alpha}{27}\tilde{V}^3\right). \tag{14}$$

Now, one can clearly see that the general relativity limit is represented by the first term of the right-hand side, whereas a correction to GR due to the nonminimal coupling between matter and nonmetricity is represented by the second term.

Now, taking the time derivative of the first equation with respect to t and using the second equation to eliminate $\dot{H}$, we obtain

$$\partial_\phi V \simeq -3H\dot\phi\left(1 + \frac{2\alpha}{3}\tilde{V}^3\right), \;\; \partial^2_{\phi\phi}V \simeq 3H^2\left(1 + \frac{2\alpha}{3}\tilde{V}^3\right)\left(-\frac{\dot{H}}{H^2} - \frac{\ddot\phi}{H\dot\phi}\right). \tag{15}$$

Using above equations, the quantities $\dot{H}/H^2$ and $\frac{\ddot\phi}{H\dot\phi}$ can be expressed as

$$\frac{\dot H}{H^2} \simeq \epsilon\left(1 - \frac{28}{27}\alpha\tilde{V}^3\right), \;\; \frac{\ddot\phi}{H\dot\phi} \simeq \epsilon\left(1 - \frac{28}{27}\alpha\tilde{V}^3\right) - \eta\left(1 - \frac{13}{27}\alpha\tilde{V}^3\right). \tag{16}$$

Now, considering the conditions on the slow-roll inflationary scenario $|\dot{H}|/H^2 \ll 1$ and $|\frac{\ddot\phi}{H\dot\phi}| \ll 1$, we find that

$$\epsilon \ll 1 + \frac{28}{27}\alpha\tilde{V}^3, \;\; \eta \ll 1 + \frac{13}{27}\alpha\tilde{V}^3. \tag{17}$$

From the above relations, we can verify that $\epsilon \ll 1$ and $\eta \ll 1$ because $\tilde{V} = V/M_p^4 \ll 1$ with assumptions $\alpha = \mathcal{O}(1)$. From these results, we conclude that the slow-roll parameter satisfies the inflation conditions $\epsilon \ll 1$ and $\eta \ll 1$. Further, these parameters are related to the parameters c_2 and c_3 (presented in (1) and (2), respectively), which evolve in the de Sitter swampland conjectures through the following relation

$$c_2^2 < 2\epsilon, \quad c_3 < |\eta|. \tag{18}$$

The above results imply that $c_2^2 \ll 1$ and $c_3 \ll 1$ during a quasi-exponential inflationary scenario. Therefore, the de-Sitter swampland conjectures can not be met for inflation under the framework of nonminimal matter-nonmetricity coupling theories of gravity.

V. Conclusions

- It is observed that the inflationary scenario discussed in this work differs from the one in general relativity, and this difference is induced by the choice of $f_2(Q)$ function.
- We found that the necessities for the inflationary potential very much control the slow-roll conditions under quite general conditions. Even if the influence of nonminimal coupling theory is determined by the model-free parameter α, which could be greater than one and cannot overcome the typical scale of the inflationary potential and its smallness in relation to the Planck scale.
- In conclusion, We infer that the de Sitter swampland criteria cannot be satisfied in the presence of nonminimal matter-nonmetricity gravity coupling. We anticipate that our findings will hold true for any number of inflation fields.
- Future work-Find compatibility between de-Sitter swampland conjectures and inflation in this theory

Find details: Sanjay Mandal and Kazuharu Bamba,*Theory of gravity with nonminimal matter -nonmetricity coupling and the de-Sitter swampland conjectures* arXiv:2312.17501