A Bayesian Estimator for Non-linear Regression with Errors-in-variables in Cosmology

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Introduction: Bayesian Inference

- We construct a Bayesian estimator for cosmological inference considering the contribution of peculiar motion in the redshift.
- Bayes' Theorem:

$$P(\boldsymbol{\theta} \mid \mathcal{D}; \mathcal{M}) = \frac{P(\mathcal{D} \mid \boldsymbol{\theta}; \mathcal{M}) \times P(\boldsymbol{\theta} \mid \mathcal{M})}{P(\mathcal{D})}$$

Posterior Likelihood Prior

Best fit ⇒ Peak of Posterior, **Uncertainty** ⇒ Width of Posterior

The Problem: Errors-in-variables

- **Errors-in-variables models:** Model fitting problems having errors in both dependent and independent variables.
  - No analytic solutions.
  - No straightforward merit function.
  - No general method for an arbitrary errors-in-variables model.

Our Method

$$y_M = f(\boldsymbol{\theta}, x)$$

Model parameters: $\boldsymbol{\theta} = (\theta_1, \theta_2, \dots, \theta_m)$

- Assume that each value of x_i has some shift, $\alpha_i \in [-\sigma_{x_i}, \sigma_{x_i}]$.
- Treat these α_i as parameters to obtain the joint posterior.
- Marginalize these parameters to get the posterior for the model parameters

$$\ln \mathcal{L}(\boldsymbol{\theta} \mid \mathcal{D}; \mathcal{M}) = -\frac{1}{2}\sum_{i=1}^{N}\left[\frac{y_i - f(\boldsymbol{\theta}, x_i - \alpha_i)}{\sigma_{y_i}}\right]^2 - \left[\frac{1}{N-m}\sum_{i=1}^{N}\left(\frac{y_i - f(\boldsymbol{\theta}, x_i - \alpha_i)}{\sigma_{y_i}}\right)^2 - 1\right]^2$$

Redshift-Magnitude Relation

$$m = M + 5\,\log_{10}\left(\frac{D_L}{\mathrm{Mpc}}\right) + 25$$

ΛCDM: $$m = M + 5\,\log_{10}\left[(1+z)\int_0^z \frac{c\,dz}{H_0\sqrt{\Omega_{m,0}(1+z)^3 + (1-\Omega_{m,0})}}\right] + 25$$

Peculiar Motion

- Velocity of galaxies relative to the Hubble flow.

z	m	σ_m
0.01012	13.90745	0.19825
0.01038	14.04959	0.20480
.	.	.
.	.	.
2.26	26.877	0.2548

$$z = z_* + z_p$$

Observed Cosmological Peculiar

Three Estimators

$\mathcal{E}_1$: Ignores peculiar motion $$\ln \mathcal{L} = -\frac{1}{2}\sum_{i=1}^{N}\left[\frac{m_i - m(z_i)}{\sigma_{m_i}}\right]^2$$

$\mathcal{E}_2$: Linear approximation $$\ln \mathcal{L} = -\frac{1}{2}\sum_{i=1}^{N}\left[\frac{(m_i - m(z_i))^2}{\sigma_{m_i}^2 + m'^2(z_i)\,\sigma_{z_i}^2}\right]$$

$\mathcal{E}_3$: Our estimator

$$\ln \mathcal{L} = -\frac{1}{2}\sum_{i=1}^{N}\left[\frac{m_i - m(z_i^*)}{\sigma_{m_i}}\right]^2 + \left(\frac{k}{2} - 1\right)\ln\left[\sum_{i=1}^{N}\left(\frac{m_i - m(z_i^*)}{\sigma_{m_i}}\right)^2\right] - \frac{1}{2}\sum_{i=1}^{N}\left[\frac{m_i - m(z_i^*)}{\sigma_{z_i}}\right]^2$$

Results: ΛCDM Model with Pantheon Data

Parameters	Priors	Best-fit values		
		$\mathcal{E}_1$	$\mathcal{E}_2$	$\mathcal{E}_3$
Ω_m	$\mathcal{U}(0.2, 0.4)$	0.286 ± 0.012	0.296 ± 0.014	0.302 ± 0.016
H_0	$\mathcal{U}(60, 80)$	73.59 ± 0.229	73.34 ± 0.287	73.13 ± 0.343

Table : Priors and best-fit values for the ΛCDM parameters

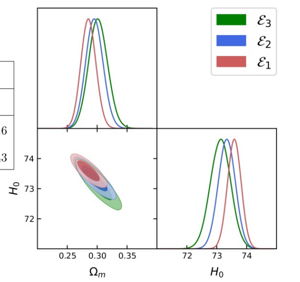

Fig.: 1 σ and 2 σ constraints on the ΛCDM parameters

Results: wCDM Model with Pantheon Data

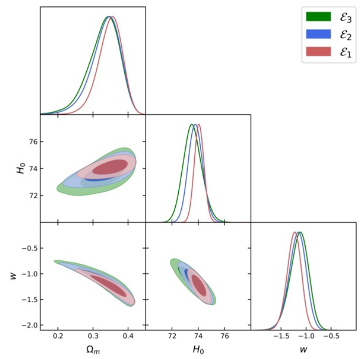

Fig. : 1 σ and 2 σ constraints on the wCDM parameters

Parameters	Priors	Best-fit values		
		$\mathcal{E}_1$	$\mathcal{E}_2$	$\mathcal{E}_3$
Ω_m	$\mathcal{U}(0.1, 0.6)$	0.348 ± 0.035	0.335 ± 0.043	0.330 ± 0.046
H_0	$\mathcal{U}(60, 80)$	74.07 ± 0.368	73.80 ± 0.546	73.58 ± 0.703
w	$\mathcal{U}(-2, 0)$	-1.24 ± 0.14	-1.17 ± 0.17	-1.14 ± 0.18

Table : Priors and best-fit values for the wCDM parameters

Summary

- We propose a Bayesian estimator for non-linear regression with errors in variables.
- We employ it to study the impact of peculiar motion on cosmological inference.
- For the ΛCDM model, we find a 5% increase in the the matter density parameter and 0.5 % decrease in the Hubble constant.
- For the wCDM model, both Ω_m and H_0 shift to lower values.
- Further, we are working on reconstruction of evolution of peculiar velocity with z.