Perturbations in $\mathbf{O}(D,D)$ string cosmology from double field theory

Stephen Angus* and Shinji Mukohyama
*presenting author

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Center for Quantum SpaceTime

Overview: $\mathbf{O}(D,D)$-complete string cosmology

In general relativity (GR) the spacetime metric $g_{\mu\nu}$ is the only gravitational field. However, in string theory it is just one of the massless modes of the closed string, appearing alongside two other fields: a skew-symmetric tensor 'B-field' $B_{\mu\nu}$, and a scalar dilaton ϕ. Moreover, different backgrounds of the three fields $\{g_{\mu\nu}, B_{\mu\nu}, \phi\}$ transform into each other under a hidden $\mathbf{O}(D,D)$ symmetry. This symmetry can be made manifest in the framework of double field theory (DFT), so called because the spacetime dimension D is formally doubled. However, physical backgrounds correspond to different D-dimensional slices of the doubled space. A doubled geometry can be constructed as well as DFT actions for additional matter (scalars, fermions, etc.), leading to a DFT generalization of Einstein's equations with an enhanced energy-momentum tensor.

As early-universe data continues to improve and cosmological tensions persist, it is pertinent to explore whether a modified theory of gravity can alleviate such tensions. Meanwhile, if string theory is really the correct theory of quantum gravity, the 'stringy gravity' of $\{g_{\mu\nu}, B_{\mu\nu}, \phi\}$ may provide a natural candidate. In the DFT framework, the $\mathbf{O}(D,D)$ symmetry uniquely prescribes the allowed interactions between the extended gravitational sector and other matter, leading to novel features beyond conventional string cosmology.

Double field theory

In double field theory [2, 3] we describe D-dimensional physics using $D+D$ coordinates, $x^A = (\tilde{x}_\mu, x^\nu)$, $A = 1, \ldots, 2D$. Doubled vector indices are raised and lowered using the $\mathbf{O}(D,D)$-invariant

$$\mathcal{J}_{AB} = \begin{pmatrix} 0 & \mathbf{1}_D \\ \mathbf{1}_D & 0 \end{pmatrix} = \mathcal{J}^{AB}\,.$$

What does it mean to 'double' the number of spacetime dimensions? Consistency requires a section condition,

$$\partial_A \partial^A = 2\,\partial_\mu \tilde{\partial}^\mu = 0\,.$$

A natural choice is $\tilde{\partial}^\mu = 0$, i.e. choosing all fields to be independent of the $\tilde{x}_\mu$ coordinates. The theory is not truly 'doubled', rather it is just a convenient repackaging of D-dimensional physics.

• **Field content of DFT:** A symmetric $\mathbf{O}(D,D)$ generalized metric $\mathcal{H}_{AB}$ and a DFT dilaton d.

• **Reduction to closed-string massless sector ('bosonic supergravity'):**
On spacetime backgrounds, under the section choice $\tilde{\partial}^\mu \equiv 0$, the fields $\{\mathcal{H}_{AB}, d\}$ reduce to $\{g_{\mu\nu}, B_{\mu\nu}, \phi\}$ as

$$\mathcal{H}_{AB} = \begin{pmatrix} g^{\mu\nu} & -\,g^{\mu\sigma}B_{\sigma\nu} \\ B_{\mu\rho}g^{\rho\nu} & g_{\mu\nu} - B_{\mu\rho}g^{\rho\sigma}B_{\sigma\nu} \end{pmatrix}\,, \qquad e^{-2d} = e^{-2\phi}\sqrt{-g}\,.$$

Doubled diffeomorphism symmetry reduces to ordinary diffeomorphisms and B-field gauge transformations. The DFT Ricci scalar $\mathcal{R}$ gives the 'stringy' gravitational Lagrangian [4],

$$\mathcal{R} = R + 4\Box\phi - 4\partial_\mu\phi\partial^\mu\phi - \tfrac{1}{12}H_{\lambda\mu\nu}H^{\lambda\mu\nu}\,,$$

where R is the usual Ricci scalar from $g_{\mu\nu}$, and $H_{\lambda\mu\nu} = 3\partial_{[\lambda}B_{\mu\nu]}$ is the field strength of $B_{\mu\nu}$ ('H-flux').

Einstein Double Field Equations

We can extend $\mathbf{O}(D,D)$ covariance to interactions with matter $\{\Upsilon_a\}$ via an action of the form

$$S = \int_\Sigma e^{-2d}\Big[\tfrac{1}{16\pi G}\mathcal{R} + L_{\rm m}(\Upsilon_a)\Big]\,,$$

where $L_{\rm m}$ is an $\mathbf{O}(D,D)$-covariant Lagrangian for the additional matter fields $\{\Upsilon_a\}$, and the integral is taken over a D-dimensional section Σ. Crucially, the $\mathbf{O}(D,D)$ symmetry fixes the covariant integration measure to be e^{-2d}, and not simply $\sqrt{-g}$ as in GR. This constrains the allowed couplings between stringy gravity and additional matter. In particular, the possibility of 'minimal coupling' in Einstein frame is not guaranteed.

Varying this action yields a DFT generalization of Einstein's equations, or 'Einstein double field equations' [5],

$$G_{AB} = 8\pi G T_{AB}\,,$$

where the DFT energy-momentum tensor T_{AB} is conserved on-shell. On spacetime backgrounds, the Einstein double field equations reduce to the usual closed-string massless sector equations of motion plus source terms,

$$R_{\mu\nu} + 2\nabla_\mu\nabla_\nu\phi - \tfrac{1}{4}H_{\mu\rho\sigma}H_\nu{}^{\rho\sigma} = 8\pi G K_{(\mu\nu)}\,,$$
$$\nabla^\rho\Big(e^{-2\phi}H_{\rho\mu\nu}\Big) = 16\pi G e^{-2\phi}K_{[\mu\nu]}\,,$$
$$R + 4\Box\phi - 4\nabla_\mu\phi\nabla^\mu\phi - \tfrac{1}{12}H_{\lambda\mu\nu}H^{\lambda\mu\nu} = 8\pi G T_{(0)}\,.$$

From the DFT perspective, $K_{\mu\nu}$ sources $\mathcal{H}_{AB}$ and $T_{(0)}$ sources d. We can arrange these into source terms for

$$\delta g_{\mu\nu}: \quad T_{\mu\nu} \equiv e^{-2\phi}\Big(K_{(\mu\nu)} - \frac{1}{2}g_{\mu\nu}T_{(0)}\Big)\,,$$
$$\delta B_{\mu\nu}: \quad \Theta_{\mu\nu} \equiv e^{-2\phi}K_{[\mu\nu]}\,,$$
$$\delta\phi: \quad \sigma \equiv e^{-2\phi}T_{(0)}\,.$$

Here $T_{\mu\nu}$ is the usual energy-momentum tensor in GR, while $\Theta_{\mu\nu}$ and σ represent sources for $B_{\mu\nu}$ and ϕ, respectively. Conservation of the DFT energy-momentum tensor gives rise to generalized conservation laws,

$$\nabla^\mu T_{\mu\nu} + \frac{1}{2}H_{\nu\mu\lambda}\Theta^{\mu\lambda} - \nabla_\nu\phi\,\sigma = 0\,, \qquad \nabla^\mu\Theta_{\mu\nu} = 0\,.$$

Homogeneous and isotropic backgrounds

Now consider homogeneous and isotropic backgrounds in $D=4$. The most general gravitational ansatz is

$$\mathrm{d}\bar{s}^2 = -N(t)^2\mathrm{d}t^2 + a(t)^2\left[\frac{\mathrm{d}r^2}{1-Kr^2} + r^2\mathrm{d}\Omega^2\right]\,, \quad \bar{B}_{(2)} = \frac{hr^2}{\sqrt{1-Kr^2}}\cos\vartheta\,\mathrm{d}r\wedge\mathrm{d}\varphi\,, \quad \bar{\phi} = \bar{\phi}(t)\,,$$

where $\bar{B}_{(2)} \equiv \frac{1}{2}\bar{B}_{\mu\nu}\,\mathrm{d}x^\mu\wedge\mathrm{d}x^\nu$. Note that the corresponding H-flux $\bar{H}_{(3)} \equiv \mathrm{d}\bar{B}_{(2)} = h\,\mathrm{d}Vol_3$, where h is constant and $\mathrm{d}Vol_3$ is the spatial volume form. Similarly the DFT energy-momentum tensor is constrained as

$$\bar{T}^\mu{}_\nu = \begin{pmatrix} -\,\bar{\rho}(t) & 0 \\ 0 & \bar{p}(t)\delta^i{}_j \end{pmatrix}\,, \qquad \bar{\Theta}_{\mu\nu} = 0\,, \qquad \bar{\sigma} = \bar{\sigma}(t)\,.$$

The generalized conservation law gives one non-trivial equation:

$$\bar{\rho}' + 3NH(\bar{\rho}+\bar{p}) + \bar{\phi}'\bar{\sigma} = 0\,,$$

where $' \equiv \mathrm{d}/\mathrm{d}t$ and $H \equiv a'/(Na)$. This contains an extra term proportional to the dilaton source, $\bar{\phi}'\bar{\sigma}$. The Einstein double field equations lead to an $\mathbf{O}(D,D)$-completion of the Friedmann equations [6],

$$\frac{8\pi G}{3}\bar{\rho}e^{2\bar{\phi}} + \frac{h^2}{12a^6} = H^2 - 2\left(\frac{\bar{\phi}'}{N}\right)H + \frac{2}{3}\left(\frac{\bar{\phi}'}{N}\right)^2 + \frac{K}{a^2}\,, \qquad (1)$$

$$\frac{4\pi G}{3}(\bar{\rho}+3\bar{p})e^{2\bar{\phi}} + \frac{h^2}{6a^6} = -H^2 - \frac{H'}{N} + \left(\frac{\bar{\phi}'}{N}\right)H - \frac{2}{3}\left(\frac{\bar{\phi}'}{N}\right)^2 + \frac{1}{N}\left(\frac{\bar{\phi}'}{N}\right)'\,, \qquad (2)$$

$$\frac{4\pi G}{3}(2\bar{\rho}-\bar{\sigma})e^{2\bar{\phi}} = -H^2 - \frac{H'}{N} + \frac{2}{3N}\left(\frac{\bar{\phi}'}{N}\right)'\,. \qquad (3)$$

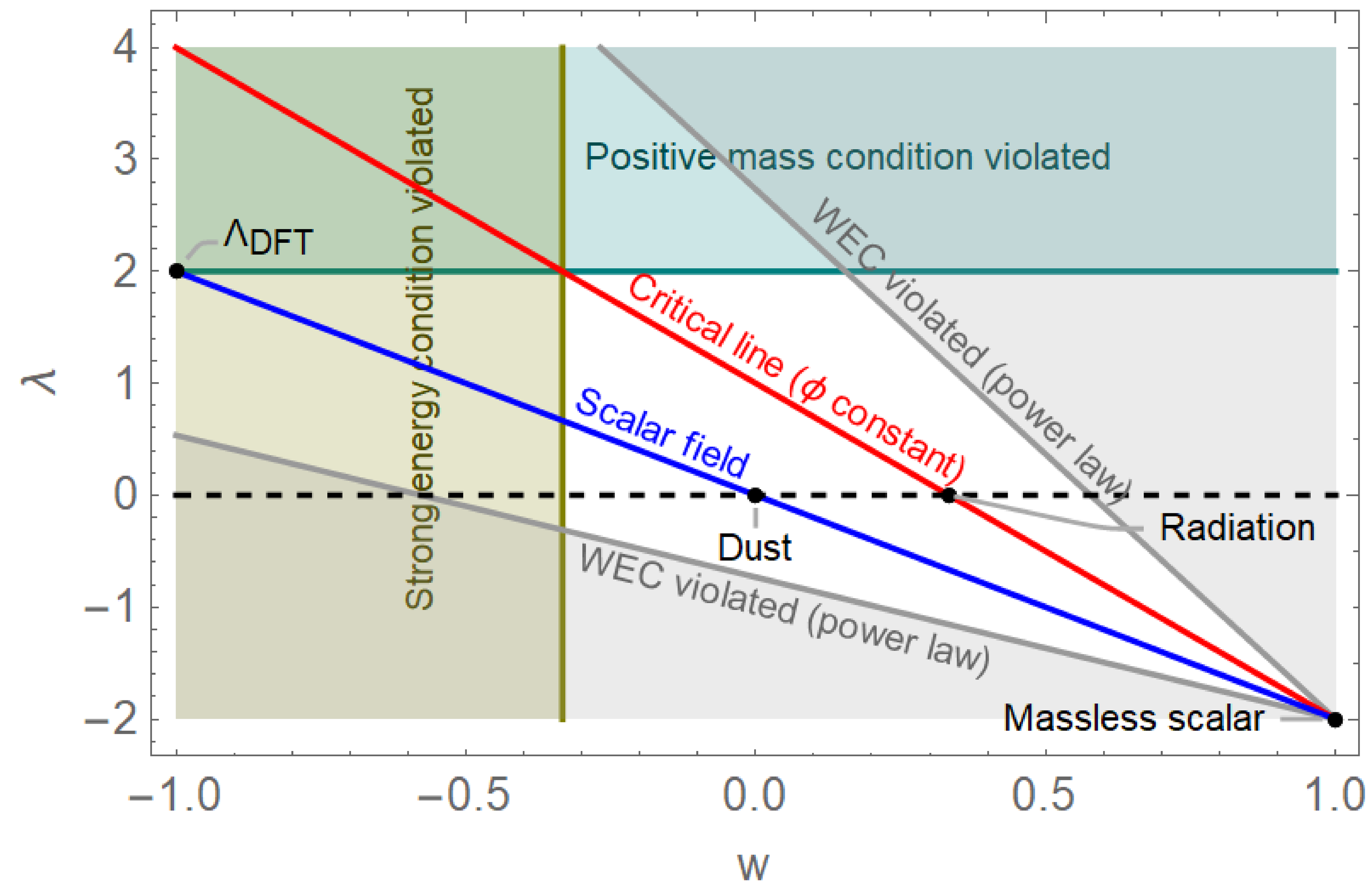
If $\bar{\phi}' = \bar{\phi}'' = 0$ and $h = 0$, we make contact with GR cosmology on the critical line, $\bar{\sigma} = \bar{\rho} - 3\bar{p}$.

Solutions

We can define two equation-of-state parameters:

$$w \equiv \frac{\bar{p}}{\bar{\rho}}\,, \qquad \lambda \equiv \frac{\bar{\sigma}}{\bar{\rho}}\,.$$

• **Power-law solutions:**
For constant w and λ ('generalized perfect fluid'), various power-law solutions with $\rho = \rho_0\,a^{-3(1+w)}e^{-\lambda\phi}$.



• **Solutions beyond power law:** Many analytic solutions, e.g. radiation plus a massless scalar field $\bar{\Phi}$:

$$\bar{\rho}_{\bar{\Phi}} = \tfrac{1}{2}\bar{\Phi}'^2a^{-2}e^{-2\bar{\phi}}\,, \qquad \bar{\rho}_{\rm r} = \bar{\rho}_{\rm r0}a^{-4}\,, \qquad \bar{\rho} = \bar{\rho}_{\bar{\Phi}} + \bar{\rho}_{\rm r} \qquad \Rightarrow \qquad w \in [1/3, 1]\,, \qquad \lambda = 1 - 3w\,;$$

$$a^2 = \frac{\tau(C_1+\mathcal{E}_0\tau)}{1+K\tau^2}e^{2\bar{\phi}}\,, \quad e^{2\bar{\phi}} = \left(\frac{C_1\tau}{\tau_*(C_1+\mathcal{E}_0\tau)}\right)^{\frac{h_{\rm o}}{C_1}} + \frac{h^2}{4h_{\rm o}^2}\left(\frac{C_1\tau}{\tau_*(C_1+\mathcal{E}_0\tau)}\right)^{-\frac{h_{\rm o}}{C_1}}\,, \quad \tau = \begin{cases} \tan(\eta-\eta_0)\,, & K = 1\,, \\ \eta - \eta_0\,, & K = 0\,, \\ \tanh(\eta-\eta_0)\,, & K = -1\,. \end{cases}$$

Here η is conformal time, η_0, C_1, τ_* and $h_{\rm o}$ are constants, and $\mathcal{E}_0 \equiv 8\pi G\bar{\rho}_0/3$. The dilaton is constant at late times for $K \in \{0,-1\}$, and the scale factor has a bounce due to the background H-flux (c.f. $h=0$ case [7]). It is pure scalar for $\mathcal{E}_0 = 0$ and pure radiation for $h_{\rm o} = \pm\sqrt{3}C_1$ (both conditions together: 'DFT vacuum' [7, 8]).

Cosmological perturbations

Many early-universe phenomena depend on the nature of small fluctuations around a homogeneous and isotropic background. From now on, consider a flat universe, $K=0$ and work in conformal gauge ($N = a$).

• **Gravitational perturbations:**
Expanding to linear order, the perturbed metric, B-field and dilaton may be written as

$$g_{\mu\nu} \equiv a^2\begin{pmatrix} -(1+2A) & B_j \\ B_i & \delta_{ij}+h_{ij} \end{pmatrix}\,, \qquad B_{(2)} \equiv \bar{B}_{(2)} + f_i\,\mathrm{d}x^i\wedge\mathrm{d}\eta + \frac{1}{2}m_{ij}\,\mathrm{d}x^i\wedge\mathrm{d}x^j\,, \qquad \phi = \bar{\phi} + \delta\phi\,.$$

Under a scalar-vector-tensor (SVT) decomposition, the perturbations can be separated as

$$B_i \equiv \hat{B}_i + \partial_i B\,, \qquad h_{ij} \equiv 2C\delta_{ij} + 2(\partial_i\partial_j - \frac{1}{3}\delta_{ij}\nabla^2)E + 2\partial_{(i}\hat{E}_{j)} + \hat{E}_{ij}\,,$$
$$f_i = \partial_i f + \hat{f}_i\,, \qquad m_{ij} = \partial_i\hat{m}_j - \partial_j\hat{m}_i + \epsilon_{ijk}\partial^k m\,,$$

where $\partial^i\hat{B}_i = \partial^i\hat{E}_i = \partial^i\hat{E}_{ij} = 0$, $\partial^i\hat{f}_i = \partial_i\hat{m}^i = 0$ and $\hat{E}^i{}_i = 0$. It turns out that f and one combination of $\hat{f}_i$ and $\hat{m}_i$ are pure gauge and drop out of the equations of motion.

• **Matter perturbations:**
The components of $T_{\mu\nu}$ have the standard perturbative expansions,

$$\rho = \bar{\rho} + \delta\rho\,, \qquad p = \bar{p} + \delta p\,, \qquad \delta T^i{}_0 \equiv (\bar{\rho}+\bar{p})\,v^i\,, \qquad p\pi^i{}_j \equiv T^i{}_j - \frac{1}{3}\delta^i{}_jT^k{}_k\,,$$

while the perturbed source terms for the B-field and dilaton take the form

$$\Theta_{(2)} \equiv \mathcal{J}_i\mathrm{d}x^i\wedge\mathrm{d}t + \frac{1}{2}\epsilon_{ijk}\mathcal{I}^k\mathrm{d}x^i\wedge\mathrm{d}x^j\,, \qquad \sigma = \bar{\sigma} + \delta\sigma\,.$$

Under an SVT decomposition, with $\partial_i\hat{v}^i = \partial^i\hat{\pi}_i = 0$, $\partial^i\hat{\mathcal{J}}_i = \partial_i\hat{\mathcal{I}}^i = 0$, $\partial^i\pi_{ij} = 0$ and $\hat{\pi}^i{}_i = 0$, expand

$$v^i = \hat{v}^i + \partial^i v\,, \qquad \pi_{ij} = (\partial_i\partial_j - \frac{1}{3}\delta_{ij}\nabla^2)\pi_{\rm T} + \partial_{(i}\hat{\pi}_{j)} + \hat{\pi}_{ij}\,, \qquad \mathcal{J}_i = \partial_i\mathcal{J} + \hat{\mathcal{J}}_i\,, \qquad \mathcal{I}^i = \partial^i\mathcal{I} + \hat{\mathcal{I}}^i\,.$$

• **Perturbed equations of motion:**
We obtained the equations of motion for linear perturbations in DFT cosmology. For the scalar perturbations, we find six non-trivial equations that separate to damped oscillator equations for $h=0$ [9], but non-vanishing H-flux induces couplings between the different components. In the superhorizon limit, the equations can be solved for various known analytic solutions, and the evolution of fluctuations is adiabatic in this limit.

• **Perturbed conservation equations:**
Expanding in Fourier modes, $\partial_i \to ik_i$, the two non-trivial perturbed conservation equations are

$$0 = \delta\rho' + 3aH(\delta\rho+\delta p) + \bar{\phi}'\delta\sigma + \delta\phi'\bar{\sigma} - k^2(\bar{\rho}+\bar{p})\,v\,,$$
$$0 = [(\bar{\rho}+\bar{p})(v+B)]' + 4aH[(\bar{\rho}+\bar{p})(v+B)] + \delta p + (\bar{\rho}+\bar{p})A - \frac{2}{3}k^2\bar{p}\pi_{\rm T} + \frac{h}{a^4}\mathcal{I} - \bar{\sigma}\delta\phi\,.$$

When $\bar{\sigma} = 0 = \delta\sigma$ and for $k \to 0$, the former implies that adiabatic superhorizon perturbations are conserved. However, when $\bar{\sigma}$ or $\delta\sigma$ are nonzero, this is no longer true in general. Nevertheless, under certain conditions (e.g. constant w and λ), a conserved variable may be identified by integrating the first equation.

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