STAR-CROSSED: MODEL-INDEPENDENT CONSISTENCY OF SUPERNOVA COSMOLOGY

William L. Matthewson

Korea Astronomy and Space Science Institute, (willmatt4th@gmail.com)

$\lesssim 1$ min

Introduction

Modern-day cosmology's description of the **dark sector** is **incomplete**.

Different observations of the Universe **disagree** on the values of parameters, e.g. H_0 **tension**.

Recent results from DESI even hint at **behaviour beyond flat ΛCDM** [3].

We take three **supernovae compilations** (Union3 [5], Pantheon+ [1], DES Year 5 [2]), and test their **mutual consistency** in an attempt to detect additional sources of **systematics** or **behaviour beyond the standard model**.

Methods I

We start by **expanding the parameter space**.

Take **dataset A**; it gives rise to **"Template Function" A** that contains information about cosmology and the dataset.

We **deform** this using a polynomial basis, **parametrized by C_i**.

Cross-checking with the **other datasets**, we constrain these parameters to determine **how consistent** dataset A is with the other two, **at the data level**.

Methods II

We use **two types** of template functions:

1) The ΛCDM model best fit to the dataset.
2) An "iterative smoothing" function.

The latter is a **model-independent** approach which produces a function containing the characteristics of the data.

This secondary analysis is useful, since **assuming one cosmological model** may **limit the sensitivity** of the consistency test if the true model is different.

Conclusion

There is a **high level of consistency** between all three datasets considered.

With both types of template functions, **no deformations** are favoured at **more than $\sim 2\,\sigma$**.

In all cases, it is possible to find regions within the extended parameter space that **can produce all three datasets simultaneously, at $1-2\sigma$**.

$\lesssim 5$ min

Supernova Cosmology

Type 1a supernova light curves can be standardised to determine their intrinsic luminosity. This allows us to measure distances in cosmology [4]:

Distance Modulus $\rightarrow \mu(z) = m(z) - M_B = 5\log\left(\frac{D_L(z)}{10\text{pc}}\right) \leftarrow$ **Luminosity Distance**

Dataset	#Supernovae	z range
Union3 [5]	2087 (22 nodes)	$0.05 < z < 2.26$
DES Year 5 [1]	$194\ (z<0.1) + 1635$	$0.025 < z < 1.3$
Pantheon+ [2]	1550	$0.01 < z < 2.26$

Fig. 1: **Details of supernova datasets used in the analysis.**

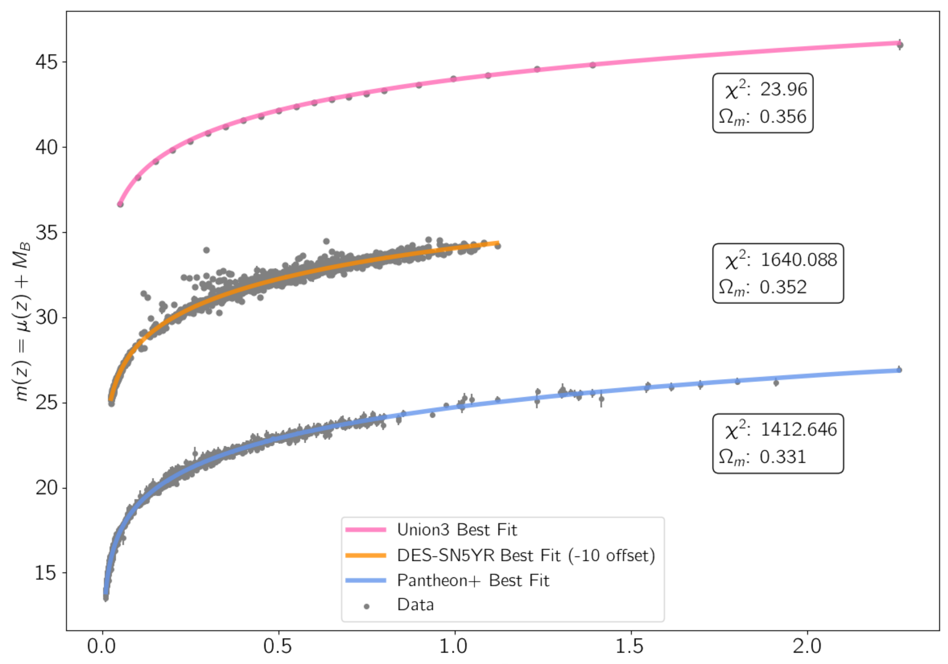

Fig. 2: **Best fit $m(z)$ results for each of the three datasets.**

Using the **flat ΛCDM model** to fit these data, we obtain our first kind of "Template Function", containing information particular to each dataset, that we **compare** in the context of the other datasets.

Expanding the Parameter Space

We use a **Chebyshev polynomial** basis $T_i(x)$ to create **Crossing Functions**, that deform the Template Functions. These are **parametrized by coefficients C_i**, as shown in Fig. 3.

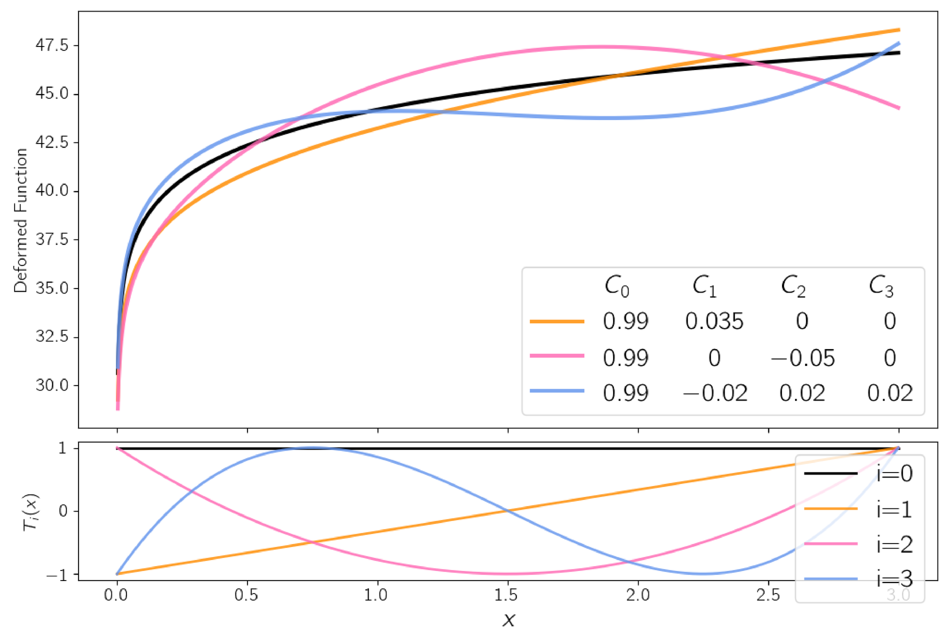

Fig. 3: **Examples of deformations to a Template Function (black) using the Chebyshev basis.**

Model-Independent Template functions

Our second Template Function is generated using a process called **Iterative Smoothing**, that produces a function following the data, but **independent of cosmological model parameter** input.

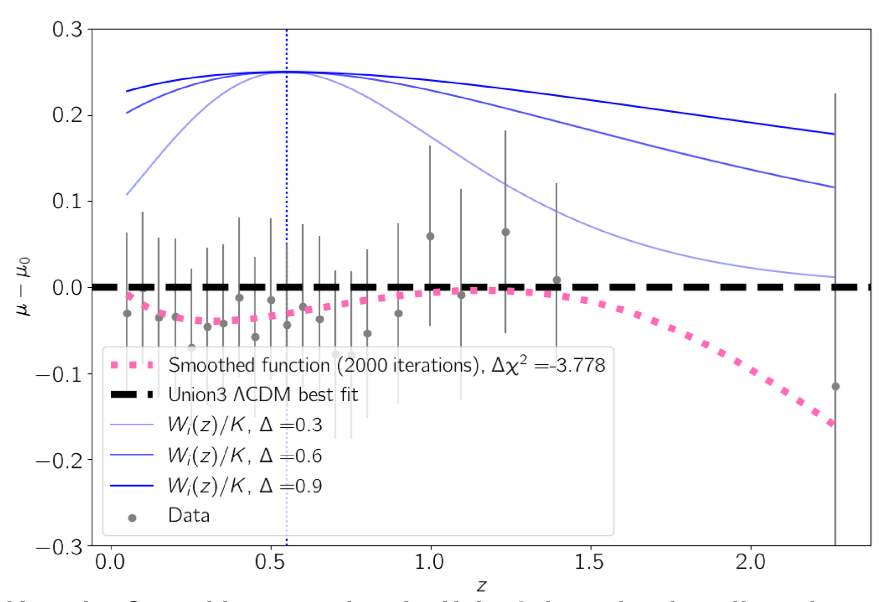

Fig. 4: **Result of Iterative Smoothing procedure for Union3 data, showing effect of smoothing width Δ.**

χ^2 - statistic of interest

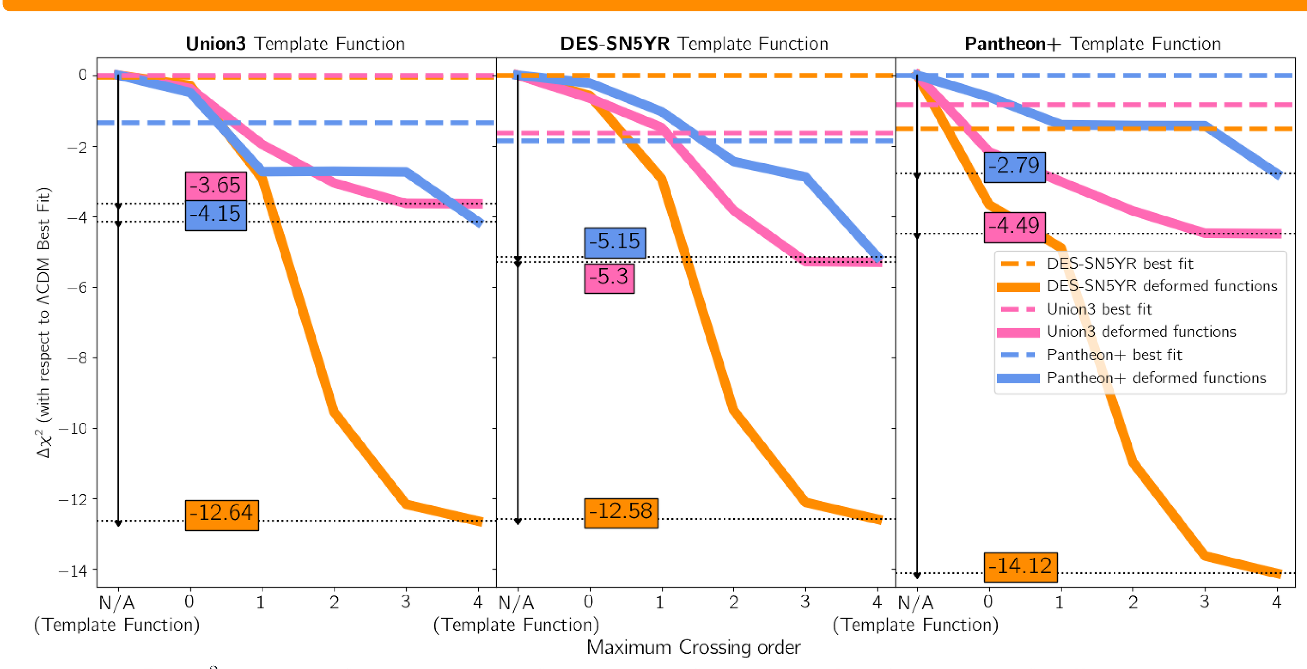

Fig. 5: $\Delta\chi^2$ **achieved from various deformations of each template function to fit all three datasets.**

The improvement in χ^2 **surpasses each dataset's own best fit** (dashed lines) at **low order** ($0 \sim 1$). This indicates the data are receptive to **different template functions**, and the parametrization is effective in revealing useful **extended behaviours** in the data. But, if we want to gauge the level of consistency, we need to compare the **types and degrees of deformations**.

> 5 min

References

[1] T. M. C. Abbott et al. "The Dark Energy Survey: Cosmology Results With ~1500 New High-redshift Type Ia Supernovae Using The DESI 2024 VI: Cosmological Constraints from the Measure-Full 5-year Dataset". In: (Jan. 2024). arXiv: 2401.02929 [astro-ph.CO].

[2] Dillon Brout et al. "The Pantheon+ Analysis: Cosmological Constraints". In: *Astrophys. J.* 938.2 (2022), p. 110. DOI: 10.3847/1538-4357/ac8e04. arXiv: 2202.04077 [astro-ph.CO].

[3] DESI Collaboration et al. *DESI 2024 VI: Cosmological Constraints from the Measurements of Baryon Acoustic Oscillations*. 2024. arXiv: 2404.03002.

[4] Adam G. Riess et al. "Observational evidence from supernovae for an accelerating universe and a cosmological constant". In: *Astron. J.* 116 (1998), pp. 1009–1038. DOI: 10.1086/300499. arXiv: astro-ph/9805201.

[5] David Rubin et al. "Union Through UNITY: Cosmology with 2,000 SNe Using a Unified Bayesian Framework". In: (Nov. 2023). arXiv: 2311.12098 [astro-ph.CO].

[6] Arman Shafieloo, Timothy Clifton, and Pedro G. Ferreira. "The Crossing Statistic: Dealing with Unknown Errors in the Dispersion of Type Ia Supernovae". In: *JCAP* 08 (2011), p. 017. DOI: 10.1088/1475-7516/2011/08/017. arXiv: 1006.2141 [astro-ph.CO].

[7] Arman Shafieloo et al. "Smoothing Supernova Data to Reconstruct the Expansion History of the Universe and its Age". In: *Mon. Not. Roy. Astron. Soc.* 366 (2006), pp. 1081–1095. DOI: 10.1111/j.1365-2966.2005.09911.x. arXiv: astro-ph/0505329.

Crossing Statistics [7]

The **deformations** are performed according to the following **prescription**, where we multiply by a linear combination of Chebyshev polynomials, truncated at finite order N:

$$\mu^{\text{DEFORMED}}(z) = \mu^{\text{TEMPLATE}}(z)\sum_{i=0}^{N} C_i^N T_i(\tilde{z}), \tag{1}$$

The results in Fig. 5 suggest that at least 0^{th} or 1^{st} order are required. We should also ensure that the resulting deformed function remains **physically realistic**.

The extended parameter space now contains a number of **Crossing Hyperparameters**, C_i, up to some chosen maximum order. These will determine the **extent and type of deformations** applied to the Template Function for some dataset A. We sample in C_i, minimising the χ^2 calculated for a particular dataset B, to determine the **posteriors of the parameter values**.

Each Template Function A is then tested in the context of all the datasets to determine the level of consistency.

Iterative Smoothing [6]

In order to produce the second, model-independent form of the Template Function, we start from an **initial guess** for the distance modulus $\mu_0(z_i)$ for datapoint redshifts z_i. In this case we use the **flat ΛCDM best fit** from the dataset; though, after sufficiently many iterations, the result is insensitive to this choice.

In the k^{th} iteration, the following calculations are performed:

The Residual Vector:
$$(\delta\mu_k)_i = \mu_k(z_i) - \mu^{\text{data}}(z_i) \tag{2}$$

The Next Guess:
$$\mu_{k+1}(z) = \mu_k(z) + \frac{\boldsymbol{\delta\mu}_k^T \cdot \mathrm{C}^{-1} \cdot \mathbf{W}(z)}{\mathbb{1}^T \cdot \mathrm{C}^{-1} \cdot \mathbf{W}(z)} \tag{3}$$

where the residuals are smoothed according to the covariance matrix C and a window function:

$$\boldsymbol{W}_i(z) = e^{\left(-\frac{\ln^2\left(\frac{1+z}{1+z_i}\right)}{2\Delta^2}\right)} \tag{4}$$

(see Fig. 4 for some examples of the window function for various values of smoothing width Δ). Iterative smoothing provides a **function passing through the data** that, while not judged for its fit, is at least a **viable functional form**, which does not rely on a cosmological model or parameters.

Contour plots (~ 0 min)

In Fig. 6 we plot the **posteriors** from one example of the analysis - we use the Template Function from **Union3**, fit to all 3 datasets over the redshift range of DES Year 5 supernovae (DES-SN5YR). From left to right, we consider **three cases** with 1, 2 and 3 added degrees of freedom, by setting the respective **maximum orders of the Crossing Functions** with non-zero C_i to $i = 0, 1$ and 2.

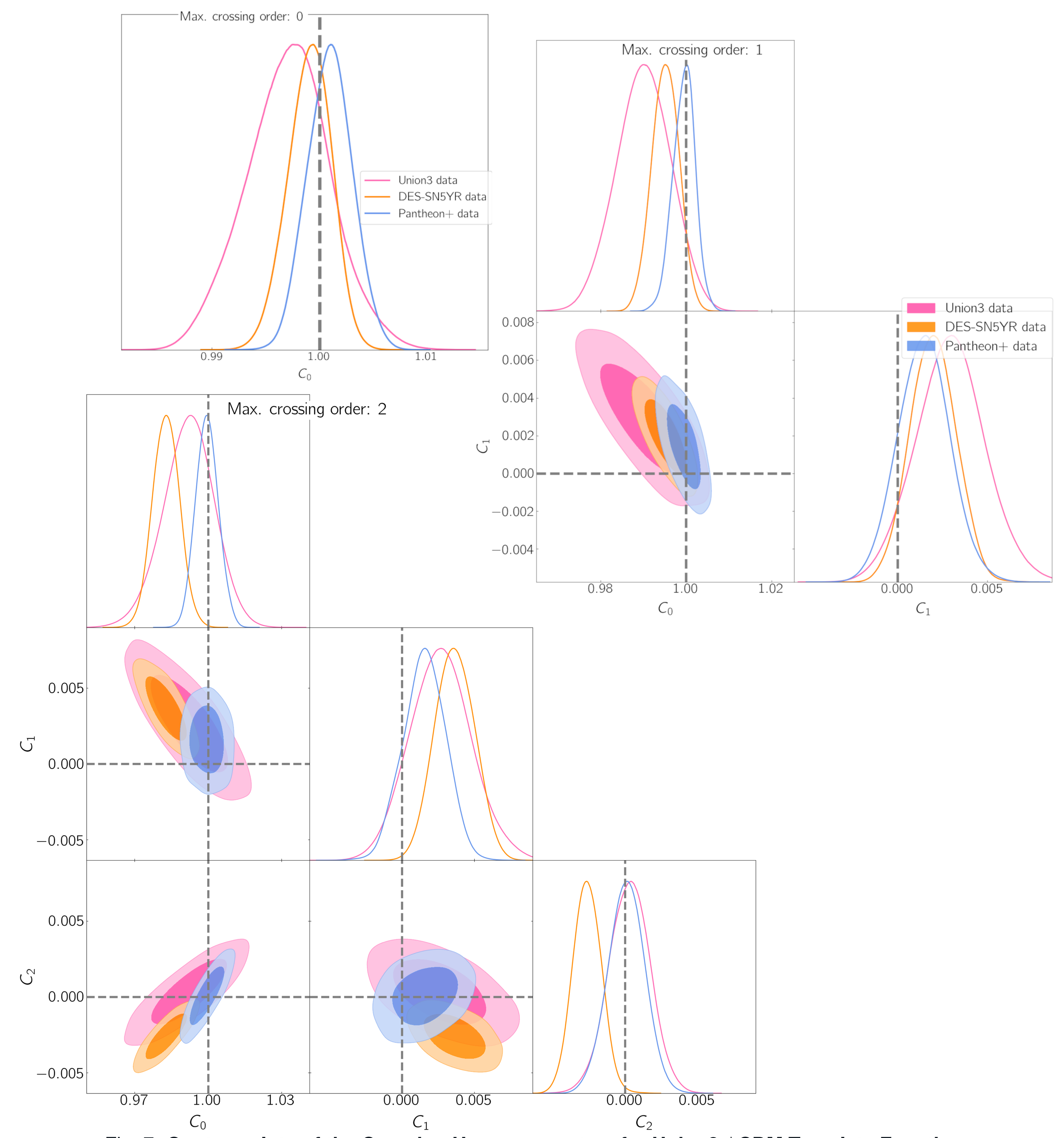

Fig. 7: **Contour plots of the Crossing Hyperparameters for Union3 ΛCDM Template Function.**

The point at $C_0 = 1$ and $C_{j\neq 0} = 0$ is the fiducial point corresponding to the **undeformed Template Function**; in this case the flat ΛCDM best fit from Union3.

The samples from all datasets are **consistent to within about 2σ** with this point, with DES being slightly further away. However, it is always possible to find an **overlapping space of preferred deformations**, where all three datasets may be simultaneously realised, at the $1-2\sigma$ level.

This suggests a **high level of mutual consistency** within the framework of our analysis.

In continuation of this work we hope to test the **sensitivity to particular forms of systematics**, as well as examine more closely the various kinds of deformations that are preferred, and what **implications** they may have for cosmology.