MSc Dissertation Project: The Plotting and Analysis of Complex Spectral Curves from Black Holes through AdS/CFT Correspondence and Hydrodynamics

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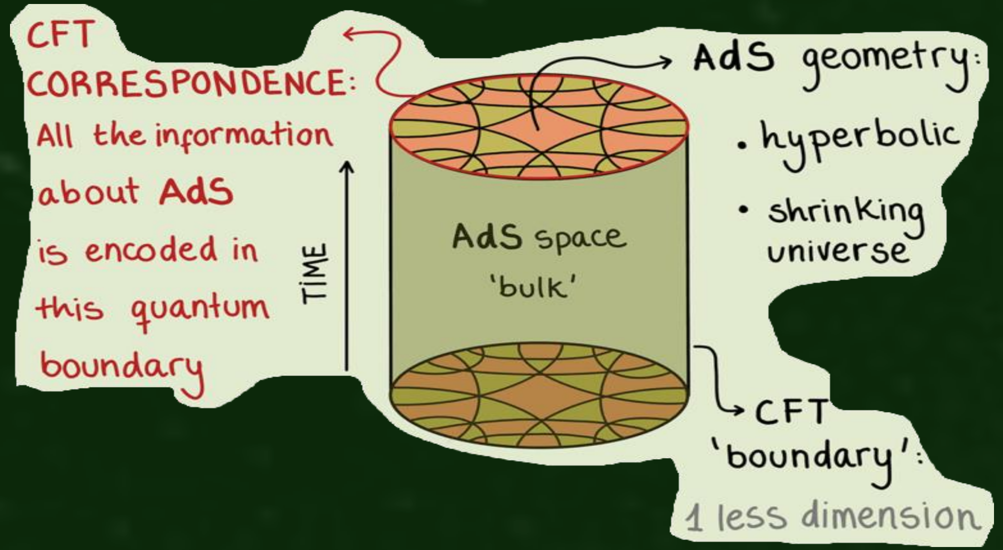
Abstract

This project aims to describe the behavior of strongly-coupled fluid-like particles, or plasma, in the vicinity of a black hole. To achieve this, we employ holography as the foundational framework, utilizing the AdS/CFT correspondence to translate the otherwise intractable large-N SYM problems into a classical approximation within gravity theory, allowing us to obtain quasinormal modes.



We then apply hydrodynamics to explore the dispersion relation and compute complex spectral curves. Through detailed plotting,we identify some concepts with physical implications such as critical points and convergence, and extend to more advanced theoretical interpretations such as pole-skipping and univalence.

Quasinormal Modes – Gravitational Perturbations

QNMs describe the oscillations of a perturbed black hole, characterized by its complex frequencies and horizon boundary condition;

- General AdS metric, black brane $p=3$ case:

$$ds^2 = \frac{(\pi T R)^2}{u}\left(-f(u)dt^2 + dx^2 + dy^2 + dz^2\right) + \frac{R^2}{4u^2 f(u)}du^2,$$

- Perturbed metric:

$g_{\mu\nu} = g^{(0)}_{\mu\nu} + h_{\mu\nu}$, all perturbations in the form: $h_{\mu\nu}(r)e^{-i\omega t + iqz}$

- Modes of perturbations: $a = x, y$

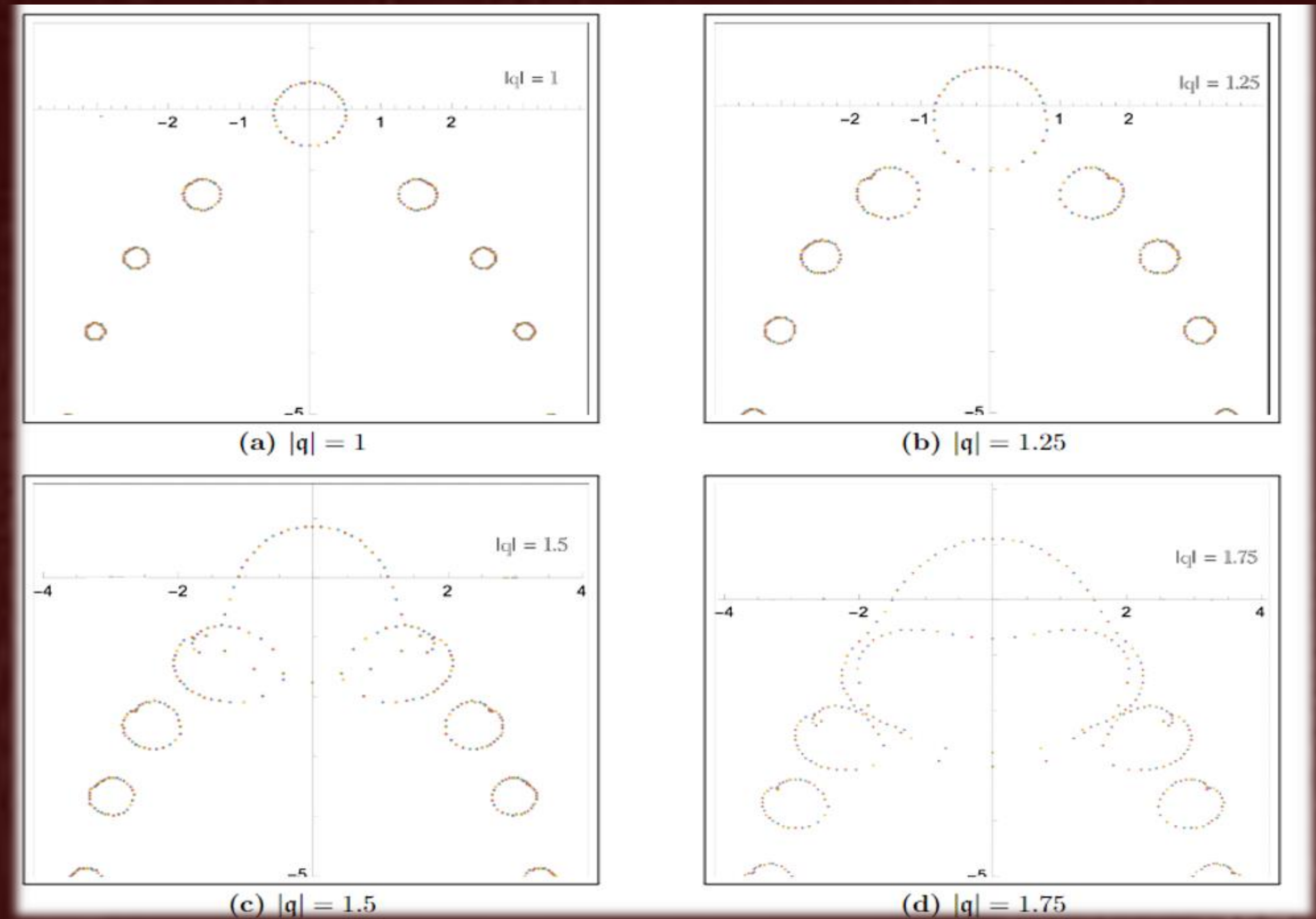
Spin 0 (sound channel): $h_{tt},\ h_{tz},\ h_{zz},\ h,\ h_{rr},\ h_{tr},\ h_{zr}$
Spin 1 (shear channel): $h_{t\alpha},\ h_{z\alpha},\ h_{r\alpha}$
Spin 2 (scalar channel): $h_{\alpha\beta} - \delta_{\alpha\beta}h/(p-1)$.

Complex Spectral Curves

To compute it: for fluctuating hydrodynamics variables - $\phi_i = \phi_i^{(0)} + \delta\phi_i$
Build up ODEs upon it, and the spectral curve P is just the determinant of such a matrix

$$K_{ij}\delta\phi_i = 0$$
$$P = \det K$$

- **Shear Channel Example:**



(a) $|\mathbf{q}| = 1$ (b) $|\mathbf{q}| = 1.25$ (c) $|\mathbf{q}| = 1.5$ (d) $|\mathbf{q}| = 1.75$

Hydrodynamics – Dispersion Relation

Shear mode: $\omega_{\text{shear}}(\mathbf{q}) = -iDq^2 + \cdots,$
Sound mode: $\omega^{\pm}_{\text{sound}}(\mathbf{q}) = \pm v_s|\mathbf{q}| - i\frac{\Gamma_s}{2}q^2 + \cdots$

The dispersion relation is the solution of this formula: $P(\mathbf{q}^2, \omega) = 0$

With transport coefficients determined:

$$\mathfrak{w}_{\text{shear}} = -i\sum_{n=1}^{\infty} c_n \mathbf{q}^{2n}$$
$$\mathfrak{w}^{\pm}_{\text{sound}} = -i\sum_{n=1}^{\infty} a_n e^{\pm(i\pi n/2)}\mathbf{q}^n$$

n	Shear Mode c_n	Sound Mode a_n
1	0.5	0.57735
2	0.0729167	−0.340278
3	0.0177951	−0.163842
4	0.00437193	0.0364583
...	...	...

Extension: Pole-skipping and Quantum Chaos

Pole-skipping refers to a special situation in Green's function:
At certain points in frequency-momentum space, instead of a well-defined pole of G, it skips a pole.

- Documented pole-skipping points in three channels:

Scalar channel: $\mathbf{q}_* = \sqrt{\frac{3}{2}}i,\quad \mathfrak{w}_* = -i$
Shear channel: $\mathbf{q}_* = \sqrt{\frac{3}{2}},\quad \mathfrak{w}_* = -i$
Sound channel: $\mathbf{q}_* = \sqrt{\frac{3}{2}}i,\quad \mathfrak{w}_* = i$

Quantum Chaos:
usually refers to a system that cannot be calculated or described analytically and exactly, different elements inside the system disturb each other in a complicated way

The pole-skipping points *(ω*, q*)* are connected to measuring chaos through the relations where *λ_L (= 2πT)* is the Lyapunov exponent and *v_b* is the butterfly velocity
→ Maximal chaotic system

$$\omega_* = i\lambda_L,$$
$$\mathbf{q}_* = \frac{i\lambda_L}{v_b},$$