Thermodynamic analysis of black holes with cloud of strings and quintessence via Barrow entropy

Usman Zafar [1], Kazuharu Bamba [1], Tabinda Rasheed [2] and Krishnakanta Bhattacharya [1,3]

[1]Faculty of Symbiotic Systems Science, Fukushima University, Fukushima 960-1296, Japan. [2]School of Mathematics and Statistics, Nanjing University of Information Science and Technology, China. [3]Department of General Science, BITS Pilani, Dubai Campus, International Academic City, Dubai, United Arab Emirates.

Abstract

In this research work, we explore a Reissner-Nordström Anti-de Sitter (RN-AdS) black hole with a cloud of string and quintessence to study thermodynamics in the presence of Barrow entropy, which is currently being used widely as the horizon of a black hole may not be a smooth surface as described in classical general relativity but instead could have a more intricate fractal-like structure. Here, we study the impact of fractal correction parameter Δ of Barrow entropy on the thermodynamics of such BHs. We computed the first law of black hole thermodynamics and the Smarr relation for RN-AdS black hole with a cloud of string and quintessence in terms of Barrow entropy by employing the generalized formula for spherically symmetric spacetime which is directly derived from the Einstein field equation.

Introduction

Black holes are among the most fascinating objects of the universe. The existence of these objects can be predicted from Einstein's GR, which is currently the best-known theory for humankind for understanding gravitational force. Over the years, studying different black holes has opened new windows to obtain a new perspective on gravitational force. As a result, people started studying various black holes with multiple features. One such example can be the study of Letelier, which focuses on solutions to the Einstein equations for a black hole (BH) surrounded by the strings cloud and quintessence (SSCQ) [1]. This approach considers nature composed of extended objects like strings instead of point particles. Letelier's work generalizes the Schwarzschild solution by incorporating a spherically symmetric cloud of strings, modifying the gravitational field. If the BH's mass is zero, only a cloud of strings remains, resulting in a naked singularity at the origin.BH thermodynamics has become a vital field in gravitational physics since Bekenstein and Hawking's 1970s work, shedding light on quantum mechanics and gravity.

Problem with the Conventional Entropy

- **Thermodynamics of RN-AdS BH SSCQ** has been discussed with the help of Hawking-Bekenstein entropy but it also has some restrictions.
- **These additional fields** complicate the thermodynamic characteristics such as phase transition analysis and stability conditions
- **If we incorporate quantum gravitational effect** It also complicates the idea of standard entropy.

Motivation

We can address these issues by employing Barrow entropy.

- Barrow introduces the concept of fractal geometry on BH's horizon, which incorporates a gravitational quantum effect [2].
- It provides a detailed mathematical framework that may produce a novel thermodynamic behavior and phase transition of BHs.
- Barrow entropy offers a more generic approach to understanding the BH thermodynamics in the existence of additional fields.

$$S_B = \left(\frac{A}{A_0}\right)^{1+\frac{\Delta}{2}}, \tag{1}$$

- Investigating this area could provide new insights into the phase transitions and thermodynamic behavior of these unique BHs, filling a significant gap in the current research and advancing our understanding of BH thermodynamics.

Reissner-Nordström Anti-de Sitter Black Holes Surrounded by String Clouds and Quintessence: A Review

The source of this BH solution can be considered to be comprised of two distinct sources, one being the quintessence and the other being the cloud of string. Also, these two sources are linearly superposed, meaning no interaction between these two distinct parts. The energy-momentum tensor for such a scenario can be given as follows [3].

$$T_t^t = T_r^r = \rho_q + \frac{1}{r^2}\ ,\ T_\theta^\theta = T_\phi^\phi = -1/2\,(1+3\omega_q)\,\rho_q\ , \tag{2}$$

where ρ_q implies the density of the quintessence and ω_q represents the quintessential state parameter. Furthermore, the equation of state can be given as $p_q = \rho_q\,\omega_q$, where the quintessential pressure (p_q) correlates the density and the state parameter. Therefore, considering the spherical symmetry ansatz in the spacetime, the line element can be given as:

$$ds^2 = f(r)dt^2 - (f(r))^{-1}dr^2 - (d\theta^2 + \sin^2\theta d\phi^2)r^2, \tag{3}$$

where

$$f(r) = -a - \frac{2M}{r} + \frac{Q^2}{r^2} - \frac{\alpha}{r^{3\omega_q+1}} - \frac{\Lambda r^2}{3} + 1, \tag{4}$$

The above solution indicates the presence of a black hole (BH) in the spacetime and its horizon is located at r_h, where r_h is determined by the condition $f(r_h) = 0$. In addition, the cosmological constant of the AdS spacetime is denoted by Λ, which is related to the AdS curvature (l) as $\Lambda = -3/l^2$.

Thermodynamics of Black Hole Surrounded by String Clouds and Quintessence

The mass, Temperature, and heat capacity of the RN-AdS BH SSCQ in terms of Barrow entropy is obtained and it is given as

1. Mass

$$M(S_B,Q,P) = \frac{S_B^{\frac{1}{\Delta+2}}}{2\sqrt{\pi}} - \frac{aS_B^{\frac{1}{\Delta+2}}}{2\sqrt{\pi}} + \frac{4PS_B^{\frac{3}{\Delta+2}}}{3\sqrt{\pi}} + \frac{Q^2\sqrt{\pi}}{2S_B^{\frac{1}{\Delta+2}}} - \frac{\alpha\ \pi^{3\omega_q/2}}{2S_B^{\frac{3\omega_q}{\Delta+2}}}. \tag{5}$$

2. Temperature

$$\begin{aligned} T_B &= \left(\frac{\partial M}{\partial S_B}\right)_{P,\ Q,\ a,\ \alpha} \\ &= \frac{1}{\Delta+2}\left(-\frac{a}{2\sqrt{\pi}S_B^{\frac{\Delta+1}{\Delta+2}}} + \frac{4PS_B^{\frac{1-\Delta}{\Delta+2}}}{\sqrt{\pi}} - \frac{Q^2}{2S_B^{\frac{\Delta+3}{\Delta+2}}} + \frac{3\alpha\omega_q\pi^{\frac{3\omega_q}{2}}}{2S_B^{\frac{3\omega_q}{\Delta+2}+1}} + \frac{1}{2\sqrt{\pi}S_B^{\frac{\Delta+1}{\Delta+2}}}\right). \end{aligned} \tag{6}$$

3. Heat Capacity

$$C_P = \frac{(\Delta+2)S_B\left(-a\varsigma + 8\pi P\varsigma^3 - Q^2\varsigma^{-1} + 3\alpha\omega_q\,\varsigma^{-3\omega_q} + \varsigma\right)}{(a-1)(\Delta+1)\varsigma - 8\pi(\Delta-1)P\varsigma^3 + (\Delta+3)Q^2\varsigma^{-1} - 3\alpha\omega_q(\Delta+3\omega_q+2)\varsigma^{-3\omega_q}}. \tag{7}$$

where $\varsigma = \left(\pi^{-\frac{\Delta}{2}-1}S_B\right)^{\frac{1}{\Delta+2}}$.

4. $P-V$ diagram

$$T_B(P,\ V) = \left(\frac{4}{3\sqrt{\pi}V}\right)^{\frac{\Delta+2}{3}}\left(\frac{Q^2}{2(2+\Delta)\chi} + \frac{\chi(1-a)}{2(2+\Delta)} + \frac{4\pi P\chi^3}{2+\Delta} + \frac{3\alpha\omega_q}{2(2+\Delta)\chi^{3\omega_q}}\right), \tag{8}$$

Graphical Behavior

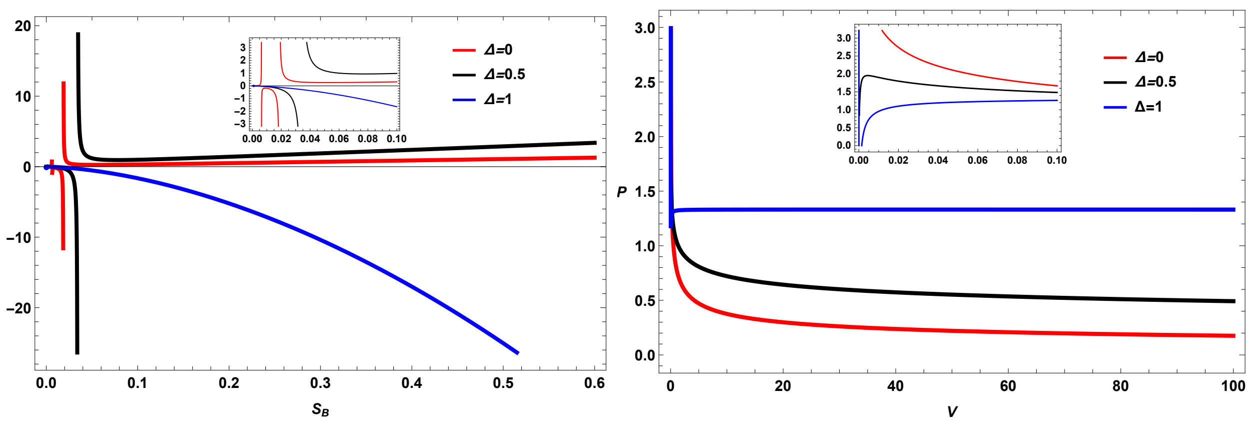

Figure 1. C_P versus S_B in left panel while in right panel we have P versus V.

- We mentioned here that $\Delta = 0$ leads us to Bekenstestein entropy while $\Delta = 1$ is for most fractile horizon structures.
- From Fig.1 left panel, it is observed that heat capacity demonstrates both stable and unstable phases.
- It can also be observed that heat capacity has a zero pint, the point at which it transits from unstable to stable phase or vice versa.
- From Fig.1 right panel, it is observed that P decreases smoothly as the volume increases for $\Delta = 0$.
- But, as the intricacy of the horizon increases, the P begins to, increase as the V increases which indicates the phase transition.

Important Conclusions

Extended First Law of BH Thermodynamics in terms of Barrow entropy

So, the generic formula to obtain the first law of BH thermodynamics is directly derived from the Einstein field equation as given in [4].

$$\delta M = T\delta S - \frac{r_h}{2}\frac{\partial f(r,M,x^i)}{\partial x^i}dx^i, \tag{9}$$

where $x^i = (x^1,\ x^2,\ x^3,\ x^4) = (P,\ Q,\ a,\ \alpha)$ in our case.

Generalized Smarr formula for Barrow entropy

The generalized Smarr formula for any spherical symmetric spacetime in the context of takes the following shape

$$M = (\Delta+2)T_BS_B - \frac{r_h^2}{2}\left(\frac{\partial f(r,\ M,\ Q,\ a,\ \alpha)}{\partial r}\right)|_{r=r_h,M=0}. \tag{10}$$

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