15th October, 2024 @Tohoku

String-wall composites from torus knot vacuum of an axionlike model and the cosmological simulations

Yuki Sakakihara

Graduate School of Science and Technology
Yamagata University

Supports

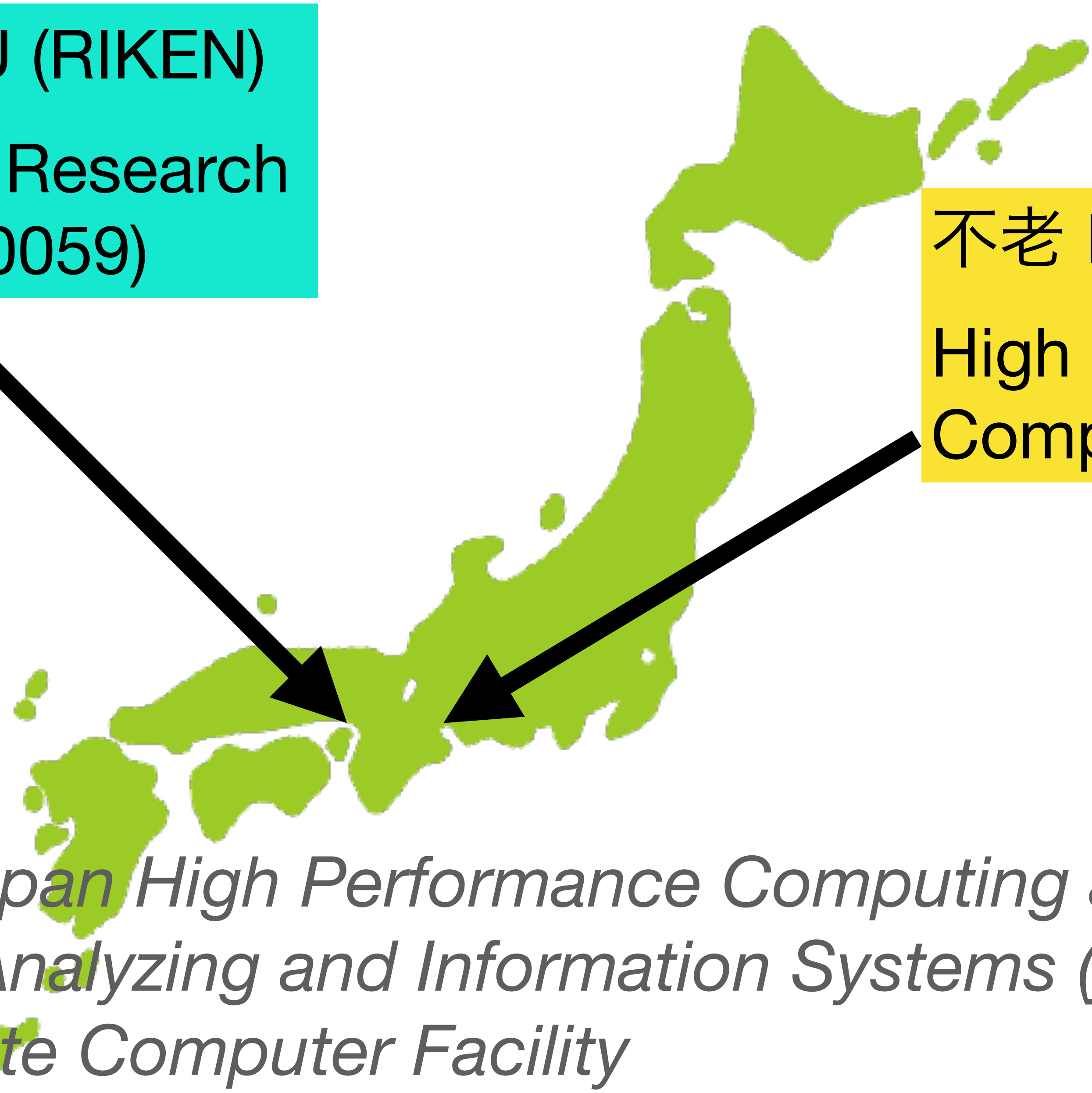
JSPS KAKENHI: JP22K20365 (YS)

富岳 FUGAKU (RIKEN)

HPCI System Research project (hp240059)

不老 Flow (Nagoya U.)

High Performance Computing (HPC2024)



Other supports: Japan High Performance Computing and Networking plus Large-scale Data Analyzing and Information Systems (jh240020), and Yukawa Institute Computer Facility

This talk is based on ...

▪#1 ***"GUTs, hybrid topological defects, and gravitational waves"***, David I. Dunsky, Anish Ghoshal, Hitoshi Murayama, Yuki Sakakihara, Graham White, PhysRevD.106.075030 (2022)

Modeling composite defects and the gravitational waves from them

▪#2 ***"String-wall composites winding around a torus knot vacuum in an axionlike model"***, Minoru Eto, Takashi Hiramatsu, Izumi Saito, Yuki Sakakihara, PhysRevD.108.116004 (2023)

Structures of the composite defects in field theoretical ground

▪#3 ***"Three-dimensional simulation of string-wall composites in an axionlike model"***, Minoru Eto, Takashi Hiramatsu, Izumi Saito, Yuki Sakakihara, Number: 86, pp. 154-161, Proceedings of JSST 2024 and AsiaSim 2024.

A benchmark simulation of the composite defects

Search for Ultimate Theory of Matter

"Standard Model (SM)" describes the law of "matter" composing us

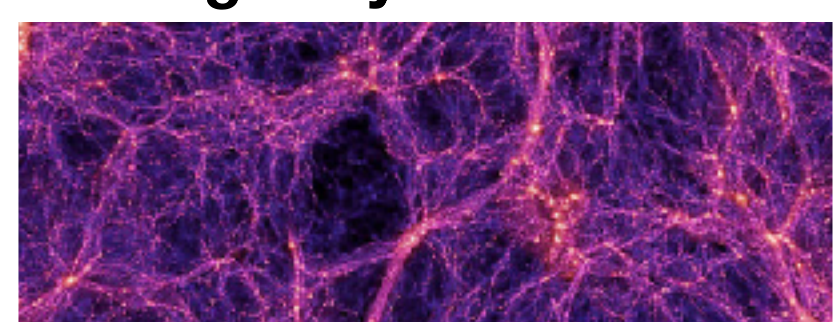
What we call "(ordinary) matter"	**Hadrons (Protons, ...), Leptons (Electrons, Neutrinos, ...), Gauge bosons (Photons, ...)**

Problem: There is a kind of "matter," not included in SM

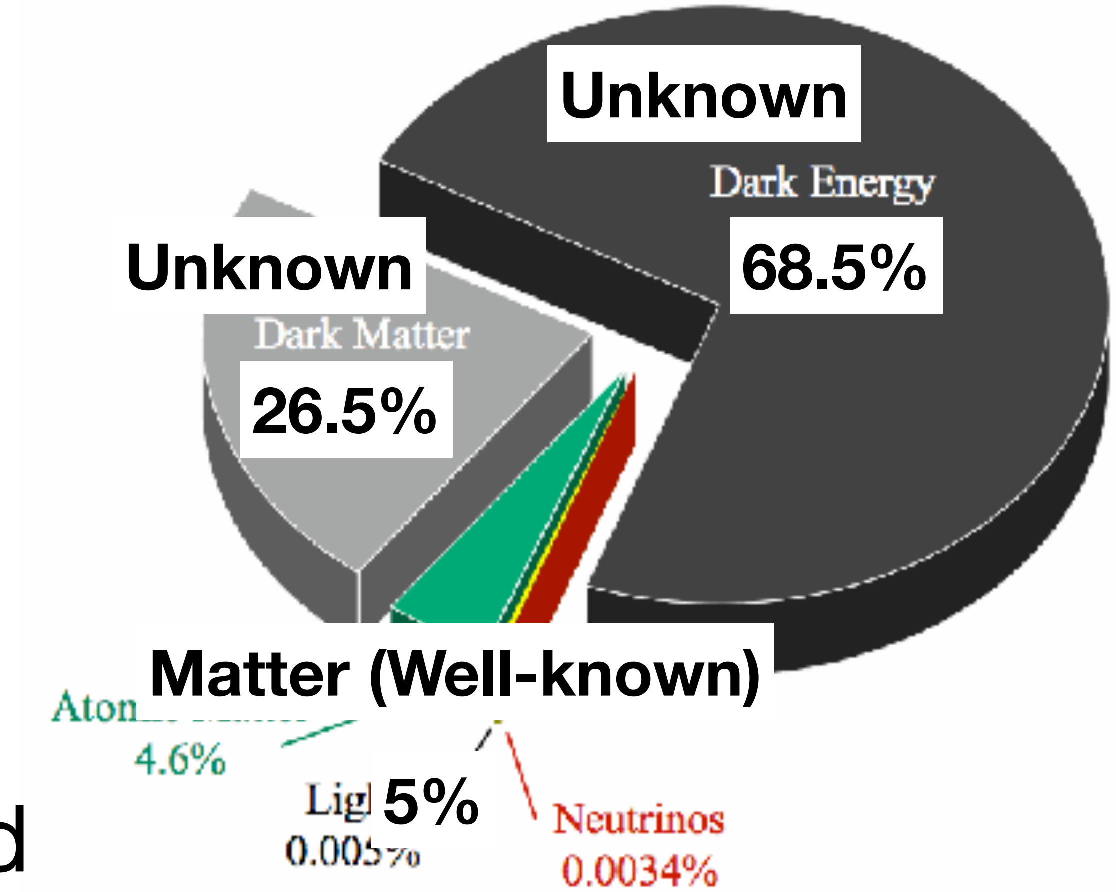
Unobservable by lights

→ Dark matter

Dark matter map estimated from galaxy observations



Dark matter



- The existence is confirmed by various observations
  - Galaxies
  - Cosmic Microwave Background

    "*Without dark matter,
    the solar system is not formed (we do not exist.)*"

- Property
  - Non-relativistic (with a low velocity << c)
  - The amount is 5 times as large as that of ordinary matter
  - Interaction with ordinary matter is weak

Dark matter candidates

- WIMPs (Weakly interacting massive particles)
  - Cross section with ordinary matter is highly constrained by direct observations
  - Natural range of the cross section to explain the amount of DM is almost excluded
- Axions
  - Domain wall (DW) problem
  - A two dimensional topological soliton (domain wall) dominates the universe today
  - Conflicts with the current observations

Our proposal: a type of scalar dark matter

- Beyond SM by two SM singlet complex scalar fields with global U(1) charges

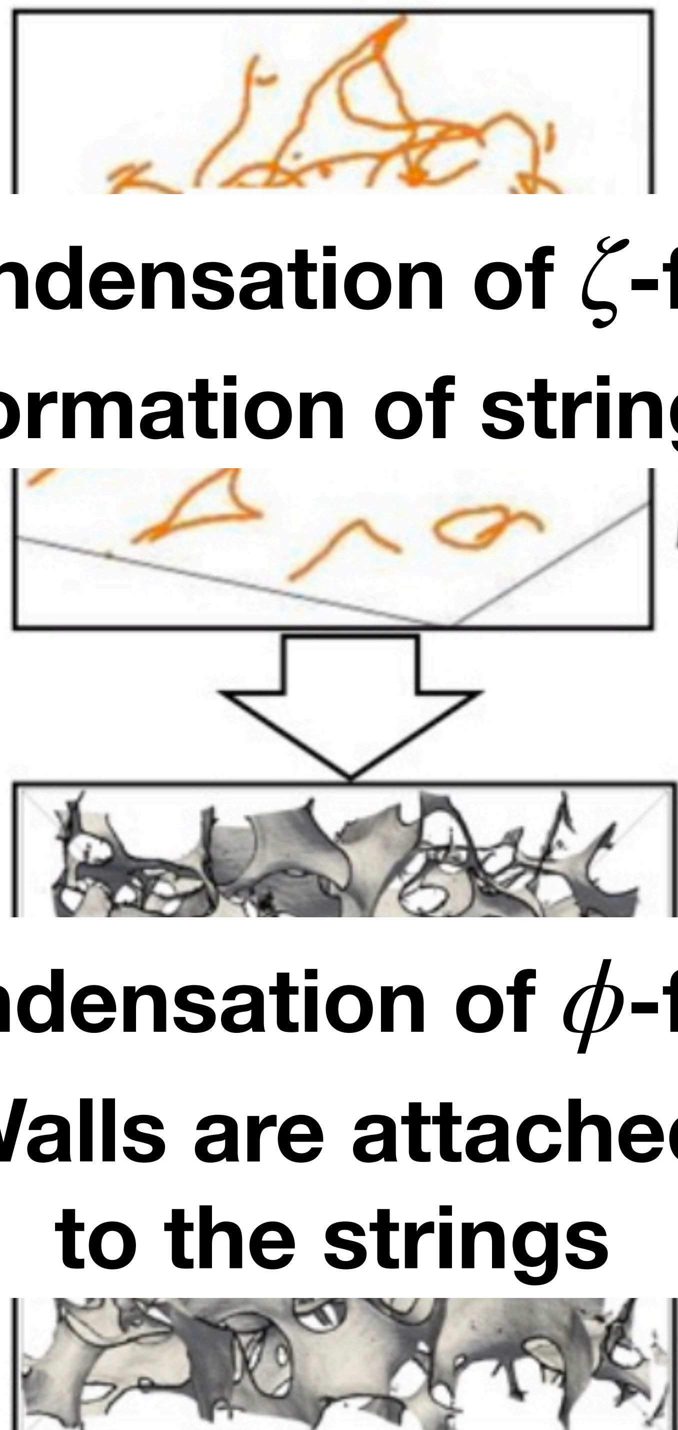
- The dark matter is a real scalar field

    - Expressed by a phase difference between the new complex scalar fields

- Coincides with the pseudo Nambu-Goldstone dark matter model *Gross, Lebedev, Toma 2017*

Walls in this model

Composite topological defects appear in the early universe

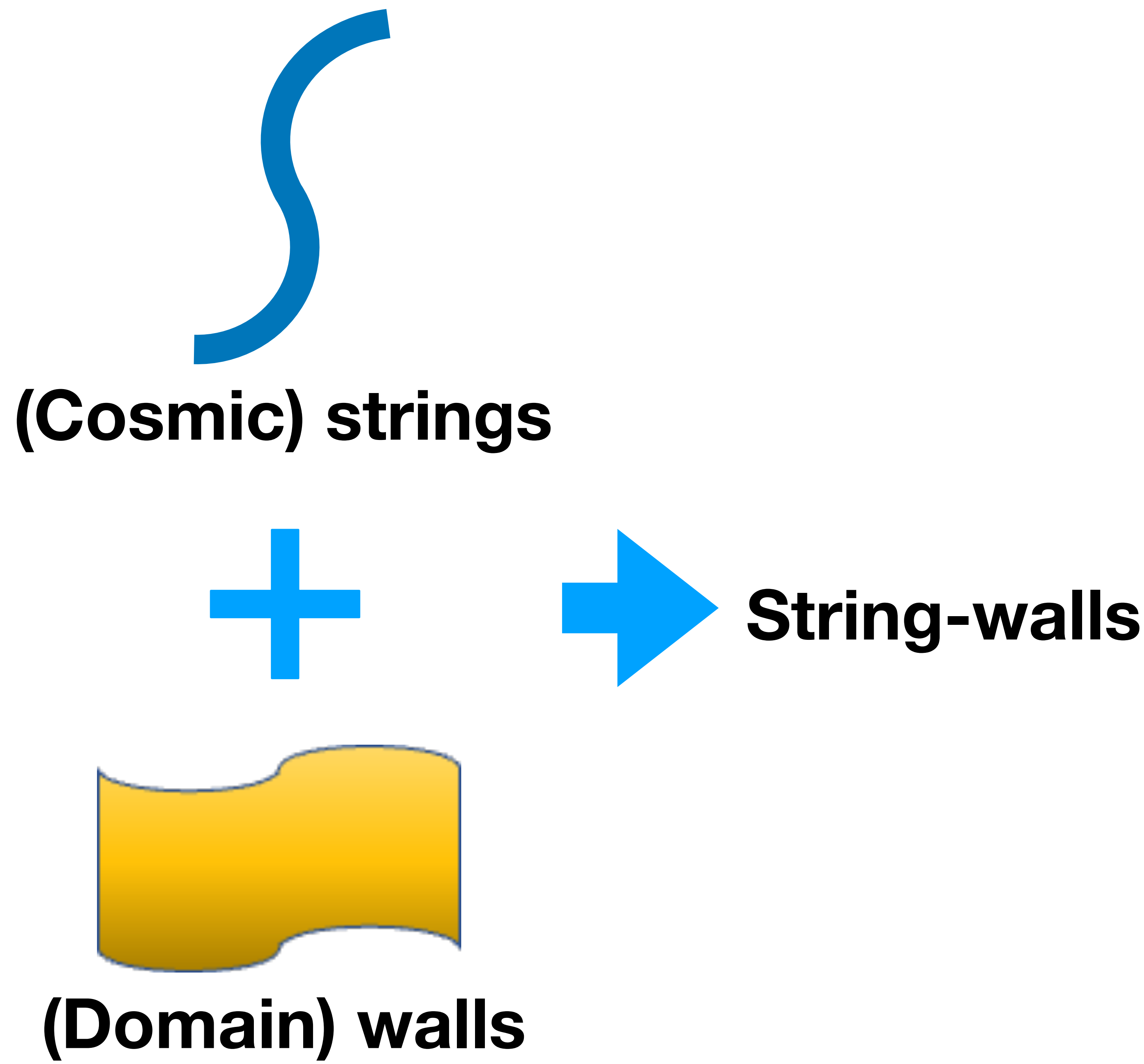


Condensation of ζ-field

Formation of strings

Condensation of ϕ-field

Walls are attached to the strings



Advantages of our scalar dark matter

- The cross section between DM and ordinary matter is highly suppressed *Gross, Lebedev, Toma, 2017*

- DW problem is avoided because of the appearance of the edges of walls made of strings

  Eto, Hiramatsu, Saito, YS, 2023, 2024

- Drawbacks of WIMPs and Axions seem not exist

- A promising candidate for DM

Advantages of our scalar dark matter

- The cross section between DM and ordinary matter is highly suppressed *Gross, Lebedev, Toma, 2017*

- DW problem is avoided because of the appearance of the edges of walls made of strings

  Eto, Hiramatsu, Saito, YS, 2023, 2024

- Drawbacks of WIMPs and Axions seem not exist

- A promising candidate for DM

What we did

- We performed three dimensional numerical simulations of string-walls in our model

- Each wall becomes confined in a narrow strip with the two edges made of the strings (a kishimen-like structure)

- Therefore, the walls behave almost like strings, which does not conflict with the current observations

- DW problem is avoided in our model!

Kishimen is a kind of Japanese noodle

Eto, Hiramatsu, Saito, YS, 2023, 2024

Our model

- Two SM singlet complex scalar fields ζ and ϕ with a $U(1)_{\mathrm{mix}}$

$$\zeta \to e^{i2\theta}\zeta\,,$$
$$\phi \to e^{i\theta}\phi\,,$$

Interaction potential $\frac{m}{2}(\zeta^*\phi^2 + \phi^{*2}\zeta)$

Explicit breaking
$U(1)_\zeta \times U(1)_\phi \to U(1)_{\mathrm{mix}}$

- Phase transition happens twice at v1 and v2 (v1>>v2)

$$\frac{\lambda_1}{4}(|\zeta|^2 - v_1^2)^2 + \frac{\lambda_2}{4}(|\phi|^2 - v_2^2)^2$$

Our model

- The condensation of ζ causes the symmetry breaking

$$U(1)_{\text{mix}} \to Z_2$$

Remnant symmetry
$\theta = \pi$

$$\zeta \to e^{i2\pi}\zeta = \zeta$$
$$\phi \to e^{i\pi}\phi = -\phi$$

- The condensation of ϕ causes the symmetry breaking:

$$Z_2 \to I$$

- In total, the symmetry breaking chain is

$$\text{SM} \times U(1)_{\text{mix}} \to \text{SM} \times Z_2 \to \text{SM}$$

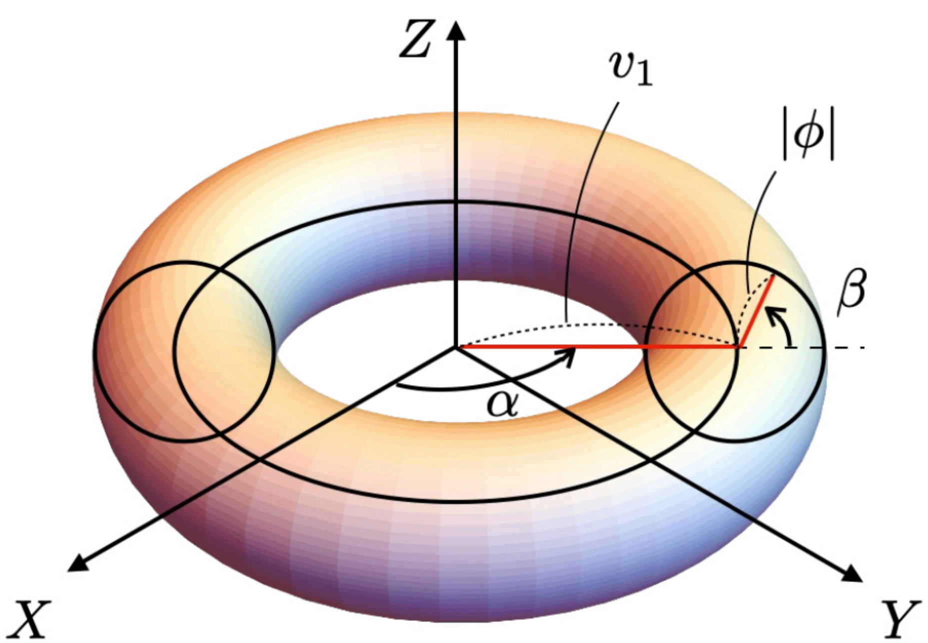
Structure of the vacuum

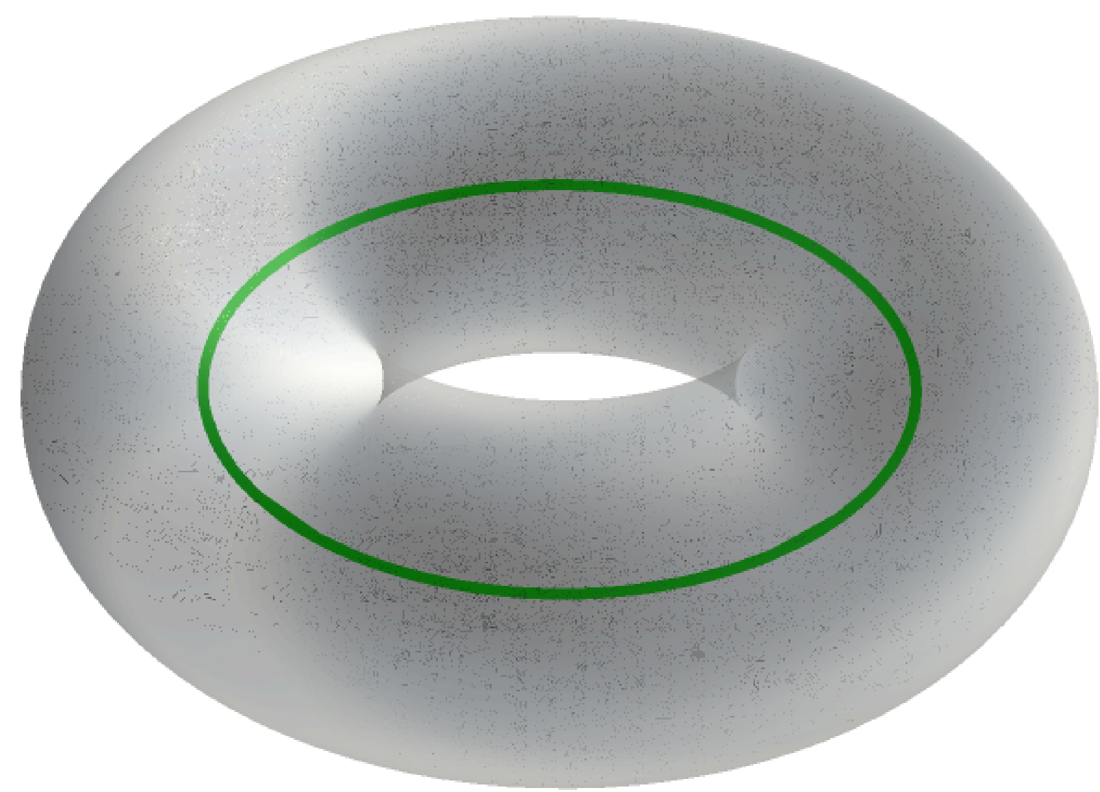
- In high energies as $v_1 > T > v_2$, where $\langle \phi \rangle = 0$

$$\zeta = v_1 e^{i\alpha} \,, \phi = |\phi| e^{i\beta}$$

The vacuum winds in α-direction twice ($\theta : 0 \to 2\pi$)
The structure is $U(1) \sim S^1$

Torus coordinate





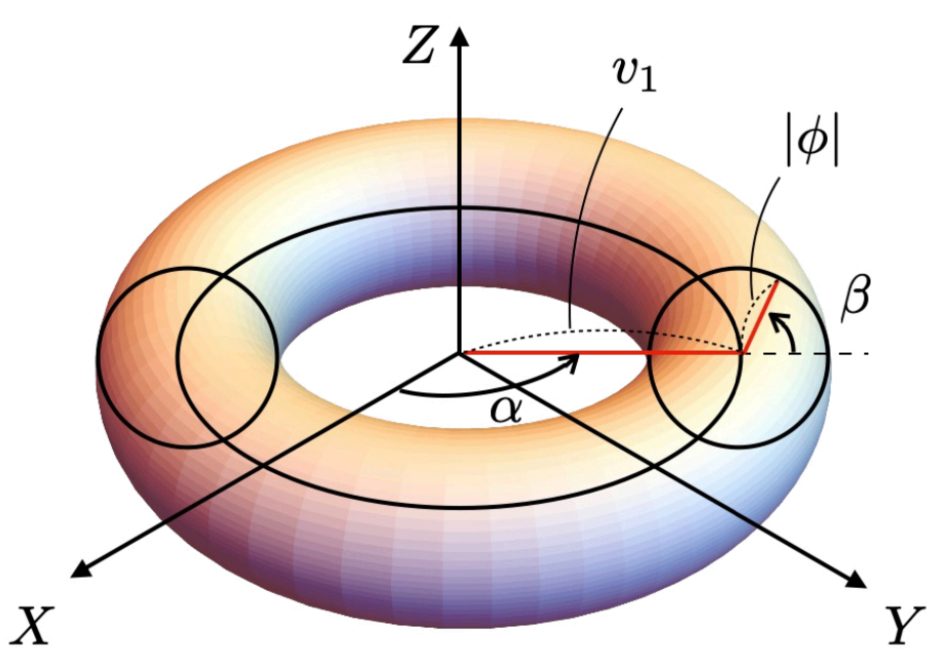
Structure of the vacuum

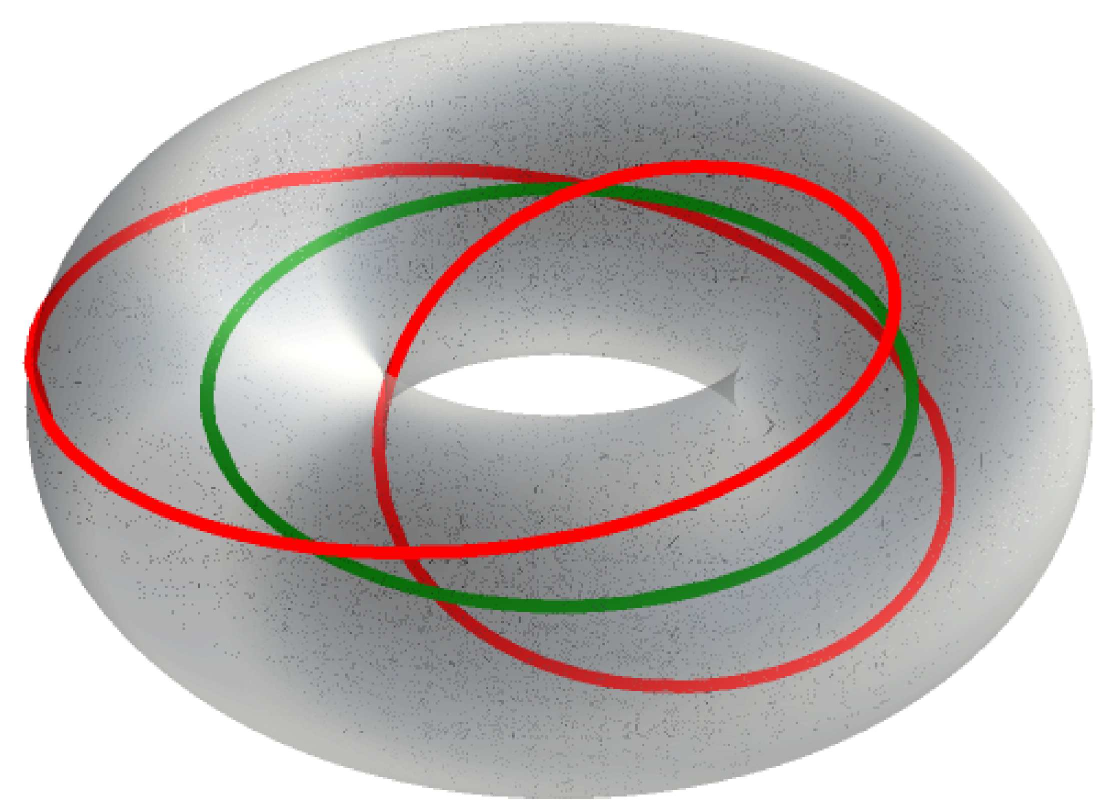
- In low energies $v_2 > T$

$$V_{\text{int}} = m v_1 |\phi|^2 \cos(2\beta - \alpha) \qquad \zeta = v_1 e^{i\alpha} \, , \quad \phi \simeq v_2 e^{i\beta}$$

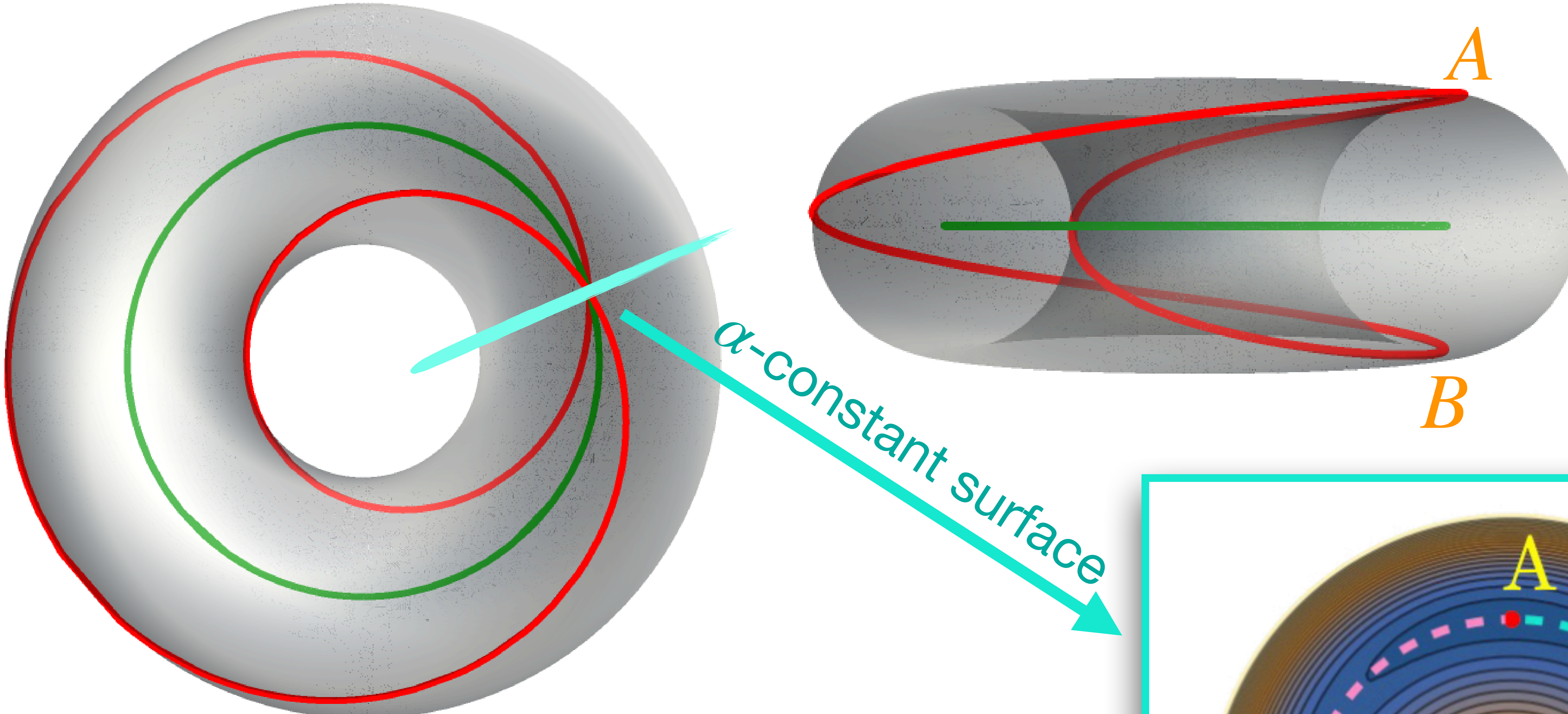
The vacuum is a **type-(2,1) torus knot** $\sim S^1$,
winding in α-direction twice and β-direction once.

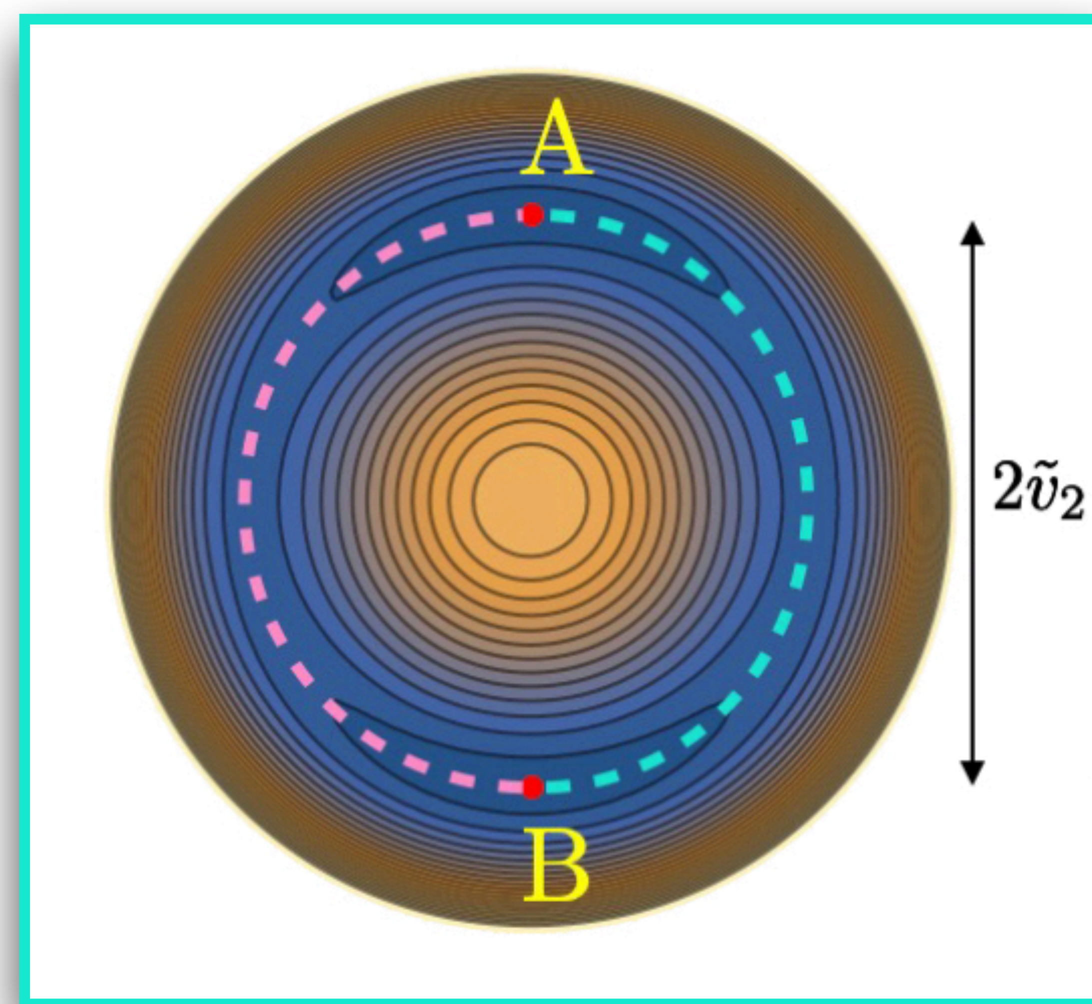
Torus coordinate





Structure of the vacuum

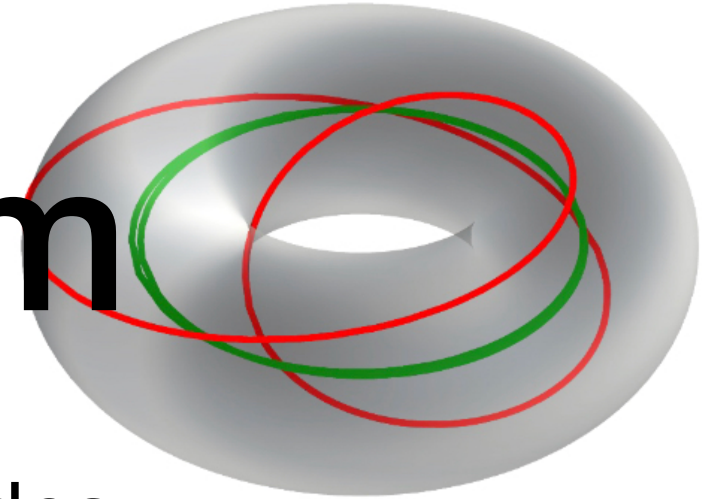




The α-constant surface has
the same structure as **N=2 axion models**
(Two Z2 symmetric vacua on a U(1))

Our model is a kind of extension of
N=2 axion models

Mass spectrum



- Radial modes: $\delta|\zeta|$ and $\delta|\phi| \Rightarrow$ Massive modes

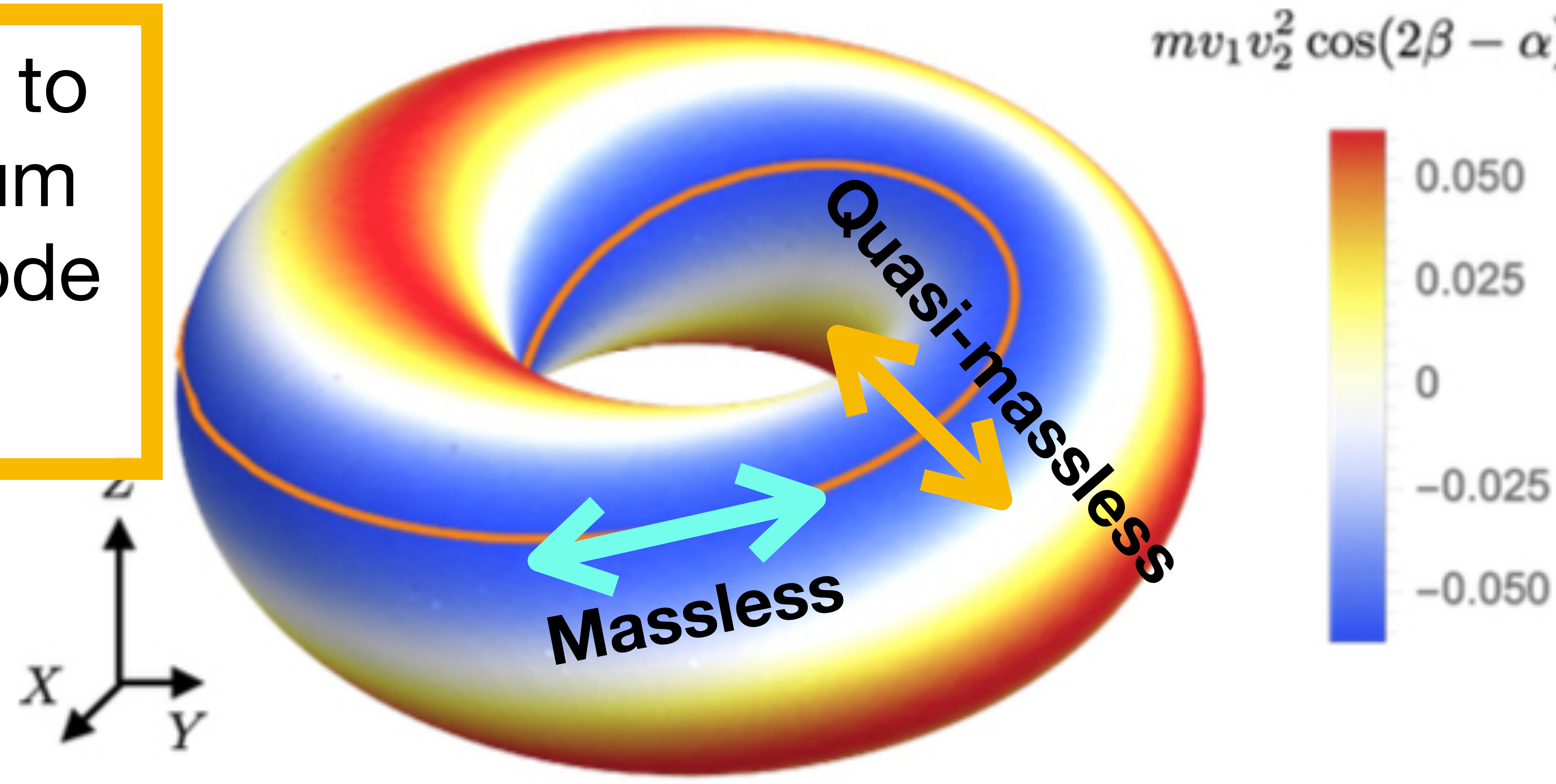
$$m_{|\zeta|} \sim \sqrt{\lambda_1} v_1 \qquad m_{|\phi|} \sim \sqrt{\lambda_2} v_2$$

- Mode along the torus knot vacuum of $\delta\alpha$ and $\delta\beta$
  $\Rightarrow$ Massless mode

- Mode perpendicular to the torus knot vacuum
  $\Rightarrow$ Quasi-massless mode

  $\Rightarrow$ Dark matter

$$m_{\rm qml} \sim m v_1$$



Strings

- Strings winding with $\underline{n_\alpha} = 2n$ where $n \in \mathbb{Z}$ are topological

  winding number in α-direction

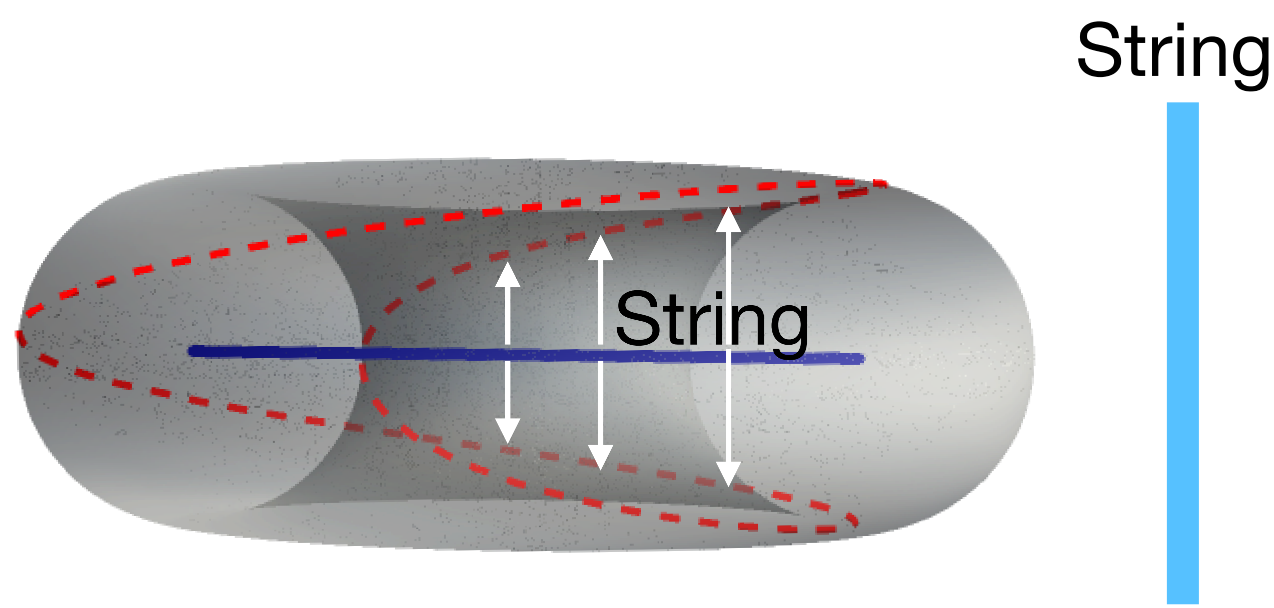
- However, strings with $n_\alpha \neq \pm 1$ are hard to be formed for cosmological initial condition

Formation of string-walls

① At a high energy $T \sim v_1$,

strings with $n_\alpha = \pm 1$

are formed



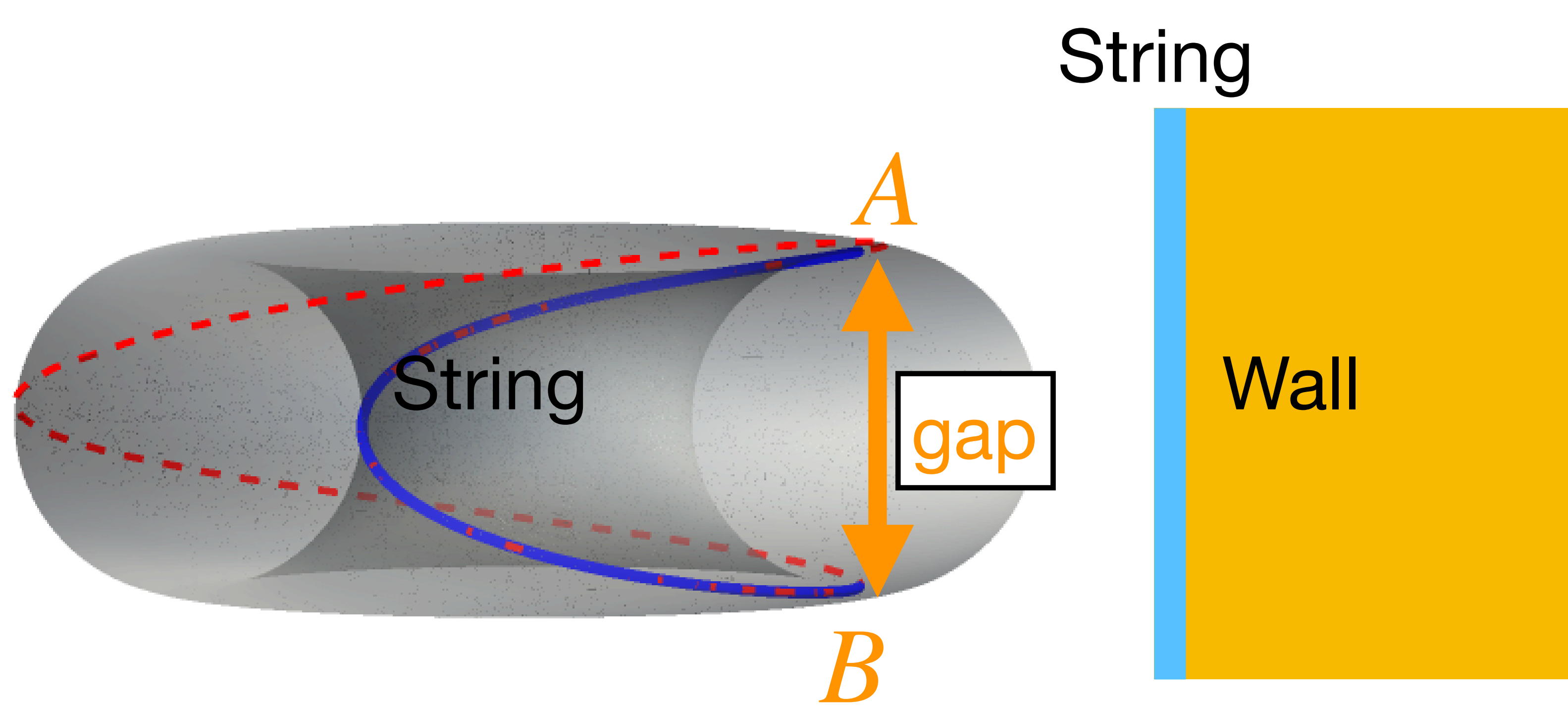
② At a low energy $T \sim v_2$,

the string tries to follow

the torus knot vacuum,

and a gap is formed

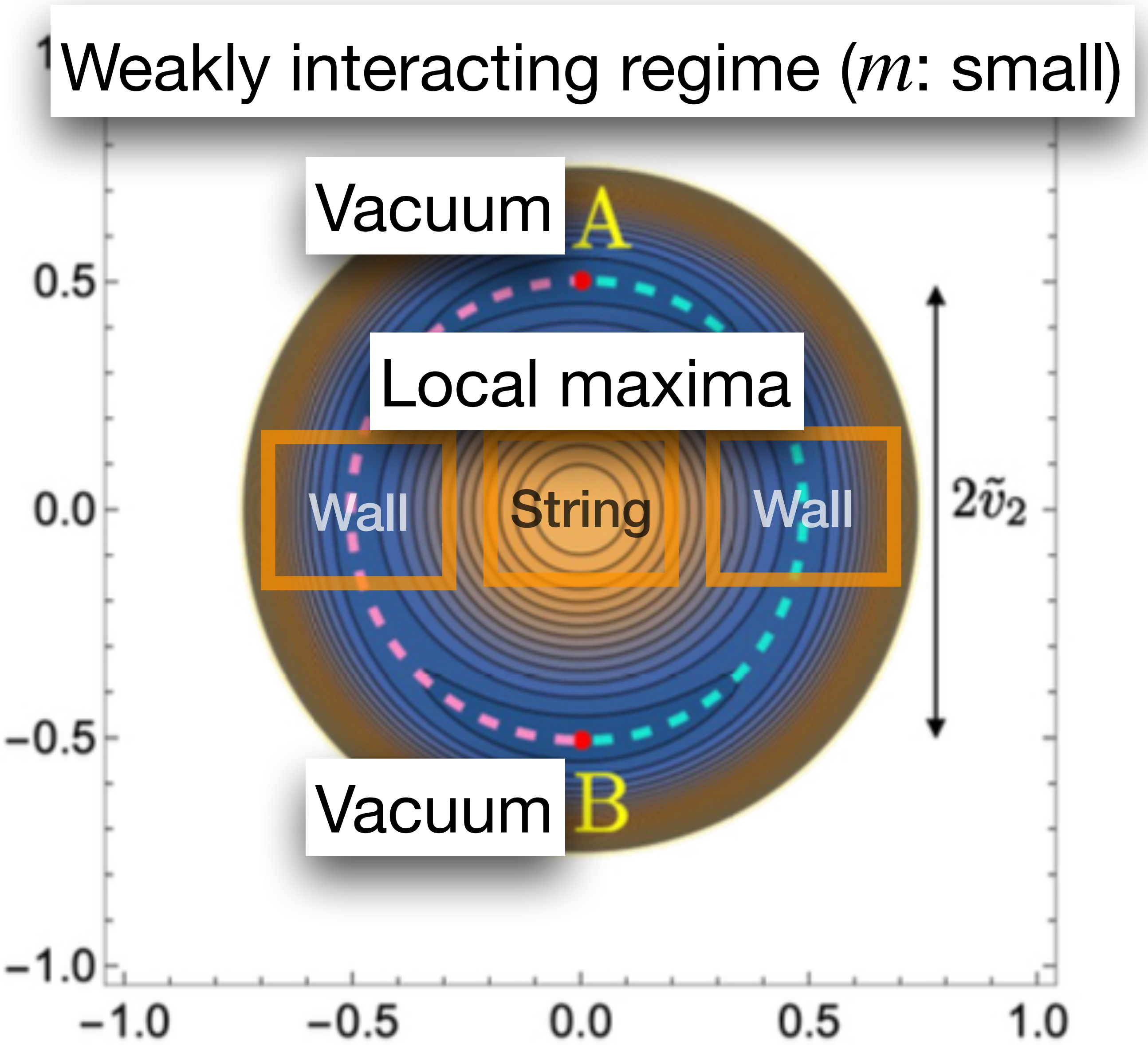
Wall



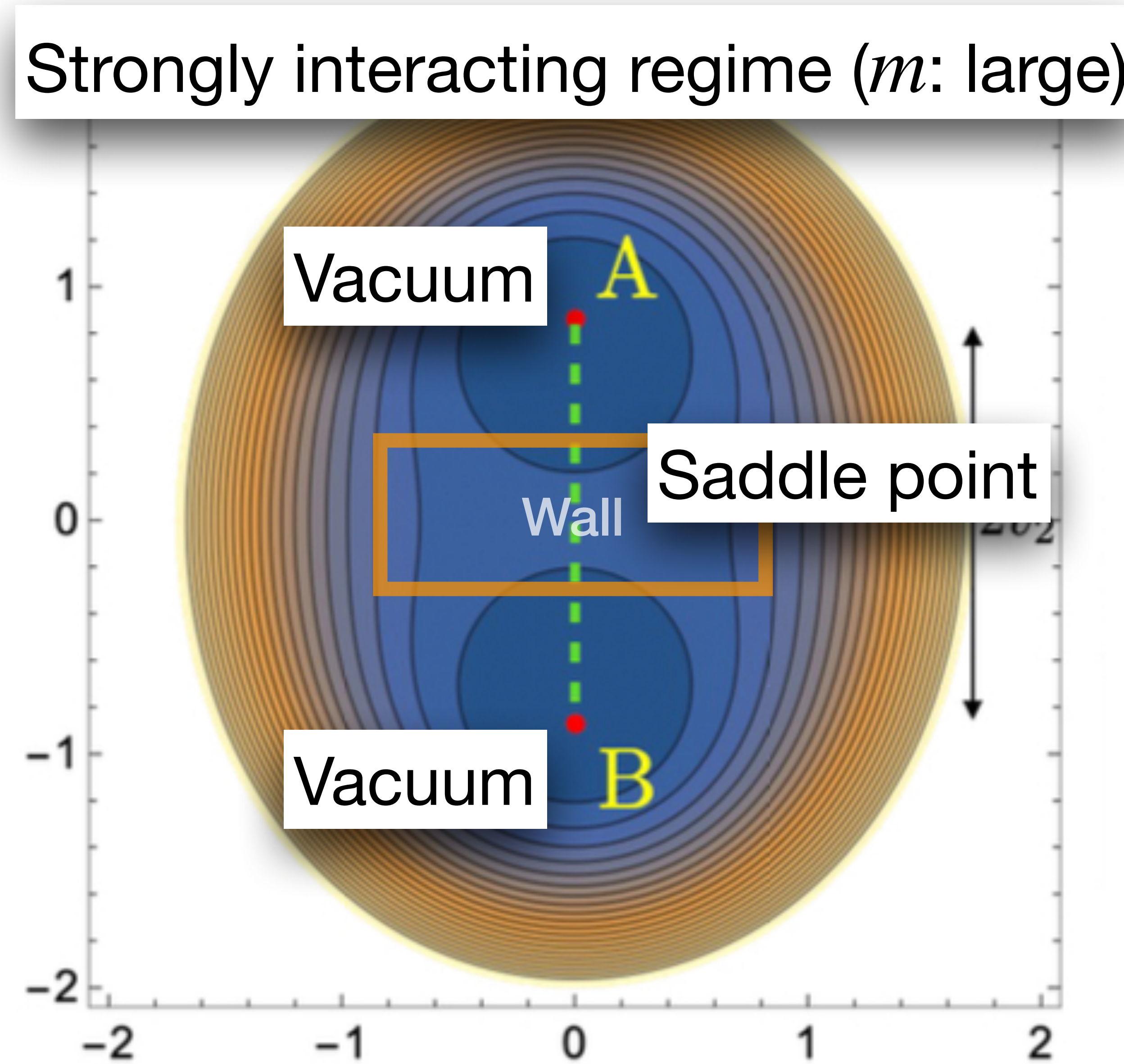
Two regimes

$$\frac{m}{2}(\zeta^* \phi^2 + \phi^{*2}\zeta)$$

- Two regimes defined by m, the strength of the interaction between ζ and ϕ

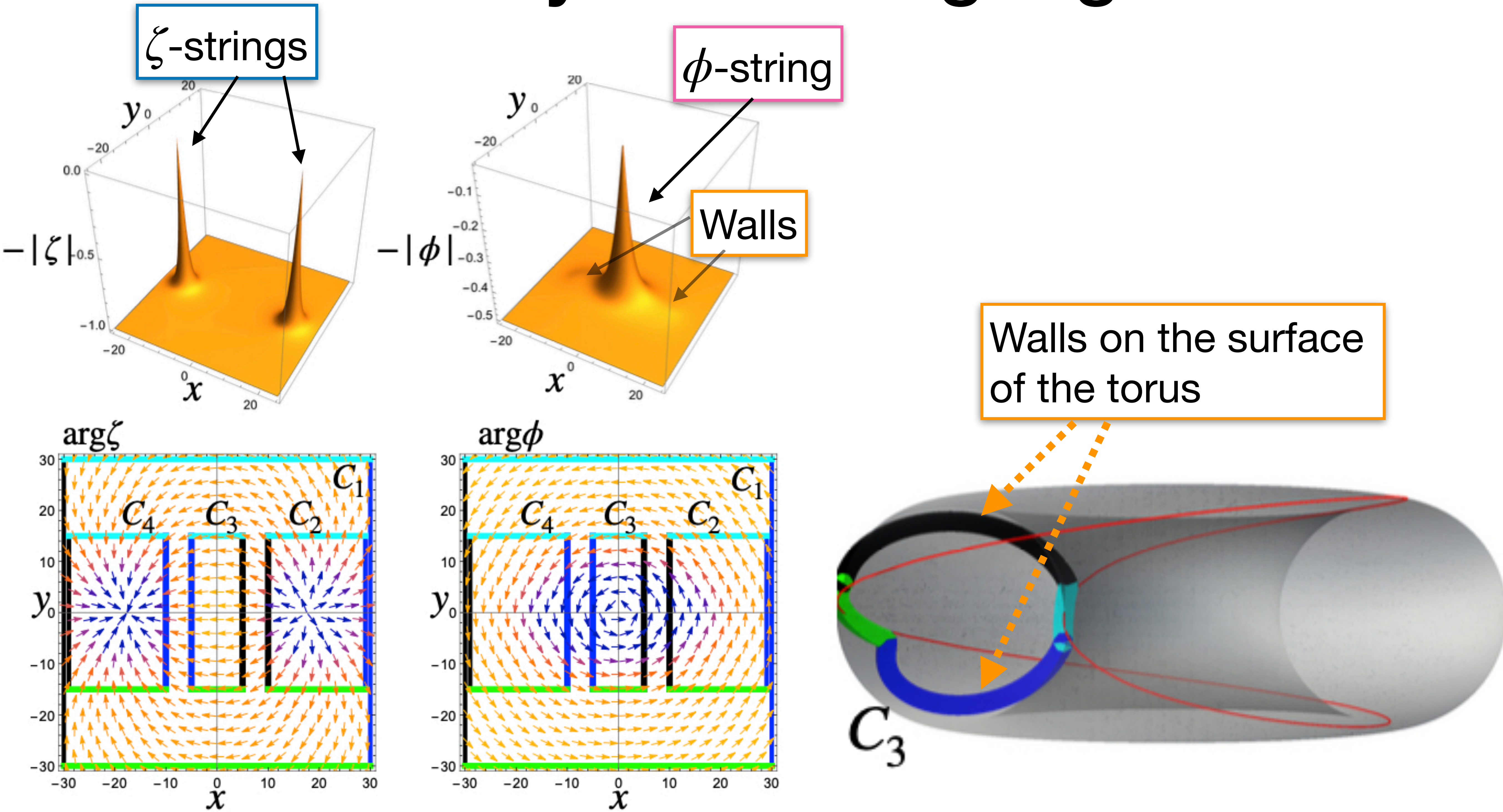


Walls are on the surface of the torus.
ϕ-strings winding in β-direction exist.



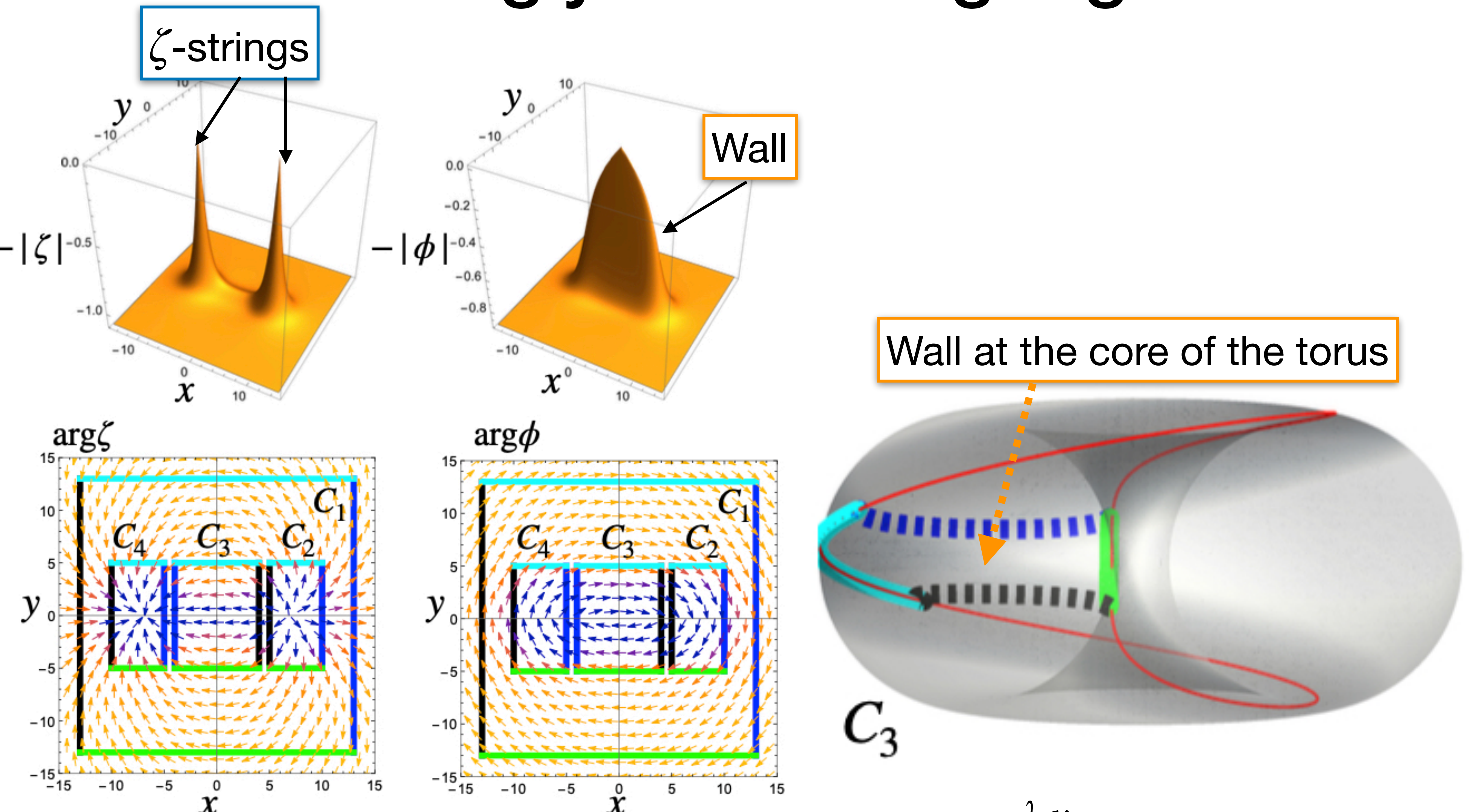
Wall is at the core of the torus.
No ϕ-strings.

Two dimensional numerical solution in weakly interacting regime



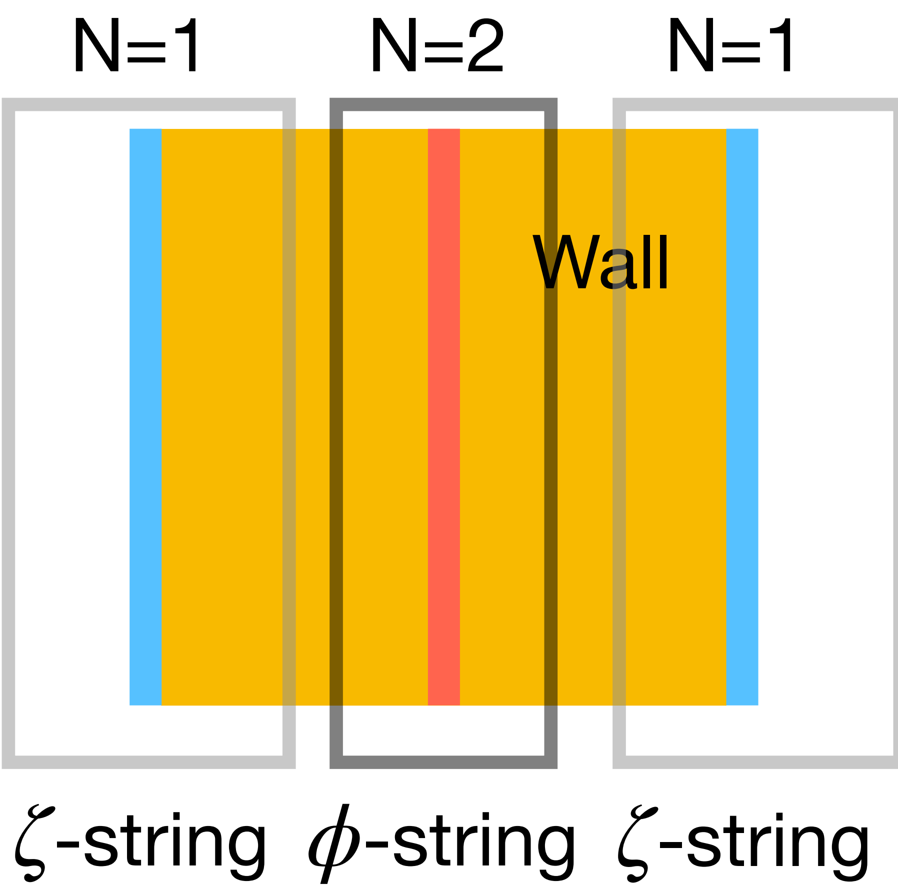
Wall tension $\sigma \sim \sqrt{m v_1} v_2^2$

Two dimensional numerical solution in strongly interacting regime



Wall tension $\sigma \sim \sqrt{\lambda_2} v_2^3 (1 + m/m_*)^{3/2}$ $\quad m_* = \dfrac{\lambda_2 v_2}{2v_1}$

Properties



$$\frac{m}{2}(\phi^2\zeta^* + \phi^{*2}\zeta)$$

ϕ ~ N=2 axion model

$$\frac{m}{2}(\phi^2\zeta^* + \phi^{*2}\zeta)$$

ζ~ N=1 axion model

- Our model has structures of N=1 and N=2 axion models

  ⇒ An extension of axion models

- Walls have the edges of ζ-strings

  ⇒ Wall property is different from axions'

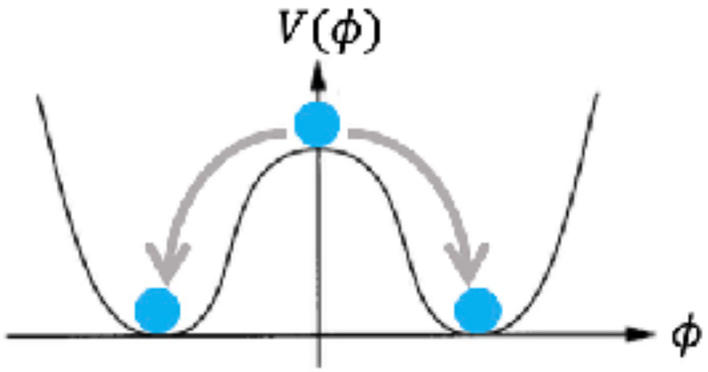
Note: For $(q_\zeta, q_\phi) = (1, n)$, where $n \geq 2$, the edges always appear

Domain wall problem

- Walls found in a simplest model (One real scalar field)

  "Two degenerate separated vacua"

  ⇒ Wall is topological



- Walls do not have the edges (infinite size)

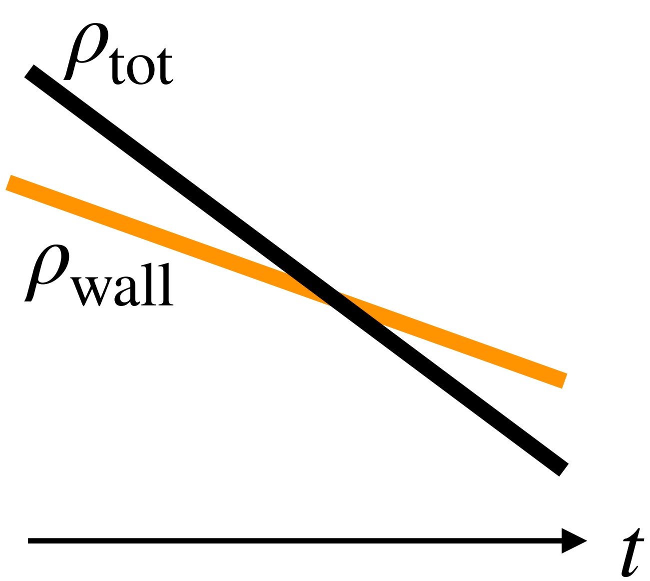
  ⇒ No mechanism to reduce the wall area

Correlation length $L \propto t$

Hubble expansion $H \propto t^{-1}$

Wall energy density $\rho_{\mathrm{wall}} \sim \dfrac{\sigma L^2}{H^{-3}} \propto \dfrac{t^2}{t^3} = \dfrac{1}{t}$

Total energy density $\rho_{\mathrm{tot}} \propto \dfrac{1}{t^2}$



⇒ Walls dominate the universe today ⇒ DW problem appears

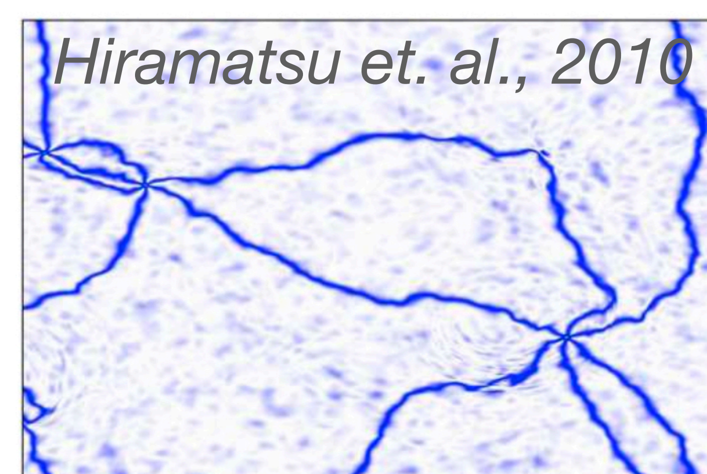
Domain wall problem

- For axion models with N>=2, DW problem appears

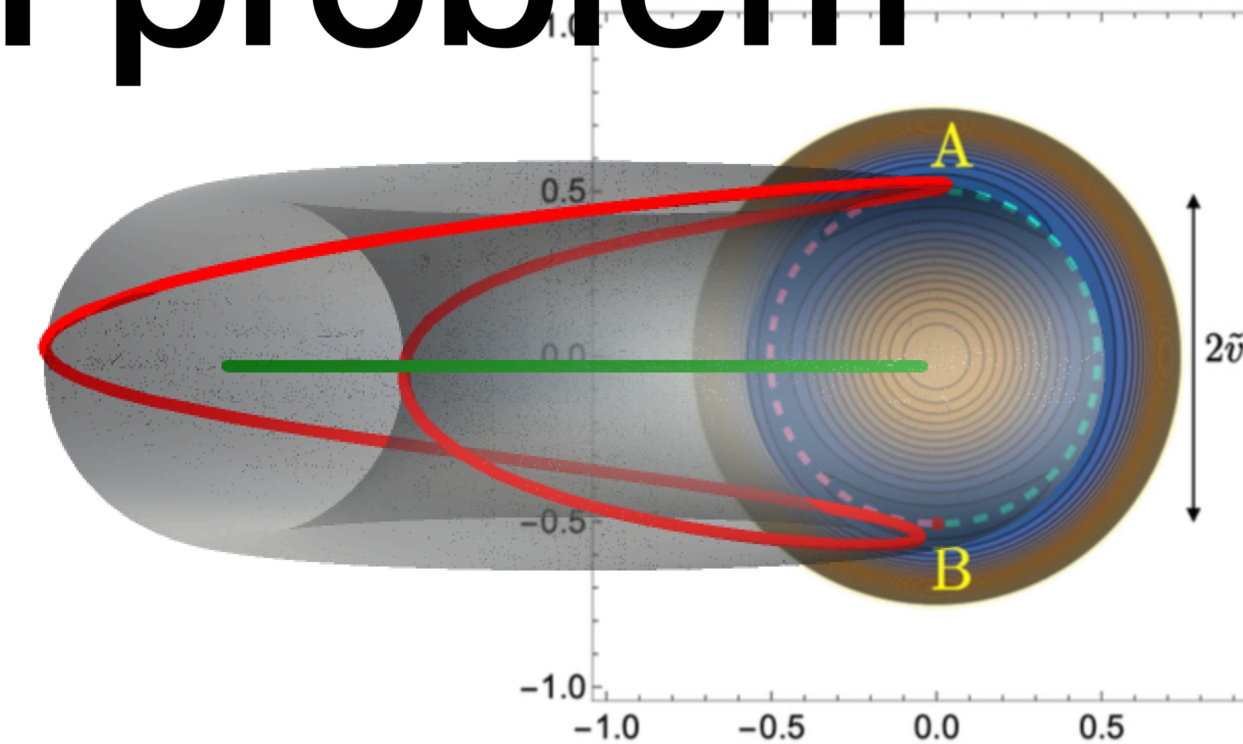
  "Two or more degenerate separated vacua"
  ⇒ Wall is topological

- Walls does not have the edges (infinite network of string-walls)
  ⇒ No mechanism to reduce the wall area

  ⇒ Walls dominate the universe today ⇒ DW problem appears



Domain wall problem



- Our model

  "Two vacua are connected"
  ⇒ Wall is non-topological

  ⇒ Walls are accompanied by strings as the edges and becomes topological

- Walls does have the edges made of strings
  ⇒ Walls seems to be able to reduce their area by their tension

  ⇒ DW problem seems to be avoided for our model

Governing equations for cosmological simulations

- Two complex scalar fields ζ, ϕ interacting with each other

$$\ddot{\zeta} + 2\mathcal{H}\dot{\zeta} - \delta^{ij}\partial_i\partial_j\zeta \ = -a^2\left(\lambda_1\zeta(|\zeta|^2 - v_1^2) + m\phi^2 \ + \frac{\lambda_1}{3}T^2\zeta\right)$$

$$\ddot{\phi} + 2\mathcal{H}\dot{\phi} - \delta^{ij}\partial_i\partial_j\phi \ = -a^2\left(\lambda_2\phi(|\phi|^2 - v_2^2) + 2m^*\zeta\phi^* + \frac{\lambda_2}{3}T^2\phi\right)$$

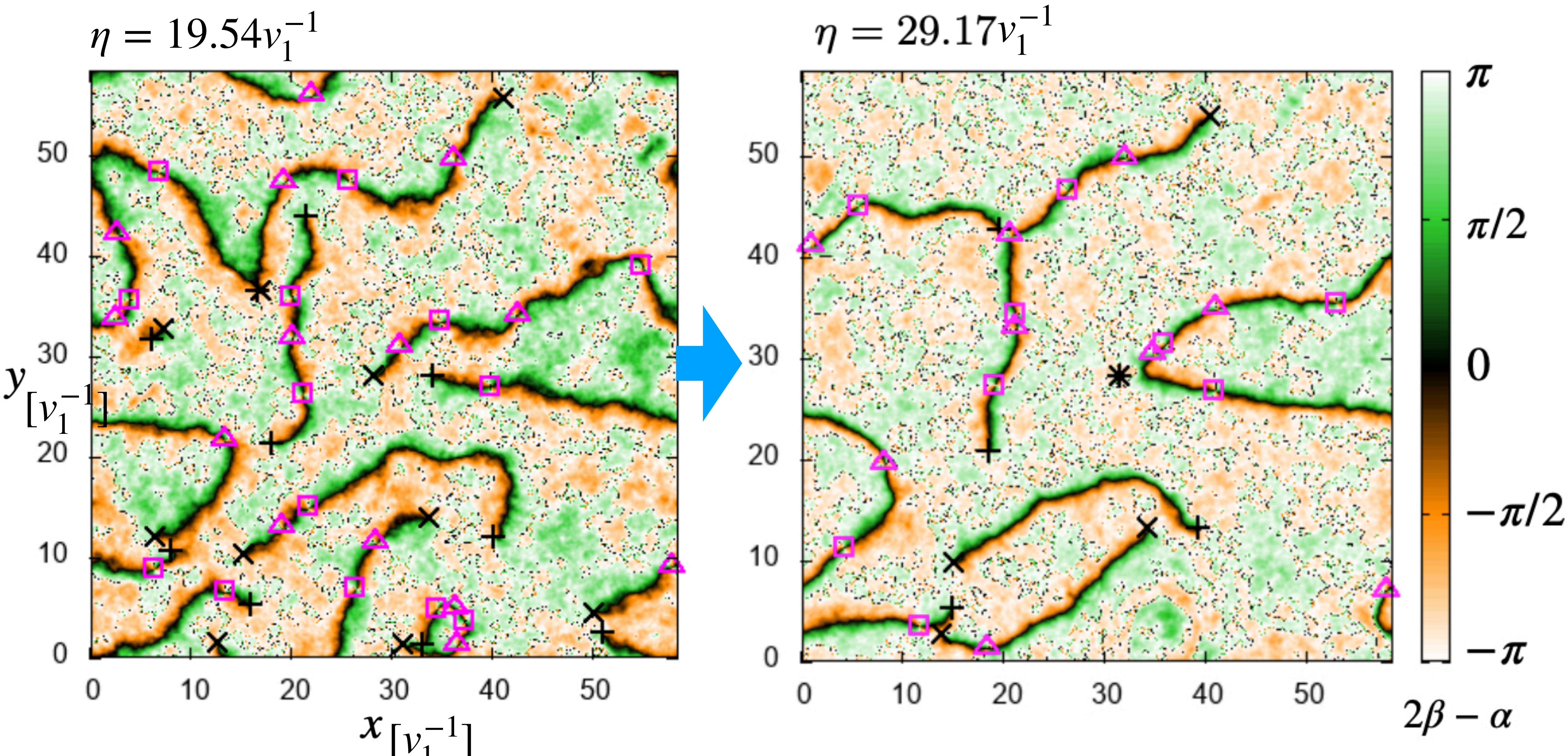
- Hyperbolic partial differential equations ~ wave equations

- Dark matter is expressed by $2\arg(\phi) - \arg(\zeta)$

Two dimensional cosmological simulations

<u>Weakly interacting regime</u>

+, × ... ζ-strings △, □ ... ϕ-strings

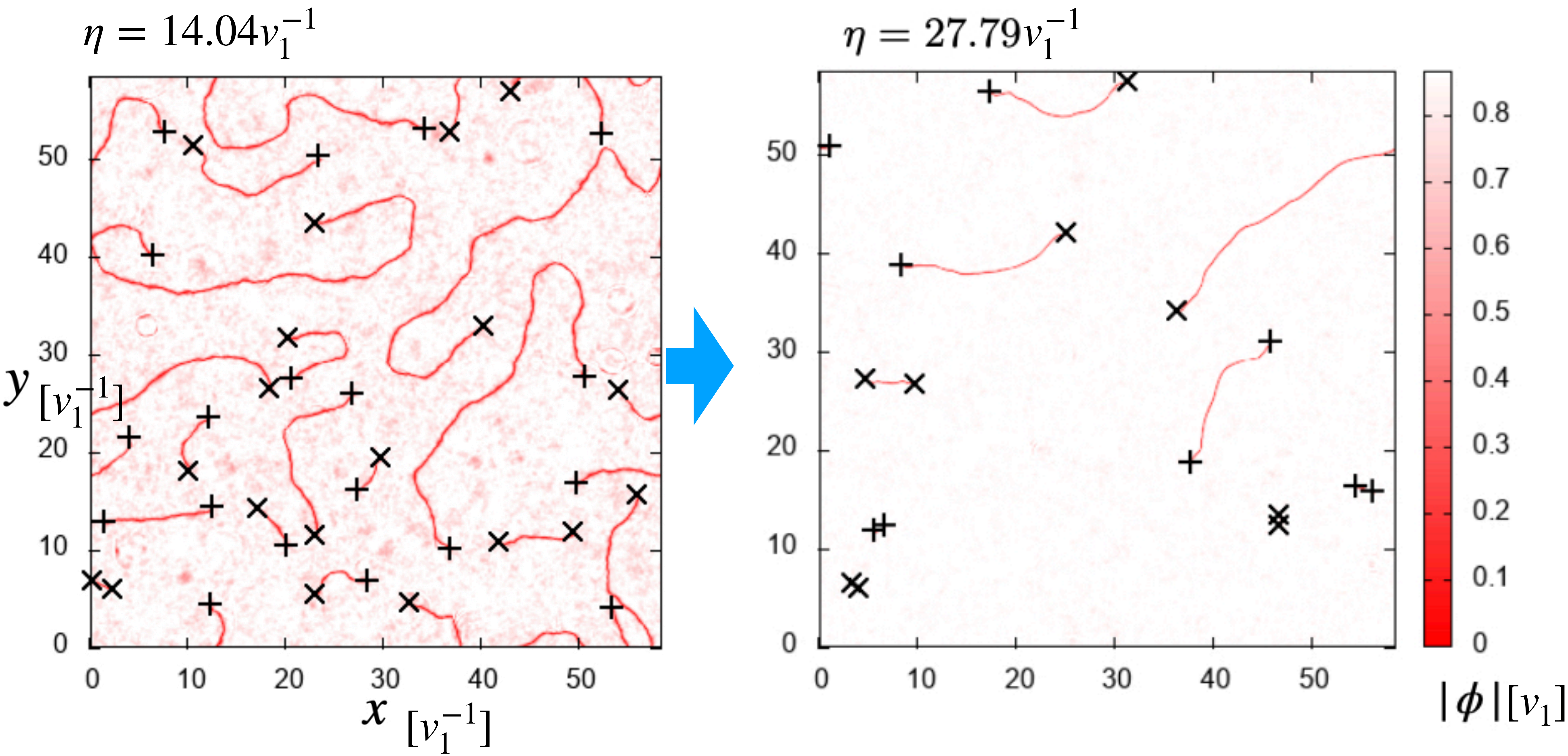


It appears the area of the wall is reducing

Two dimensional cosmological simulations

Strongly interacting regime

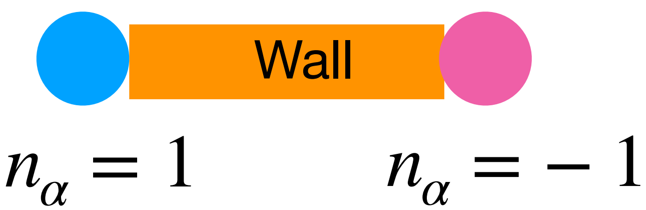
$+$, $\times$ ... ζ-strings



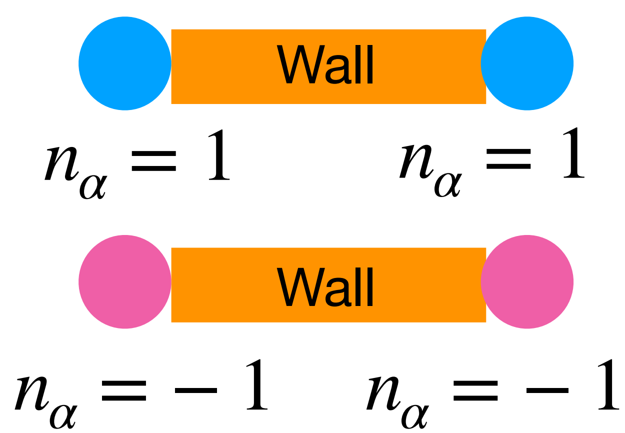
It appears the area of the wall is reducing

Late time behavior

String-walls made of ζ-strings with the different winding numbers
⇒ non-topological
⇒ vanish



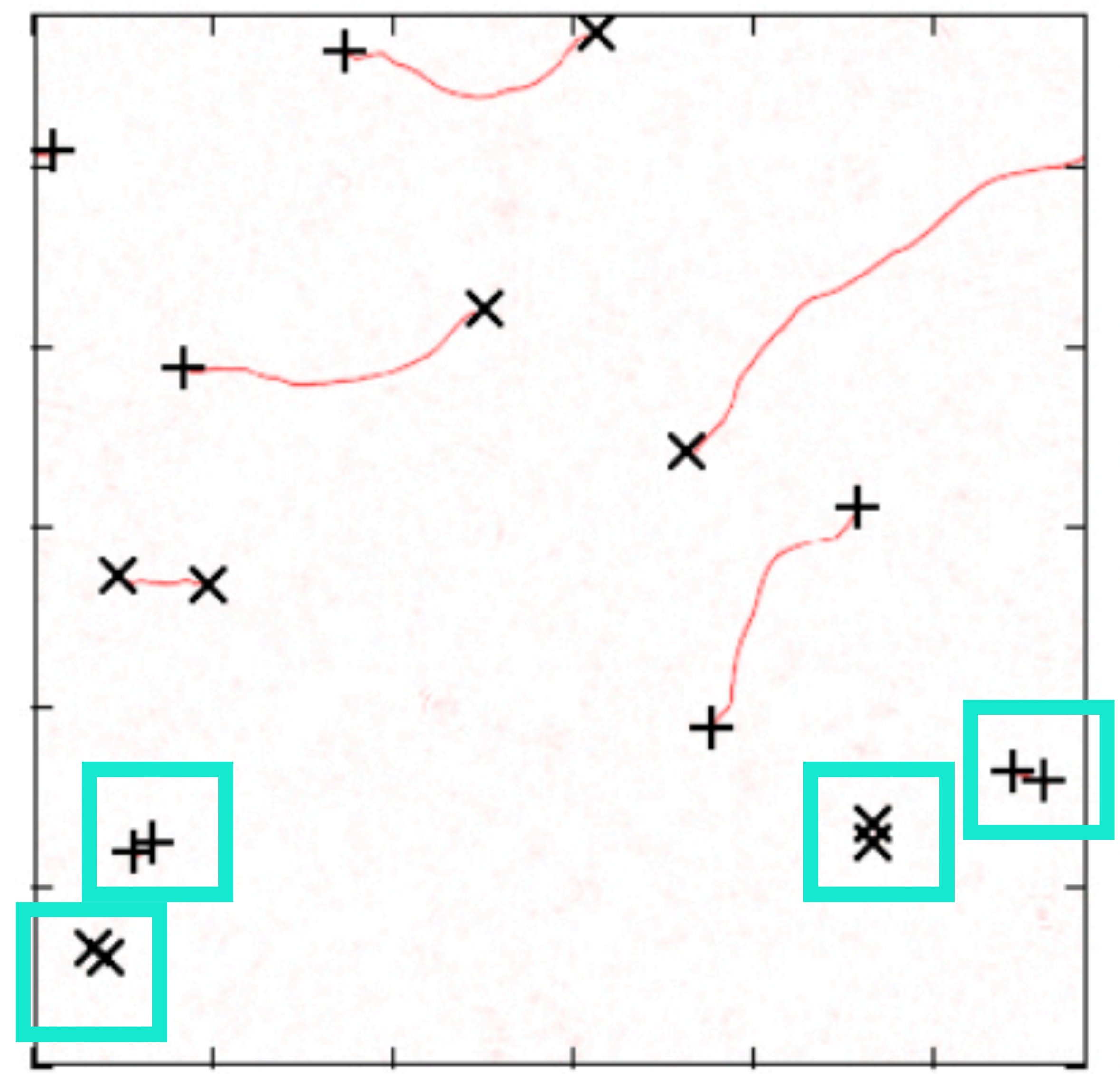
String-walls made of ζ-strings with the same winding number
⇒ topological
⇒ do not vanish



Two strings connected by a wall rotate around each other

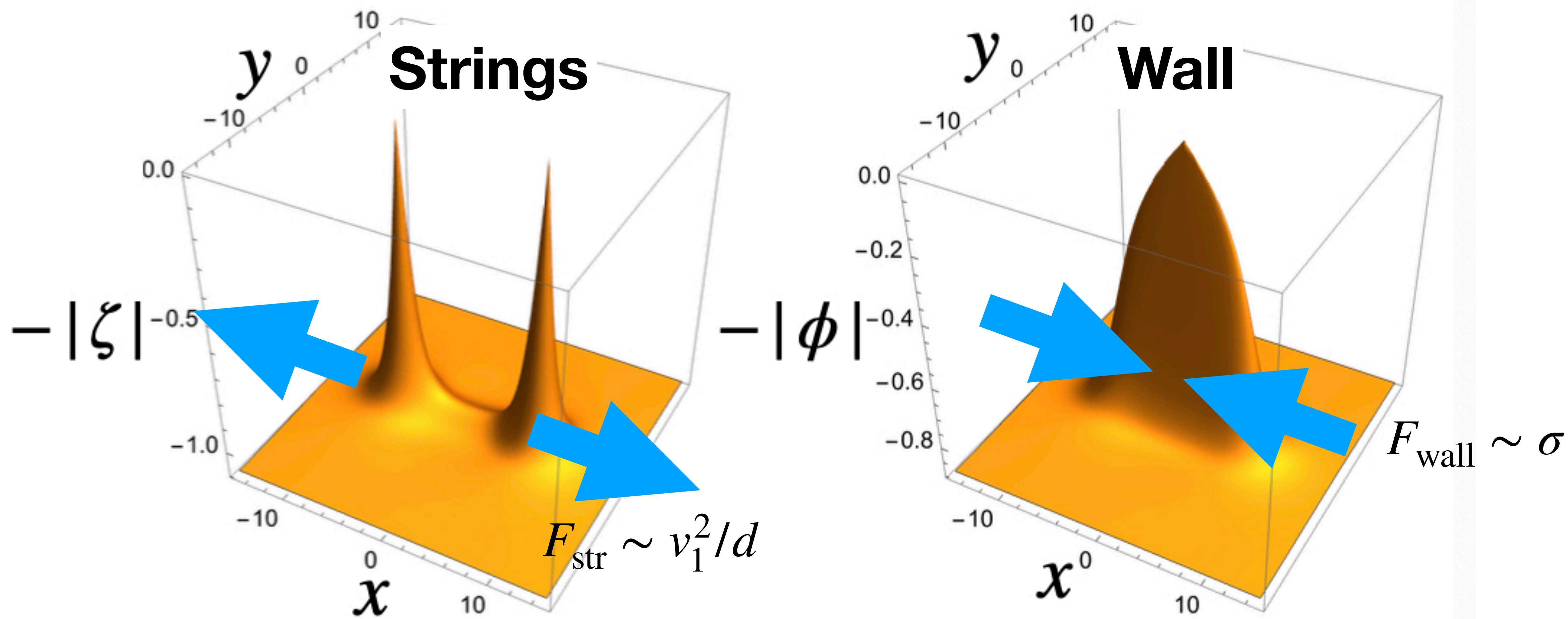
$+$... ζ-strings with $n_\alpha = 1$

$\times$ ...ζ-strings with $n_\alpha = -1$



$\eta = 27.79 v_1^{-1}$

Equilibrium configuration



Repulsive force between the strings

Wall tension = pull the strings

At $d \sim v_1^2/\sigma$, the repulsive/attracting forces balance

⇒ Remaining string-walls are expected to become narrow strips

Equilibrium configuration

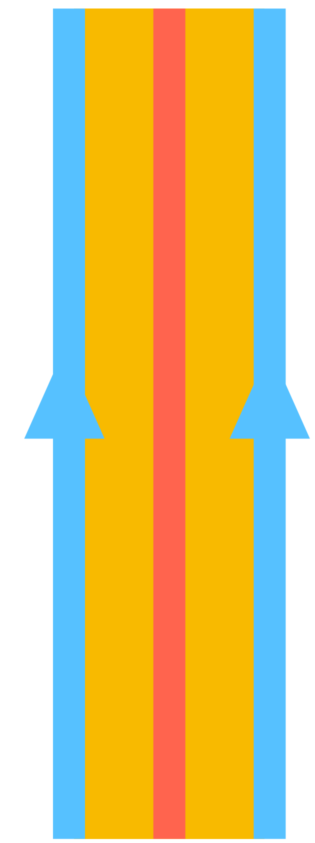
In three dimensional space,

Equilibrium configuration

If a string-wall is unclosed,

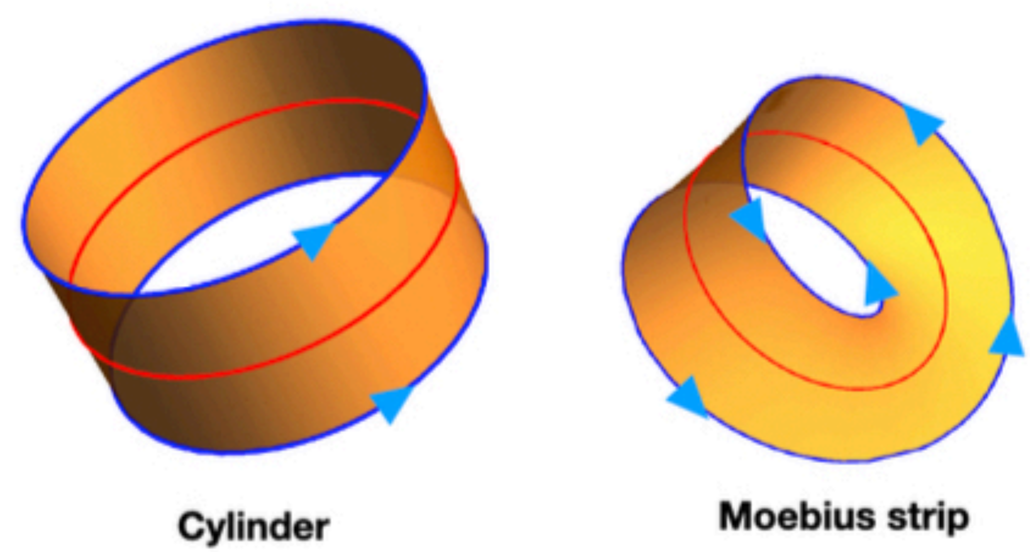
narrow strips

Kishimen-like structure

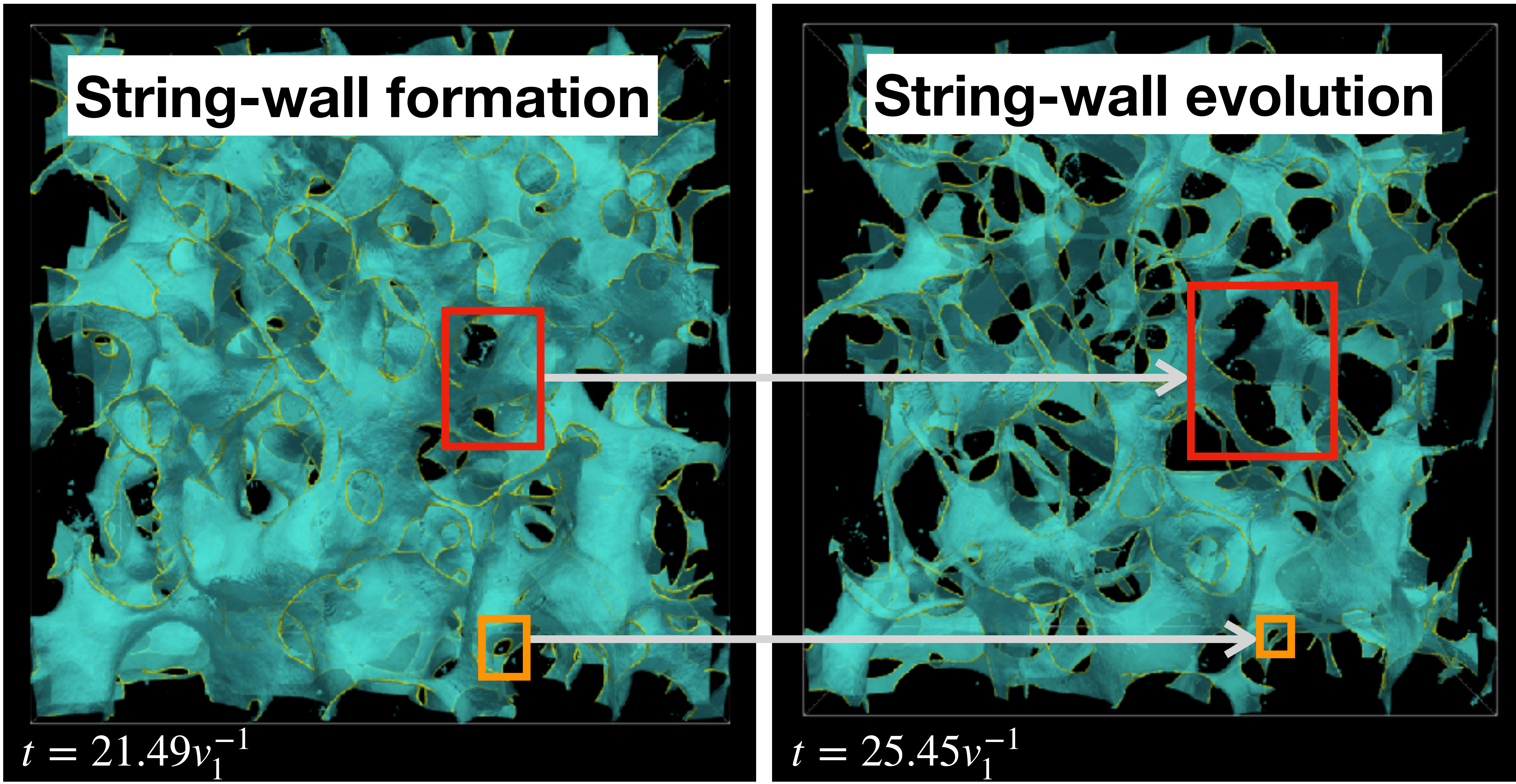


If a string-wall is closed,

cylinders and moebius strips



Three dimensional simulation



Yellow: Strings

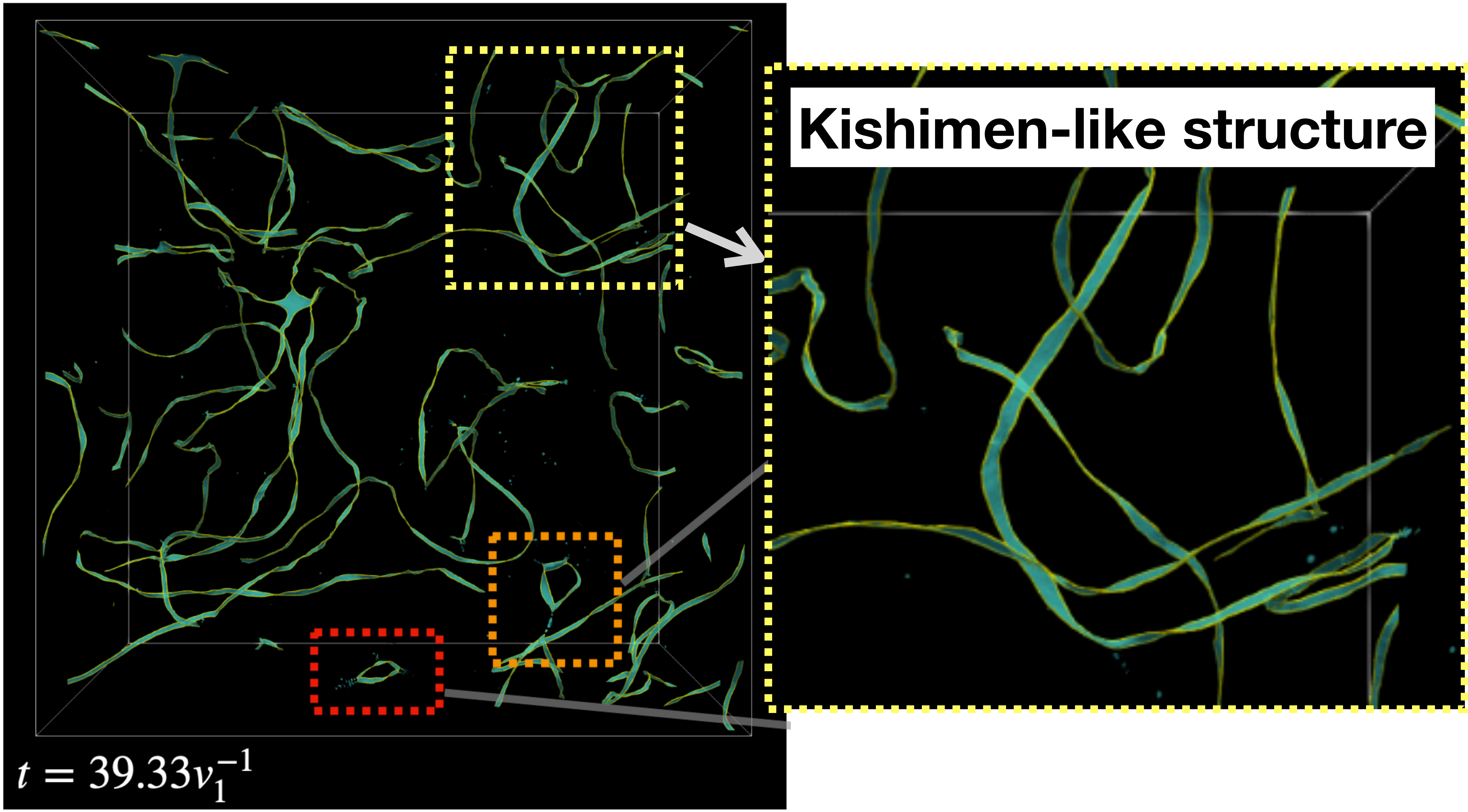
Green: Walls

Number of grids: 2048^3

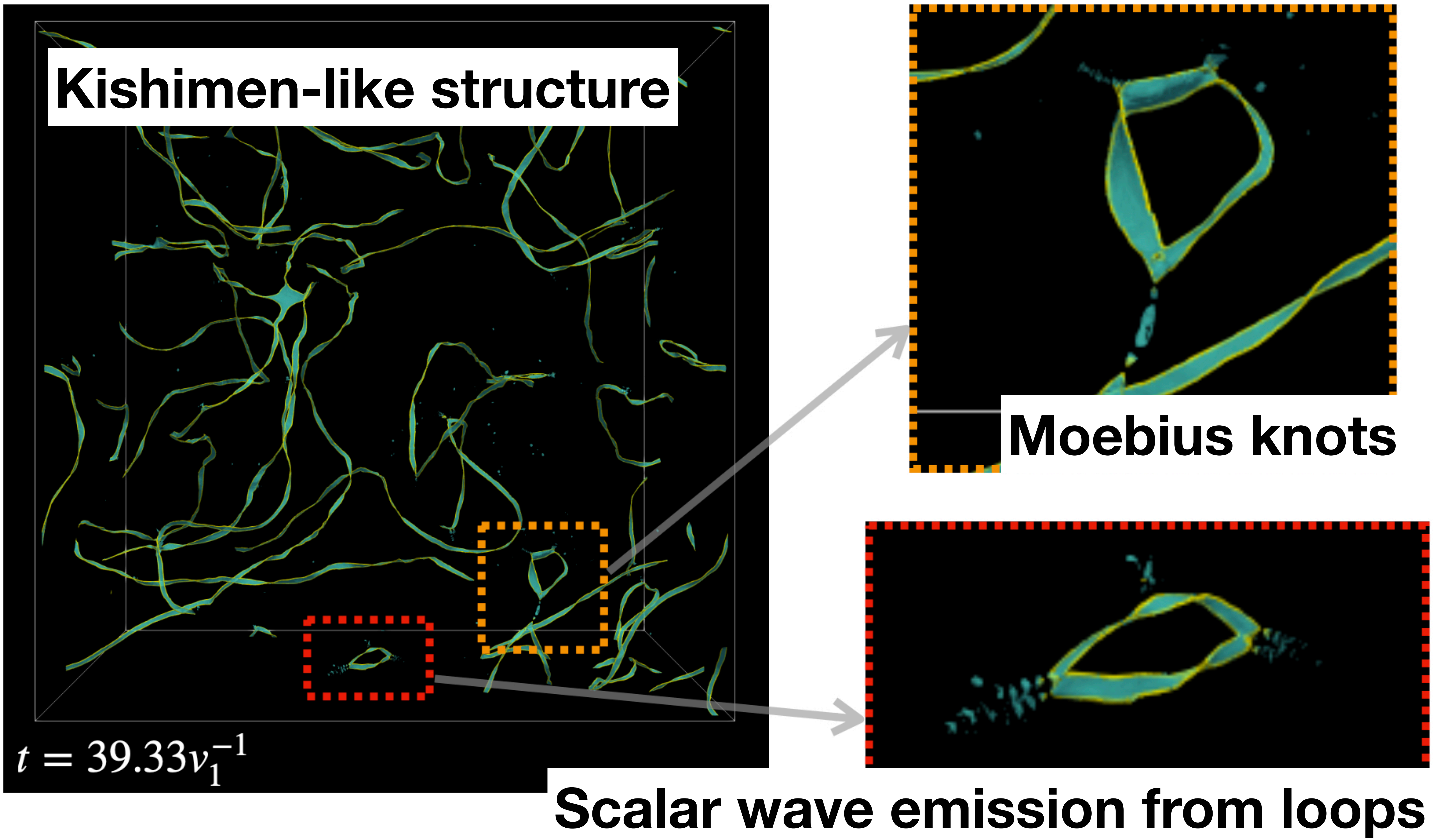
64 nodes on Flow@Nagoya

30 mins

Late time behavior



Late time behavior



Conclusion

- Proposed a dark matter model based on a global U(1)

    - Understood as an extension of N=2 axion models

- Examined the formation and the time evolution of the string-walls in our dark matter model

    - Walls get confined in narrow strips

- Performed a large scale cosmological simulation for longer time integration

    - Wall energy density evolves as t^{-2} at late time