Interferometer Searches for Dark Matter

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Workshop on Cosmic Indicators of Dark Matter 2024
October 16, 2024, Tohoku University

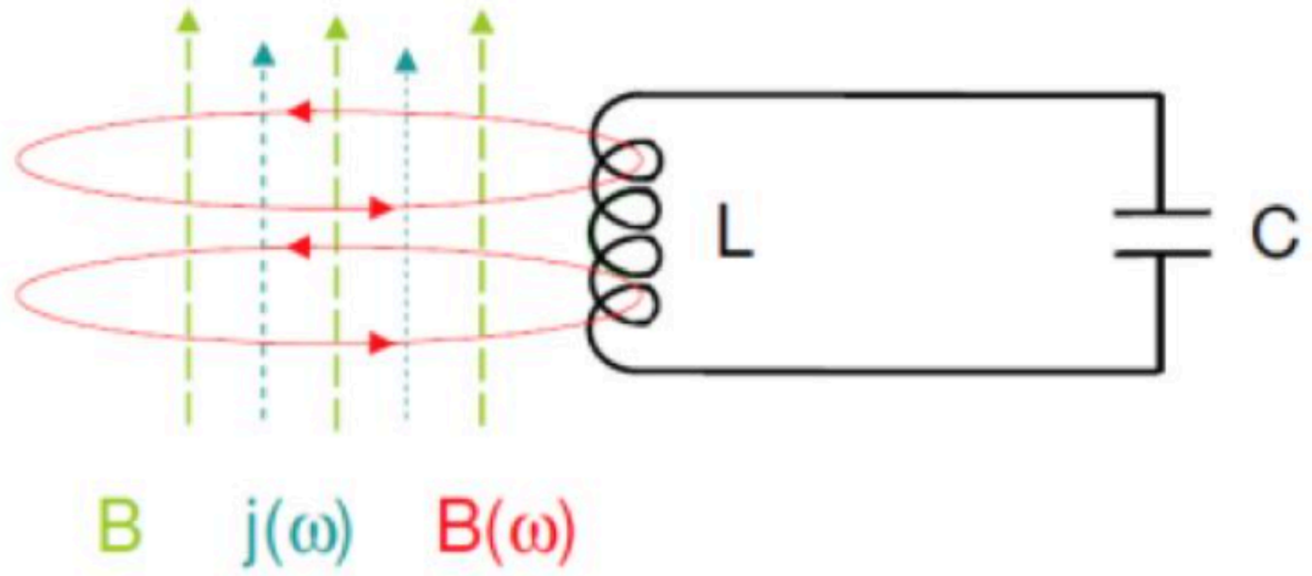
In Collaboration with: Leonardo Badurina, Yufeng Du, Vincent Lee, Clara Murgui, Kris Pardo, and Kathryn M. Zurek

Dark matter direction detection

The next discovery?

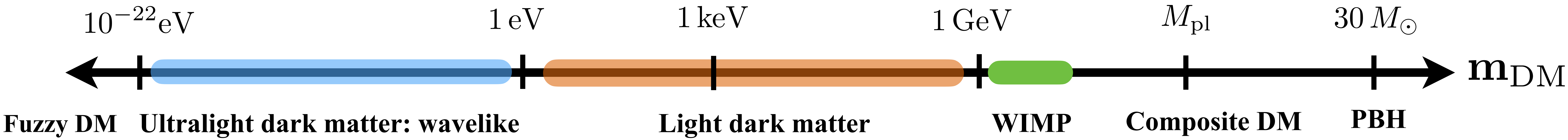
10^{-22}eV   1 eV   1 keV   1 GeV $M_{\rm pl}$ $30\,M_\odot$ $m_{\rm DM}$

Fuzzy DM   **Ultralight dark matter: wavelike**   **Light dark matter**   **WIMP**   **Composite DM**   **PBH**



Dark matter direction detection

The next discovery?



10^{-22}eV

1 eV

1 keV

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$M_{\rm pl}$

30 $M_\odot$

$\mathbf{m}_{\rm DM}$

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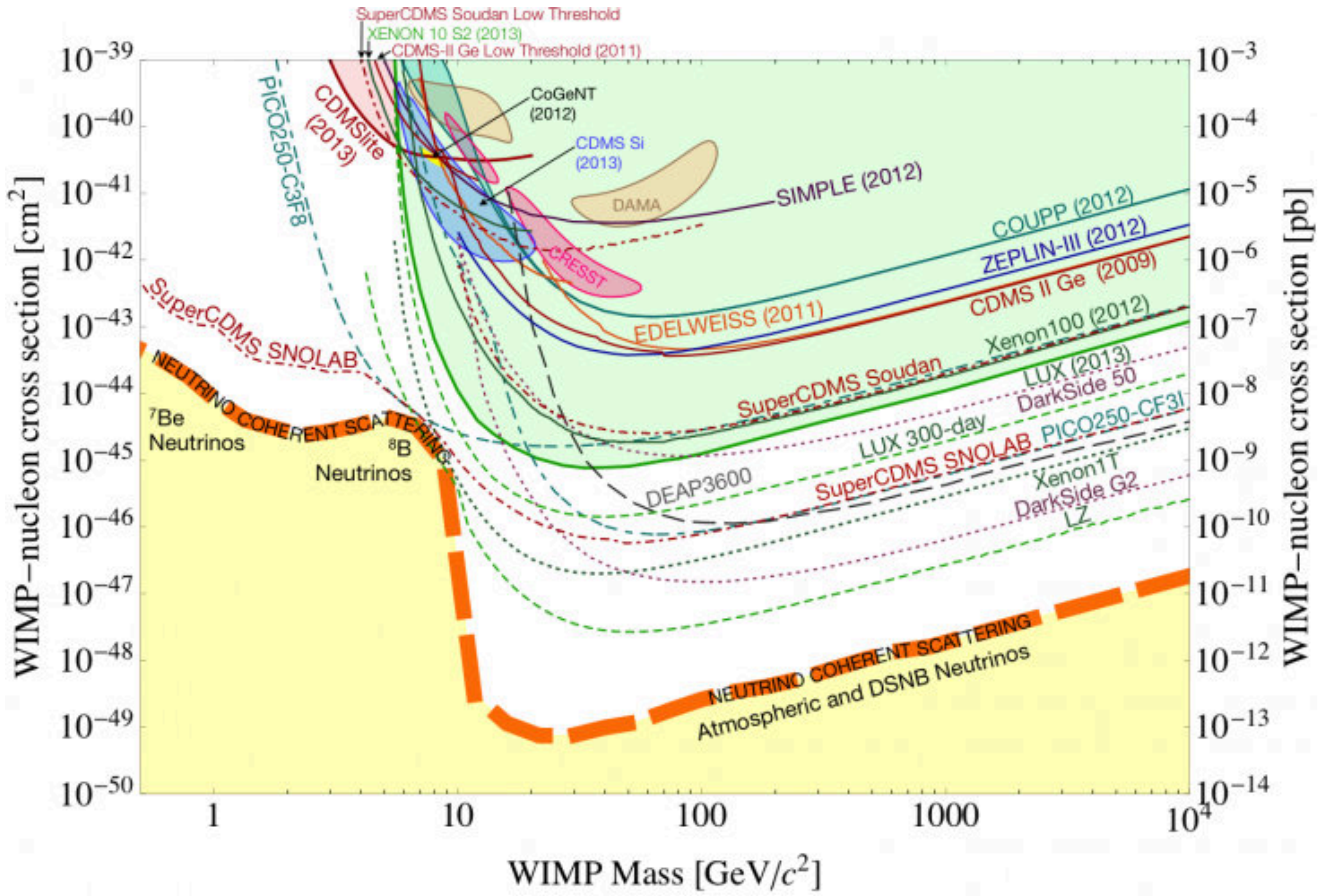

Dark matter direction detection

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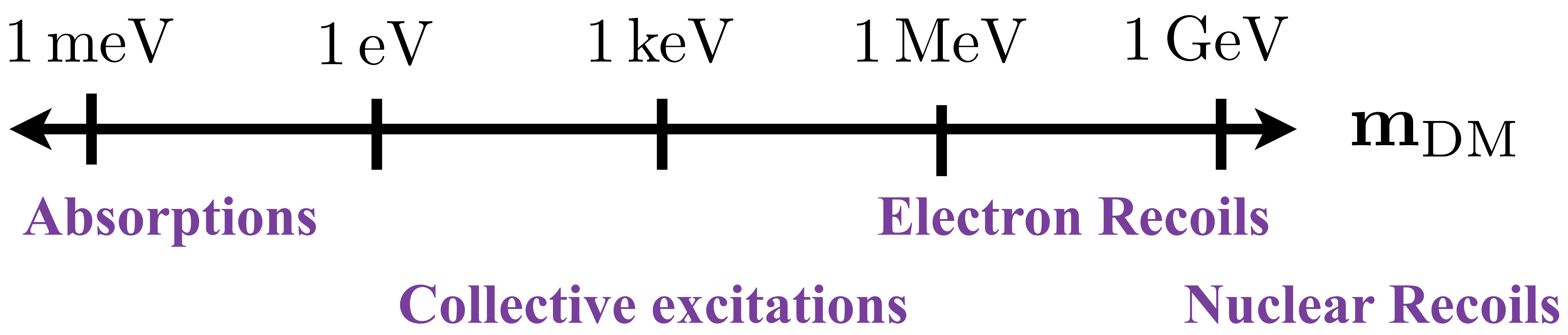
Direct detection



$1\,\mathrm{meV}$ $1\,\mathrm{eV}$ $1\,\mathrm{keV}$ $1\,\mathrm{MeV}$ $1\,\mathrm{GeV}$ $\mathbf{m}_{\mathrm{DM}}$

Absorptions **Electron Recoils**

Collective excitations **Nuclear Recoils**

- **Low energy threshold**

$$E_{\mathrm{R}}^{\mathrm{max}} = \frac{2\mu_{\chi N}^2 v^2}{m_N} \simeq 20 - 200\,\mathrm{keV} \quad \text{for WIMP mass range}$$

- **Small momentum deposition can boost the rate**

$$\bar{\sigma} \propto \alpha_\chi \alpha_n \frac{\mu^2}{q^4} \quad \text{for light mediator}$$

Dark matter direction detection

The next discovery?

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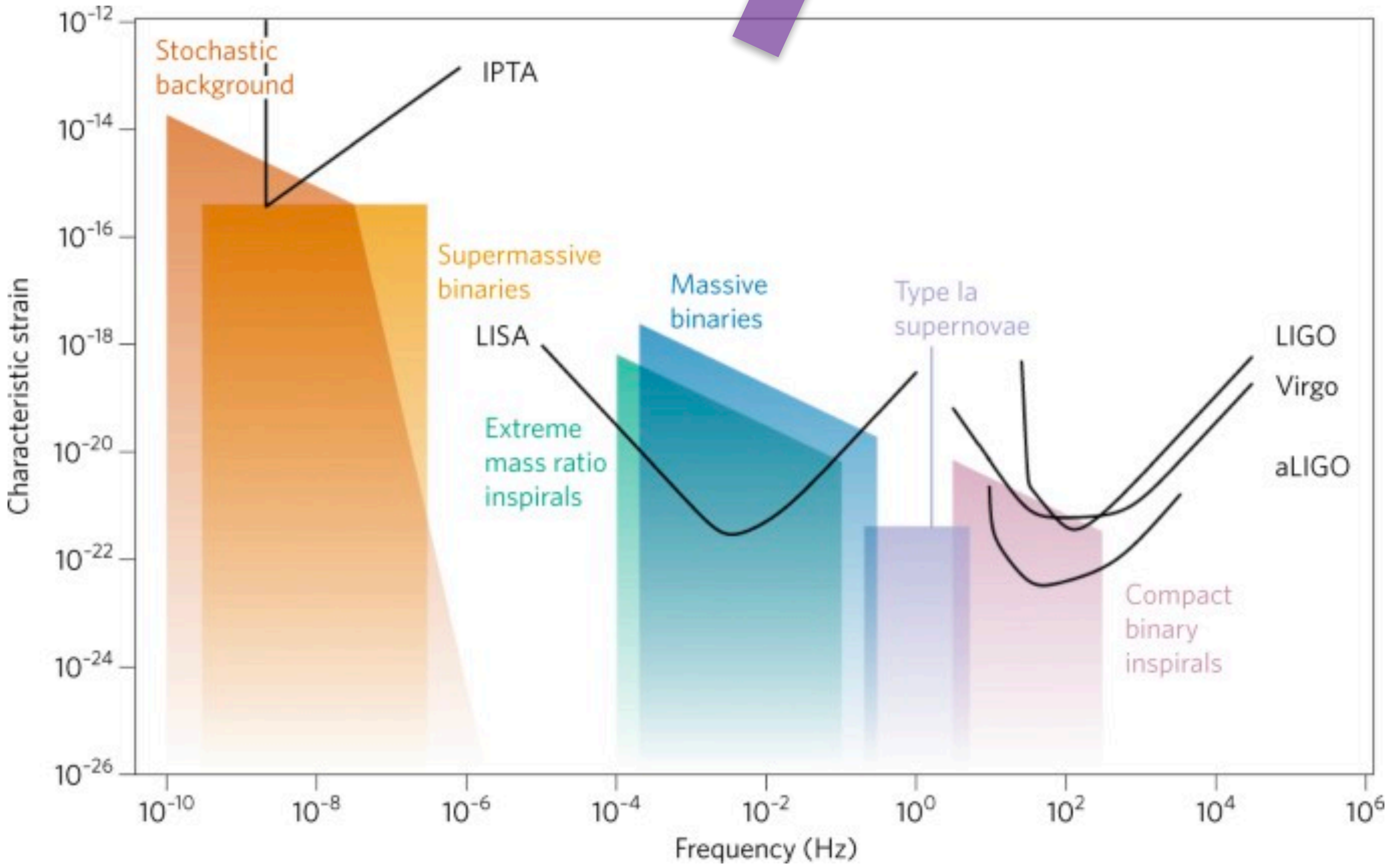


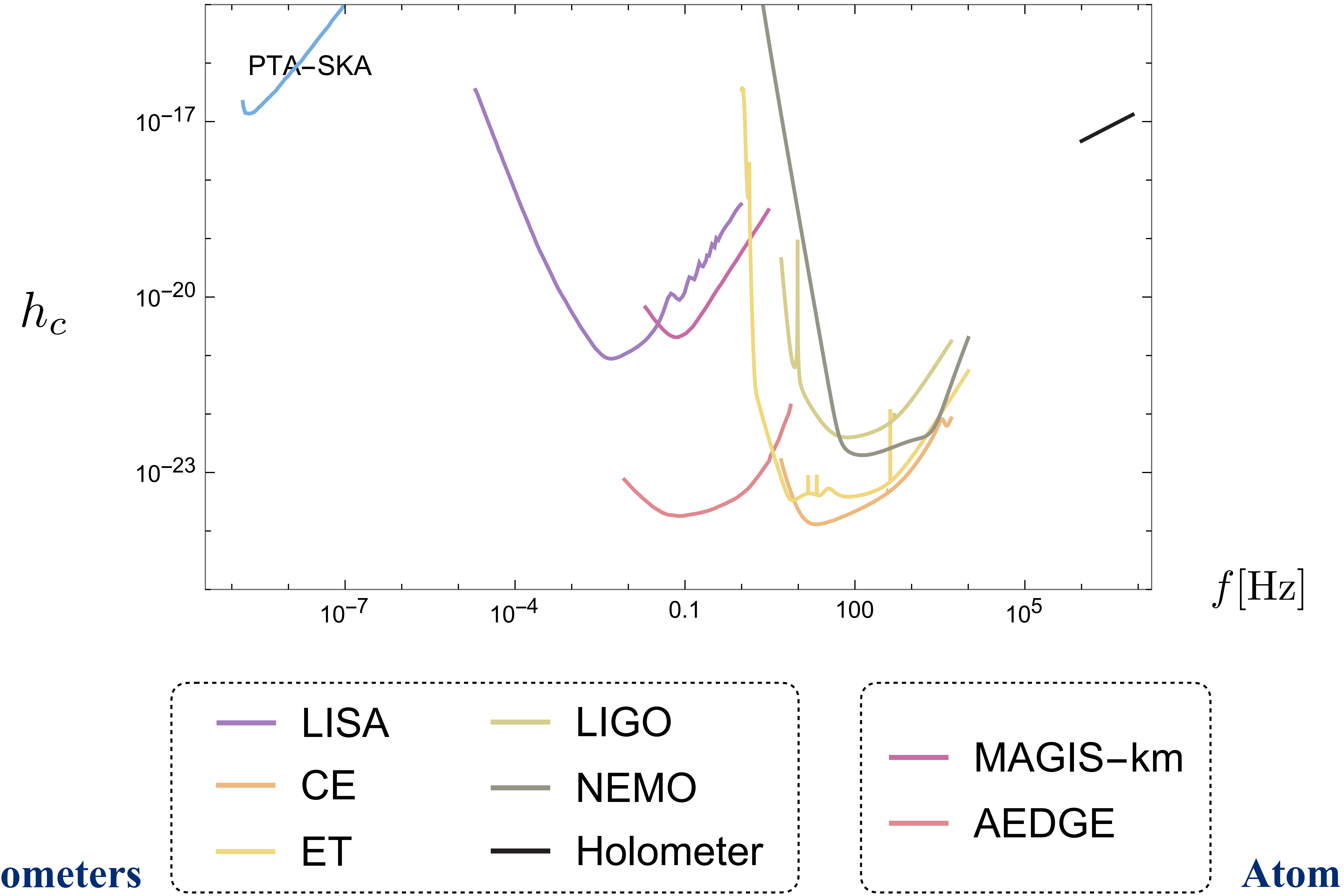
Dark matter direction detection

The next discovery?

Devices with super high sensitivity to classical and quantum fluctuations.

“sub-attometer position sensing over multi-kilometer baselines” - LIGO
And yet to be continued…

Interferometers as high precision sensors

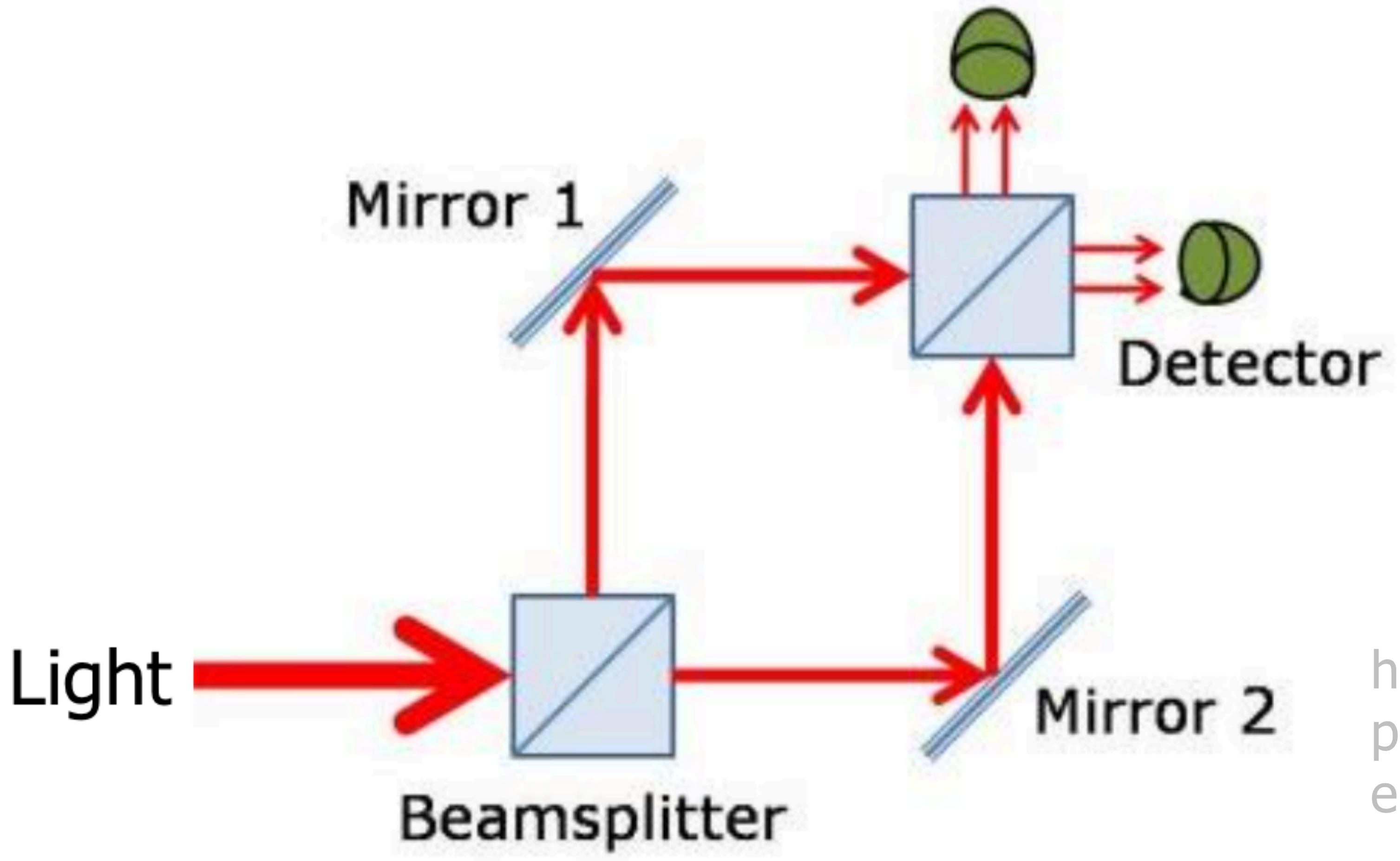


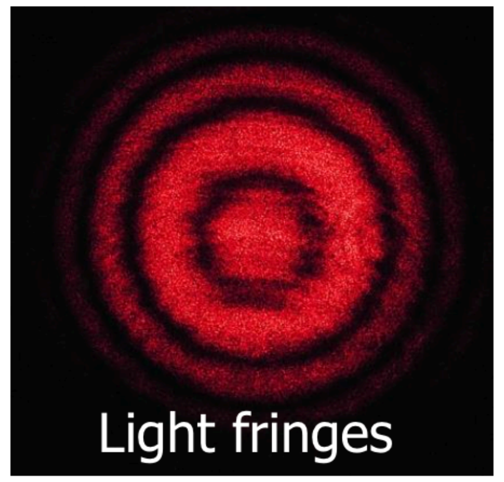
Laser Interferometers

Atom Interferometers

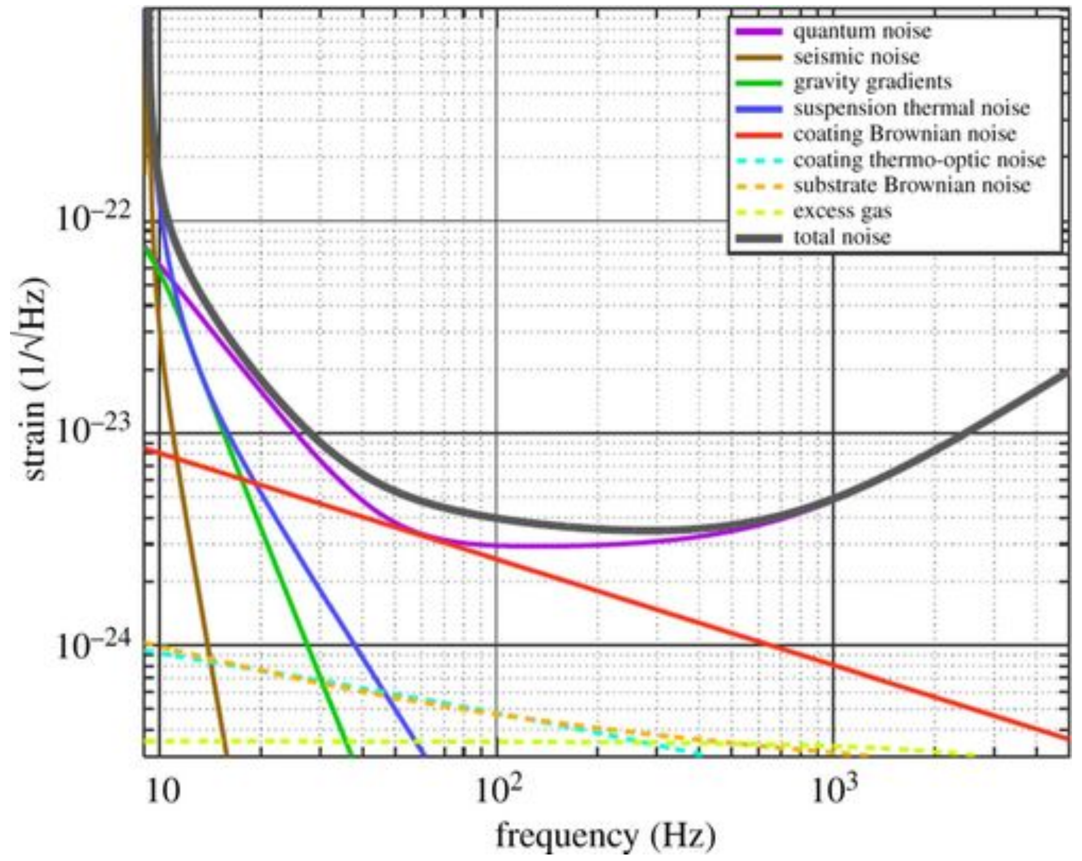
Interferometers as high precision sensors

Laser Interferometers





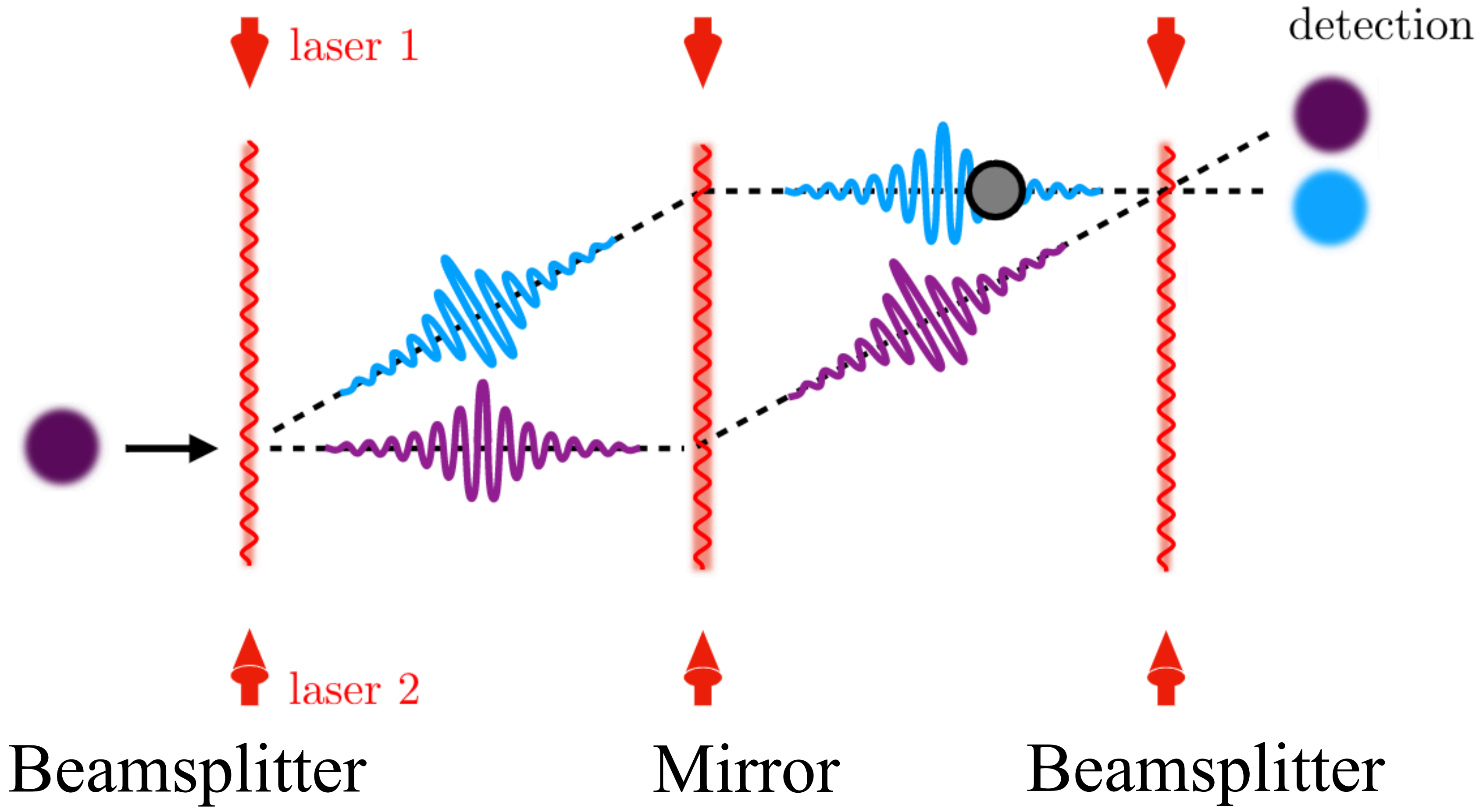
http://scienceblogs.com/principles/2013/10/22/quantum-erasure/

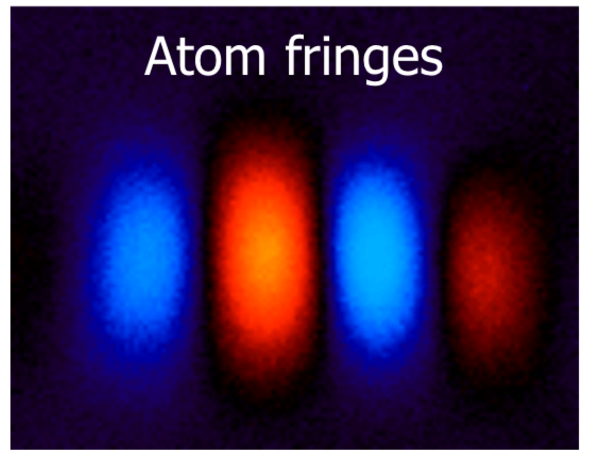


The LIGO Scientific Collaboration 2015

Gravitational wave
quantum geometry, fifth force,
dark matter direct detection…

Matter-Wave Interferometers





http://www.cobolt.se/interferometry.html

$$\sigma_\phi \equiv \frac{1\ \mathrm{rad}}{\sqrt{N_{\rm ind}}}$$

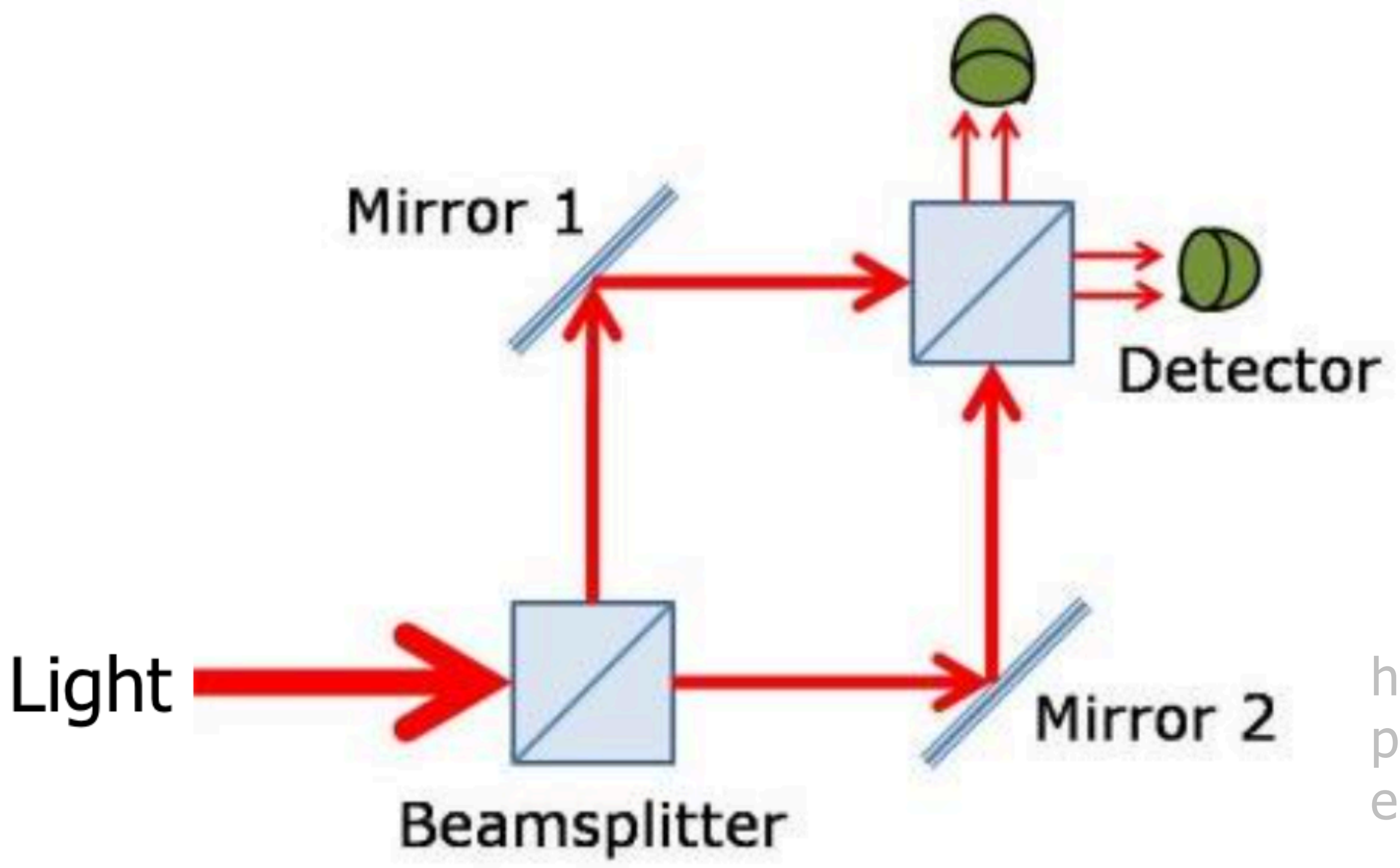
$$\sigma_\phi = \frac{1}{4} m_{\rm ind} \Delta x \, t_{\rm exp} a_{\rm min};$$

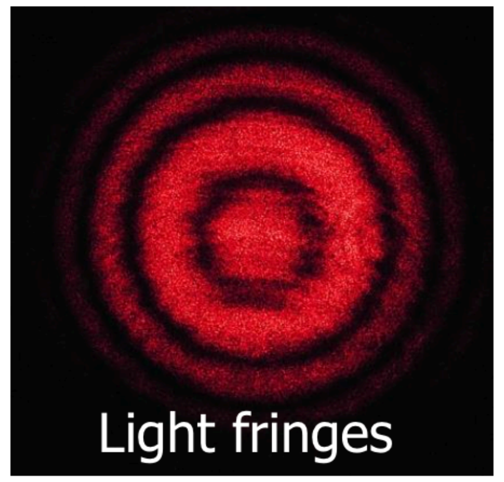
Gravitational wave
fine structure constant, equivalent principle
dark matter direct detection …

For a review, see e.g. Bertone et al, SciPost Phys.Core 3 (2020) 007

Laser interferometer as Dark Matter Detectors

Laser Interferometers





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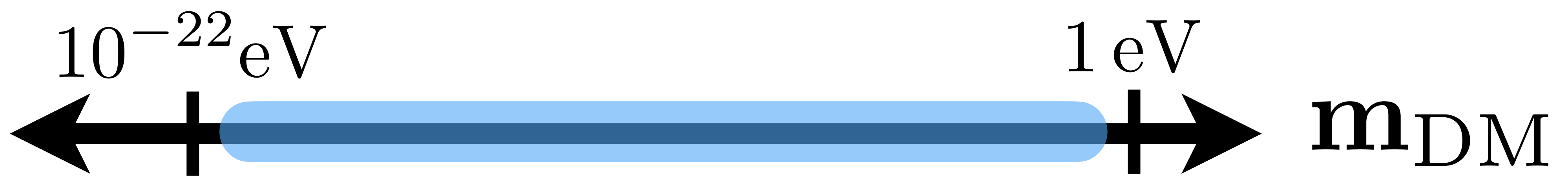
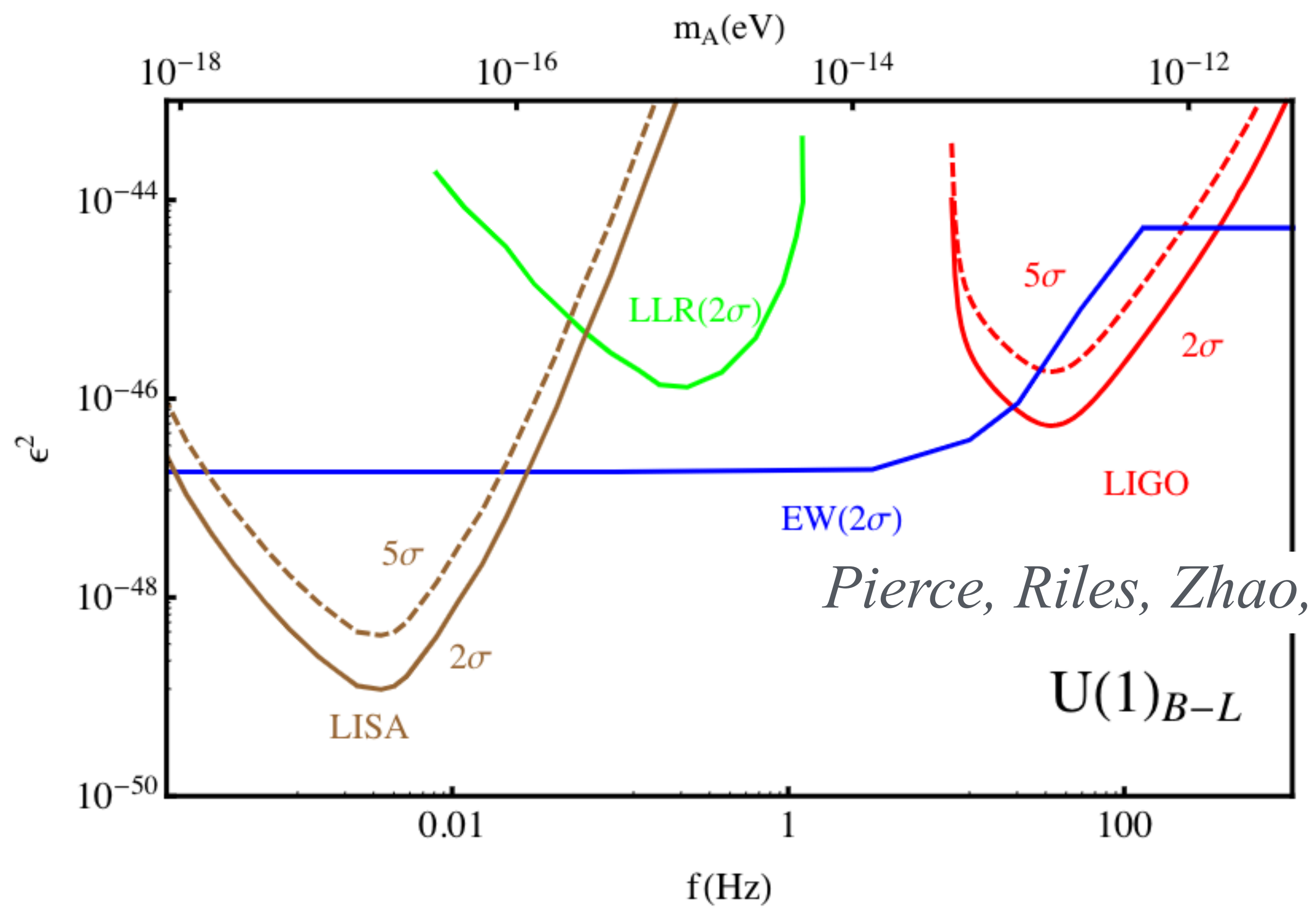
Ultralight DM: wavelike

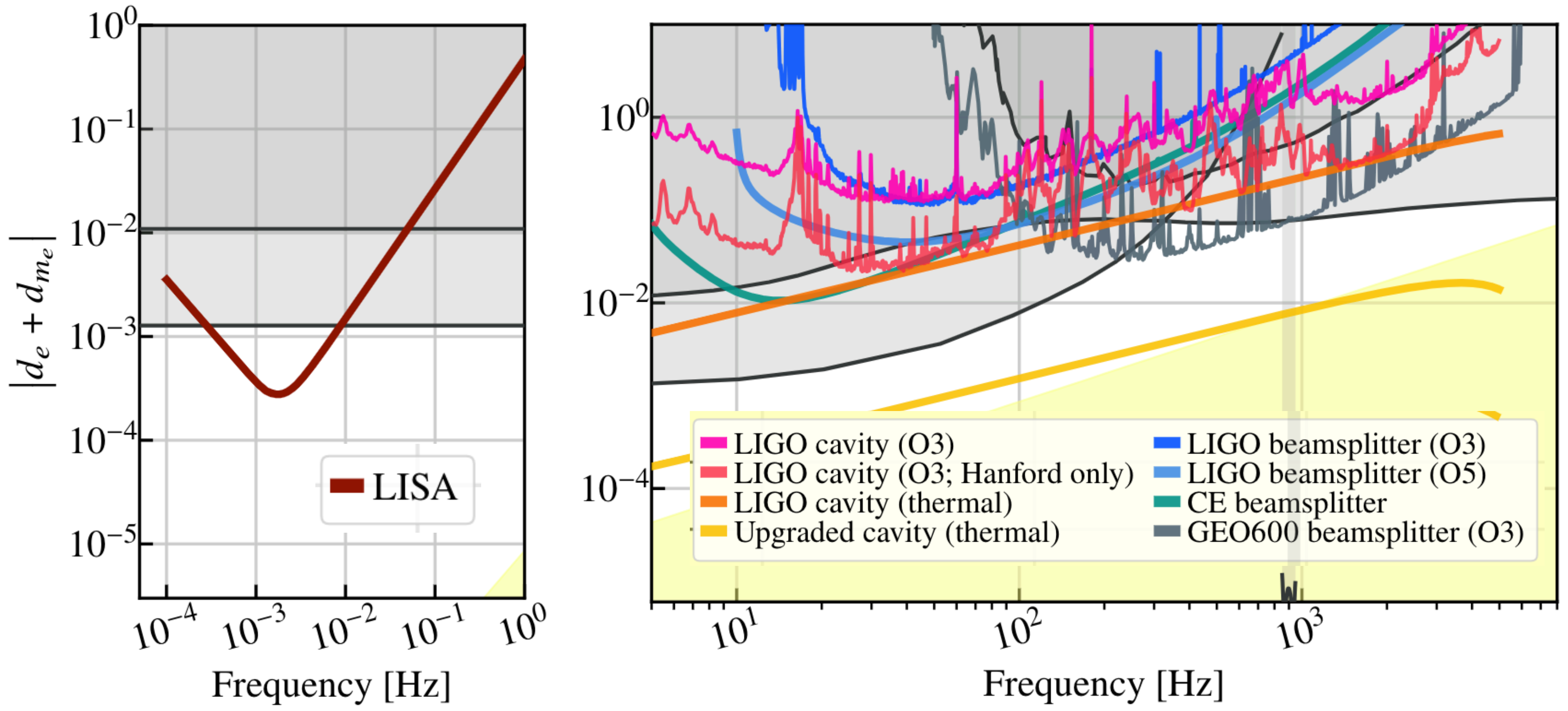


$$m_{\rm DM} \sim 4 \times 10^{-13}{\rm eV}\,\frac{f_{\rm GW}}{100\,{\rm Hz}}$$



Pierce, Riles, Zhao, 18'

$U(1)_B$ and $U(1)_{B-L}$ dark photon DM acts as a fifth force on mirrors.



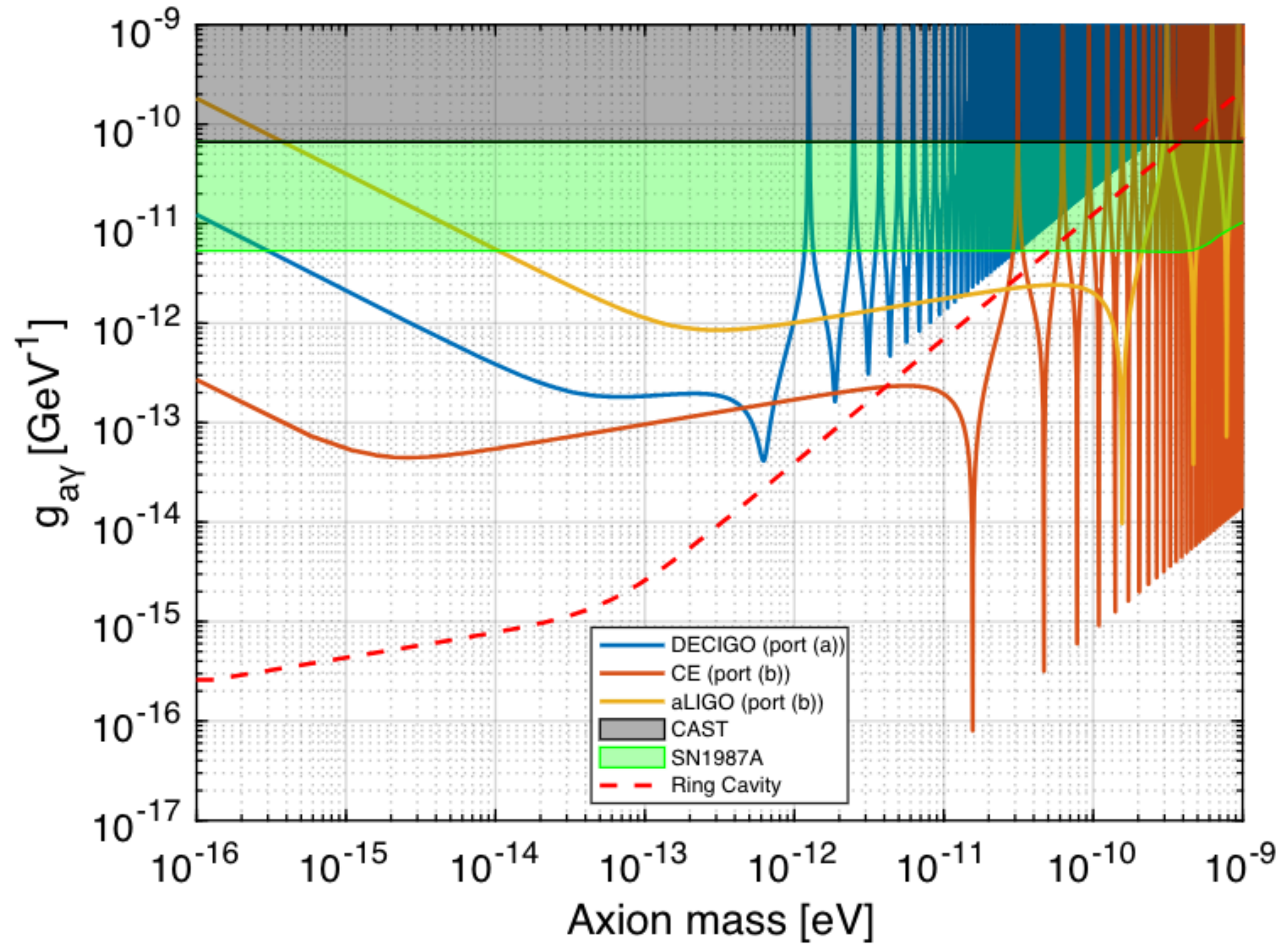
$$\frac{\Delta\alpha(t)}{\alpha} = A_e \cos\left(\Omega_{\rm DM} t\right)$$

$$\frac{\Delta m_e(t)}{m_e} = A_{m_e} \cos\left(\Omega_{\rm DM} t\right)$$

Stadnik, Flambaum '15
Morisaki1, Suyama '18
Hall, Aggarwal, '22 ...

Temporal variations on physical parameters (coupling to the photon or fermions)

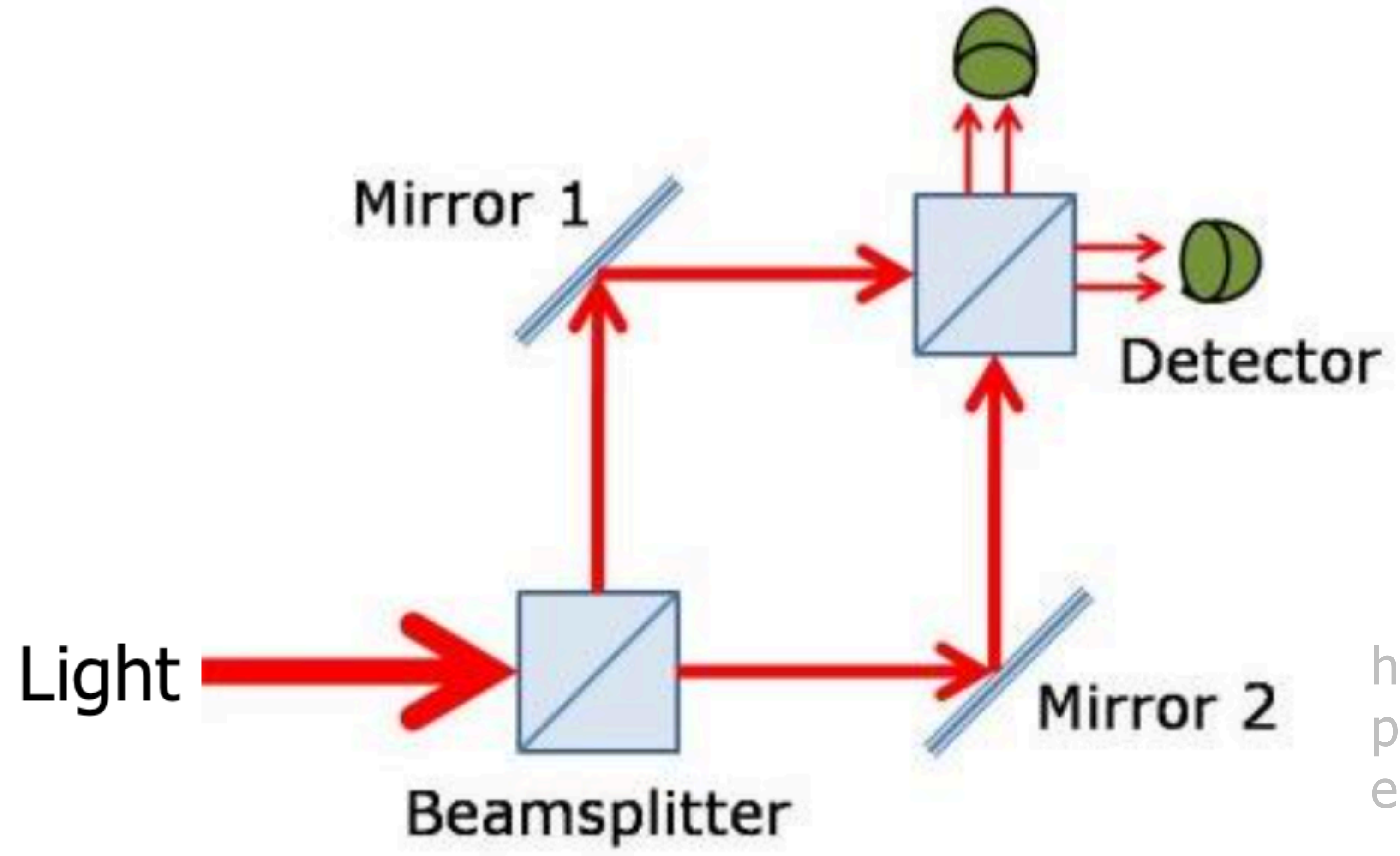
Nagano, Fujita, Michimura, Obata '19

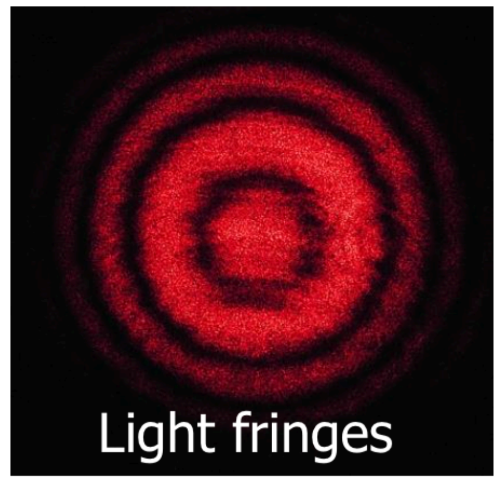


Axions can induce polarization change of photons circulating in FP cavities.

Laser interferometer as Dark Matter Detectors

Laser Interferometers





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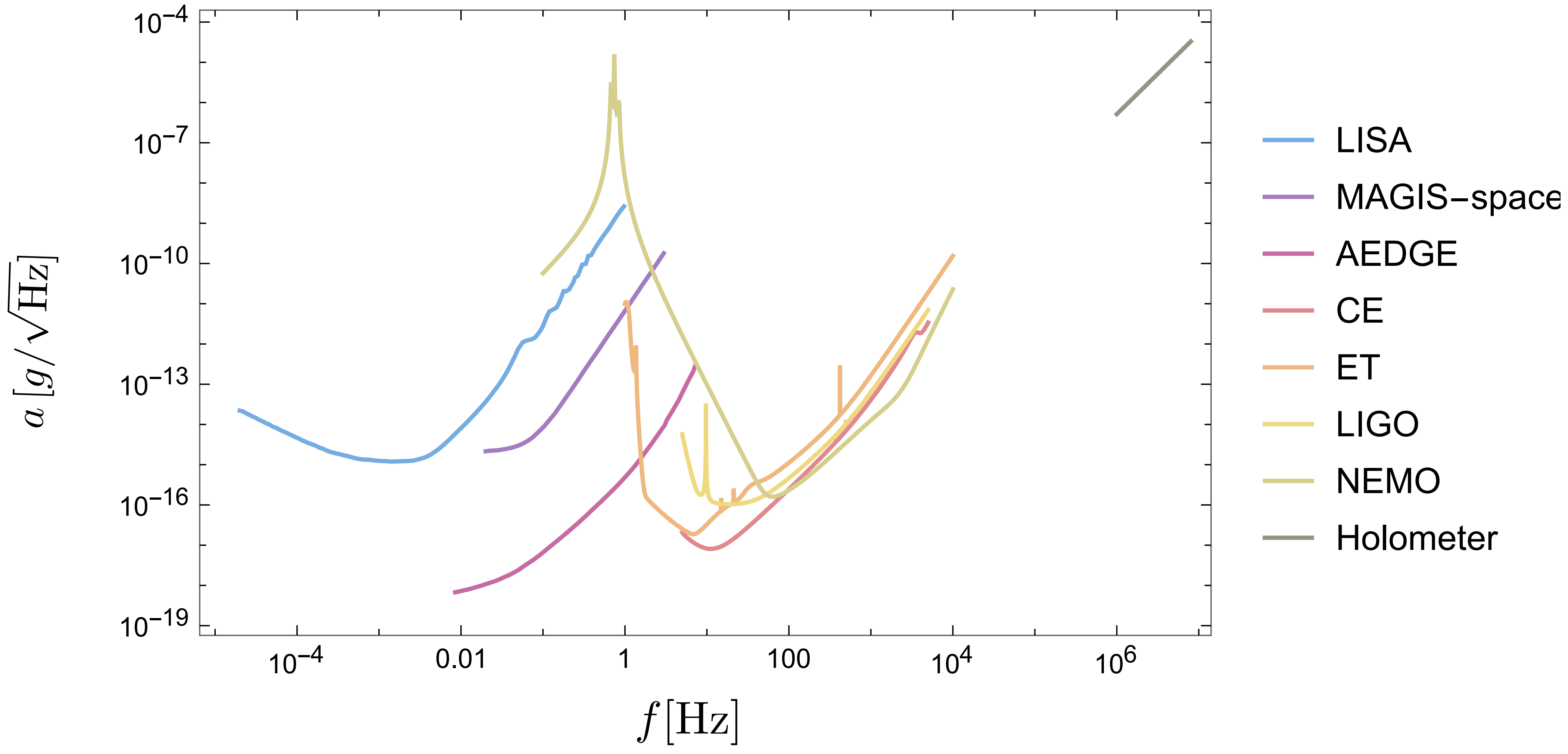
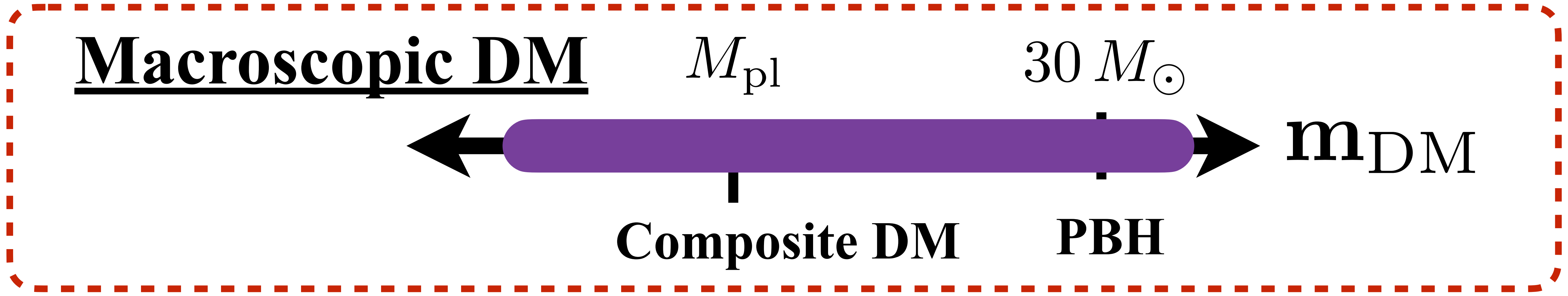
Accelerometers of ultra high sensitivity.

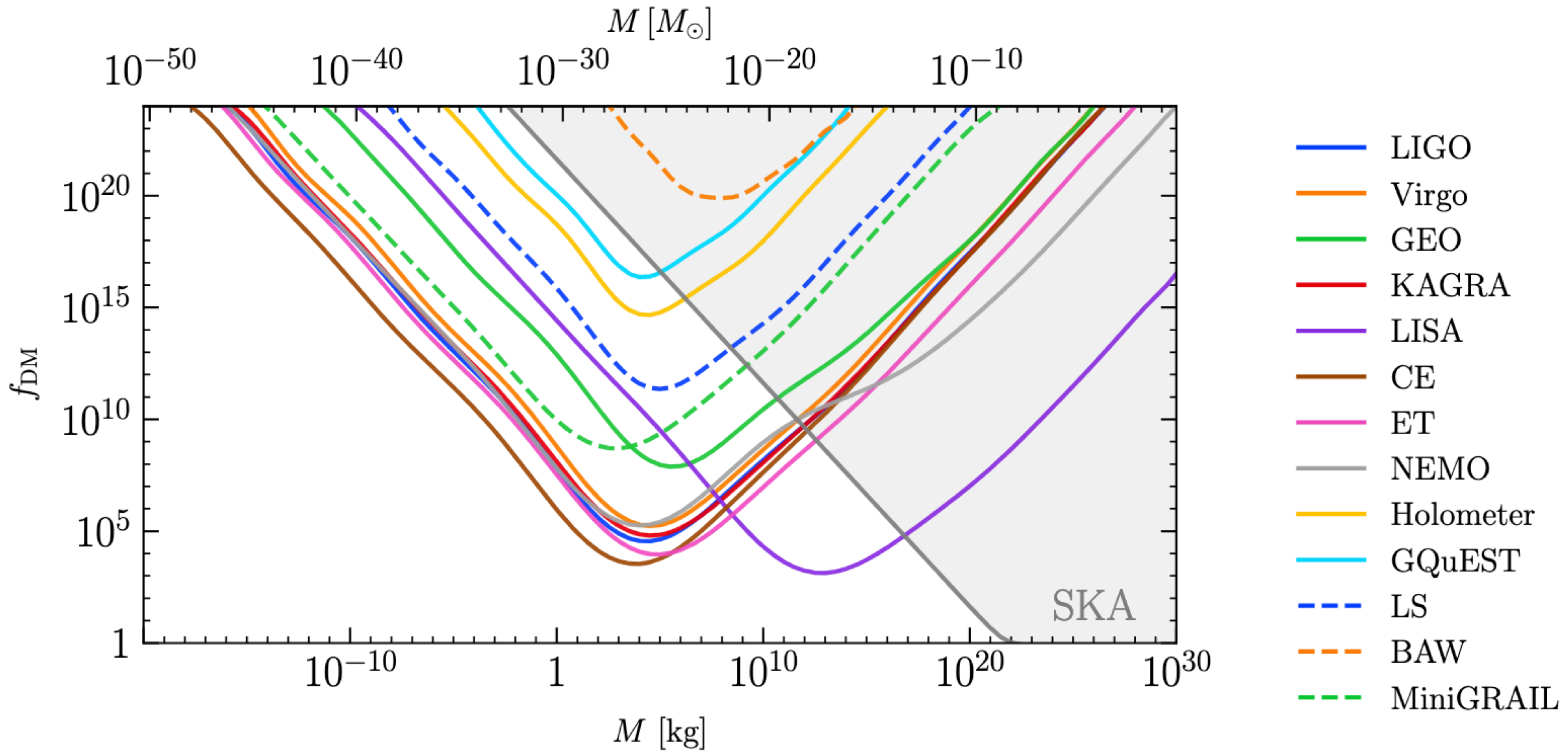




Ultra-heavy clumpy DM can induce acceleration through pure **gravity**.

$$a_{\rm DM} \sim 10^{-18}{\rm m/s^2}\,\frac{G}{6.7\times 10^{-11}\,{\rm N\,m^2/kg^2}}\,\frac{M}{10^7\,{\rm kg}}\left(\frac{4.4\times 10^7\,{\rm m}}{L}\right)^2$$

$$R \sim (\rho_{\rm DM}/M)L^2 v_{\rm DM} \sim 1/{\rm year}$$



Projected 90th-percentile upper limits on transiting DM fraction. One year observation time.

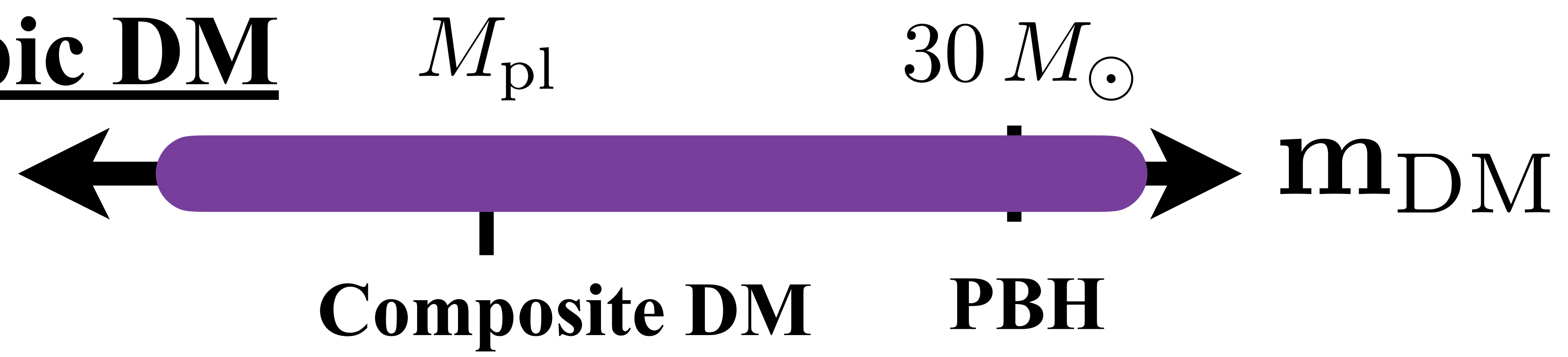
Du, Lee, YW, Zurek 23'

Laser interferometer as Dark Matter Detectors

Du, Lee, YW, Zurek 23'

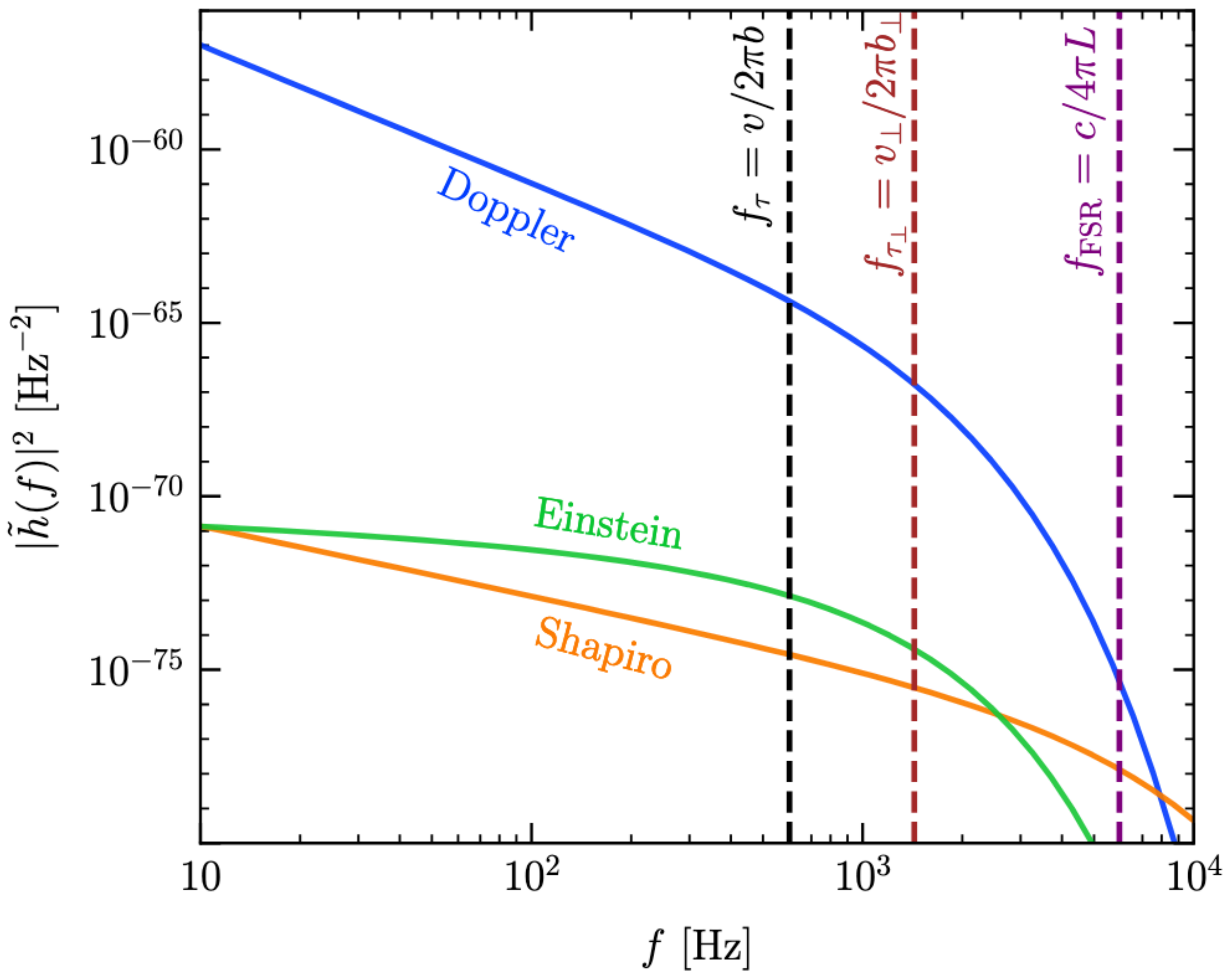
Beyond a deterministic accelerometer model.

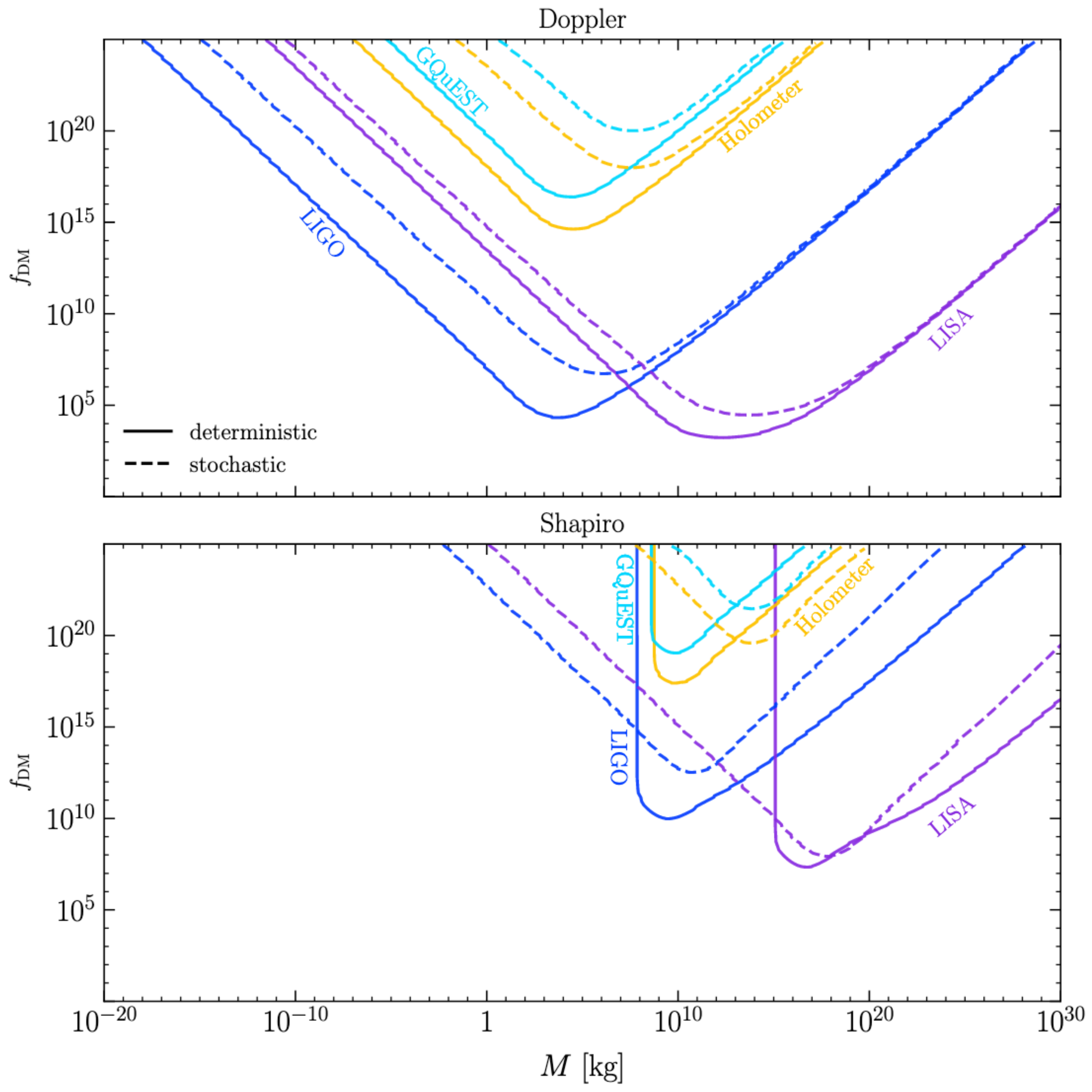
Macroscopic DM



$$c\delta T_\gamma \equiv cT_\gamma - 2L = \underbrace{c\delta\tau}_{\text{Einstein}} + \underbrace{2r_{\rm M}\left(t+\frac{L}{c},L\right) - r_{\rm M}\left(t,0\right) - r_{\rm M}\left(t+\frac{2L}{c},0\right)}_{\text{Doppler}}$$

$$\underbrace{+\frac{1}{2}\int_0^L dr\mathcal{H}^{\rm out}\left(t+\frac{r}{c},r\right) - \frac{1}{2}\int_L^0 dr\mathcal{H}^{\rm in}\left(t+\frac{2L-r}{c},r\right)}_{\text{Shapiro}}.$$





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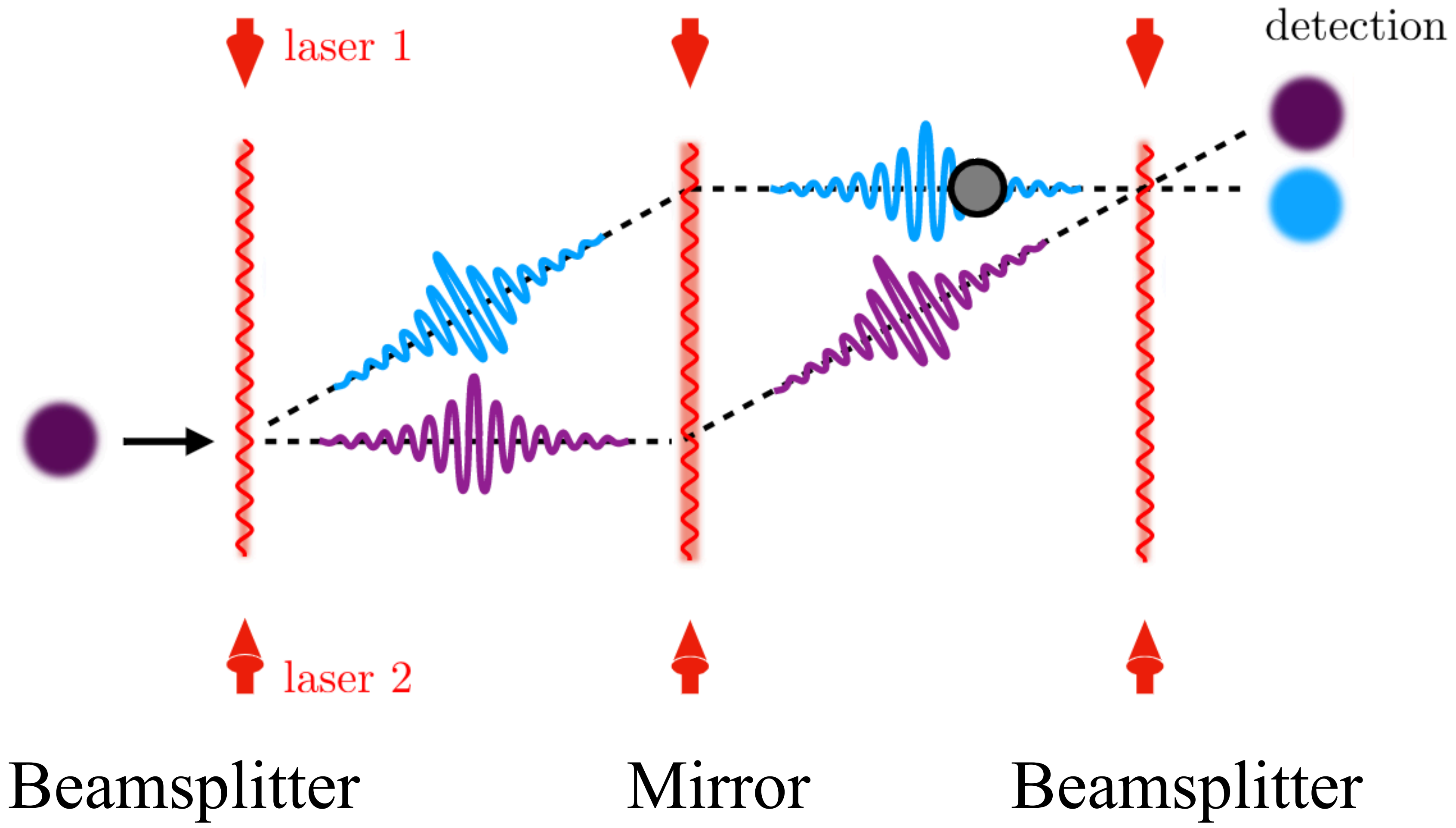
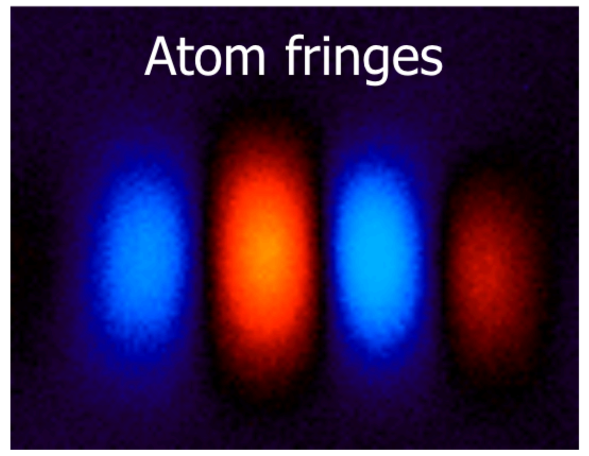
Matter-Wave Interferometers as Dark Matter Detectors

Matter-Wave Interferometers





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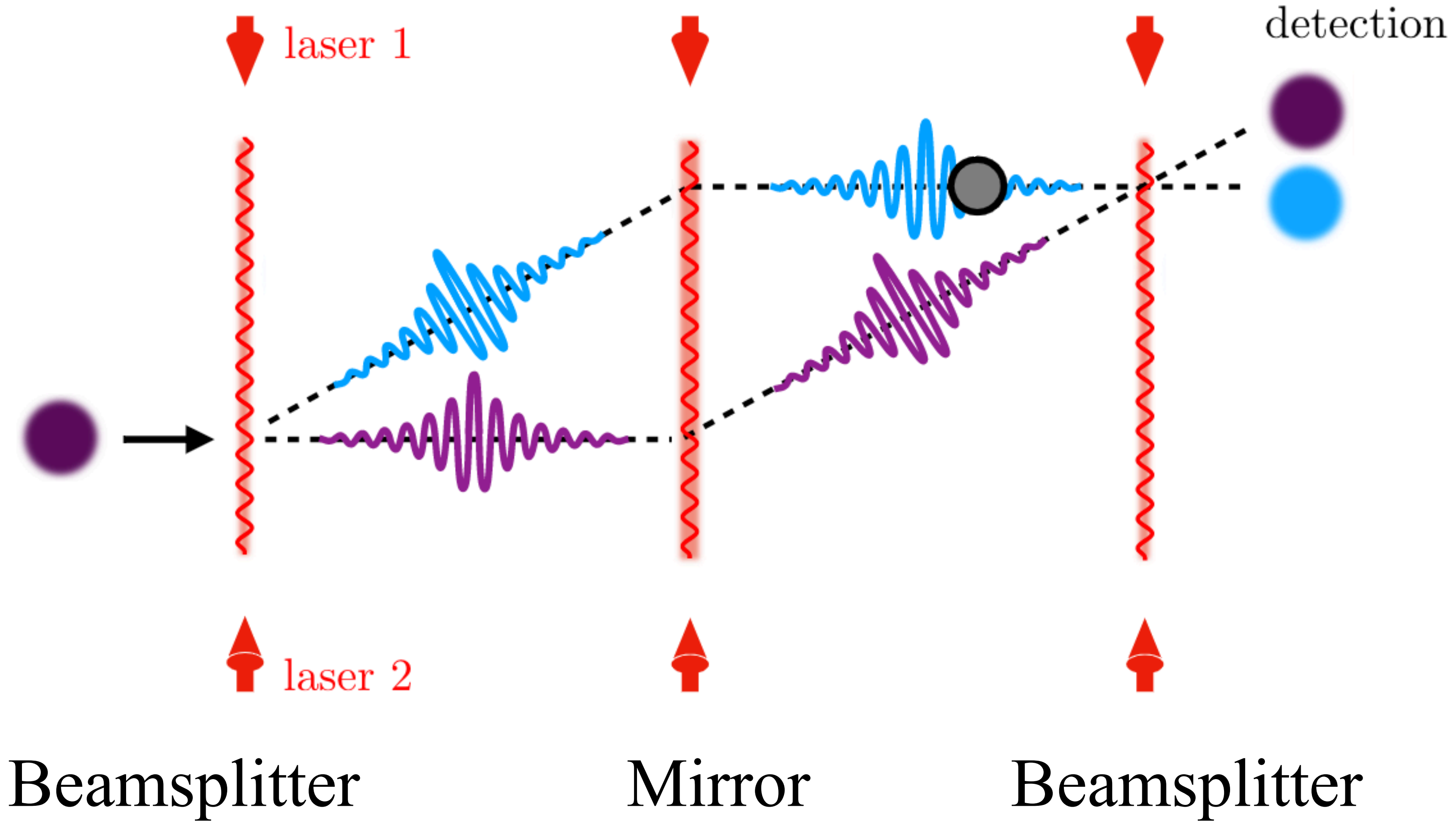

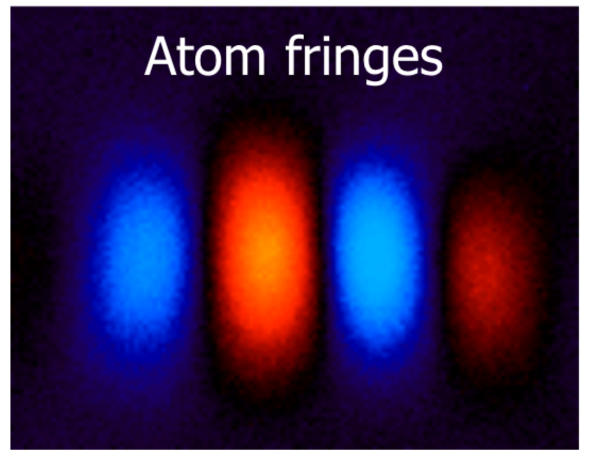

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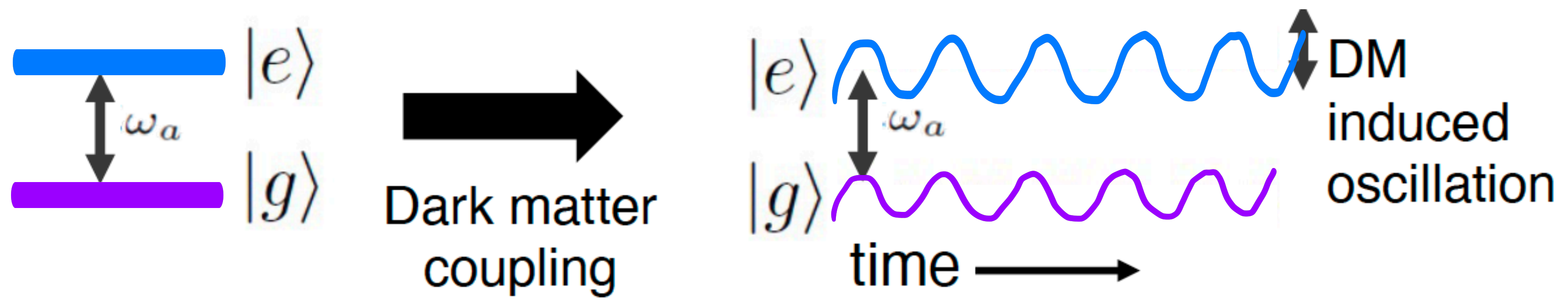
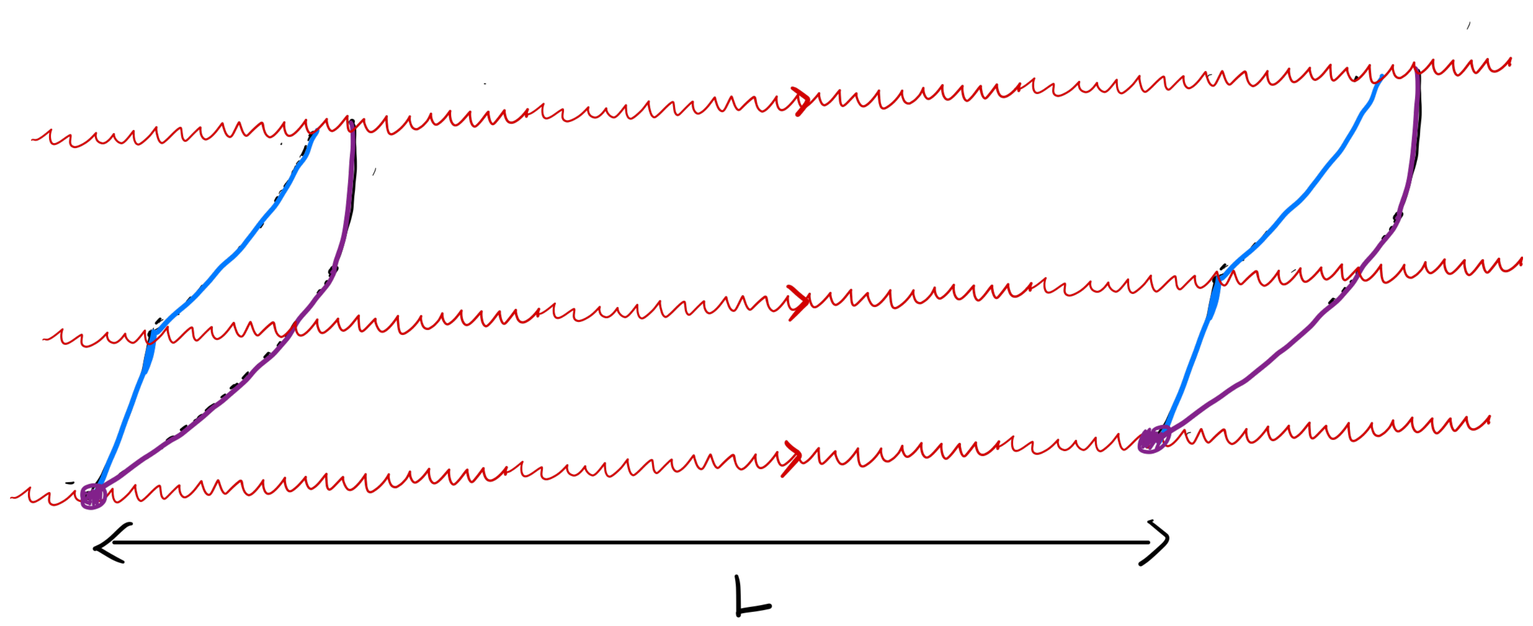


From Oliver Buchmueller's talk

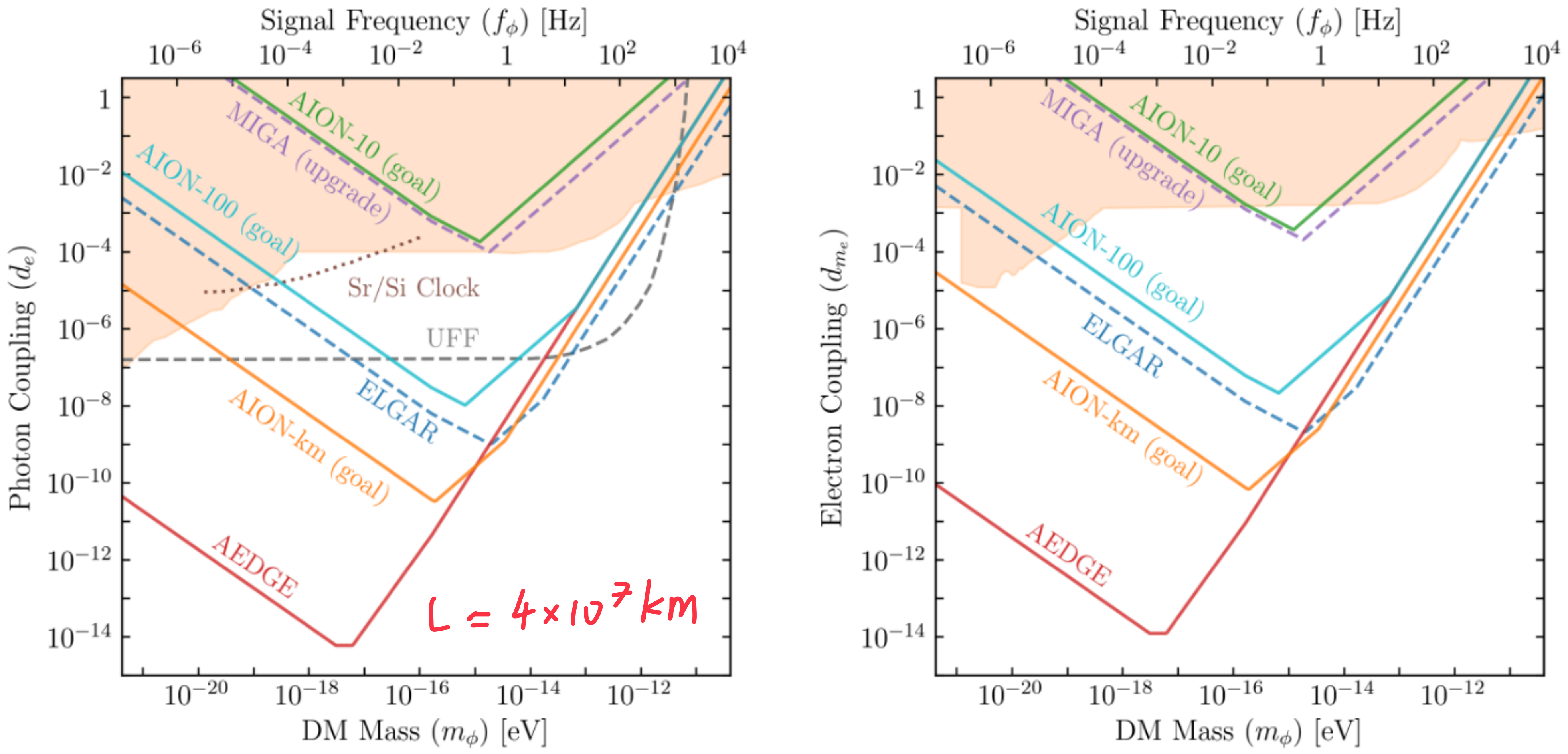


Long baseline large scale experiment

Snowmass

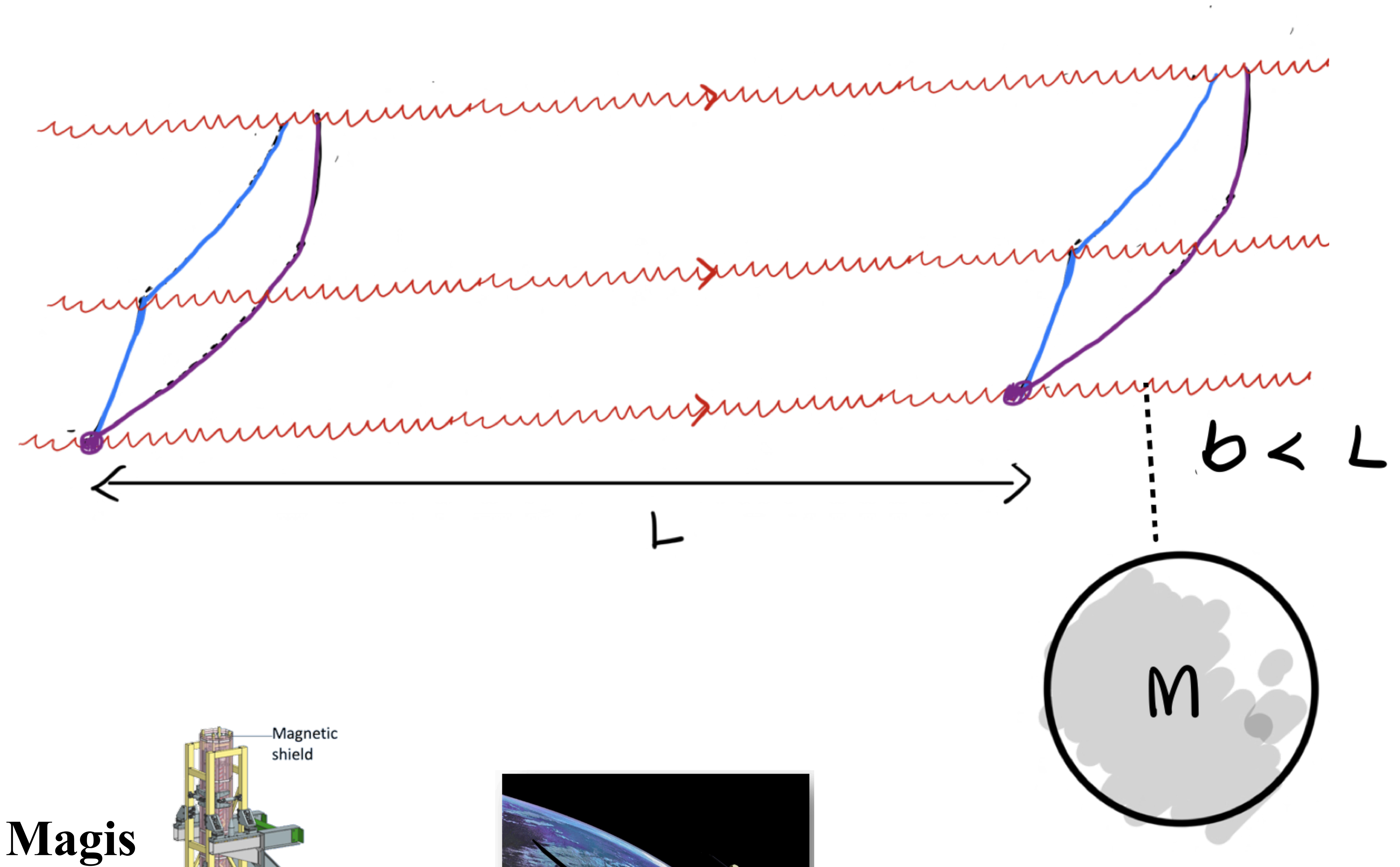
2203.07250

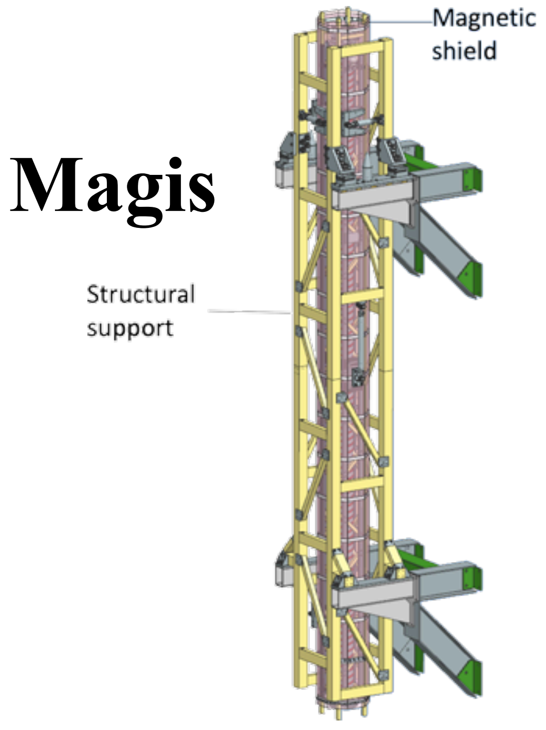
2201.07789



Matter-wave interferometers as dark matter detectors

Long-baseline Atom Interferometers





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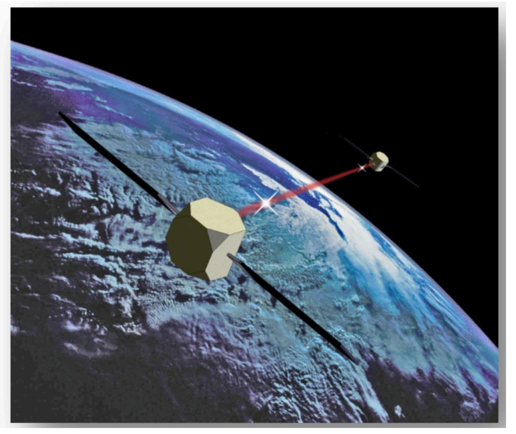
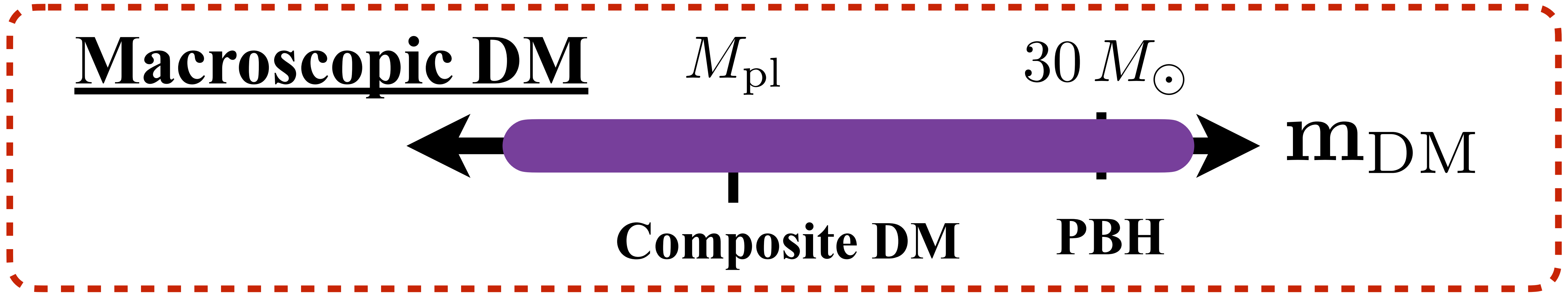


AEDGE

$$\Delta\phi \sim k_{\rm eff}\, a\, t_{\rm exp}^2$$

$$a_{\rm sens} \sim 10^{-18}{\rm m/s^2}$$



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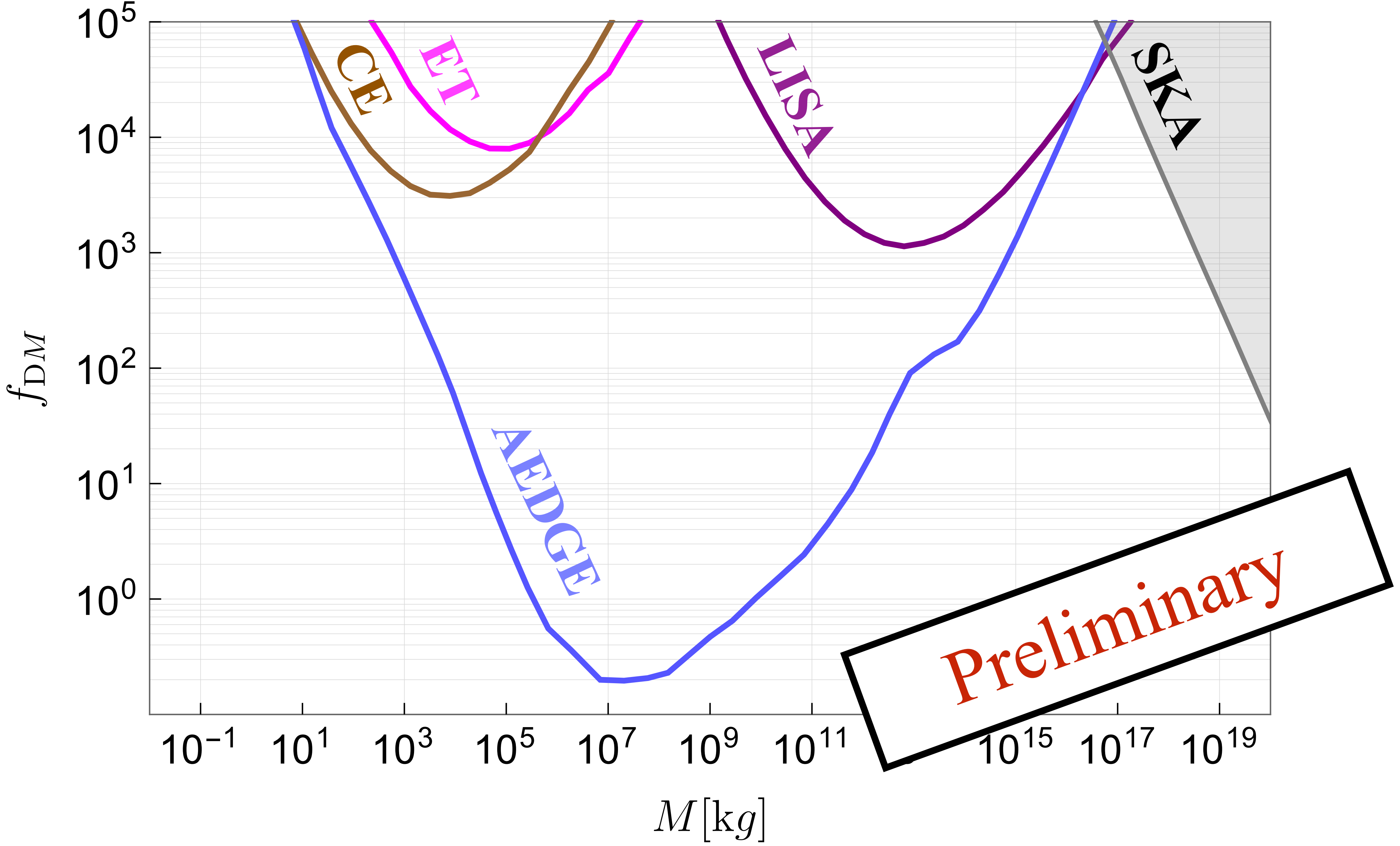
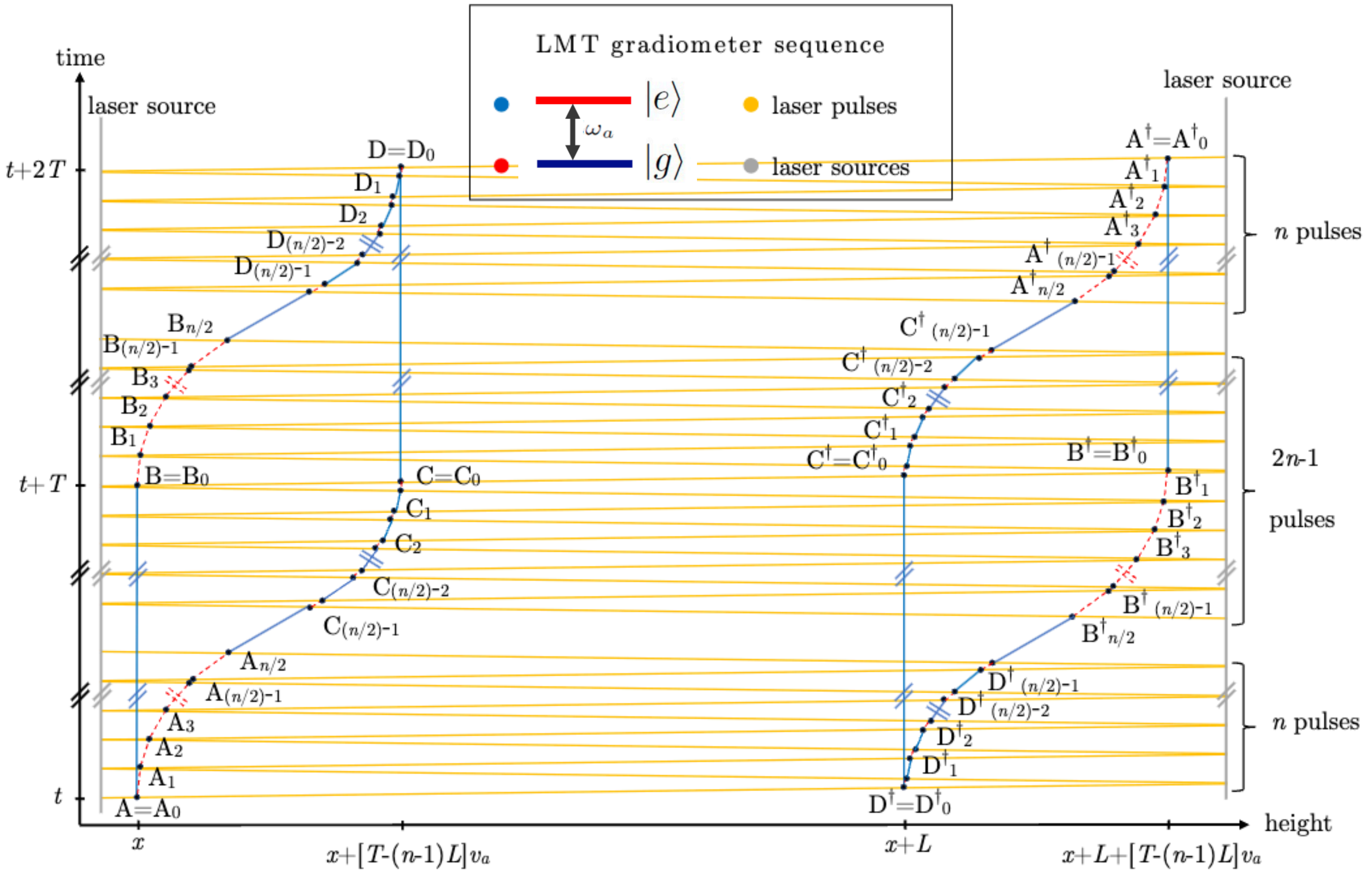


Projected 90th-percentile upper limits on transiting DM fraction.

Atom interferometer phase shift from linearized gravity and a weak potential

Long-baseline Atom Interferometer Gradiometer



$$\Delta\phi_{\rm tot} = \oint m\sqrt{-g_{\mu\nu}\frac{dx^\mu}{dt}\frac{dx^\nu}{dt}}\,dt$$

[Dimopoulos, Graham, Hogan, and Kasevich1, 08'] [Roura, 20']

Badurina, Du, Lee, YW, Zurek 23'

$$\Delta\phi_{\rm tot} = \underbrace{-\frac{1}{2}\oint m\left(1+\int dt'\,\mathbf{v}_a\cdot\nabla\right)h_{00}\,dt}_{\textbf{Einstein delay}} \;\underbrace{+\,\omega_a\int_{|e\rangle} dt}_{\textbf{Doppler shift, Shapiro delay}} \;+\mathcal{O}(h^2 v_a, h v_a^2)$$

Einstein delay **Doppler shift, Shapiro delay**

Example: a weak potential

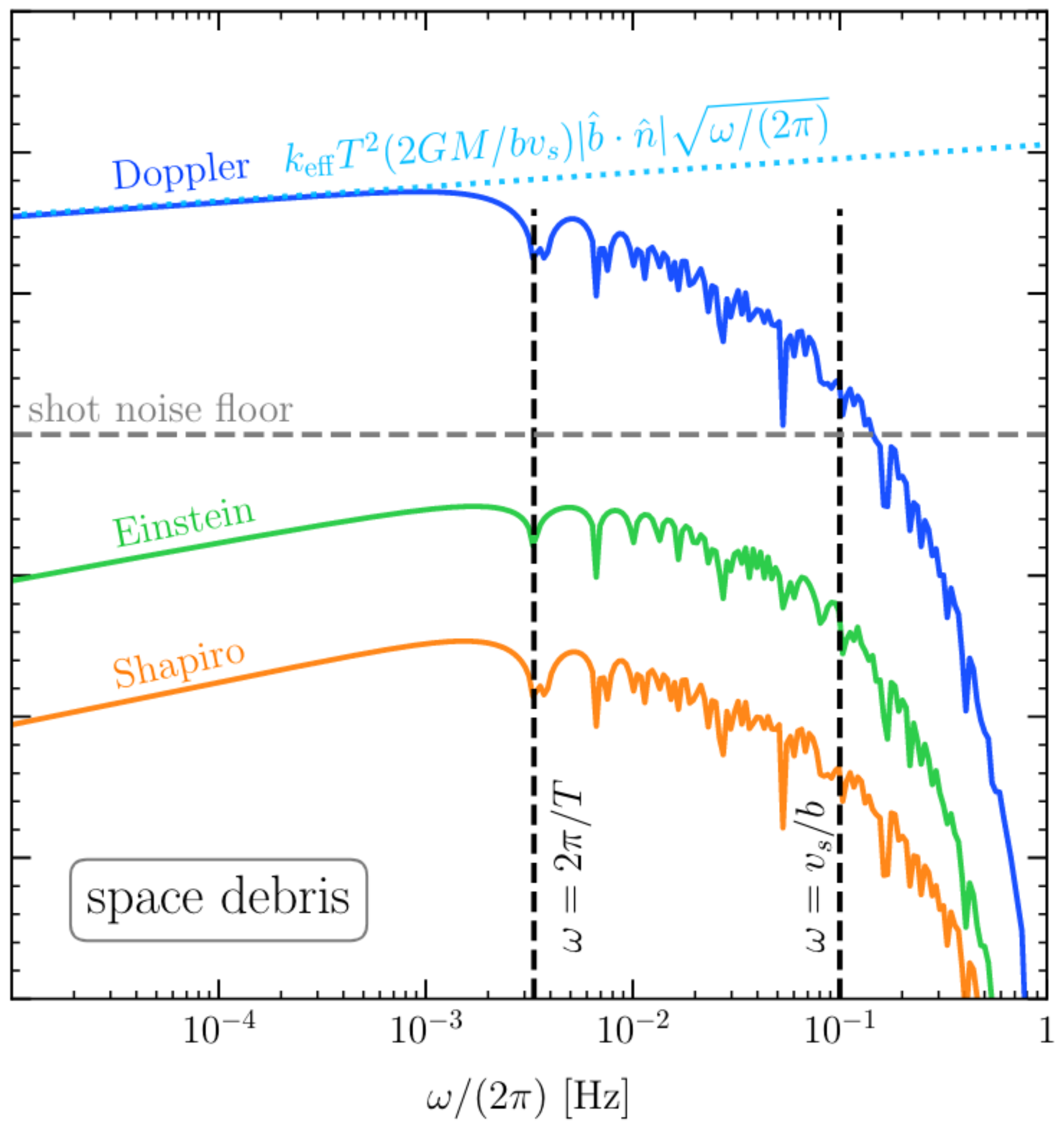
$$ds^2 = -(1+2\Phi)dt^2 - 8\Phi v_i dx_i dt + (1-2\Phi)dx_i dx_i$$

gives

$$\widetilde{\Delta\phi}_{\mathcal{E}}(\omega,\mathbf{x}_0) = T^2\,\mathbf{k}_{\rm eff}\cdot\nabla\tilde{\Phi}(\omega,\mathbf{x}_0)\,K_1(\omega) + T^2\,\omega_a\tilde{\Phi}(\omega,\mathbf{x}_0)\,K_2(\omega)$$

$$\widetilde{\Delta\phi}_{\mathcal{D}}(\omega,\mathbf{x}_0) \simeq T^2\,\mathbf{k}_{\rm eff}\cdot\nabla\tilde{\Phi}(\omega,\mathbf{x}_0)\,K_D(\omega)$$

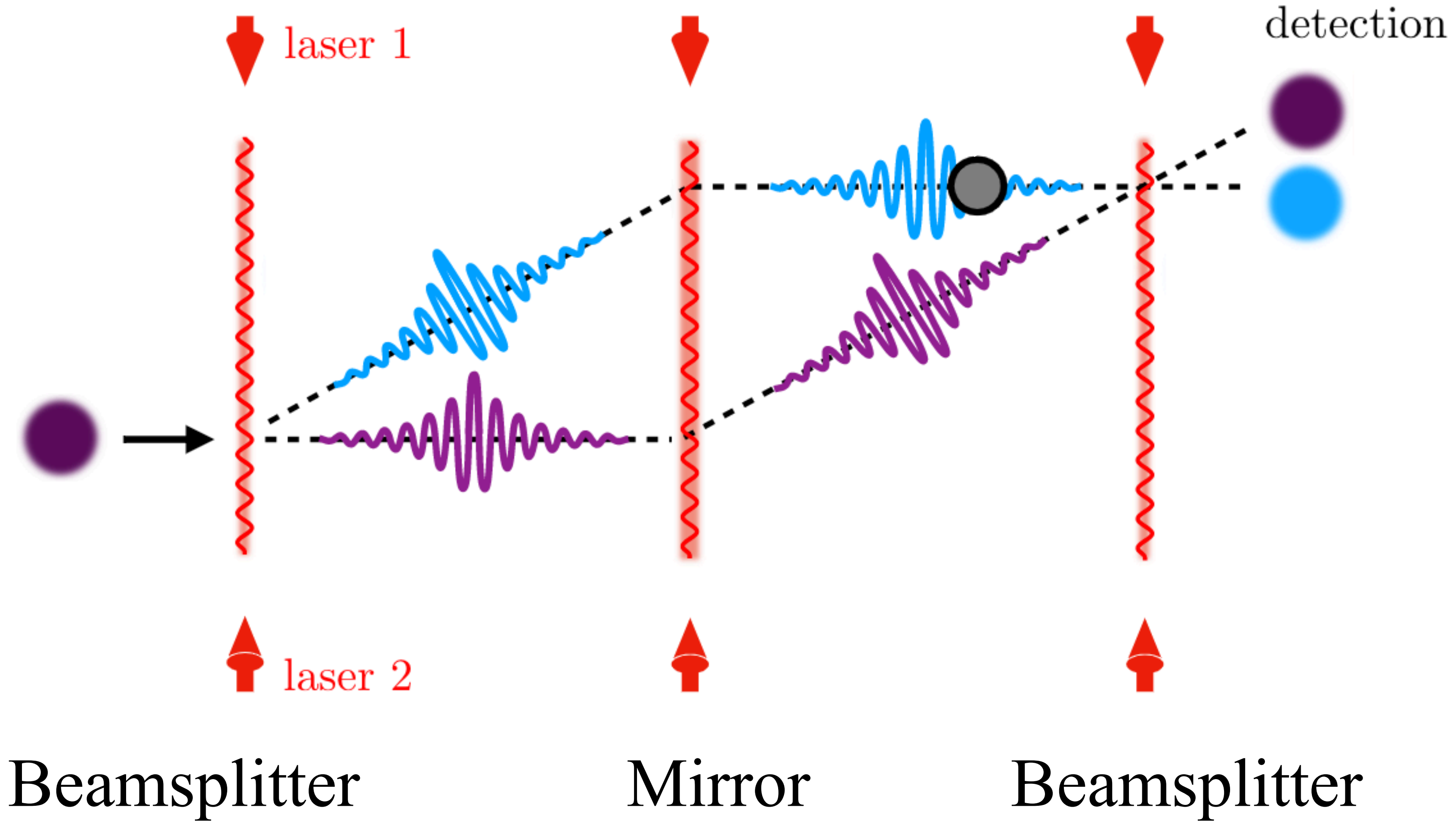
$$\widetilde{\Delta\phi}_{\mathcal{S}}(\omega,\mathbf{x}_1\to\mathbf{x}_2) \simeq T^2\,|\mathbf{k}_{\rm eff}|\mathcal{FT}_{t\to\omega}\left\{\int_{|\mathbf{x}_2-\mathbf{x}_1|/2}^{-|\mathbf{x}_2-\mathbf{x}_1|/2} dx\,\Phi\Big(t\pm x, \frac{\mathbf{x}_1+\mathbf{x}_2}{2} + x\frac{\mathbf{x}_2-\mathbf{x}_1}{|\mathbf{x}_2+\mathbf{x}_1|}\Big)\right\}K_S(\omega)$$

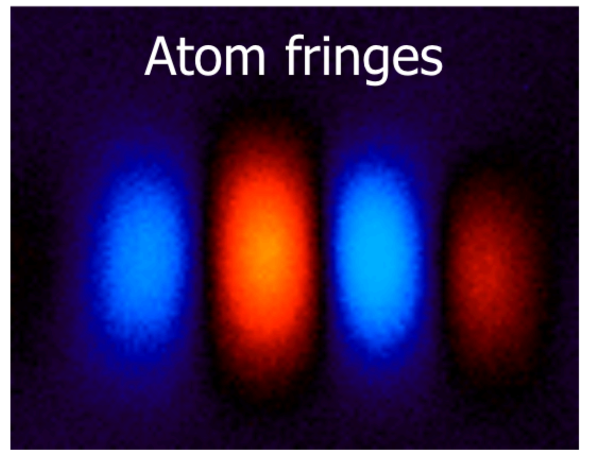


Different contributions to the phase shift of a benchmark space debri.

Matter-Wave Interferometers as Dark Matter Detectors

Matter-Wave Interferometers



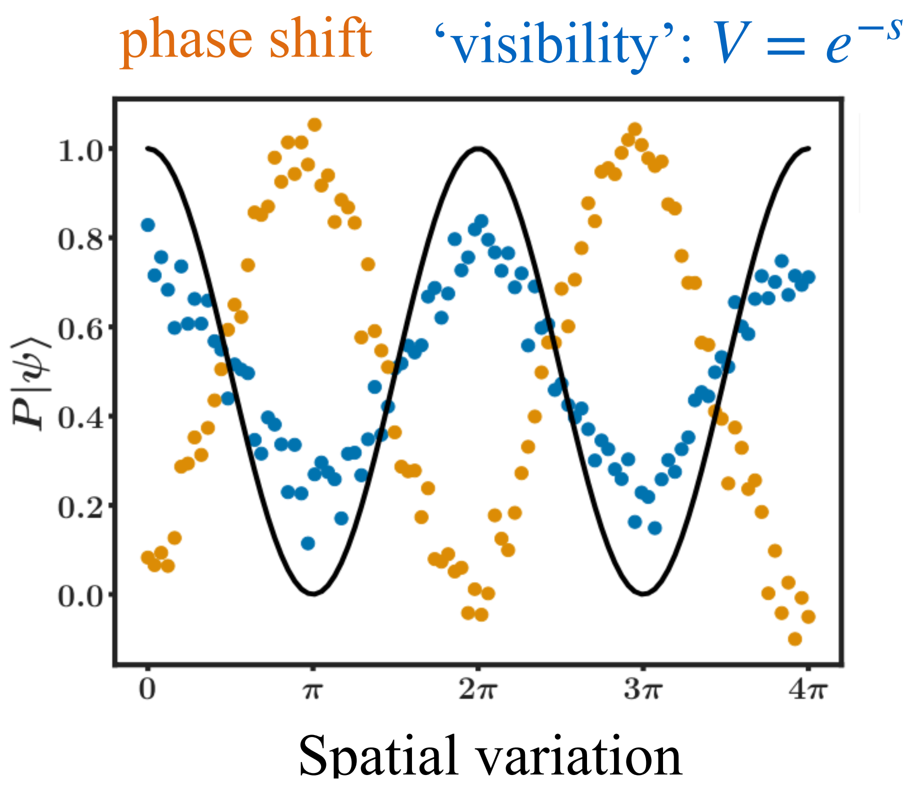


http://www.cobolt.se/interferometry.html

$$\Psi = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\end{pmatrix}\begin{pmatrix}\bullet \\ \bullet\end{pmatrix}$$

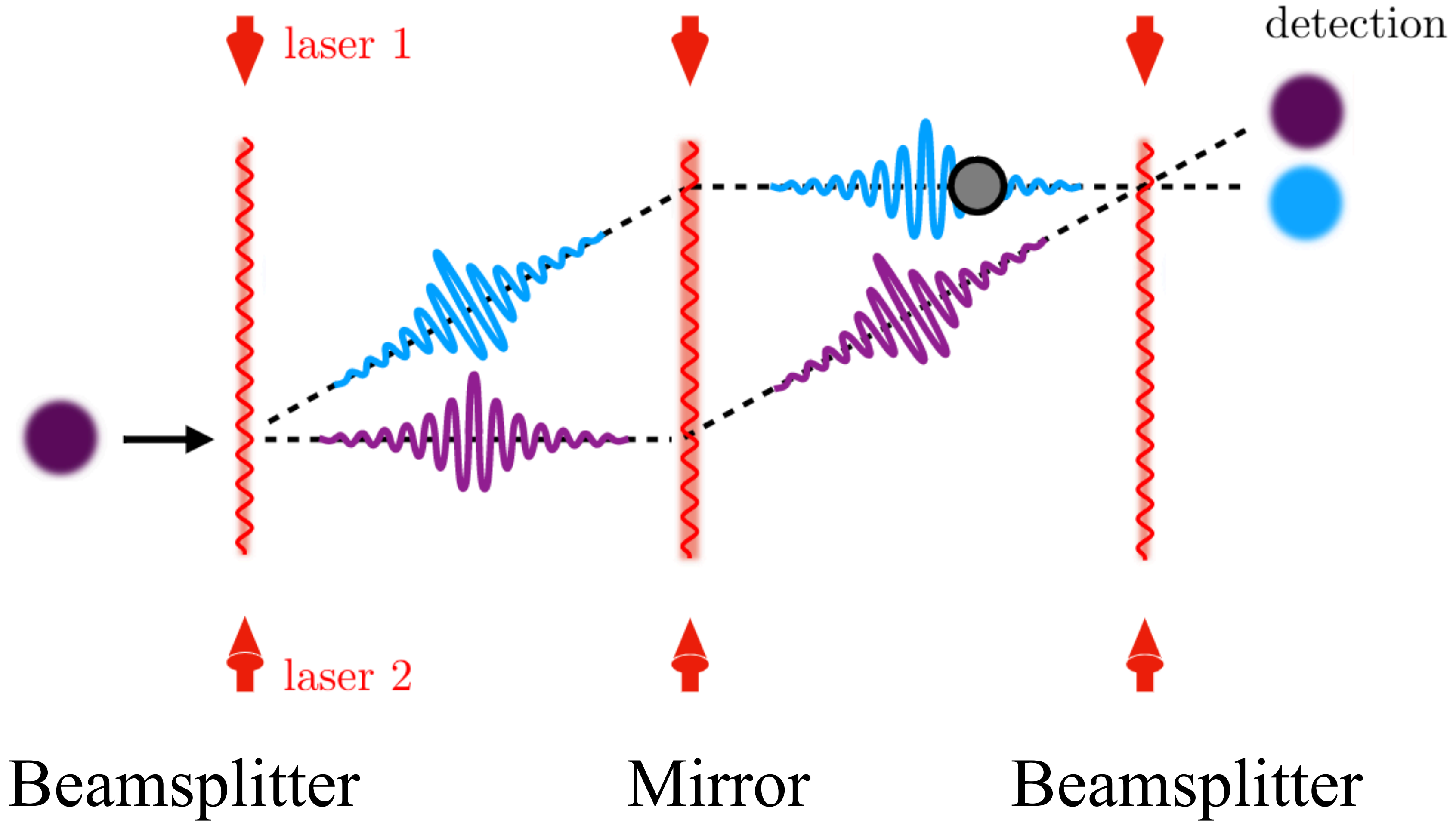
$$\tilde{\Psi} = \rho\,\Psi = \frac{1}{2}\begin{pmatrix}1 & \gamma \\ \gamma^* & 1\end{pmatrix}\Psi \qquad \gamma = e^{-s+i\phi}$$

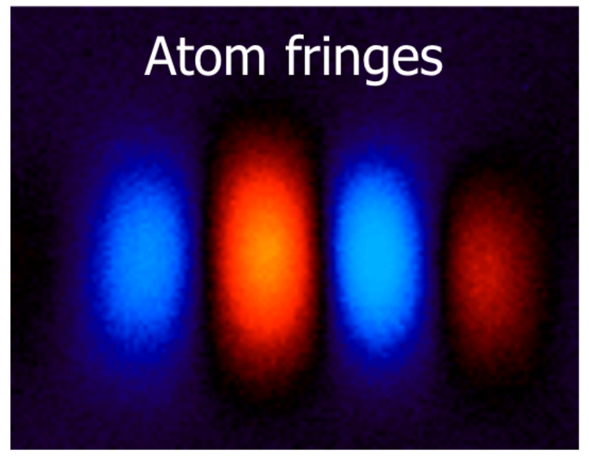
The fringe: $\langle\tilde{\Psi}|\Psi\rangle = \frac{1}{2}\left(1 + e^{-s}\cos\phi\right)$



Matter-Wave Interferometers as Dark Matter Detectors

Matter-Wave Interferometers





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$$\Psi = \frac{1}{\sqrt{2}} \left(1 \quad 1\right) \begin{pmatrix} \bullet \\ \bullet \end{pmatrix}$$

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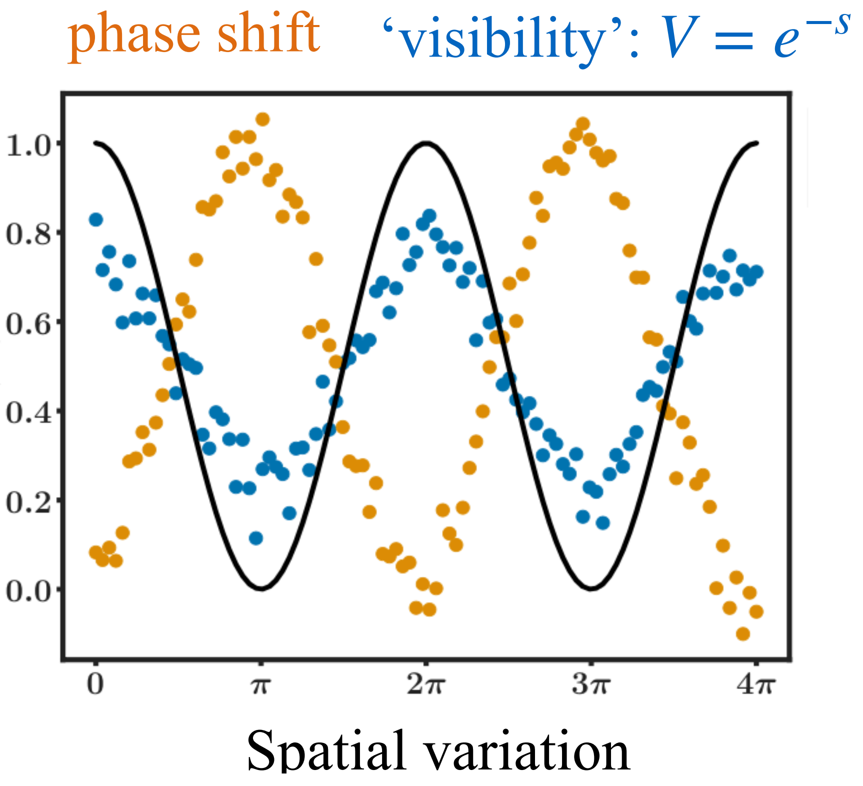
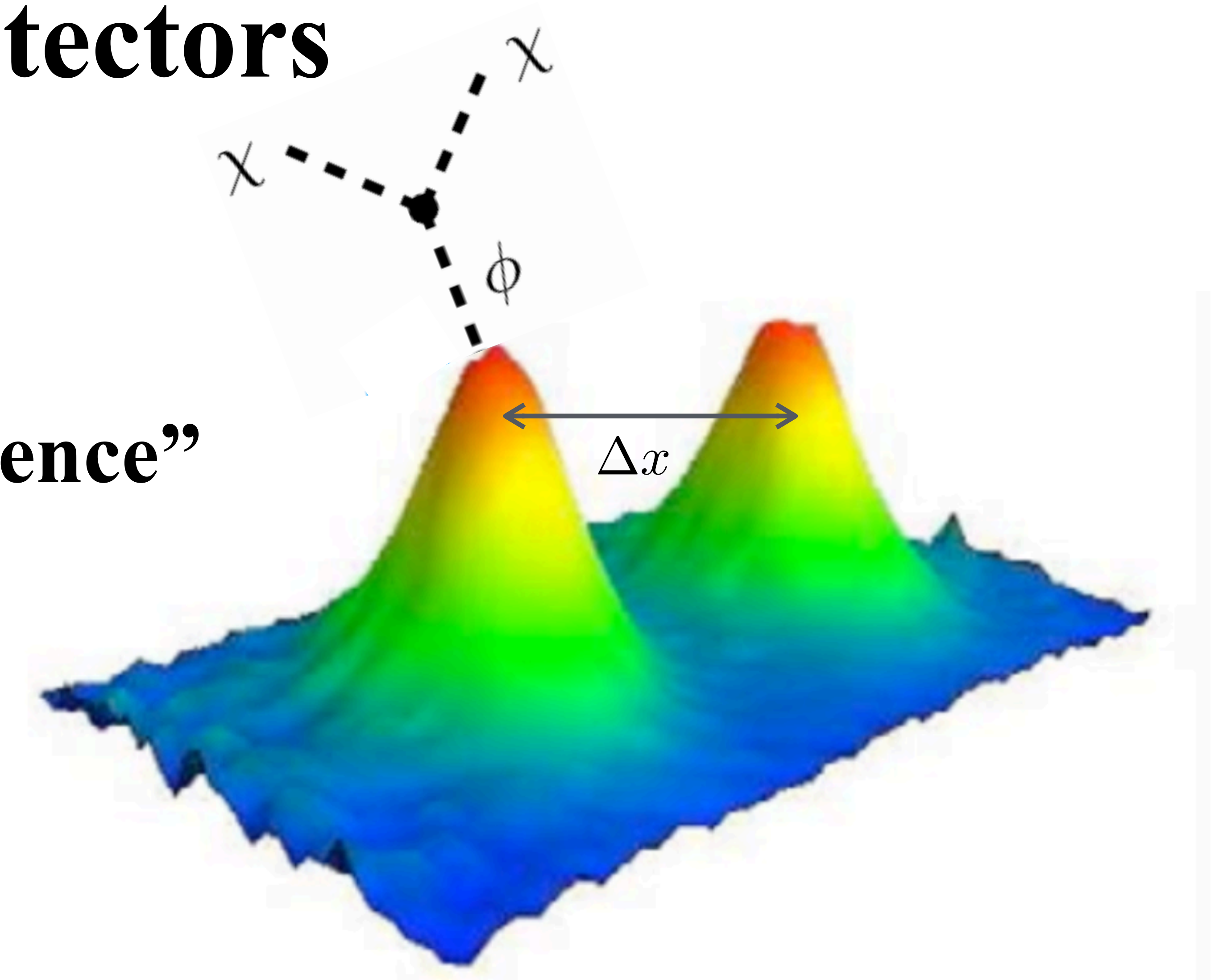
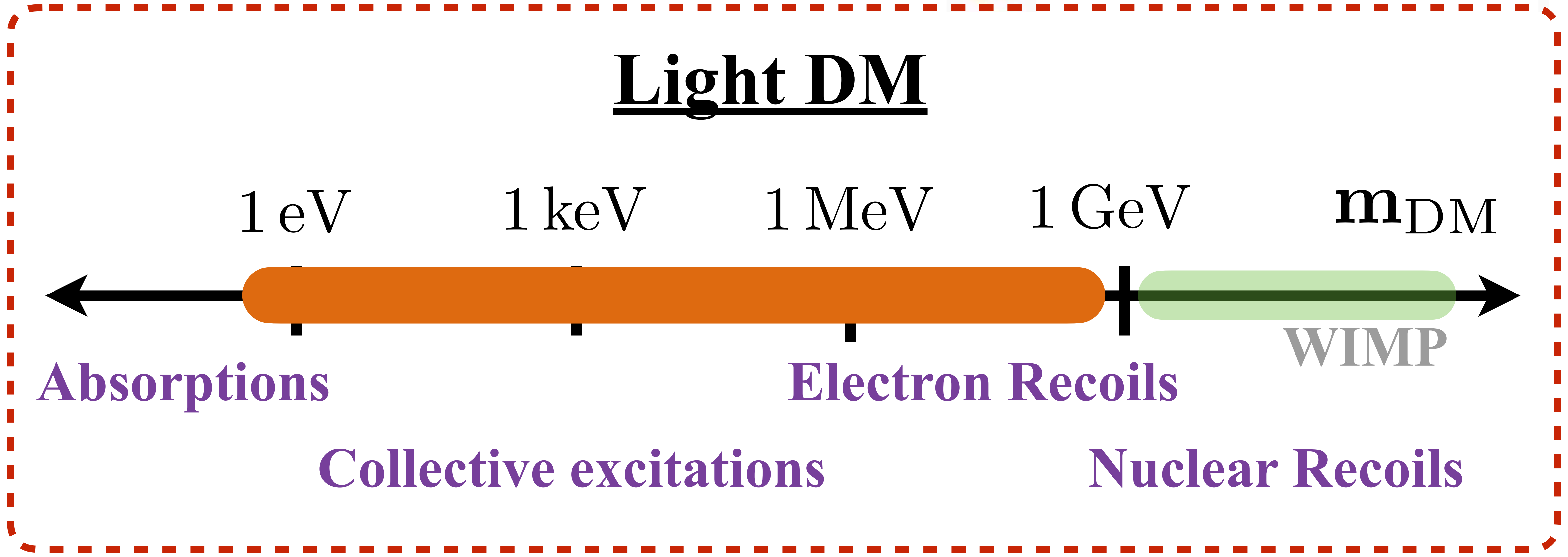


"Collisional decoherence"





Light DM

- **'Zero' energy threshold**

Decoherence depends on momentum transfer

- **Coherent enhancement**

$$R \propto N_{\rm atm}^2 \quad \text{for } q^{-1} \gtrsim r_C$$

A general formalism for decoherence caused by dark matter scattering

Riedel '13, Riedel, Yavin'17
Du, Murgui, Pardo, Y.W. and Zurek '22

Decoherence factor from DM events

$$\gamma = \exp\left[-\frac{m_T}{N_{\rm ind}}\int_0^{t_{\rm exp}} R\, dt\right]$$

$t_{\rm exp}$: measurement time
$m_{\rm T}$: target mass
$N_{\rm ind}$: number independently detected targets

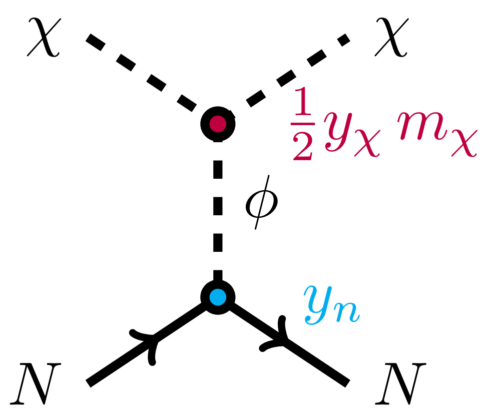
Event rate per target mass $R = \dfrac{1}{\rho_T}\dfrac{\rho_\chi}{m_\chi}\displaystyle\int d^3\mathbf{v}\, f(\mathbf{v})\,\dfrac{\pi\bar{\sigma}}{\mu^2}\int \dfrac{d^3\mathbf{q}}{(2\pi)^3}\mathcal{F}^2_{\rm med}(\mathbf{q})S(\mathbf{q},\omega_{\mathbf{q}})$

m_χ : DM mass $\rho_{\mathrm{T},\chi}$: target/DM mass density $\mu \approx m_\chi$: DM − nucleon reduced mass

DM velocity distribution: $f(\mathbf{v}) = \dfrac{1}{N_0}\exp\left(-\dfrac{(\mathbf{v}+\mathbf{v}_e)^2}{v_0^2}\right)\Theta(v_{\rm esc} - \|\mathbf{v}+\mathbf{v}_e\|)$

$\mathbf{q}$: momentum deposition $\omega_{\mathbf{q}} = \mathbf{q}\cdot\mathbf{v} - \dfrac{q^2}{2m_\chi}$: energy deposition

spin-independent DM **scattering** processes



: $\bar{\sigma} = \dfrac{y_\chi^2\, y_n^2}{4\pi}\dfrac{\mu^2}{(m_\chi^2 v_0^2 + m_\phi^2)^2}$

$$\mathcal{F}_{\rm med}(q) = \frac{(m_\chi v_0)^2 + m_\phi^2}{q^2 + m_\phi^2} = \begin{cases} 1, & \text{heavy mediator} \\ (m_\chi v_0/q)^2, & \text{light mediator}\end{cases}$$

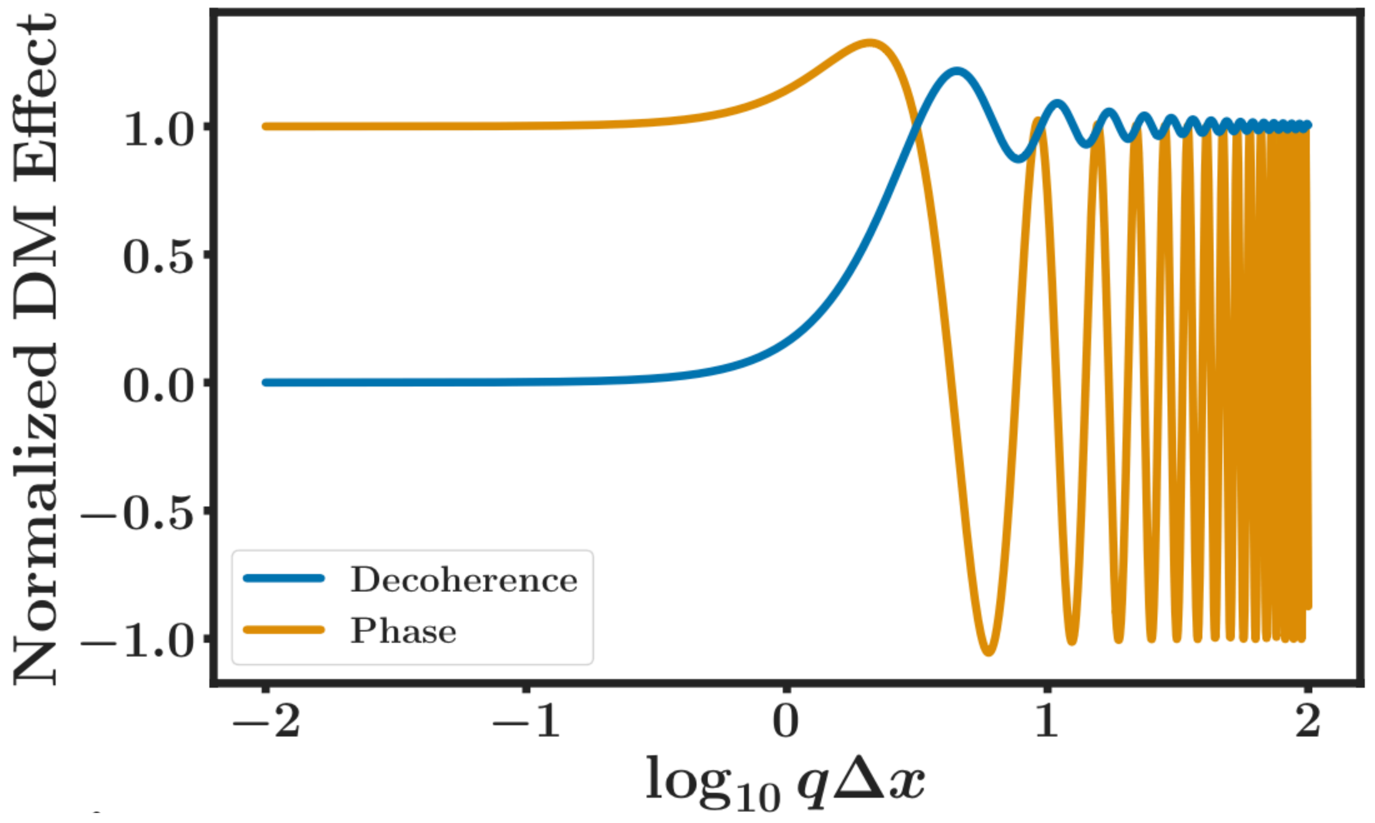
A general formalism for decoherence caused by dark matter scattering

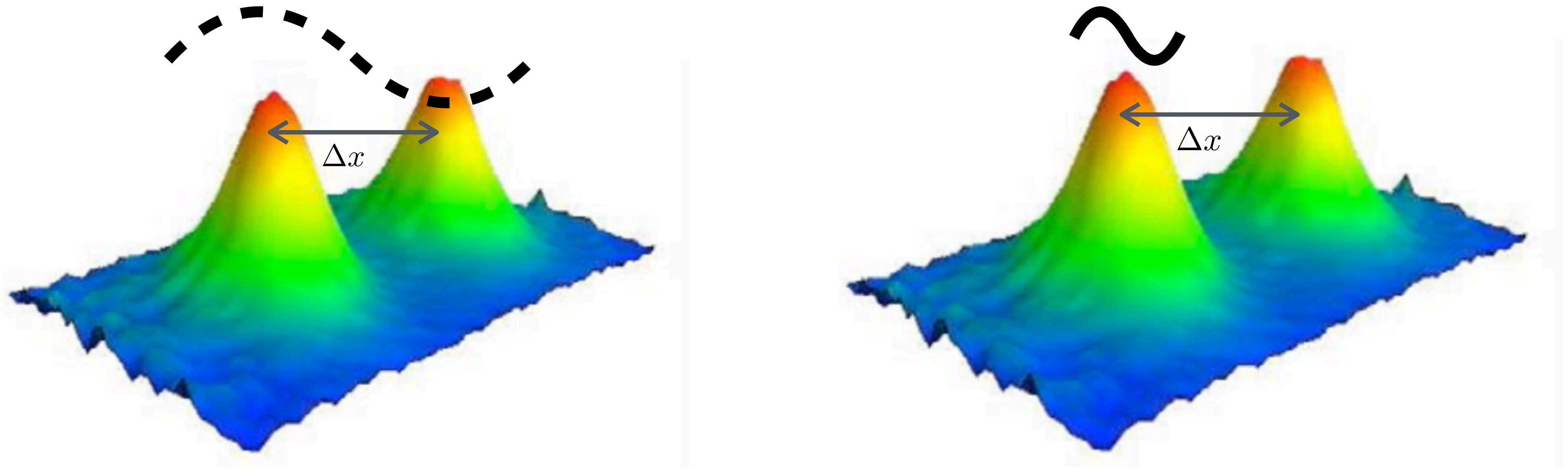
$$R = \frac{1}{\rho_T}\frac{\rho_\chi}{m_\chi}\int d^3\mathbf{v}\; f(\mathbf{v})\,\frac{\pi\bar{\sigma}}{\mu^2}\int \frac{d^3\mathbf{q}}{(2\pi)^3}\mathcal{F}^2_{\rm med}(\mathbf{q})S(\mathbf{q},\omega_\mathbf{q})$$

Dynamical structure factor for target response: $S(\mathbf{q},\omega) \equiv \frac{p_{\rm decoh}}{V}\sum_f |\langle f|\mathcal{F}_T(\mathbf{q})|i\rangle|^2\; 2\pi\delta(E_f - E_i - \omega_\mathbf{q})$

Response from two coherent atom clouds

$$p_{\rm decoh} = 1 - \exp\left[i\mathbf{q}\cdot\mathbf{\Delta x}\right]$$





$q \gtrsim 1/\Delta x$: large cloud seperation

$$\Delta x = 10^{-7}\,\mathrm{m} \rightarrow q \gtrsim \;\mathrm{eV}$$

$$\Delta x = 1\,\mathrm{m} \rightarrow q \gtrsim 0.1\,\mu\mathrm{eV}$$

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$$R = \frac{1}{\rho_T}\frac{\rho_\chi}{m_\chi}\int d^3\mathbf{v}\; f(\mathbf{v})\,\frac{\pi\bar{\sigma}}{\mu^2}\int \frac{d^3\mathbf{q}}{(2\pi)^3}\mathcal{F}^2_{\rm med}(\mathbf{q})S(\mathbf{q},\omega_\mathbf{q})$$

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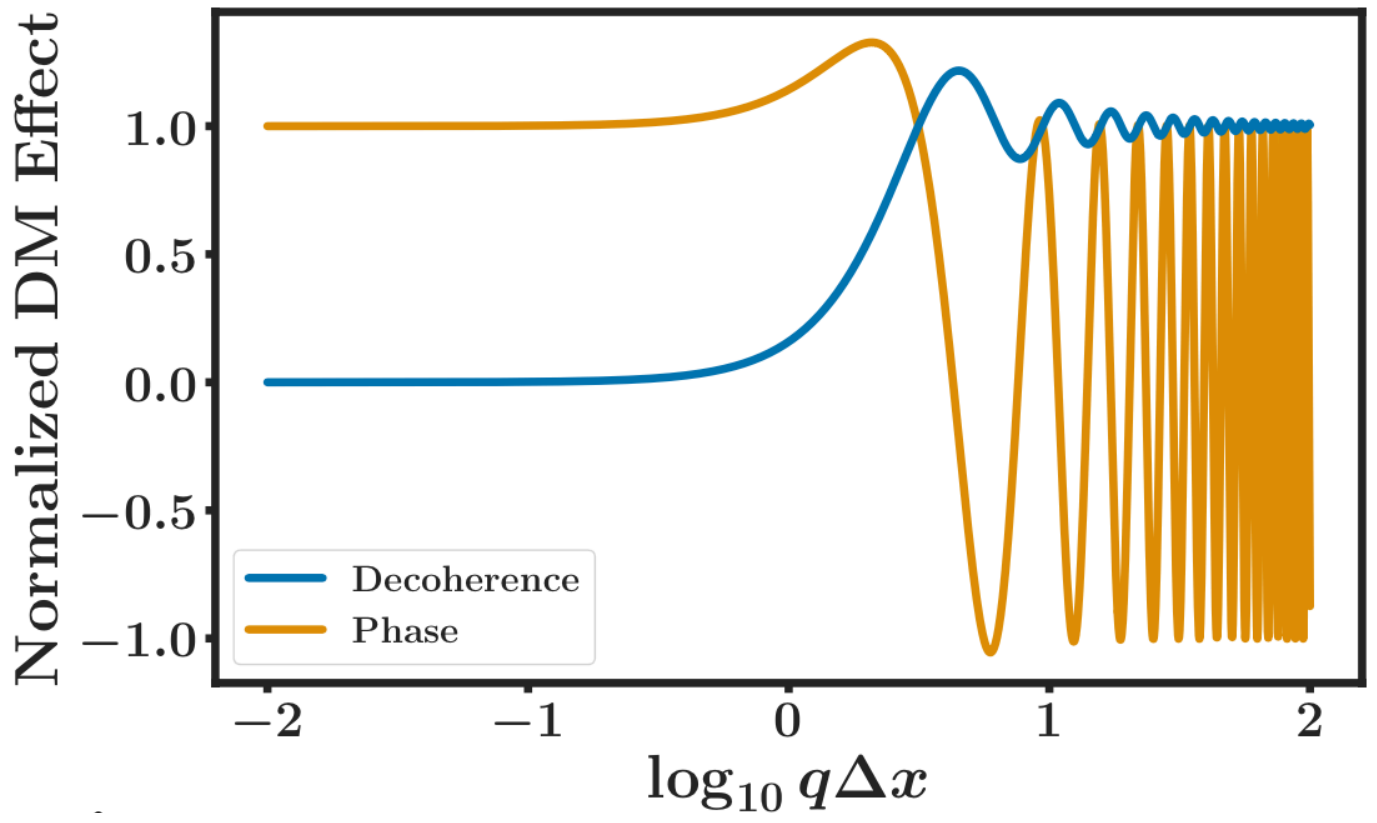
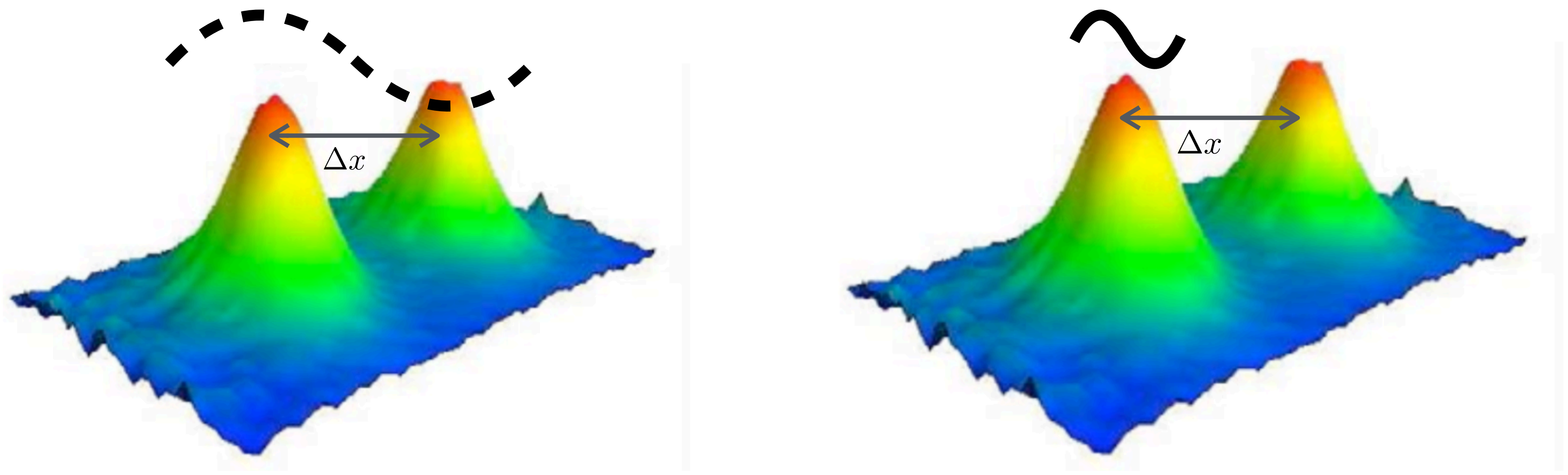




$q \gtrsim 1/\Delta x$: **large cloud seperation**

$\Delta x = 10^{-7}\,\mathrm{m} \to q \gtrsim \mathrm{eV}$

$\Delta x = 1\,\mathrm{m} \to q \gtrsim 0.1\,\mu\mathrm{eV}$

Response from each cloud: process dependent

Example: nuclear recoil (phase shift)

$$\sum_f |\langle f|\mathcal{F}_T(\mathbf{q})|i\rangle|^2 = N_{\rm atm}(N_{\rm atm}-1)F^2(qr_{\rm C}) + N_{\rm atm}F_A^2(qr_{\rm A}) + N_{\rm atm}/A$$

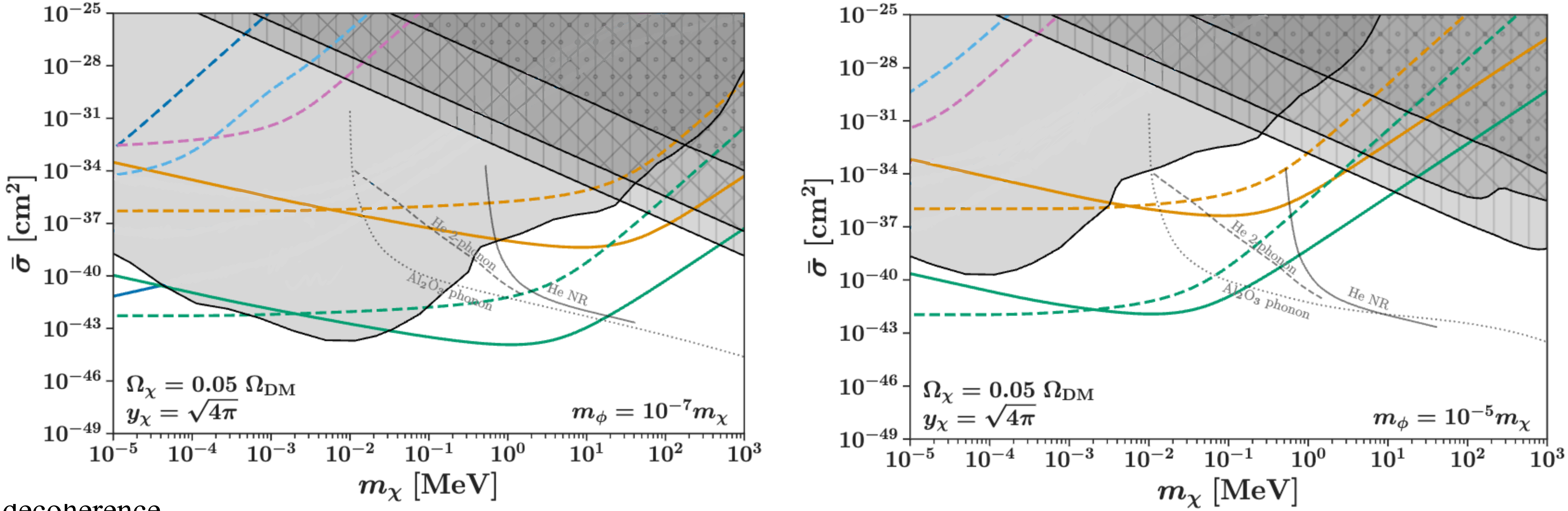
- **Cloud form factor:** $F(qr_C) = \frac{3j_1(qr_C)}{qr_C} \xrightarrow{1/q\gtrsim r_C} 1$
- **Atom form factor:** $F_A(qr_A)) = \frac{3j_1(qr_A)}{qr_A}e^{-q^2s_p^2/2} \xrightarrow{1/q\gtrsim r_A} 1$

Coherent scattering: $q\,r_C \lesssim 1 \to \sum_f |\langle f|\mathcal{F}_T(\mathbf{q})|i\rangle|^2 \approx N_{\rm atm}^2$

$q \lesssim 1/r_{\rm C}$, $S(q,\omega) \propto N_{\rm atm}^2$: **small cloud size and large number of atoms**

$r_{\rm C} = \mathrm{cm} \to q \lesssim 10\mu\mathrm{ev}$

Reach for light mediator



Solid: decoherence
Dashed: phase shift

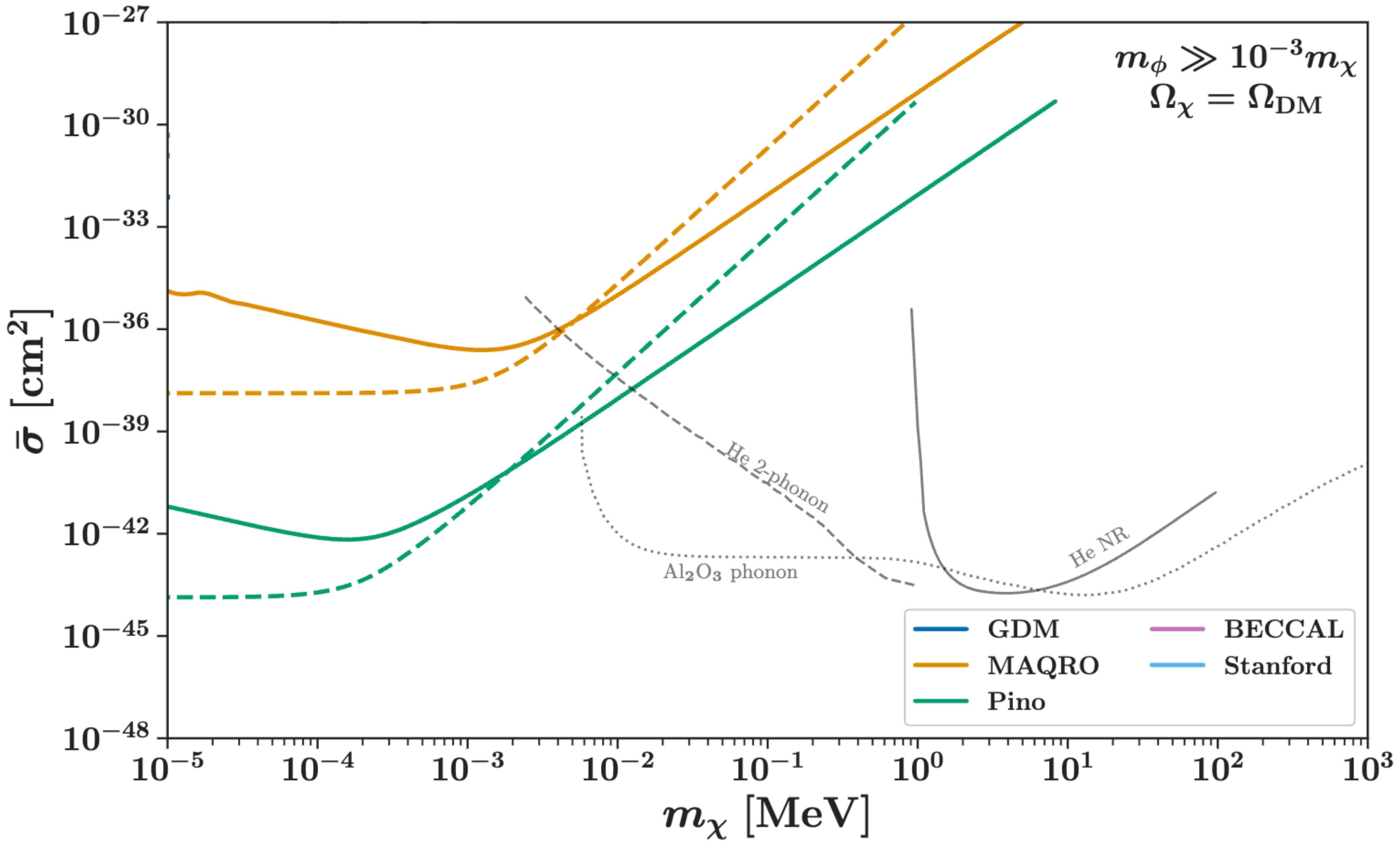
Mission	Target	$r_{\rm cloud}$ [m]	$N_{\rm nucleon}$	Δx [m]	$t_{\rm exp}$ [s]	σ_ϕ [rad]
BECCAL —	^{87}Rb	1.5×10^{-4}	8.7×10^{7}	3×10^{-3}	2.6	1.0×10^{-3}
MAQRO —	SiO$_2$	1.2×10^{-7}	10^{10}	10^{-7}	100	1.0
GDM —	^{87}Rb	10^{-3}	8.7×10^{9}	25	20	1.0×10^{-4}
Pino —	Nb	10^{-6}	2.2×10^{13}	2.9×10^{-7}	0.483	1.0
Stanford —	^{87}Rb	2×10^{-4}	3.5×10^{8}	6.7×10^{-2}	1.91	5.0×10^{-4}

Conclusion

- Interferometer experiments are sensors of ultra high sensitivity;
- Dark matter candidates over a broad mass range can induce potential signals in interferometer experiments;
- Laser interferometers: ultralight DM oscillations and acceleration induced by macroscopic dark matter
- Matter-wave interferometers: ultralight DM, macroscopic dark matter and decoherence effect of sub-GeV dark matter scattering
- Active field for more ideas

Thank you!

Heavy mediator and other processes



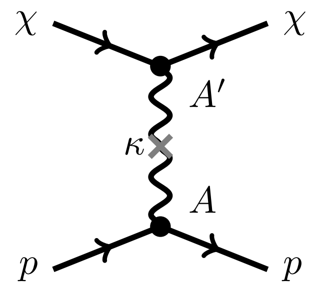
$$\mathcal{F}_{\rm med}(q) = \frac{(m_\chi v_0)^2 + m_\phi^2}{q^2 + m_\phi^2} = \begin{cases} 1, & \text{heavy mediator} \\ (m_\chi v_0/q)^2, & \text{light mediator} \end{cases}$$

- The rate prefers **large momentum transfer**: no coherent enhancement;
- Most of the parameter space, for scalar mediator, has been excluded by various collider, astrophysical, and cosmological constraints.

……

- Hidden photon processes
  - Kinematic mixing

$$H_{\rm I} = -\alpha\,\kappa \int d^3\mathbf{r}\, n(\mathbf{r})\mathbf{E}(\mathbf{r}) \cdot \mathbf{E}'(\mathbf{r})$$



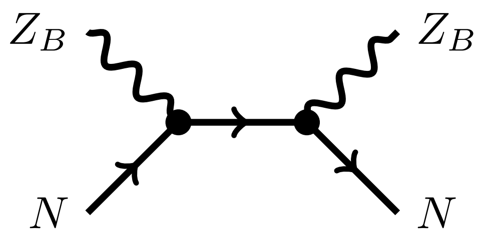
$$\bar{\sigma} = \kappa^2 \frac{\mu^2}{\pi} \frac{1}{(m_\chi^2 v_0^2 + m_{A'}^2)^2}$$

$$\sum_f |\langle f|\mathcal{F}_T(\mathbf{q})|i\rangle|^2 g(\mathbf{q},\omega_1) = a_0^6 \frac{q^2}{2} \int \frac{d^3\mathbf{k}_1}{(2\pi)^3} \omega_1 S(\mathbf{q}-\mathbf{k}_1) g(\mathbf{q},\omega_1)$$

Suppressed by poor polarizibility of atom clouds

- baryon and lepton number couplings

$$\mathcal{L} \supset \frac{g_B}{3} \bar{q}\gamma_\mu q Z_B^\mu$$



$$\bar{\sigma} = \frac{3}{16\pi m_N^2} \left(\frac{g_B}{3}\right)^4$$

Heavy mediator scenario

- Axion scattering

Fukuda, Shirai '21

$$\mathcal{L} \supset \frac{a^2}{8 f_a^2} \sum_{N=p,n} \delta m_N \bar{N} N$$

$$\bar{\sigma} = \frac{\mu^2}{256\pi} \left(\frac{\delta m_N}{f_a^2}\right)^2 \frac{1}{m_a^2}$$

Heavy mediator scenario

Only probes f_a up to 100GeV

- and more…

Statistics and sensitivity

$$\mathrm{SNR} = \frac{(X - X_{\mathrm{bkg}})^2}{\sigma_X^2}$$

Decoherence s $V = e^{-s}$

- Quantum noise limit (QNL)

  See e.g. Bize et. al. 05'

$$\sigma_V \equiv \frac{1}{\sqrt{N_{\mathrm{ind}}}}$$

- **Atom interferometers:** $N_{\mathrm{ind}} = N_{\mathrm{atom}}$
- **Matter interferometers:** $N_{\mathrm{ind}} = 1$

- Over the experiment run time (1 year), the noise scales with the number of measurement:

$$\sigma_V \propto N_{\mathrm{meas}}^{-1/2} \quad \text{with} \quad N_{\mathrm{meas}} = T_{\mathrm{run}}/t_{\mathrm{exp}}$$

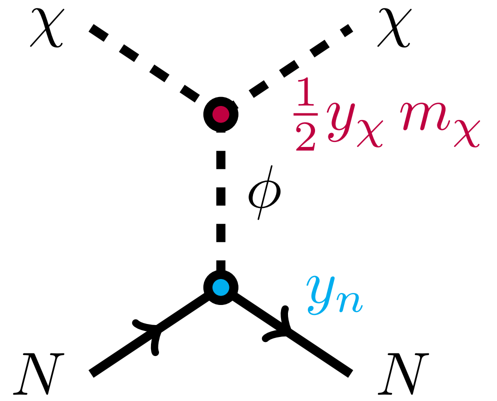
Phase ϕ

- Quantum noise limit

$$\sigma_\phi \equiv \frac{1\ \mathrm{rad}}{\sqrt{N_{\mathrm{ind}}}}$$

- Some experiments phrase the phase sensitivity as the minimal measurable acceleration $\sigma_\phi = \frac{1}{4} m_{\mathrm{ind}} \Delta x \, t_{\mathrm{exp}} a_{\mathrm{min}}$,

Other bounds and projections



$$\bar{\sigma} = \frac{y_\chi^2\, y_n^2}{4\pi}\, \frac{\mu^2}{(m_\chi^2 v_0^2 + m_\phi^2)^2}$$

Direct Detections

- Prospects from superfluid helium for the NR and 2-phonon excitations, and the Al_2O_3 for phonon excitations, are compatible with certain AI proposals above 10 keV mass;

Terrestrial bounds

- Meson decays $K \to \pi\, \phi, \quad B \to K\phi$
- Fifth force

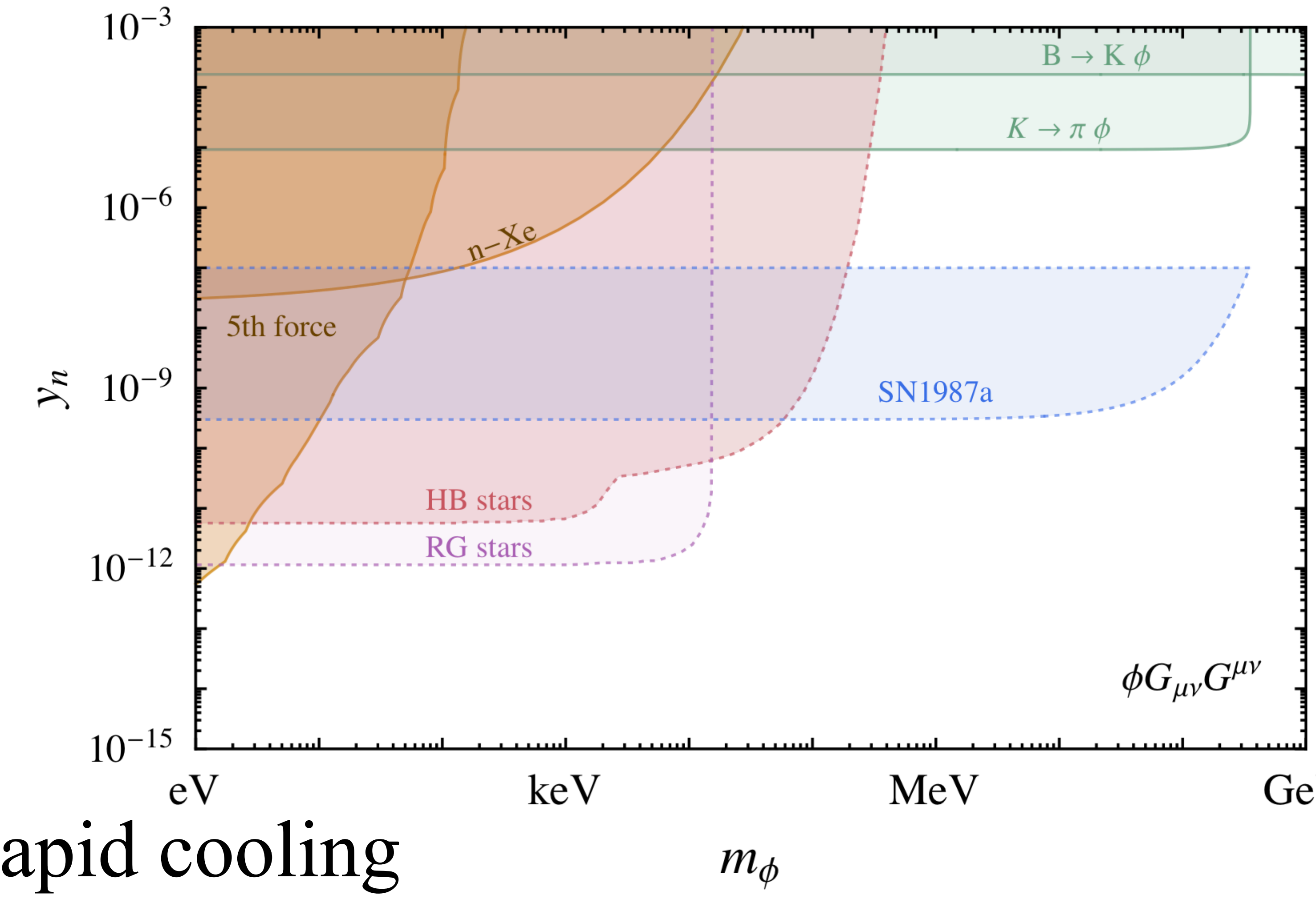
  Hadrophilic massive mediator induces Yukawa potential $V(r) = -\frac{y_n^2}{4\pi}\frac{1}{r}e^{-m_\phi r}$

Astrophysical bounds

strongest bound on y_n for light mediator

- Stellar emission

  Light bosons coupling to electrons or nucleons can be emitted by stars, resulting in rapid cooling



Knapen, Lin, Zurek '22

- Dark matter self interaction (DMSI)

  If χ composes all of the DM in the universe, DMSI are constrained by cluster mergers and halo shaped observations to satisfy

$$\sigma_{\rm DMSI}/m_\chi < 1 \;-\; 10\,{\rm cm}^2/{\rm g}$$

The DMSI bound can be considerably relaxed if χ is a subcomponent of the total DM.

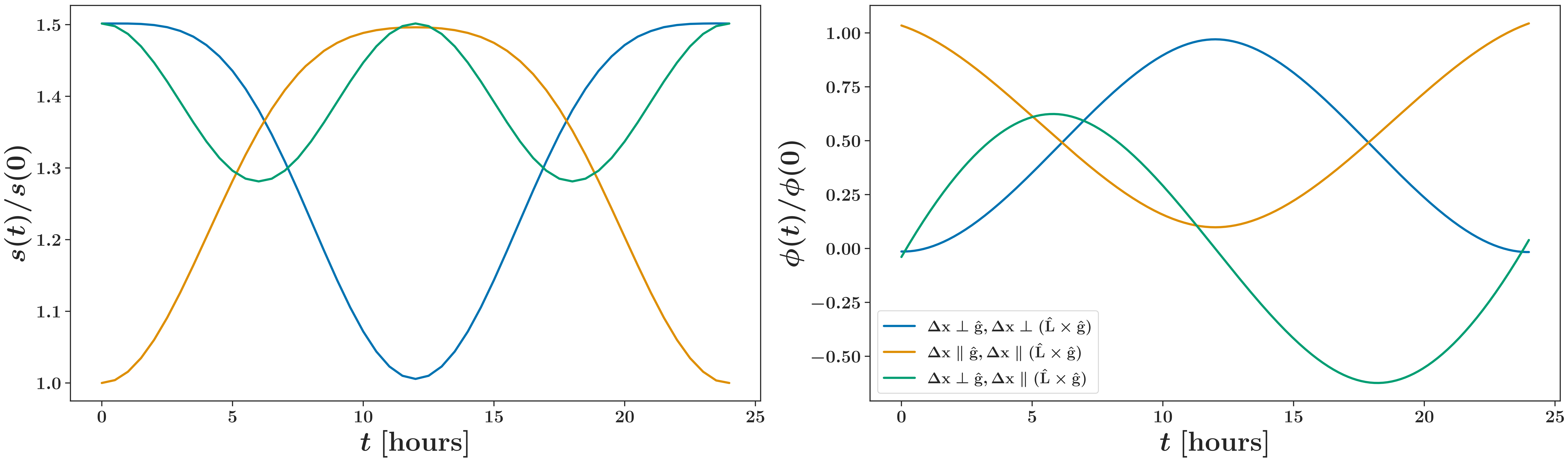
we assume DM sub-component for light mediator

Cosmological bounds

$$\Delta N_{\rm eff} = \frac{4}{7}\sum_i g_i \left(\frac{g(T_{\nu L}^{\rm dec})}{g(T_i)}\right)^{4/3} \xrightarrow[\chi \text{ cannot be produced thermally}]{\text{with } y_n < 10^{-9}\text{, } \phi \text{ decouples before QCD}} \Delta N_{\rm eff} \simeq 0.06\sum_i g_i$$

Not in tension with current measurements, but will be easily probed by upcoming experiments.

Daily modulation



Directional signal: $p_{\mathrm{decoh}} = 1 - \exp\left[i\mathbf{q} \cdot \boldsymbol{\Delta}\mathbf{x}\right]$

DM velocity distribution in the lab frame: $f(\mathbf{v}) = \dfrac{1}{N_0} \exp\left(-\dfrac{(\mathbf{v} + \mathbf{v}_e(t))^2}{v_0^2}\right) \Theta(v_{\mathrm{esc}} - \|\mathbf{v} + \mathbf{v}_e(t)\|)$

$$\mathbf{v}_{\mathrm{e}}(t) = \|\mathbf{v}_e\| \begin{pmatrix} \mathrm{s}\theta_{\mathrm{e}}\,\mathrm{c}\theta_{\mathrm{x}}\,\mathrm{s}\phi(t) - \mathrm{s}\theta_{\mathrm{x}}\,\mathrm{s}\theta_{\mathrm{e}}\,\mathrm{c}\theta_{\mathrm{l}}\,\mathrm{c}\phi(t) + \mathrm{s}\theta_{\mathrm{x}}\,\mathrm{c}\theta_{\mathrm{e}}\,\mathrm{s}\theta_{\mathrm{l}} \\ \mathrm{c}\theta_{\mathrm{e}}\,\mathrm{s}\theta_{\mathrm{g}}\,\mathrm{s}\theta_{\mathrm{l}}\,\mathrm{c}\theta_{\mathrm{x}} - \mathrm{s}\theta_{\mathrm{e}}\,\mathrm{s}\theta_{\mathrm{g}}\,\mathrm{c}\theta_{\mathrm{l}}\,\mathrm{c}\theta_{\mathrm{x}}\,\mathrm{c}\phi(t) - \mathrm{s}\theta_{\mathrm{e}}\,\mathrm{c}\theta_{\mathrm{g}}\,\mathrm{s}\theta_{\mathrm{l}}\,\mathrm{c}\phi(t) - \mathrm{c}\theta_{\mathrm{e}}\,\mathrm{c}\theta_{\mathrm{g}}\,\mathrm{c}\theta_{\mathrm{l}} - \mathrm{s}\theta_{\mathrm{e}}\,\mathrm{s}\theta_{\mathrm{g}}\,\mathrm{s}\theta_{\mathrm{x}}\,\mathrm{s}\phi(t) \\ \mathrm{s}\theta_{\mathrm{e}}\,\mathrm{c}\theta_{\mathrm{g}}\,\mathrm{c}\theta_{\mathrm{l}}\,\mathrm{c}\theta_{\mathrm{x}}\,\mathrm{c}\phi(t) - \mathrm{c}\theta_{\mathrm{e}}\,\mathrm{c}\theta_{\mathrm{g}}\,\mathrm{s}\theta_{\mathrm{l}}\,\mathrm{c}\theta_{\mathrm{x}} - \mathrm{s}\theta_{\mathrm{e}}\,\mathrm{s}\theta_{\mathrm{g}}\,\mathrm{s}\theta_{\mathrm{l}}\,\mathrm{c}\phi(t) - \mathrm{c}\theta_{\mathrm{e}}\,\mathrm{s}\theta_{\mathrm{g}}\,\mathrm{c}\theta_{\mathrm{l}} + \mathrm{s}\theta_{\mathrm{e}}\,\mathrm{c}\theta_{\mathrm{g}}\,\mathrm{s}\theta_{\mathrm{x}}\,\mathrm{s}\phi(t) \end{pmatrix}$$

- Choose the optimal oriental of $\boldsymbol{\Delta}\mathbf{x}$;
- Can be used to isolate DM signal from isotropic backgrounds.