A dark photon search with a gravitational wave detector and the effect of the relative motion of detectors

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Dark matter search with a laser interferometer

Ultra-light dark matter is motivated by high energy theory, cosmology, etc…

A new approach with a Gravitational wave detector

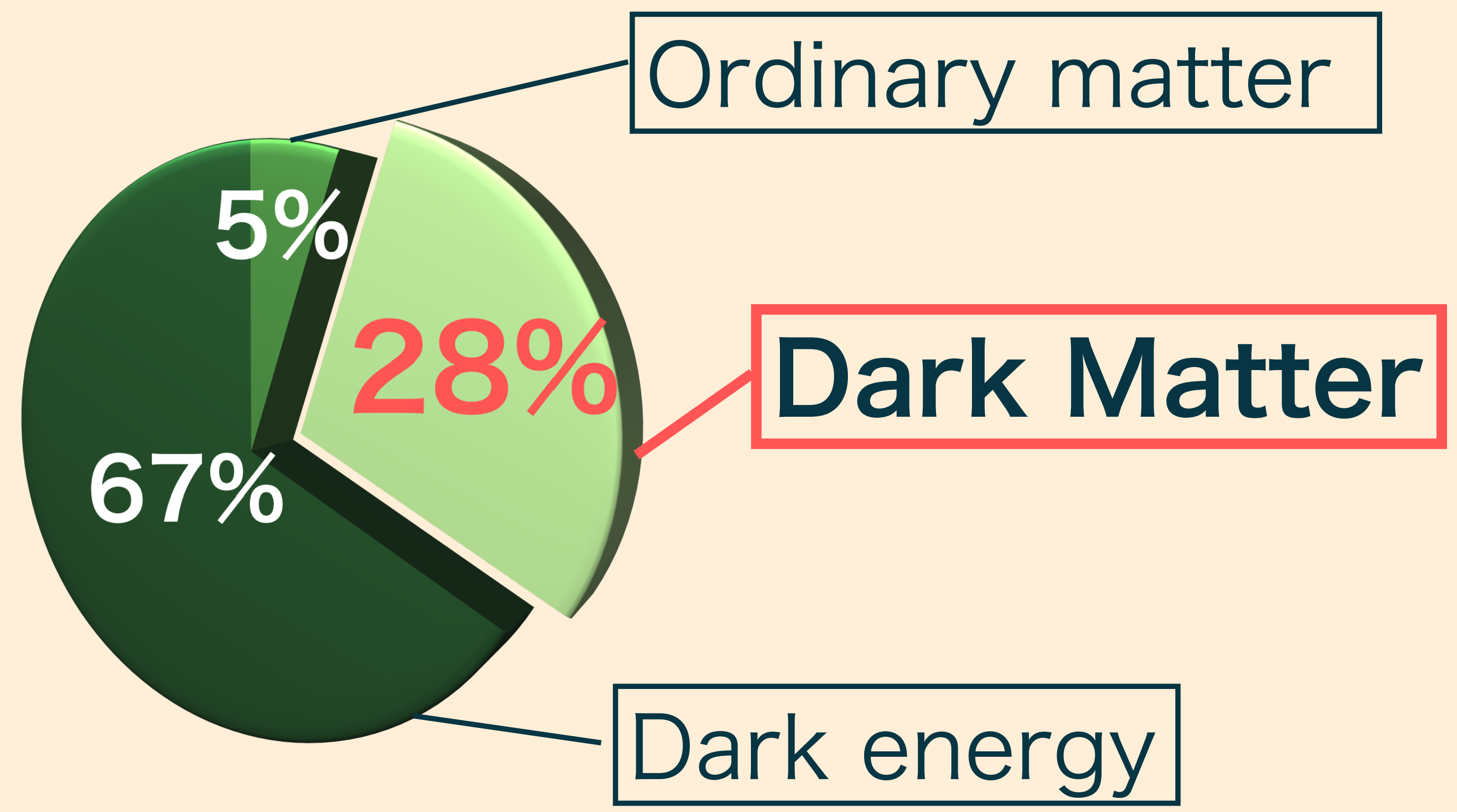
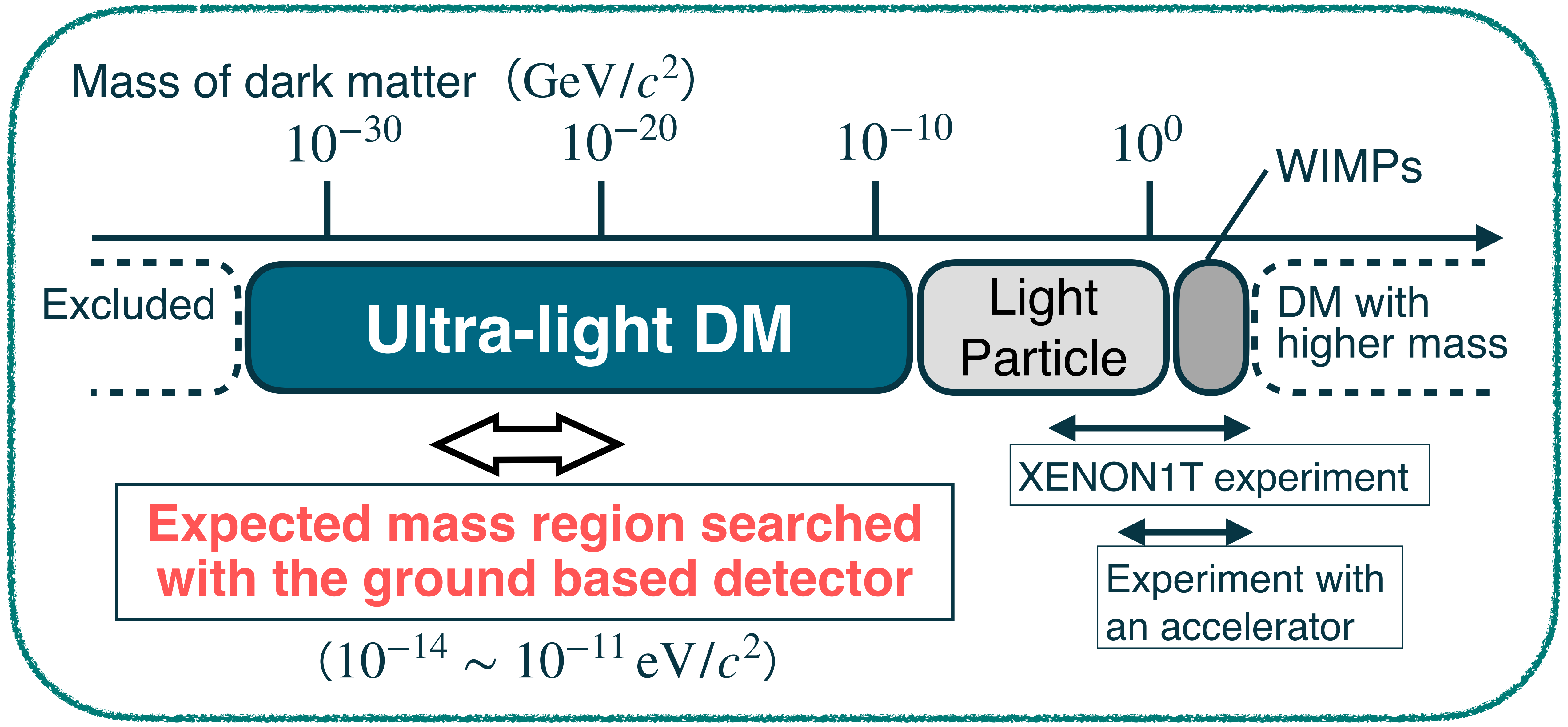


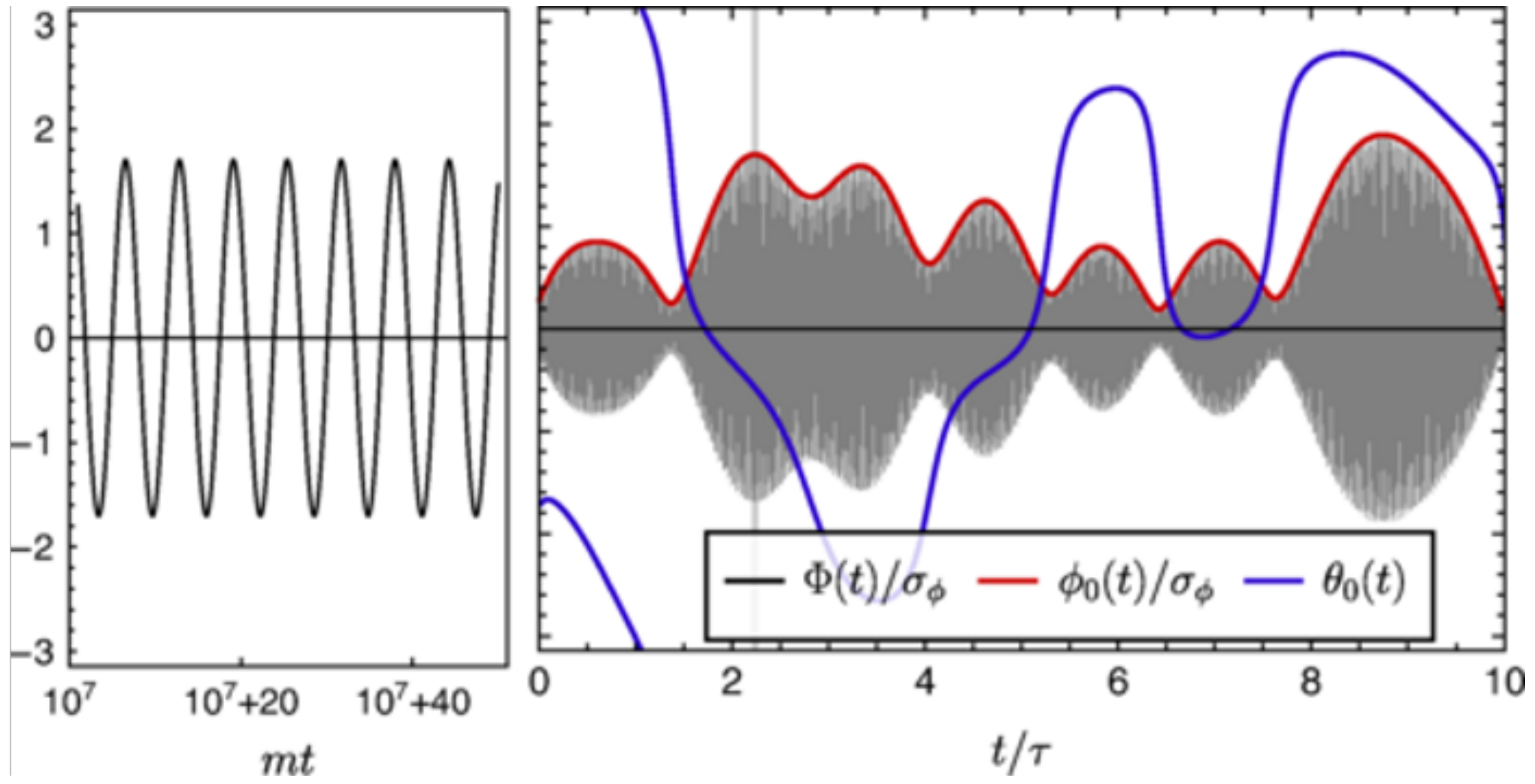
Figure: Fraction of Matter and Energy in the Universe



Ultra-Light Dark Matter $m_{\mathrm{DM}} < 1\ \mathrm{eV/c^2}$

- **Wave nature predominates** due to low mass
- Number density: $\mathcal{O}(\mathbf{10^{20}})\ /\mathrm{cm}^3$ → particles are **bosons**
- **Forms a field with frequency proportional to mass** as a superposition of many waves
- The amplitude of the field changes stochastically

The amplitude of the field formed by ULDM
from H. Nakatsuka et. al., (2023)



$$A_i(t) = \frac{A}{\sqrt{N}} \sum_{n=1}^{N} \cos\left(2\pi f_{\mathrm{DM}}\left(1+\frac{v_{(i,n)}^2}{2}\right)t - 2\pi f_{\mathrm{DM}}\vec{v}_{(i,n)}\cdot\vec{x} + \theta_{(i,n)}\right)$$

$$f_{\mathrm{DM}} \sim \frac{m}{2\pi} = 242\ \mathrm{Hz}\left(\frac{m}{10^{-12}\ \mathrm{eV}}\right)$$

$$\tau = \frac{2\pi}{m_{\mathrm{DM}} v_{\mathrm{vir}}^2} \sim \frac{10^6}{f_{\mathrm{DM}}}$$ Coherent time

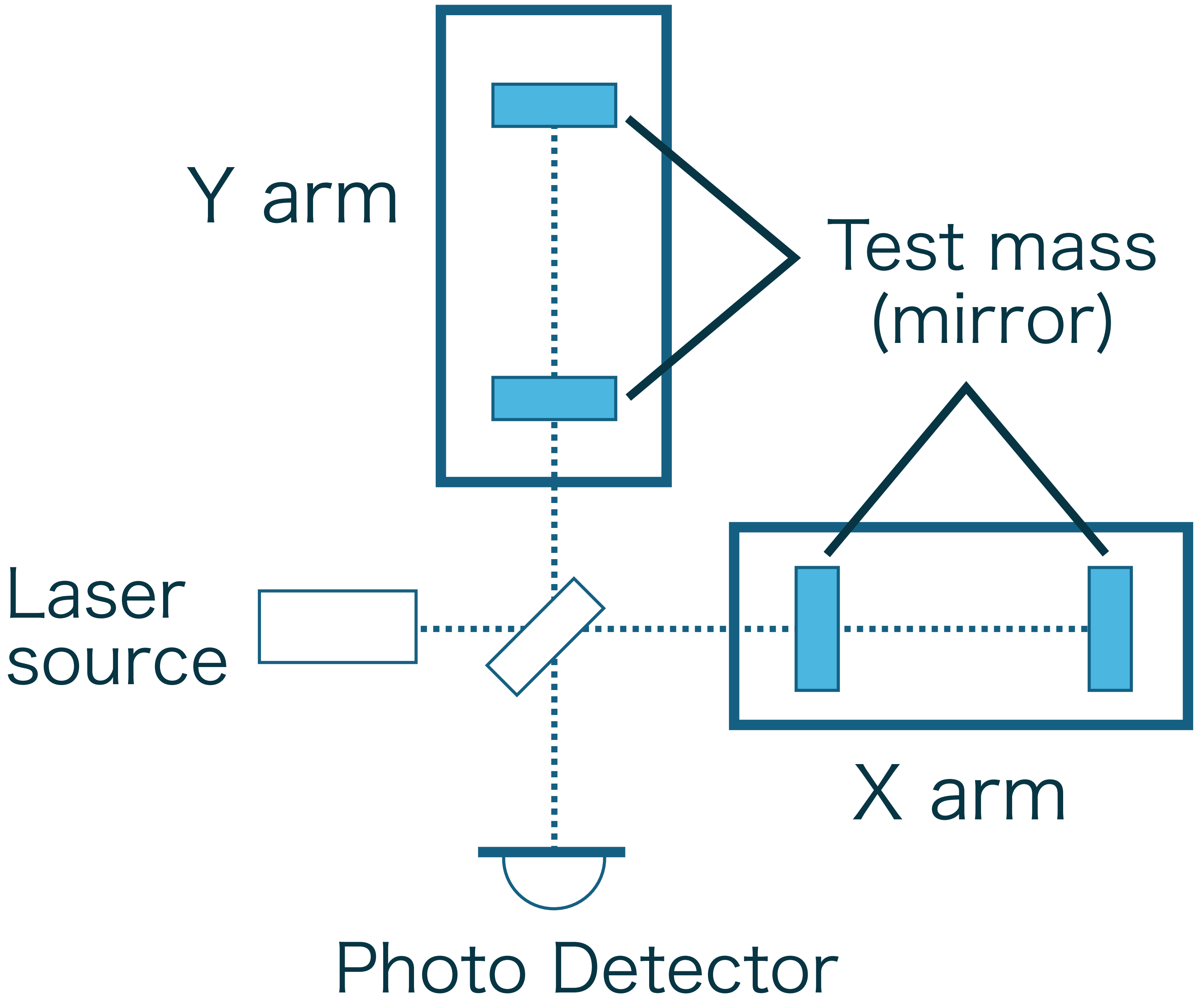
We assumed
Dark Photon (a vector dark matter)
as the dark matter model
to be searched.

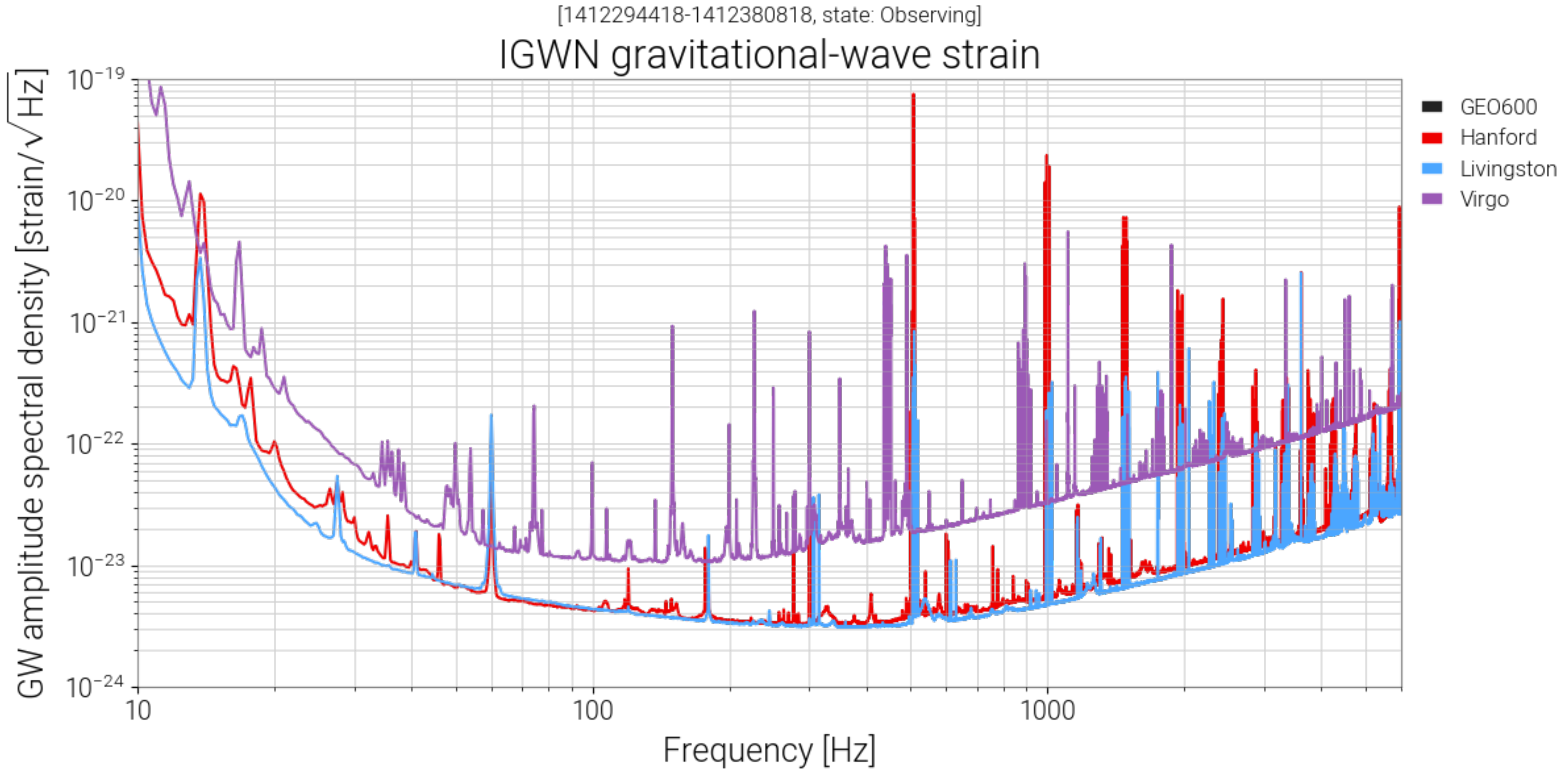
Gravitational wave detector

- Laser Michelson interferometer
- Observe the change in the difference between the lengths of the two arms.

$$\frac{\Delta L}{L} \sim 10^{-23}$$

- Arm length: 4 km (LIGO, Virgo), 3 km(KAGRA)





Interaction between DPDM and detector

- **Lagrangian of DPDM** ($D = B$ or $B - L$)

$$\mathcal{L}_A = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m^2 A_\mu A^\mu - e\epsilon_D J^\mu_D A_\mu$$

$$\delta\vec{x}(t,\vec{x}) = -e\epsilon_{\mathrm{D}}\frac{Q_D}{M}\int^t dt' \vec{A}(t',\vec{x})$$

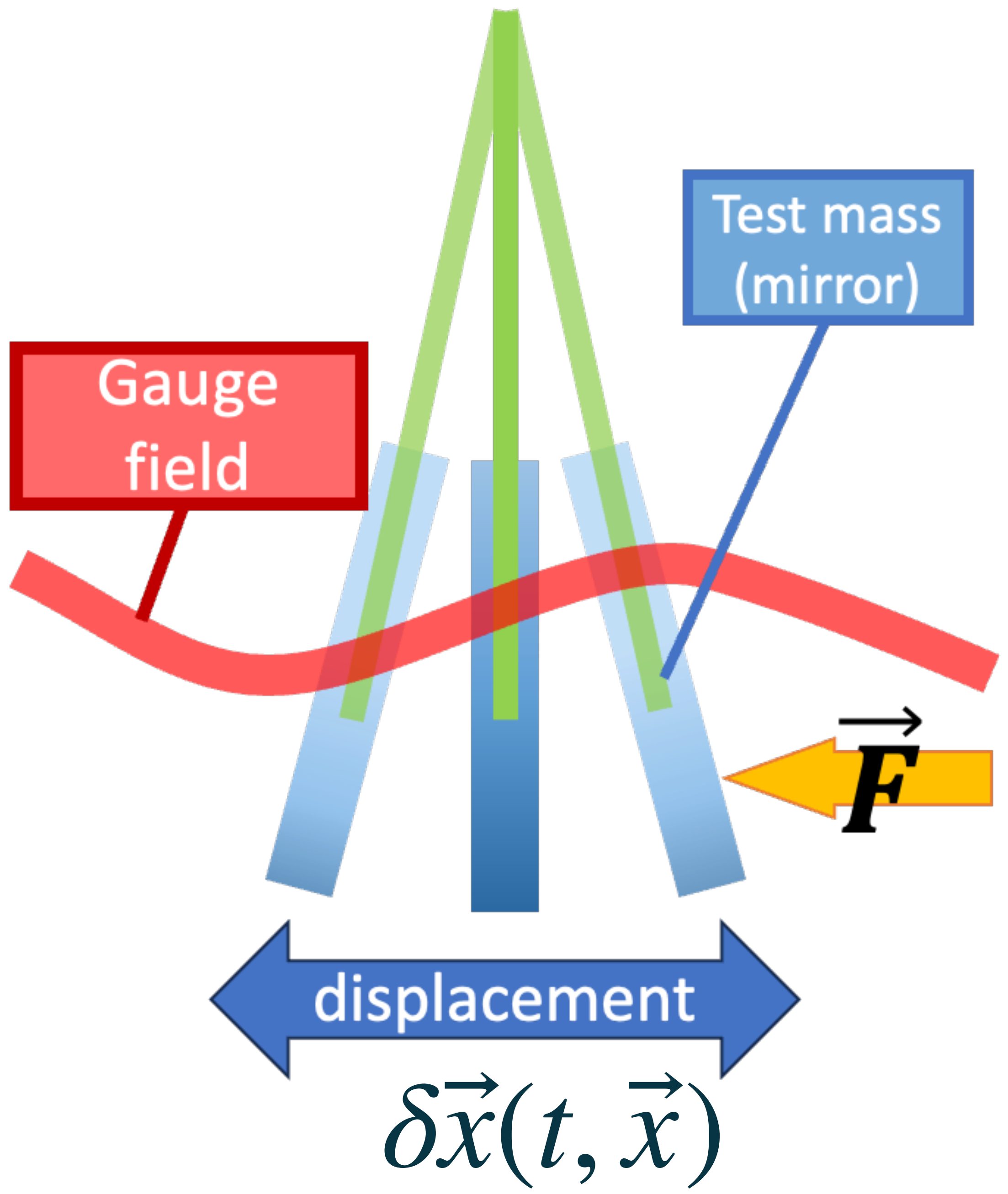
m_{DM} : mass of DM

J^μ_D : $U(1)_D$ current

$$\frac{Q_{B-L}}{M} \sim \begin{cases} 0.501 & \text{(silica) in LIGO, Virgo} \\ 0.510 & \text{(sapphire) in KAGRA} \end{cases}$$

signal frequency ~ Compton frequency

proportional to DM mass



$e\epsilon_{\mathrm{D}}$: **Coupling Constant**

The parameter characterizing
the magnitude of the interaction

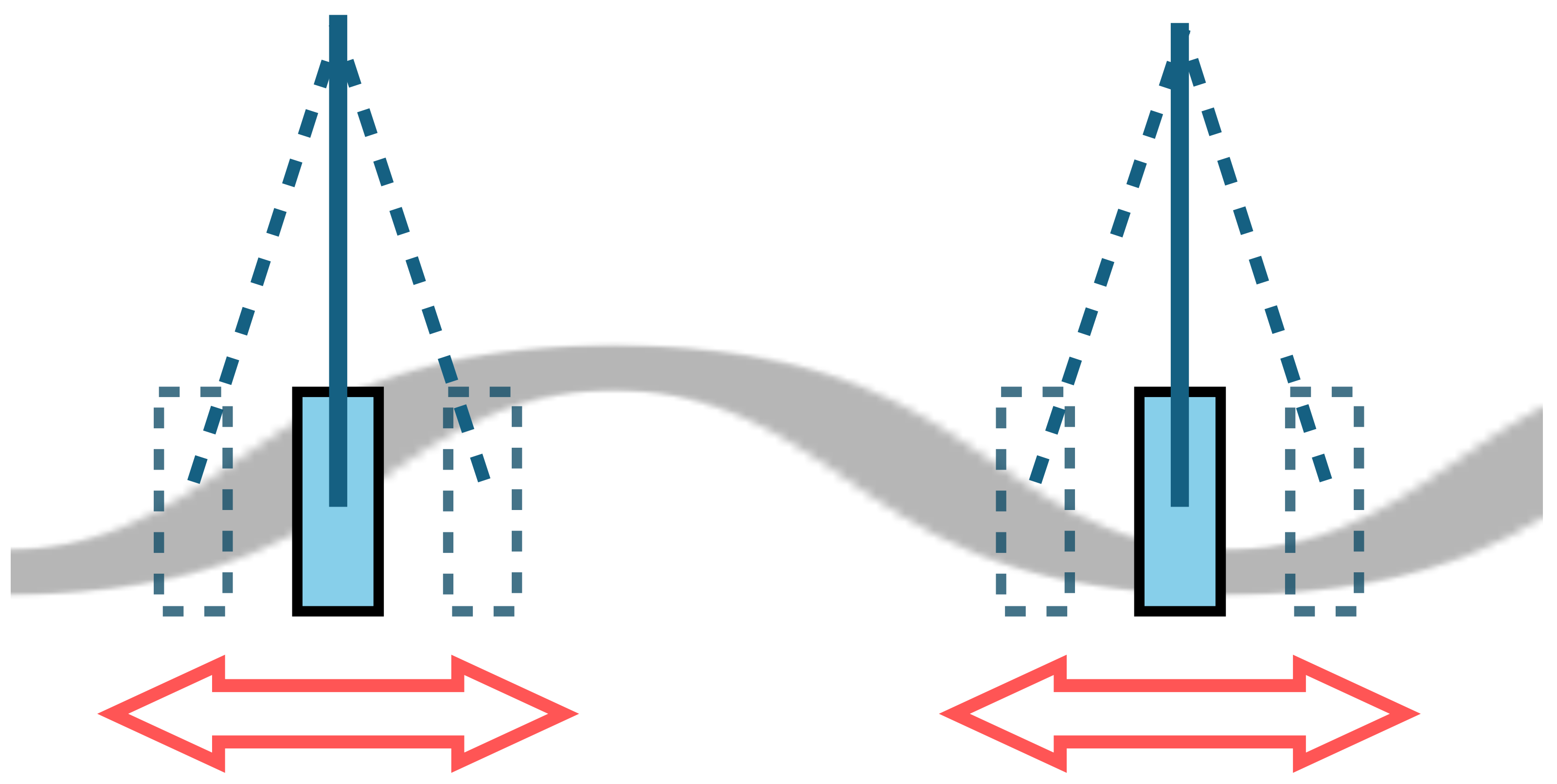
Signal generated in the detector

$$h(t) = h_1(t) + h_2(t)$$

h_1 : Signal by the finite light-traveling time effect [S. Morisaki, et.al, (2021)]

h_2 : Signal appearing as a result of the phase difference of DPDM field (Including the space derivative)

$$h_2(t) = \frac{\epsilon_{\mathrm{D}} e}{m_{\mathrm{DM}}} \frac{Q_{\mathrm{D}}}{M} \sum_i A_i((\boldsymbol{n} \cdot \boldsymbol{e}_i)(\boldsymbol{n} \cdot \underline{\boldsymbol{k}_i}) - (\boldsymbol{m} \cdot \boldsymbol{e}_i)(\boldsymbol{n} \cdot \underline{\boldsymbol{k}_i})) \cos(\omega_i(t - L) + \phi_i)$$



$$\frac{h_1(t)}{h_2(t)} \sim \frac{m_{\mathrm{DM}} L}{v_{\mathrm{DM}}} \sim 8 \left(\frac{m_{\mathrm{DM}}}{2\pi \times 100\ \mathrm{Hz}} \right)$$

h_2 is dominant in **the lower mass regions**

Signal generated in the detector

$$h(t) = h_1(t) + h_2(t)$$

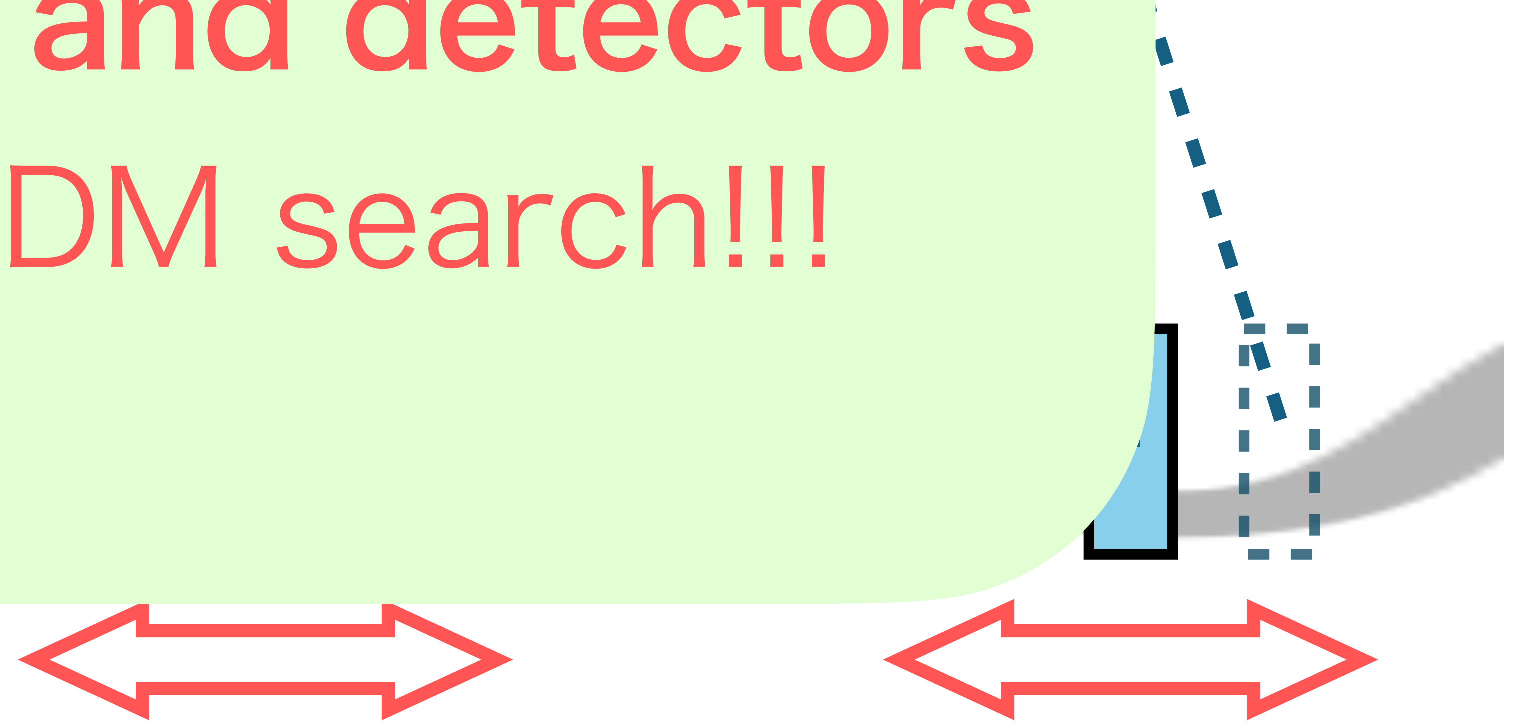
h_1 : Si … (2021)]

h_2 : S … mirrors

(I

$h_2(t)$ =

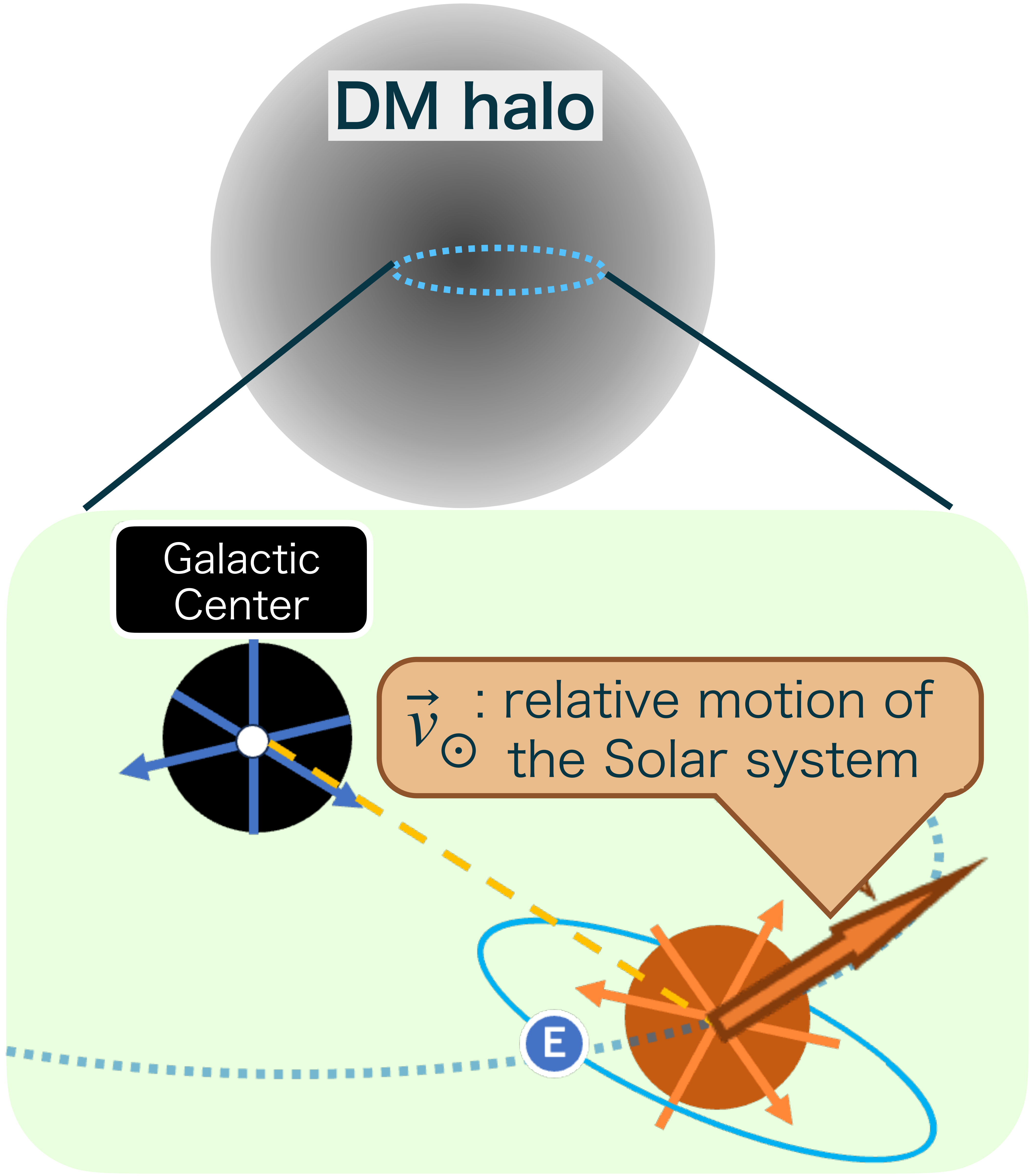
The magnitude and direction of
the relative velocity between DM and detectors
are important for lower mass DPDM search!!!



$$\frac{h_1(t)}{h_2(t)} \sim \frac{m_{\rm DM} L}{v_{\rm DM}} \sim 8 \left(\frac{m_{\rm DM}}{2\pi \times 100\ {\rm Hz}} \right)$$

h_2 is dominant in
the lower mass regions

Note on the relative velocity



- DM distribute isotropically around GC
- Velocity distribution follows the Standard Halo Model:

$$f_{\mathrm{SHM}}(\vec{v})\,\mathrm{d}^3\vec{v} = \frac{1}{(\pi v_{\mathrm{vir}}^2)^{3/2}} \exp\left[-\frac{(\vec{v}+\vec{v}_\odot)^2}{\vec{v}_{\mathrm{vir}}^2}\right]\mathrm{d}^3\vec{v}$$

- Proper motion of the Solar system causes **a bias in the velocity dispersion**
  → **Directional dependence of signal magnitude!**
- For the relative velocity, we only considered **the velocity between the galactic center and the solar system's barycenter**.

Data Analysis

Detection of dark matter and constraints

Detection of the signal from dark matter

$$\rho(f) = \sum_{i}^{N_{\mathrm{chunk}}} \frac{4|\tilde{d}(f;t_i)|^2}{T_{\mathrm{ch}} S(f;t_i)}$$

Incoherently sum up the spectra to detect non-Gaussian power excess.

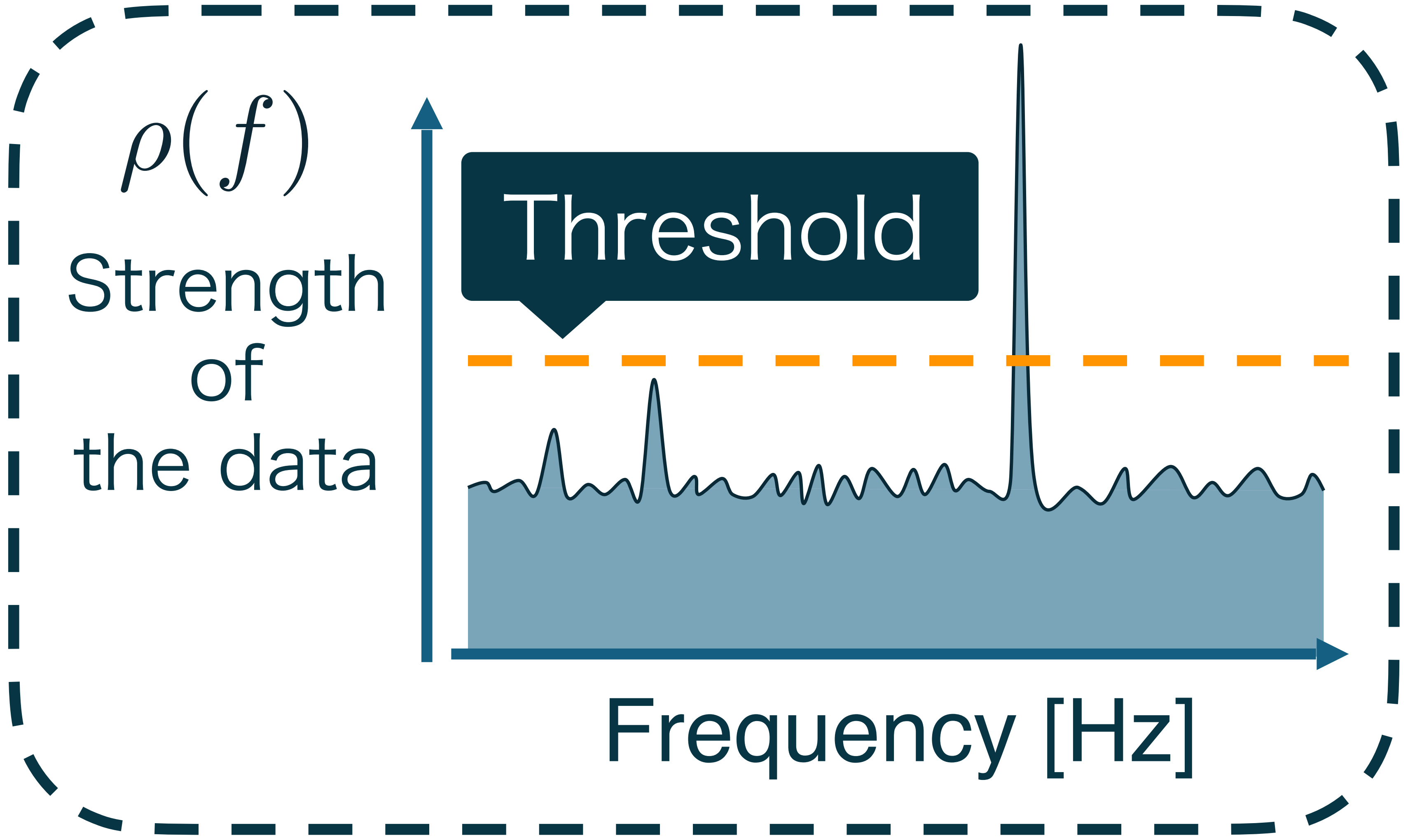
Then, test whether the signal originates from DM

(narrowband spectra, persistency)

Estimation of the upper bound

If there's no power excess,

we can set the upper limit on the coupling constant.



To appropriately estimate the upper limits,

we must address **specific features of DPDM**

Estimation of upper bound and Likelihood

- **Frequentist method** (confidence level β %)

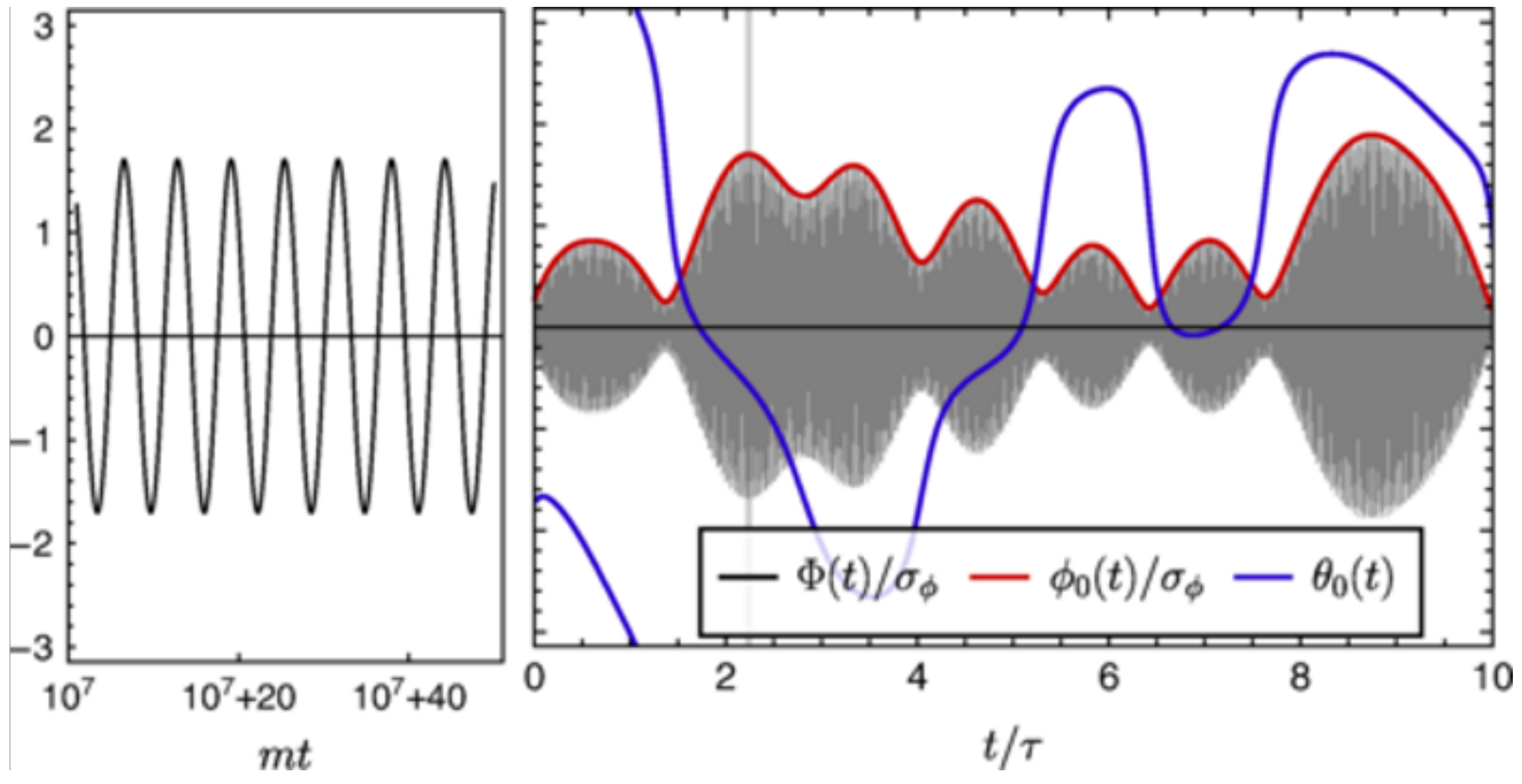
$$1 - \frac{\beta}{100} = \int_0^{\rho_{\mathrm{obs}}} \mathcal{L}(\rho(f)|\epsilon^{\beta\%})d\rho$$

Estimate 95% upper bound $\epsilon^{95\%}$ with observed value ρ_{obs}

- **The likelihood function** $\mathcal{L}(\rho(f)|\epsilon^{\beta\%})$ (detection statistic: ρ)

Upper bound estimation is affected by two terms;

Amplitude fluctuation



The relative motion

ρ (quantity relative to the power of the signal) varies along with the direction of the relative motion in the lower mass search.

Not considered in previous studies!!!

Analysis -Settings-

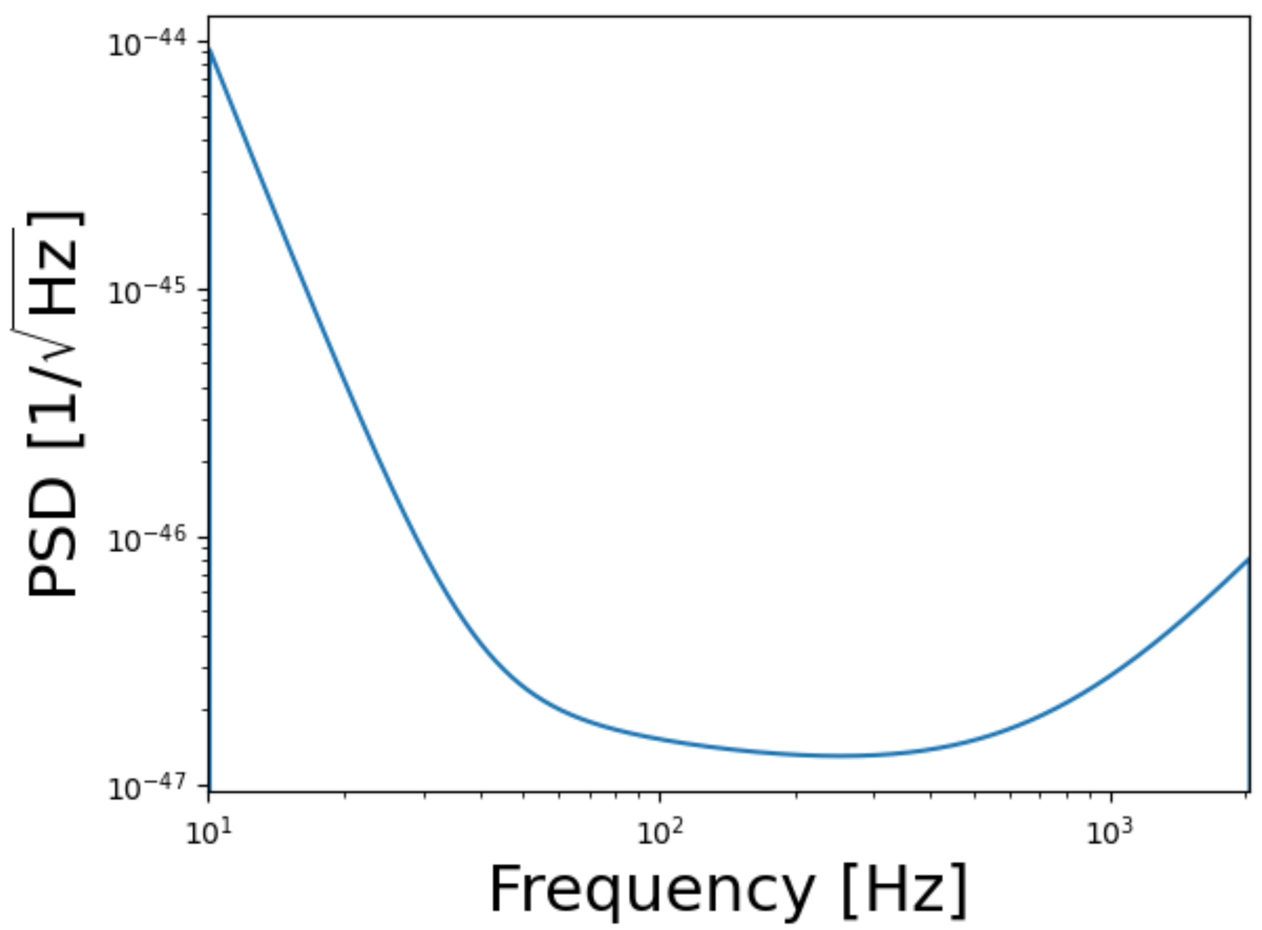
$$d(t) = n(t) + \epsilon_{\text{true}} \times h(t)$$

Apply the method for the simulated data and analyze as described below:

- **Estimate the upper bound of the coupling constants**
- **Compare the results in different directions of relative velocity**

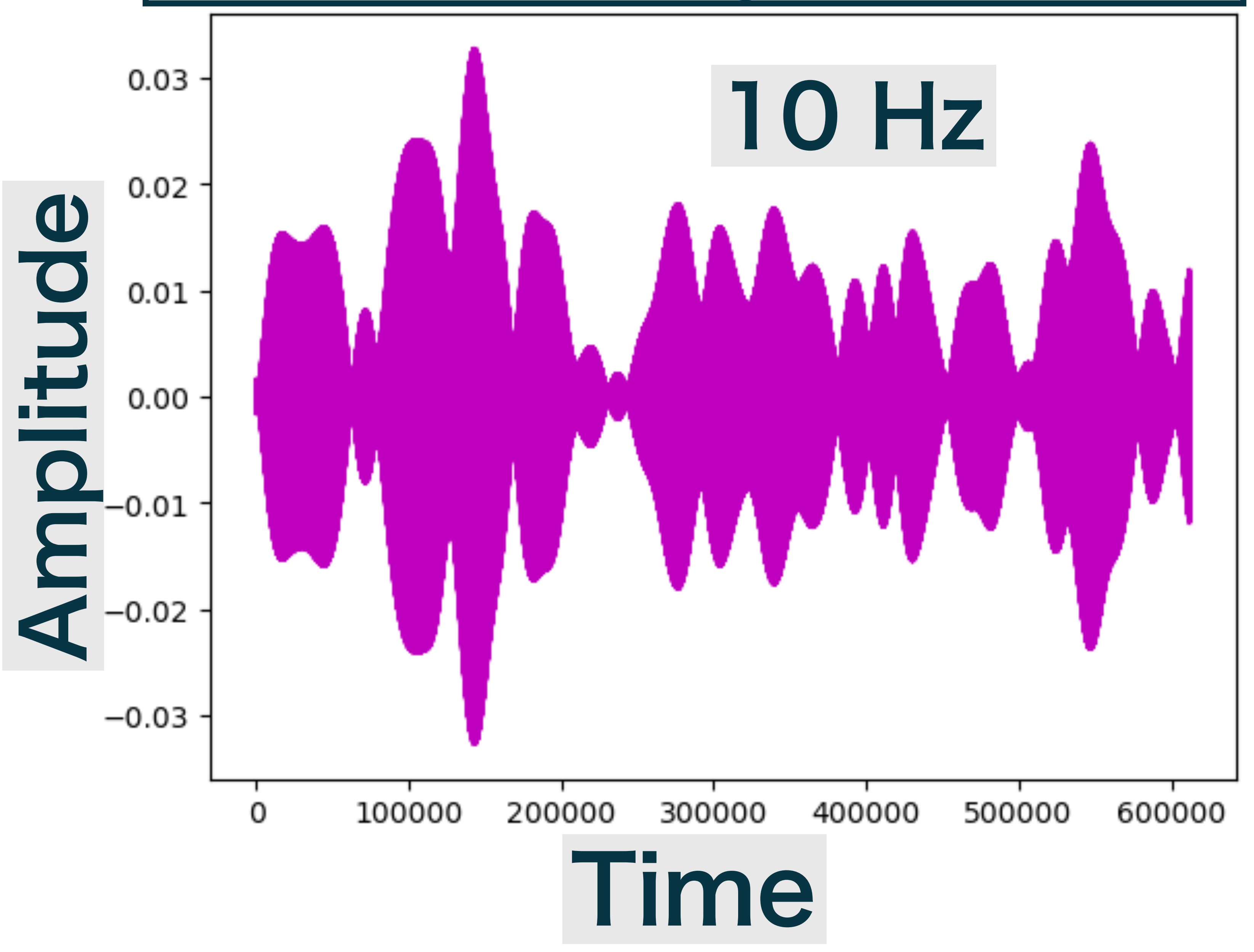
Conditions

- Detector: LIGO (Hanford)
- DM model: $U(1)_{B-L}$ gauge boson



Design sensitivity of LIGO

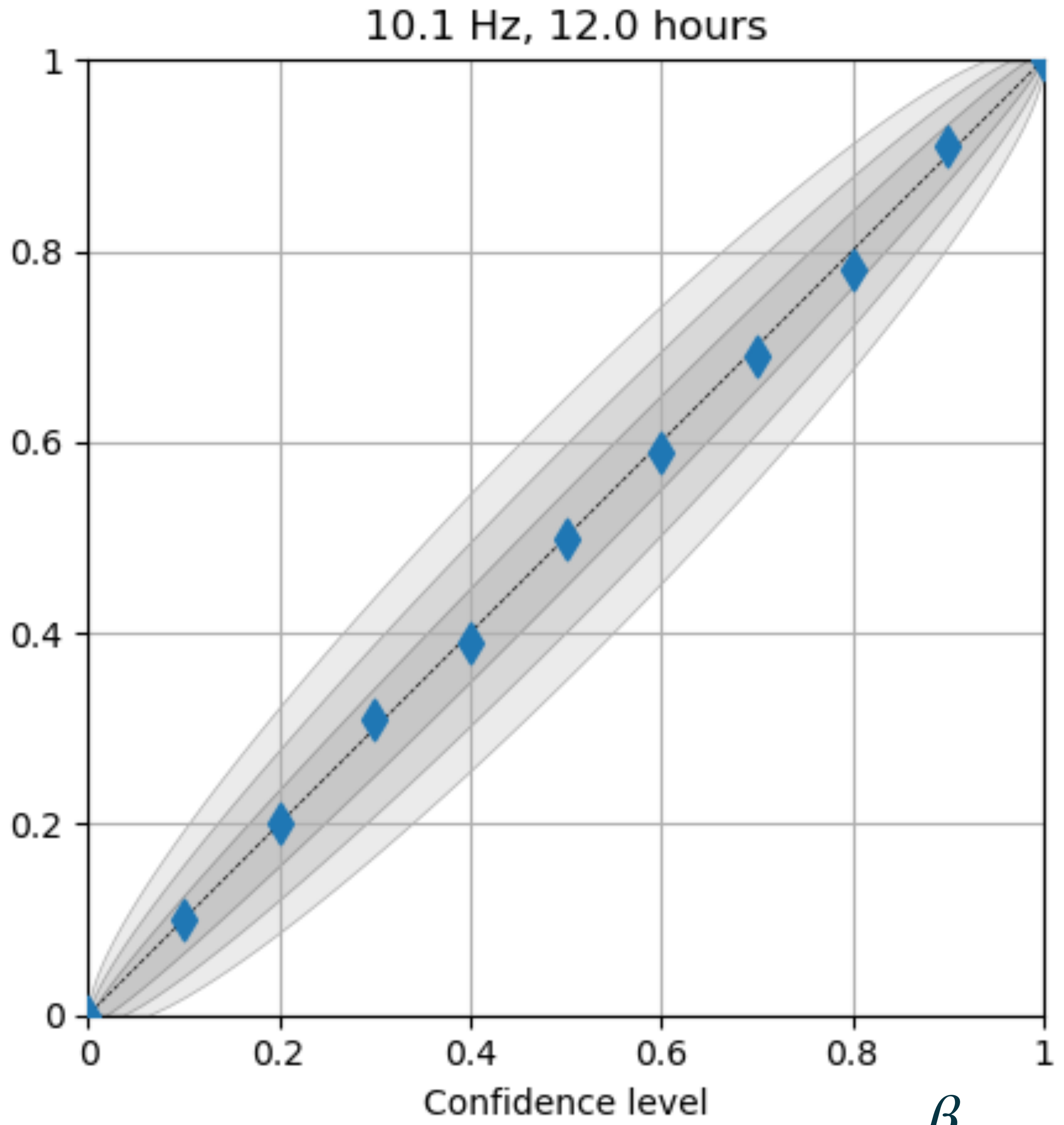
Simulated signal of DM



Analysis -Results-

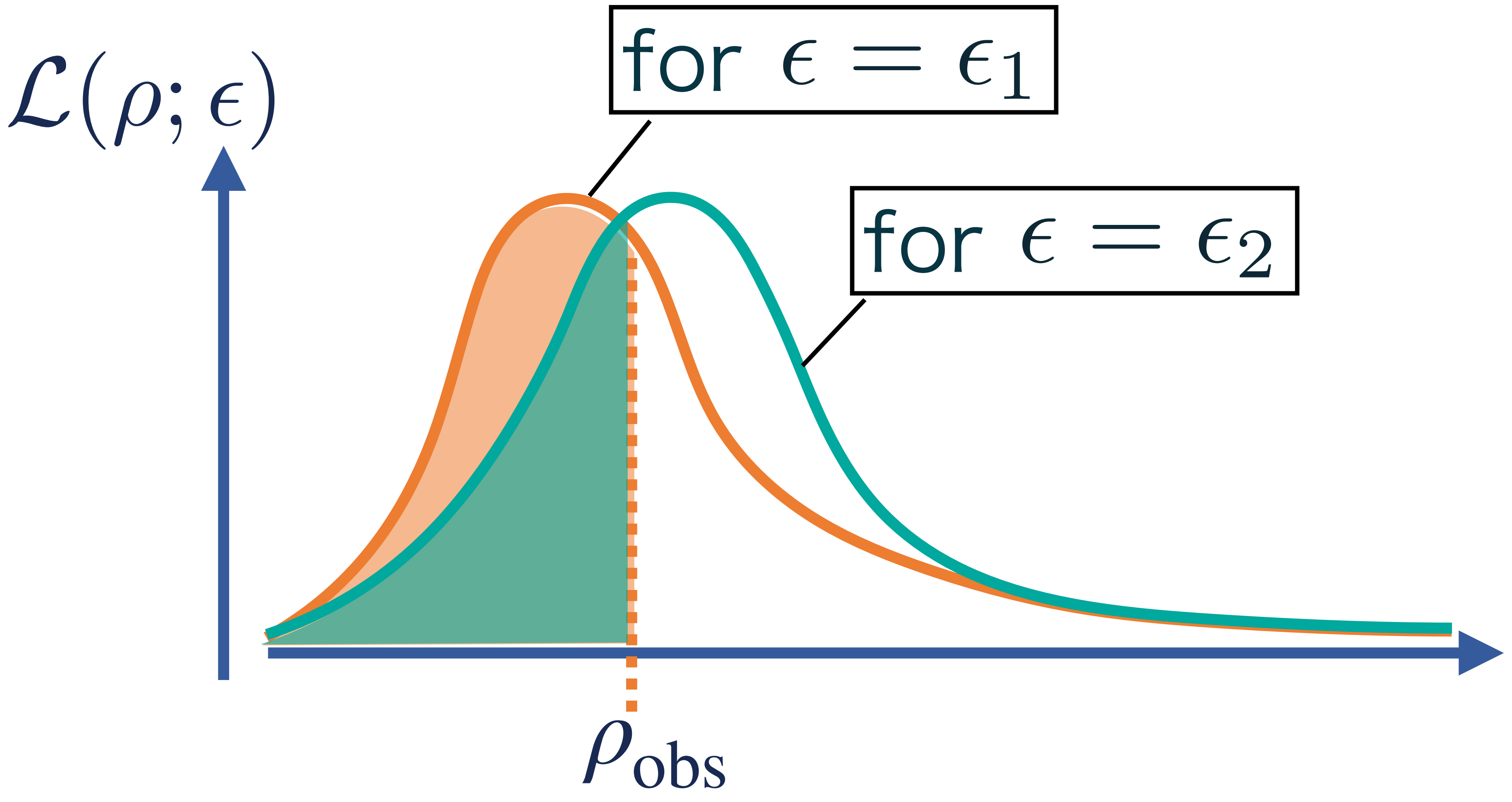
- **Tested against 100 simulated data**
  - Estimate upper bound from each data with various confidence level: $\epsilon_{\mathrm{est}}^{\beta\%}$
  - β % out of all $\epsilon_{\mathrm{est}}^{\beta\%}$ will be below ϵ_{true} .

Fraction of the number of $\epsilon_{\mathrm{est}}^{\beta\%} \geq \epsilon_{\mathrm{true}}$



Confidence level: $\frac{\beta}{100}$

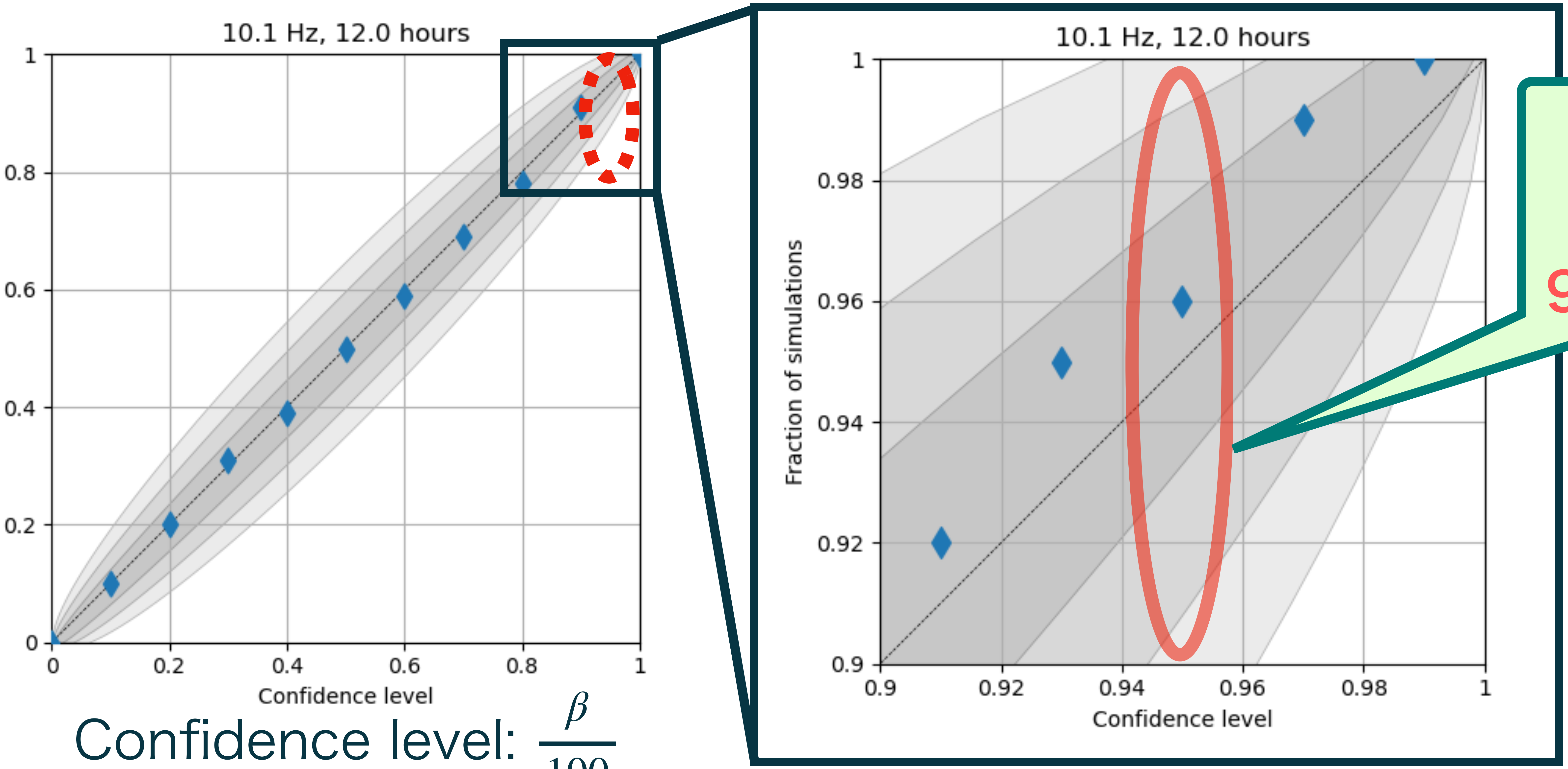
$$1 - \frac{\beta}{100} = \int_0^{\rho_{\mathrm{obs}}} \mathcal{L}(\rho(f)|\epsilon^{\beta\%})d\rho$$



Analysis -Results-

- **Tested against 100 simulated data**
  - Estimate upper bound from each data with various confidence level: $\epsilon_{\rm est}^{\beta\%}$
  - β % out of all $\epsilon_{\rm est}^{\beta\%}$ will be below $\epsilon_{\rm true}$.

Fraction of the number of $\epsilon_{\rm est}^{\beta\%} \geq \epsilon_{\rm true}$

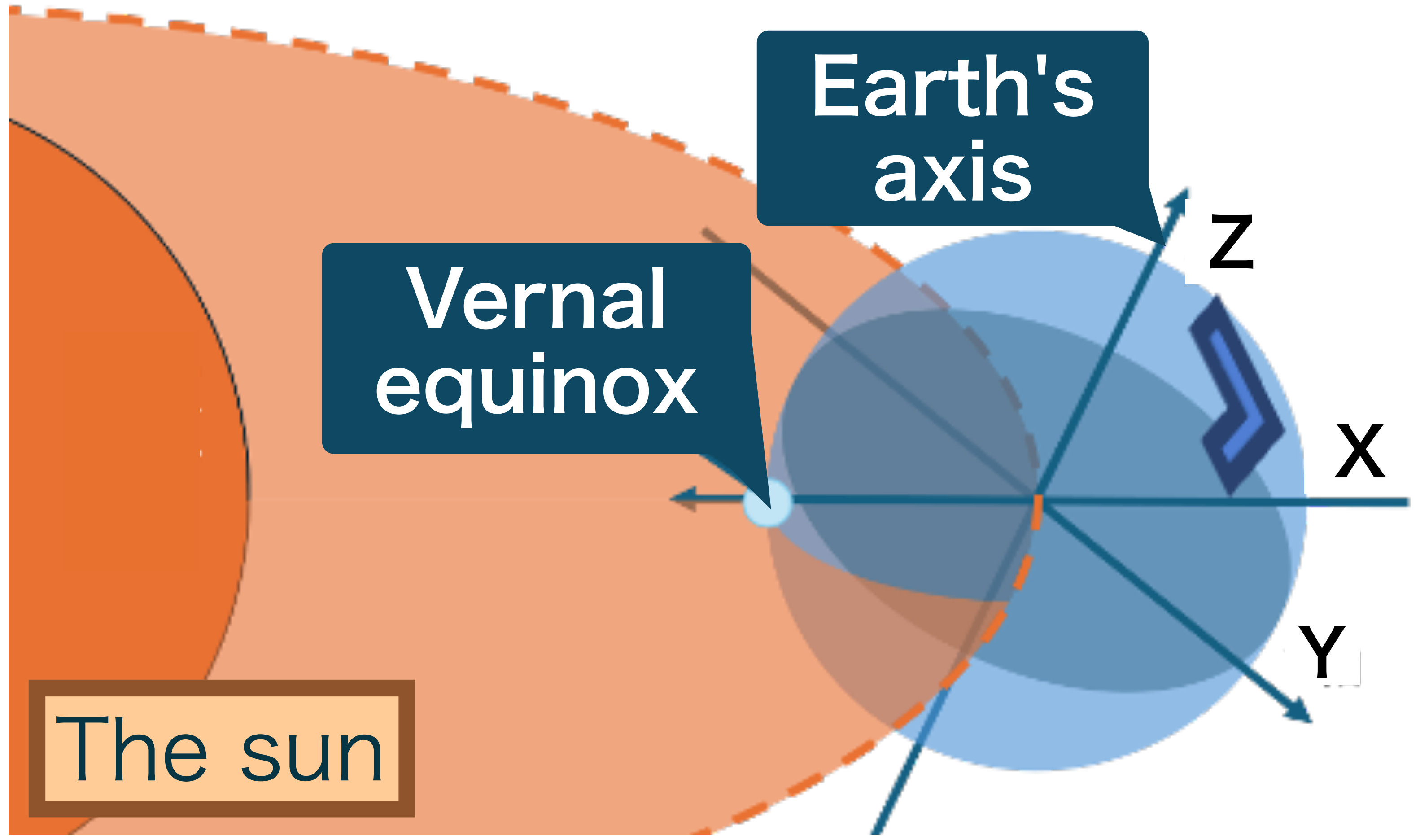


Confidence level: $\frac{\beta}{100}$

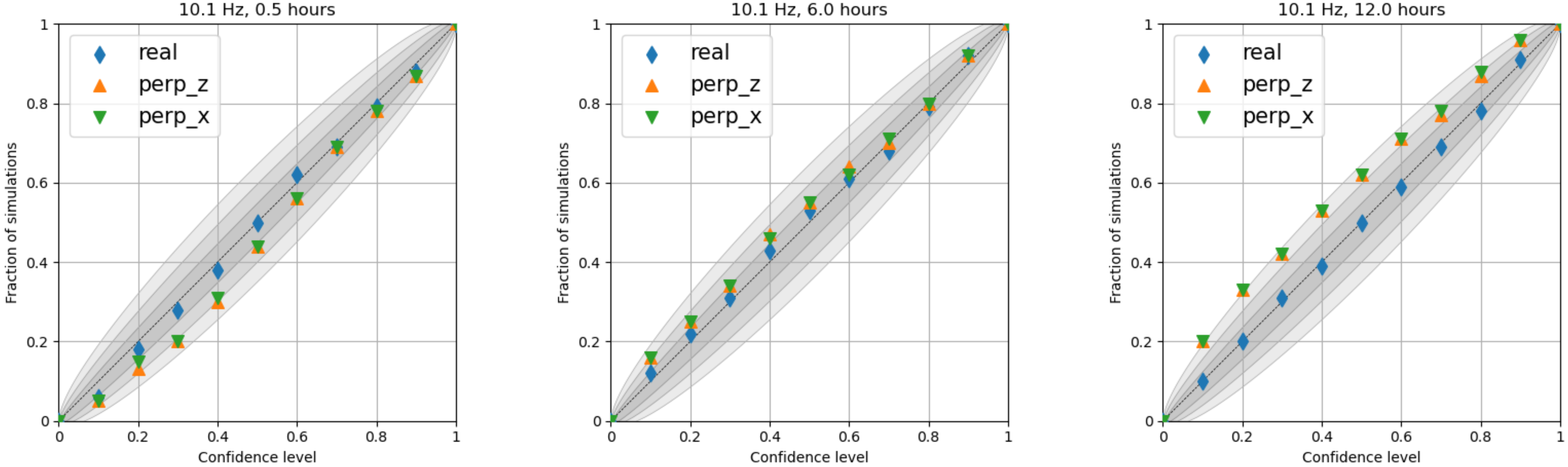
Here represents the result of 95% upper bound

Results

Plot with true direction & perpendicular direction to the true;
perp_z: with largest z-component
perp_x: lying in the x-y plane**

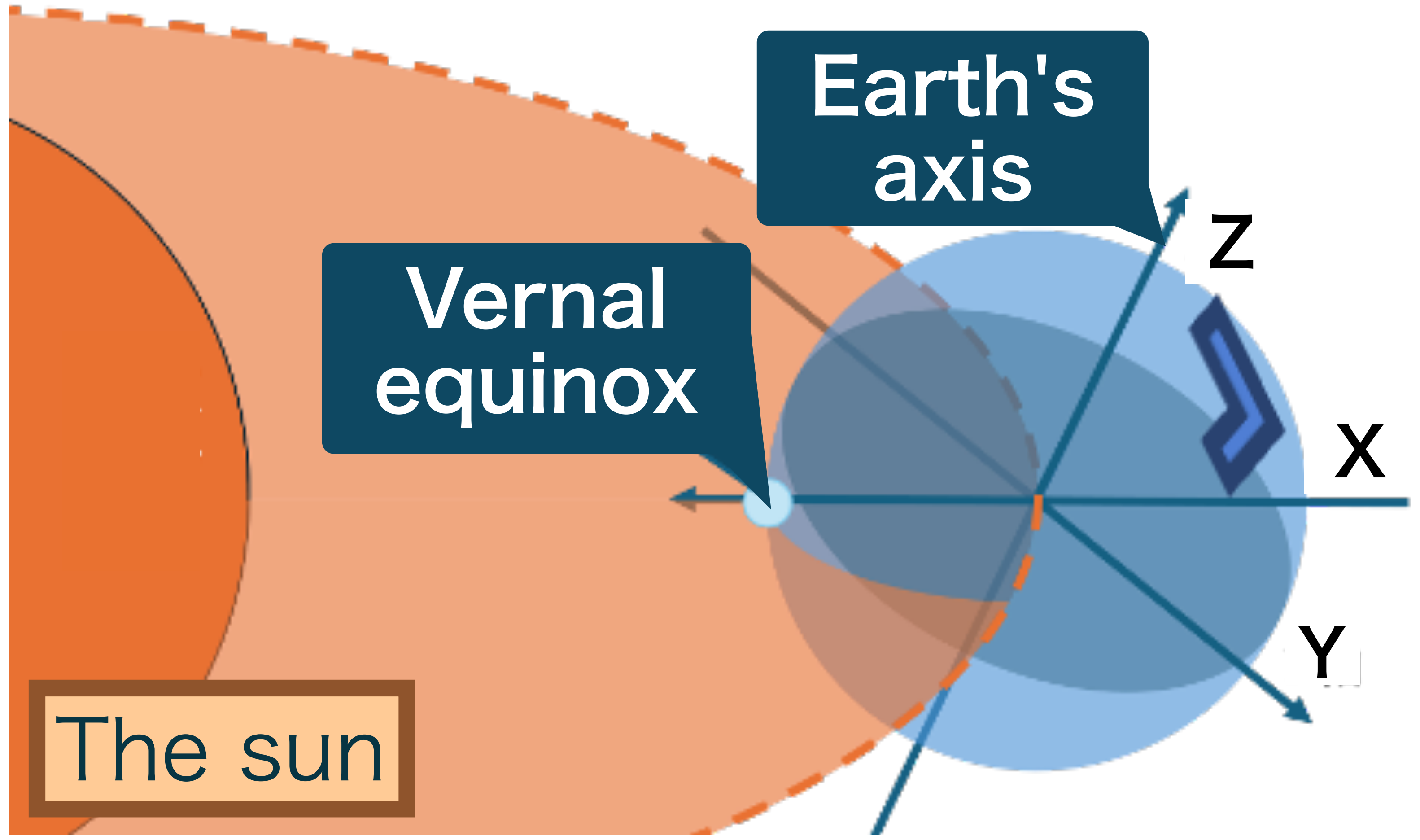


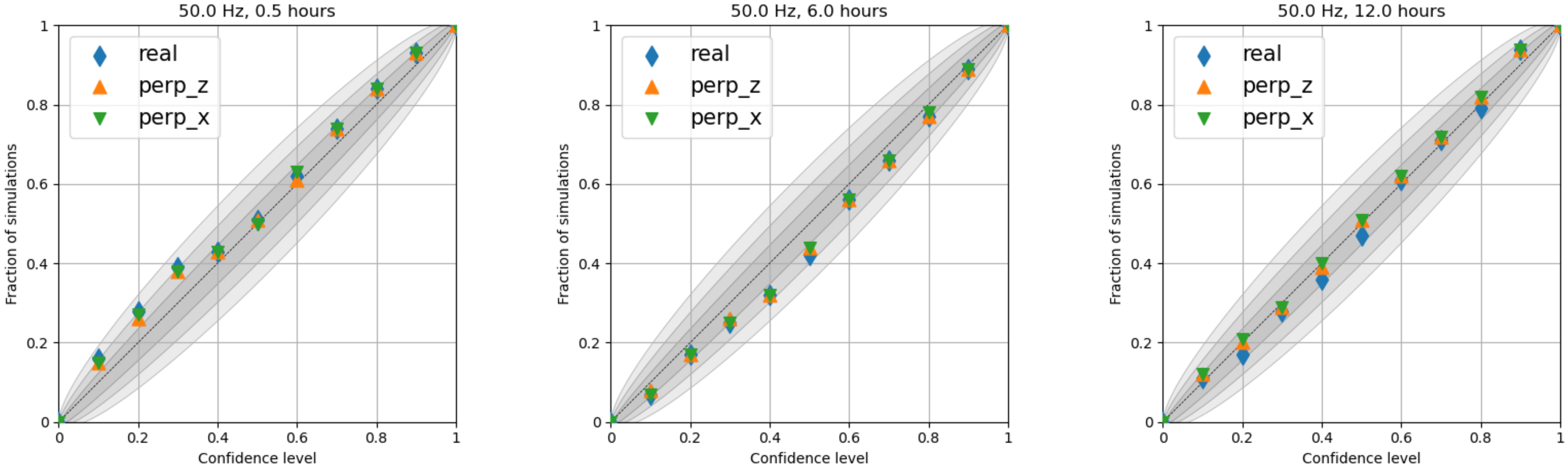
- Injected coupling constant: $\epsilon = 2 \times 10^{-20}$
- Signal frequency: 10.1 Hz
- Upper bounds are properly estimated in the proper condition.



Results

- Injected coupling constant: $\epsilon = 2 \times 10^{-20}$
- Signal frequency: 50 Hz
- The effect of direction of $\vec{v}_{\odot}$ is barely observable.





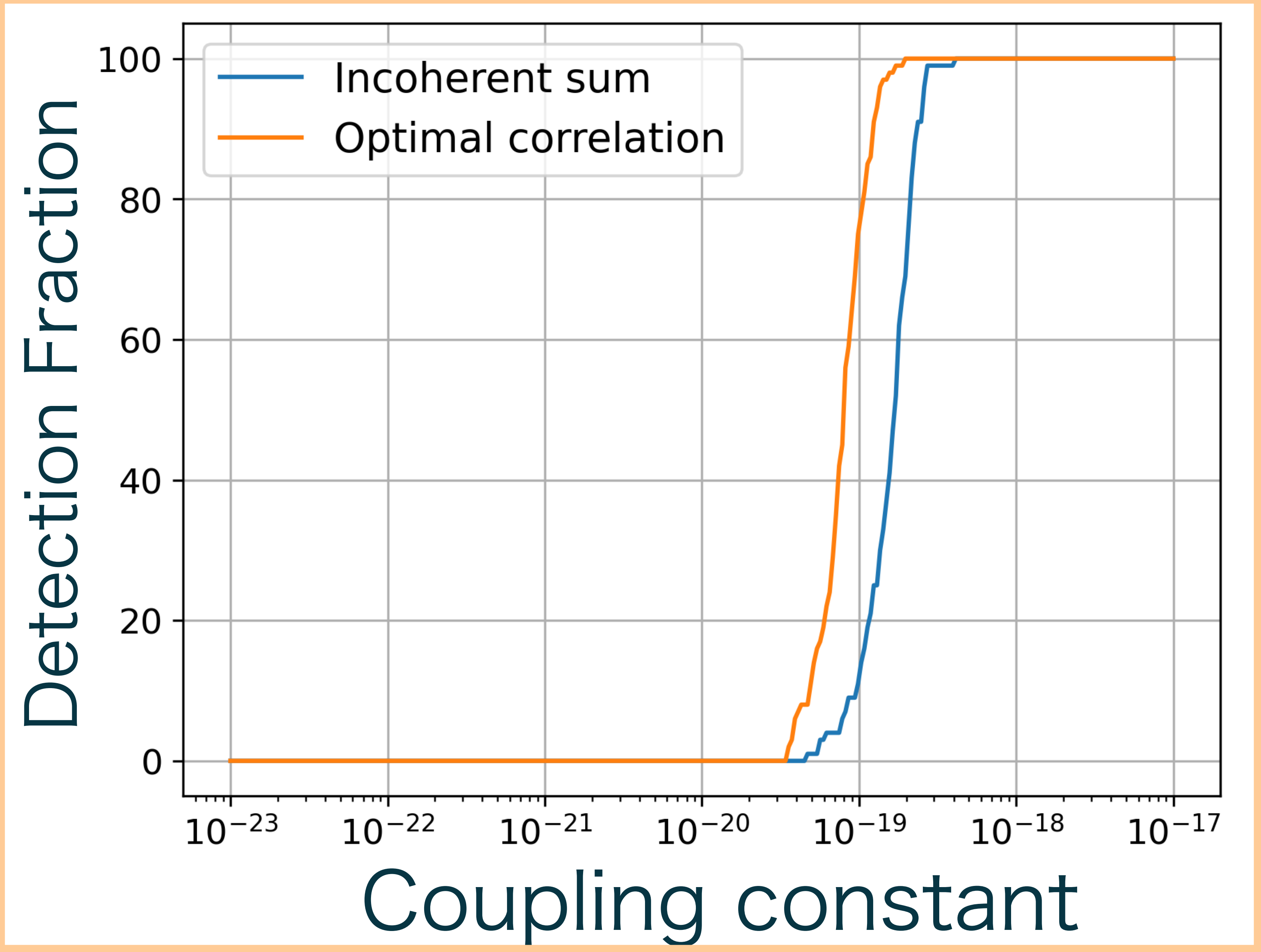
Summary of Results and Future Work

Summary

- Described about the DPDM search with a gravitational wave detector and the effect of the relative velocity.
- **The relative velocity** has to be considered in the analysis.
- Confirmed that **the upper bound of the coupling constant is appropriately estimated** in the lower-frequency region.

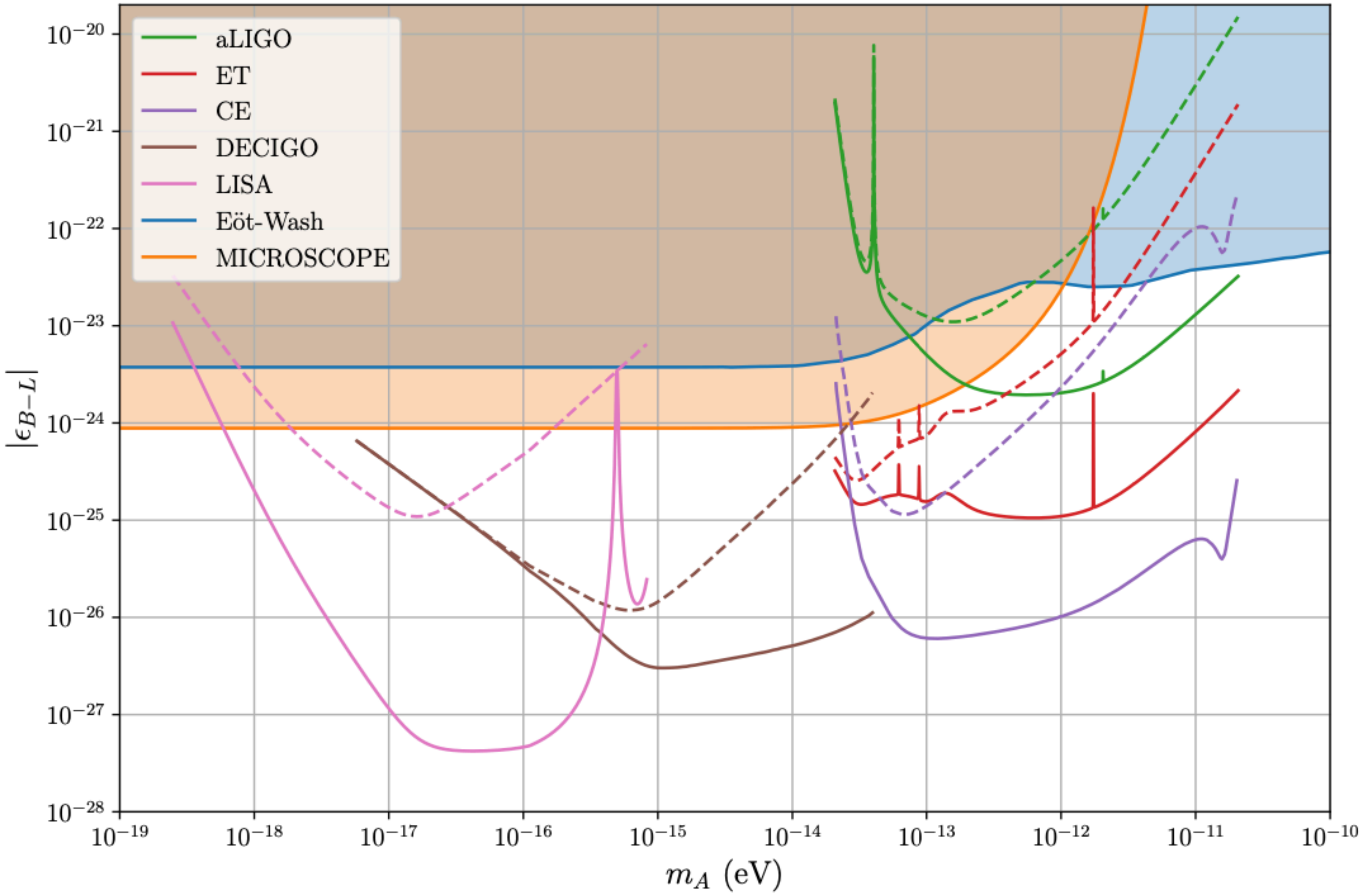
Future work

- Analysis of real data
- Construct the detection statistic with **the optimal filtering method**
- Extension to **other ULDM models**



Appendix

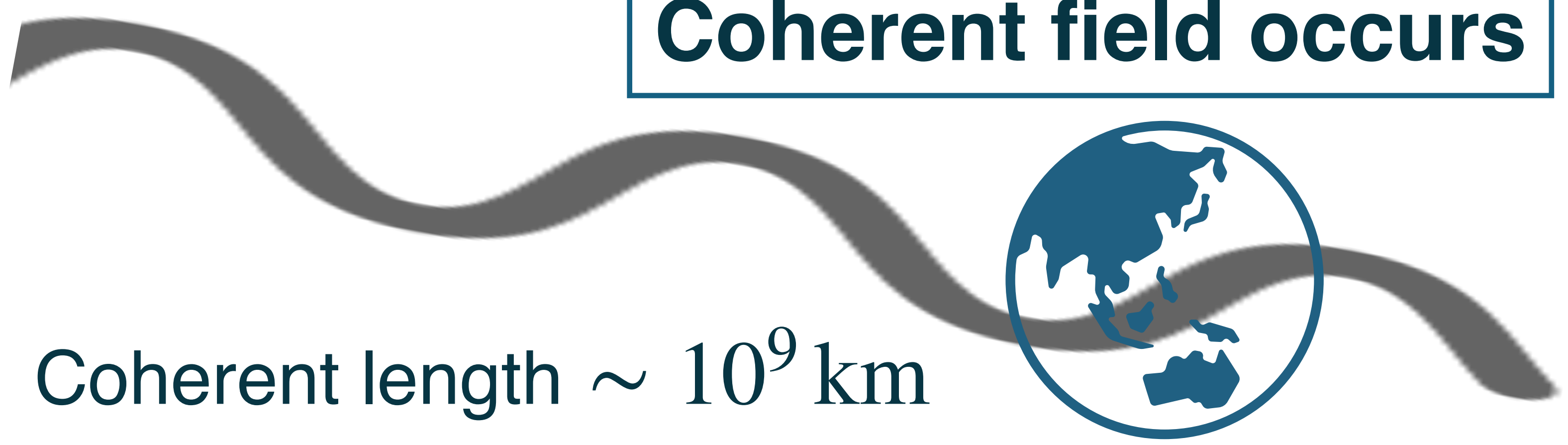
The expected sensitivity



Coupling Constant

Mass of DM

S. Morisaki et. al., Phys. Rev. D 103, 051702 (2021)

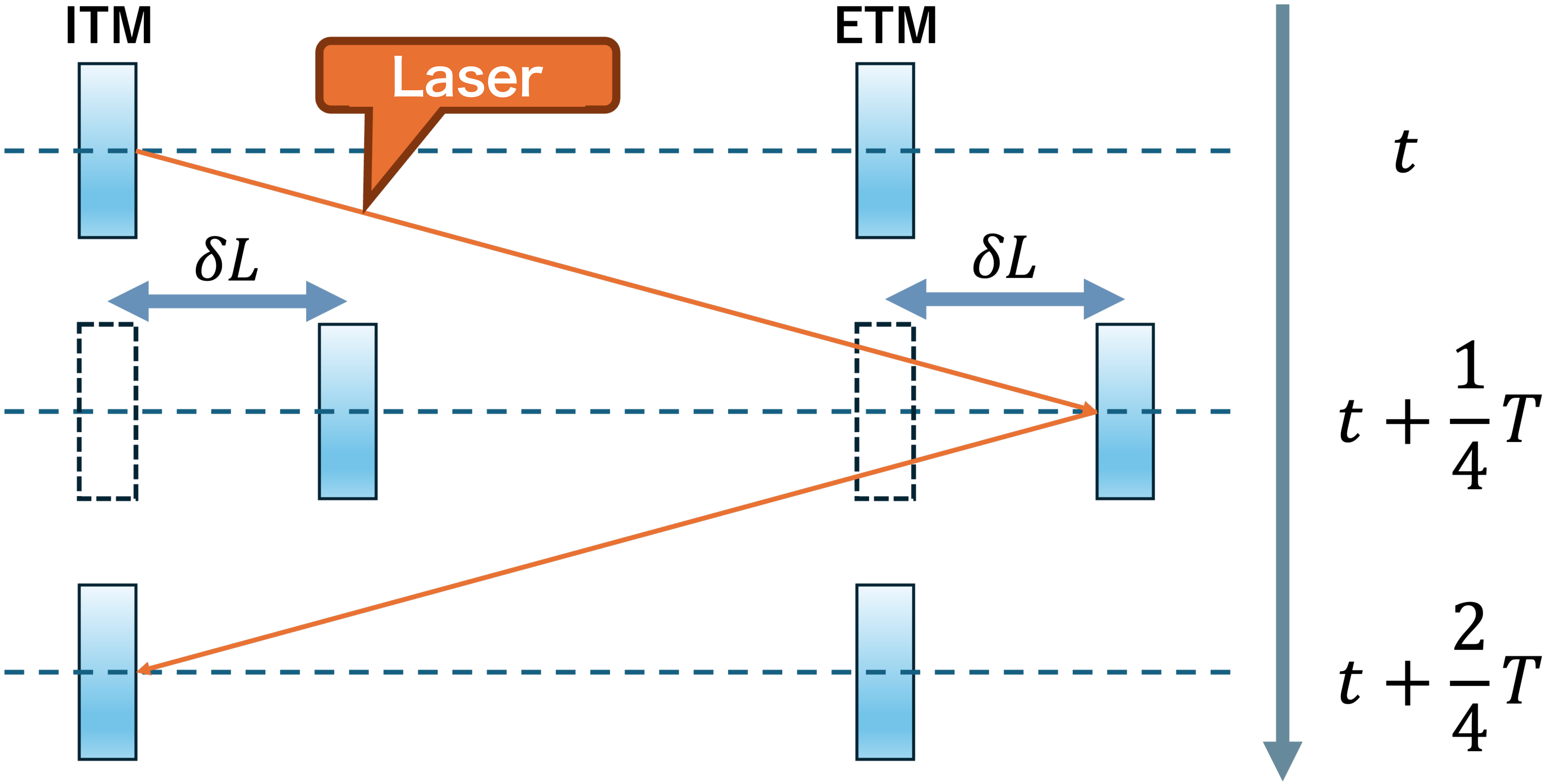


$$f_{\mathrm{DM}} \sim \frac{m}{2\pi} = 242\ \mathrm{Hz}\left(\frac{m}{10^{-12}\ \mathrm{eV}}\right)$$

The finite light-traveling time effect

$h_{\rm time}$:

(cf. Morisaki *et. al.*, Phys. Rev. D 103, L051702 (2021))



Optical path length : $L + 2\delta L$

※T: Period of the field

Concrete way to get the likelihood

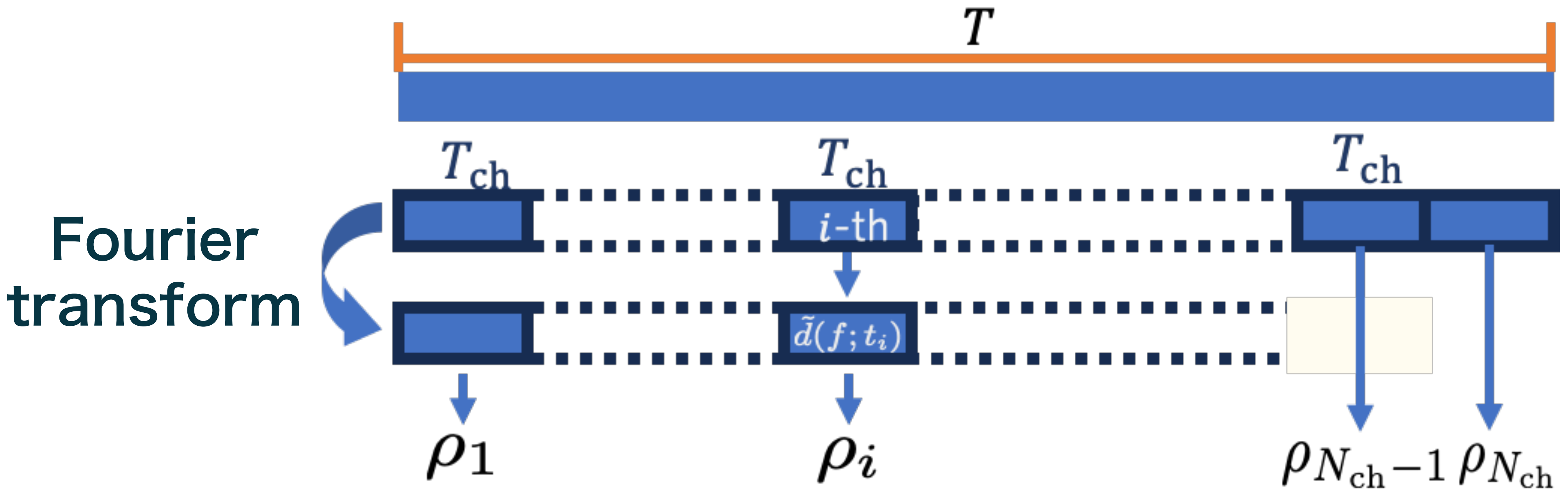
- **We utilize ρ as the detection statistic**

…we separate the data into 30min. chunks and FT by each chunk

$$\rho(f) = \sum_{i}^{N_{\mathrm{chunk}}} \rho_i(f), \qquad \rho_i(f) = \frac{4|\tilde{d}(f;t_i)|^2}{T_{\mathrm{ch}} S(f;t_i)}$$

T_{ch} : duration of chunks

$S(f;t_i)$: one-sided PSD of each chunk



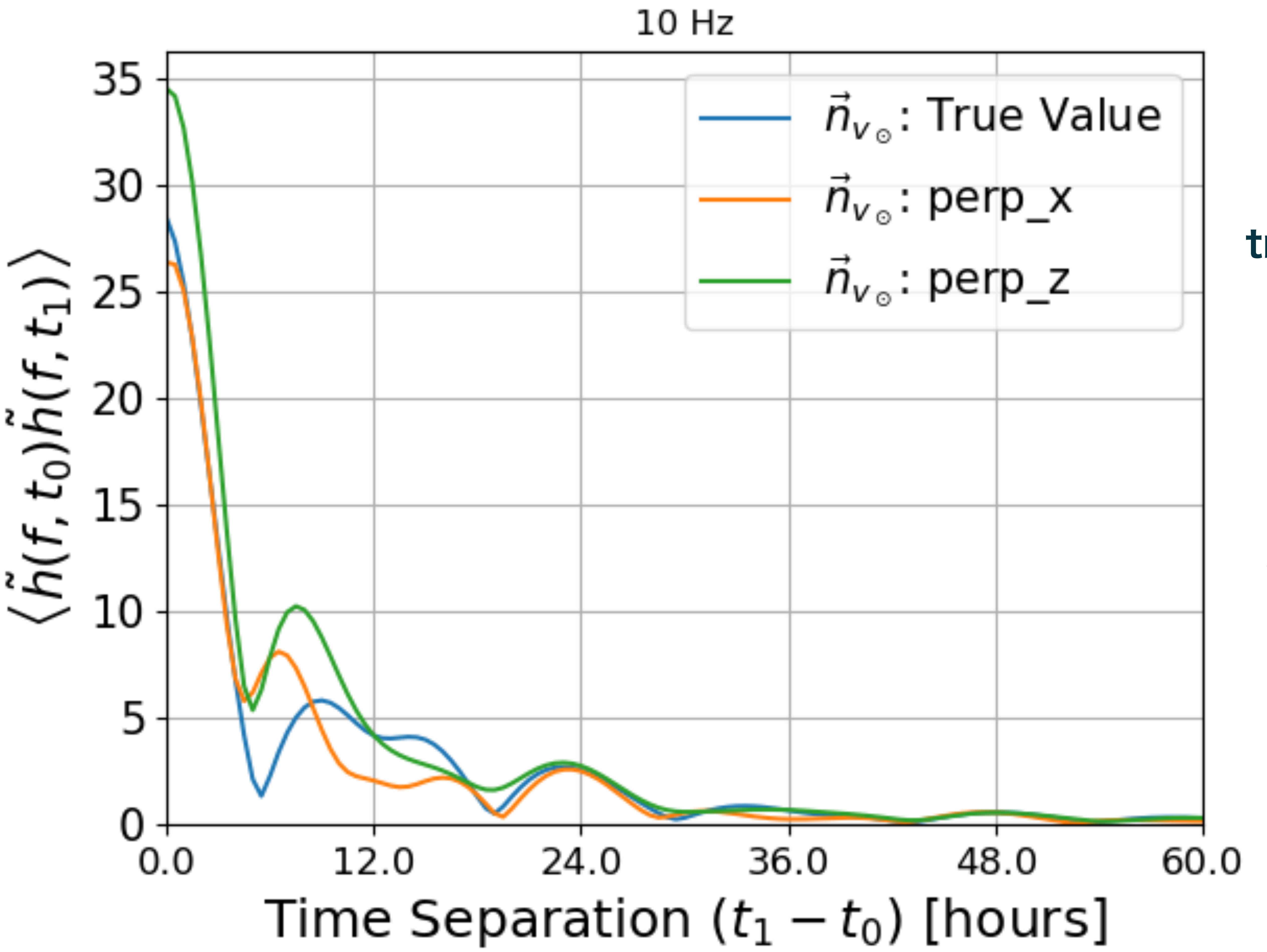
Each ρ_i has the correlation depending on the direction of $\vec{v}_{\odot}$.

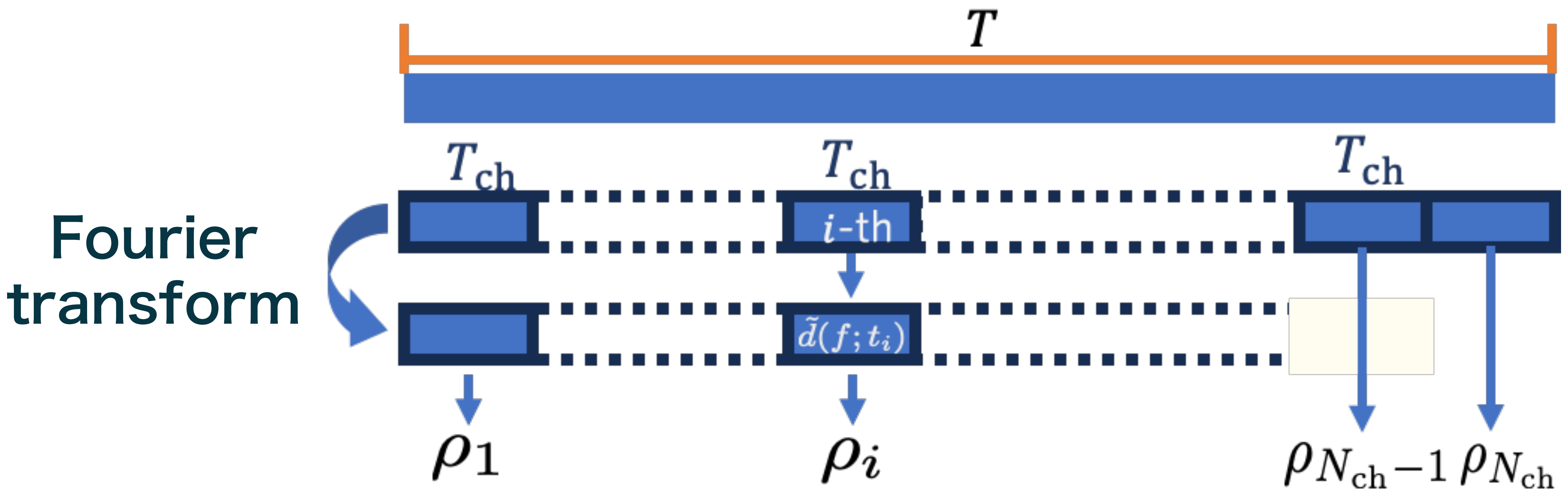
So we cannot obtain the likelihood analytically.

We generated 10^5 ρ by simulation and then use the distribution as the likelihood $\mathscr{L}(\rho(f)\,|\,\epsilon)$.

The correlation of the signal

Each ρ_i has the correlation depending on the direction of $\vec{v}_\odot$





$$\rho(f) = \sum_i^{N_{\rm chunk}} \rho_i(f), \qquad \rho_i(f) = \frac{4|\tilde{d}(f;t_i)|^2}{T_{\rm ch} S(f;t_i)}$$

$$\tilde{d}(f;t_i) = \tilde{n}(f;t_i) + \tilde{h}(f;t_i)$$

The simulation of ρ

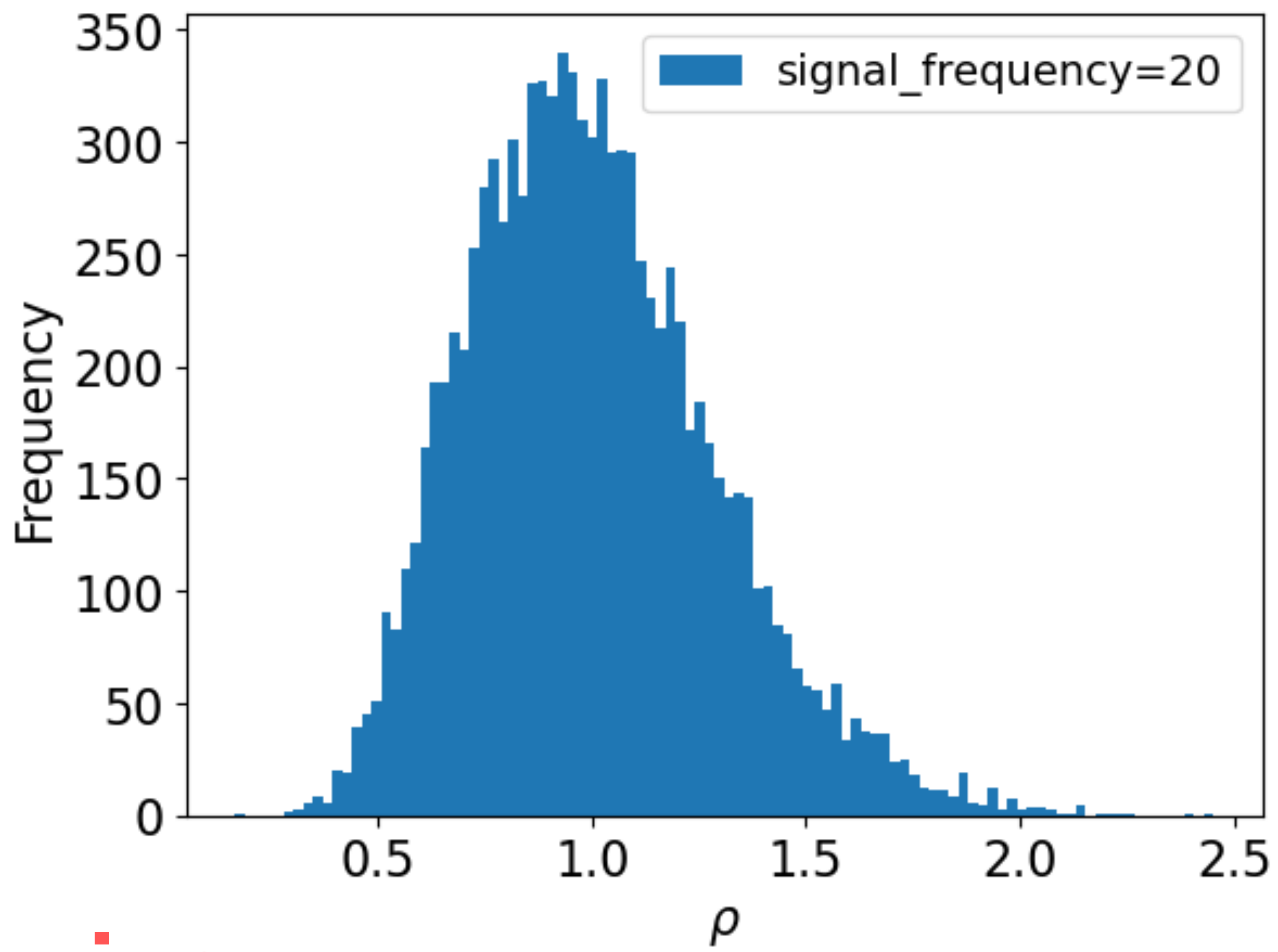
$$\rho(f) = \sum_i^{N_{\rm chunk}} \rho_i(f),$$

$$\rho_i(f) = \frac{4|\tilde{d}(f;t_i)|^2}{T_{\rm ch} S(f;t_i)}$$

$$\tilde{d}(f;t_i) = \tilde{n}(f;t_i) + \tilde{h}(f;t_i)$$

Detector noise

Signal from DM



DM signals: generated as random value of multivariate normal dist. $\mathcal{S}_i$ with 0 mean and variance;

$$\mathrm{Cov}(f) = \begin{pmatrix} c_{00} & c_{01} & \cdots & c_{0n} \\ c_{10} & c_{11} & \cdots & c_{1n} \\ \vdots & \vdots & c_{ij} & \vdots \\ c_{n0} & c_{n1} & \cdots & c_{nn} \end{pmatrix}$$

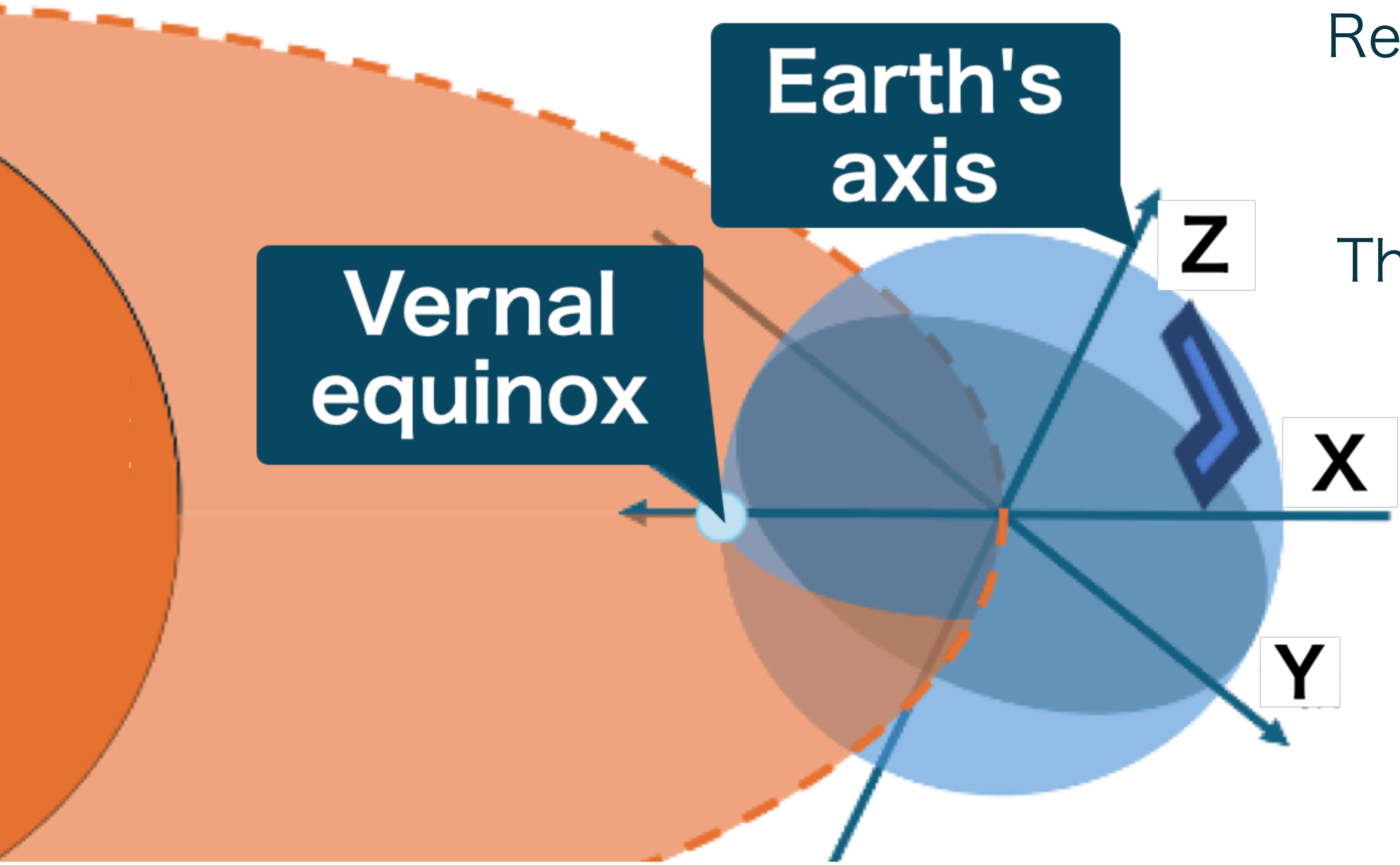
$$c_{ij} = c*_{ji} = \left\langle \tilde{h}^* (f;t_i)\, \tilde{h}\,(f;t_j) \right\rangle$$

Detector noise:

Gaussian random number with 0 mean and 1 variance.
(no correlation between chunks)

$$\rho_i(f_{\rm DM};\epsilon_{\rm D}) = \mathcal{N}_i^2 + 2\epsilon_{\rm D}\mathrm{Re}[\mathcal{N}_i^* \mathcal{S}_i] + \epsilon_{\rm D}^2 \mathcal{S}_i^2$$

The coordinate and relative velocity



Real direction of relative velocity

[0.463, -0.496, 0.735]

The amplitude of relative velosity

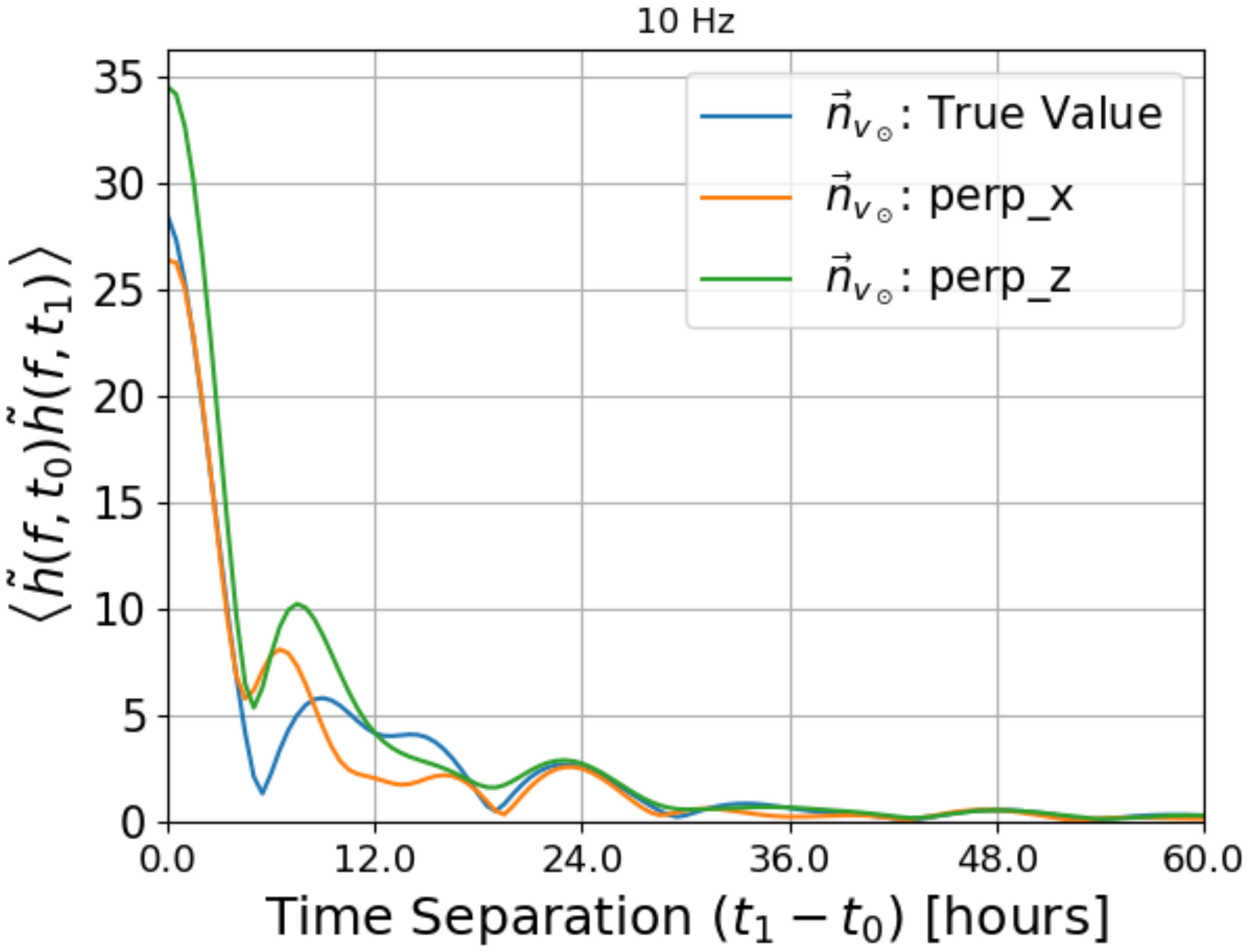
$230 \sim 240$ [km/s]

Correlation function

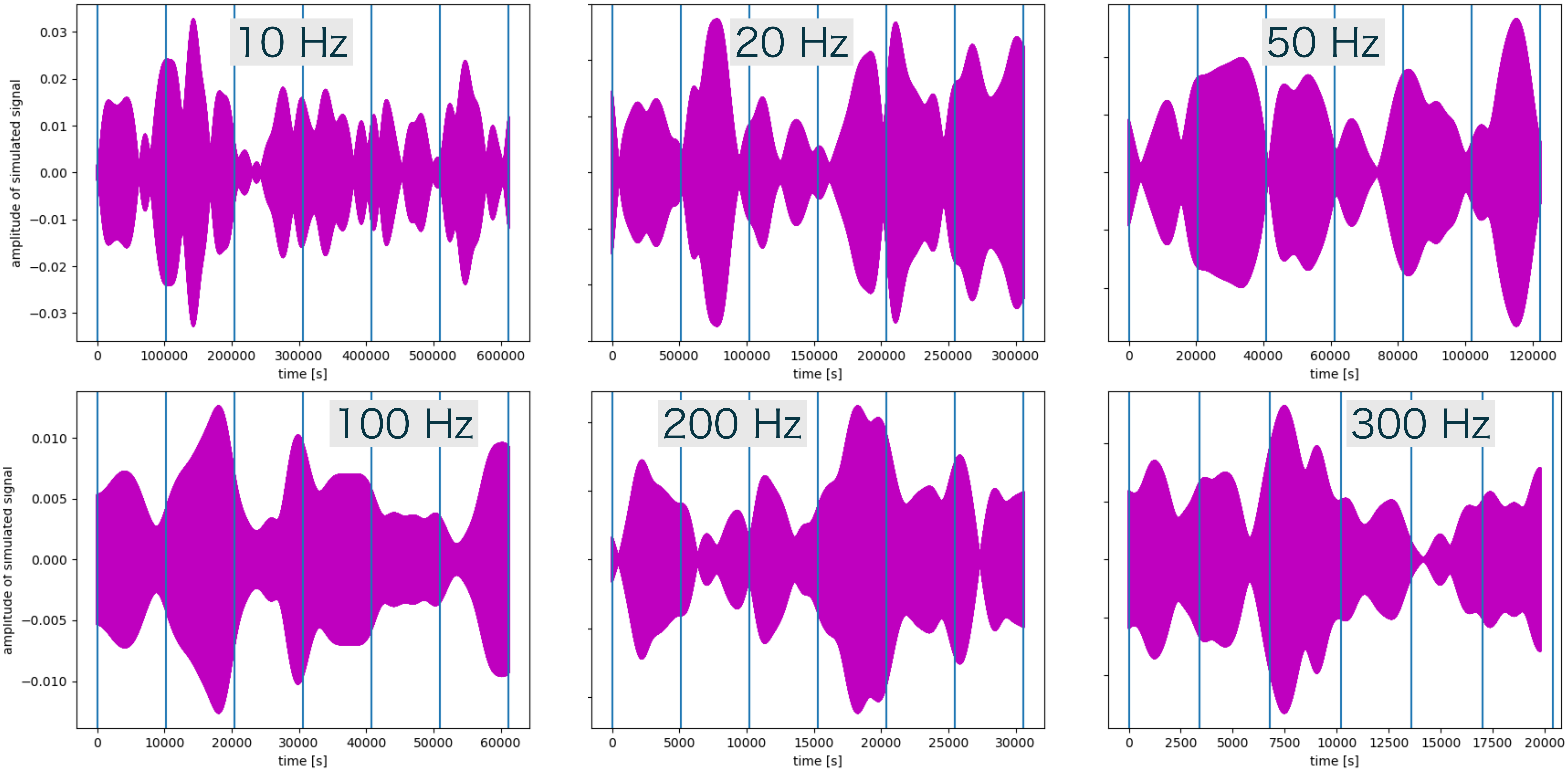
$$\left\langle \tilde{h}_1^*(f;t_0)\tilde{h}_1(f;t_1)\right\rangle =$$
$$\frac{\epsilon^2 e^2 A^2 T^2 v_{\rm vir}^3}{8\sqrt{\pi}V^3}\left(\frac{Q}{M}\right)^2 \frac{\sin^4(\pi f L)}{(\pi f L)^2} \mathrm{e}^{-V^2/v_{\rm vir}^2+2\pi \mathrm{i} f_{\rm DM}(t_1-t_0)} d_1^i(t_0) d_{1,i}(t_1)\left(I(x_+)-I(x_-)\right),$$

$$\left\langle \tilde{h}_2^*(f;t_0)\tilde{h}_2(f;t_1)\right\rangle =$$
$$\frac{\epsilon^2 e^2 A^2 f_{\rm DM}^2 T^2 v_{\rm vir}^7}{8\sqrt{\pi}f^2V^5}\left(\frac{Q}{M}\right)^2 \mathrm{e}^{-V^2/v_{\rm vir}^2+2\pi \mathrm{i} f_{\rm DM}(t_1-t_0)} D^{ij}(t_0) D_j^k(t_1)\times$$
$$\left[\left(J_\perp(x_+)-J_\perp(x_-)\right)\left(\delta_{ik}-\frac{V_iV_k}{V^2}\right)+\left(J_\parallel(x_+)-J_\parallel(x_-)\right)\frac{V_iV_k}{V^2}\right],$$

$$\left\langle \tilde{h}_1^*(f;t_0)\tilde{h}_2(f;t_1)\right\rangle + \left\langle \tilde{h}_2^*(f;t_0)\tilde{h}_1(f;t_1)\right\rangle =$$
$$-\mathrm{i}\frac{\epsilon^2 e^2 A^2 f_{\rm DM} T^2 v_{\rm vir}^5 \sin^2(\pi f L)}{8\pi^{\frac{3}{2}}V^4 f^2 L}\left(\frac{Q}{M}\right)^2 \mathrm{e}^{-V^2/v_{\rm vir}^2+2\pi \mathrm{i} f_{\rm DM}(t_1-t_0)}\times$$
$$\left[\mathrm{e}^{2\pi \mathrm{i} f L} d^i(t_0) D_i^j(t_1) + \mathrm{e}^{-2\pi \mathrm{i} f L} d^i(t_1) D_i^j(t_0)\right]\frac{V_j}{V}\left(K(x_+)-K(x_-)\right).$$

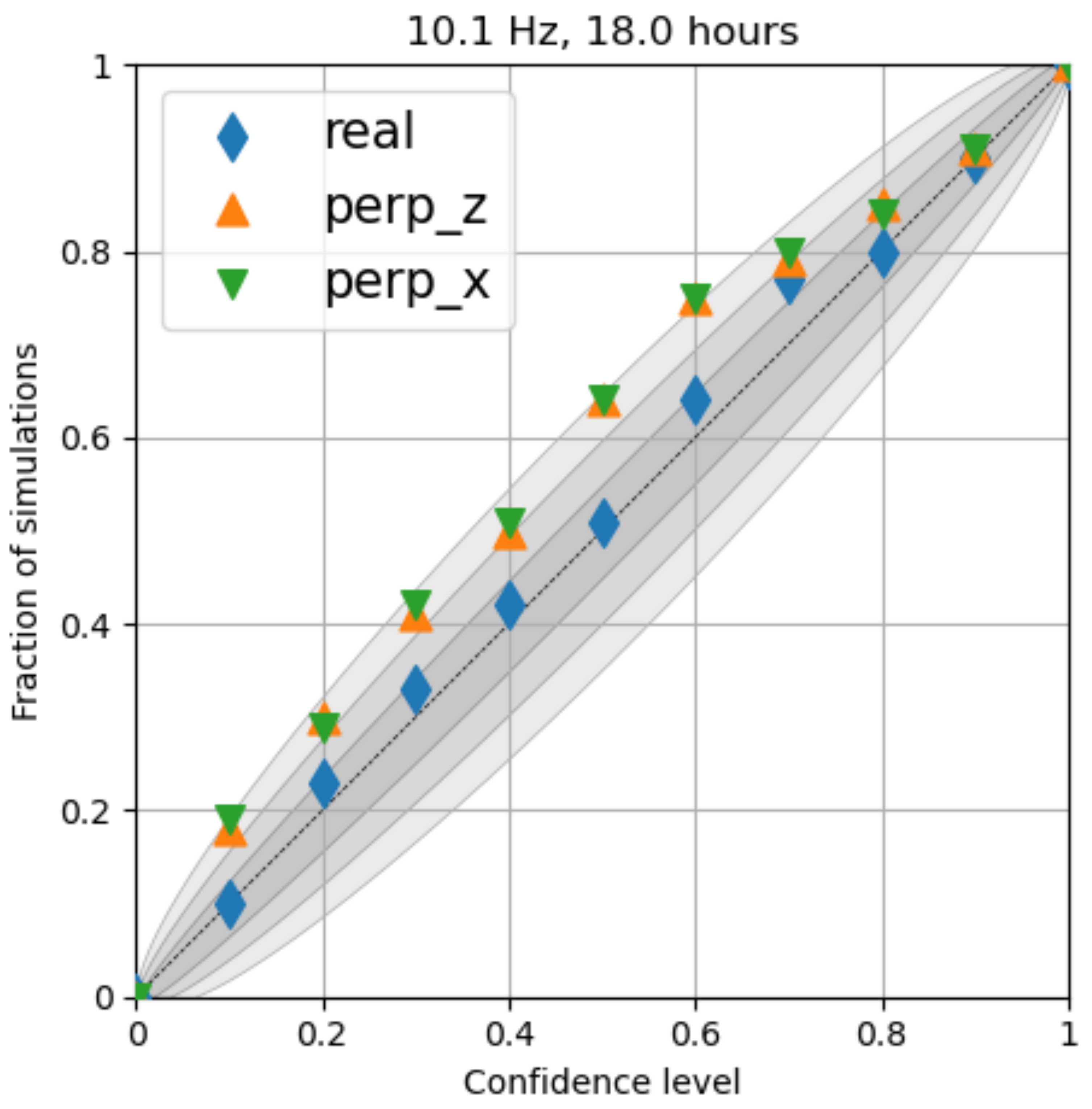


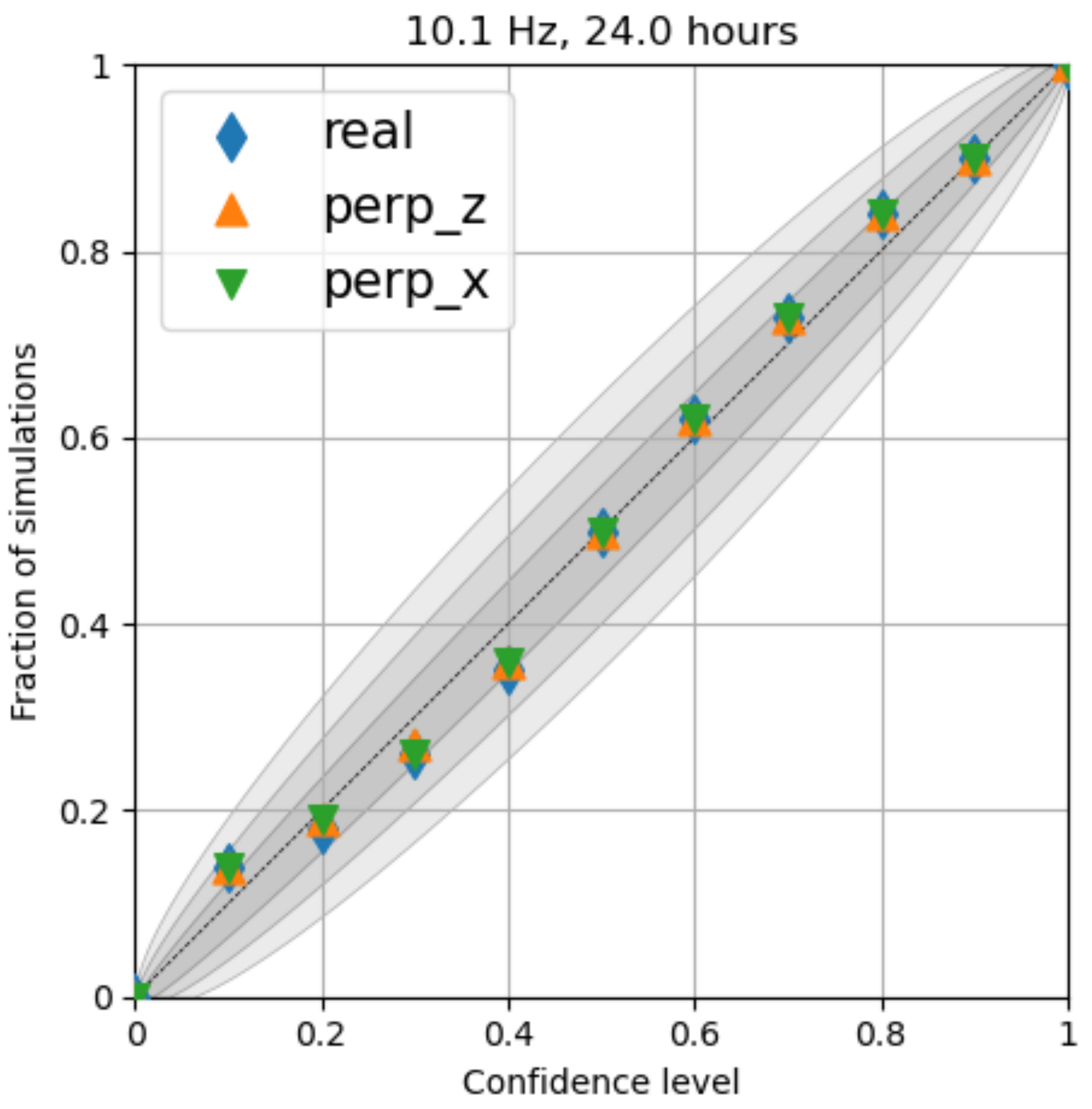
Simulated signals



Longer observation time

- Signal frequency: 10.1 Hz
- Observation for **18, 24 hours**





Analysis for higher frequency signal

- 100 Hz