Non-Gaussianity in PBH Formation

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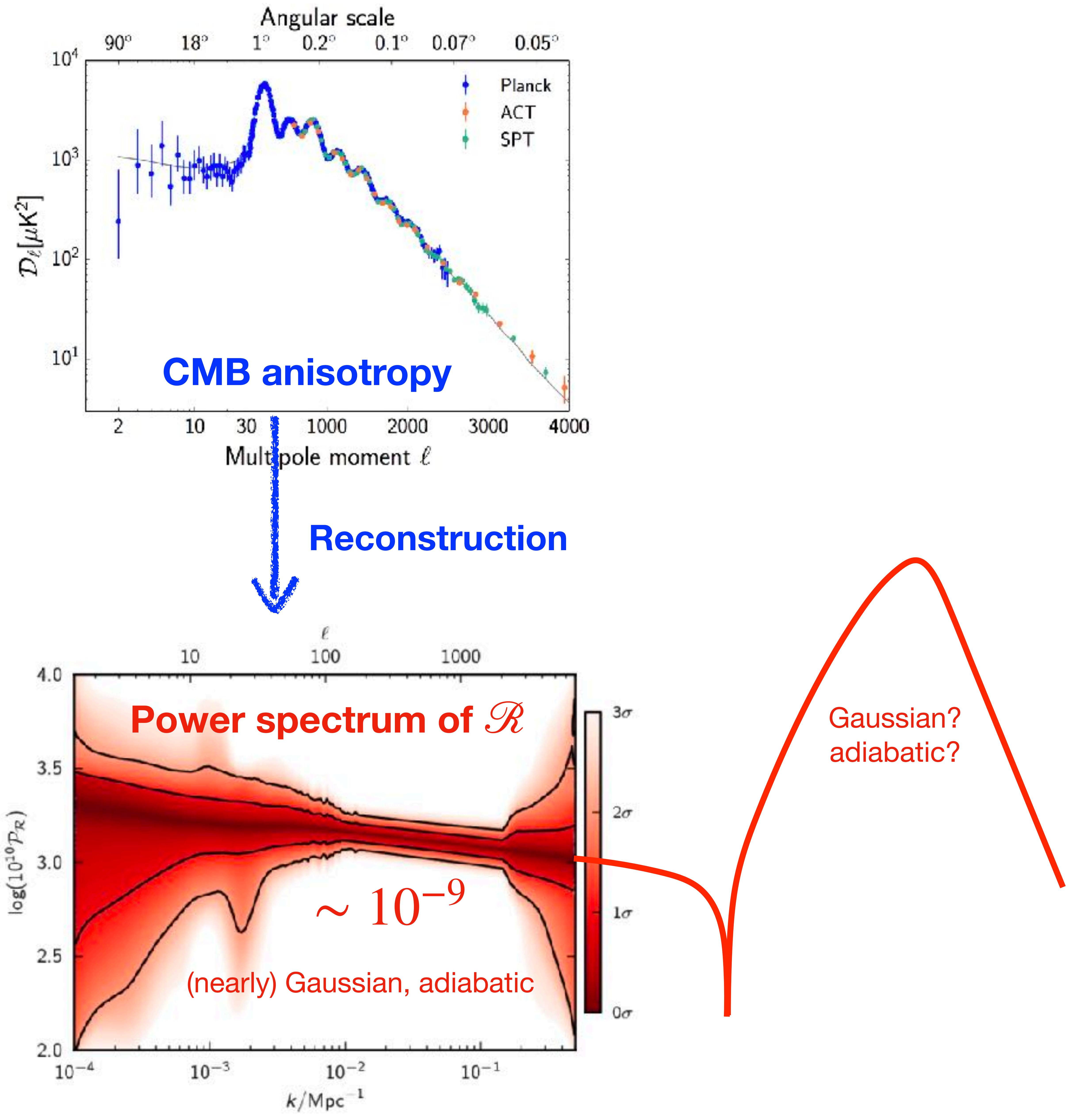
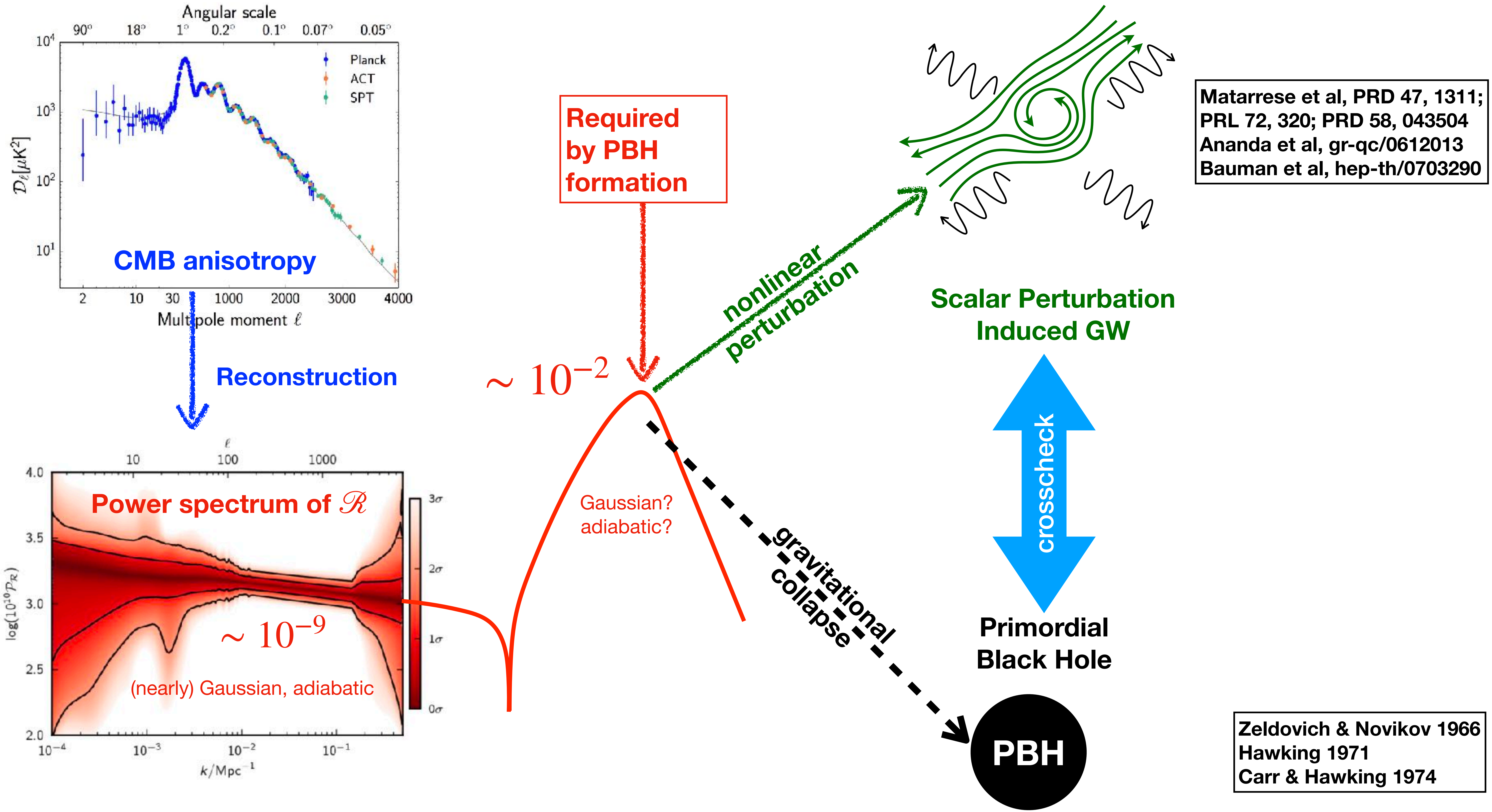
PBH focus week
Kavli IPMU, Nov 15, 2024

CONTENT

- Introduction
- PBH formation and observational constraints
- Non-Gaussianity and their impact on PBHs
- Prediction in mHz and nHz



Angular scale
90° 18° 1° 0.2° 0.1° 0.07° 0.05°
Planck
ACT
SPT
$\mathcal{D}_\ell [\mu K^2]$
10^4 10^3 10^2 10^1
CMB anisotropy
2 10 30 1000 2000 3000 4000
Multipole moment ℓ
Reconstruction
ℓ
10 100 1000
Power spectrum of $\mathscr{R}$
$\log(10^{10}\mathcal{P}_\mathcal{R})$
4.0 3.5 3.0 2.5 2.0
$\sim 10^{-9}$
(nearly) Gaussian, adiabatic
10^{-4} 10^{-3} 10^{-2} 10^{-1}
k/Mpc^{-1}
3σ 2σ 1σ 0σ
Gaussian?
adiabatic?



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Planck
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$\mathcal{D}_\ell [\mu K^2]$
CMB anisotropy
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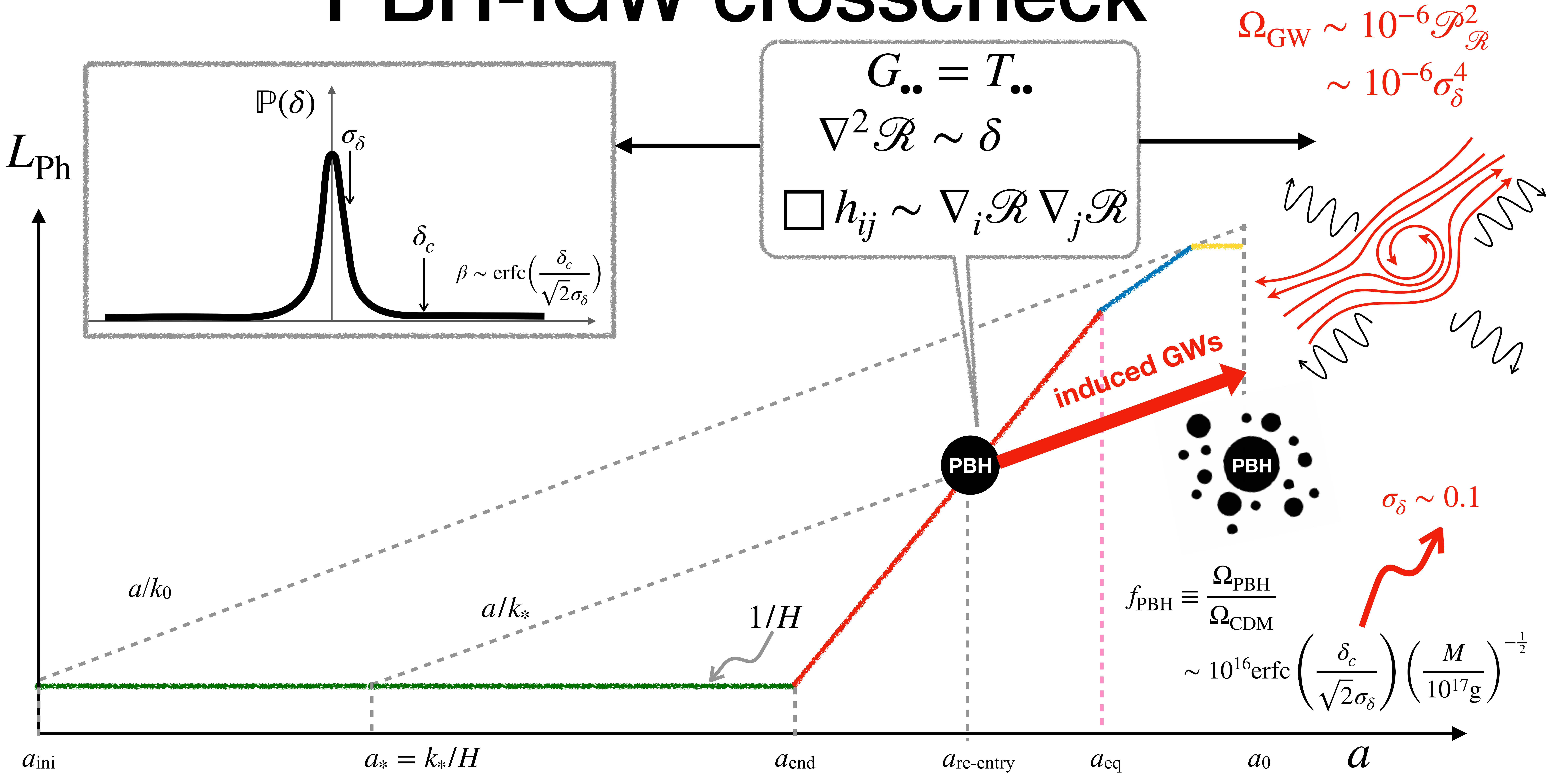
Power spectrum of $\mathscr{R}$
$\sim 10^{-9}$
(nearly) Gaussian, adiabatic
$\log(10^{10}\mathcal{P}_\mathcal{R})$
k/Mpc^{-1}
Required by PBH formation
$\sim 10^{-2}$
Gaussian?
adiabatic?
nonlinear perturbation
gravitational collapse
Scalar Perturbation Induced GW
crosscheck
Primordial Black Hole
PBH
Matarrese et al, PRD 47, 1311; PRL 72, 320; PRD 58, 043504
Ananda et al, gr-qc/0612013
Bauman et al, hep-th/0703290
Zeldovich & Novikov 1966
Hawking 1971
Carr & Hawking 1974

PBH-IGW crosscheck



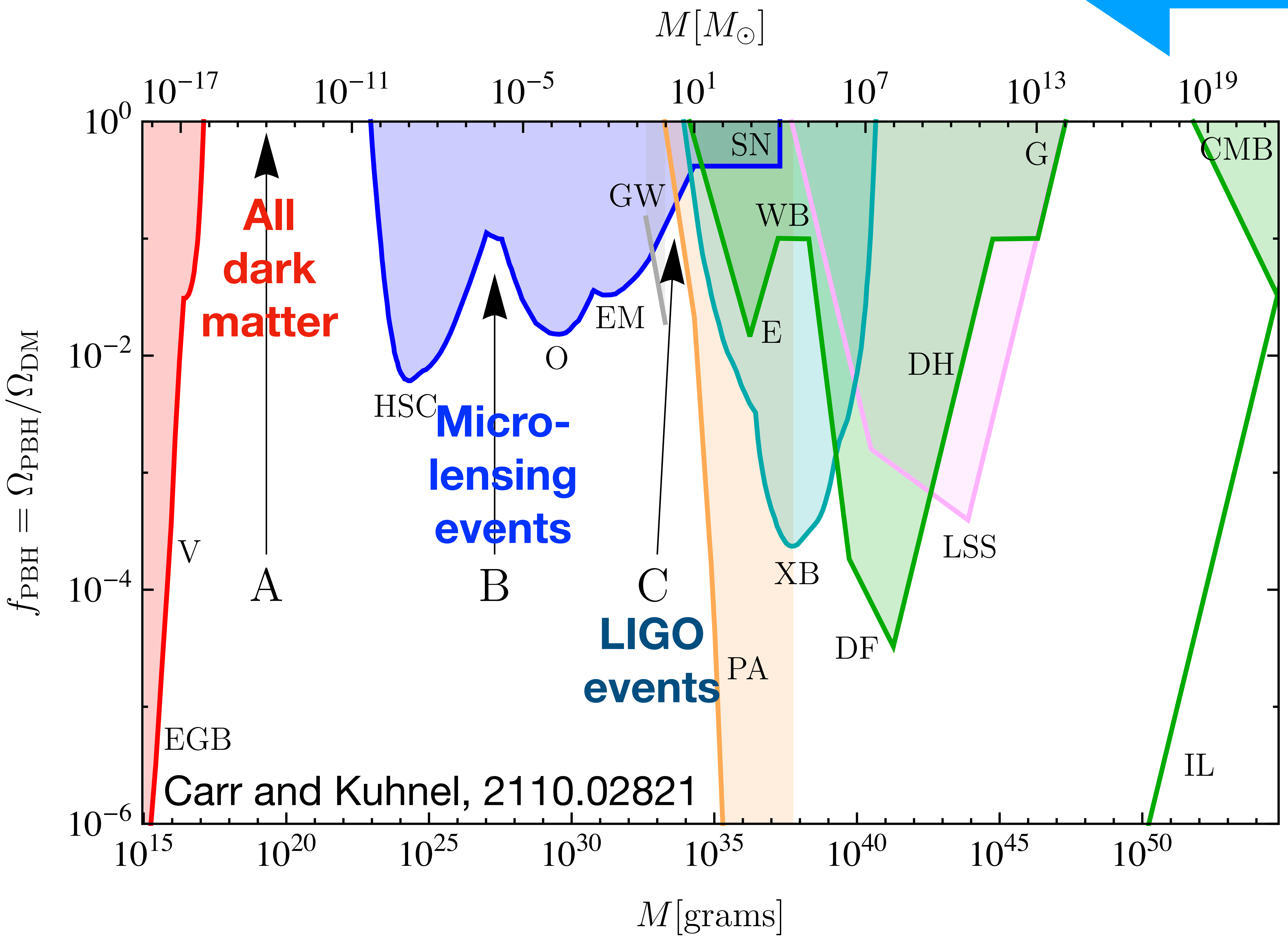
PBH-IGW crosscheck

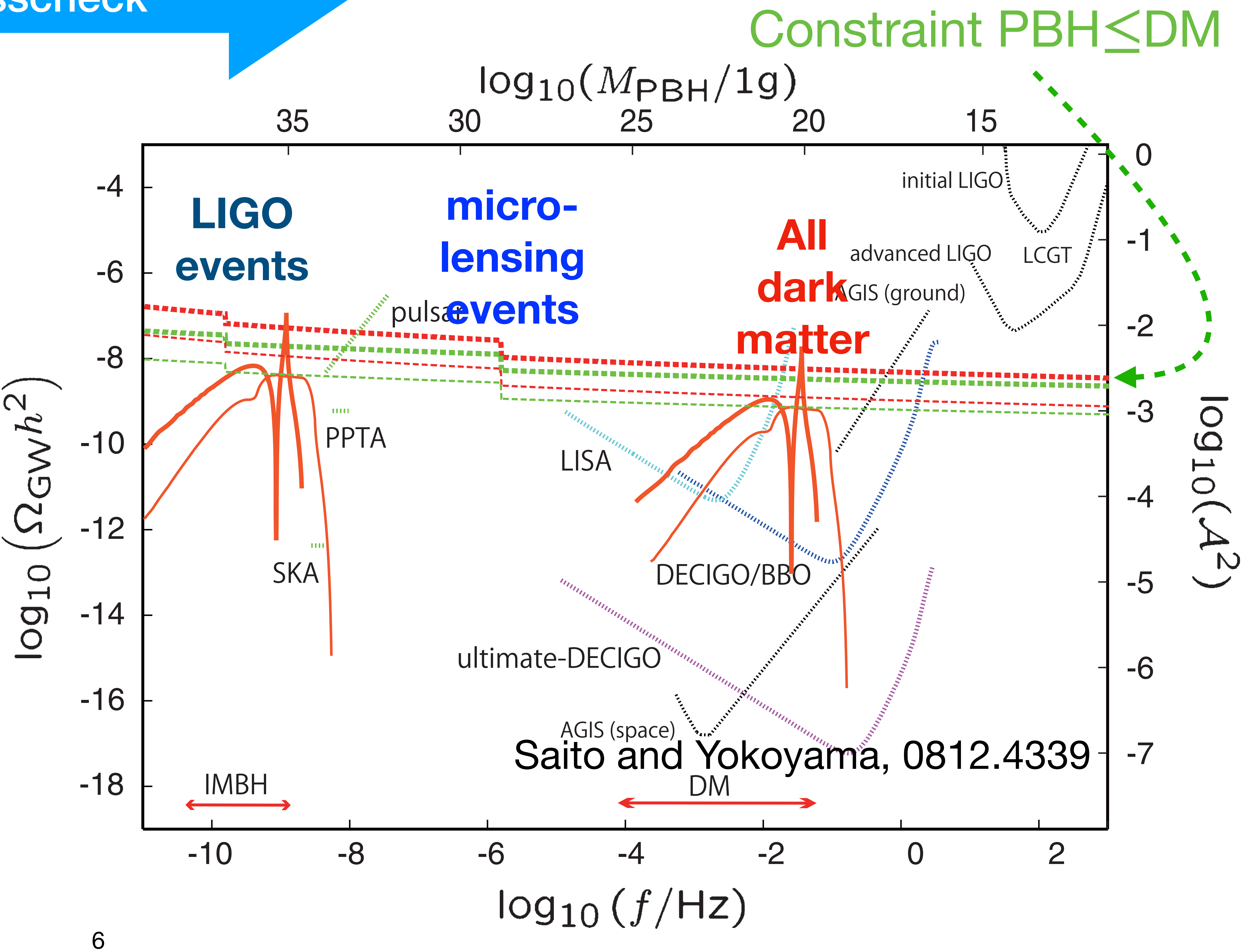
$$f_{\mathrm{IGW}} \sim 3\mathrm{Hz}\left(\frac{M_{\mathrm{PBH}}}{10^{16}\mathrm{g}}\right)^{-\frac{1}{2}}$$

PBH constraints

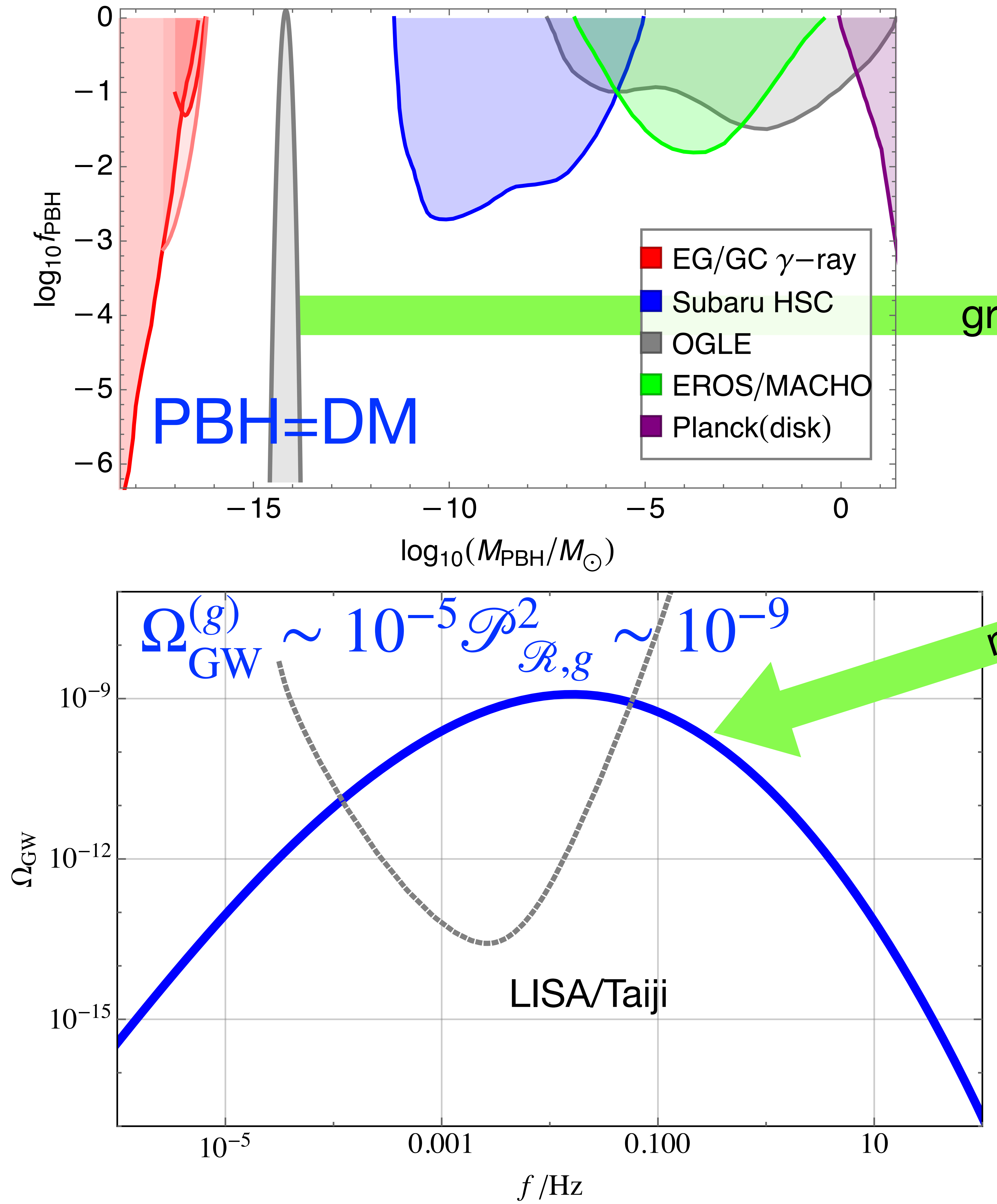
crosscheck

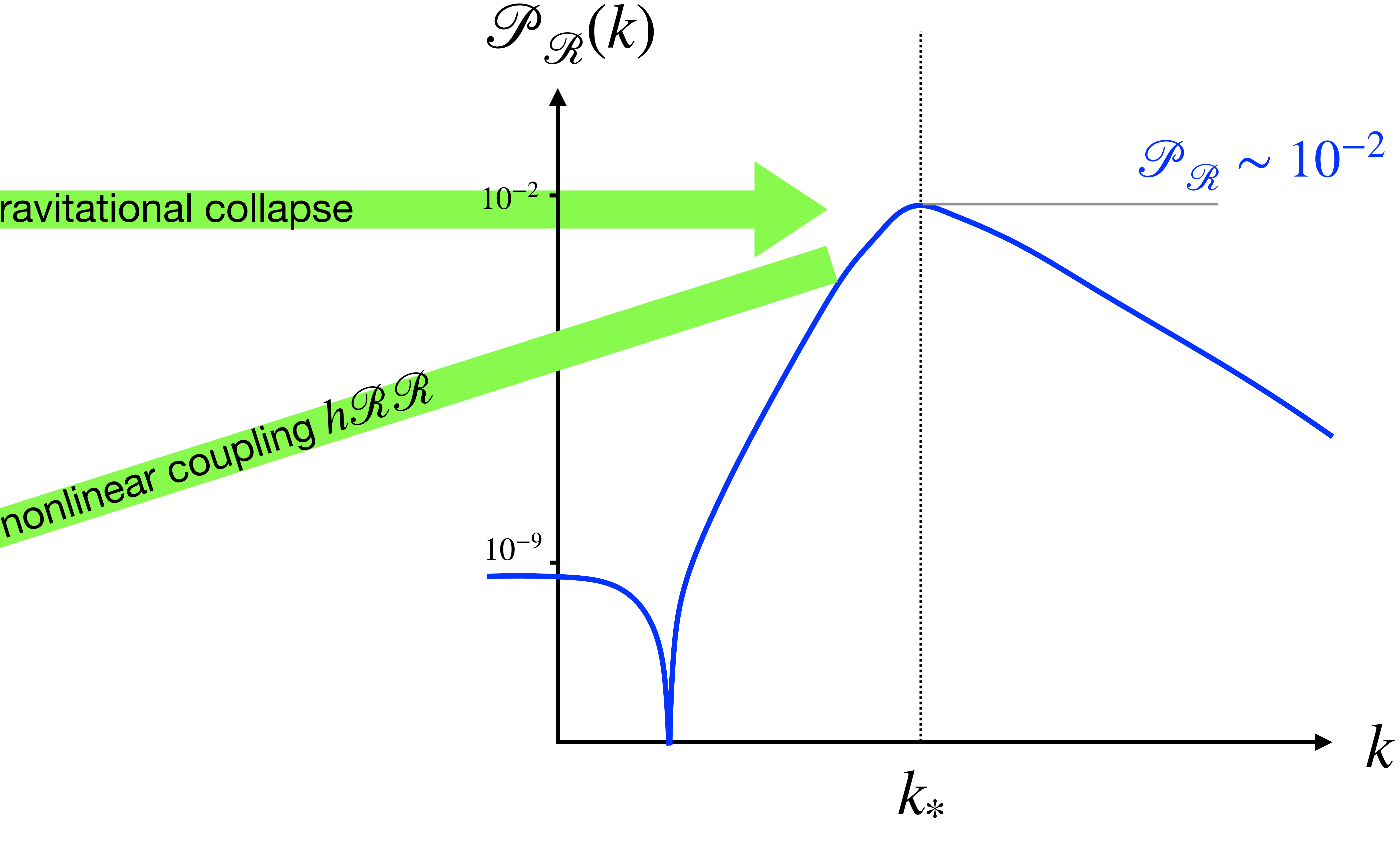
Induced GW predictions





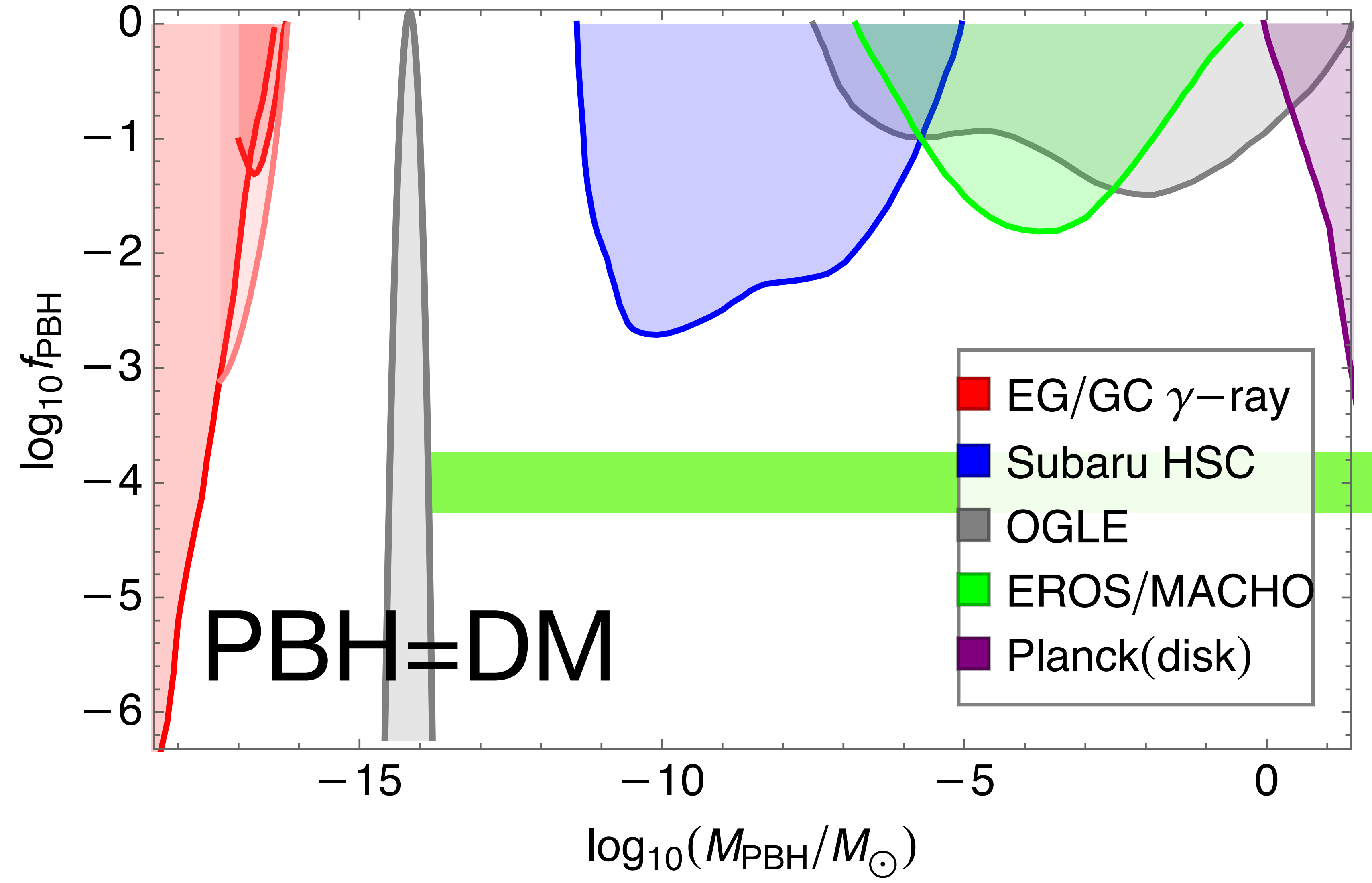
PBH-IGW crosscheck





Saito & Yokoyama 0812.4339; 0912.5317
Bugaev & Klimai 0908.0664; 1012.4697
Escriva et al, 2211.05767

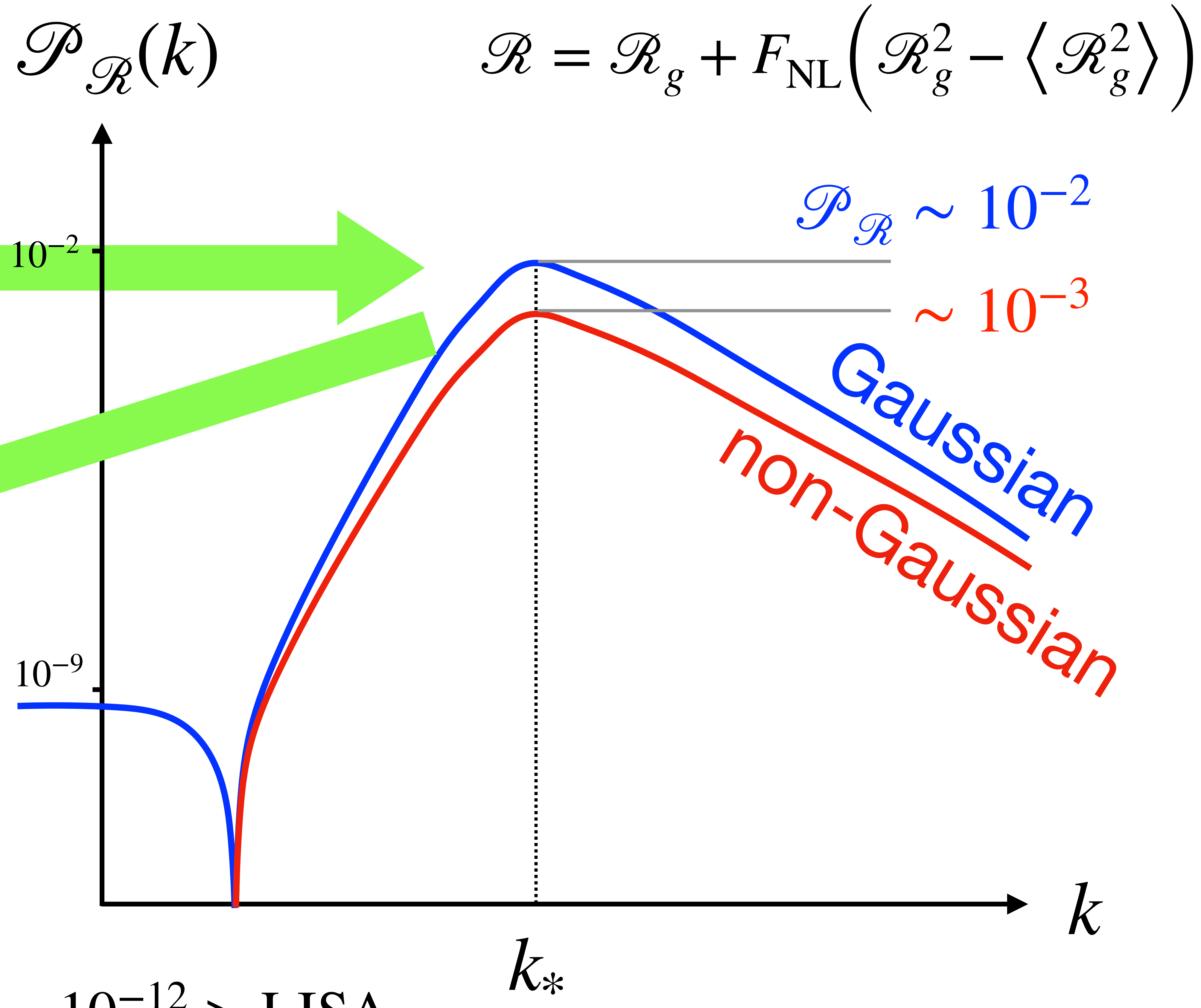
Including non-Gaussianity

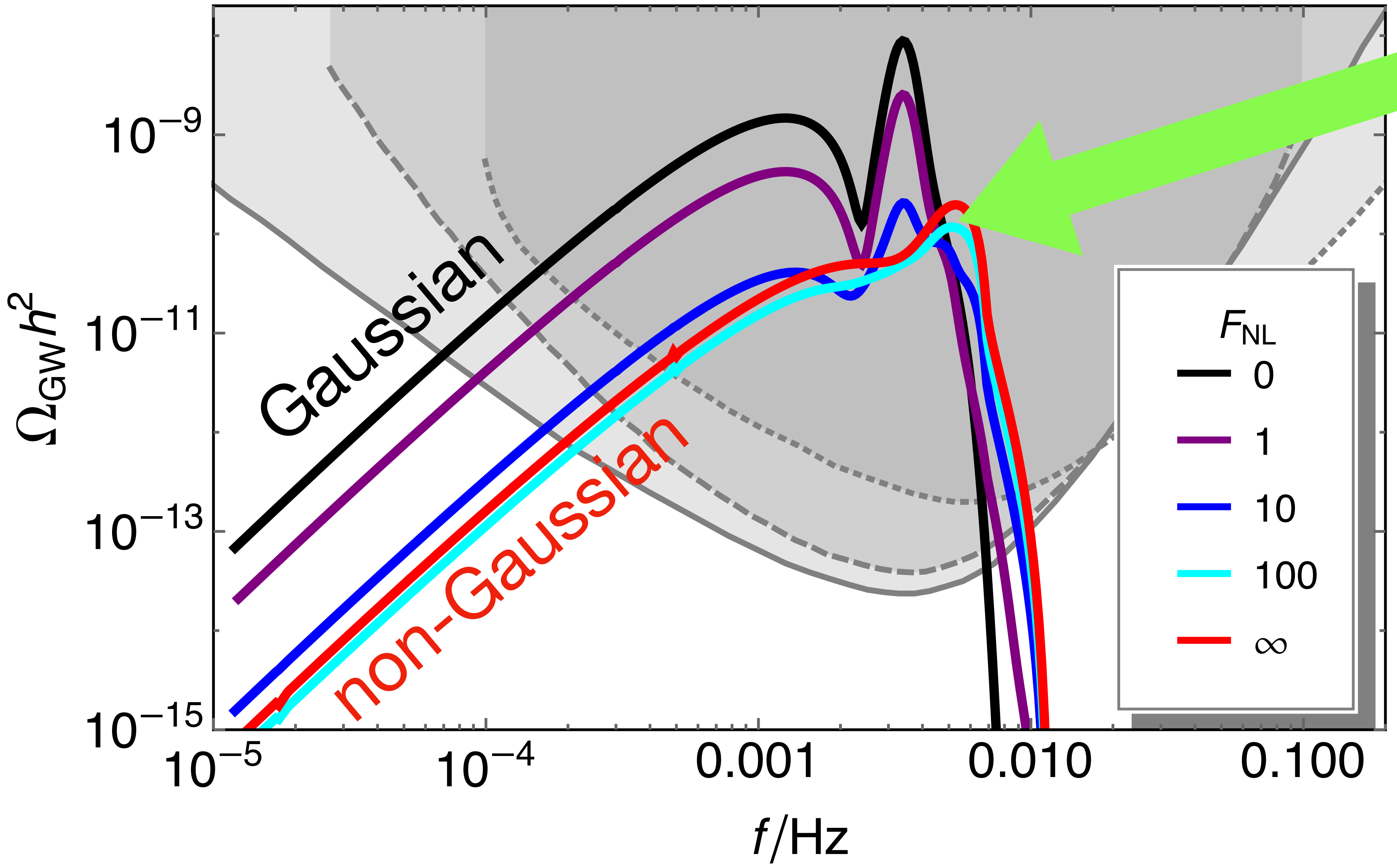


Press-Schechter

nonlinear coupling $h\mathscr{R}\mathscr{R}$

with $F_{NL} > 0$





$F_{NL}\mathscr{P}_{\mathscr{R}} \sim 10^{-2}$

$\Omega_{GW}^{(NG)} \sim 10^{-5} F_{NL}^4 \mathscr{P}_{\mathscr{R},g}^4 \sim 10^{-12} > \text{LISA}$

LISA/Taiji/TianQin can probe PBH-DM.

Cai, SP, Sasaki, 1810.11000

(Simplest) Press-Schechter

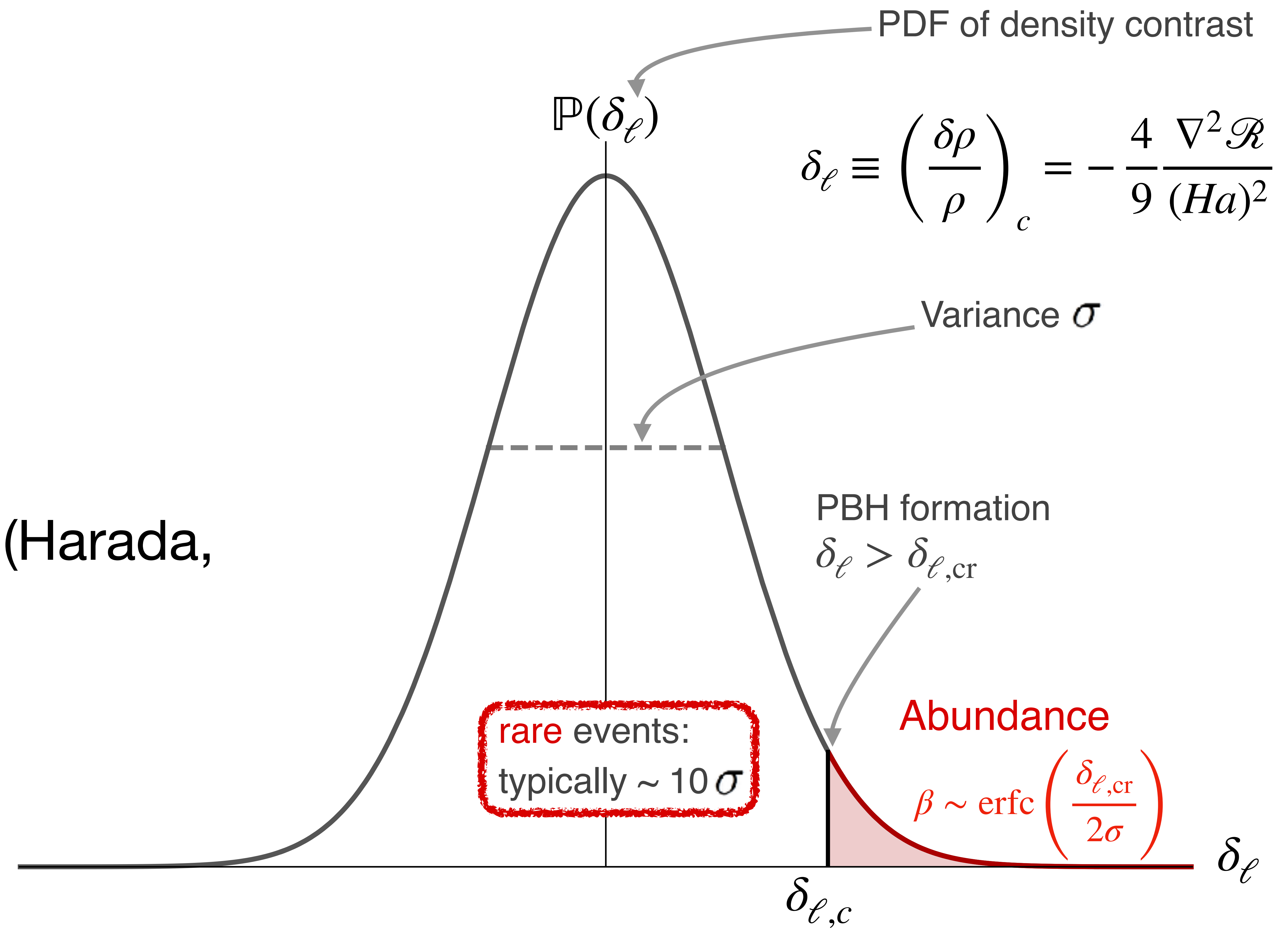
$$\begin{rmatrix} \mathscr{R} \xrightarrow{(1)} \delta_\ell \\ \mathbb{P}(\mathscr{R}) \xrightarrow{(2)} \mathbb{P}(\delta_\ell) \end{rmatrix}$$

$$\left.\begin{matrix} \mathscr{R} \xrightarrow{\quad(1)\quad} \delta_\ell \\ \mathbb{P}(\mathscr{R}) \xrightarrow{\quad(2)\quad} \mathbb{P}(\delta_\ell) \end{matrix}\right\} \xrightarrow[\text{(4) Window function}]{\text{(3) given } \delta_{\ell,\mathrm{cr}}} \beta = \int_{\delta_{\ell,\mathrm{cr}}} \mathbb{P}(\delta_\ell)\frac{M(\delta_\ell)}{M_H}\mathrm{d}\delta_\ell$$

Every step is linear/Gaussian:

- (1) Linear Poisson equation.
- (2) Gaussian PDF $\mathbb{P}(\mathscr{R})$ gives Gauss PDF $\mathbb{P}(\delta_\ell)$:

  $\mathbb{P}(\mathscr{R})d\mathscr{R} = \mathbb{P}(\delta_\ell)d\delta_\ell$
- (3) Critical density contrast $\delta_{\ell,\mathrm{cr}}$ given by HYK limit (Harada, Yoo, Kohri, 1309.4201).
- (4) Window function.



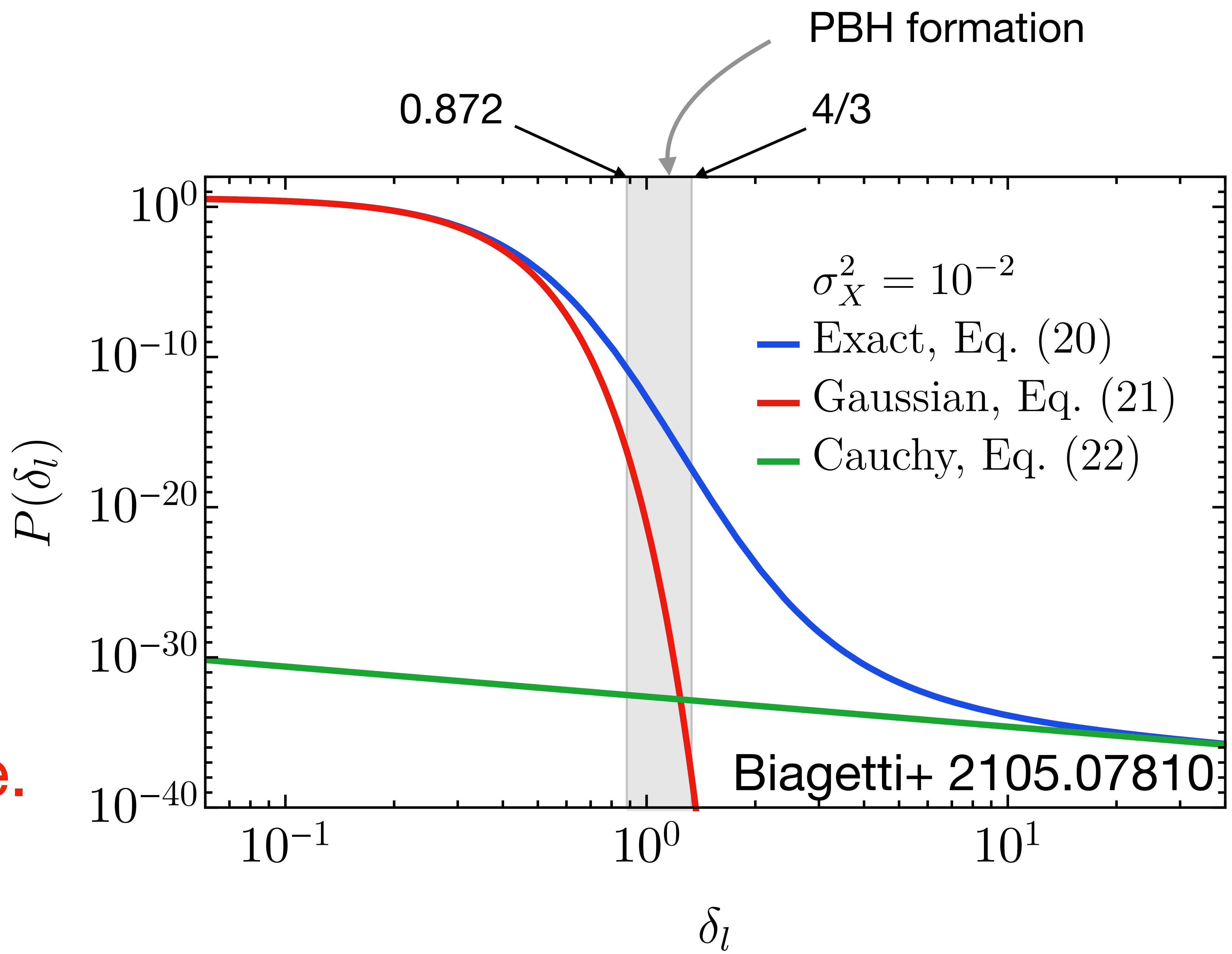
Why non-Gaussianity?

$$\left.\begin{array}{l}\mathscr{R} \xrightarrow{(1)} \mathscr{C}_\ell \\ \mathbb{P}(\mathscr{R}) \xrightarrow{(2)} \mathbb{P}(\mathscr{C}_\ell)\end{array}\right\} \xrightarrow[\text{(4) Window function}]{\text{(3) given } \mathscr{C}_{\rm cr}} \beta = \int_{\mathscr{C}_{\ell,\rm cr}}^{4/3} \mathbb{P}(\mathscr{C}_\ell)\frac{M(\mathscr{C}_\ell)}{M_H}\mathrm{d}\mathscr{C}_\ell$$

Non-Gaussianity must be taken into account:

- (1) Use compaction function $\mathscr{C}$ which nonlinearly depends on $\mathscr{R}$. (Harada et al 1503.03934; De Luca et al 1904.00970.)
- (2) Primordial non-Gaussianity of $\mathscr{R}$.
- (3) $\mathscr{C}_{\rm cr}$ depends on profile. (Musco 1809.02127; Escrivà et al 1907.13311)
- (4) Window function

Remaining caveat: Profile.



Peak Theory

- Instead, in peak theory, BBKS gives the profile of a local peak, from which the critical value of $\overline{\mathscr{C}_c}$ can be calculated analytically. Then we transfer it to the critical value of the Laplacian of the curvature perturbation, μ_2.

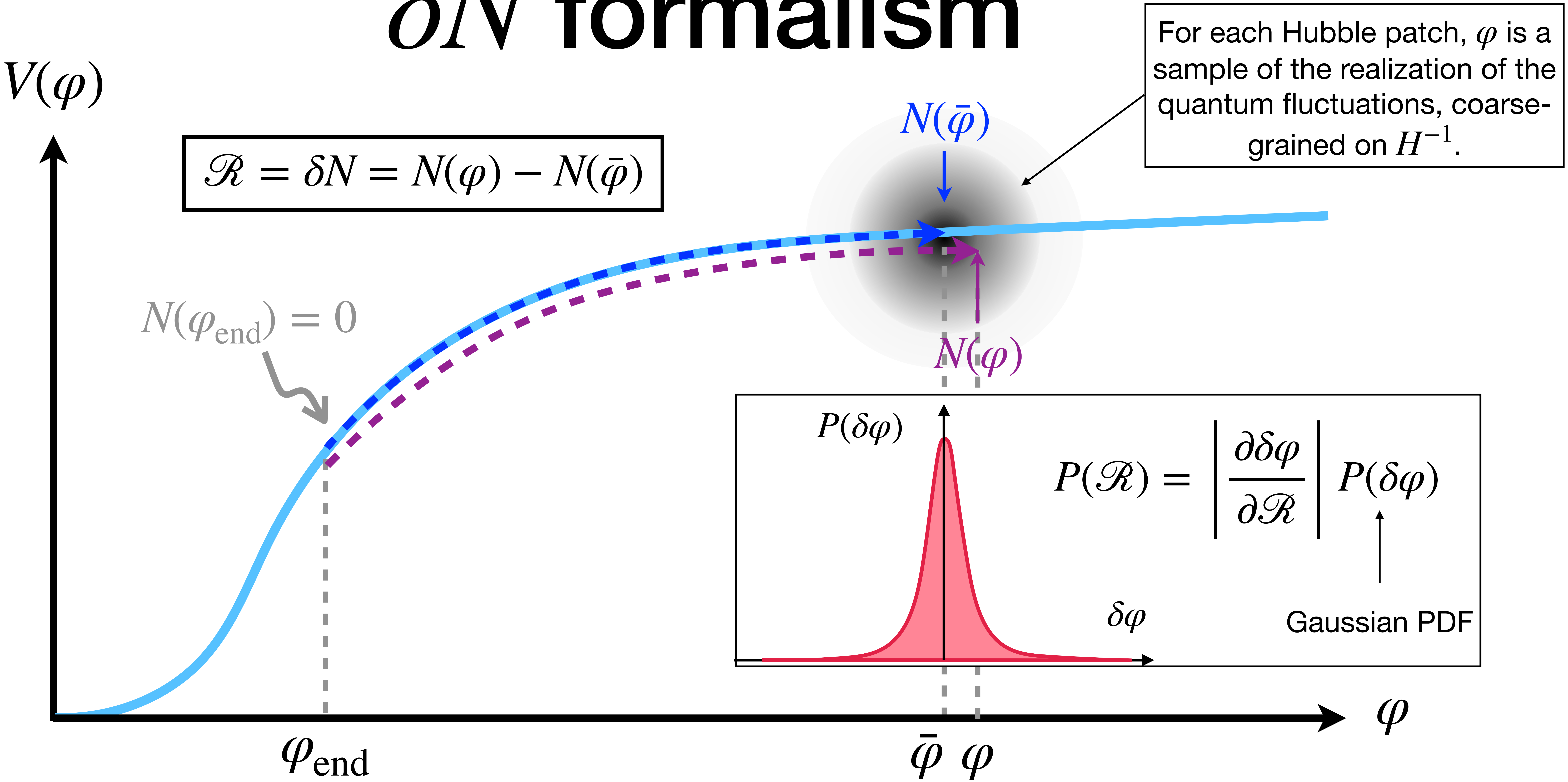
- The statistic quantities are μ_2 and its dispersion, μ_4.

- The PBH mass function is then

$$\beta(M) = \int_{\mu_2 \geq \mu_{2,th}} d\mu_2 d\mu_4 \cdot n_{\text{peak}}(\mu_2(M, \mu_4), \mu_4) \left| \frac{d \ln M}{d\mu_2} \right|^{-1} M(\mu_2, \mu_4)$$

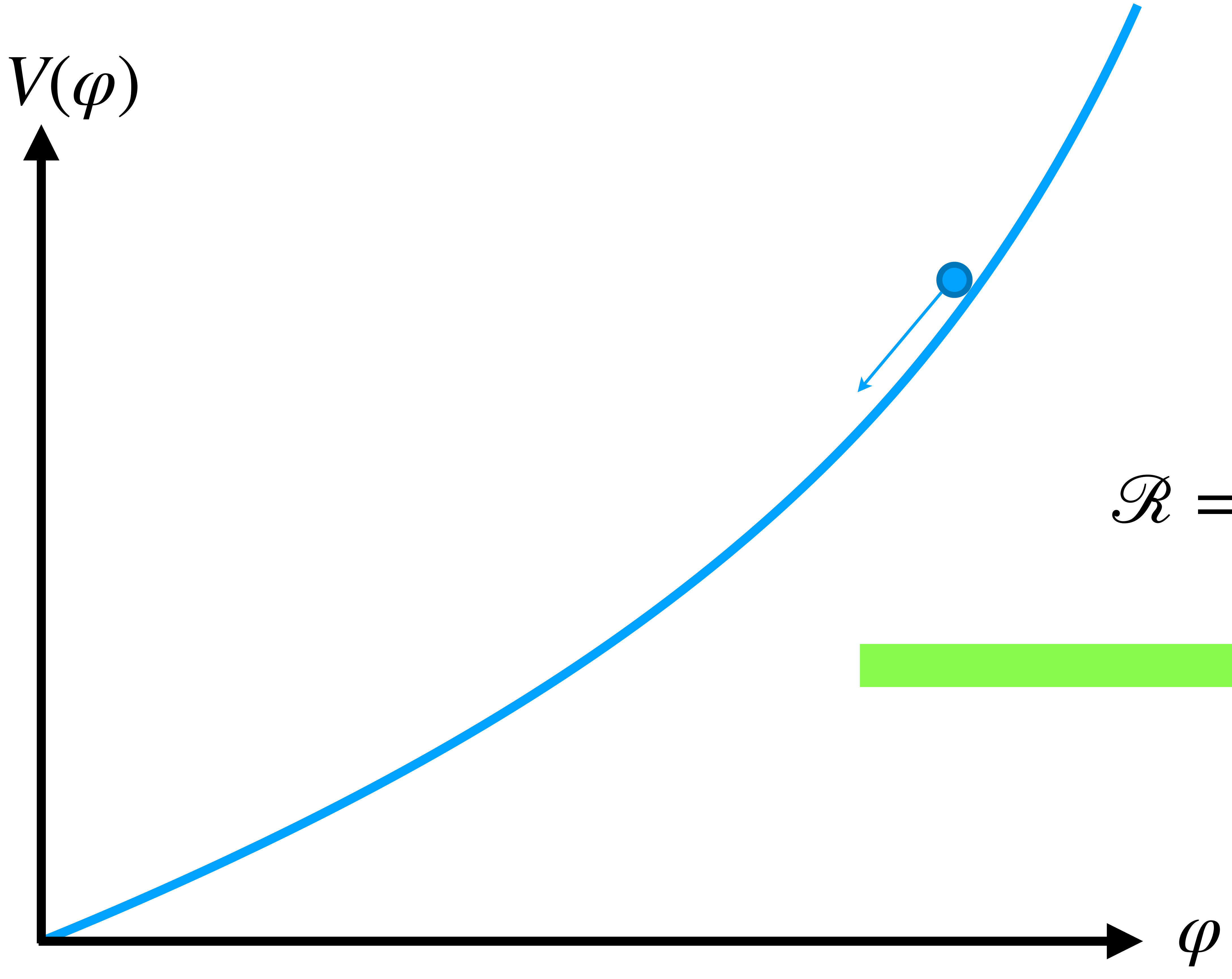
Kitajima et al, 2109.00791
SP, Sasaki, Takhistov, Jianing Wang, in prep

δN formalism

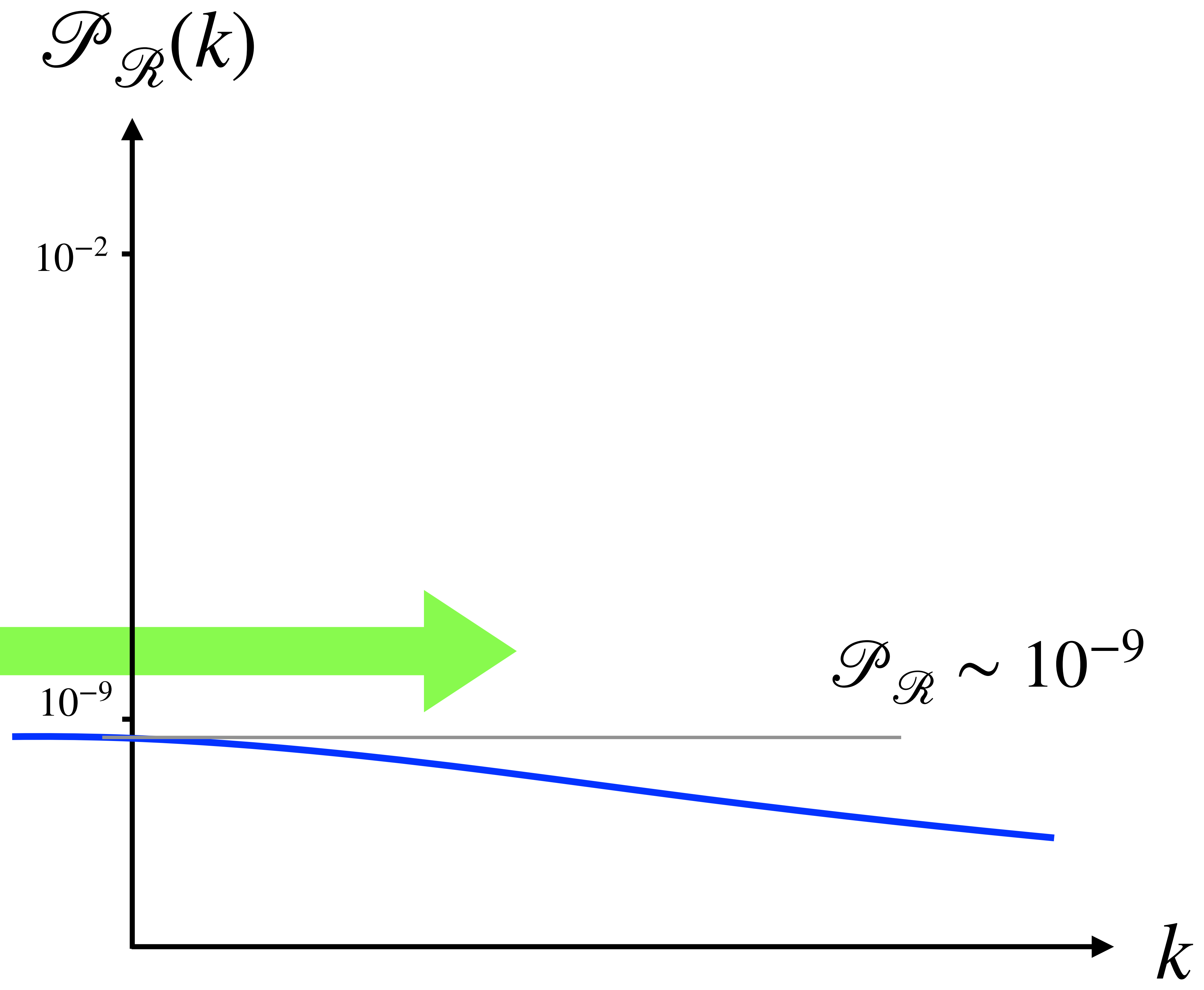


Sasaki and Stewart, astro-ph/9507001
Lyth, Malik, Sasaki, astro-ph/0411220

Gaussian Curvature Perturbation

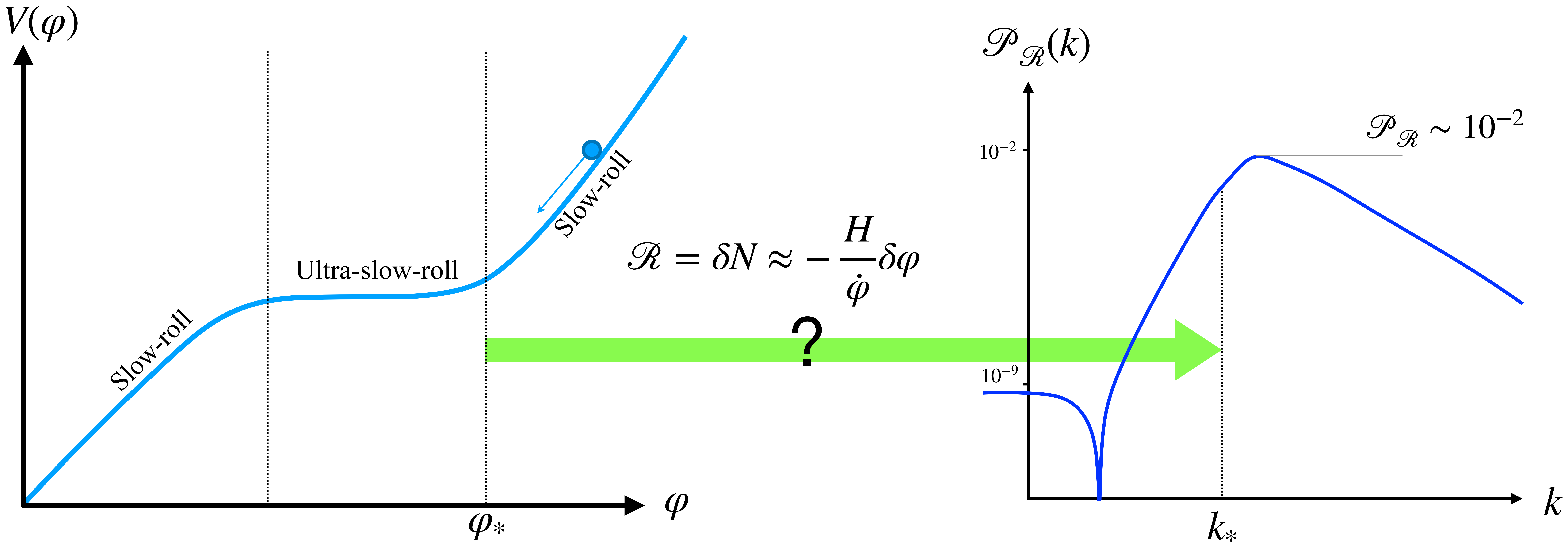


$$\mathcal{R} = \delta N \approx -\frac{H}{\dot{\varphi}}\delta\varphi$$



Stewart and Sasaki, astro-ph/9507001
Lyth and Roquigez, astro-ph/0504045

Ultra-slow-roll Inflation



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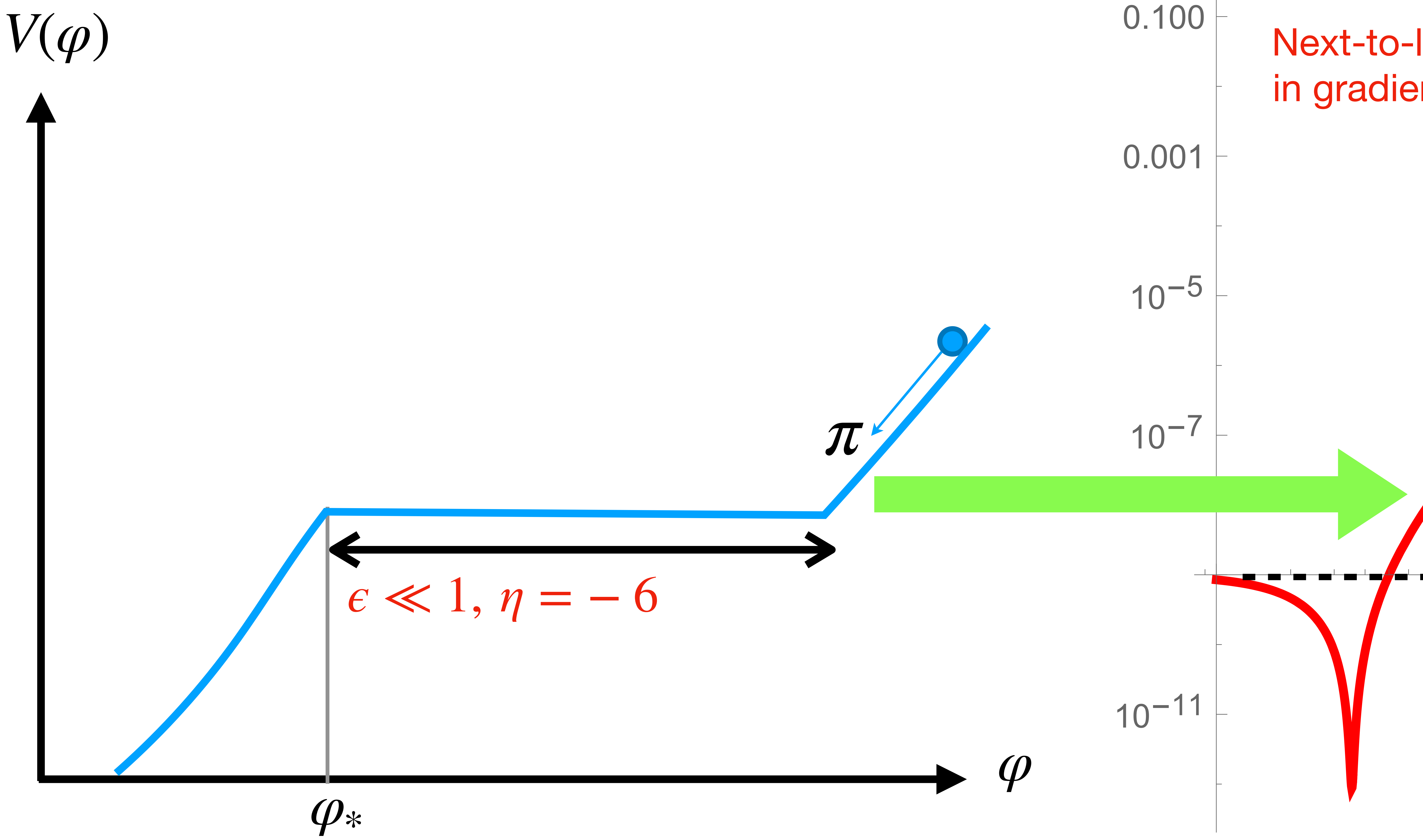
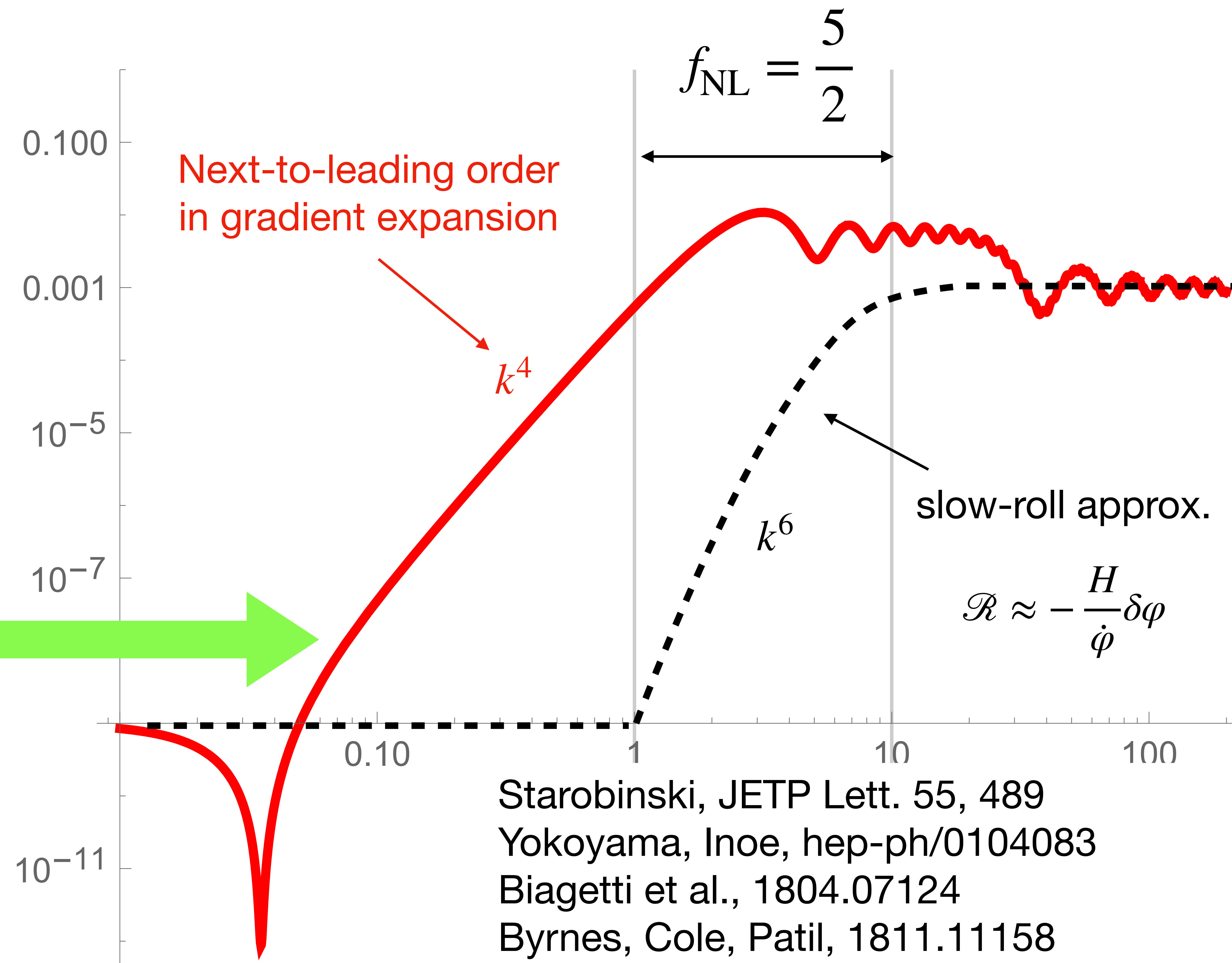
Starobinski, JETP Lett. 55, 489
Byrnes, Cole, Patil, 1811.11158
Cole, Gow, Byrnes, Patil, 2204.07573
SP & Jianing Wang, 2209.14183

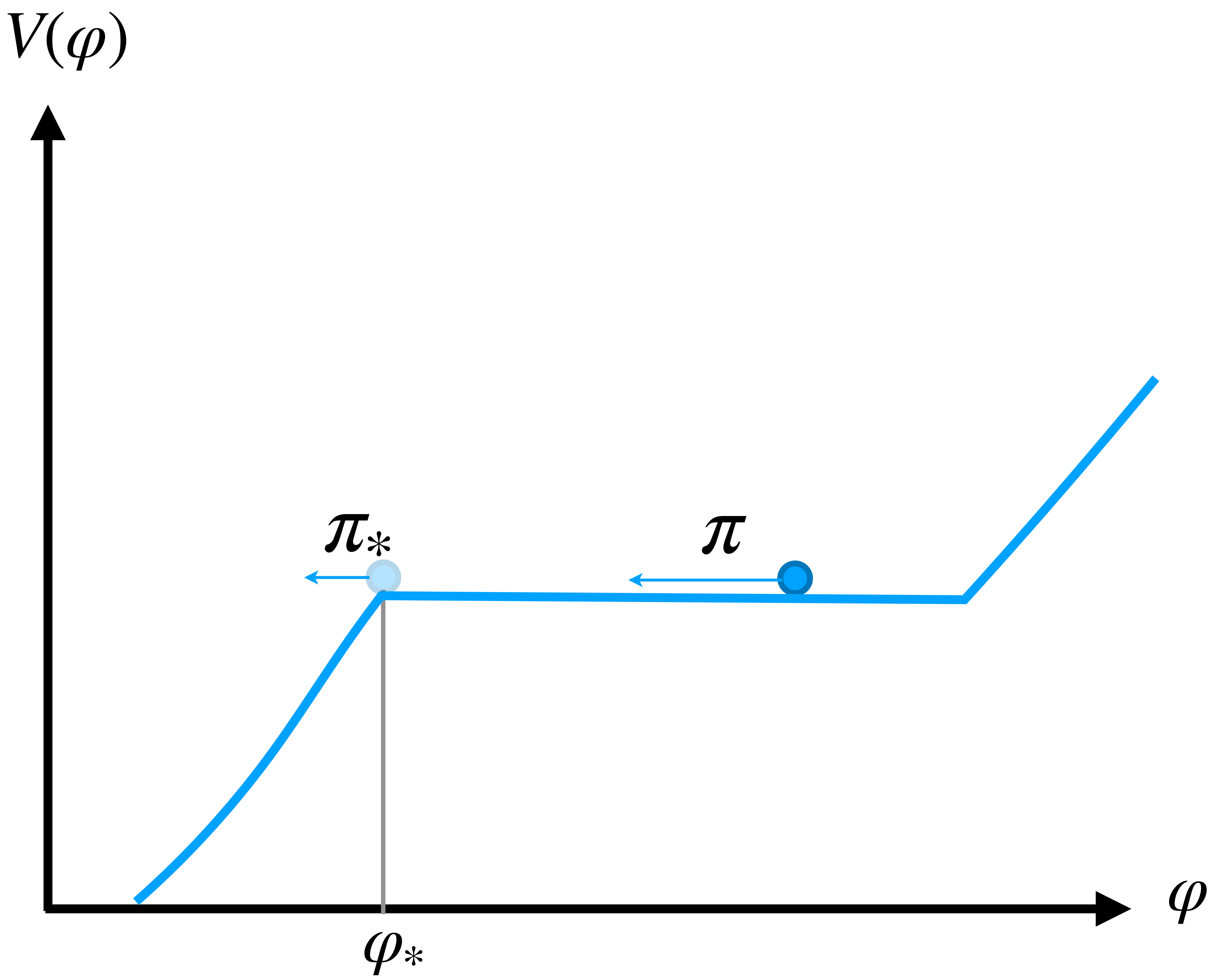
Ultra-slow-roll inflation





Starobinski, JETP Lett. 55, 489
Yokoyama, Inoe, hep-ph/0104083
Biagetti et al., 1804.07124
Byrnes, Cole, Patil, 1811.11158
Cole, Gow, Byrnes, Patil, 2204.07573
SP, Jianing Wang, 2209.14183
Domenech, Vargas, Vargas, 2309.05750

Ultra-slow-roll inflation



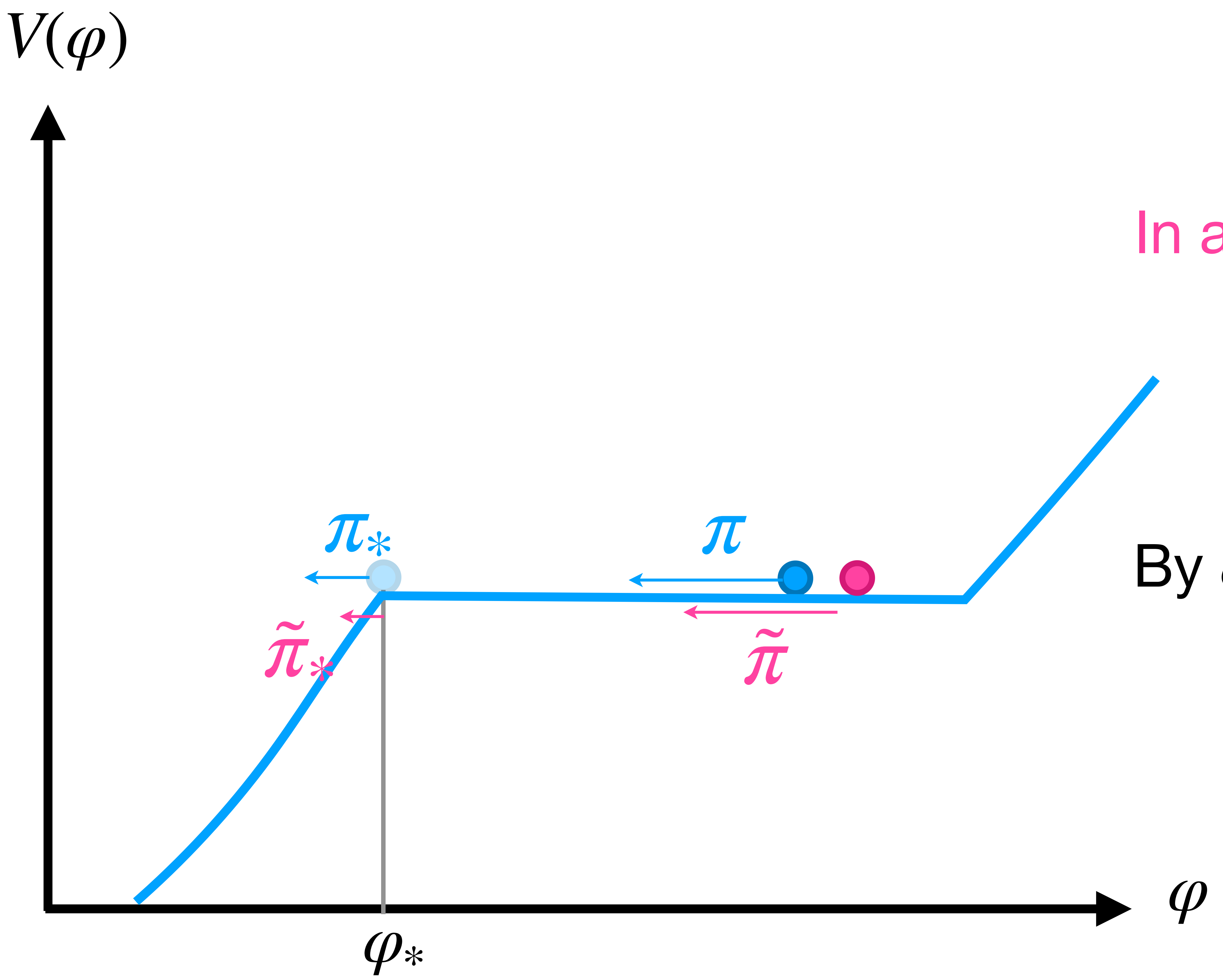
$$\frac{\mathrm{d}^2\varphi}{\mathrm{d}N^2} - 3\frac{\mathrm{d}\varphi}{\mathrm{d}N} = 0 \qquad N = \int_{t_*}^{t} H\mathrm{d}t$$

$$\varphi(N) = \varphi_* + \frac{\pi_*}{3}\left(1 - e^{3N}\right)$$

$$\pi(N) \equiv -\frac{\mathrm{d}\varphi}{\mathrm{d}N} = \pi_* e^{3N}$$

$$N = -\frac{1}{3}\ln\frac{\pi_*}{\pi}$$

Ultra-slow-roll inflation



In the “fiducial” patch

$$N = -\frac{1}{3}\ln\frac{\pi_*}{\pi}$$

In a perturbed patch

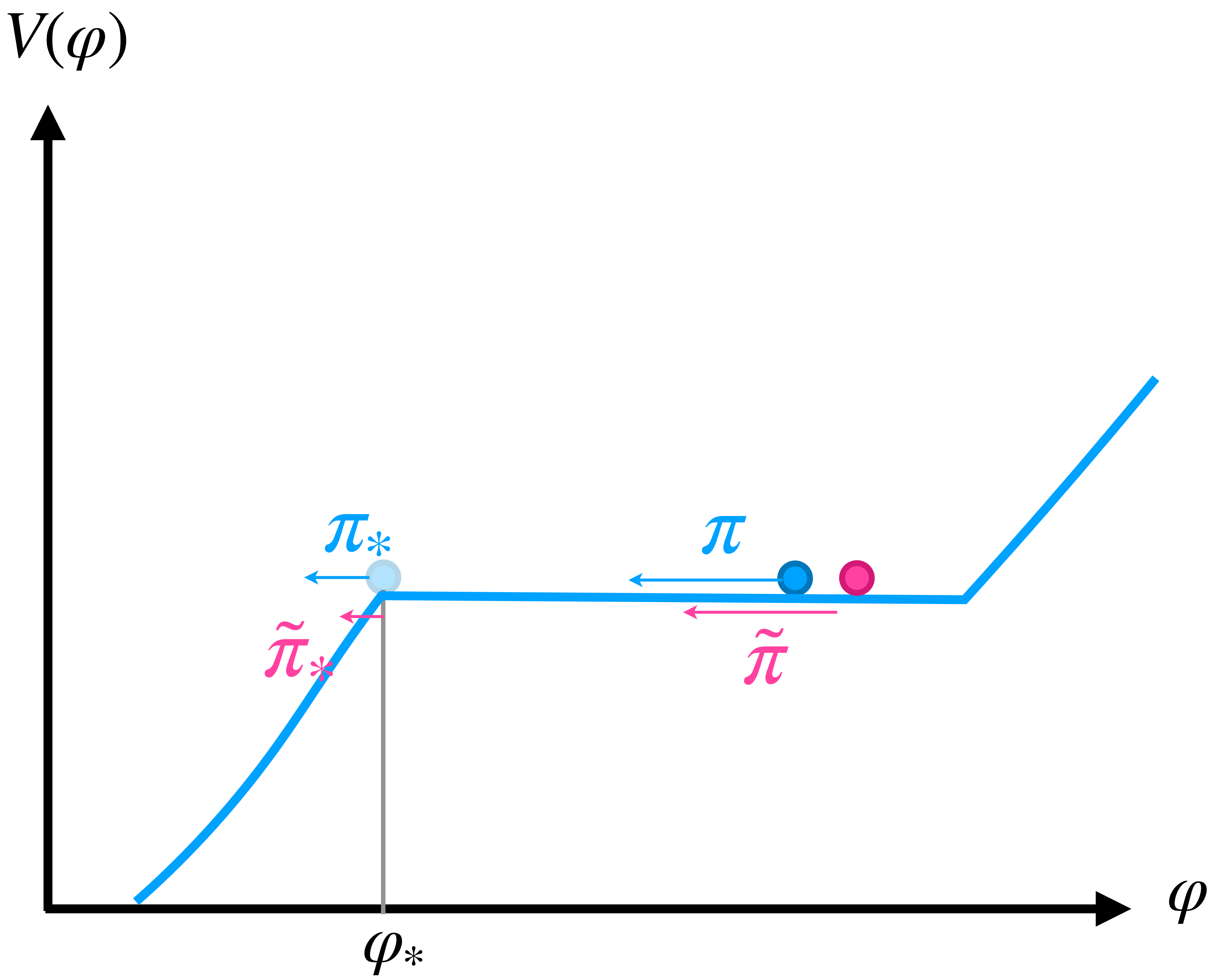
$$\tilde{N} = -\frac{1}{3}\ln\frac{\tilde{\pi}_*}{\tilde{\pi}}$$

By δN formalism, the curvature perturbation is

$$\mathcal{R} = \delta N = \tilde{N} - N = \frac{1}{3}\ln\frac{\tilde{\pi}}{\pi}\frac{\pi_*}{\tilde{\pi}_*}$$

$$= \frac{1}{3}\ln\left(1 - \frac{\delta\pi}{\pi}\right) - \frac{1}{3}\ln\left(1 - \frac{\delta\pi_*}{\pi_*}\right)$$

Ultra-slow-roll inflation

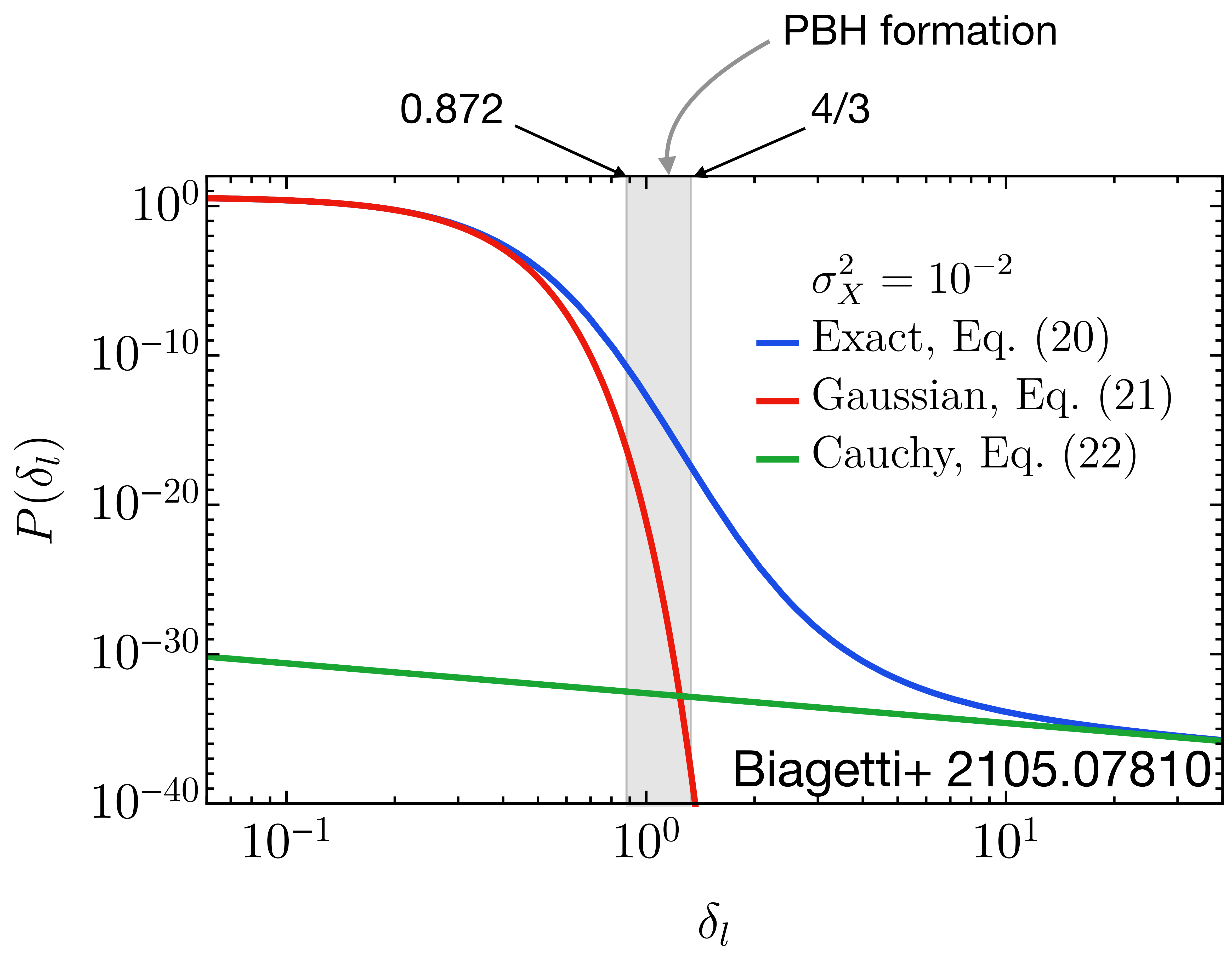


$$\mathscr{R} = -\frac{1}{3}\ln\left(1 - \frac{\delta\pi_*}{\pi_*}\right)$$

$$\left(f_{\rm NL} = \frac{5}{2}, \quad g_{\rm NL} = -\frac{25}{3}, \ \cdots\right)$$

Namjoo, Firouzjahi, Sasaki, 1210.3692
Chen, Firouzjahi, Komatsu, Namjoo, Sasaki, 1308.5341
Cai, Chen, Namjoo, Sasaki, Wang, Wang, 1712.09998
Biagetti, Franciolini, Kehagias, Riotto, 1804.07124
Passaglia, Hu, Motohashi, 1812.08243
SP and Sasaki, 2211.13932
SP, 2404.06151

Ultra-slow-roll inflation



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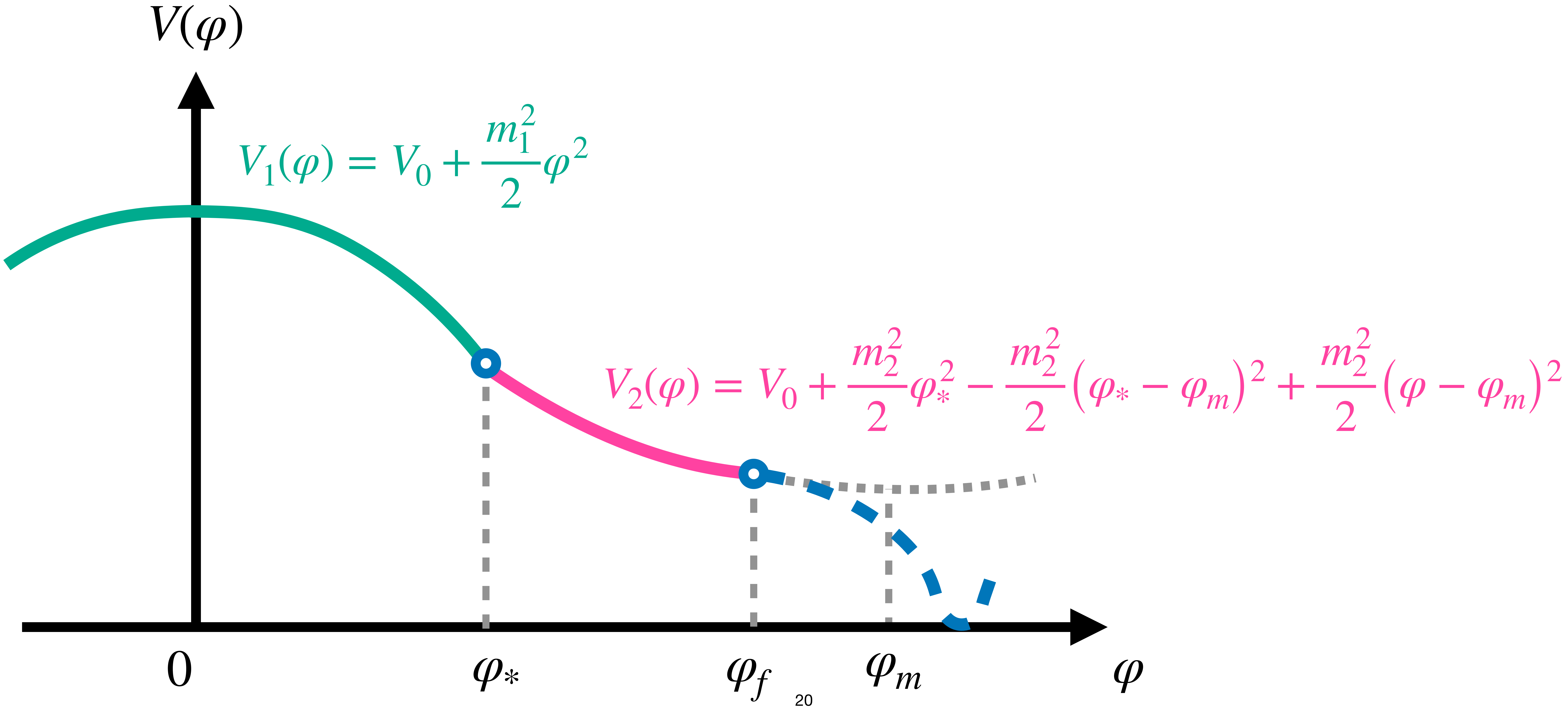
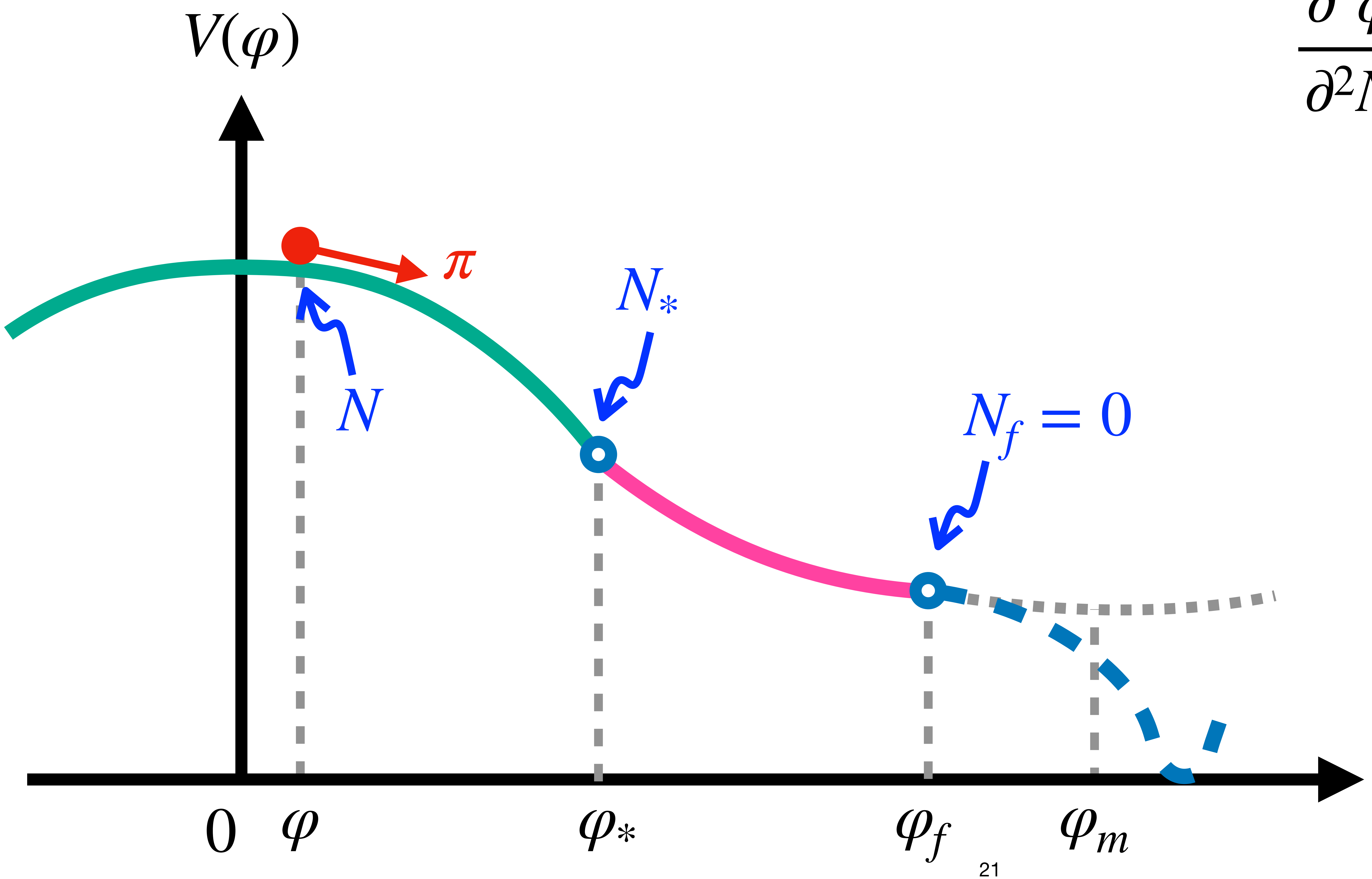
Biagetti, Franciolini, Kehagias, Riotto, 1804.07124
Passaglia, Hu, Motohashi, 1812.08243
SP and Sasaki, 2211.13932
SP, 2404.06151
This exponential tail is basically different from stochastic approach, see e.g.
Jackson et al 2410.13683, Cruces et al 2410.17987
Cruces, SP, Sasaki in prep

piecewise quadratic potential



piecewise quadratic potential



$$\frac{\partial^2\varphi}{\partial^2 N} - 3\frac{\partial\varphi}{\partial N} + 3\eta_V\varphi = 0$$

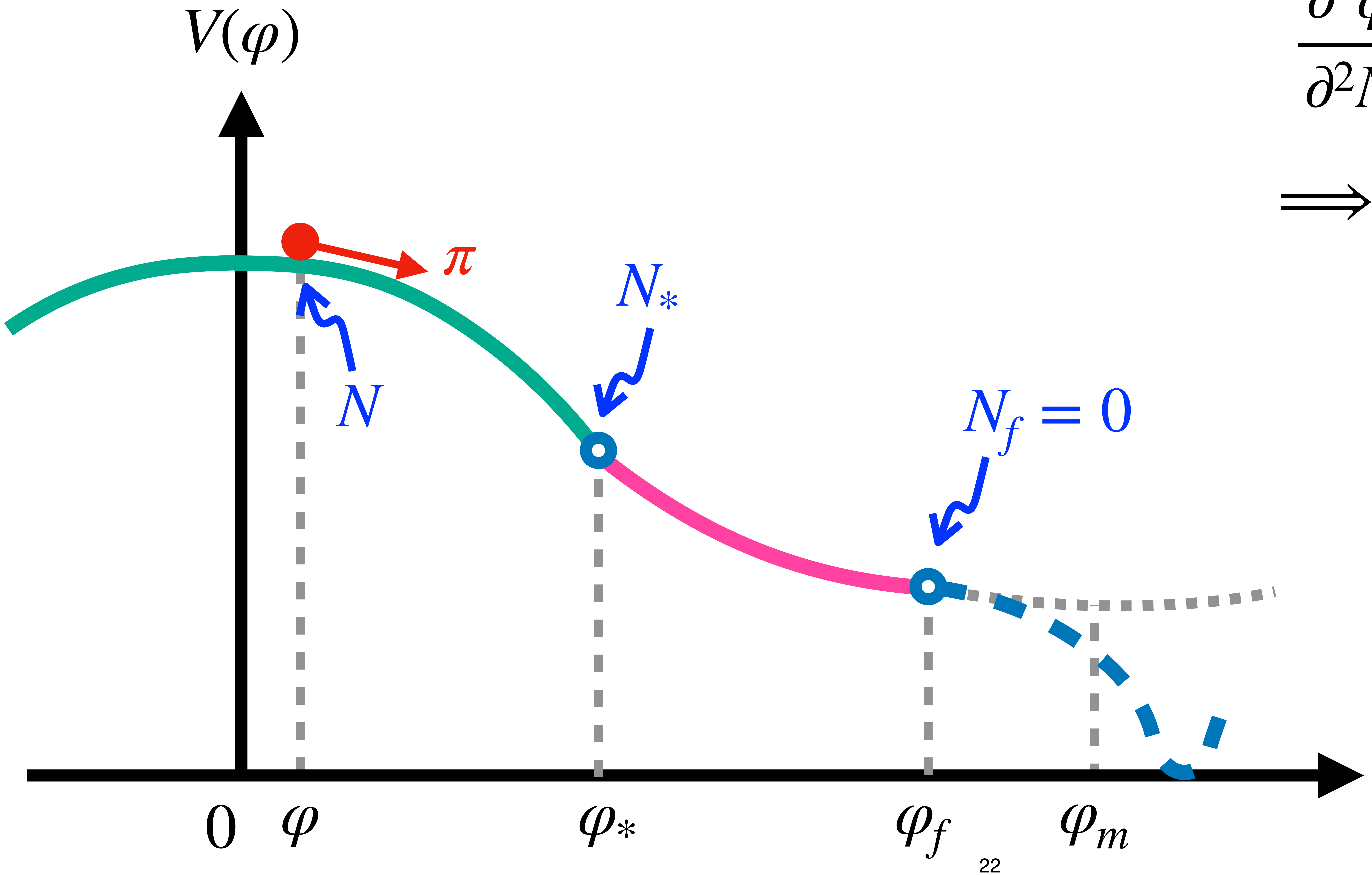
$$\eta_V = \frac{m_1^2}{3H^2}$$

$$N = \int_t^{t_f} H dt$$

$$V_1(\varphi) = V_0 + \frac{m_1^2}{2}\varphi^2$$

$$V_2(\varphi) = V_0 + \frac{m_2^2}{2}\varphi_*^2 - \frac{m_2^2}{2}(\varphi_* - \varphi_m)^2 + \frac{m_2^2}{2}(\varphi - \varphi_m)^2$$

background solution



$$\frac{\partial^2\varphi}{\partial^2 N} - 3\frac{\partial\varphi}{\partial N} + 3\eta_V\varphi = 0$$

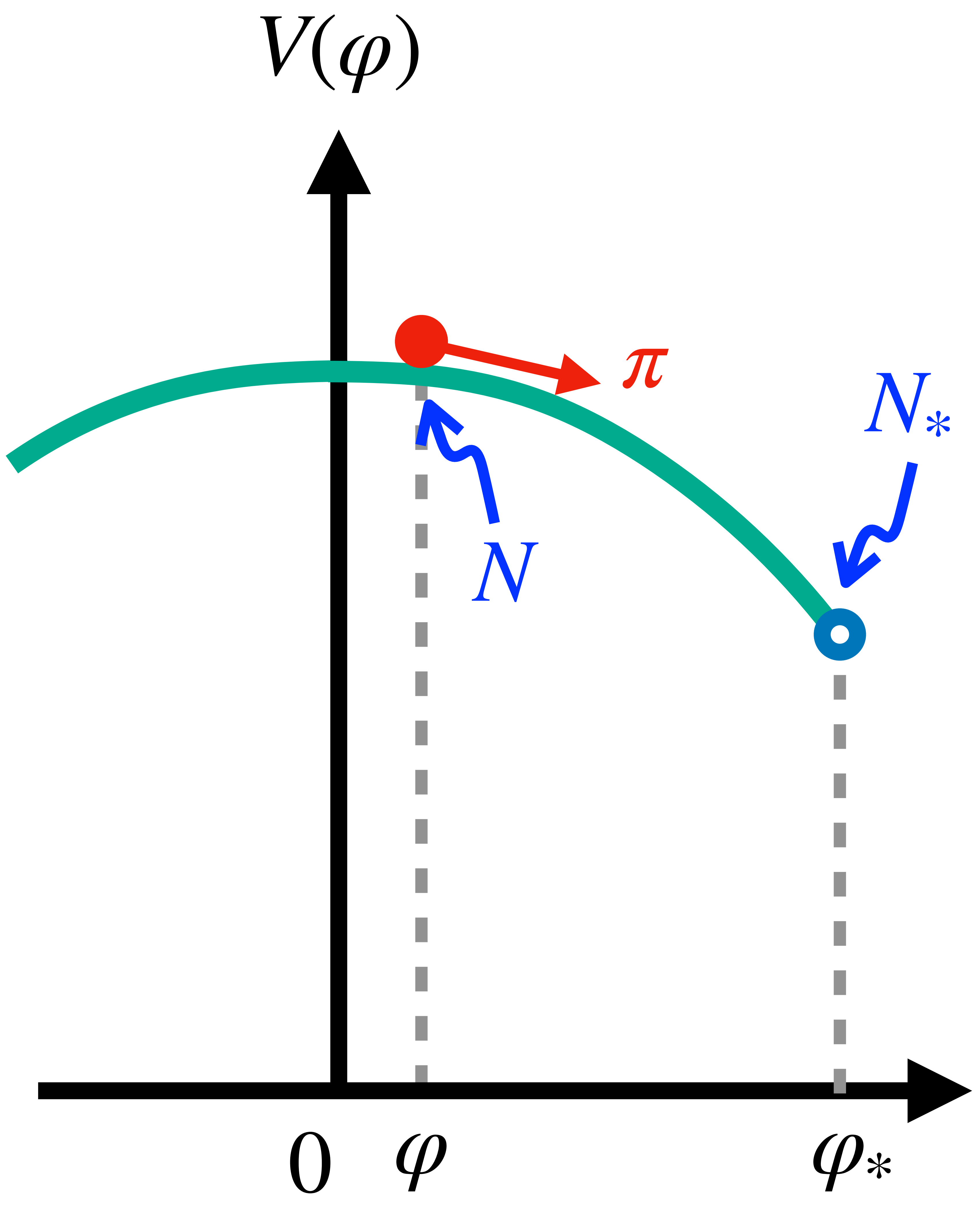
$$\Longrightarrow \quad \varphi = c_+ e^{\lambda_+ N} + c_- e^{\lambda_- N}$$

$$\lambda_\pm = \frac{3 \pm \sqrt{9 - 12\eta_V}}{2}$$

$$V_1(\varphi) = V_0 + \frac{m_1^2}{2}\varphi^2$$

$$V_2(\varphi) = V_0 + \frac{m_2^2}{2}\varphi_*^2 - \frac{m_2^2}{2}(\varphi_* - \varphi_m)^2 + \frac{m_2^2}{2}(\varphi - \varphi_m)^2$$

background solution



$$\varphi(N) = c_+ e^{\lambda_+(N-N_*)} + c_- e^{\lambda_-(N-N_*)}$$

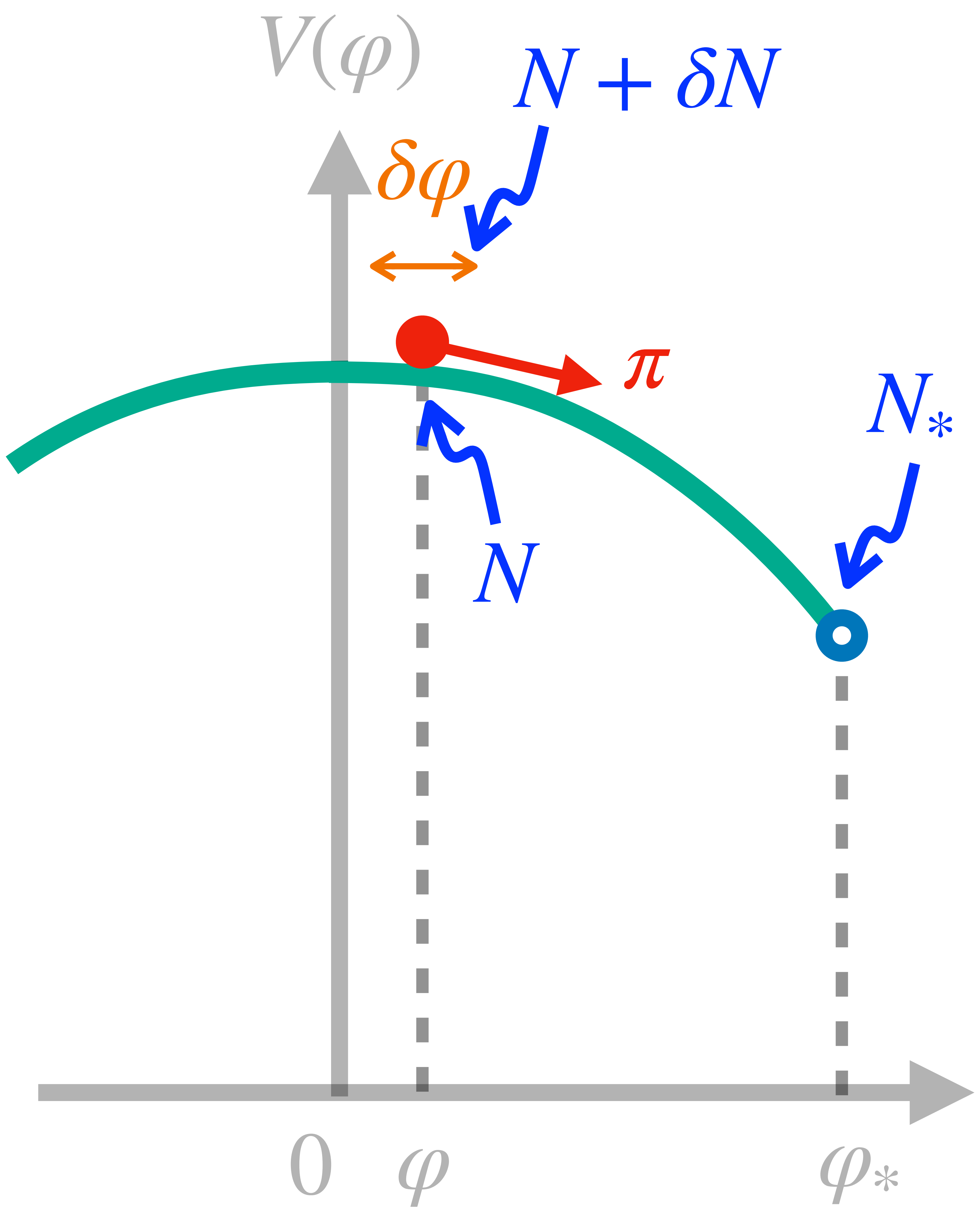
$$-\pi(N) \equiv \frac{\partial\varphi}{\partial N} = \lambda_+ c_+ e^{\lambda_+(N-N_*)} + \lambda_- c_- e^{\lambda_-(N-N_*)}$$

$$\varphi(N_*) \equiv \varphi_* = c_+ + c_-$$

$$-\pi(N_*) \equiv \pi_* = \lambda_+ c_+ + \lambda_- c_-$$

$$\Longrightarrow \quad c_\pm = \mp \frac{\pi_* + \lambda_\mp \varphi_*}{\lambda_+ - \lambda_-}$$

Logarithmic Duality



The (fiducial) e-folding number can be expressed by (φ, π) and their values on the boundary (φ_*, π_*).

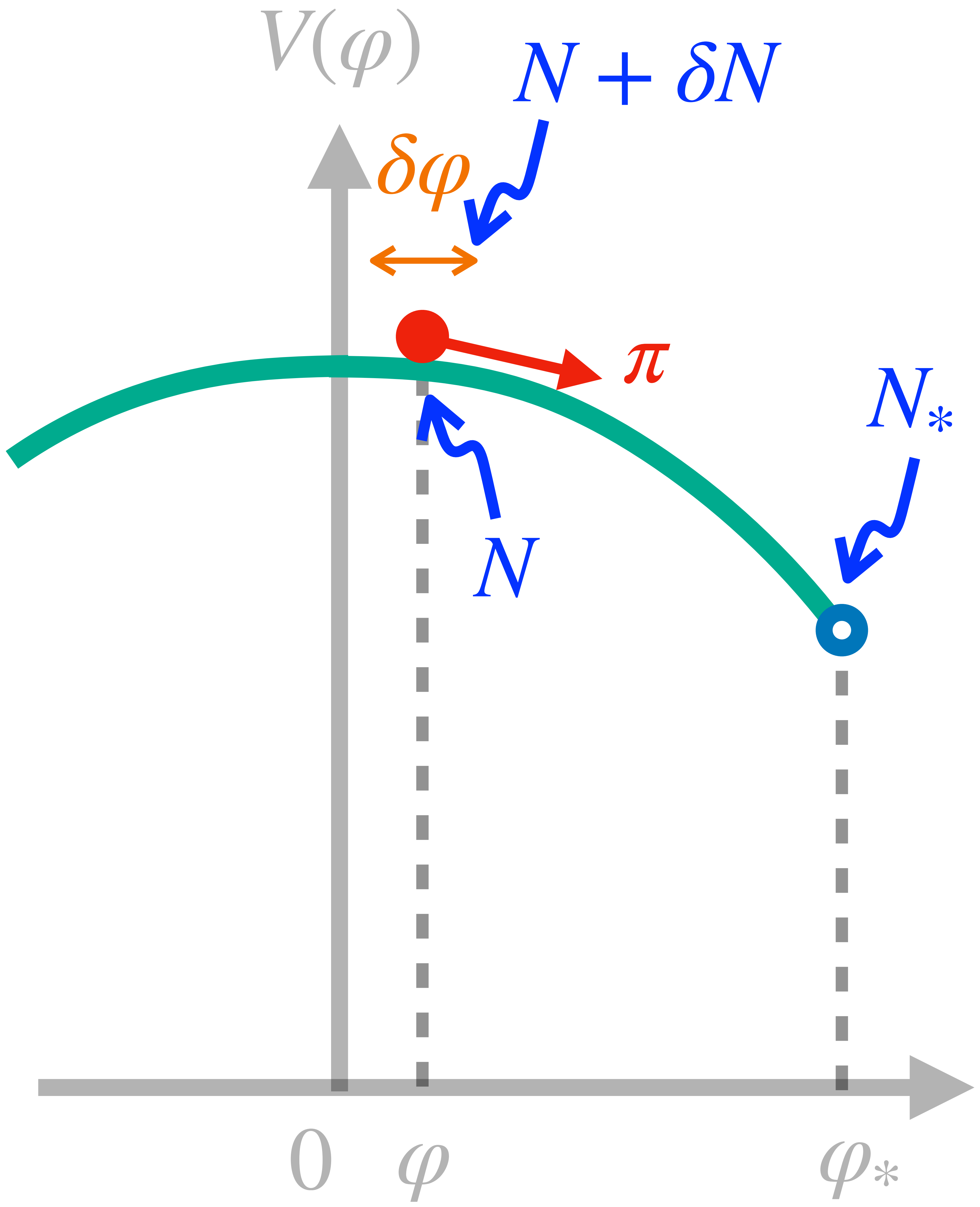
$$\left.\begin{aligned} \frac{\pi + \lambda_+\varphi}{\pi_* + \lambda_+\varphi_*} &= e^{\lambda_+(N-N_*)} \\ \frac{\pi + \lambda_-\varphi}{\pi_* + \lambda_-\varphi_*} &= e^{\lambda_-(N-N_*)} \end{aligned}\right\} \Longrightarrow N - N_* = \frac{1}{\lambda_\pm} \ln \frac{\pi + \lambda_\mp\varphi}{\pi_* + \lambda_\mp\varphi_*}$$

For another trajectory, we take the perturbation as

$$\left.\begin{aligned} N &\to N + \delta N \\ \varphi &\to \varphi + \delta\varphi \\ \pi &\to \pi + \delta\pi \\ \pi_* &\to \pi_* + \delta\pi_* \end{aligned}\right\} \quad N - N_* + \delta(N - N_*) = \frac{1}{\lambda_\pm} \ln \frac{\pi + \delta\pi + \lambda_\mp(\varphi + \delta\varphi)}{\pi_* + \delta\pi_* + \lambda_\mp\varphi_*}$$

And then subtract the fiducial N from $N + \delta N$:

Logarithmic Duality



$$\left.\begin{aligned} \frac{\pi+\lambda_+\varphi}{\pi_*+\lambda_+\varphi_*} &= e^{\lambda_+(N-N_*)} \\ \frac{\pi+\lambda_-\varphi}{\pi_*+\lambda_-\varphi_*} &= e^{\lambda_-(N-N_*)} \end{aligned}\right\} \Longrightarrow N-N_* = \frac{1}{\lambda_\pm}\ln\frac{\pi+\lambda_\mp\varphi}{\pi_*+\lambda_\mp\varphi_*}$$

$$\Longrightarrow \mathscr{R} = \delta(N-N_*)$$

$$= \frac{1}{\lambda_\pm}\ln\left(1+\frac{\delta\pi+\lambda_\mp\delta\varphi}{\pi+\lambda_\mp\varphi}\right) - \frac{1}{\lambda_\pm}\ln\left(1+\frac{\delta\pi_*}{\pi_*+\lambda_\mp\varphi_*}\right)$$

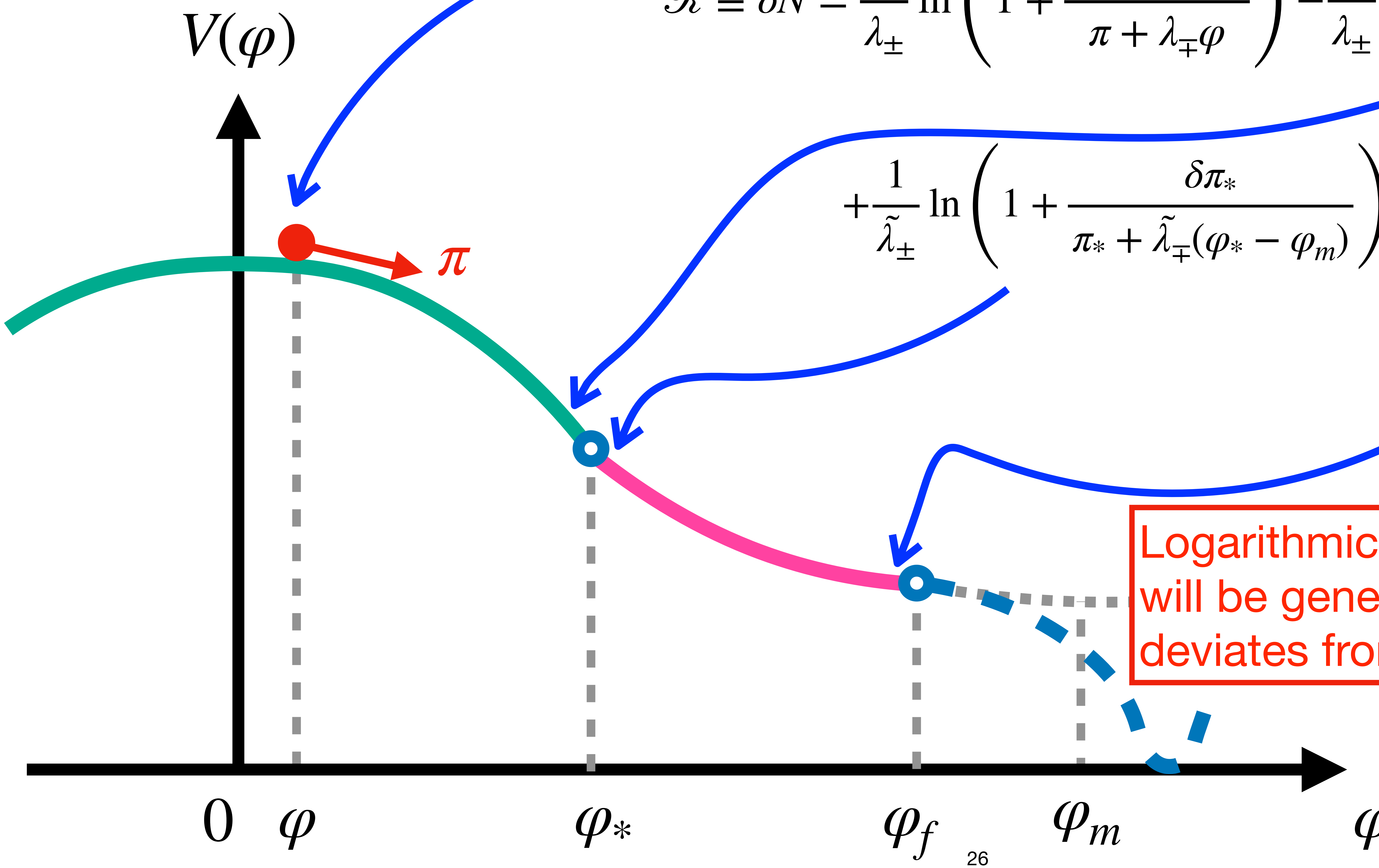
Logarithmic duality of the curvature perturbation

SP and Sasaki, 2211.13932

Logarithmic Duality

$$\mathscr{R} \equiv \delta N = \frac{1}{\lambda_\pm}\ln\left(1+\frac{\delta\pi+\lambda_\mp\delta\varphi}{\pi+\lambda_\mp\varphi}\right) - \frac{1}{\lambda_\pm}\ln\left(1+\frac{\delta\pi_*}{\pi_*+\lambda_\mp\varphi_*}\right)$$

$$+\frac{1}{\tilde\lambda_\pm}\ln\left(1+\frac{\delta\pi_*}{\pi_*+\tilde\lambda_\mp(\varphi_*-\varphi_m)}\right) - \frac{1}{\tilde\lambda_\pm}\ln\left(1+\frac{\delta\pi_f}{\pi_f+\tilde\lambda_\mp(\varphi_f-\varphi_m)}\right)$$



Logarithmic relation (i.e. non-Gaussianity) will be generated when the evolution deviates from attractor.

SP and Sasaki, 2211.13932
SP, 2404.06151

Logarithmic Duality

$\mathscr{R}(\delta\varphi, \delta\pi)$

SP and Sasaki, 2211.13932

$$\mathscr{R} = \frac{1}{\lambda_\pm}\ln\left(1+\frac{\delta\pi+\lambda_\mp\delta\varphi}{\pi+\lambda_\mp\varphi}\right) - \frac{1}{\lambda_\pm}\ln\left(1+\frac{\delta\pi_*}{\pi_*+\lambda_\mp\varphi_*}\right)+\cdots$$

$\lambda_- \ll 1$

$(f_{NL} = -\frac{5}{6}\lambda_-)$

$$\mathscr{R} = -H\frac{\delta\varphi}{\dot\varphi} + \frac{3}{5}f_{\rm NL}\left(-H\frac{\delta\varphi}{\dot\varphi}\right)^2$$

Slow-roll inflation
Stewart and Sasaki, 1995
Lyth and Roquigez, 2005

$\lambda_- = -1/\mu$

$$\mathscr{R} = -\mu\ln\left(1-\frac{\mathscr{R}_g}{\mu}\right)$$

Constant-roll
Atal, Garriga, Marcos-Caballero, 1905.13202
Atal, Cid, Escrivà, Garriga, 1908.11357
Escrivà, Atal, Garriga, 2306.09990
Inui, Motohashi, SP, et al, 2409.13500

$\lambda_- = 0$

$$\mathscr{R} = -\frac{1}{3}\ln\left(1+\frac{\delta\pi_*}{\pi_*}\right)$$

Ultra-slow-roll
Namjoo, Firouzjahi, Sasaki, 1210.3692
Cai, Chen, et al 1712.09998
Biagetti et al 1804.07124
Passaglia et al 1812.08243

$\lambda_- = 3/2$

$$\mathscr{R} = \frac{2}{3}\ln(1+\delta)$$

Curvaton scenario,
SP and Sasaki, 2112.12680
Ferrante et al, 2211.01728
Hooper et al. 2308.00756

$$\mathscr{R} = -\frac{1}{\lambda}\ln\left(f(\mathscr{R}_{\rm G})\right)$$

Extensions,
Kawaguchi et al, 2305.18140
SP and Yokoyama, in prep

Probability Distribution Function

For the simplest single-logarithm case: $\mathscr{R} \equiv \delta N = \frac{1}{\lambda_-} \ln \left(1 + \frac{\delta\pi + \lambda_+ \delta\varphi}{\pi + \lambda_+ \varphi} \right)$

$$\Downarrow \quad P(\mathscr{R})d\mathscr{R} = P(\delta\varphi)d\delta\varphi$$

$P(\delta\varphi)$: Gaussian PDF with variance $\sigma^2_{\delta\varphi}$

$$P(\mathscr{R}) = \frac{e^{\lambda_- \mathscr{R}}}{\sqrt{2\pi}\sigma_{\delta\varphi}} \left|\lambda_-\right| \varphi \exp\left[-\frac{\varphi^2}{2\sigma^2_{\delta\varphi}} \left(e^{\lambda_- \mathscr{R}} - 1\right)^2 \right]$$

$\lambda_- < 0$: $P(\mathscr{R}) \sim e^{\lambda_- \mathscr{R}}$

exponential tail

$\lambda_- > 0$: $P(\mathscr{R}) \sim \exp\left(-c^2 e^{2\lambda_- \mathscr{R}}\right)$

Gumbel-distribution-like tail

SP and Sasaki, 2211.13932

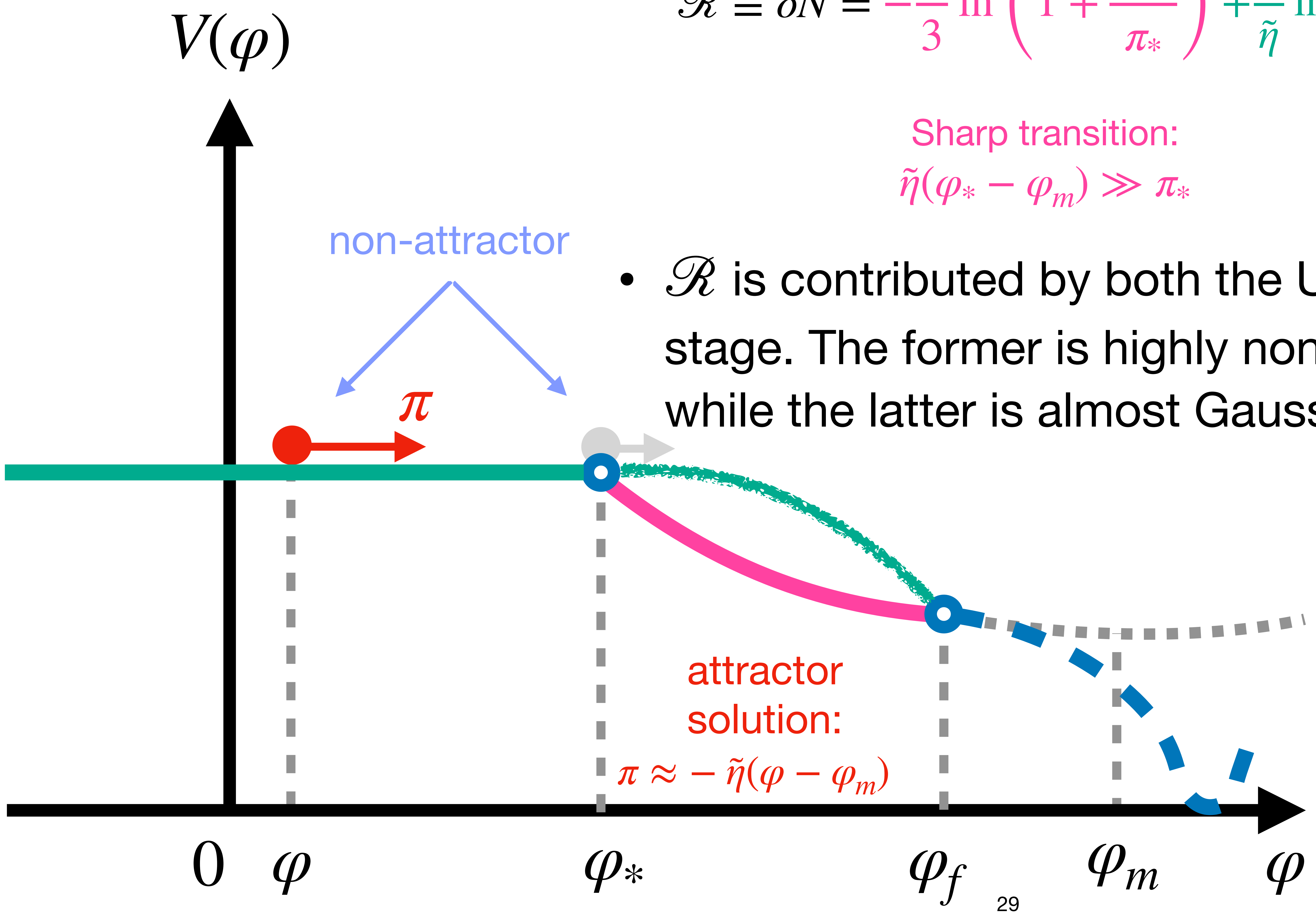
USR and NG

$(\lambda_- = 0, \quad \lambda_+ = 3)$
$(\tilde{\lambda}_- = \tilde{\eta}, \quad \tilde{\lambda}_+ = 3 - \tilde{\eta})$

$$\mathcal{R} \equiv \delta N = -\frac{1}{3}\ln\left(1 + \frac{\delta\pi_*}{\pi_*}\right) + \frac{1}{\tilde{\eta}}\ln\left(1 + \frac{\delta\pi_*}{\pi_* + (3-\tilde{\eta})(\varphi_* - \varphi_m)}\right)$$

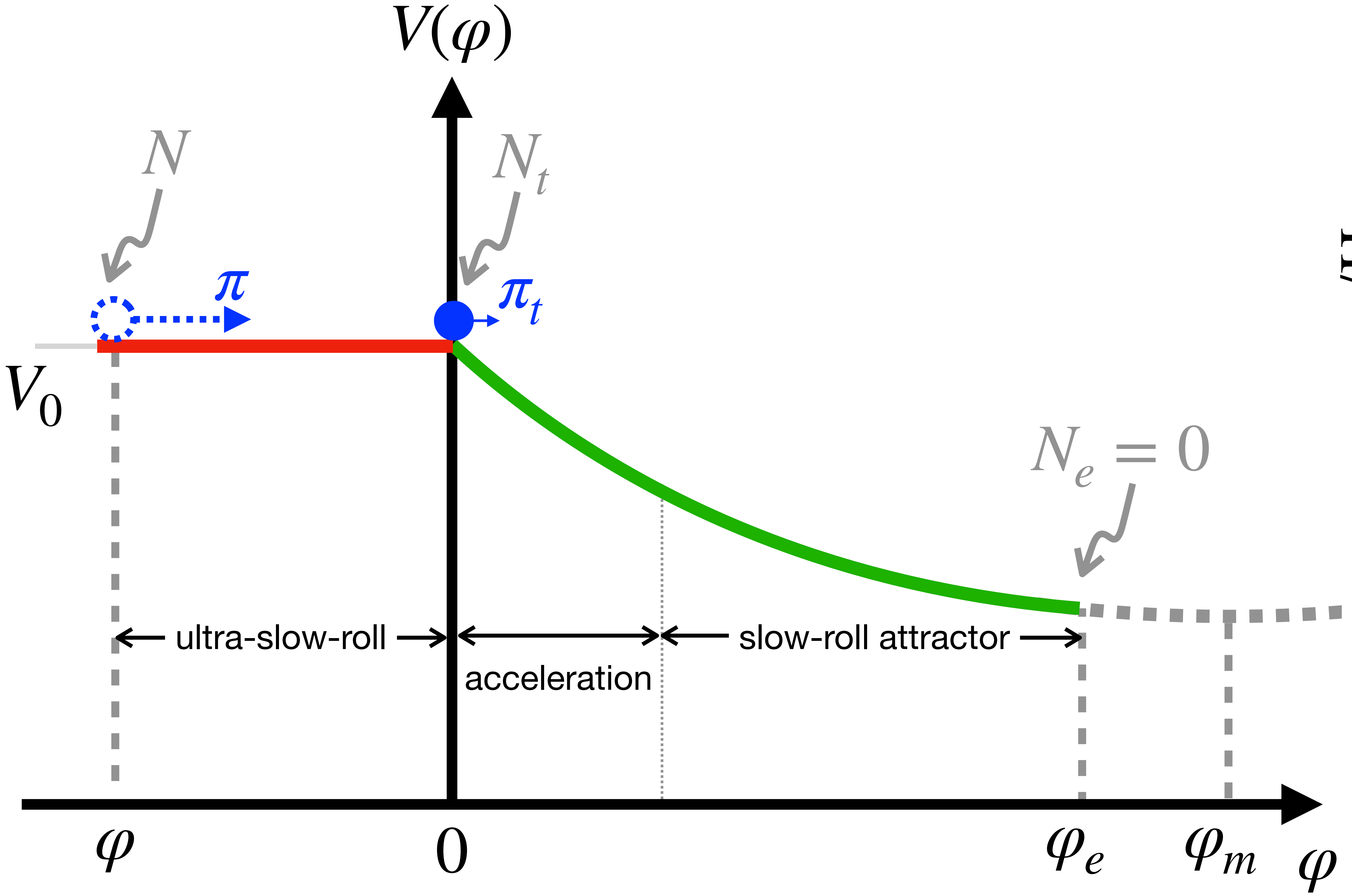
Sharp transition:
$\tilde{\eta}(\varphi_* - \varphi_m) \gg \pi_*$

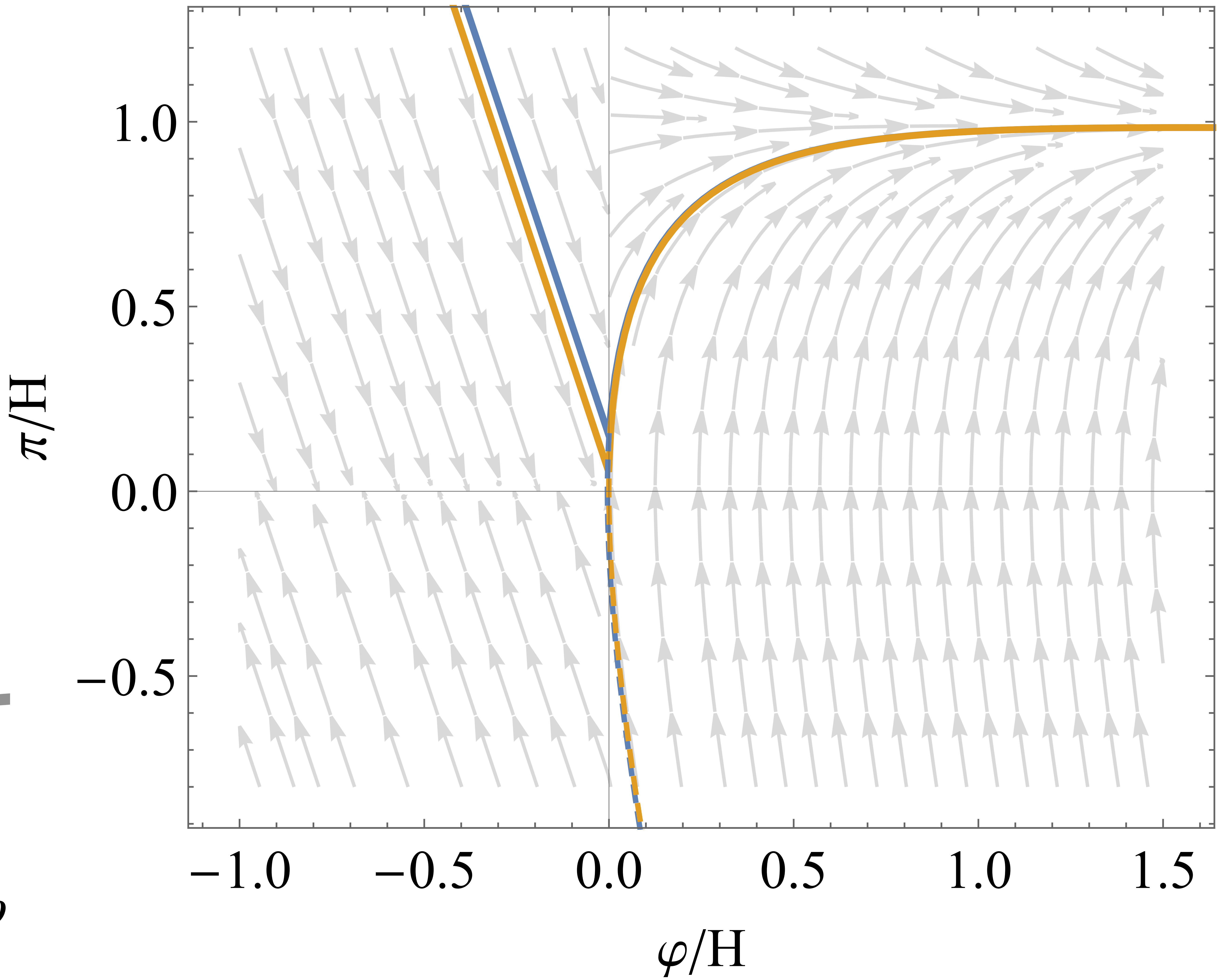
Smooth transition
$\tilde{\eta}(\varphi_* - \varphi_m) \ll \pi_*$



- $\mathcal{R}$ is contributed by both the USR stage and the later slow-roll stage. The former is highly non-Gaussian (i.e. exp tail, $f_{\rm NL} = 5/2$), while the latter is almost Gaussian.

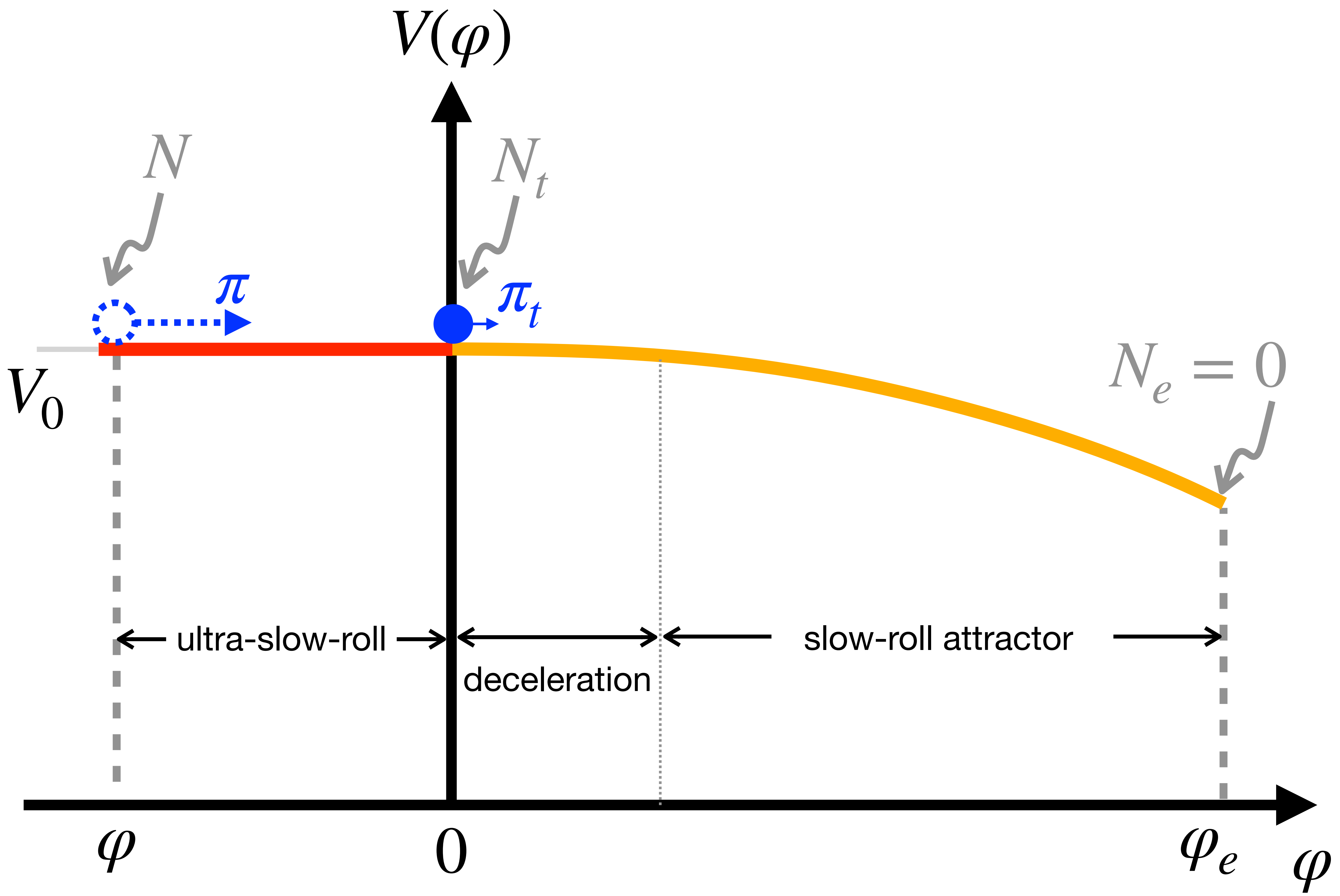
USR: Sharp end

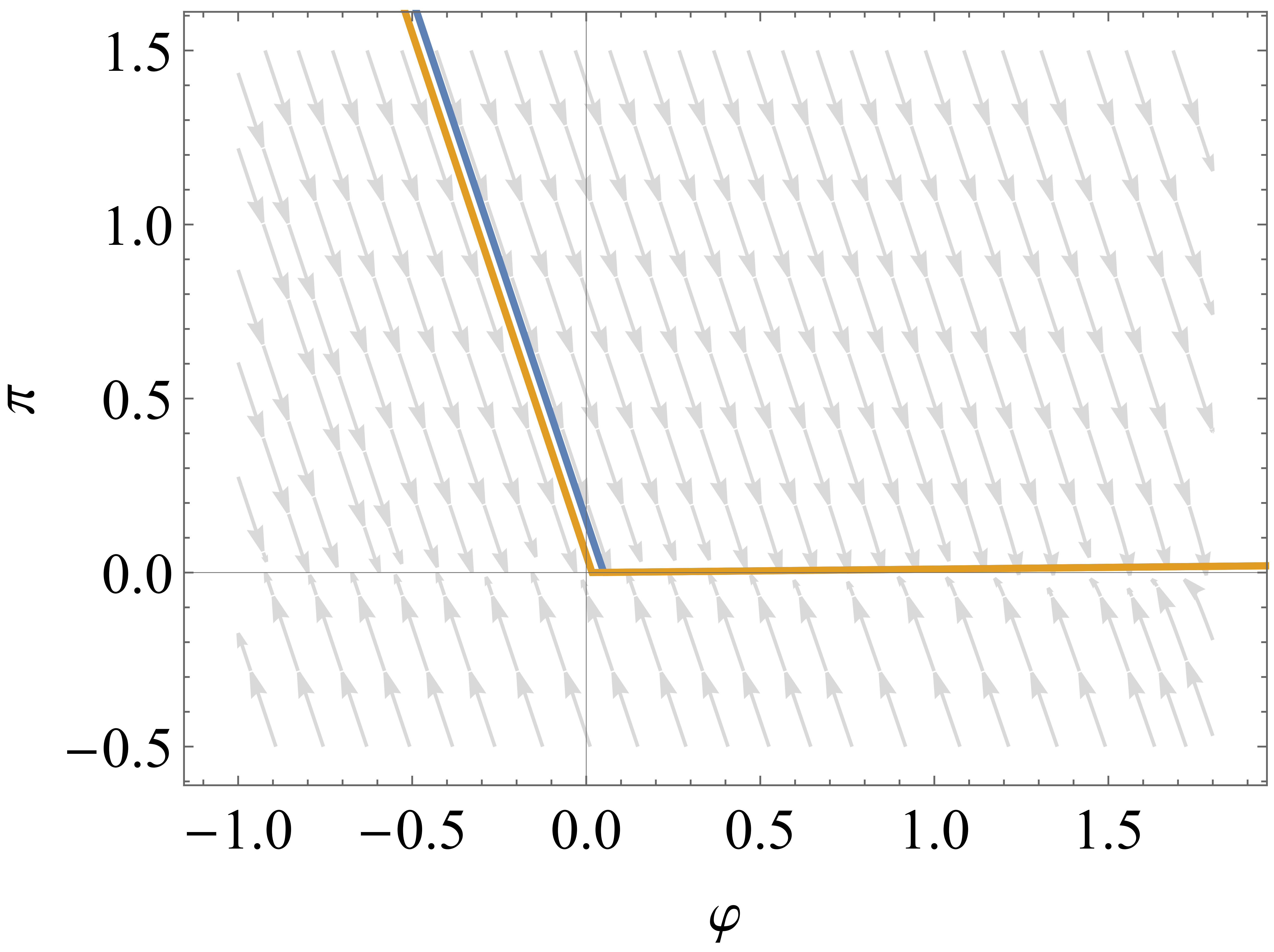




SP and Sasaki, 2211.13932
SP, 2404.06151
c.f. Cai et al, 1712.09998

USR: Smooth end





SP and Sasaki, 2211.13932
SP, 2404.06151
c.f. Cai et al, 1712.09998

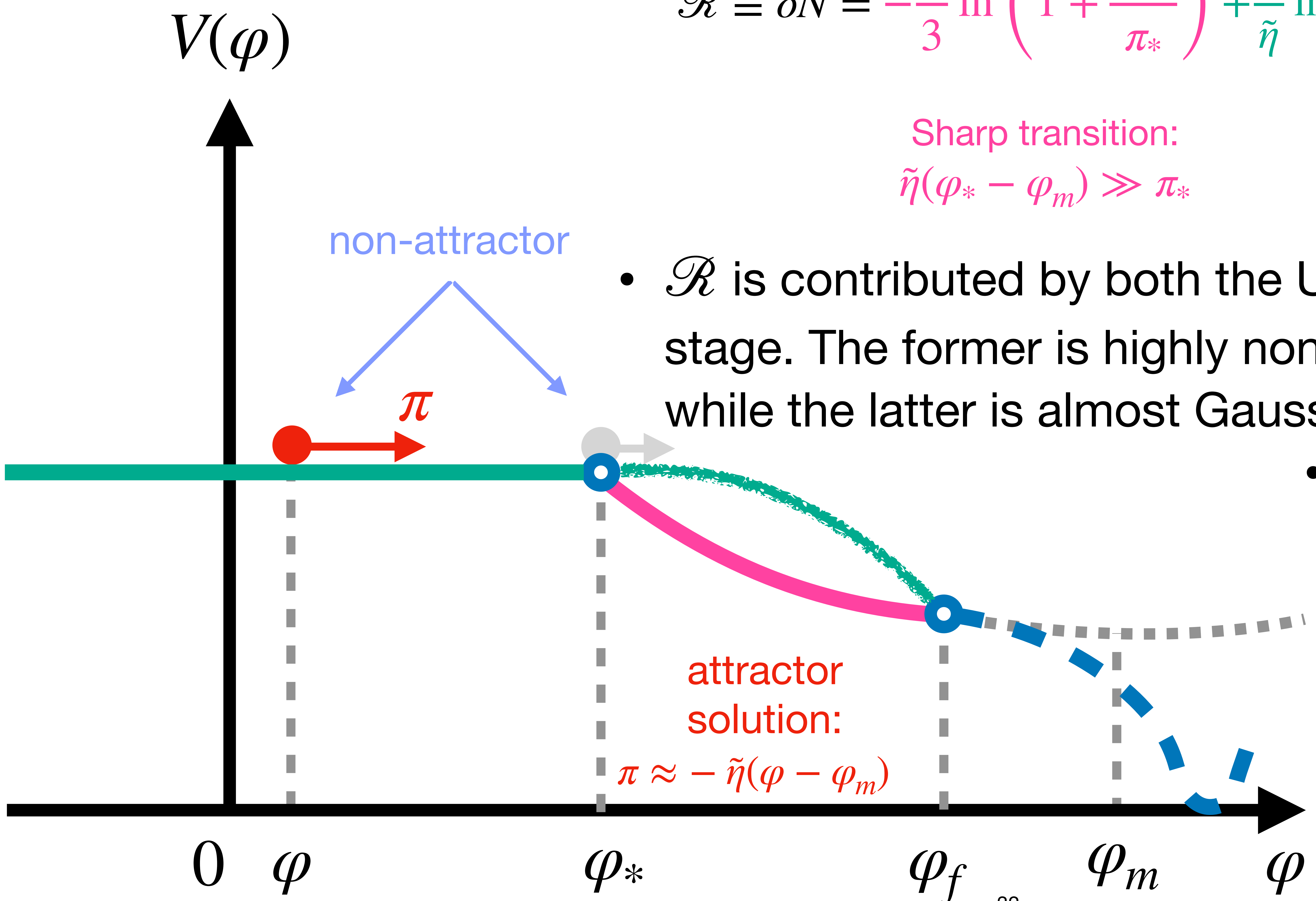
USR and NG

$(\lambda_- = 0, \quad \lambda_+ = 3)$
$(\tilde{\lambda}_- = \tilde{\eta}, \quad \tilde{\lambda}_+ = 3 - \tilde{\eta})$

$$\mathcal{R} \equiv \delta N = -\frac{1}{3}\ln\left(1 + \frac{\delta\pi_*}{\pi_*}\right) + \frac{1}{\tilde{\eta}}\ln\left(1 + \frac{\delta\pi_*}{\pi_* + (3-\tilde{\eta})(\varphi_* - \varphi_m)}\right)$$

Sharp transition:
$\tilde{\eta}(\varphi_* - \varphi_m) \gg \pi_*$

Smooth transition
$\tilde{\eta}(\varphi_* - \varphi_m) \ll \pi_*$



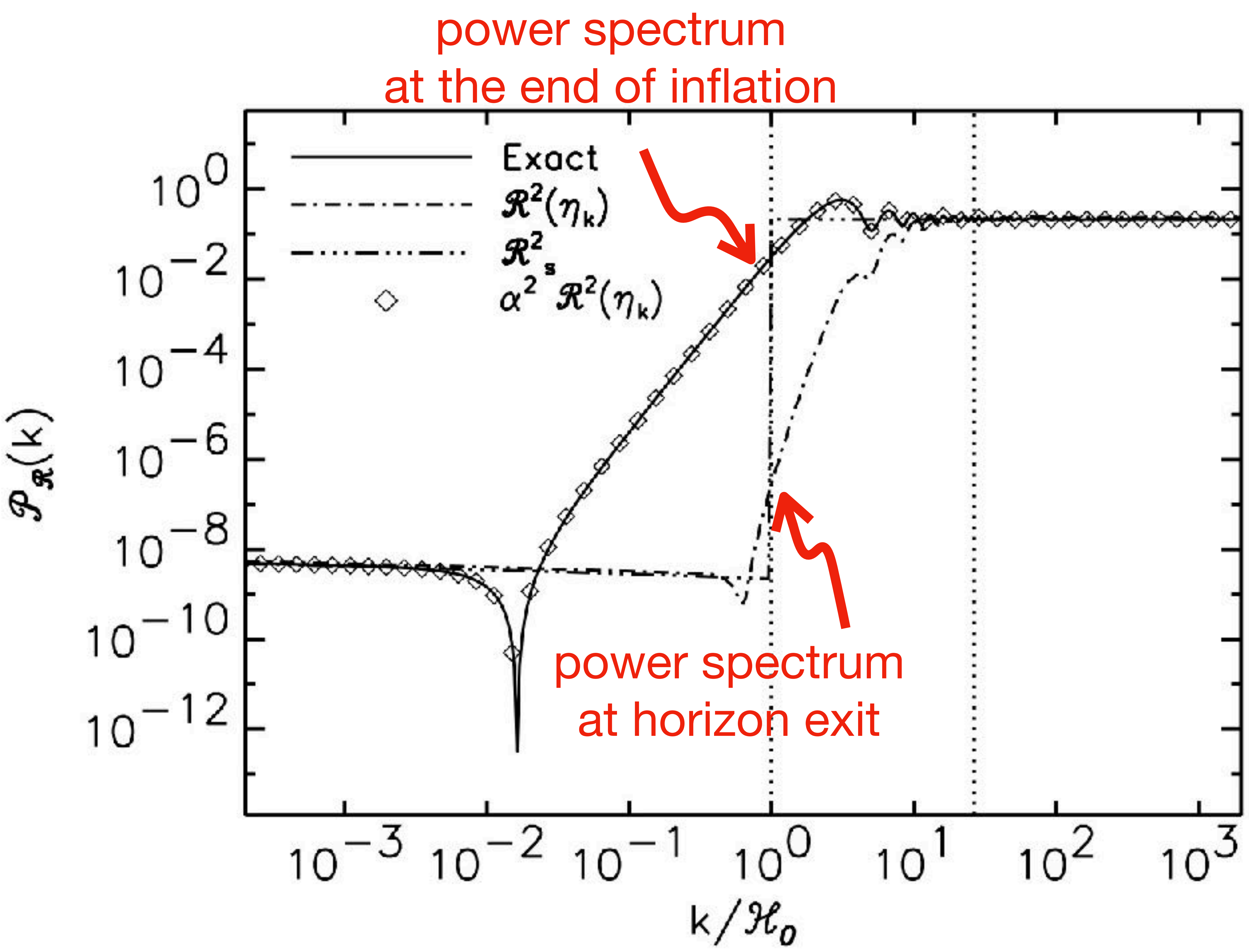
- $\mathcal{R}$ is contributed by both the USR stage and the later slow-roll stage. The former is highly non-Gaussian (i.e. exp tail, $f_{\rm NL} = 5/2$), while the latter is almost Gaussian.
- Compare with stochastic approach

Jackson et al 2410.13683,
Cruces, SP, Sasaki in prep

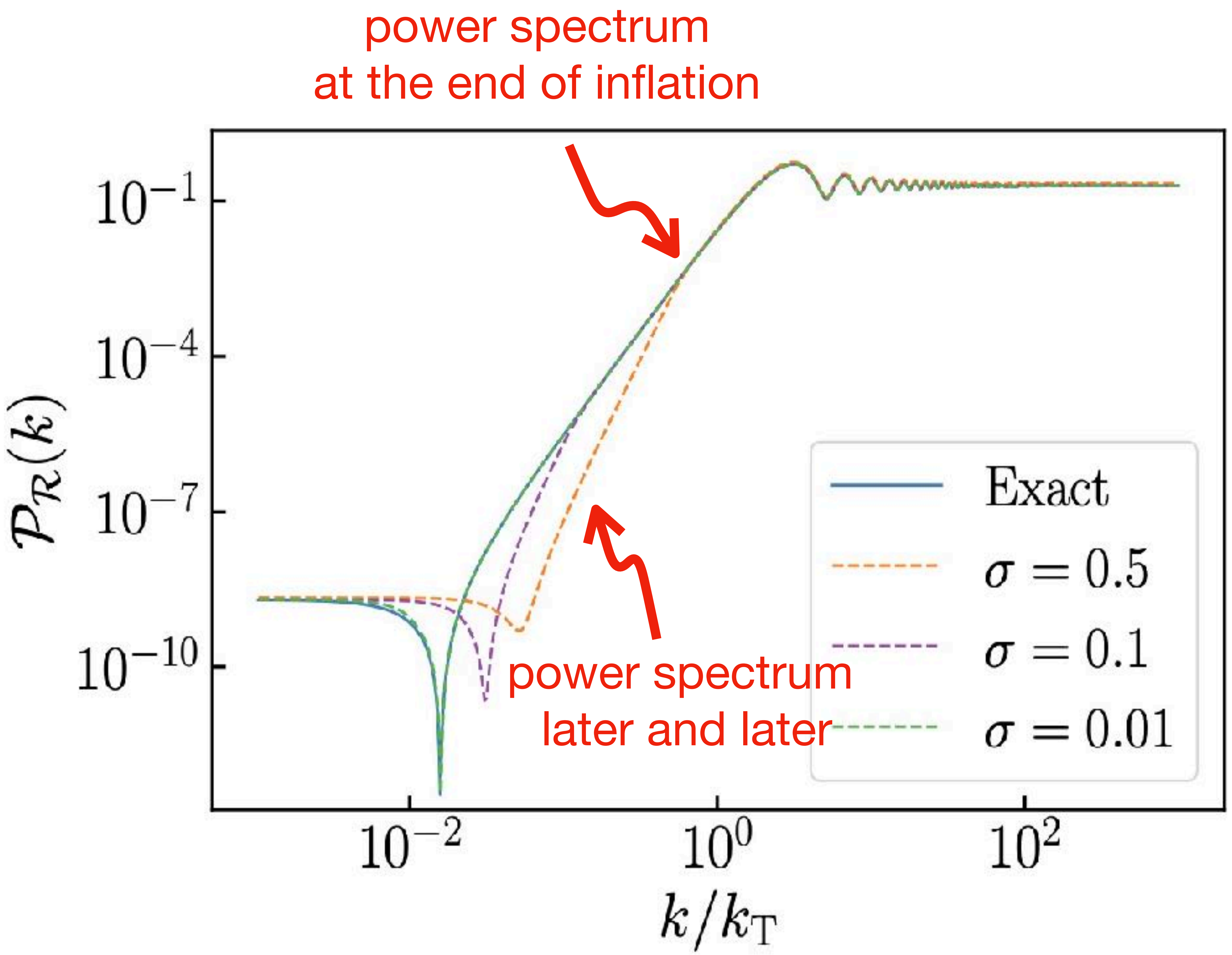
- Sharp transition will make the separate universe approach (thus δN formalism) invalid transiently.

Domenech et al., 2309.05750
Jackson et al., 2311.03281
Artigas, SP, Tanaka, 2408.09964

Separate Universe

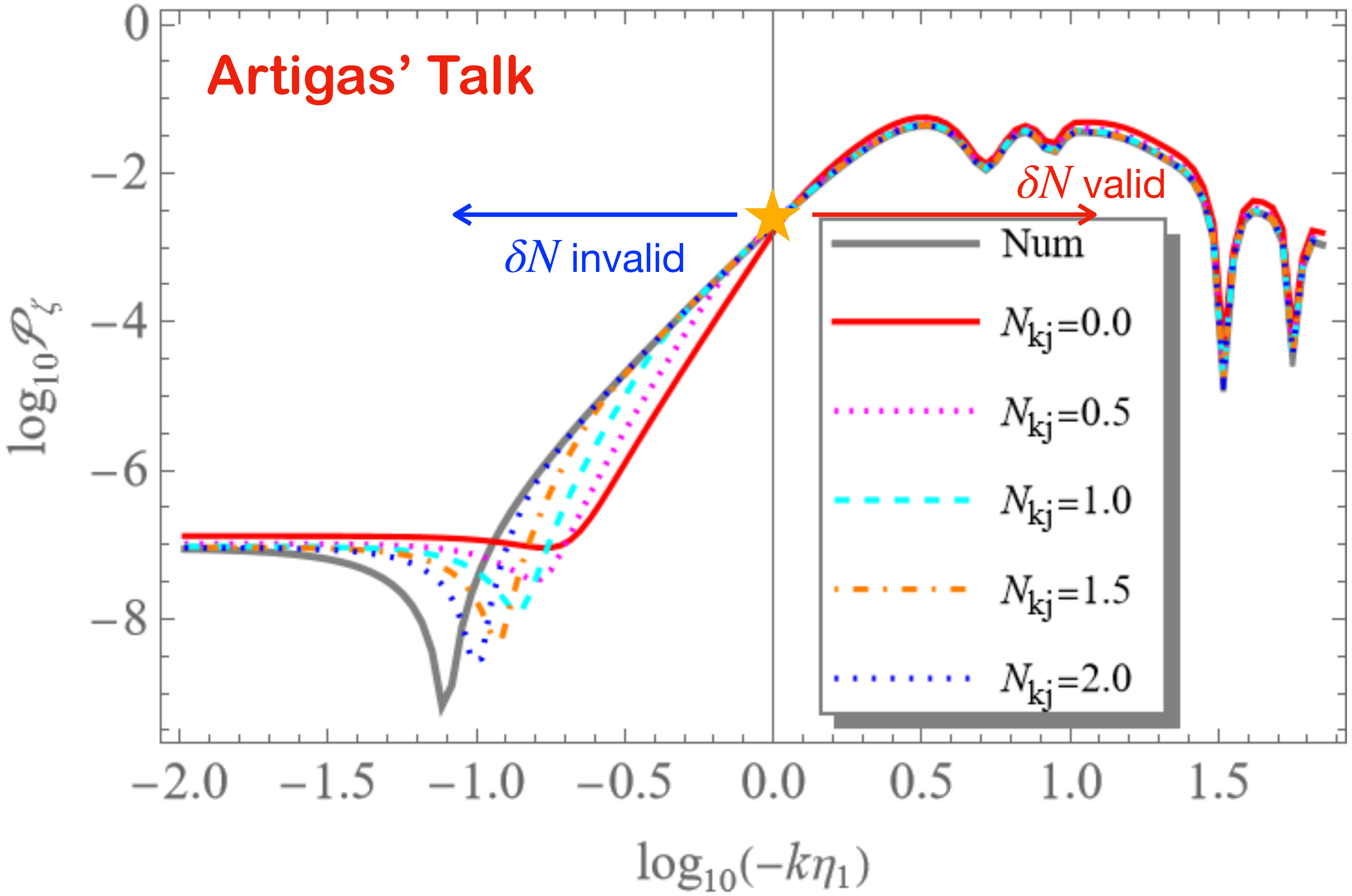


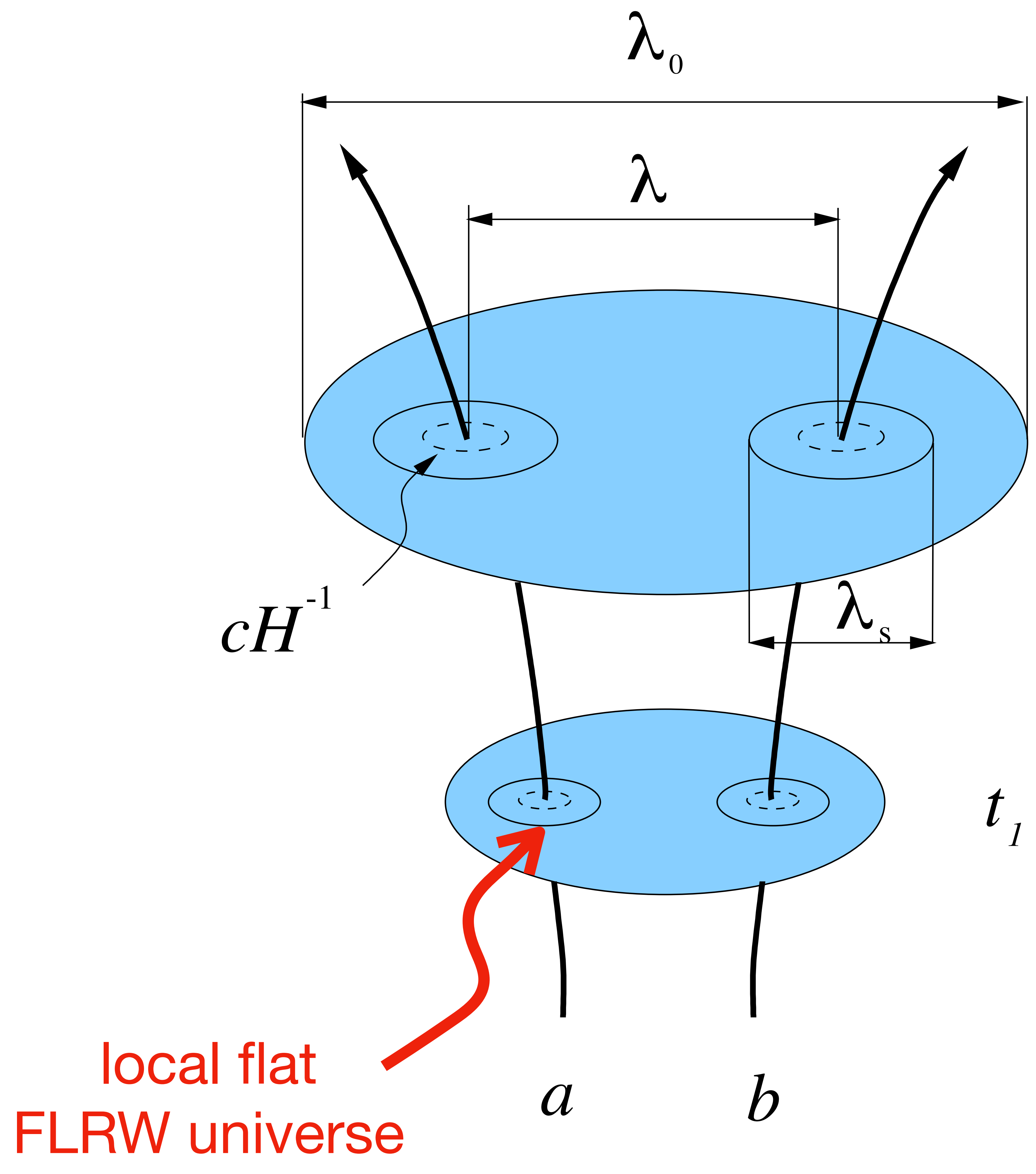
Leach, Sasaki, Wands, Liddle, astro-ph/0101406
SP and Jianing Wang, 2209.14183



Domenech et al., 2309.05750
Jackson et al., 2311.03281

Separate Universe

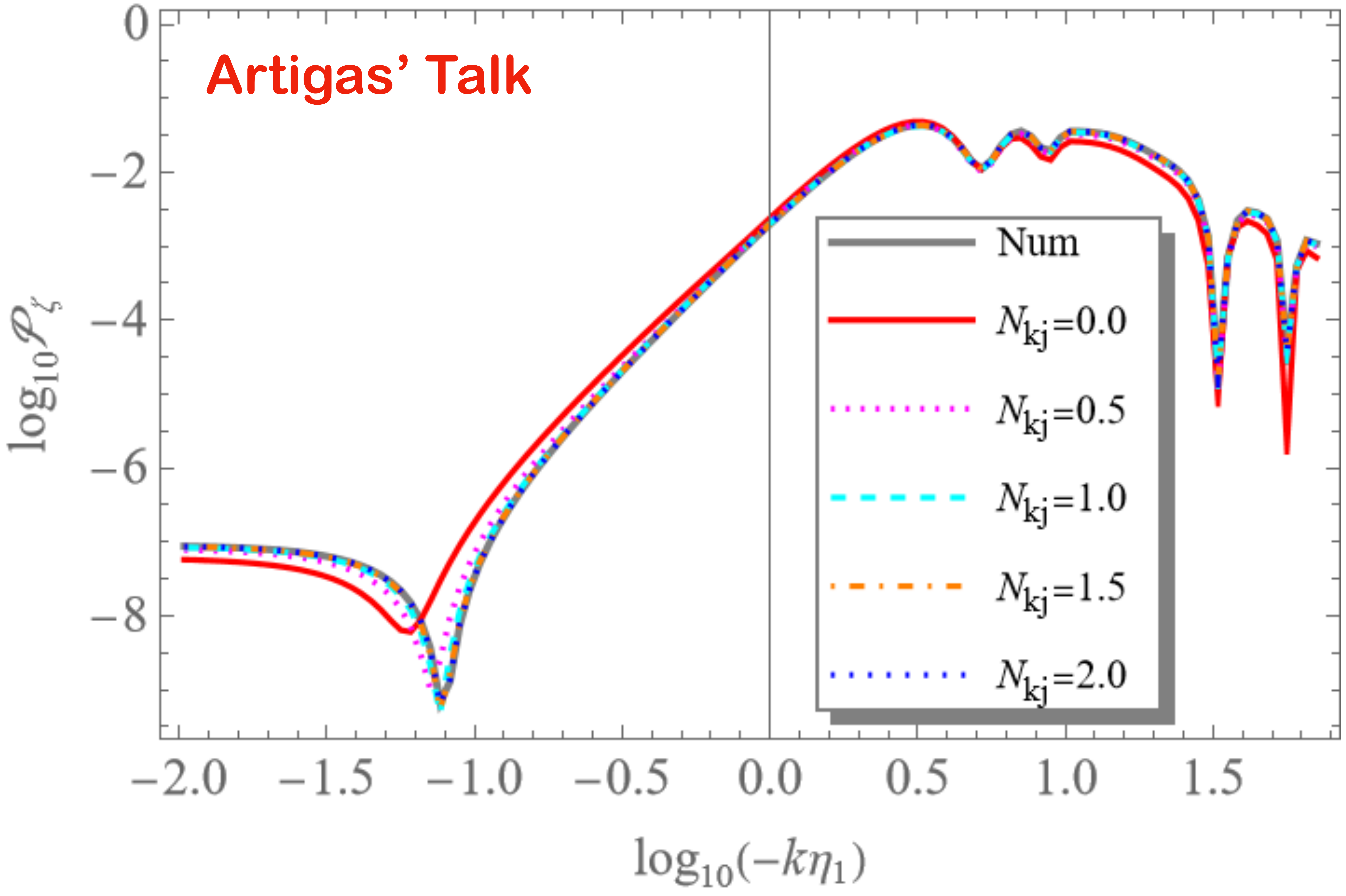


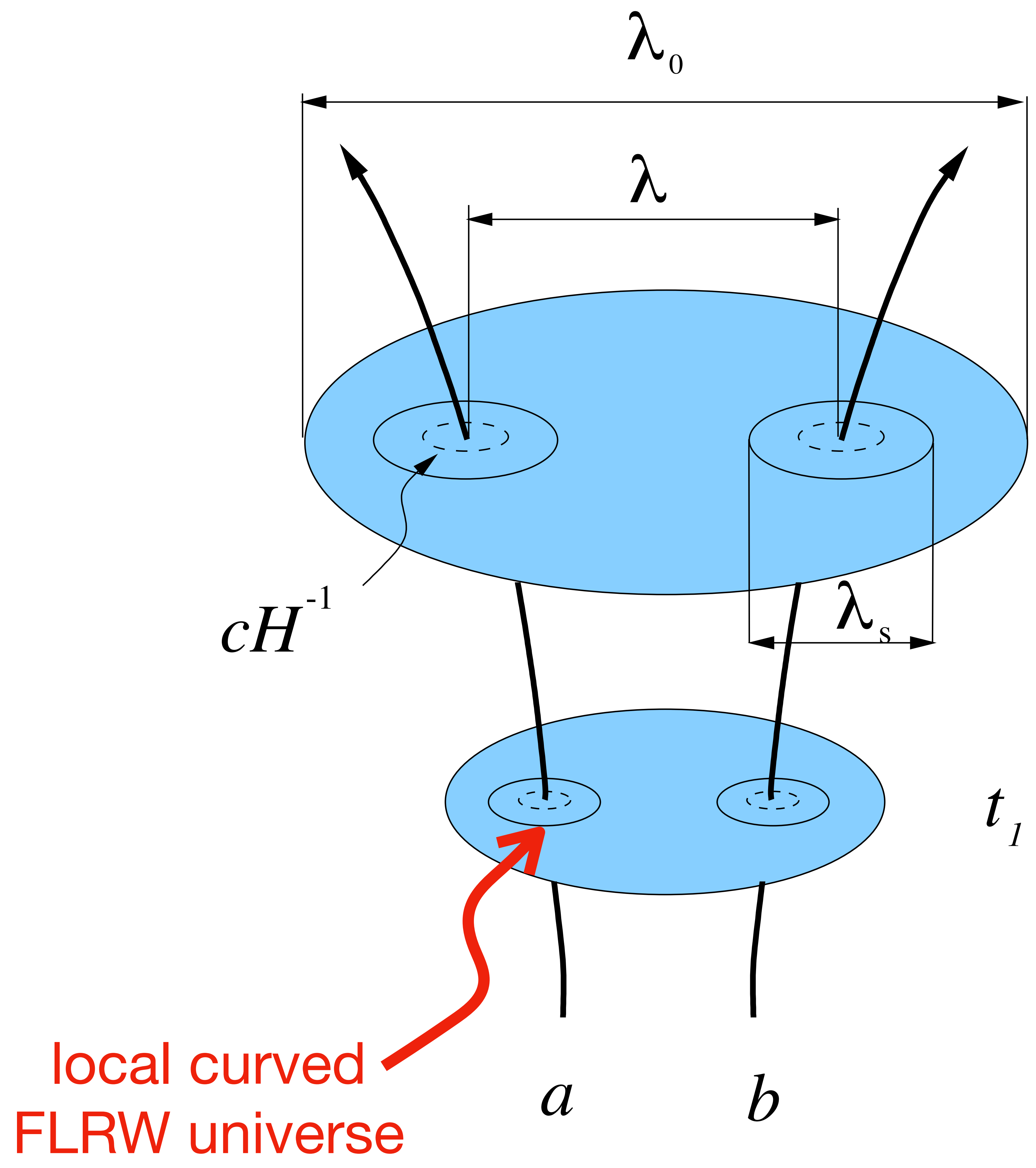


$\mathscr{R}(N_e) = \delta N(\delta\varphi(N_i))$

Domenech et al., 2309.05750
Jackson et al., 2311.03281
Artigas, SP, Tanaka, 2408.09964

Separate Universe

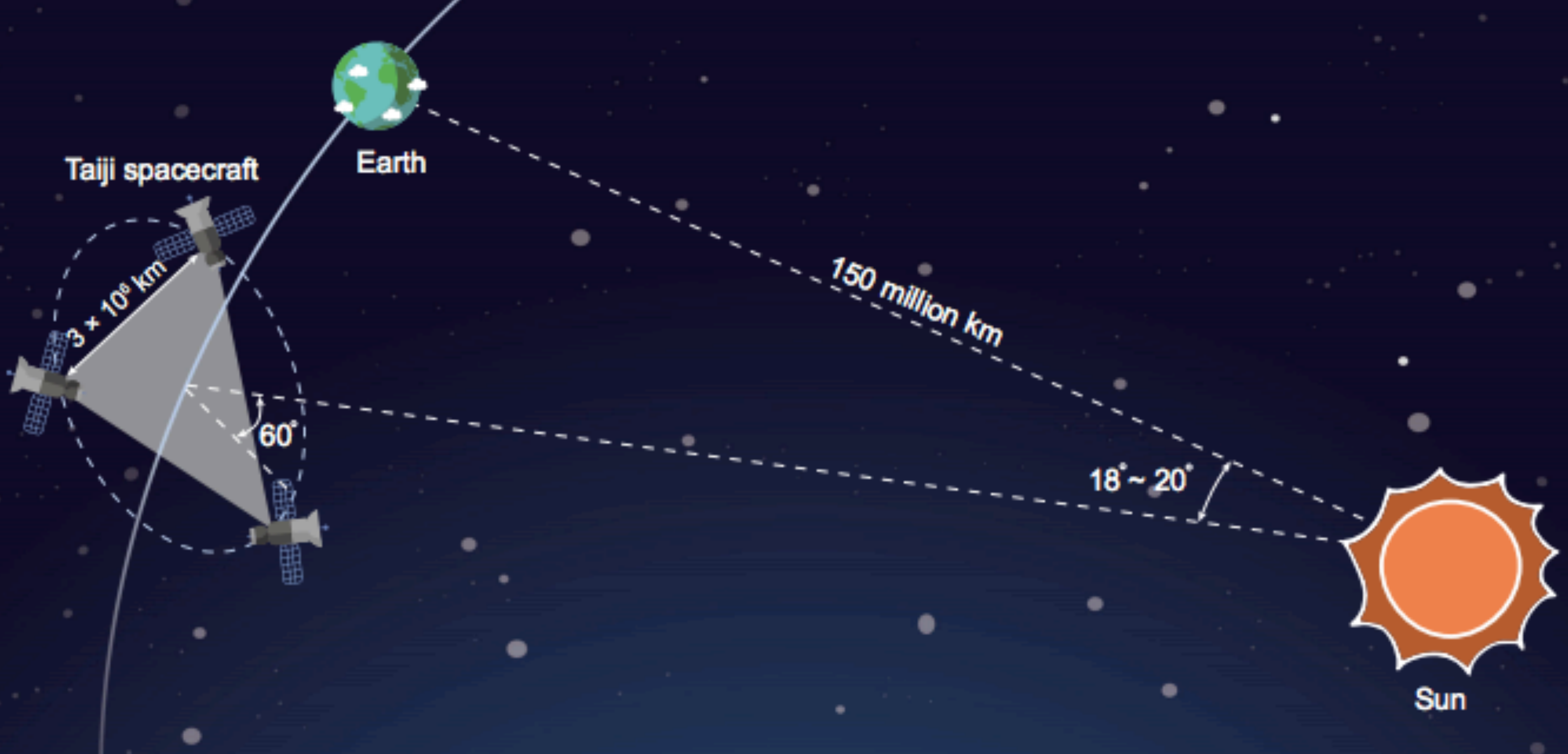




$$\mathcal{R}(N_e) = \mathcal{R}(N_i) + \delta N(k^2\mathcal{R}(N_i))$$

Artigas, SP, Tanaka, 2408.09964

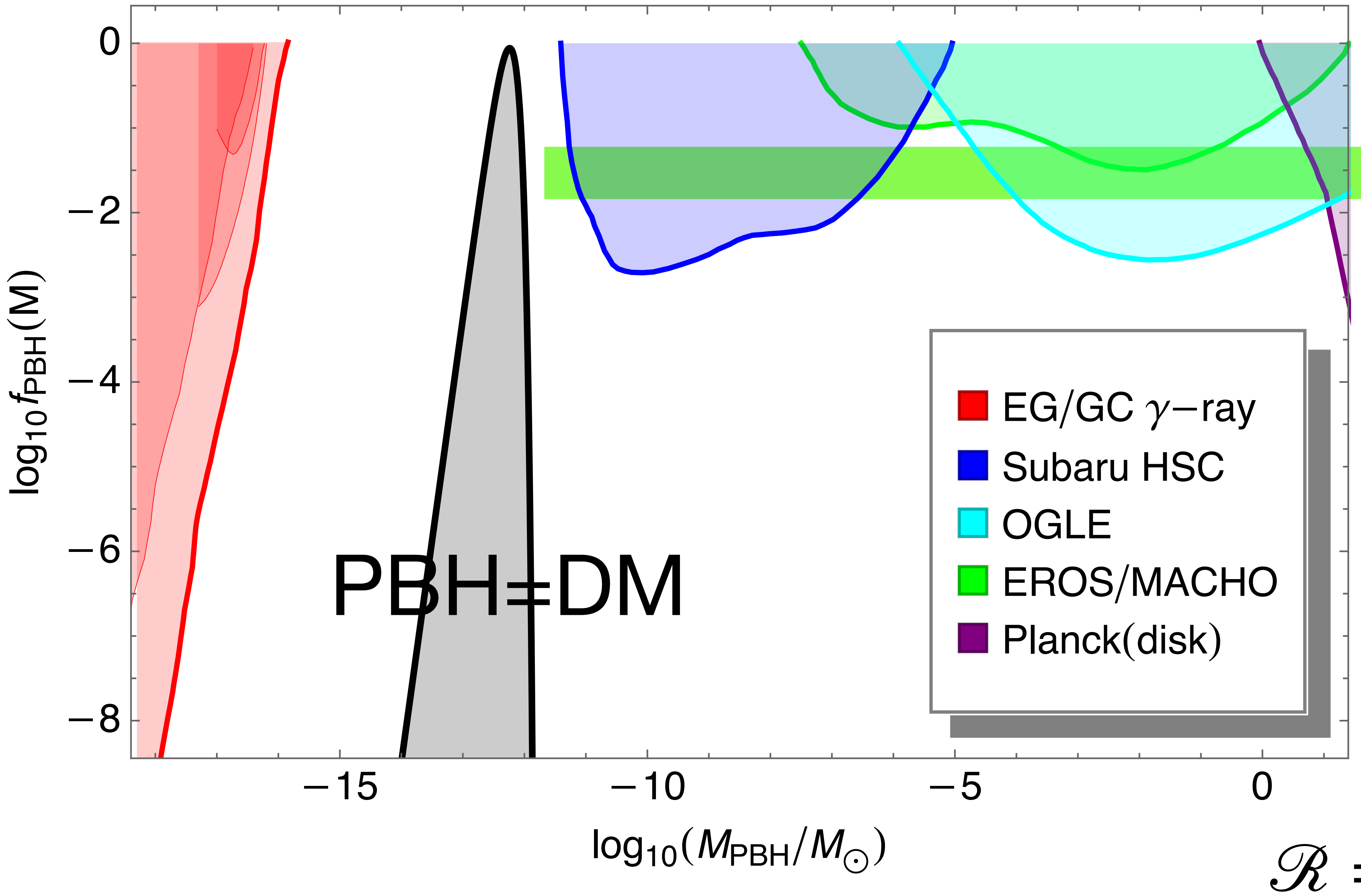
Predictions on mHz and nHz GWs



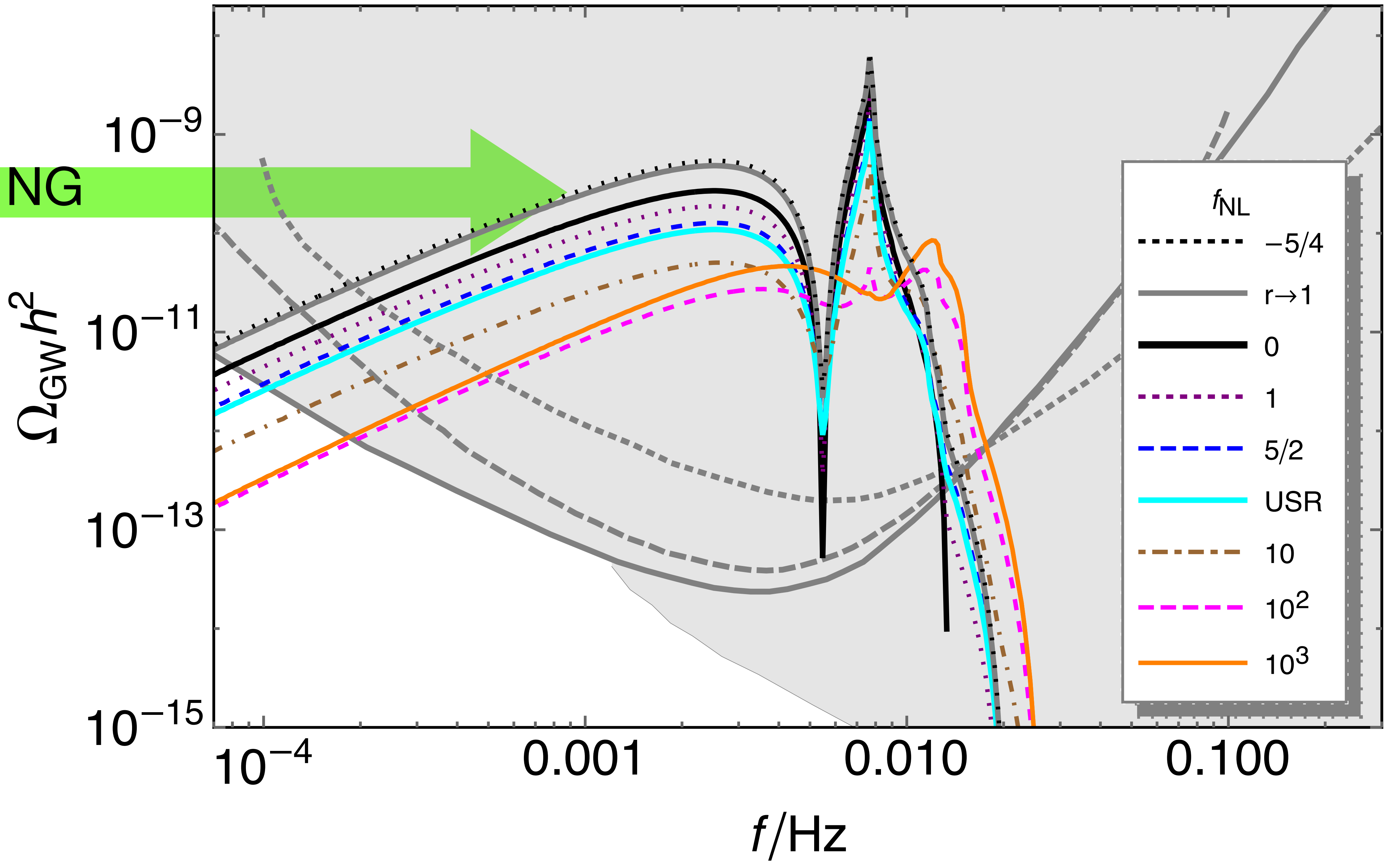
Credit: Natl.Sci.Rev. 4, 5, 685-686

PBH as DM

Cai, SP and Sasaki, 1810.11000
SP, 2404.06151



Press-Schechter-type w/. NG



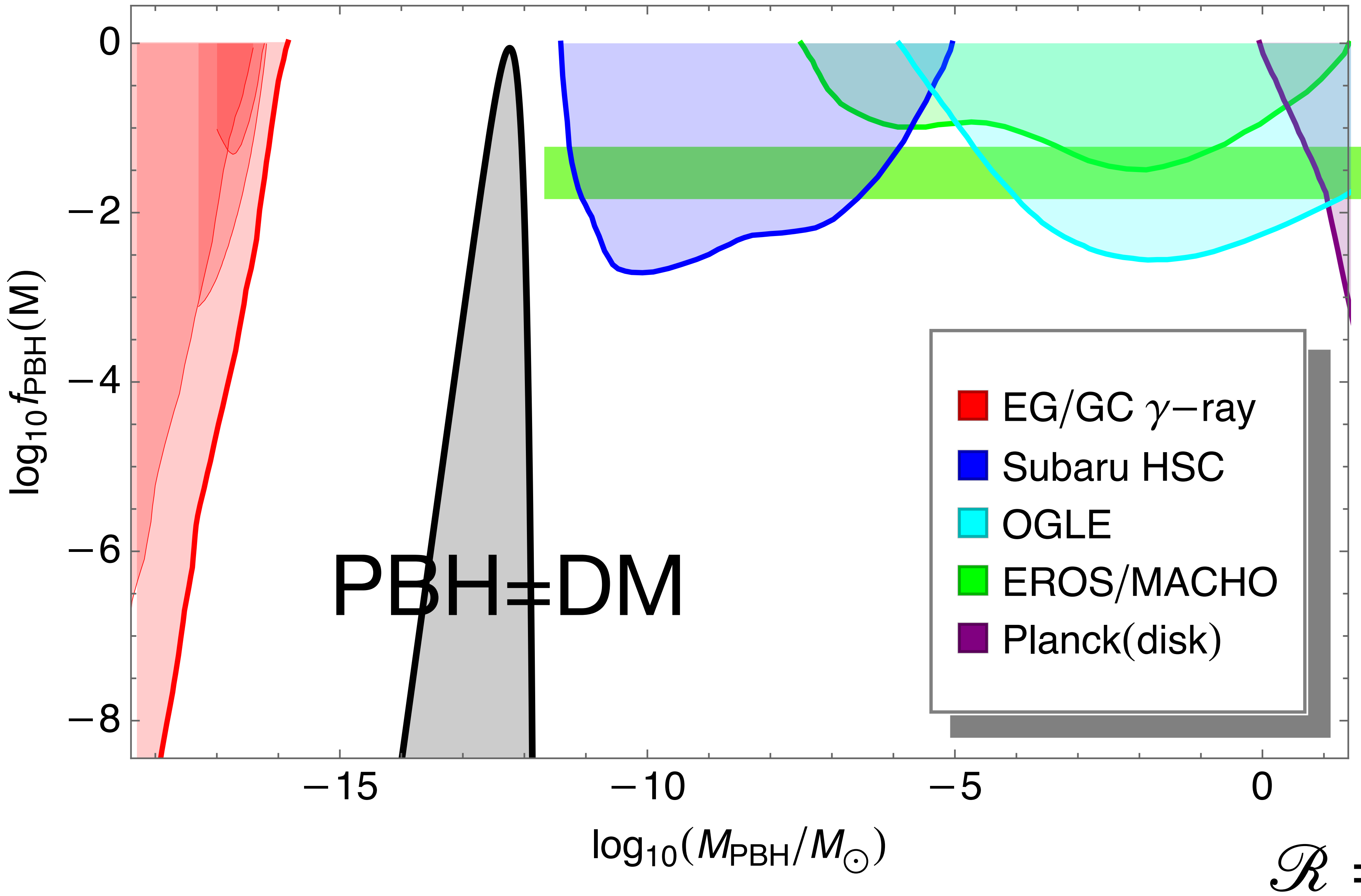
$$\mathscr{R} = \mathscr{R}_g + \frac{3}{5} f_{\rm NL} \mathscr{R}_g^2$$

$$\mathscr{R} = -\frac{1}{3} \ln\left(1 - 3\mathscr{R}_g\right)$$

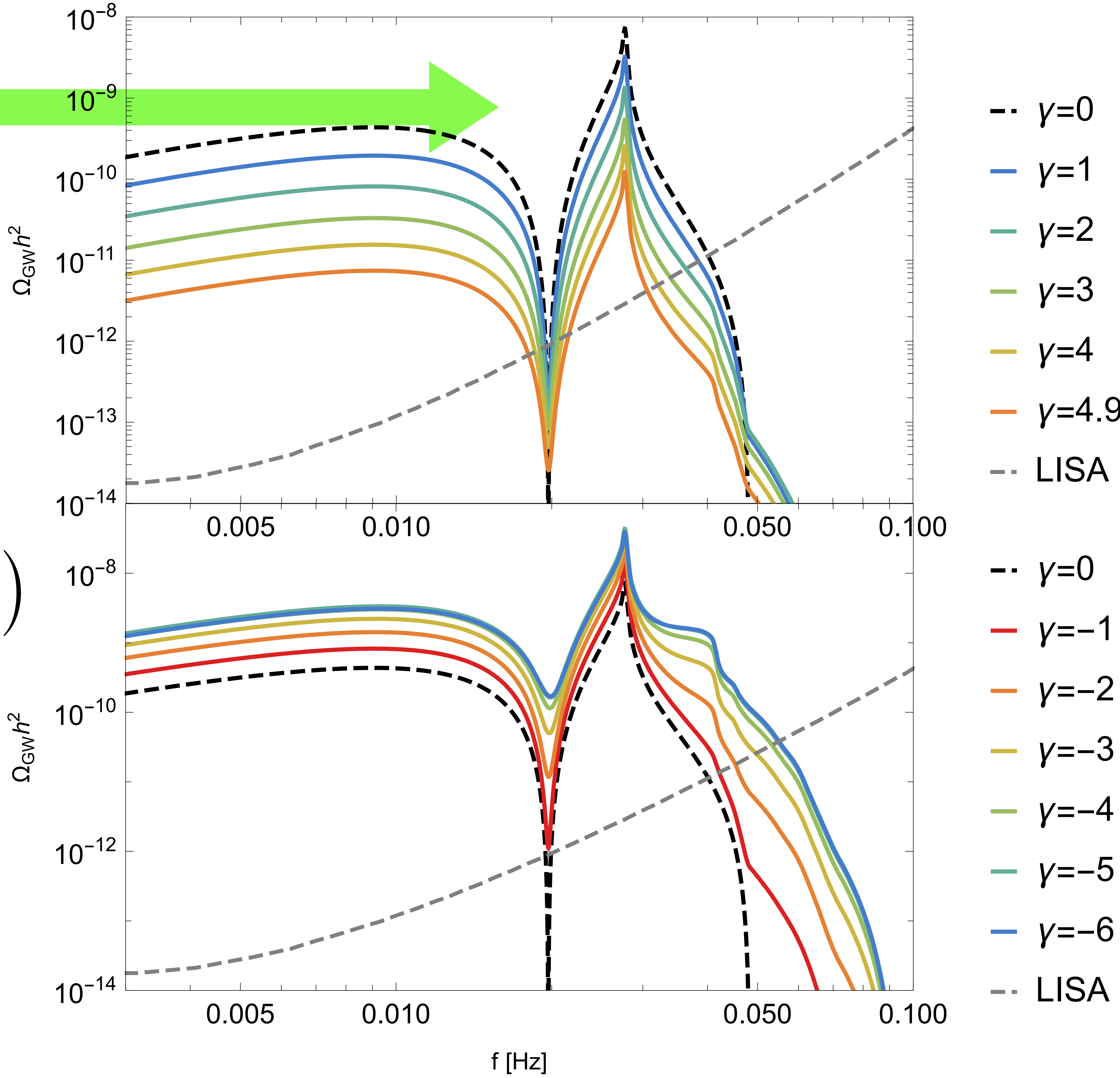
- Primordial NG must be taken into account when calculating PBH abundance
- When fixing PBH abundance, NG impact on SGWB is mild
- LISA/Taiji/TianQin can probe the induced GW when PBH=DM

PBH as DM

Inui, Joana, Motohashi,
SP, Tada, Yokoyama, 2411.07647



peak theory w/. NG

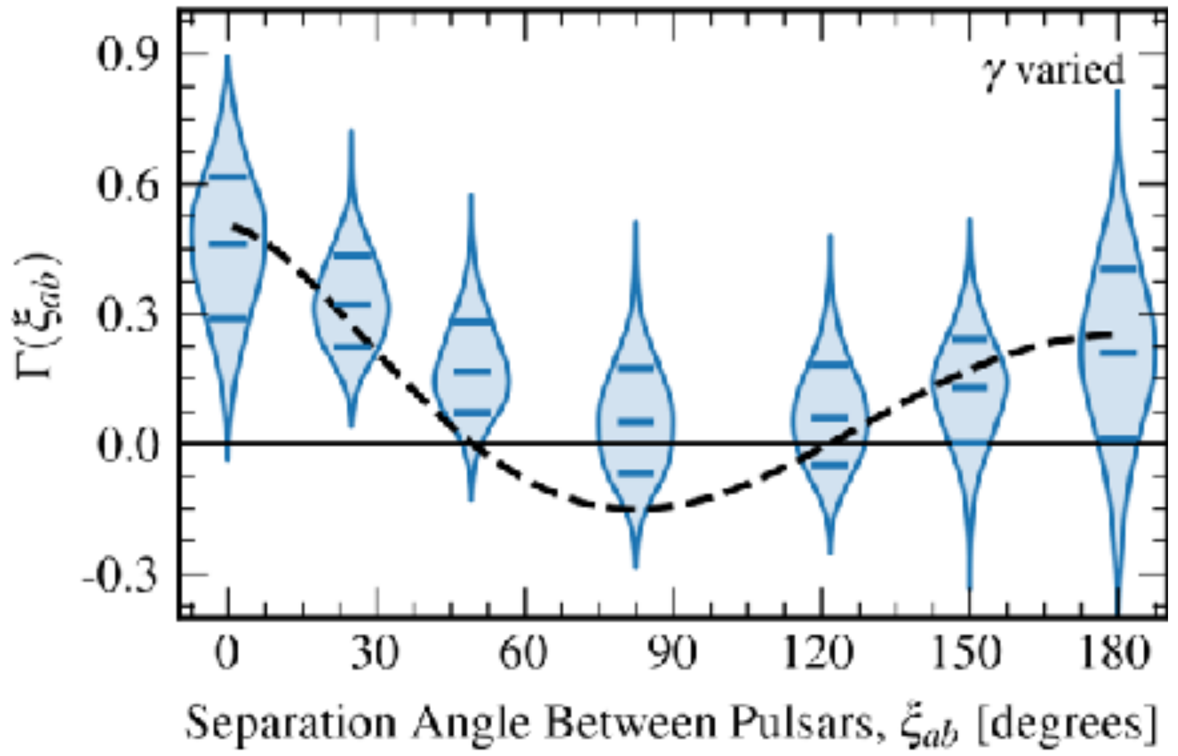


$$\mathscr{R} = -\frac{1}{\gamma}\ln\left(1-\gamma\mathscr{R}_g\right)$$

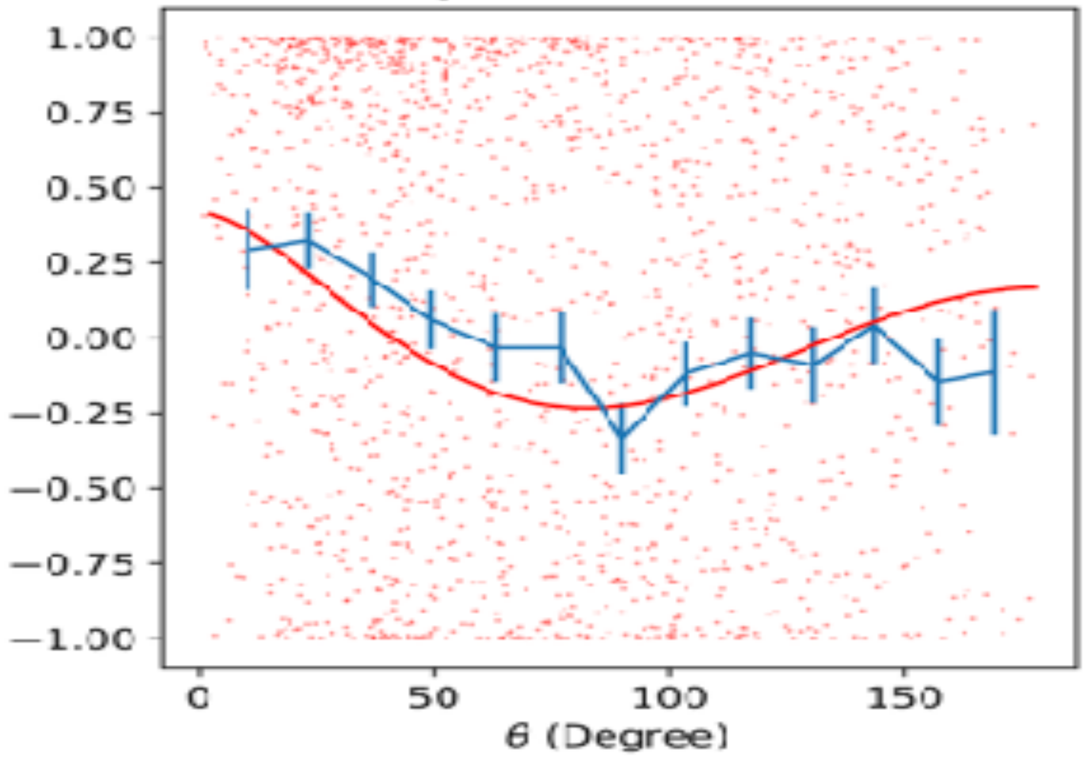
- Primordial NG must be taken into account when calculating PBH abundance
- When fixing PBH abundance, NG impact on SGWB is mild
- LISA/Taiji/TianQin can probe the induced GW when PBH=DM

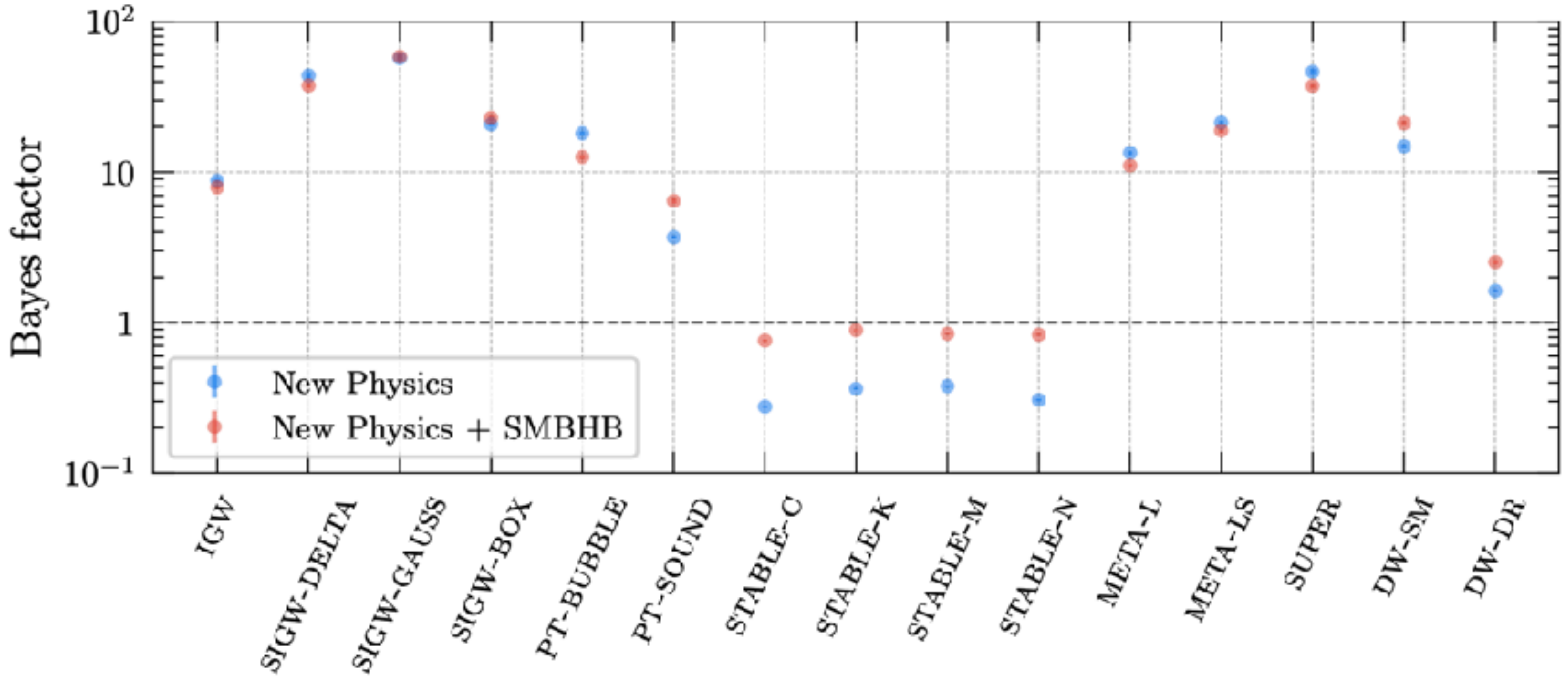
Application: nHz SGWB

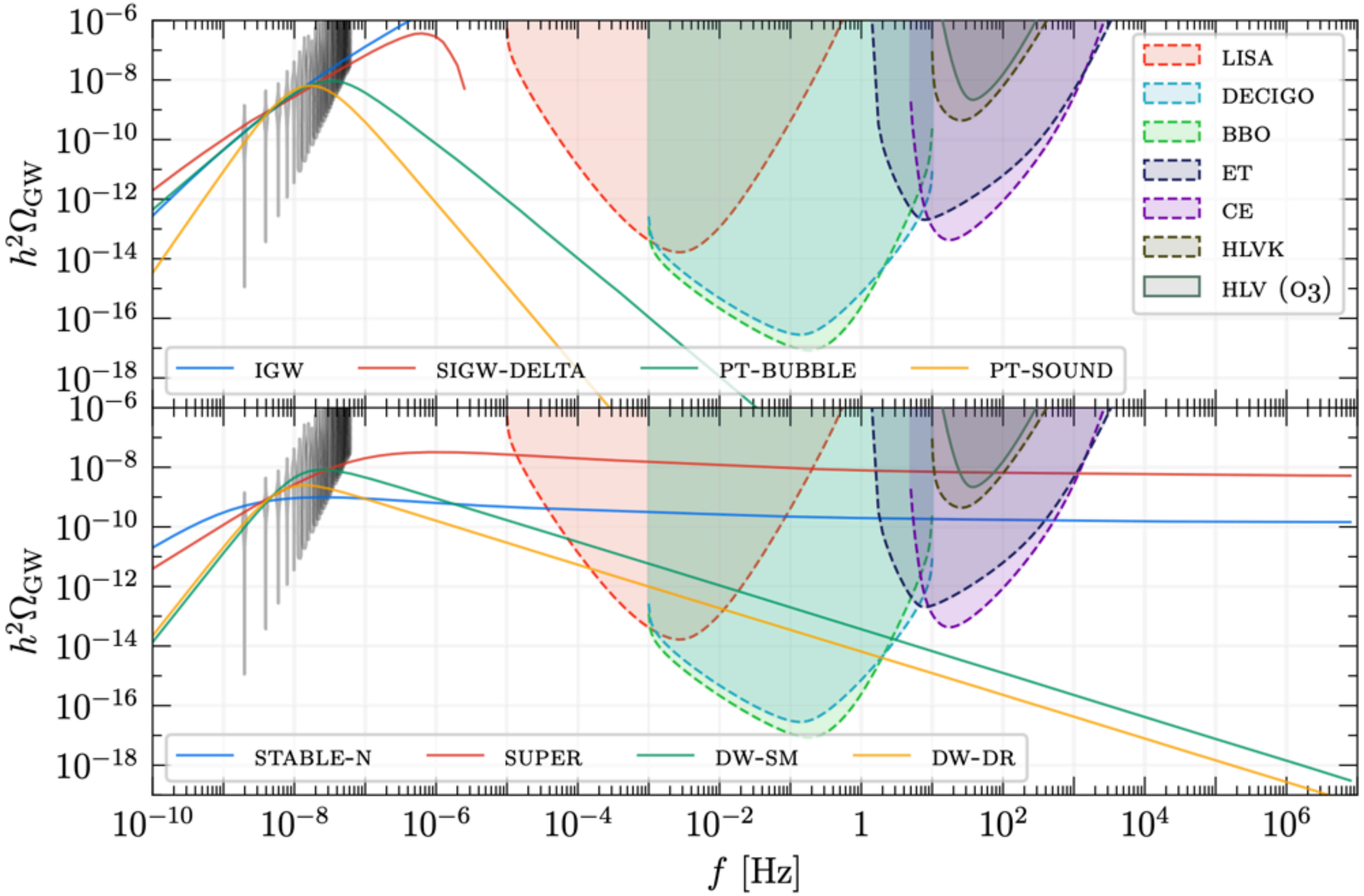
NANOGrav, 2306.16213



CPTA, 2306.16216



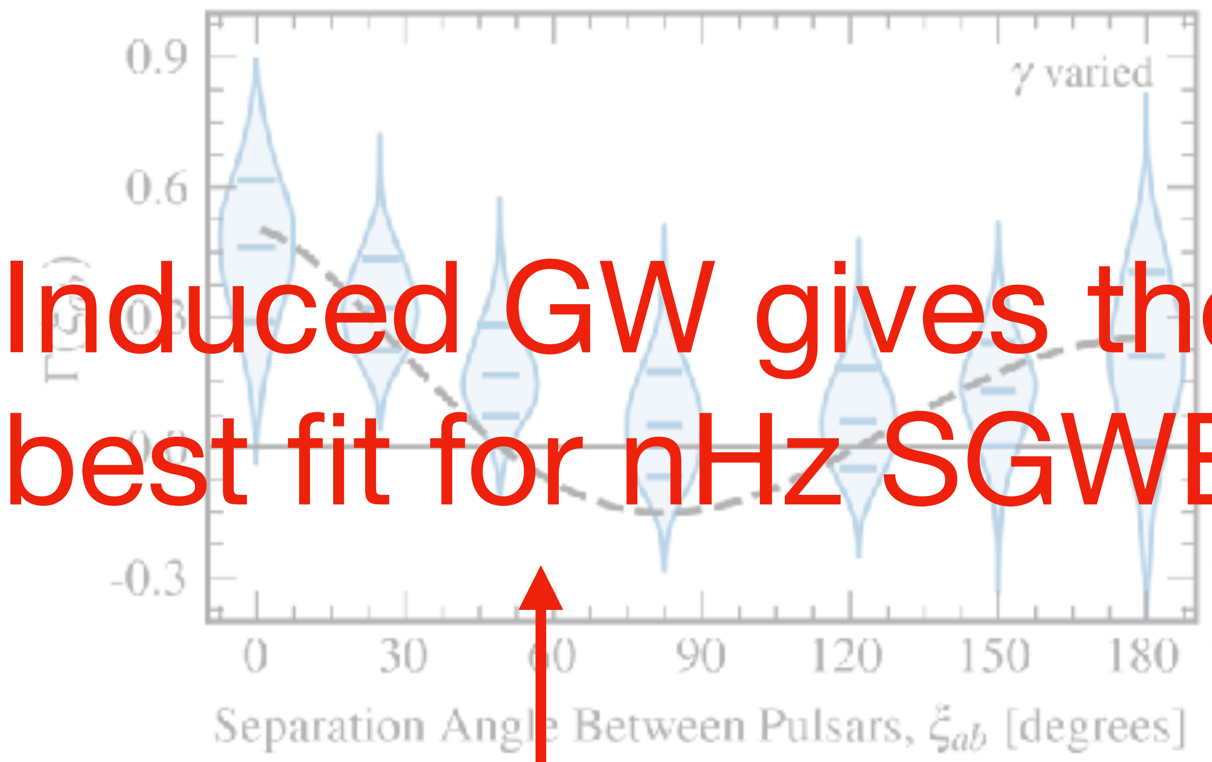




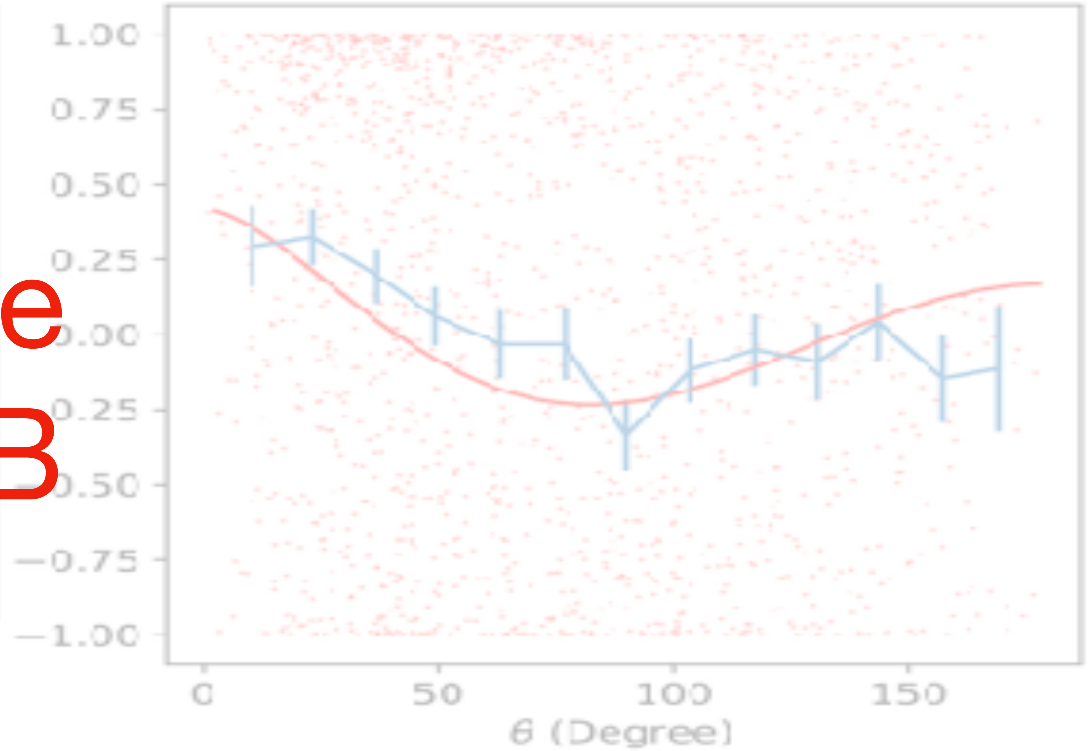
NANOGrav, 2306.16219

Application: nHz SGWB

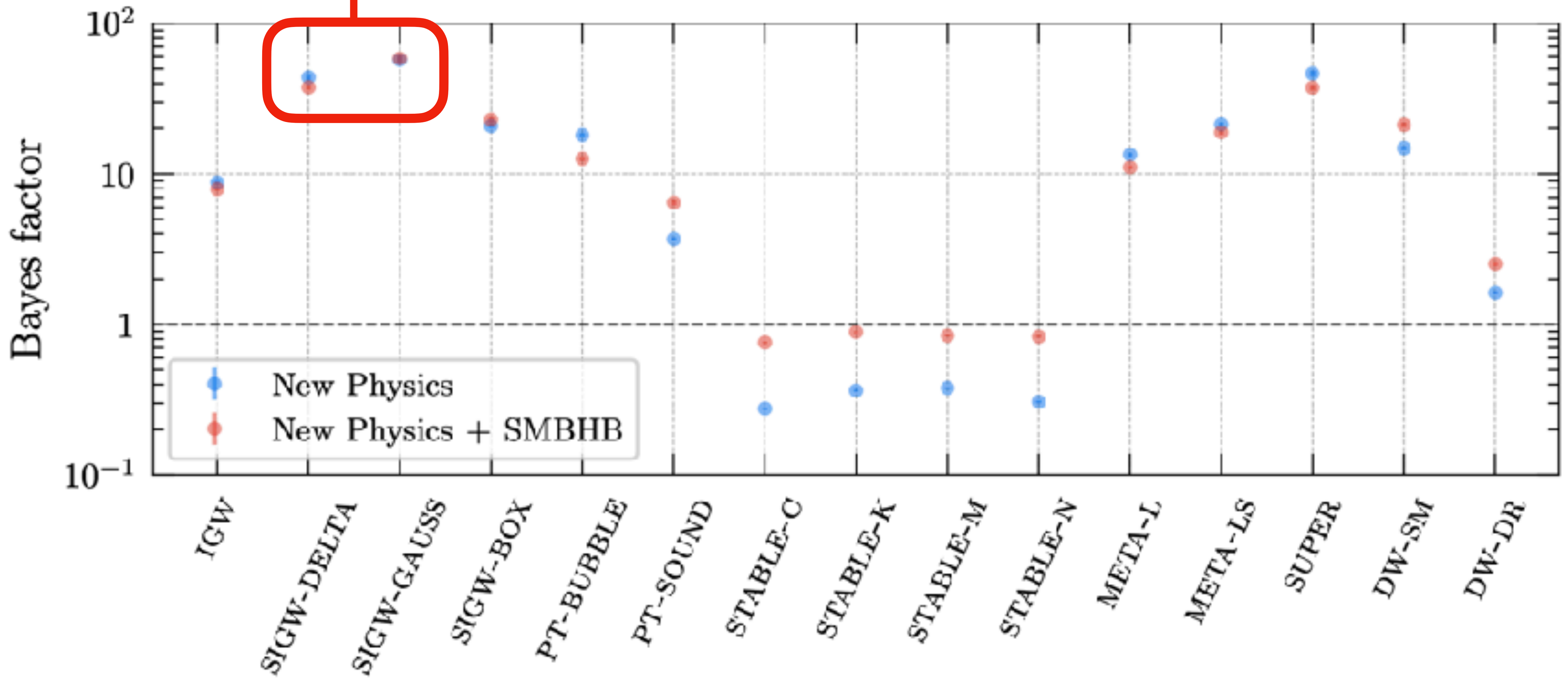
NANOGrav, 2306.16213



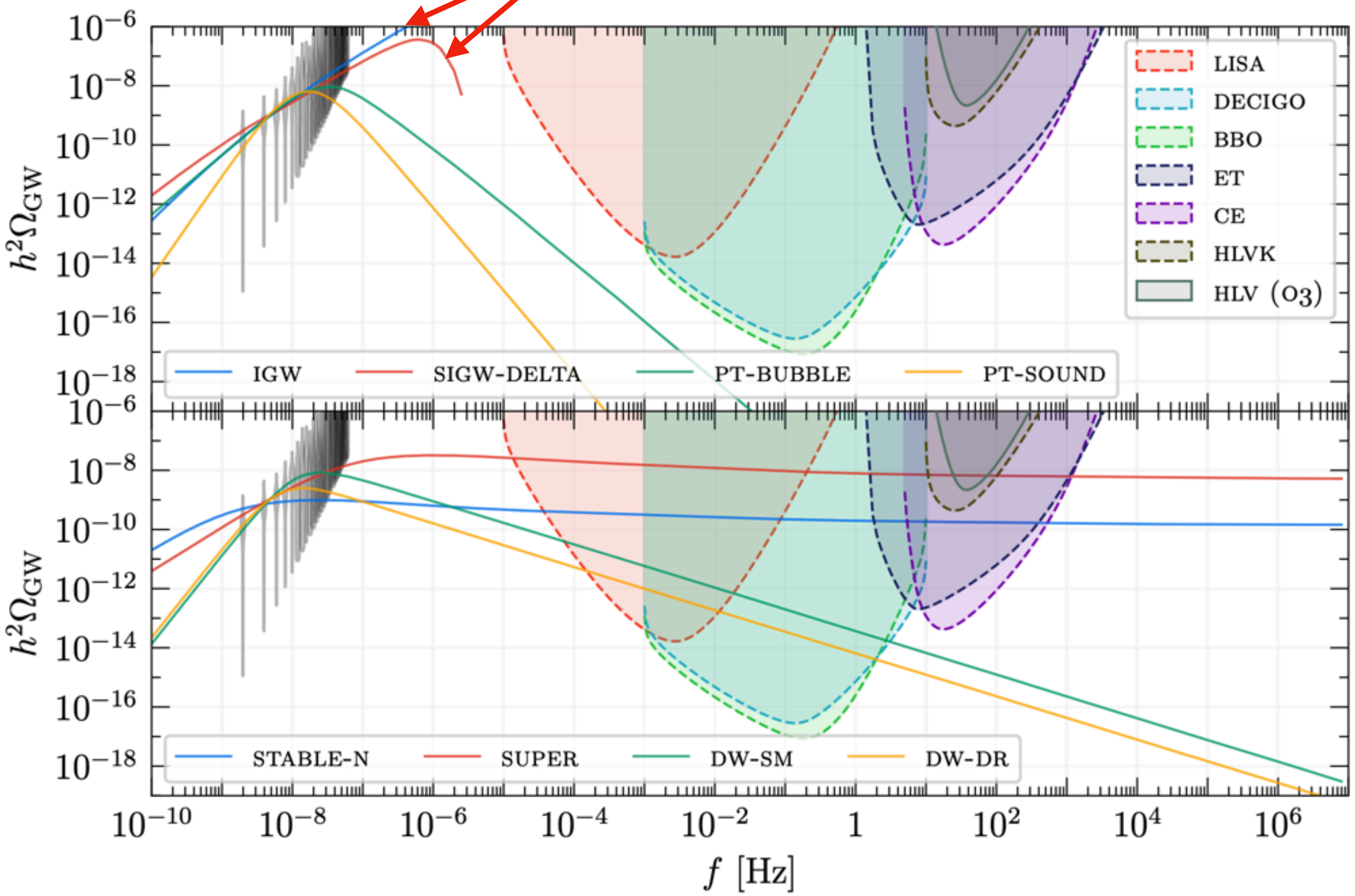
CPTA, 2306.16216



Induced GW gives the best fit for nHz SGWB

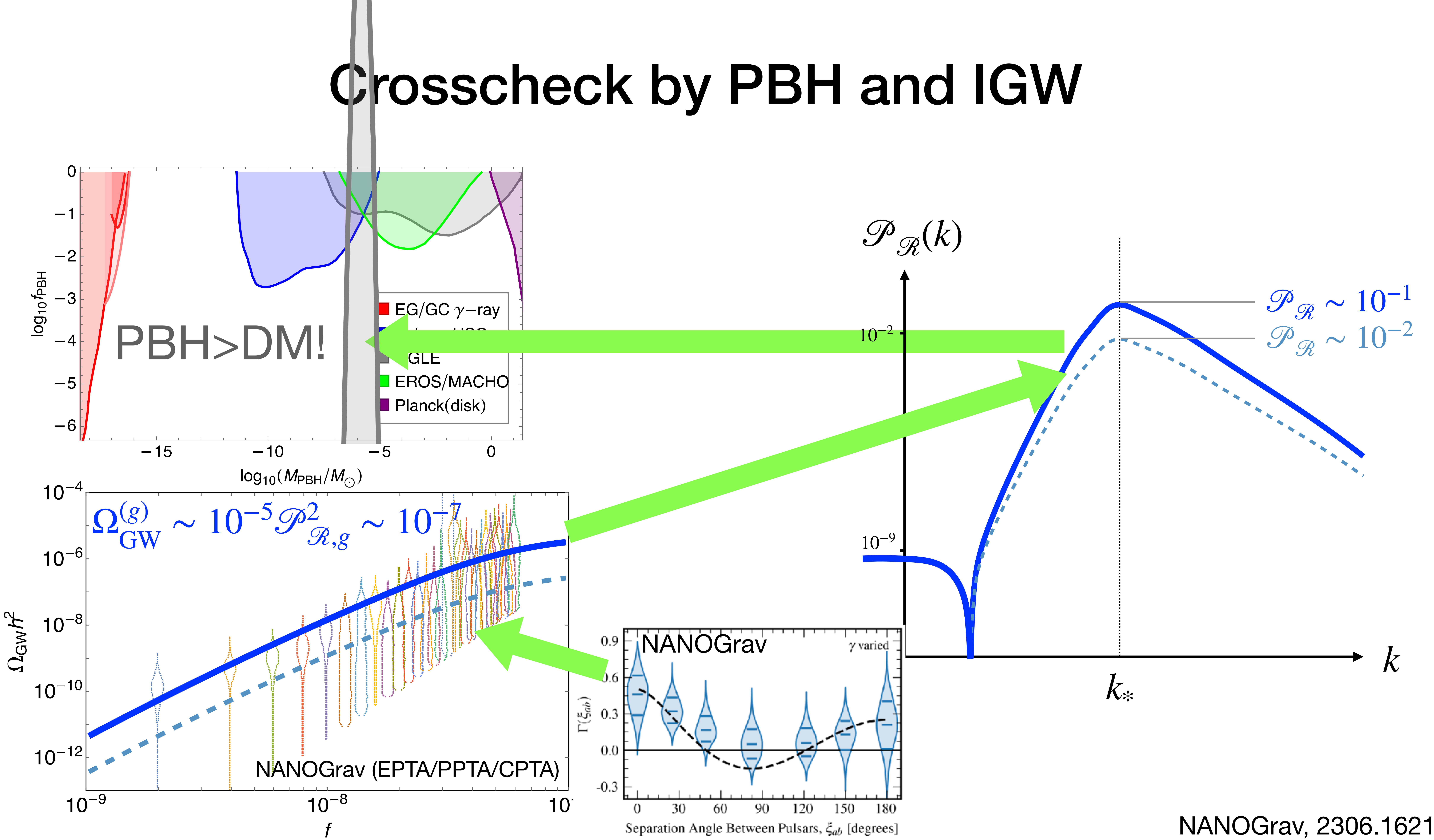


Induced GW



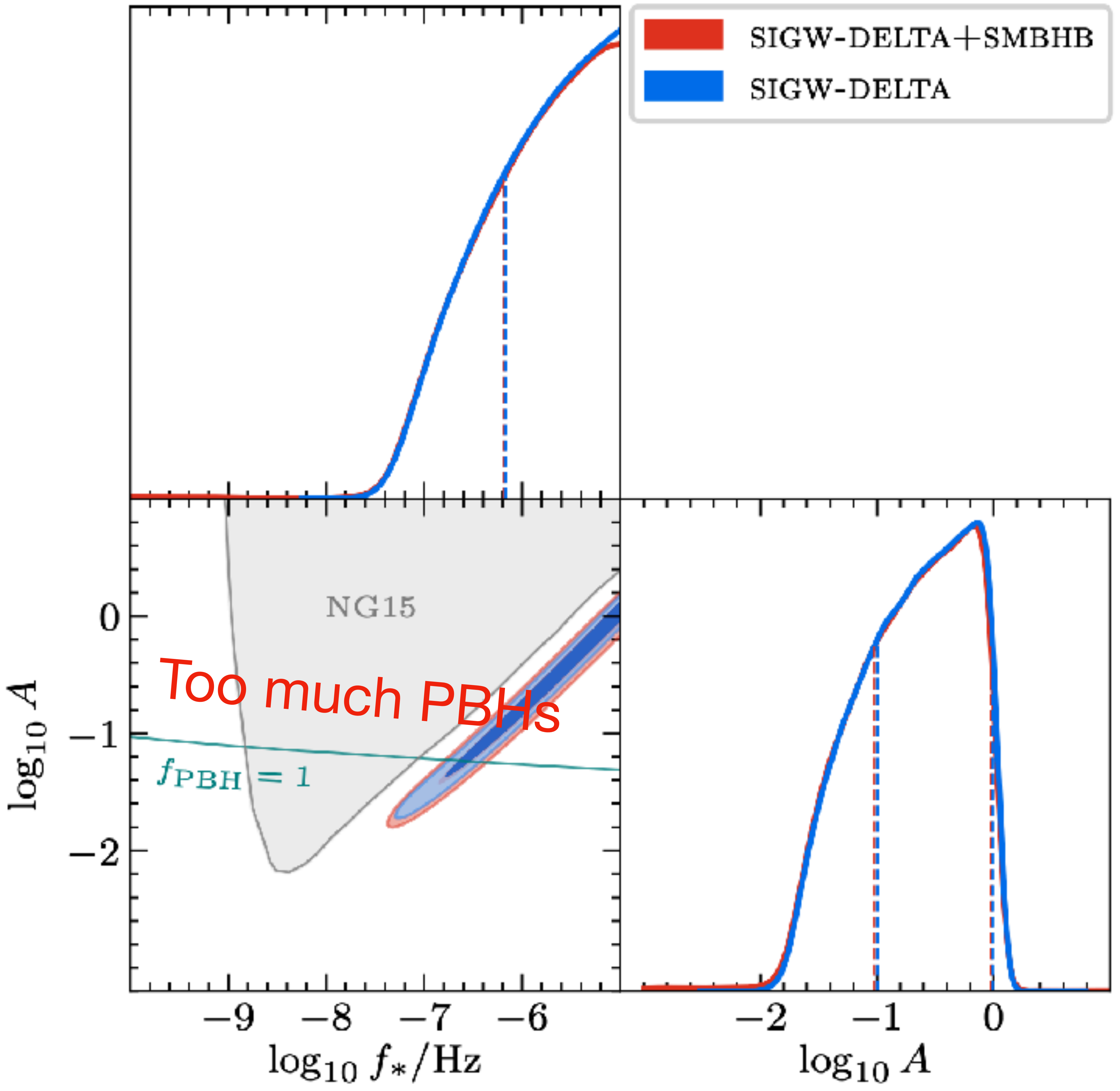
NANOGrav, 2306.16219

Crosscheck by PBH and IGW

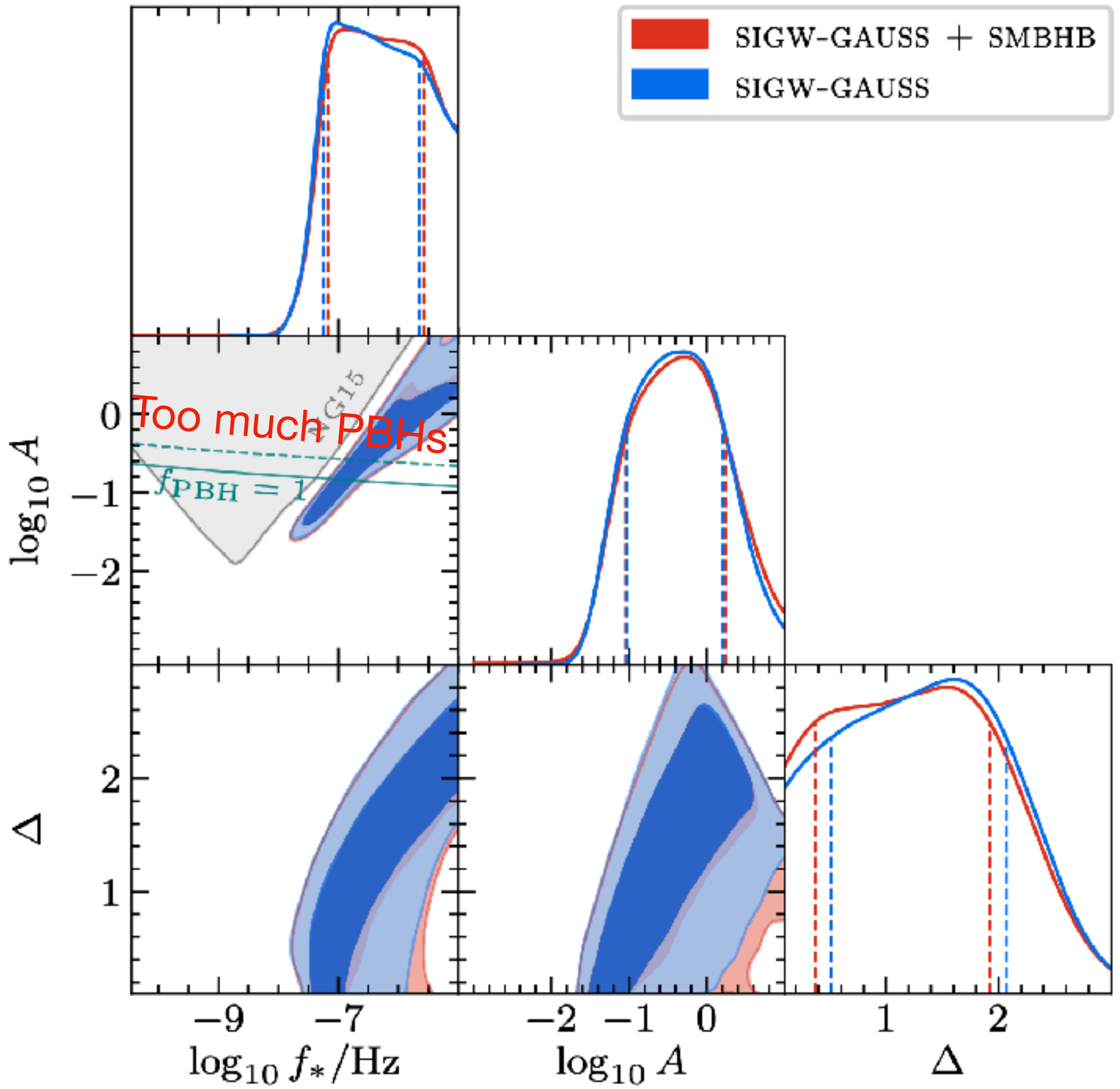


NANOGrav, 2306.16219

IGW as nHz SGWB



$$\mathscr{P}_{\mathscr{R}} = A\delta(\ln k - \ln k_*)$$



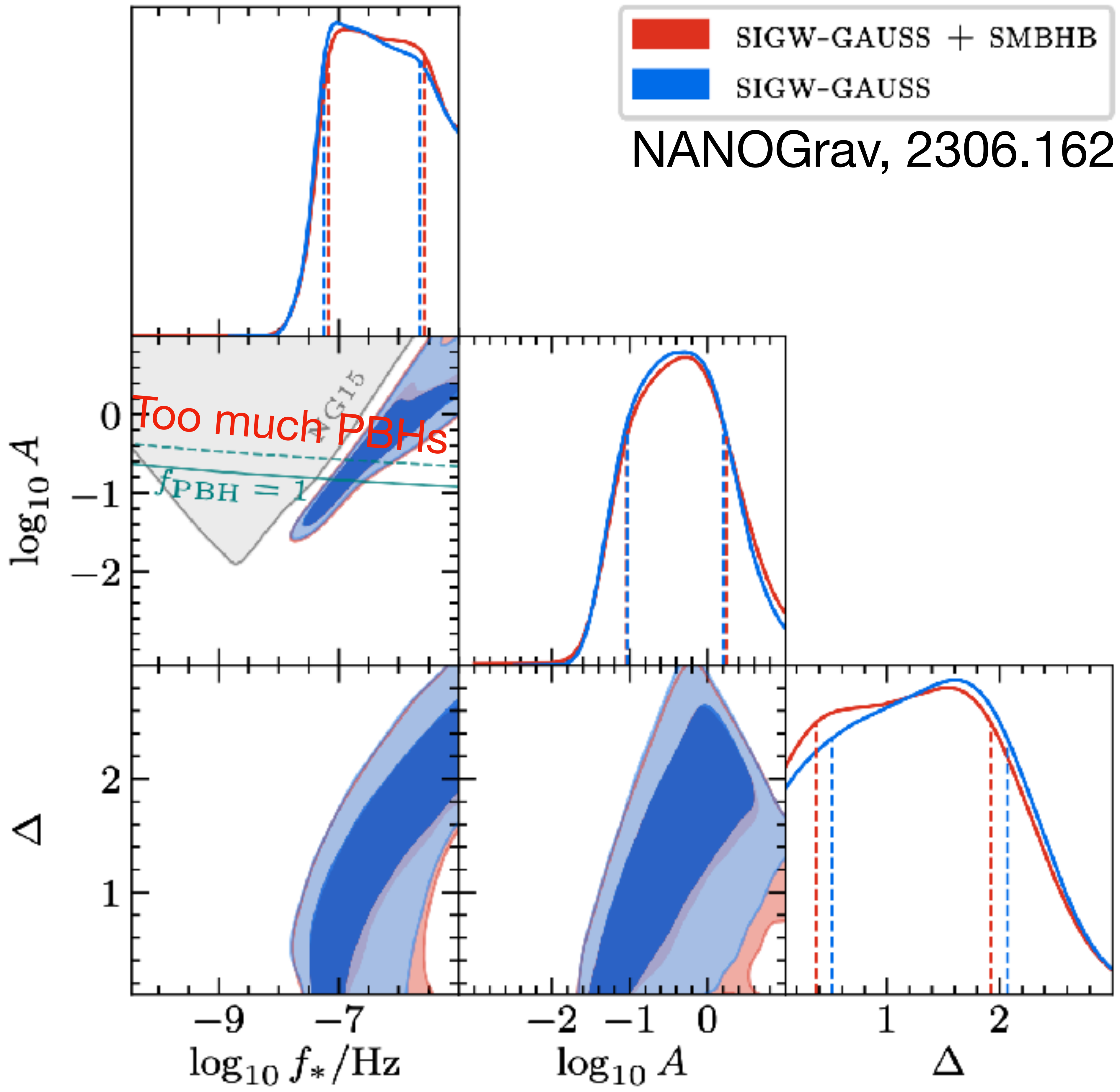
$$\mathscr{P}_{\mathscr{R}} = \frac{A}{\sqrt{2\pi}\Delta} \exp\left(-\frac{(\ln k - \ln k_*)^2}{2\Delta^2}\right)$$

NANOGrav, 2306.16219

IGW as nHz SGWB

How to solve the PBH overproduction

- (1) Use more conservative method to calculate. (Inomata et al 2306.17834; Iovino et al 2406.20089)
- (2) Suppress PBH abundance by increase the threshold, usually by changing the equation-of-state. (Domenech and SP, 2010.03976; Domenech, SP, et al, 2402.18965)
- (3) Suppress PBH abundance by negative non-Gaussianity, where the logarithmic $\mathscr{R}$ is the only known fully nonlinear expression.



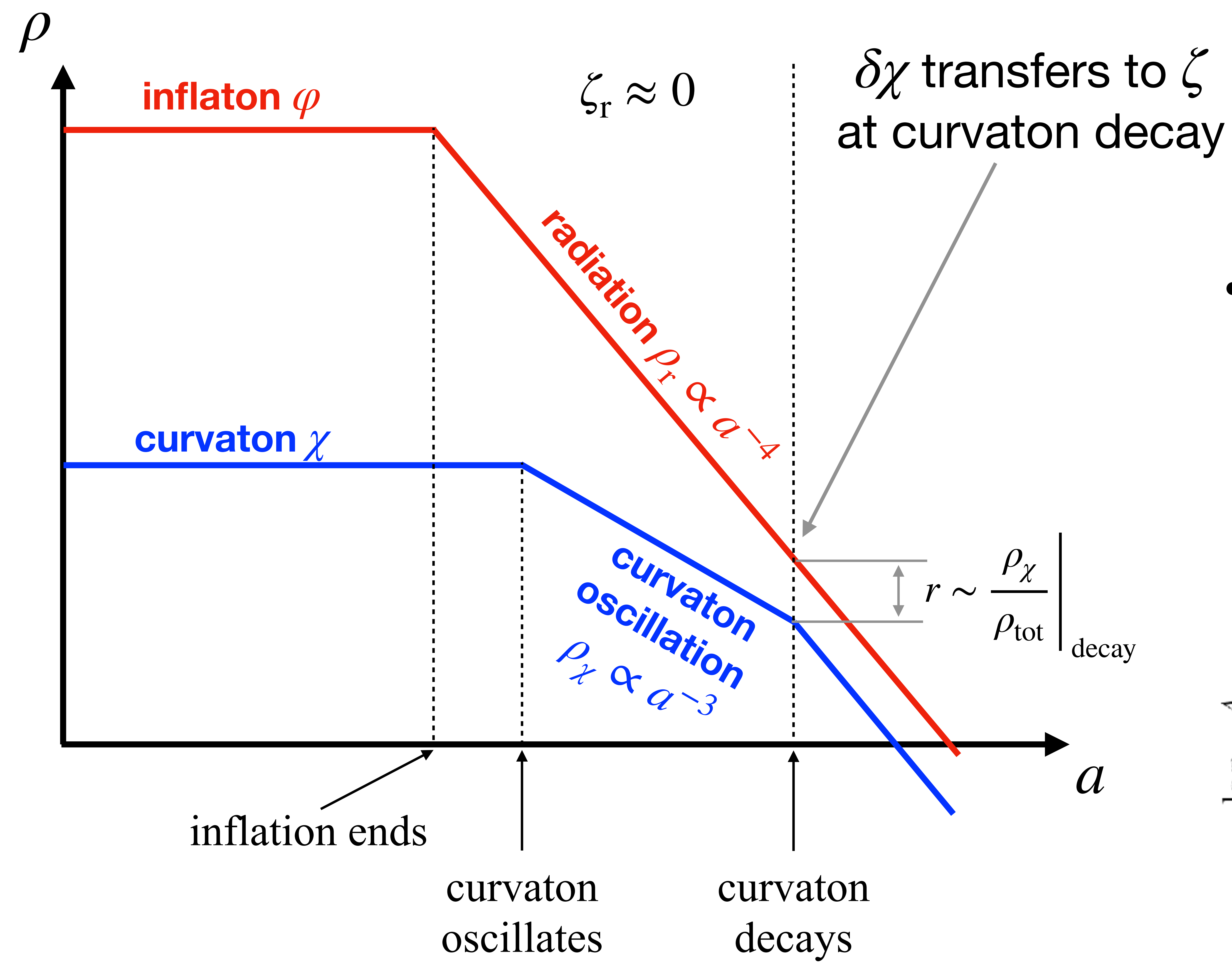
NANOGrav, 2306.16219

$$\mathscr{P}_{\mathscr{R}} = \frac{A}{\sqrt{2\pi}\Delta} \exp\left(-\frac{(\ln k - \ln k_*)^2}{2\Delta^2}\right)$$

lognormal [SP and Sasaki 2005.12306]

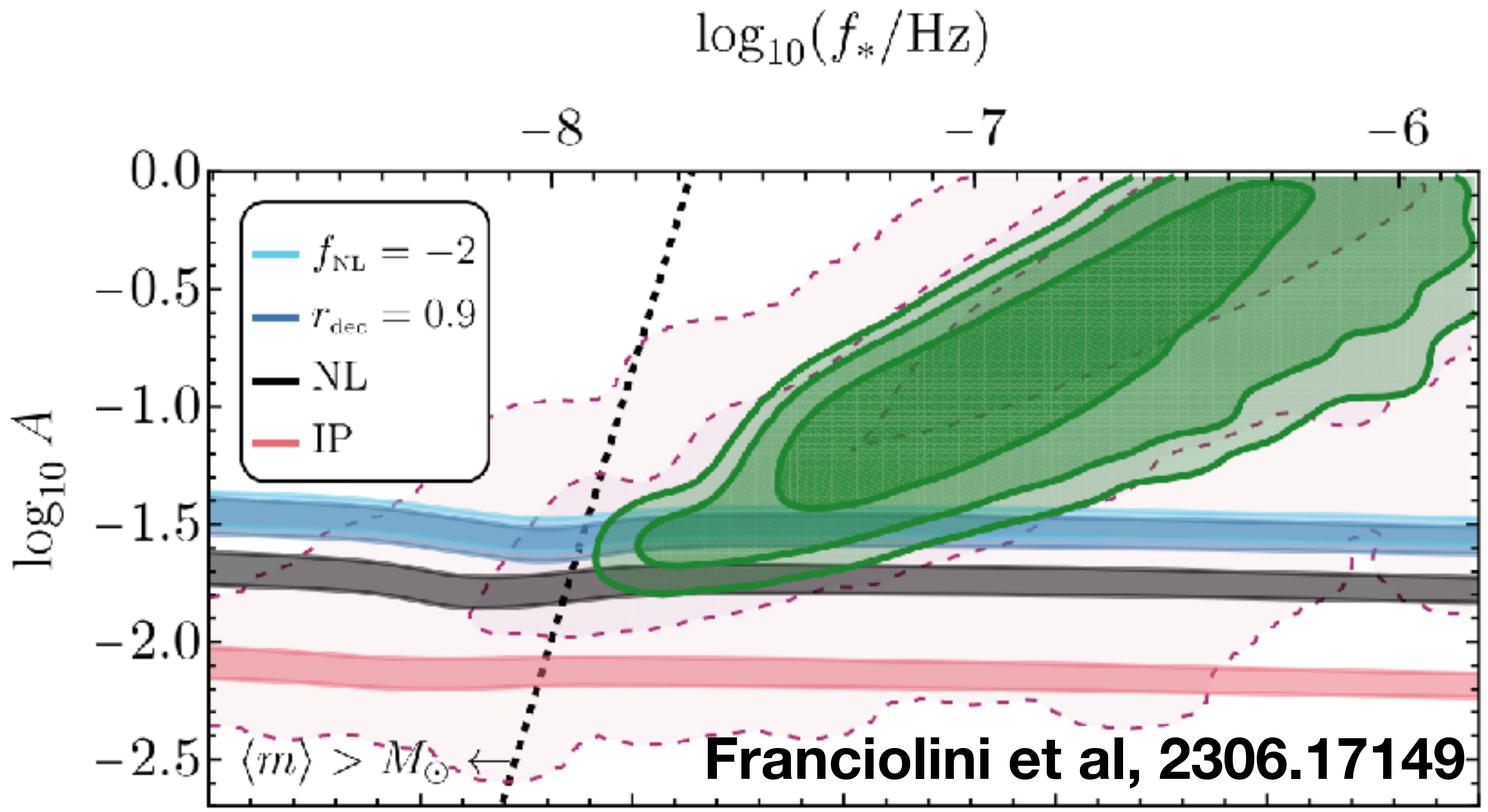
IGW as nHz SGWB

SP and Sasaki, 2112.12680

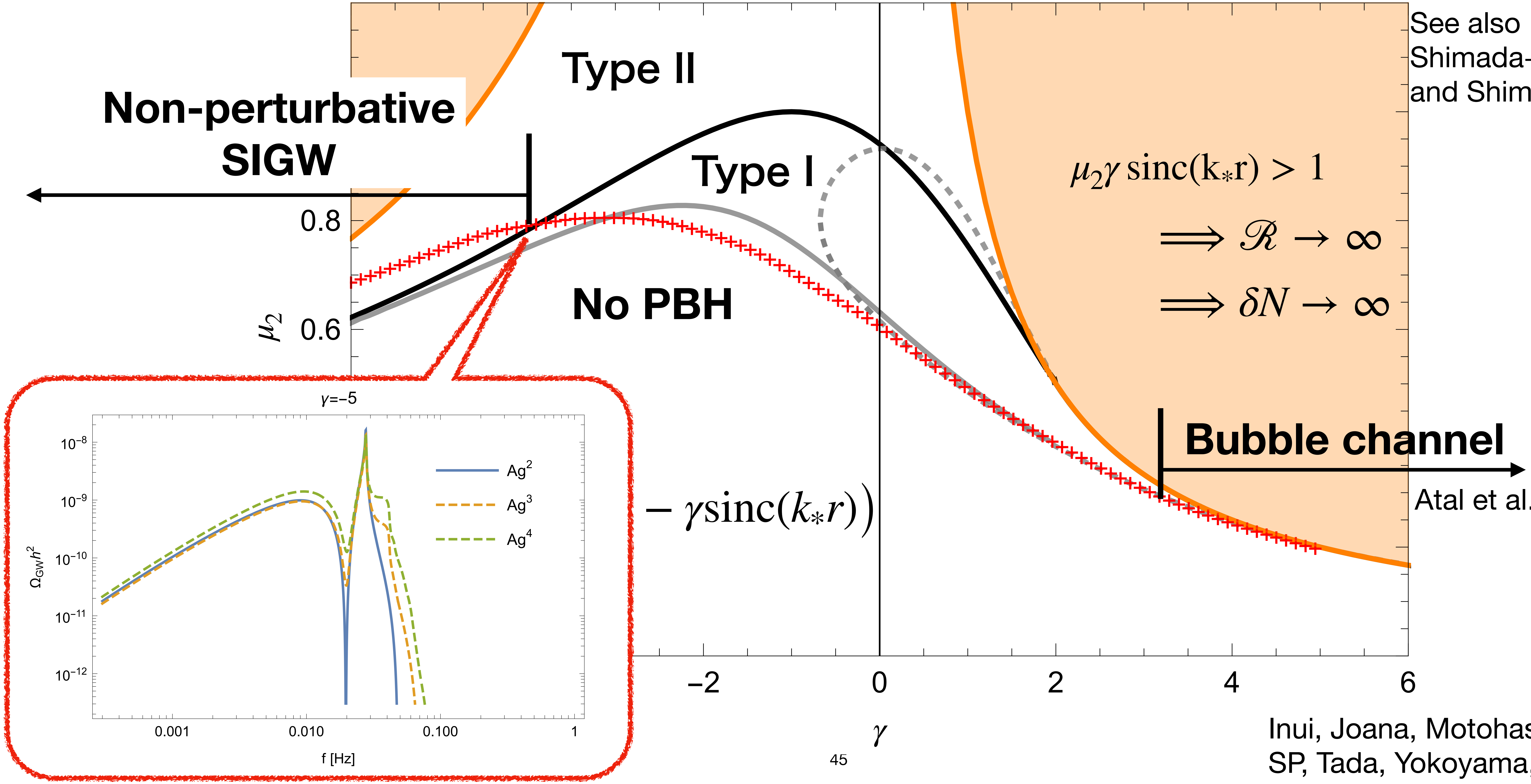


$$\zeta = \zeta(\delta\chi/\chi) \longrightarrow \begin{cases} \frac{r}{3}\left[2\frac{\delta\chi}{\chi} + \left(\frac{\delta\chi}{\chi}\right)^2\right] & \text{when } r \ll 1 \\ \boxed{\frac{2}{3}\ln\left|1+\frac{\delta\chi}{\chi}\right|} & \text{when } r \sim 1 \end{cases}$$

- $\zeta(\delta\chi)$ degenerates to a logarithmic relation ($f_{\rm NL} = -5/4$) when the curvaton dominates.



PBH in logarithmic $\mathscr{R}$

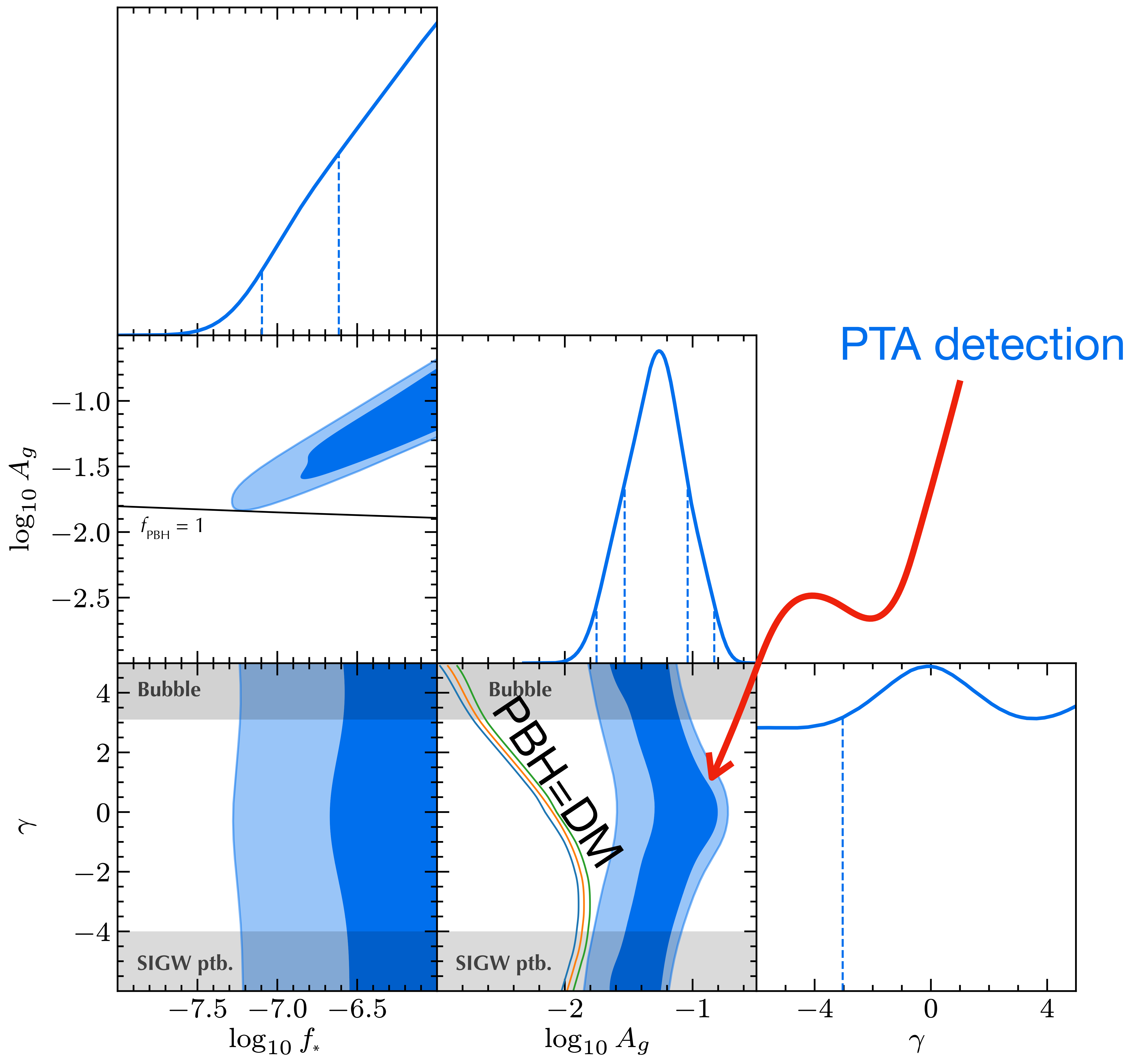


See also
Shimada+ 2411.07648
and Shimada's talk

Atal et al. 1908.11357

Inui, Joana, Motohashi,
SP, Tada, Yokoyama, 2411.07647

PTA implication



- PBH overproduction is a serious problem, mainly because we use state-od-art peak theory.
- Considering bubble channel, more PBH will form.
- Negative non-Gaussianity up to $\gamma \geq -4$ can not help.
- Considering higher-order contribution to induced GW, the required $\mathscr{A}_g$ is smaller, which may alleviate the tension. However we do not have a systematic method of calculating.

Inui, Joana, Motohashi, SP, Tada, Yokoyama, 2411.07647

Conclusion

- Primordial non-Gaussianity must be taken into account when calculating PBH abundance.
- The mHz induced GW of PBH-DM is robust against non-Gaussianity, which is an important scientific goal of LISA/Taiji/TianQin.
- In logarithmic form of non-Gaussianity, PBH overproduction seems to be a serious problem, unless we enter the non-perturbative regime of induced GW.