Spins of Primordial Black Holes in a matter-dominated era

Daiki Saito (Nagoya U.)

with T.Harada (Rikkyo U.), Y.Koga (YITP), C-M.Yoo (Nagoya U.)

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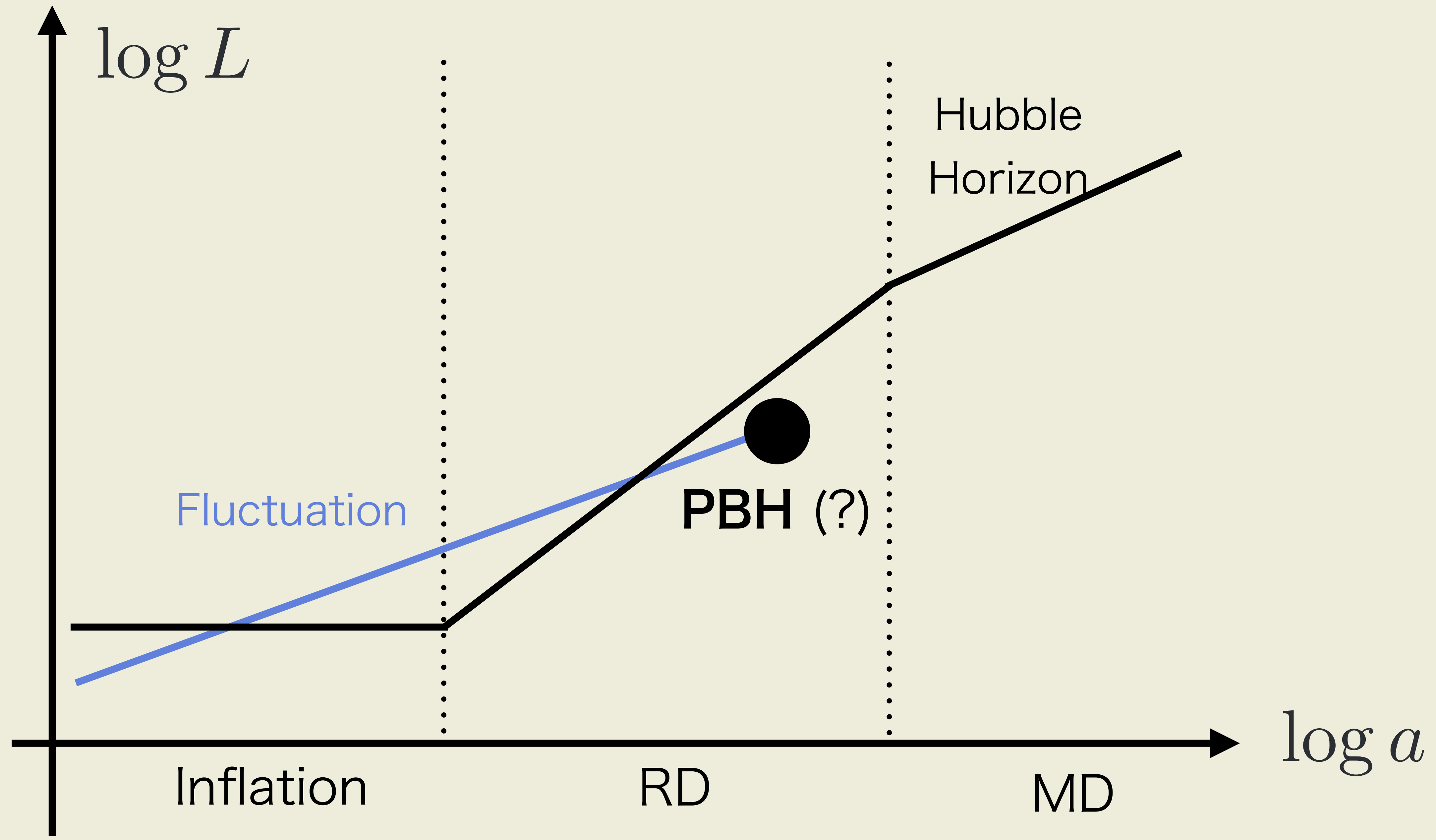
Introduction

PBH: BH formed in the early universe (Needless to say …)

- Remnant of primordial inhomogeneity (Primordial scalar perturbation)
- Candidate for DM
- Candidate for supermassive BH
- Source for GW (Binary, 2nd order GW)

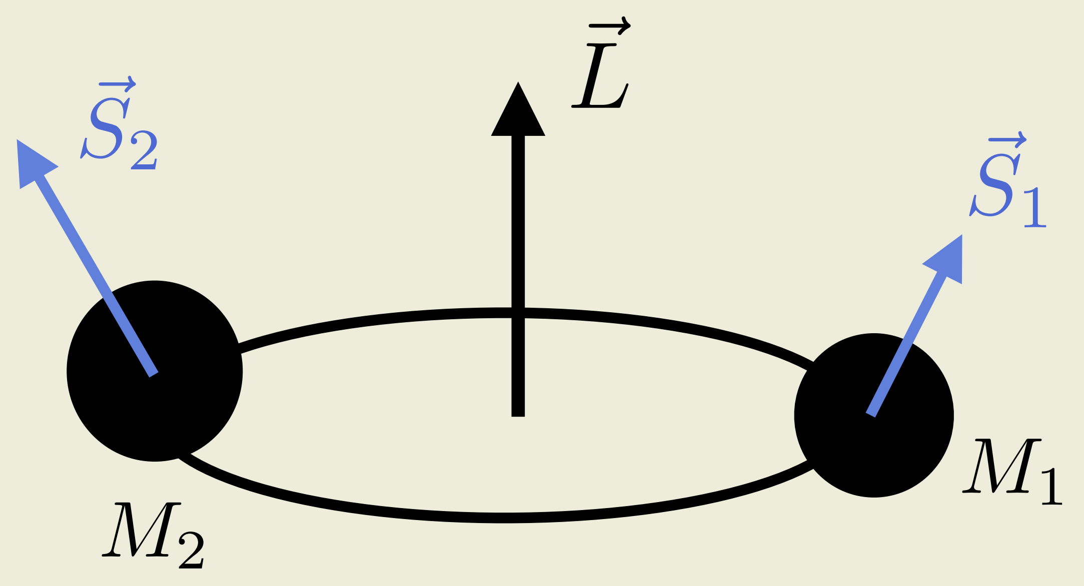
How PBHs form?

- (typically) formed in the RD era
- Formed from e.g. collapse of density fluctuation



Introduction

Q. What characterize formed PBH? What is observable ?



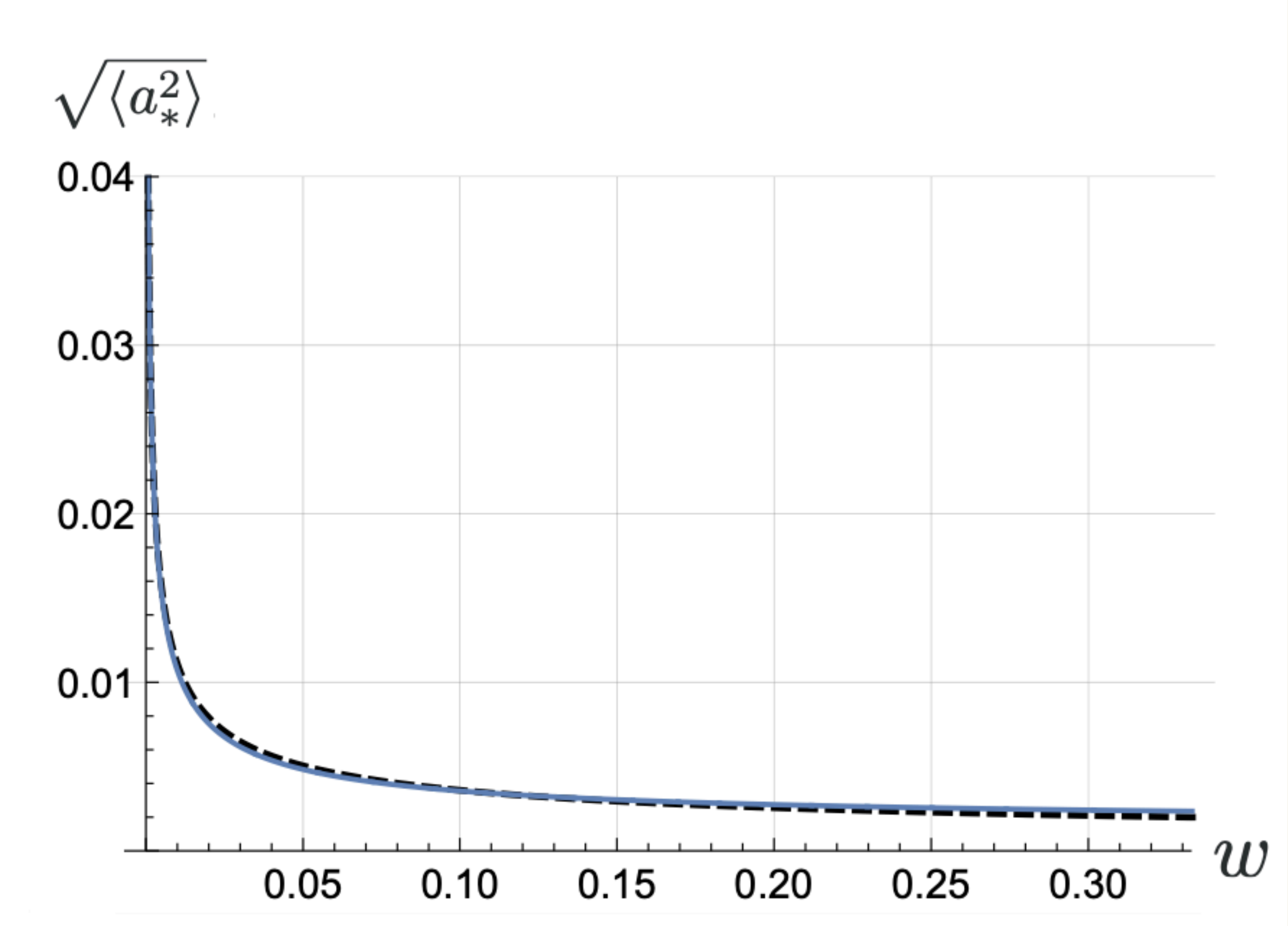
A. Mass & Angular momentum

Theory: No-hair theorem

Obs. : GW from BBH

$$\mathcal{M} = \frac{(M_1 M_2)^{3/5}}{(M_1 + M_2)^{1/5}} \qquad q = \frac{M_2}{M_1} \qquad \boxed{\chi_{\mathrm{eff}} = \frac{M_1 \vec{S}_1 + M_2 \vec{S}_2}{M_1 + M_2} \cdot \vec{L}}$$

Our interest: Estimate spin of a single PBH



In RD era: spin is small $a_* \lesssim O(10^{-3})$

$a_* = \dfrac{a}{M}$: non-dim. Kerr parameter

Fluctuation with high peak $\longrightarrow$ near-spherical collapse

($\because$ Peak theory)

Soft EoS: spin is amplified [DS, Harada, Koga, and Yoo 2023]

$\sqrt{\langle a_*^2 \rangle} = O(0.1)$ for $w \leq 0.06$

Introduction

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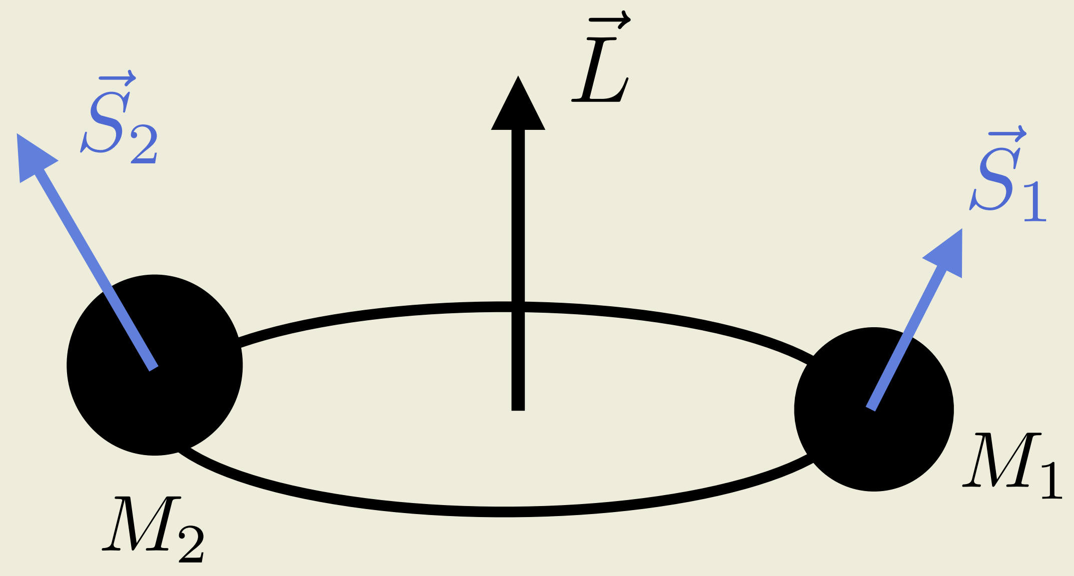

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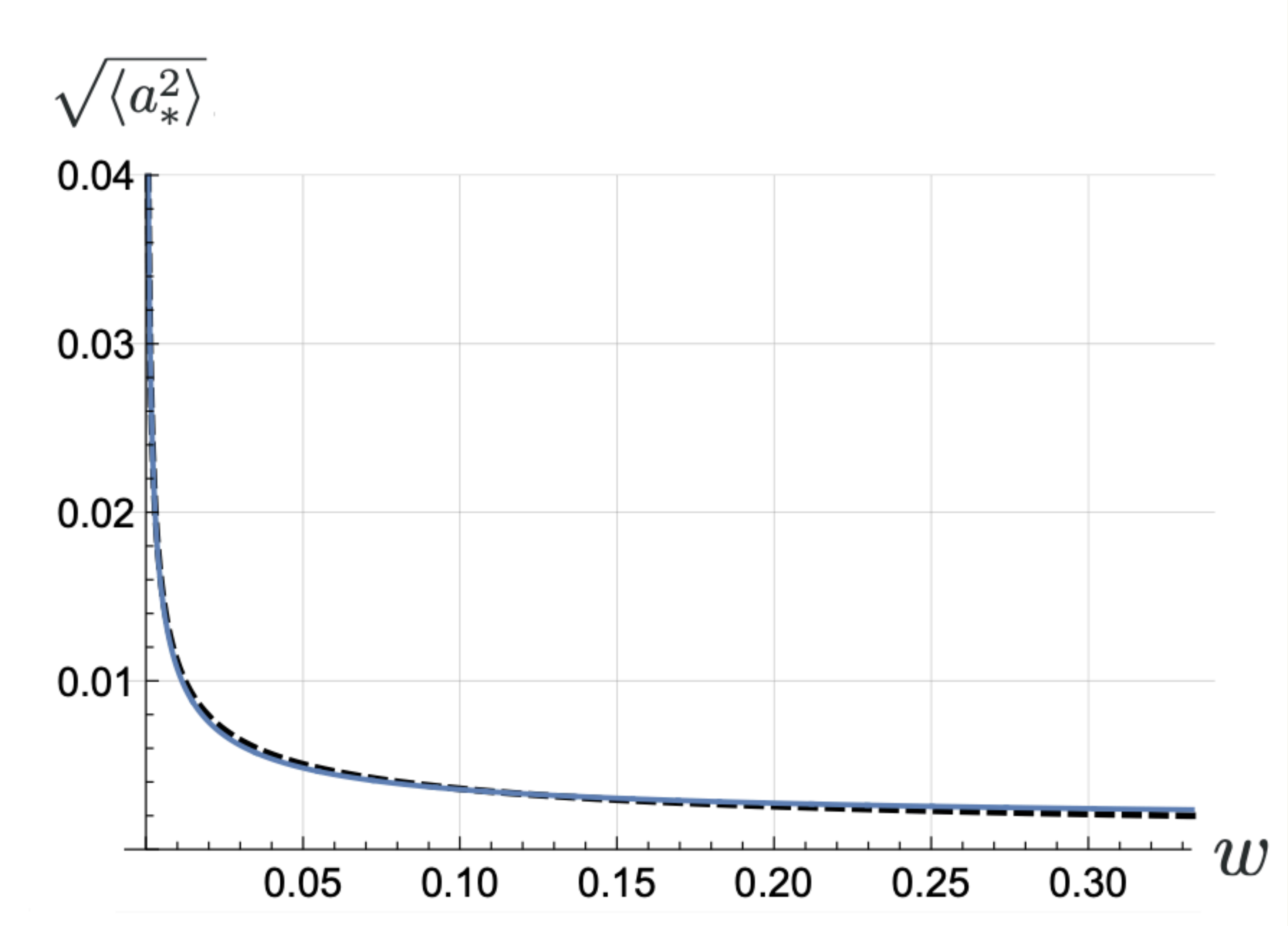
How about other scenarios?

In this work, we discuss PBH spin in MD era

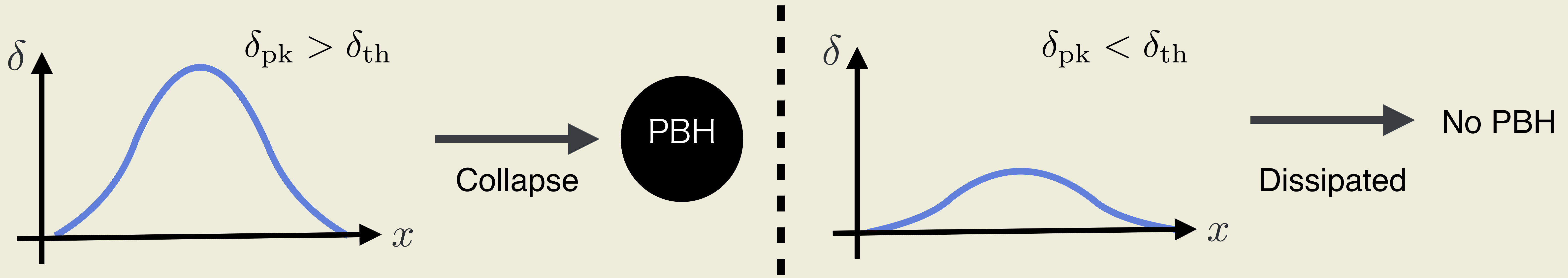


Introduction

early

PBH formation in MD era

(Inflaton oscilation etc..)



MD era: no pressure gradient

- fluctuation with small amp. may collapse *can be highly non-spherical*
- other mechanisms e.g.
  - non-sphericity [Khlopov and Pornalev 1980, Harada Yoo, Kohri, Nakao, and Jhingan 2016]
  - velocity dispersion [Harada, Kohri, Sasaki, Terada, and Yoo 2022]
  - spin [Harada, Yoo, Kohri, and Nakao 2017]

  of fluctuation may impede collpase

Rotation could be important for PBH formation (even before it forms!)

This work: focus on spin effect of fluid on PBH formation

Introduction

Aim:

- Discuss spin effect in overdense region on PBH formation
- Evaluate spin of resulting PBH

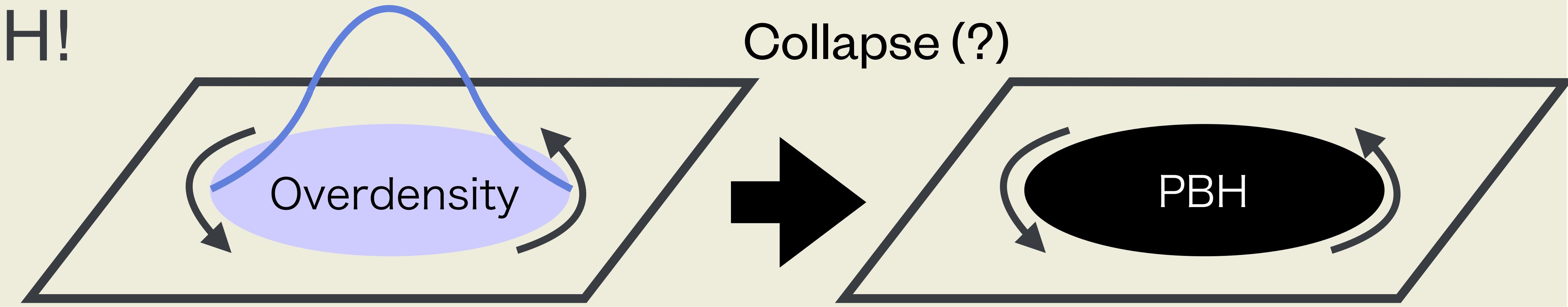
Strategy:

Compute angular momentum inside closed region based on perturbative method

Generated by perturbation (Tidal torque) — Around density peak

Spin is too large ⟶ Do not collapse, No PBH!

Spin is not so large ⟶ PBH forms



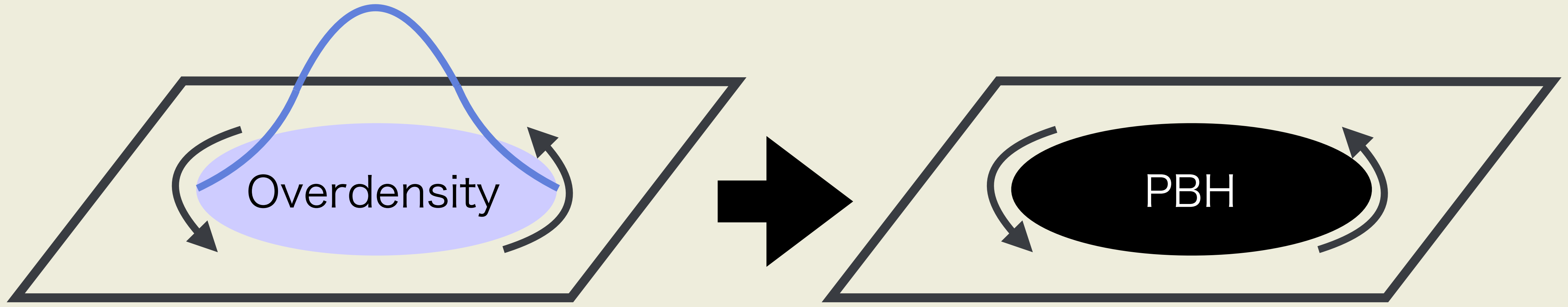
We may investigate PBH formation threshold & typical spin of PBH

Assumptions

Strategy:

Compute angular momentum inside closed region

(Tidal torque of fluid)



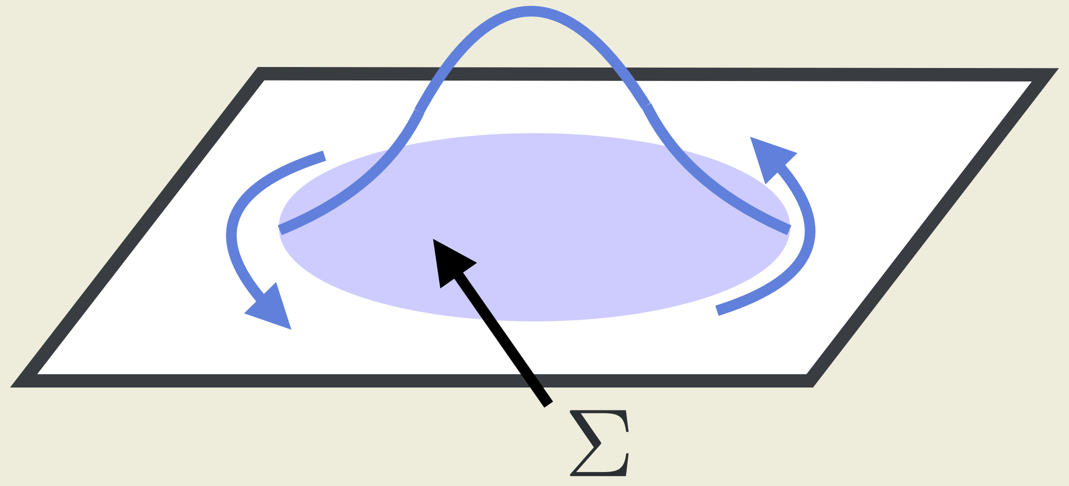
Assumptions:

- Matter: dust fluid $\longrightarrow$ Newtonian approximation
- Background : Einstein de Sitter (Spatially flat, matter dominated)
- Apply the Zel'dovich approx. Discuss in Lagrange coord. $\vec{x} = \vec{q} + \vec{D}(t, \vec{q})$

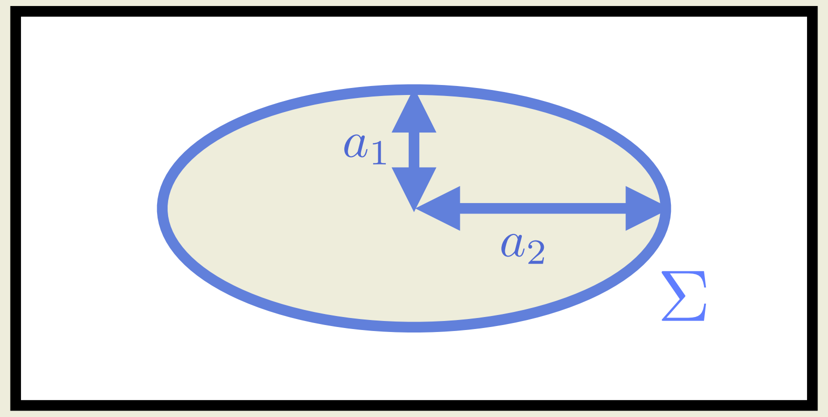
  Linearize displacement $\vec{D}(t, \vec{q}) \propto -\nabla_{\vec{q}} \underbrace{\psi(t, \vec{q})}_{\text{Grav. Pot.}}$
- Focus on linear order effects in perturbation $\left(\vec{D}(t, \vec{q})\right)$ on spin
- Gravitational potential $\psi(t, \vec{q})$: random Gaussian field $\langle \psi_{\vec{k}} \psi^*_{\vec{k}'} \rangle \propto \delta^3\left(\vec{k} - \vec{k}'\right) P_\psi(k)$

Spin in a closed region

Σ: collapsing region (overdense region: $\delta(\vec{x}) > 0$) $\sim$ ellipsoid aroud density peak

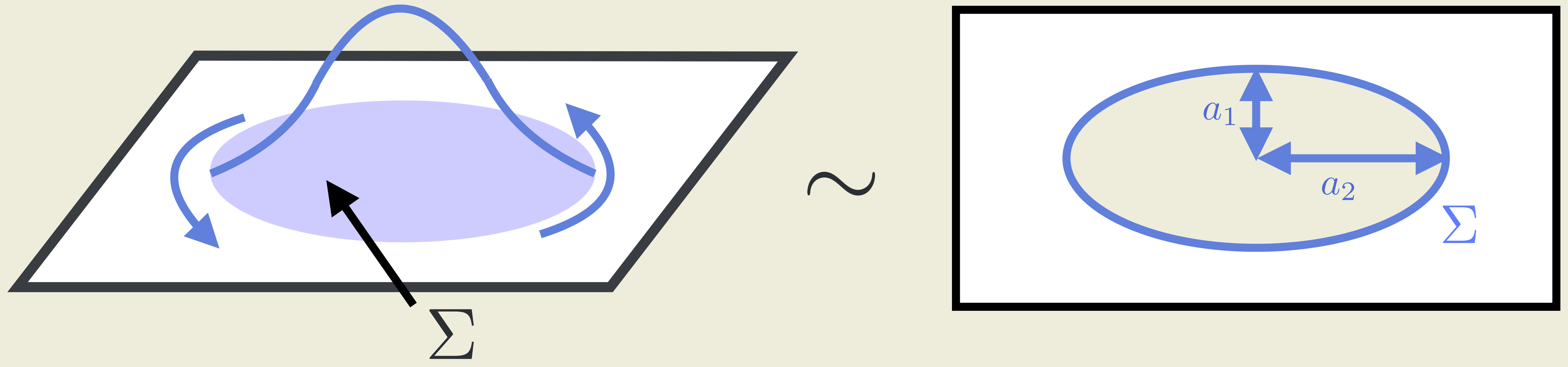


$\sim$



Spin in a closed region

Σ: collapsing region (overdense region) $\sim$ ellipsoid



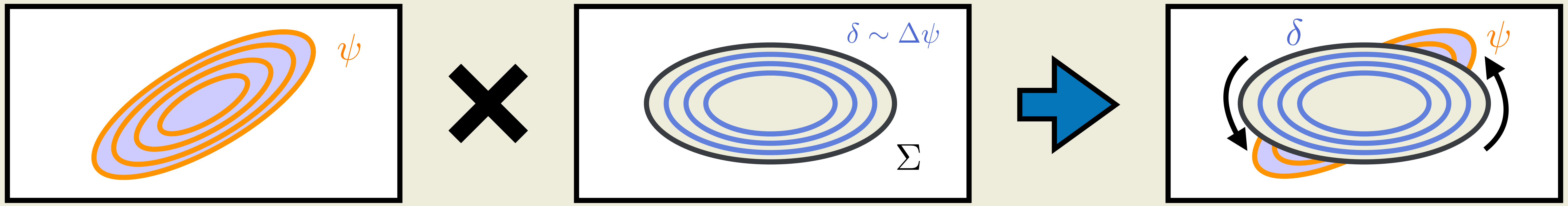
Spin inside Σ : $\vec{S} := \int_\Sigma d^3(a\vec{x})\rho(a\vec{x}\times\vec{v})$

$$\sim \quad ([A_2^2 - A_3^2]v_{23}, [A_3^2 - A_1^2]v_{13}, [A_1^2 - A_2^2]v_{12})$$

Linear order effect

$A_i \sim \frac{\partial^2\delta}{\partial q^i \partial q^j}$: "curvature" of δ $\qquad$ $v_{kl} \sim \frac{\partial^2\psi}{\partial q^l \partial q^k}$: "curvature" of ψ $\qquad$ $\vec{S} \sim$ Products of curvature of ψ and δ

$S_i \neq 0 \Rightarrow$ non-spherical Σ & misallignment of ψ and δ ***(tidal torque)***



M : mass inside $\Sigma \longrightarrow$ Non-dimensional spin $a_{*i} = \frac{S_i}{M^2}$ depends on $\delta, \delta_{ij}, v_{ij}$

Peak theory and RMS of spin

$\sigma_j^2 := \int \frac{d^3\vec{k}}{(2\pi)^3} k^{2j} |\delta_{\vec{k}}|^2$ $\sigma_{\rm H} := \sigma_0(t_{\rm H})$ $t_{\rm H}$: horizon reentry

$\nu = \frac{\delta_{\rm pk}}{\sigma_0}$ $\lambda_i \sim -\frac{\partial_i^2 \delta_{\rm pk}}{\sigma_2}$ $w_i \sim -\frac{\psi_{,ij}}{\sigma_0}$ (Non-diag.)

$Q(\lambda_i)$: (non-dim) quadrupole moment

We want to know RMS of spin $a_*(\delta, \delta_{ij}, v_{ij}, t)$

Statistics of fluctuation $V_i = \{\delta, \partial_i\delta, \partial_{ij}\delta, v_{ij}\} \longrightarrow \sqrt{\langle a_*^2 \rangle}$

$\psi(\vec{q})$: Gaussian ➡ V_i are Gaussian at linear order

Peak theory: theory for statistics of peaks of Gaussian variables [Bardeen, Bond, Kaiser, and Szalay, 1984]

Gaussian distribution $f(V_i) d^{16}V_i = \frac{1}{(2\pi)^8 \sqrt{\det M}} \exp\left[-\frac{1}{2} V_i (M^{-1})^{ij} V_j\right] d^{16}V_i$ $M_{ij} = \langle V_i V_j \rangle$: correlation

$\xrightarrow[\text{Evaluate at peak}]{\text{Integration,}}$ # density of density peak $\mathcal{N}_{\rm pk}(\nu, \lambda_i, w_i)$

Average of a_* over shapes $\lambda_i \sim \partial_i\partial_j\delta, \quad w_i \sim \partial_i\partial_j\psi$

$$\longrightarrow \bar{a}_*(\nu, t) = \frac{2 \cdot 3^{1/2}}{5^{3/2}} \sqrt{1-\gamma^2}\,\sigma_{\rm H} \left(\frac{t}{t_{\rm H}}\right) \sqrt{\langle Q^2 \rangle_\lambda}$$

RMS of spin with fixed amp. $\nu = \frac{\delta_{\rm pk}}{\sqrt{\langle \delta^2 \rangle}}$

evaluate at turn-around time $t_{\rm ta}$: fluctuation turn to collapse ~ maximum expansion in dust universe

$$\longrightarrow \boxed{\bar{a}_*(\nu, t_{\rm ta}) = \frac{2^{1/2} \cdot 3}{5^{3/2}} \sqrt{1-\gamma^2}\,\sigma_{\rm H}^{-1/2} \sqrt{\langle Q^2 \rangle_\lambda}}$$

Result: spin of overdensity

RMS of spin at turn-around time

$$\boxed{\bar{a}_*(\nu, t_{\rm ta}) = \frac{2^{1/2}\cdot 3}{5^{3/2}}\sqrt{1-\gamma^2}\sigma_{\rm H}^{-1/2}\sqrt{\langle Q^2\rangle_\lambda}}$$

$\nu = \dfrac{\delta_{pk}}{\sigma_0}$ $\gamma := \dfrac{\sigma_1^2}{\sigma_0\sigma_2}$ $\sigma_j^2 := \displaystyle\int \frac{d^3\vec{k}}{(2\pi)^3} k^{2j}|\delta_{\vec{k}}|^2$

$\sigma_{\rm H} := \sigma_0(t_{\rm H})$ $t_{\rm H}$: horizon reentry time

Q : (non-dim) quadrupole moment

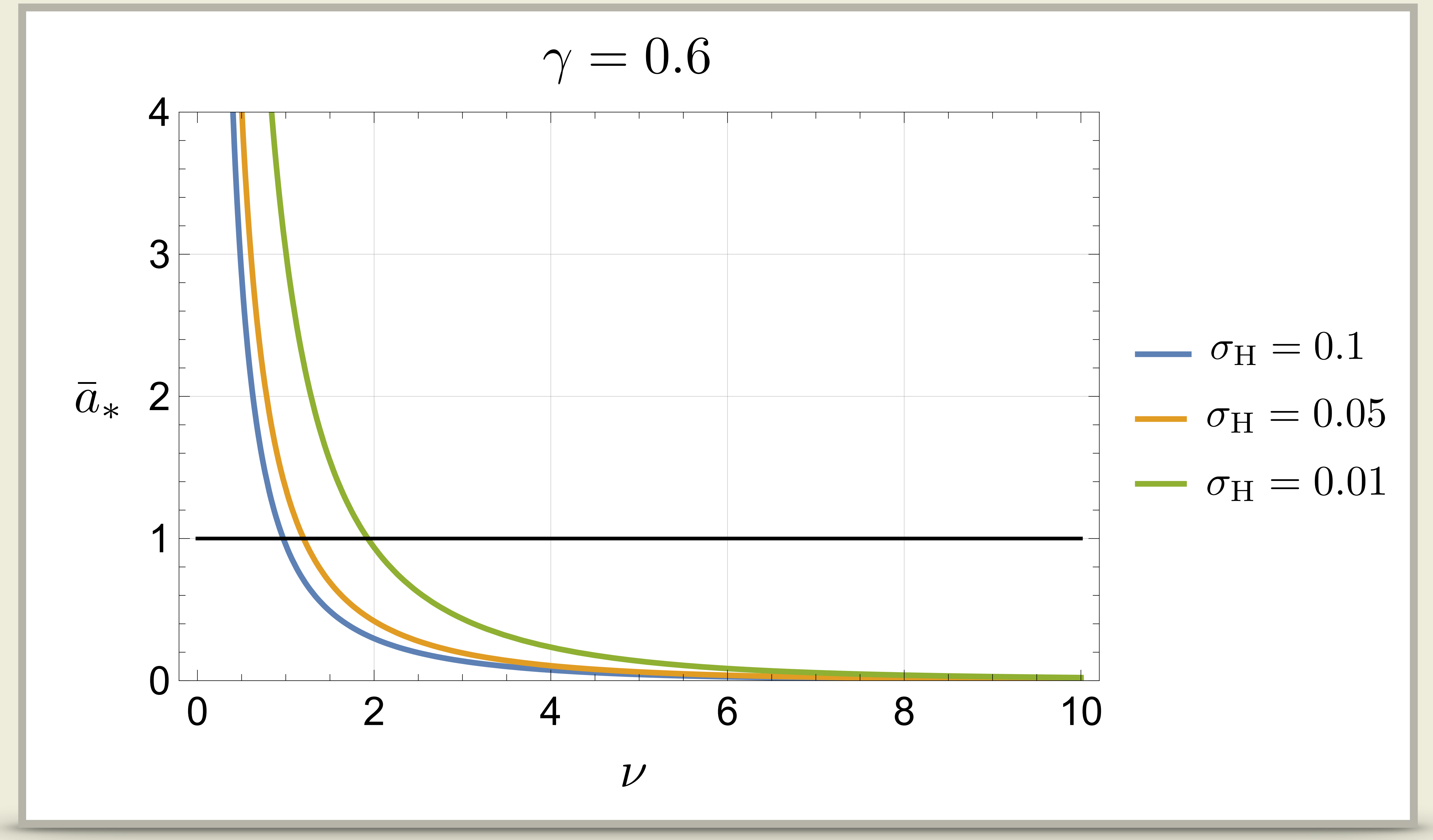
- Smaller $\sigma_{\rm H} = \langle \delta(t_{\rm H})^2\rangle^{\frac{1}{2}}$ gives larger spin

  Smaller initial fluctuation grows larger until turn-around (collapse)

$$\frac{t_{\rm ta}}{t_{\rm H}} \sim (\nu\sigma_{\rm H})^{-3/2} = \underline{\delta_{\rm H}^{-3/2}}$$

- $\gamma := \dfrac{\sigma_1^2}{\sigma_0\sigma_2}$: narrowness of power spectrum

  Wider power spectrum gives larger spin



Result: spin of overdensity

RMS of spin at turn-around time

$$\boxed{\bar{a}_*(\nu, t_{\mathrm{ta}}) = \frac{2^{1/2}\cdot 3}{5^{3/2}}\sqrt{1-\gamma^2}\sigma_{\mathrm{H}}^{-1/2}\sqrt{\langle Q^2\rangle_\lambda}}$$

Decreases with amplitude ν

For small ν, we have $\underline{\bar{a}_*(\nu) > 1}$

Prohibited for BH

Can't regard as BH spin

Rotation should prevent collapse

$\nu = \dfrac{\delta_{pk}}{\sigma_0}$ $\quad \gamma := \dfrac{\sigma_1^2}{\sigma_0\sigma_2}$ $\quad \sigma_j^2 := \displaystyle\int \frac{d^3\vec{k}}{(2\pi)^3} k^{2j}|\delta_{\vec{k}}|^2$

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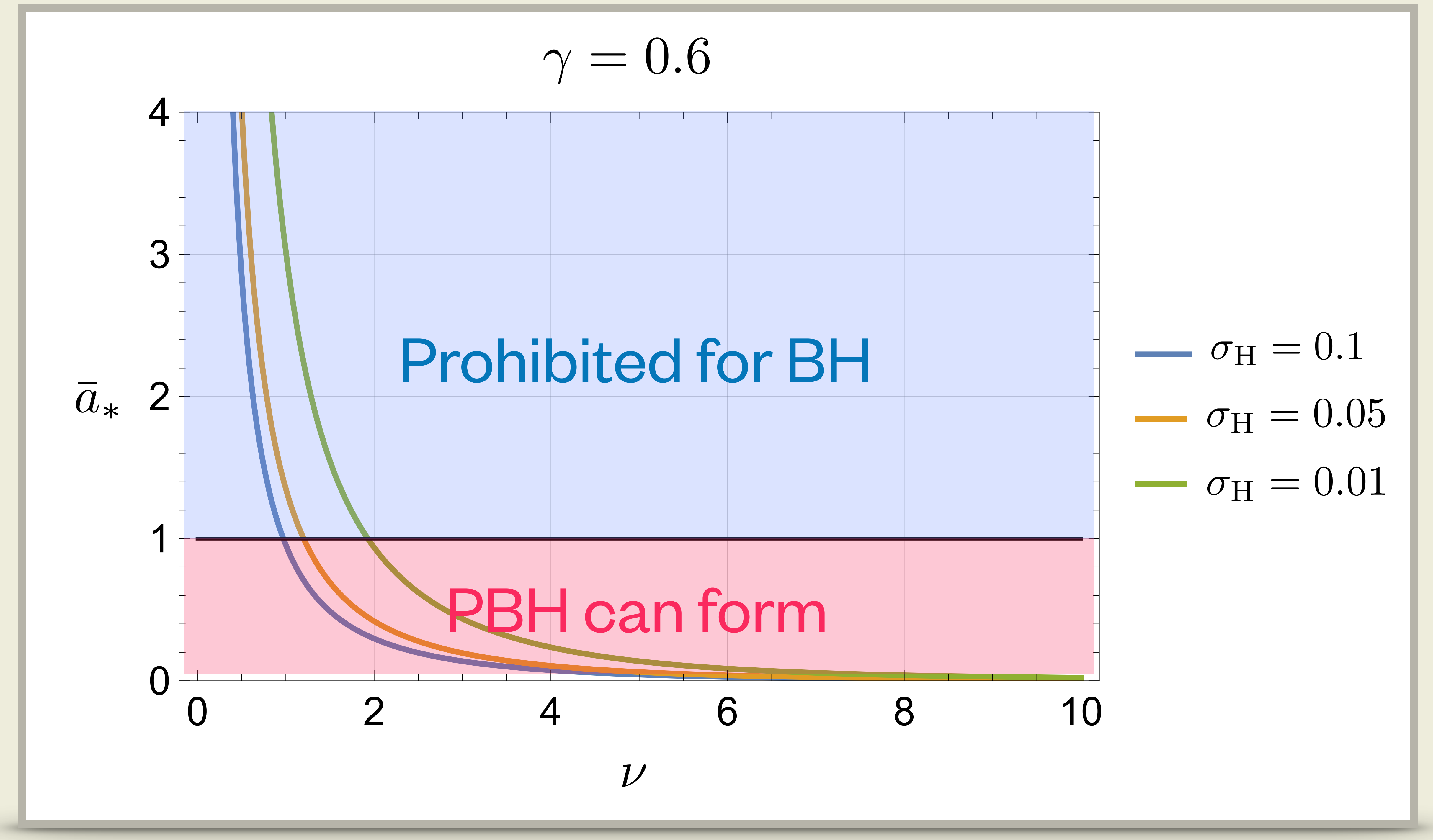

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Decreases with amplitude ν

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↓

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↓

Threshold of ν / δ for PBH formation

$$\nu = \frac{\delta_{pk}}{\sigma_0} \qquad \gamma := \frac{\sigma_1^2}{\sigma_0\sigma_2} \qquad \sigma_j^2 := \int \frac{d^3\vec{k}}{(2\pi)^3} k^{2j} |\delta_{\vec{k}}|^2$$

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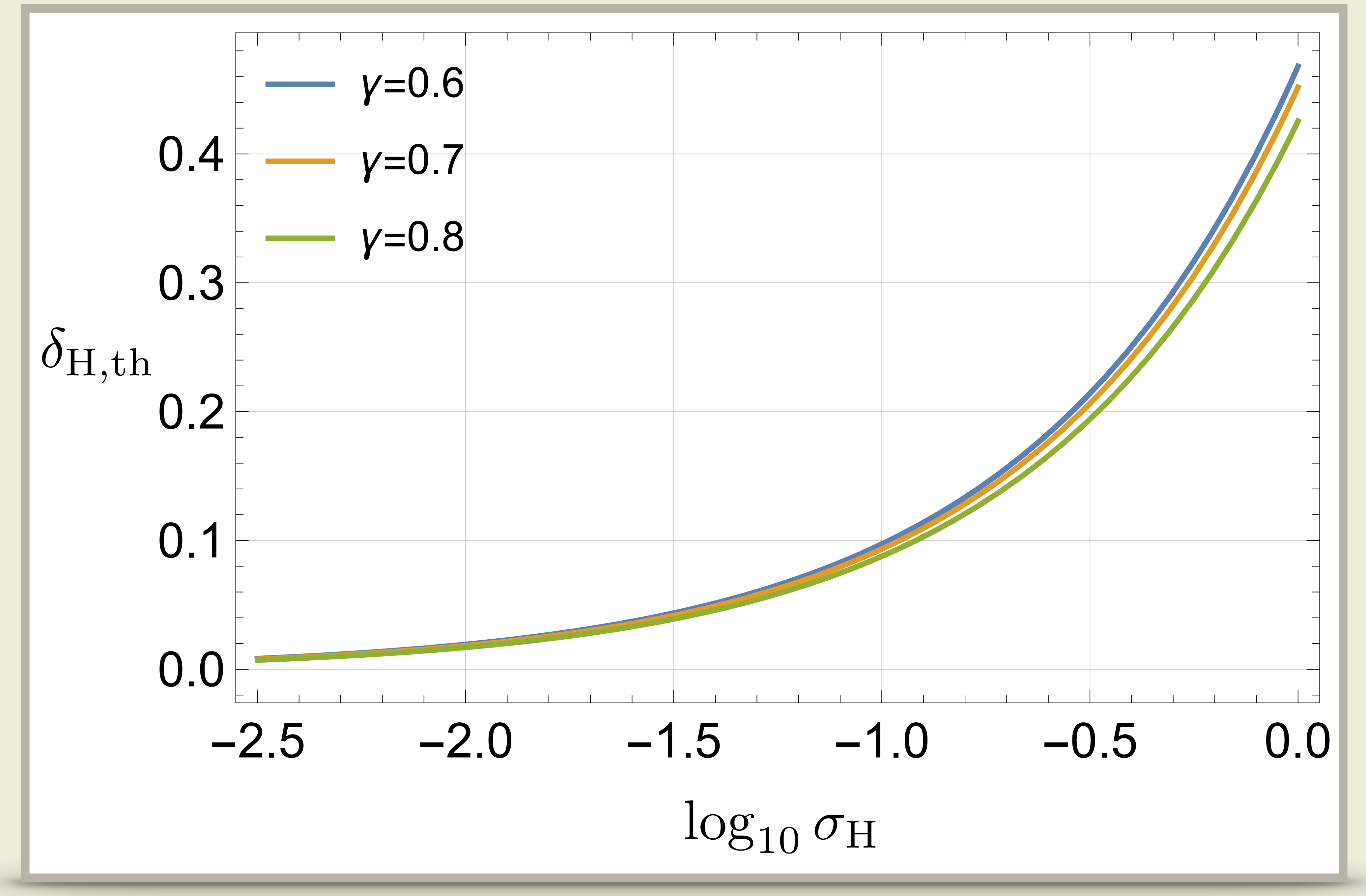


Result: spin vs anisotropy

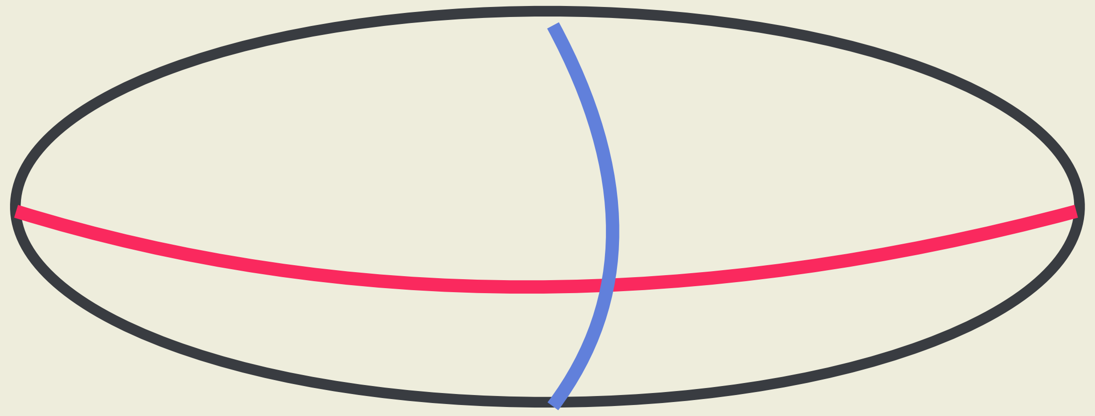
What we've discussed: Rapid rotation of fluid impede PBH formation

Q: How significant this on PBH abundance?

Comparison with effect of anisotropy

- Anisotropy and PBH collapse [Harada Yoo, Kohri, Nakao, and Jhingan 2016]

Fluctuation with large anisotropy can't collapse



Condition for collapse: Hoop conjecture [Thorne 1972]

$$\underbrace{\mathcal{C}}_{\text{Circumference}} \leq 4\pi \underbrace{M}_{\text{Mass}}$$

In terms of fluctuation,

$$h(\alpha,\beta,\gamma) \leq 1 \qquad h(\alpha,\beta,\gamma) := \frac{2}{\pi}\frac{\alpha-\gamma}{\alpha^2} E\left(\sqrt{1-\left(\frac{\alpha-\beta}{\alpha-\gamma}\right)^2}\right)$$

(α,β,γ) : Eigenvalue of $-\frac{\partial^2\psi}{\partial q^l \partial q^k}(t_{\rm H})$

Note: We have not consider anisotropy effect in evaluating $\bar{a}_*(\nu)$

Result: spin vs anisotropy

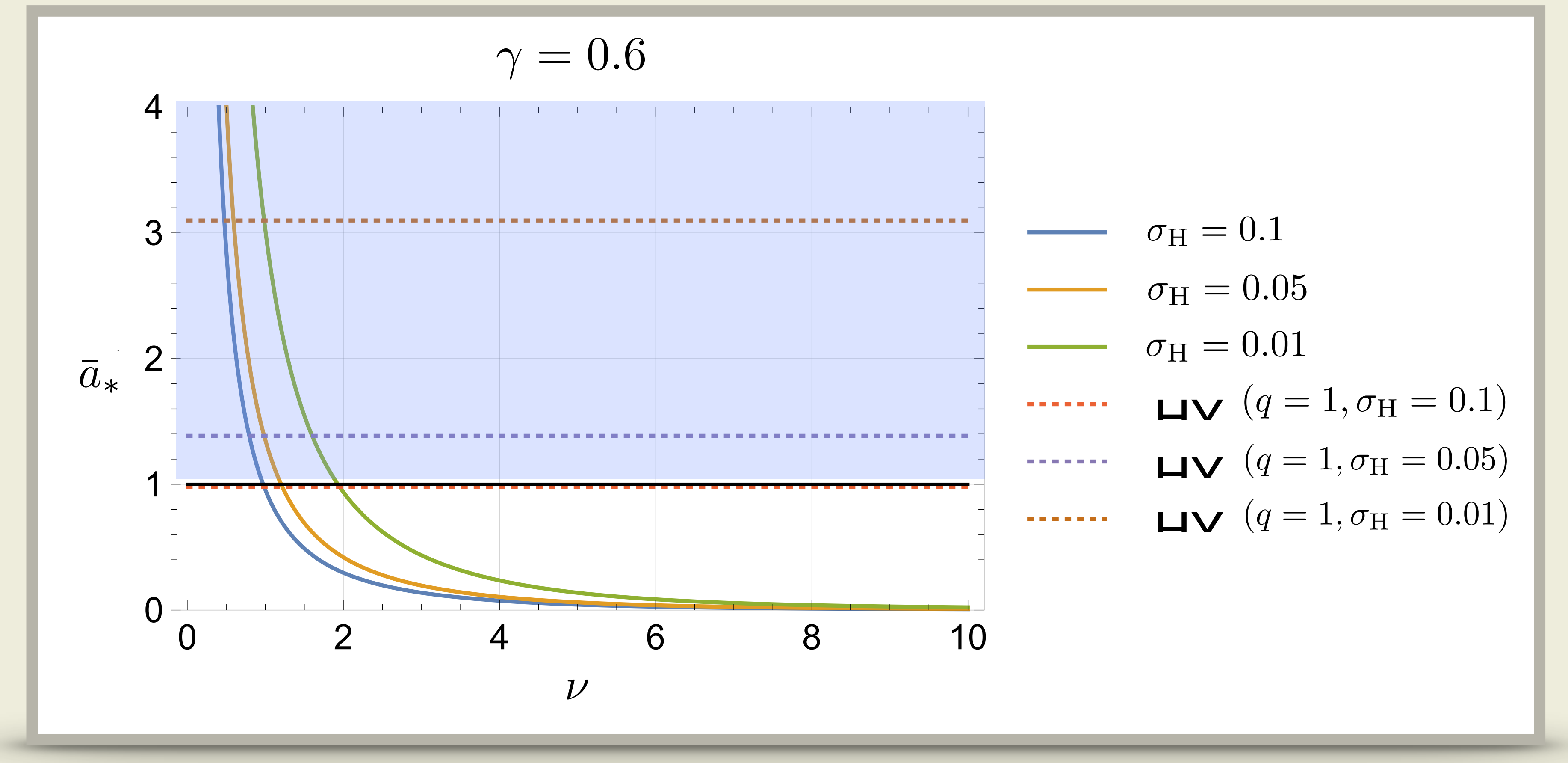
PBH abundance with spin & non-spherical effect

$$\beta_0 \simeq \int d\alpha d\beta d\gamma \theta[\delta_{\mathrm{H}}(\alpha,\beta,\gamma) - \delta_{\mathrm{H,th}}]\theta[1 - h(\alpha,\beta,\gamma)]w(\alpha,\beta,\gamma)$$

Distribution of fluctuation

Threshold from spin

$$\bar{a}_*(\nu) \leq 1$$



Threshold from anisotropy [Harada Yoo, Kohri, Nakao, and Jhingan 2016]

Hoop conjecture [Thorne 1972]

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$$\sigma_j^2 := \int \frac{d^3\vec{k}}{(2\pi)^3} k^{2j} |\delta_{\vec{k}}|^2 \quad \sigma_{\rm H} := \sigma_0(t_{\rm H}) = \sqrt{\langle \delta(t_{\rm H})^2 \rangle}$$

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- Large variance of fluctuation

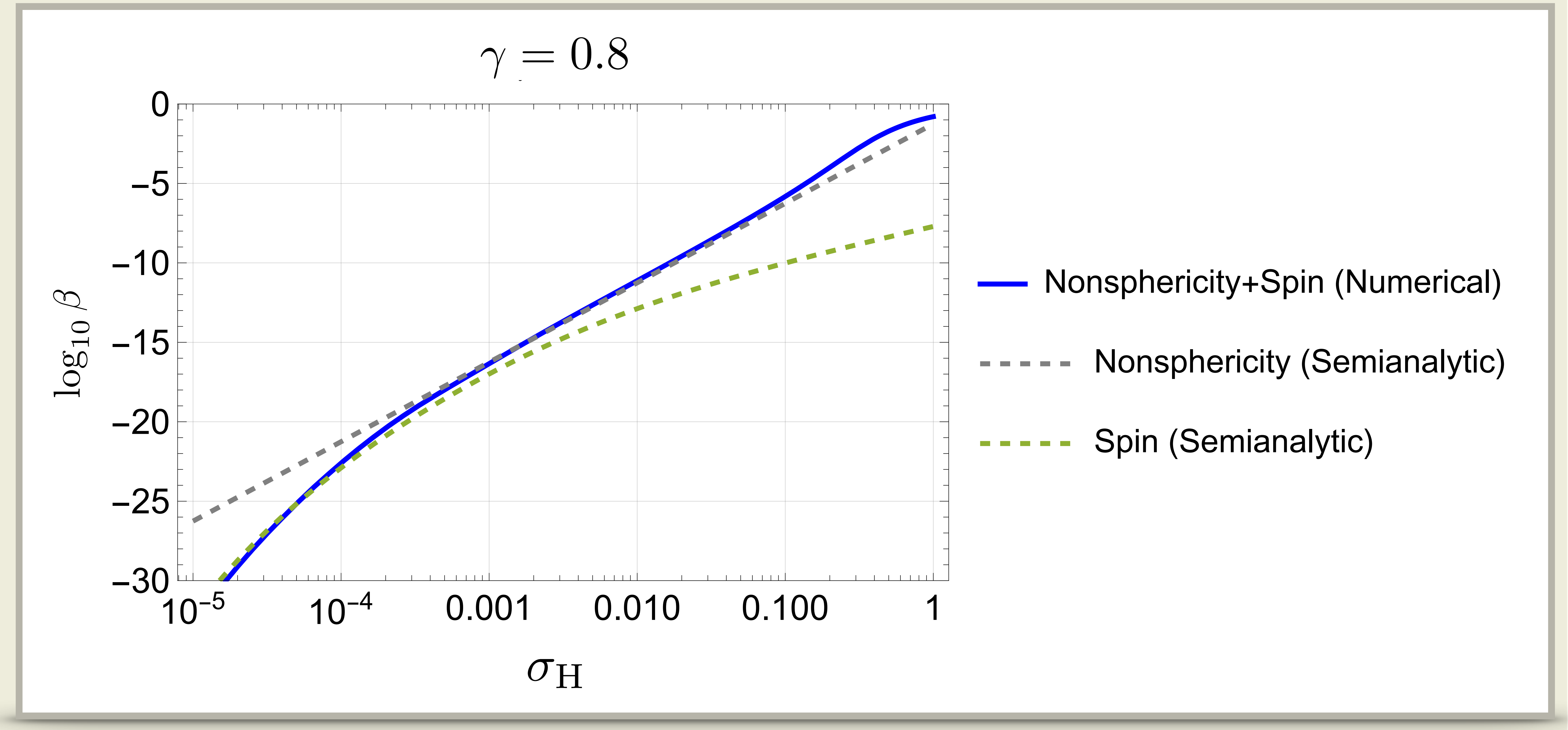
  $(\sigma_{\rm H} > 10^{-3})$

  Effect of ellipticity is significant

- Small variance of fluctuation

  $(\sigma_{\rm H} \leq 10^{-3})$

  Effect of spin is significant



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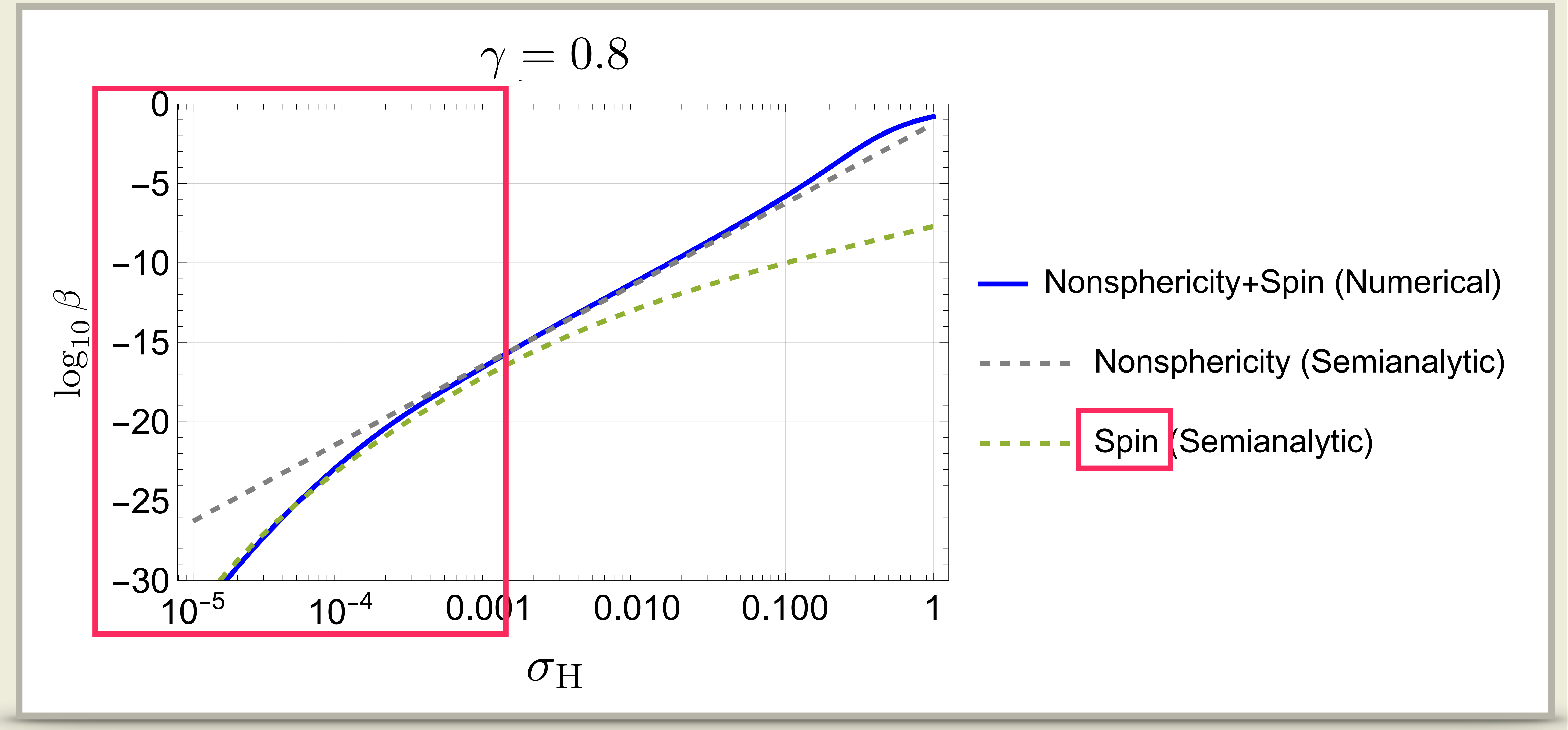
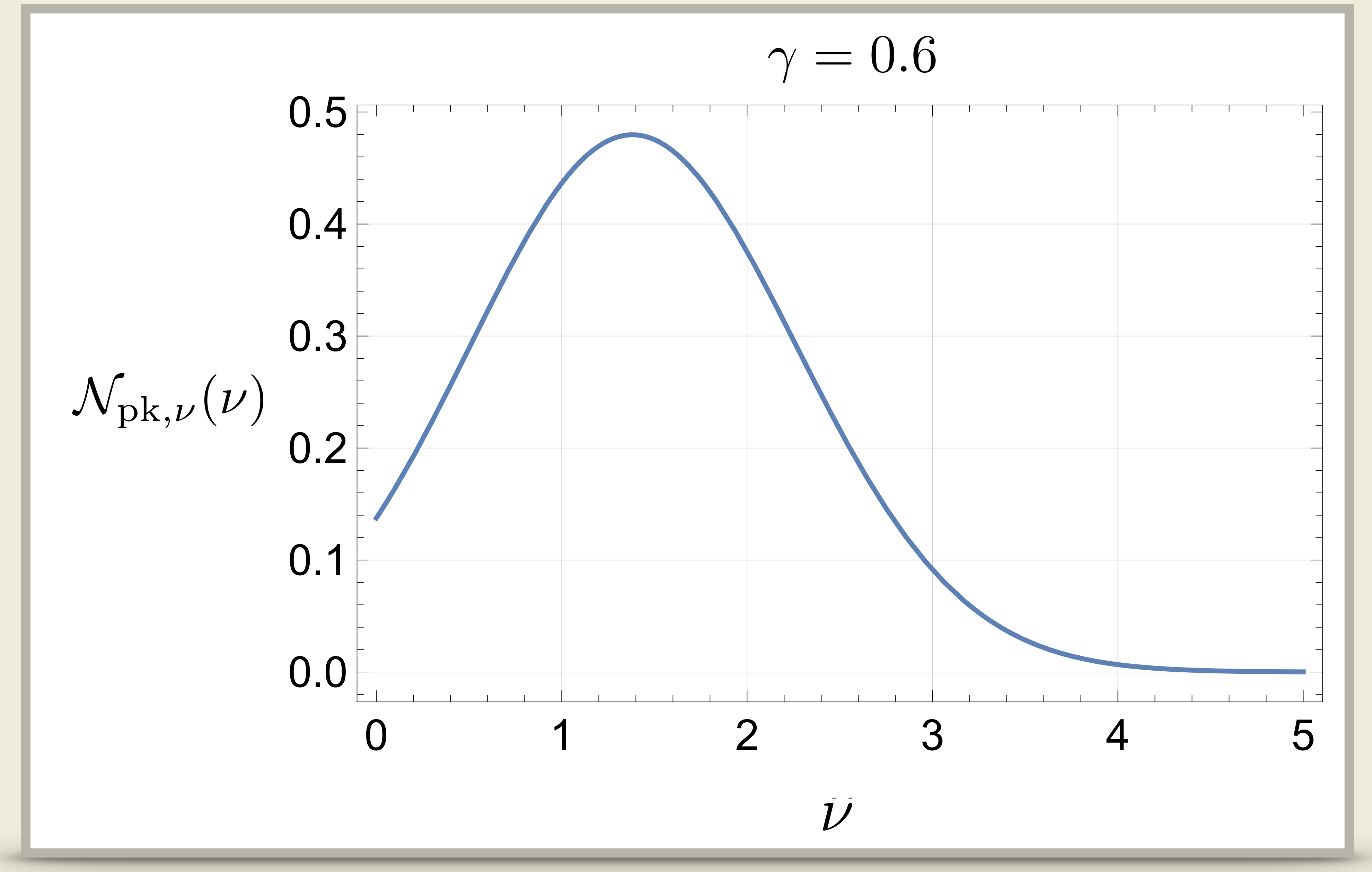


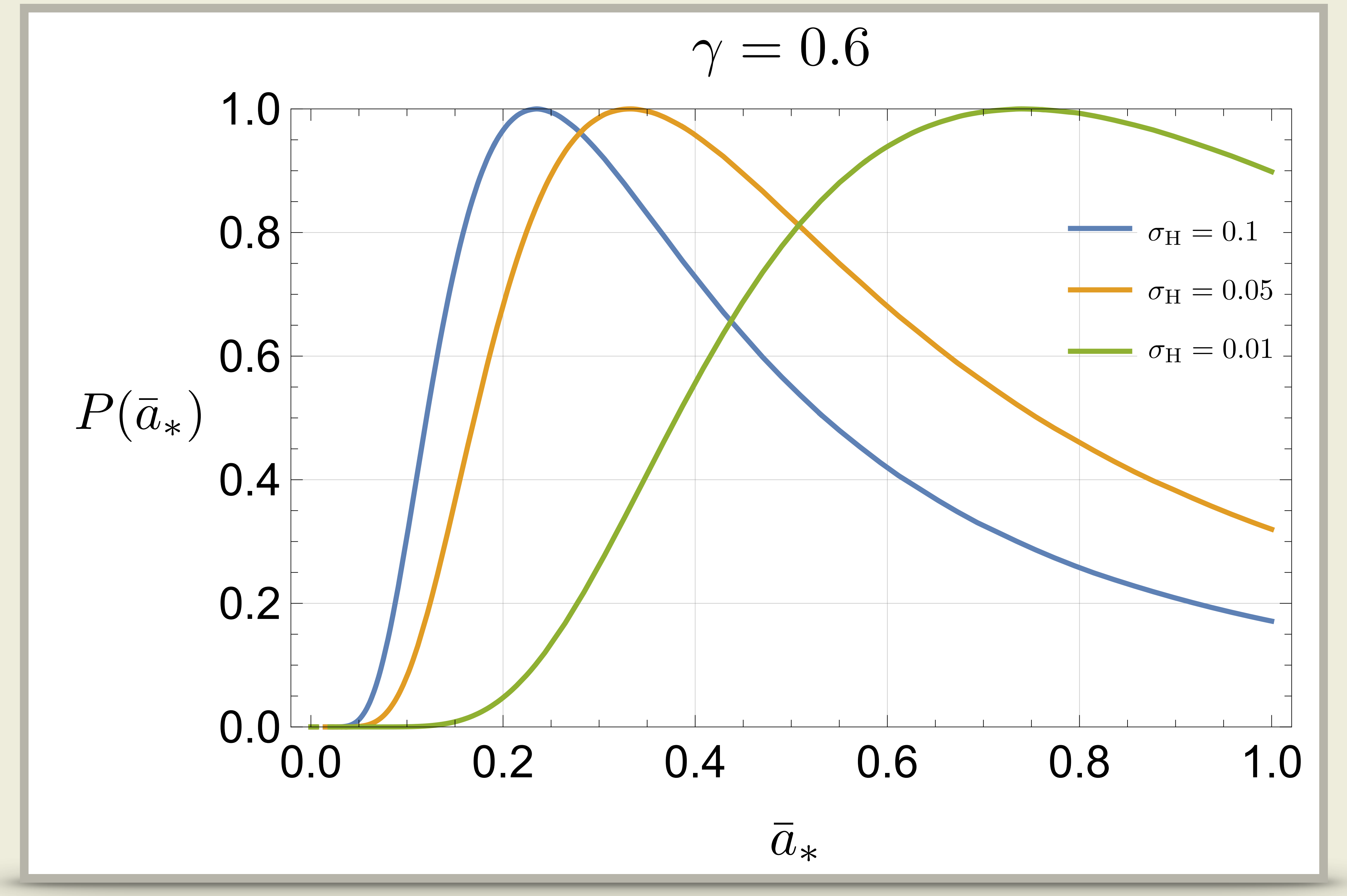
Result: probability for spin

$\mathcal{N}_{\mathrm{pk},\nu}(\nu)$:number density of peak with ν (∵peak theory) $\xrightarrow{\bar{a}_* = \bar{a}_*(\nu)}$ Probability distribution for PBH spin $P(\bar{a}_*)$

- Peaks at $\bar{a}_* = O(10^{-1})$: larger than RD era ($\sqrt{\langle a_*^2 \rangle} \sim O(10^{-2})$)
- Larger peak for smaller σ_{H}



$\xrightarrow{\bar{a}_* = \bar{a}_*(\nu)}$



Summary

- We have estimated the PBH spin in MD era

  perturbatively, using peak theory

- For small amplitude of fluctuation, we have $a_* > 1$

  $\longrightarrow$ Threshold for amplitude (Spin prevents gravitational collapse)

- Suppress PBH abundance for small variance of fluctuation

- $P(\bar{a}_*)$ peaks at $\bar{a}_* = O(10^{-1})$

Thank you for attention!!