# Primordial Black Hole formation from Curvature perturbation enhanced during inflation


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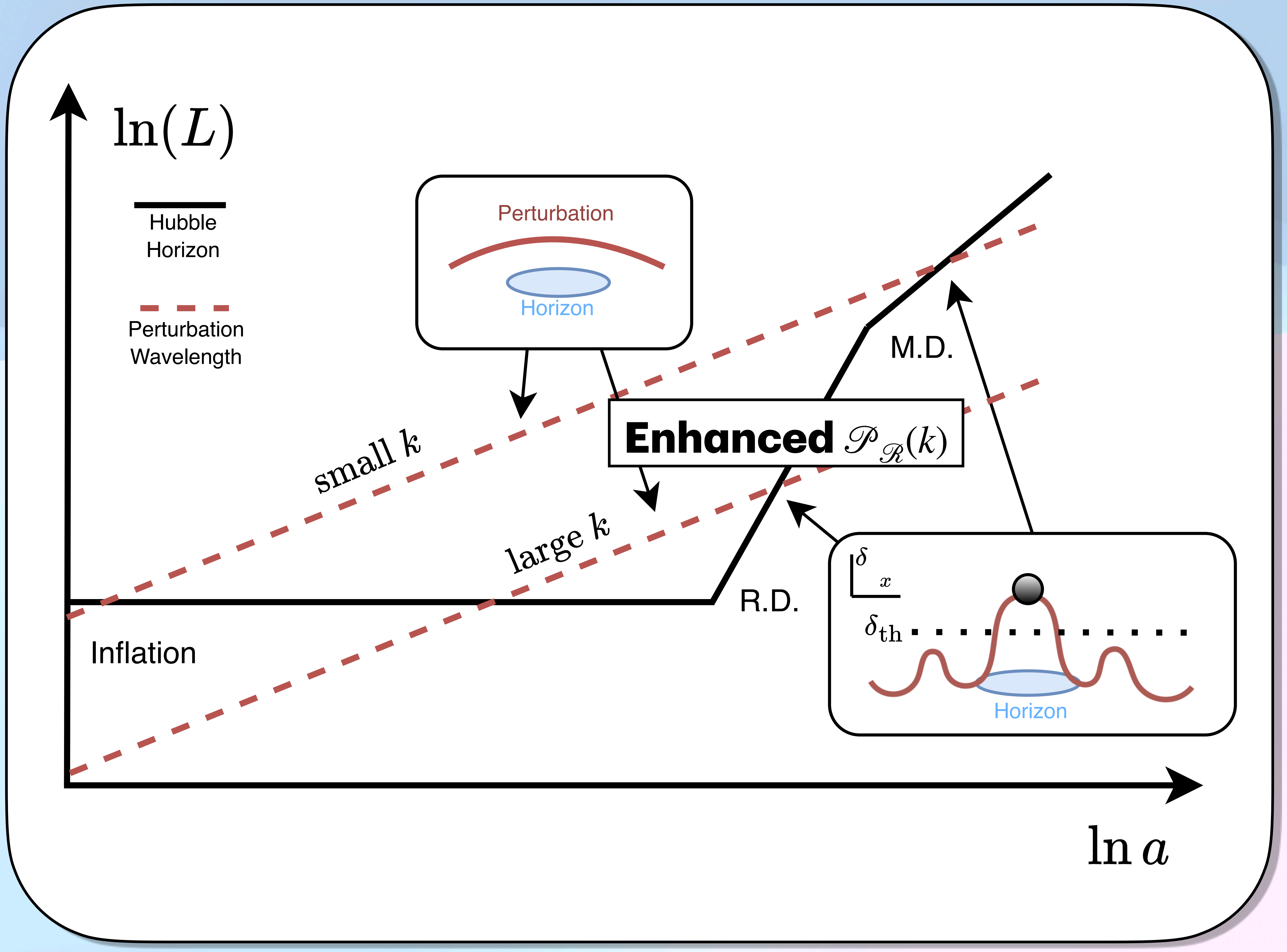
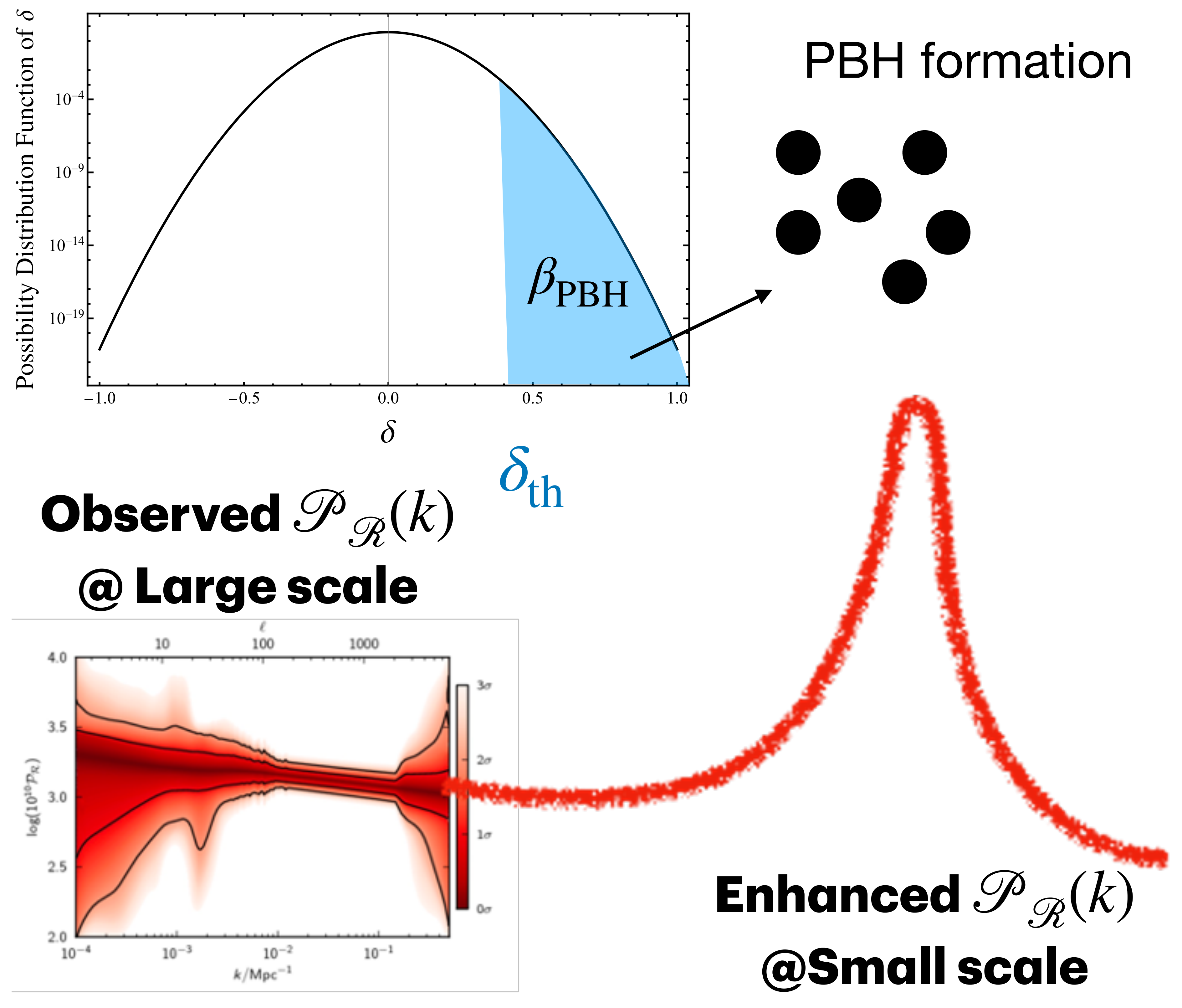
Based on **XW**, YL. Zhang, M. Sasaki, 2404.02492, **XW**, XH. Ma, M.Sasaki, 2411. XXXXX, and **XW,** M. Sasaki in prep.

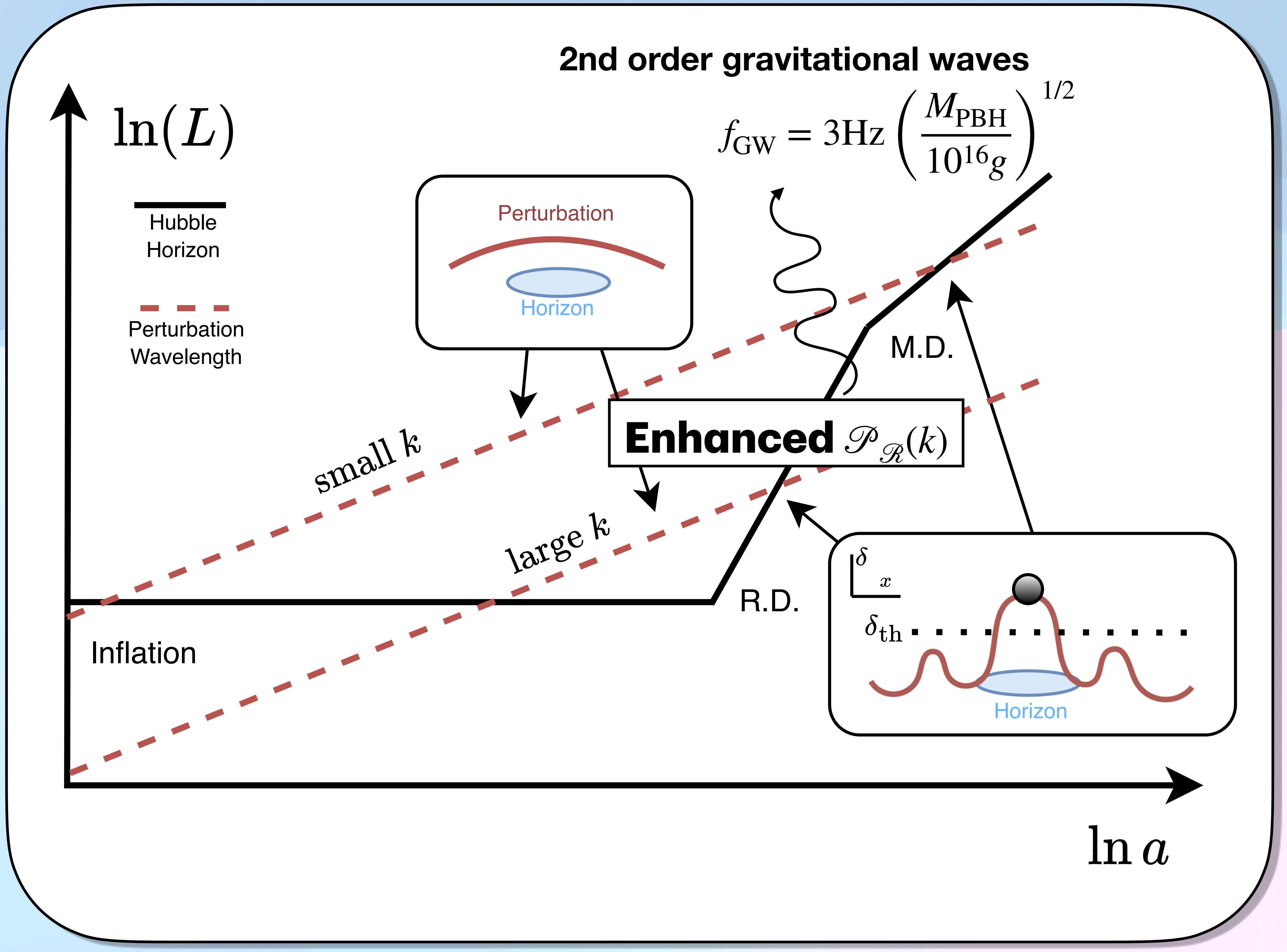
# Primordial Black holes from inflation




The Planck Collaboration, 2018 (1807.06209)

# Primordial Black holes from inflation




**CDM PBHs:** $10^{17} \sim 10^{21} g$

**Ultra light PBHs:** $< 10^9 g$

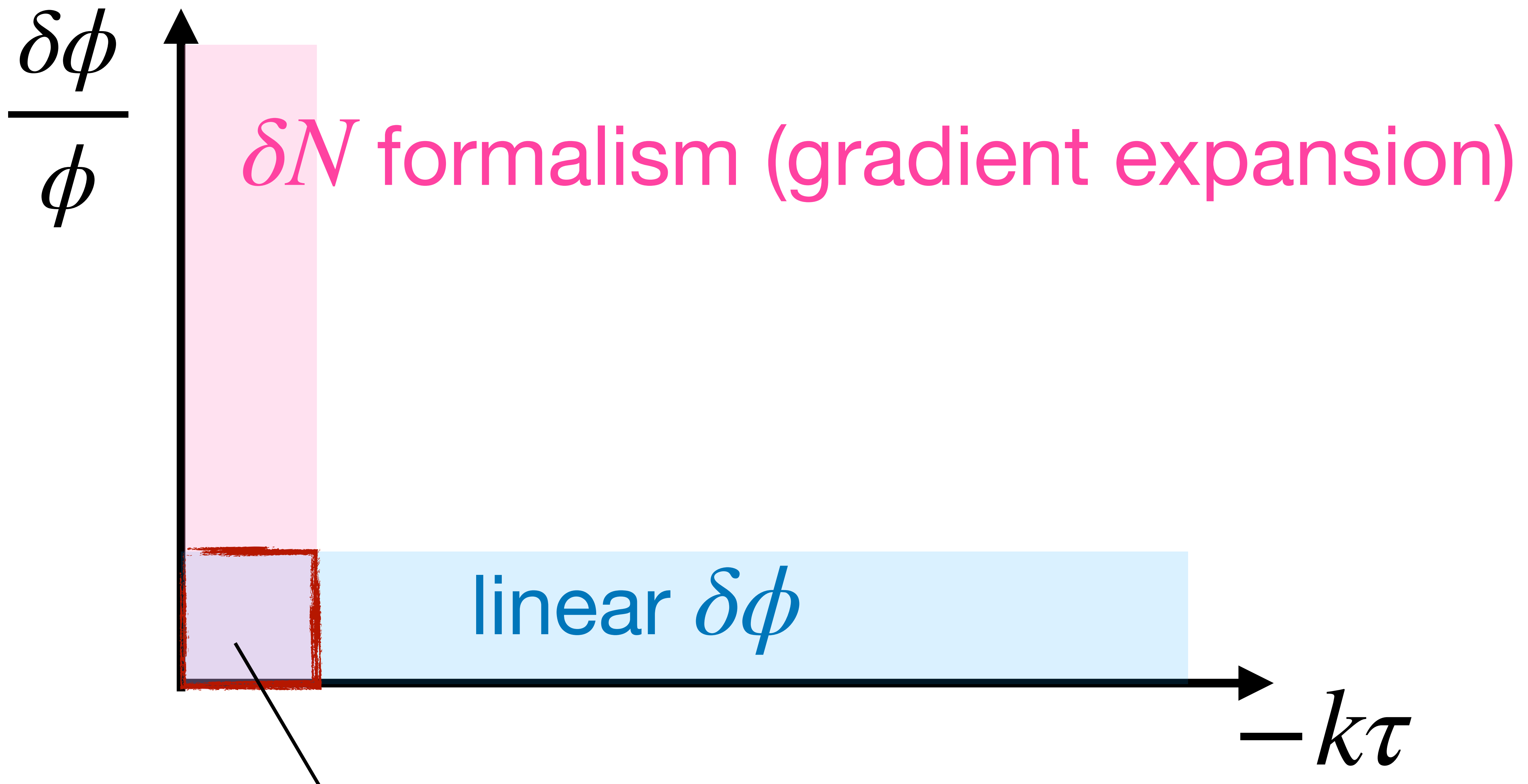
**(Sub-)Solar mass PBHs:** $\lesssim 1 M_\odot$

**LIGO PBH mergers:** $10 \sim 100 M_\odot$

**Seeds of Supermassive BHs:** $> 10^4 M_\odot$
**(More talks in NEXT WEEK?)**

# How to enhance the small scale power spectrum?

Single field slow roll inflation:




The solution in small k limit: adiabatic constant mode and decaying mode.

$$\mathscr{R}_0(\eta_f) = \mathscr{R}(\eta_k) + \mathscr{R}'(\eta_k)\int_{\eta_k}^{\eta_f}\frac{z^2(\eta_k)}{z^2(\eta')}d\eta'$$

$\propto (a/a_k)^{-3}$

Negligible outside horizon

“The co-moving curvature is frozen after horizon crossing”

# Ways to amplify the power spectrum (Multi-field)

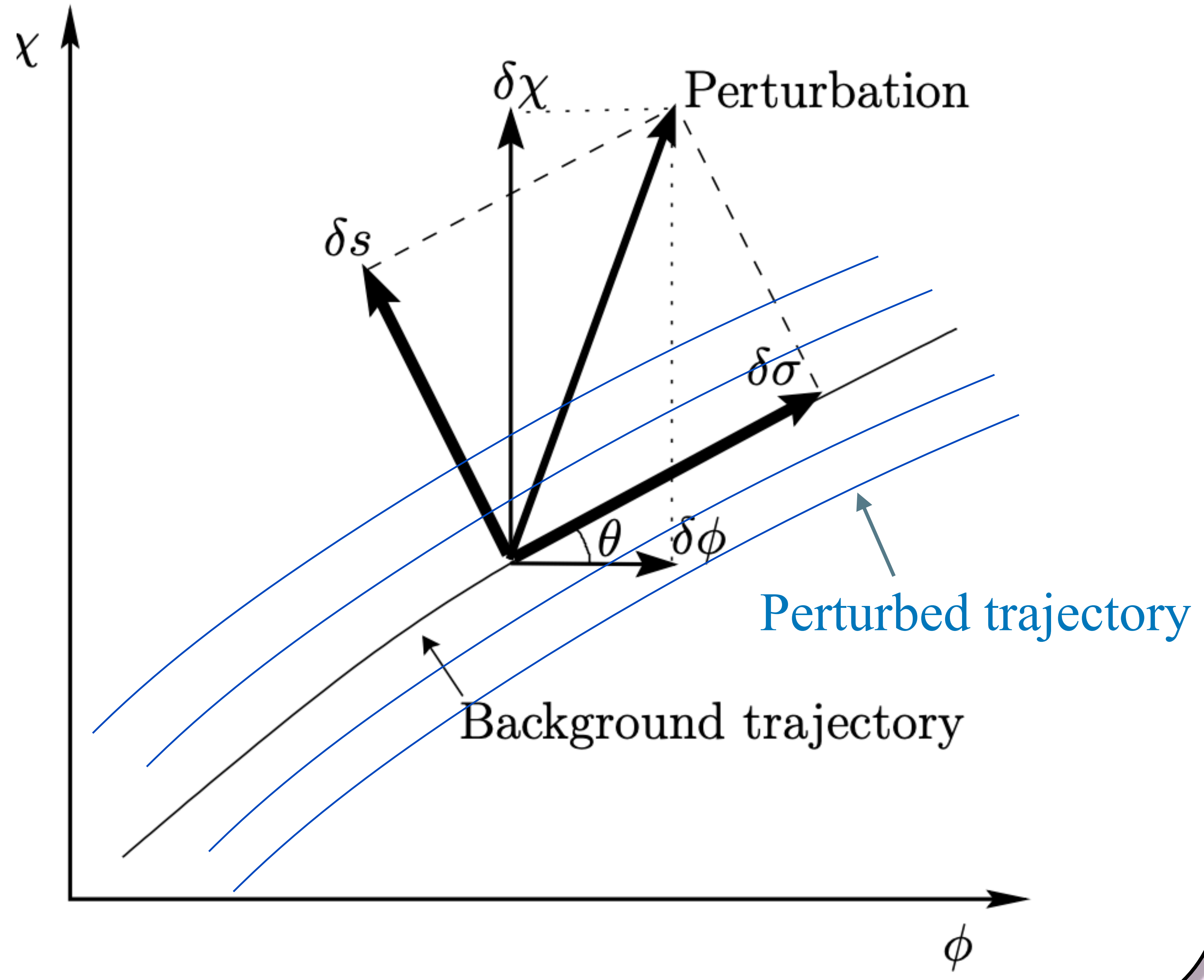
## Contributions of the iso-curvature fields

C. Gordon, D. Wands, B. Bassett and R. Maartens. 2000, (astro-ph/0009131)

$$\mathcal{S} = -3H\left(\frac{\delta p}{\dot{p}} - \frac{\delta\rho}{\dot{\rho}}\right)$$

$$\dot{\mathcal{R}} \approx -3H\frac{\dot{p}}{\dot{\rho}}\mathcal{S} \approx \sqrt{\frac{2}{\epsilon}}\dot{\theta}\delta s + \mathcal{O}\left(\frac{k^2}{a^2H^2}\right)$$

$\propto \frac{\dot{\chi}\delta\phi - \dot{\phi}\delta\chi}{\epsilon_H}$ "Field perturbations orthogonal to the background"




Isocurvature sources the time evolution of curvature perturbation

# Ways to amplify the power spectrum (Multi-field)

## Contributions of the iso-curvature fields (sharp turn case)

XW, YL. Zhang, M. Sasaki, 2024 (2404.02492)

$R^2$ inflation + non-minimally coupling $\chi$

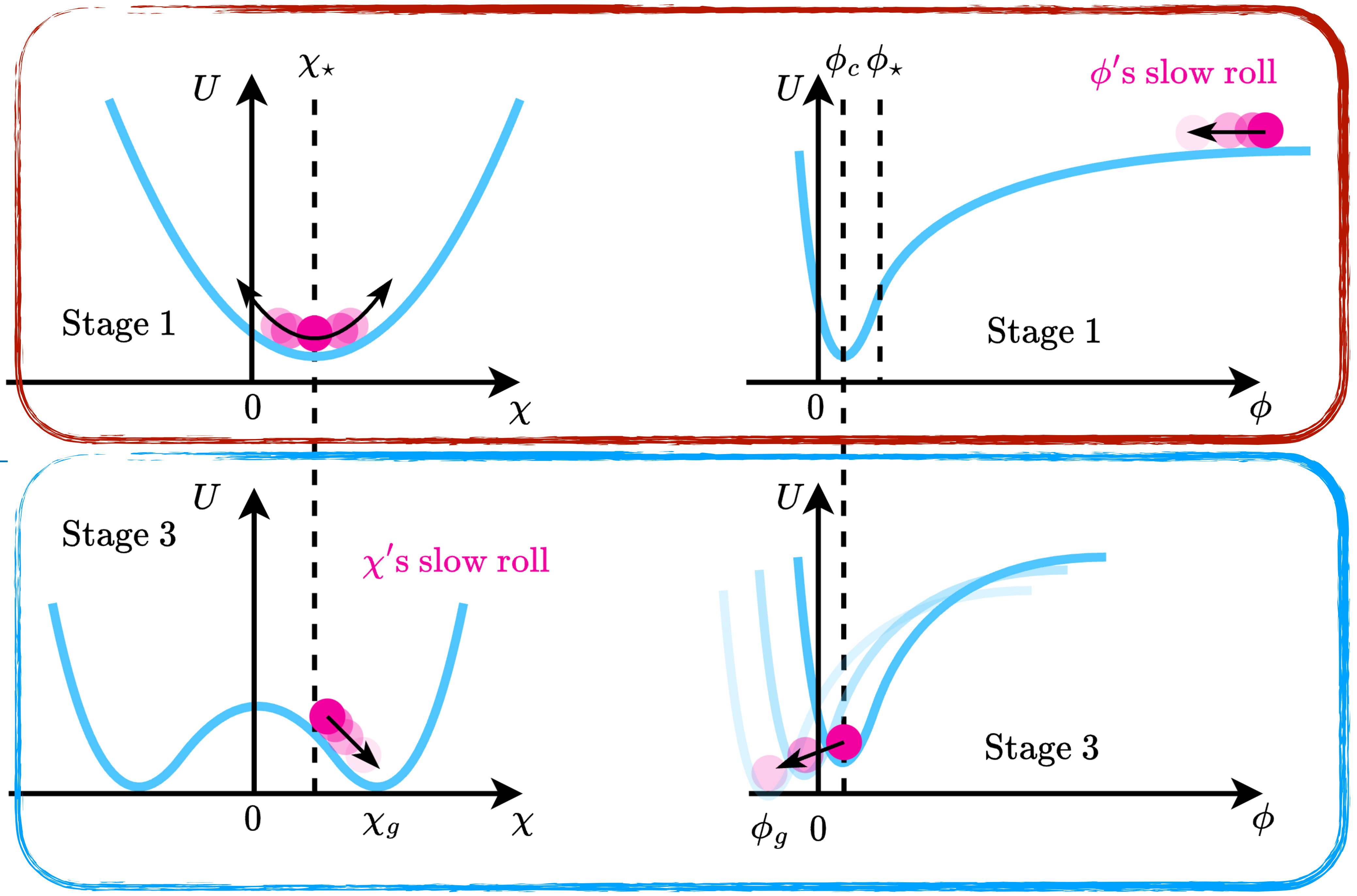
$$S_J = \int d^4x\sqrt{-g}\left[\frac{M_{\rm pl}^2}{2}f(R,\chi) - \frac{1}{2}g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi - V(\chi)\right],$$

$$f(R,\chi) = R + \frac{R^2}{6M^2} - \frac{\xi R}{M_{\rm pl}^2}(\chi - \chi_0)^2,$$

To break $Z_2$ symmetry

$$V(\chi) = V_0 - \frac{1}{2}m^2\chi^2 + \frac{1}{4}\lambda\chi^4.$$

To end inflation



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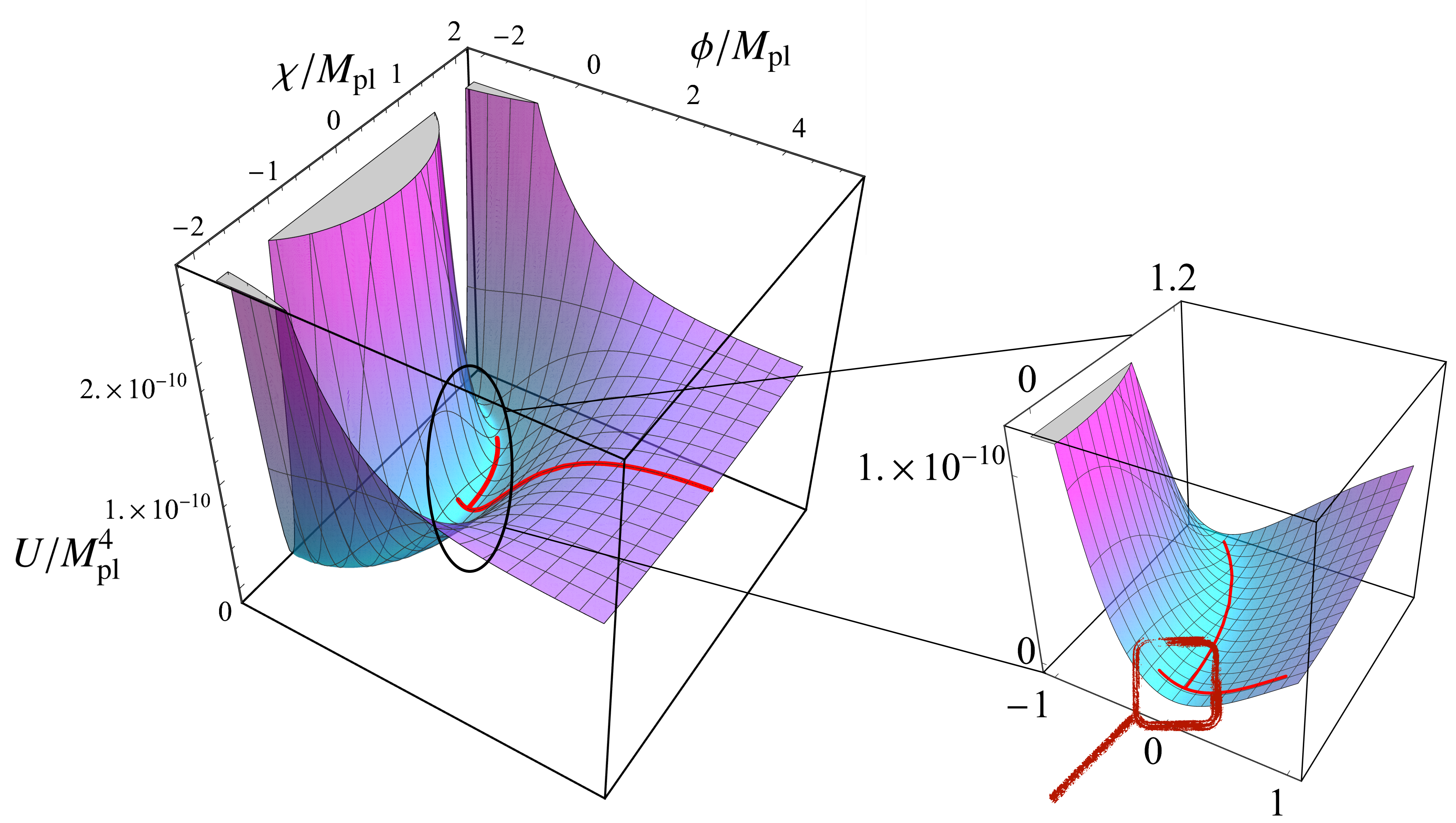


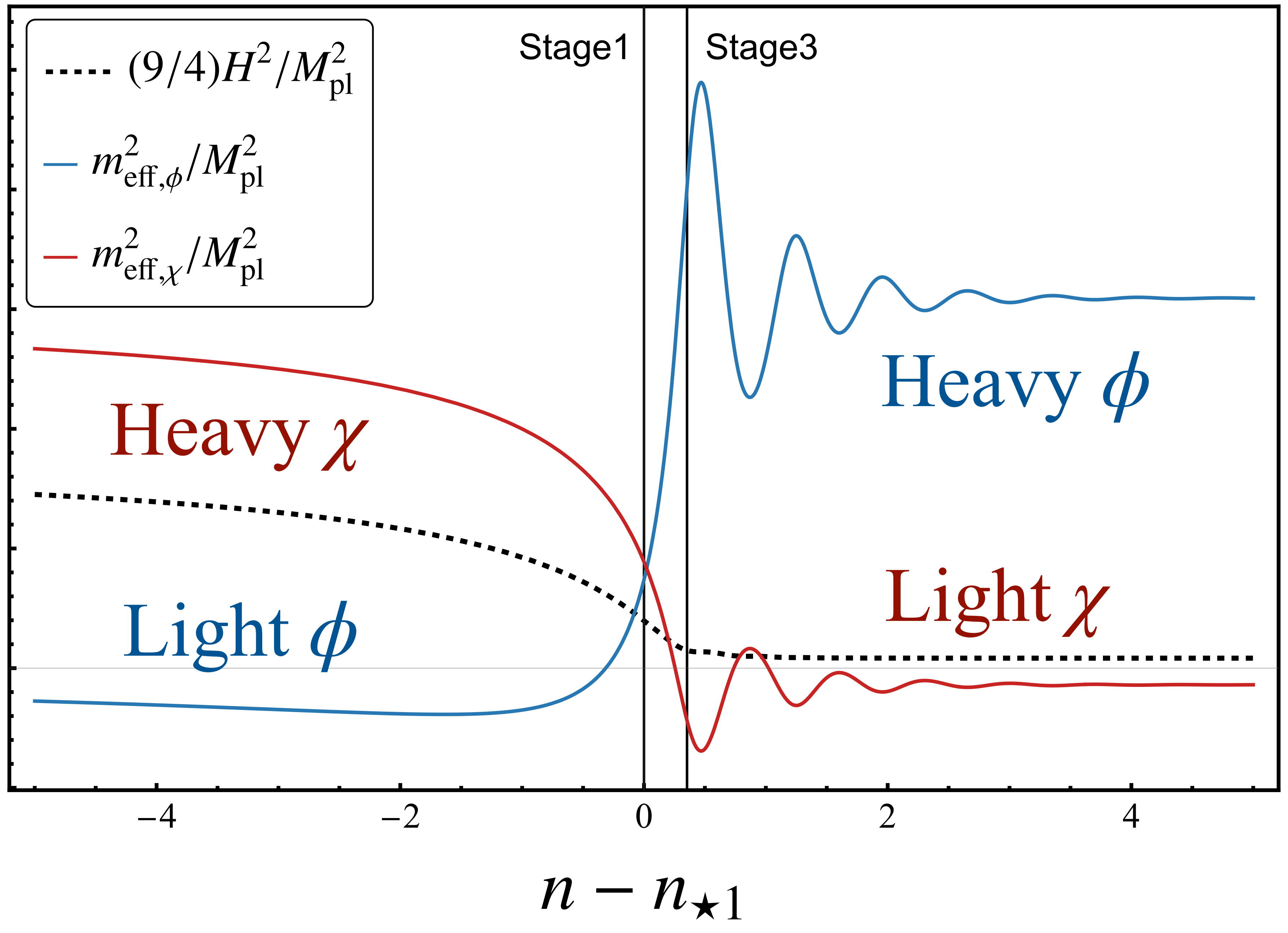

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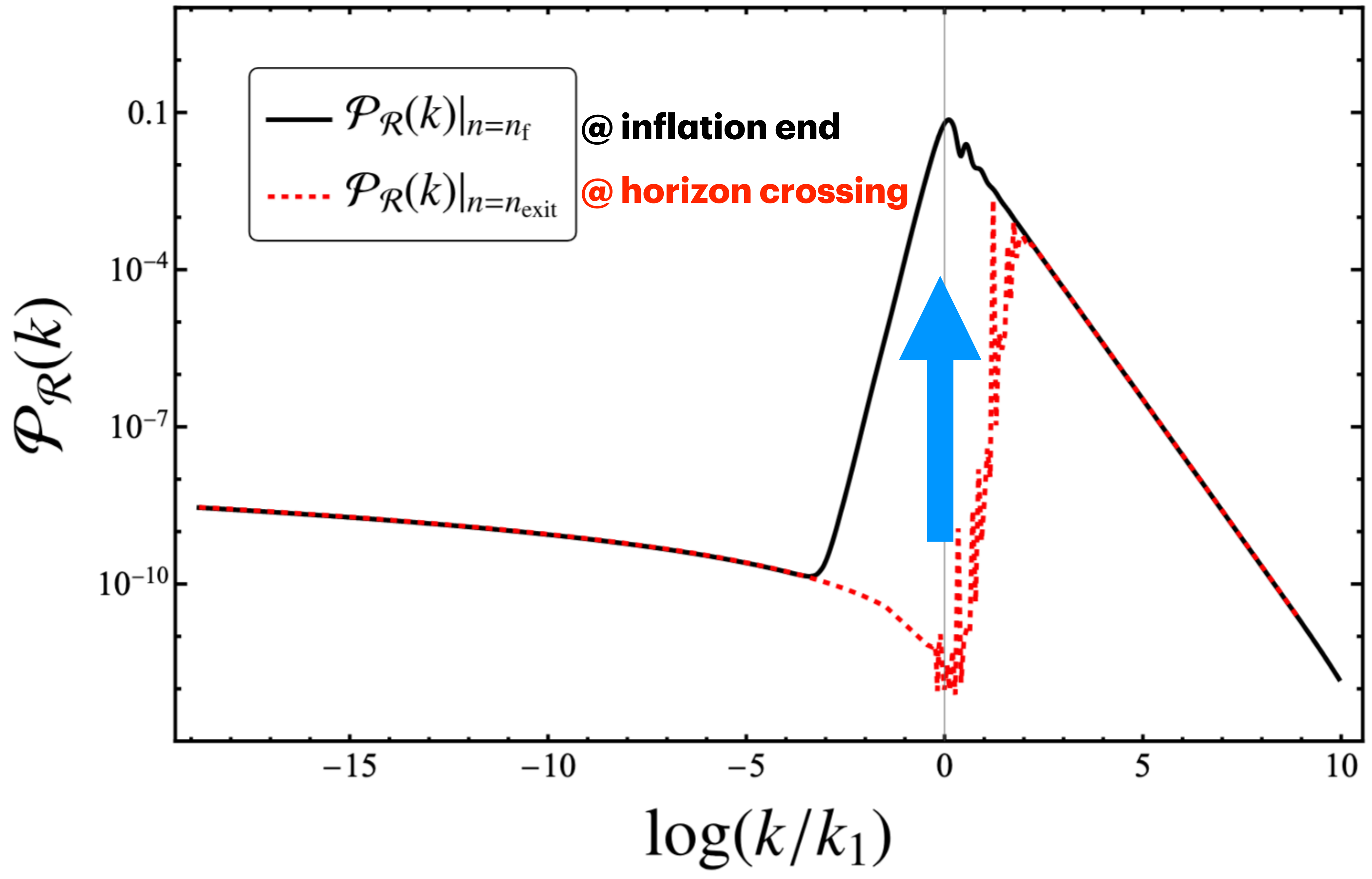


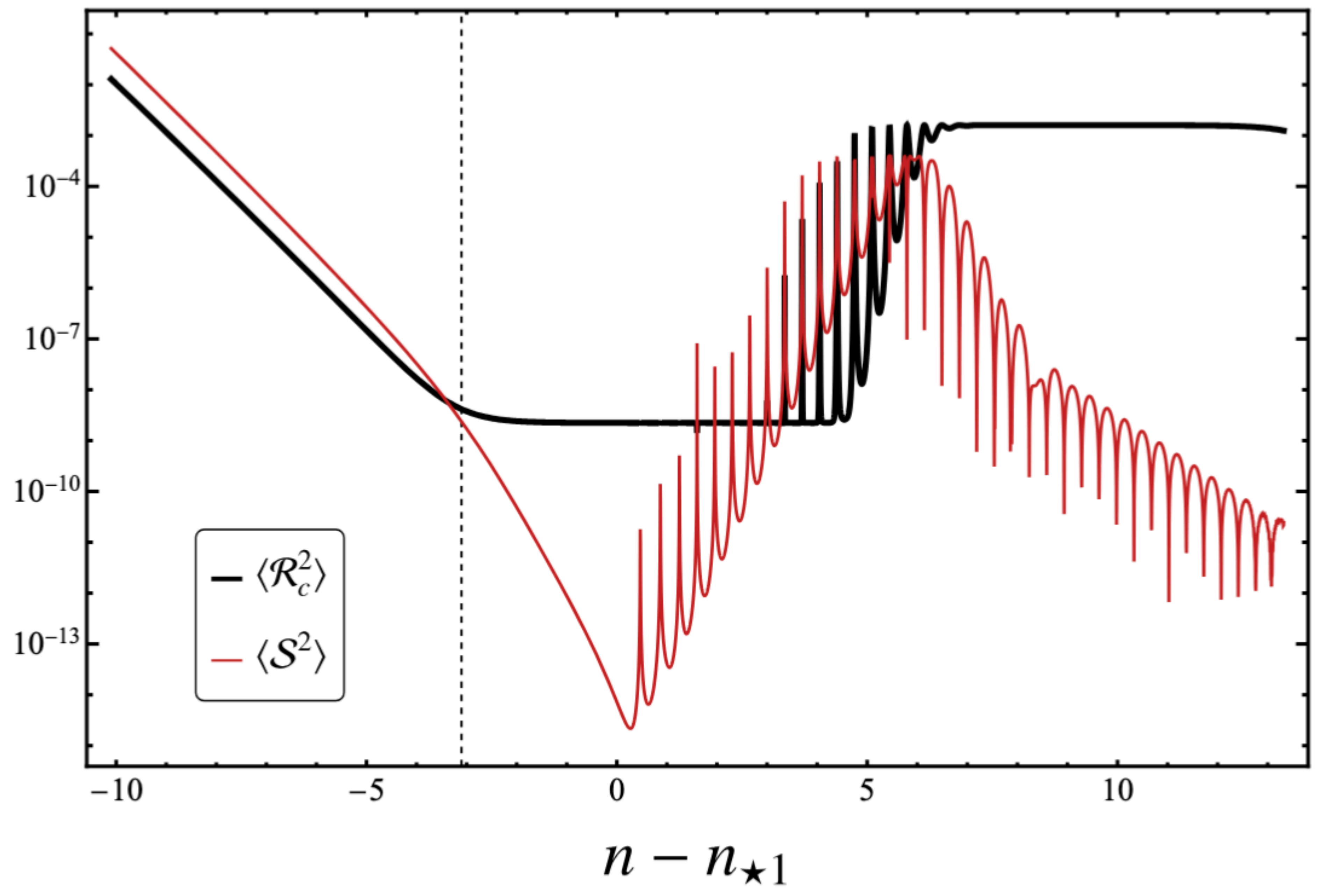

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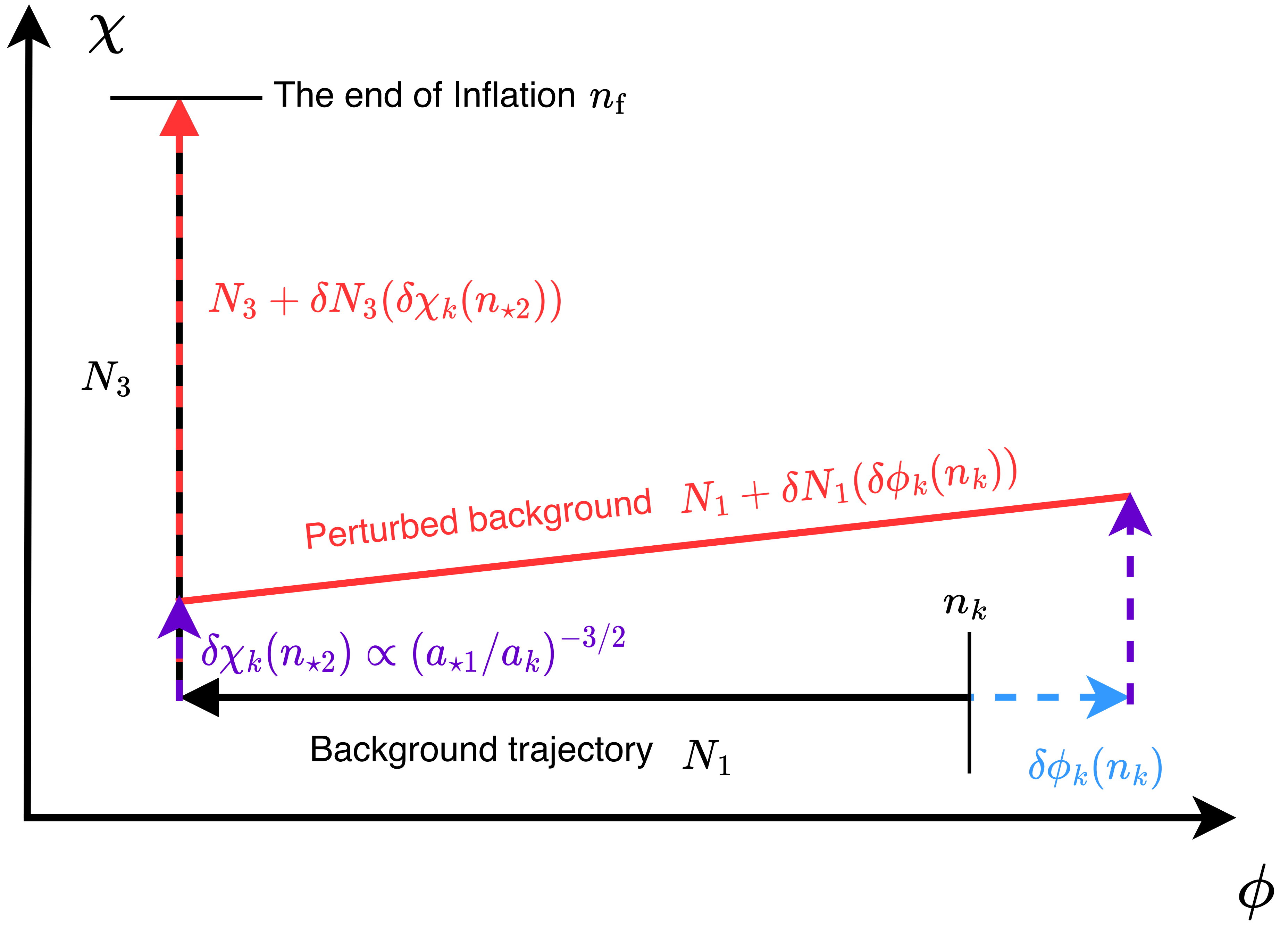
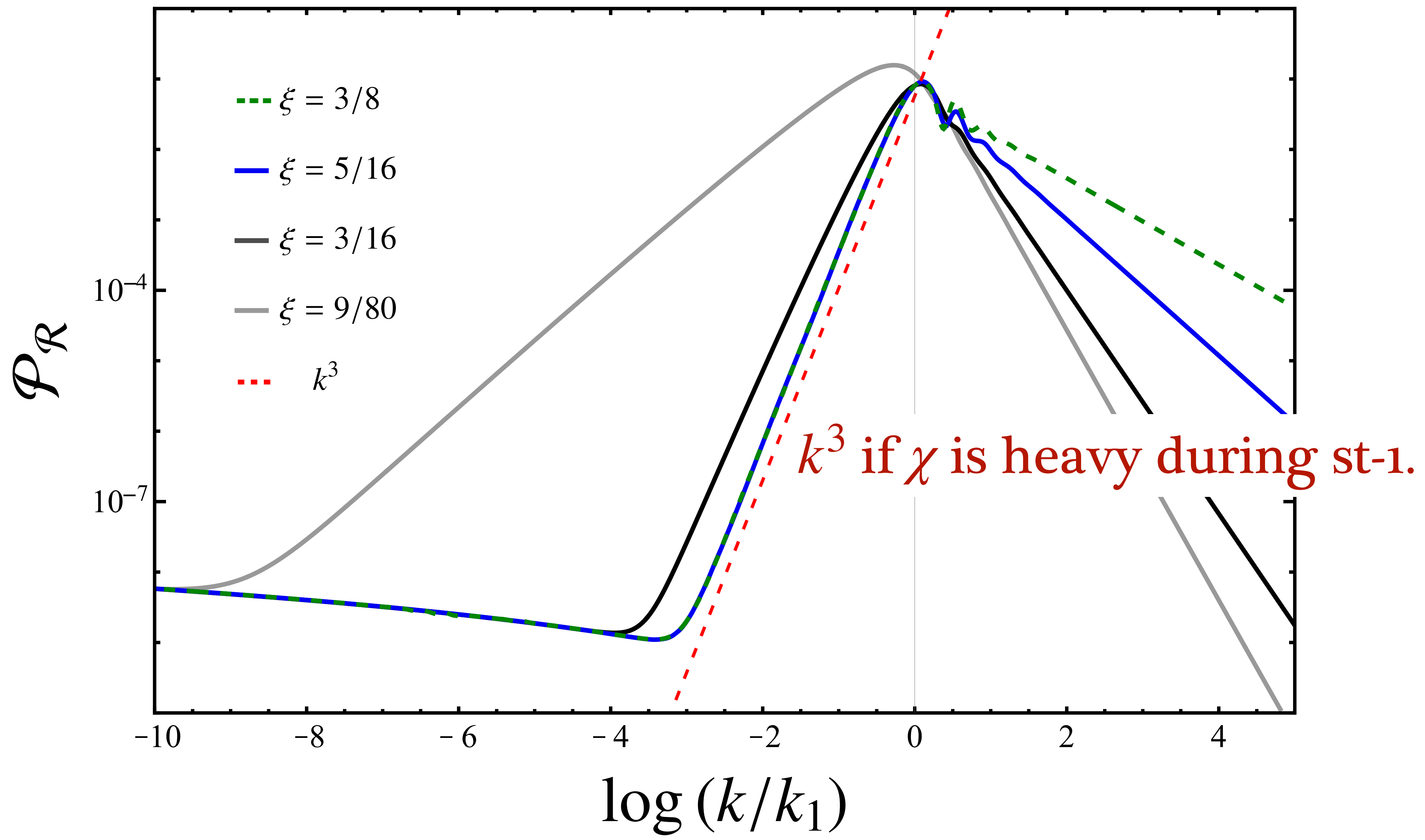
$R^2$ inflation + non-minimally coupling $\chi^4$

The growth rate depends on the effective mass of early isocurvature field that dominates the latter inflation.



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XW, YL. Zhang, M. Sasaki, 2024 (2404.02492)

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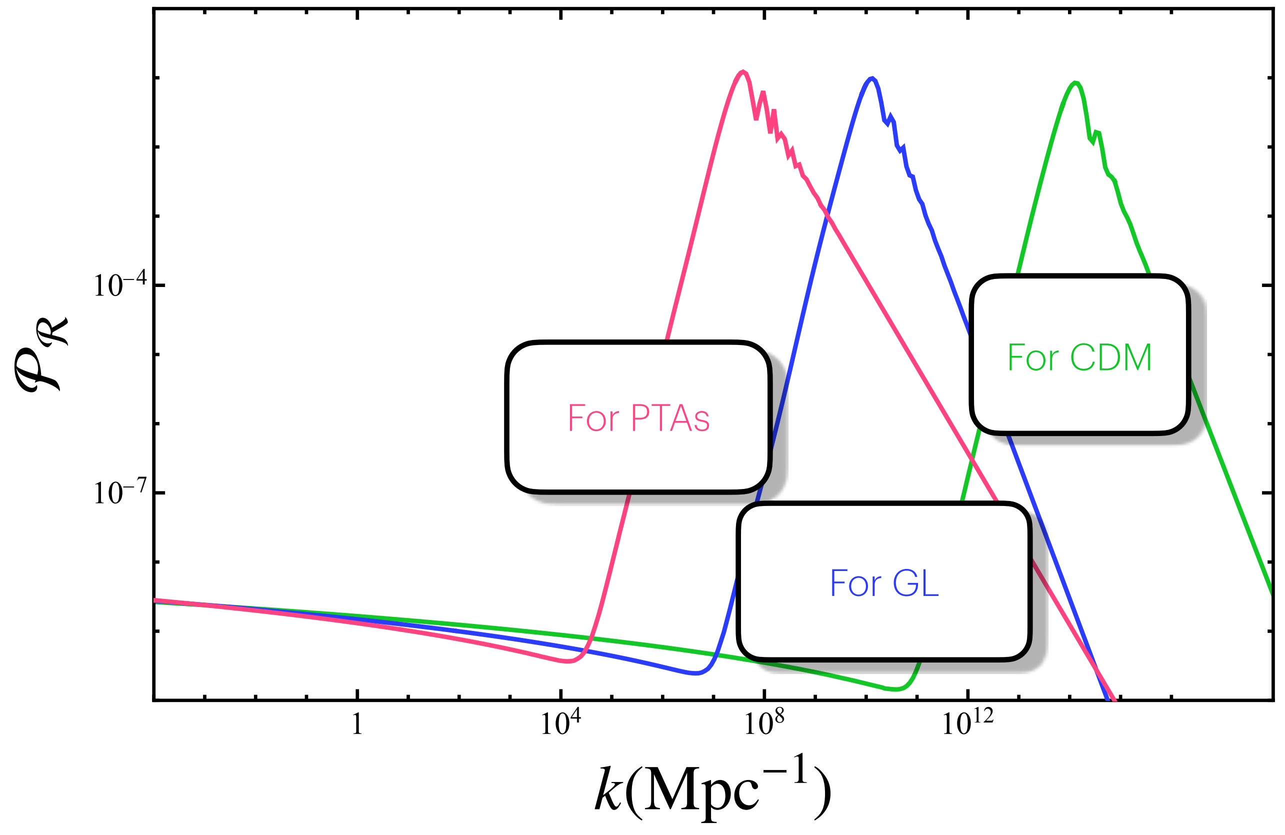
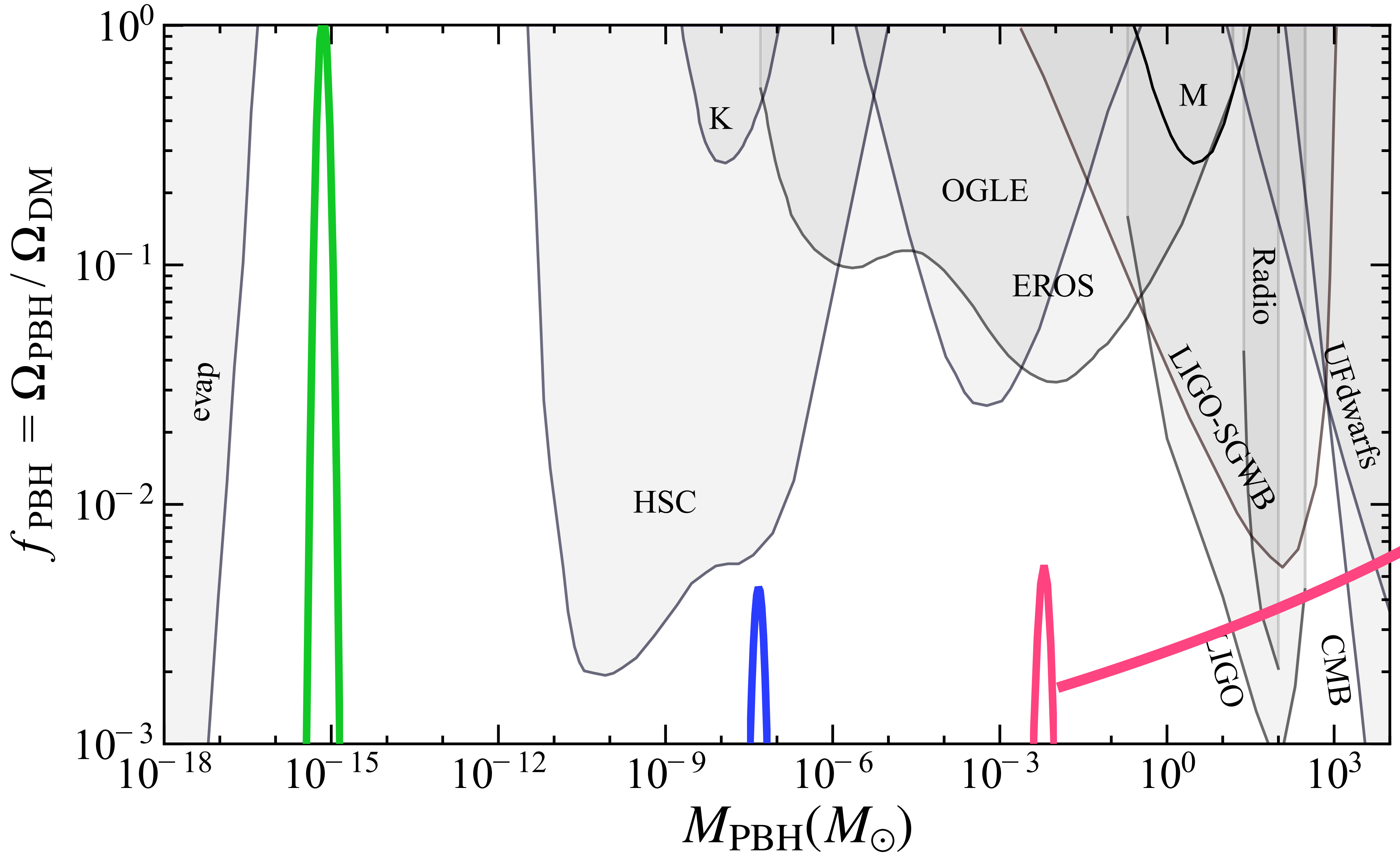
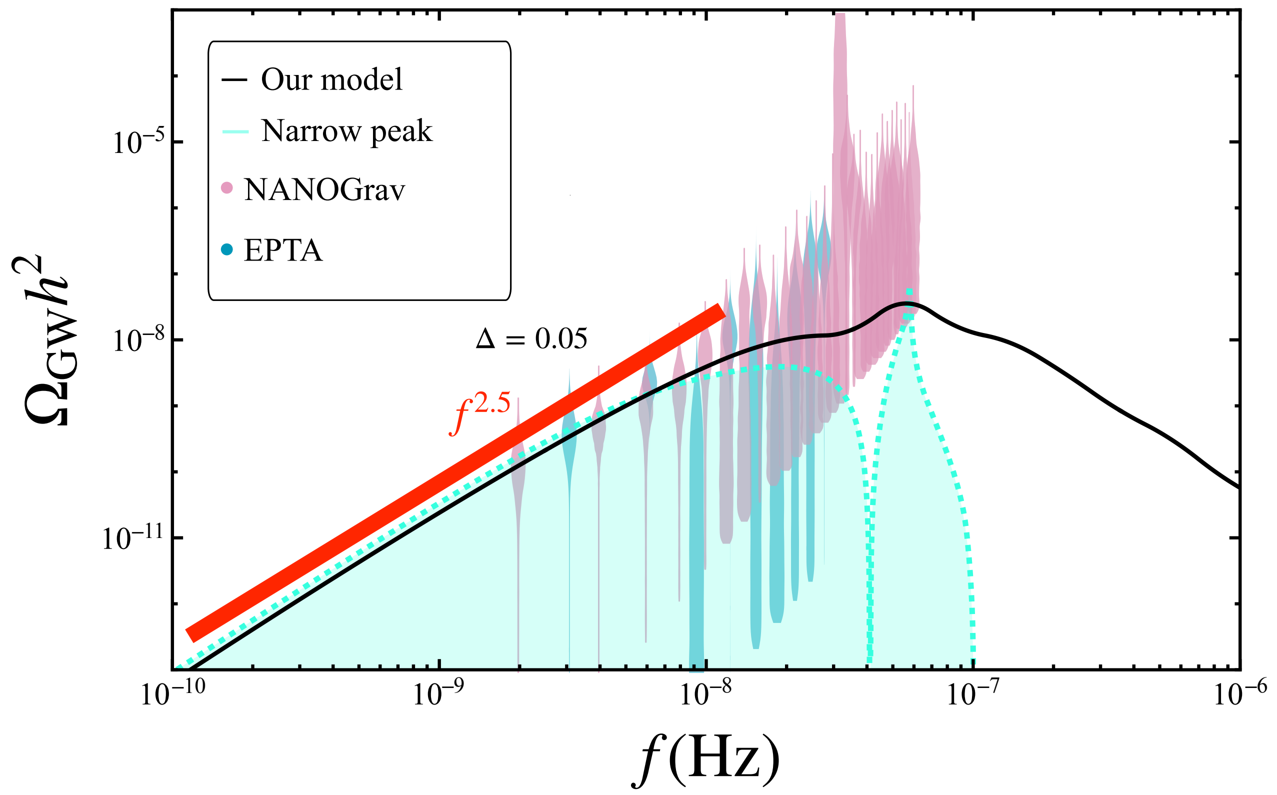
Scalar induced GW




Press-Schechter Formalism



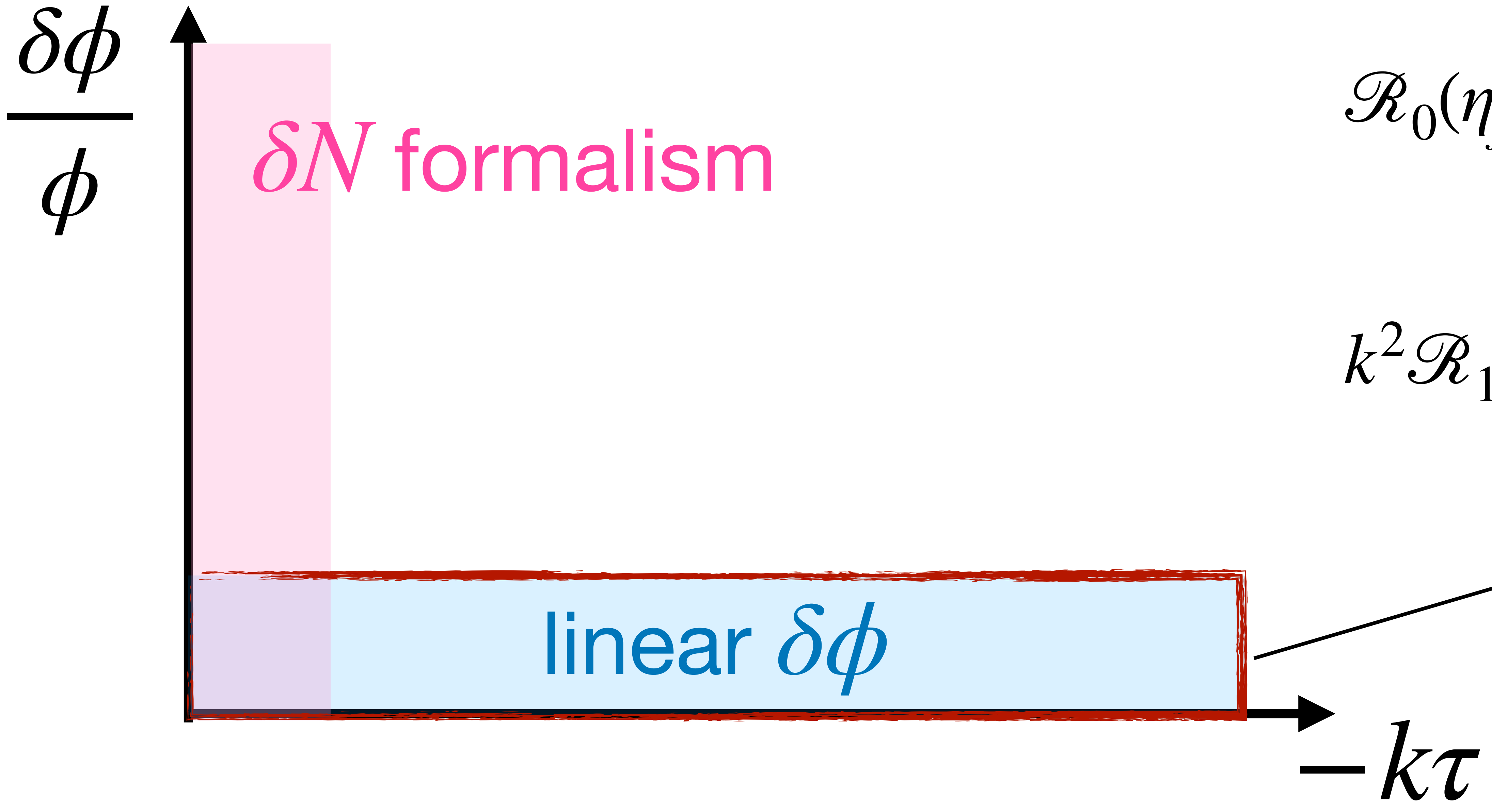



# Ways to amplify the power spectrum (Single field)

## The super horizon behavior———Decay modes and $k^2$ correction

S.Leach , M. Sasaki, D. Wands and A. Liddle , 2001 (astro-ph/0101406)

Given $\mathcal{R}_k, \mathcal{R}'_k$ at(shortly after) horizon crossing $\eta = \eta_k$




Adiabatic Decay mode

$$\mathcal{R}_0(\eta_f) = \mathcal{R}(\eta_k) + \mathcal{R}'(\eta_k)\int_{\eta_k}^{\eta_f} \frac{z^2(\eta_k)}{z^2(\eta')} d\eta'$$

$$k^2\mathcal{R}_1(\eta_f) = k^2\mathcal{R}(\eta_k)\int_{\eta_f}^{\eta_k}\int_{\eta_f}^{\eta'} \frac{z^2(\eta'')}{z^2(\eta')} d\eta'' d\eta'$$

1st order $k^2$ correction

Superhorizon behavior can be important for single field inflation.

# Ways to amplify the power spectrum (Single field)

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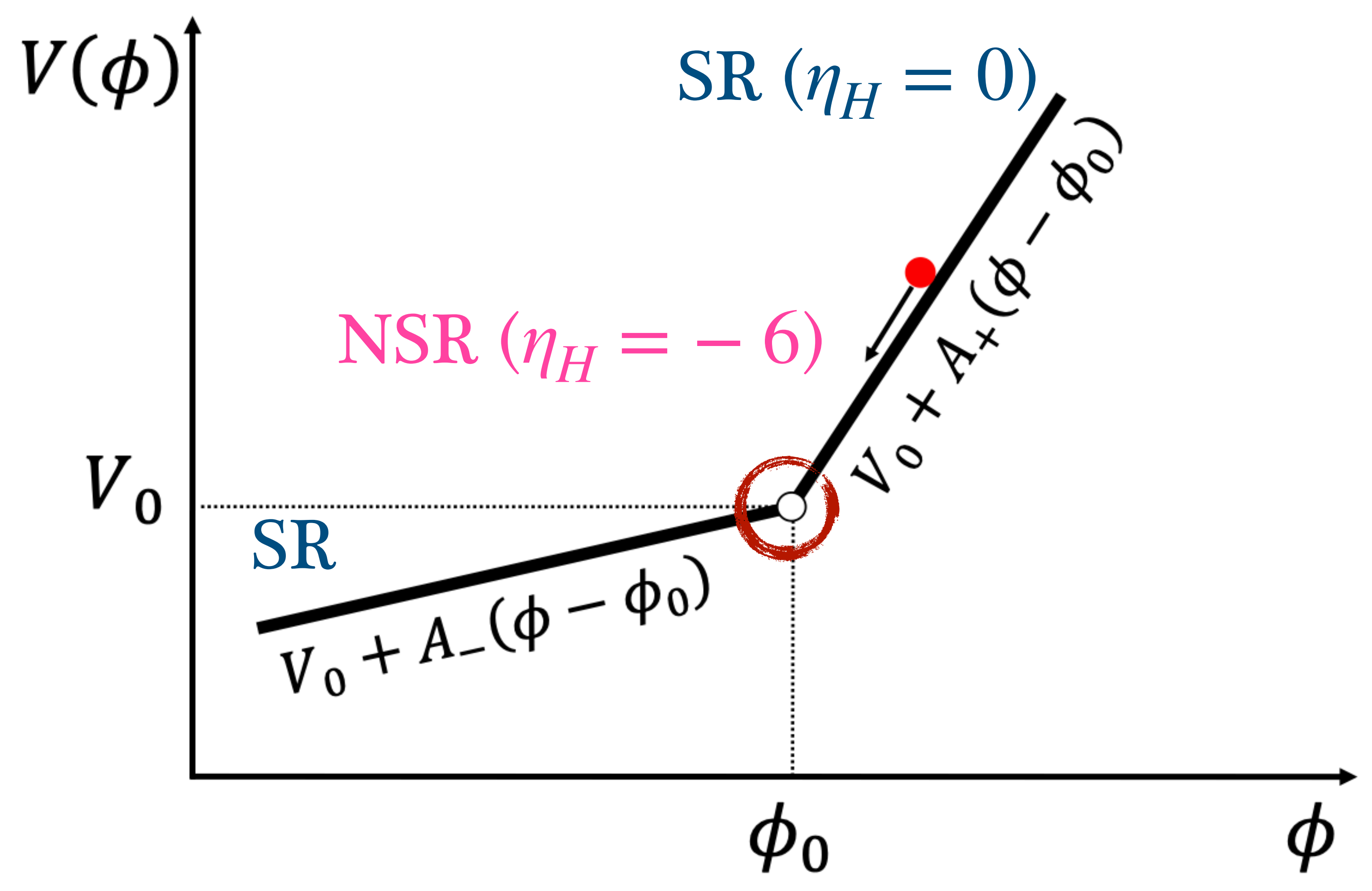
S.Leach , M. Sasaki, D. Wands and A. Liddle , 2001 (astro-ph/0101406), Shi Pi, Jianing Wang, 2022 (2209.14183);

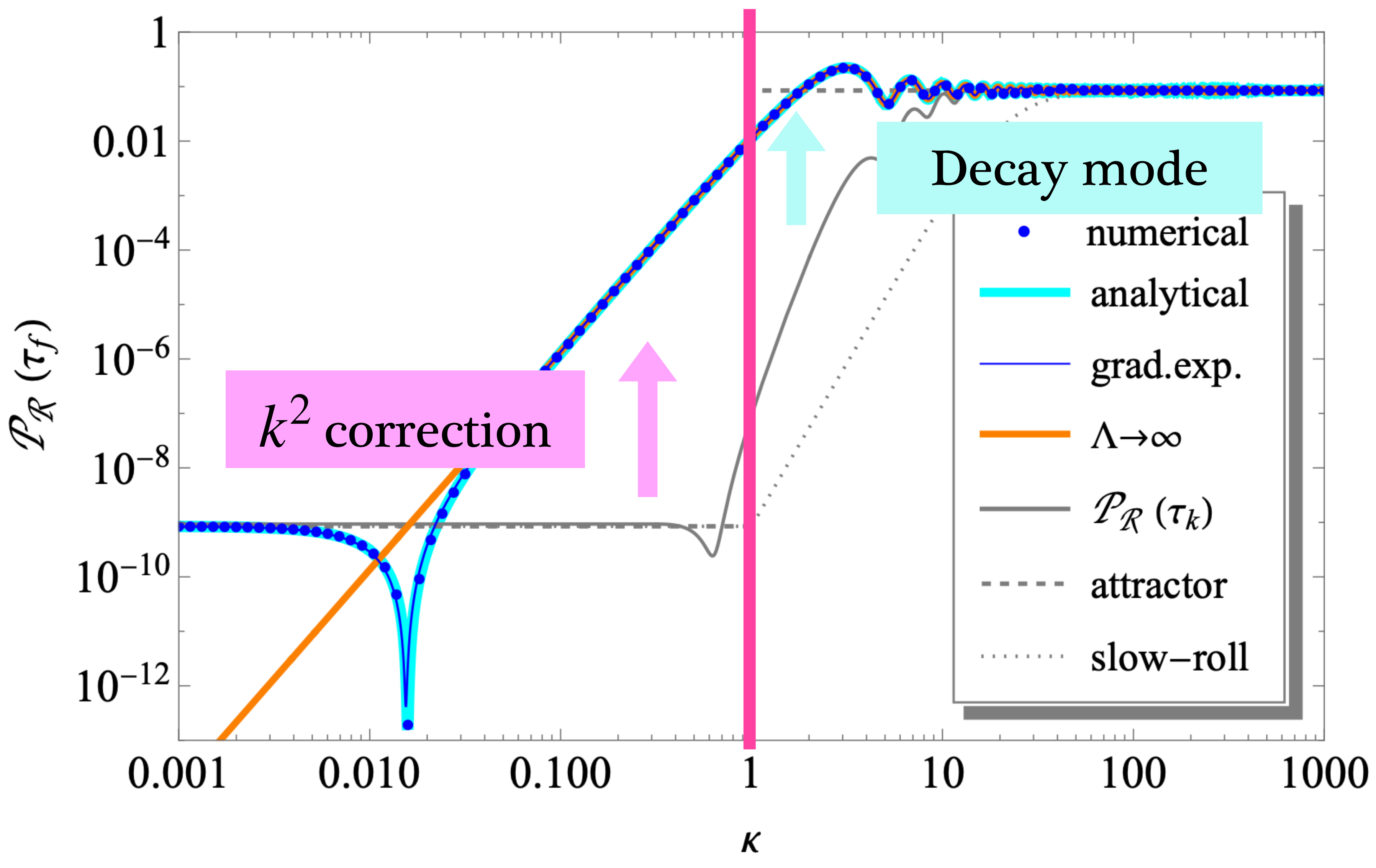
C. T. Byrnes, P. S. Cole, and S. P. Patil, 1811.11158 **The "Steepest" growth: $\mathcal{P}_{\mathcal{R}} \propto k^4$ for single field inflation**

Starobinsky's linear model




$\eta \ll \epsilon$ during SR phases is assumed

$k_0 = a_0 H_0$



# Ways to amplify the power spectrum (Single field)

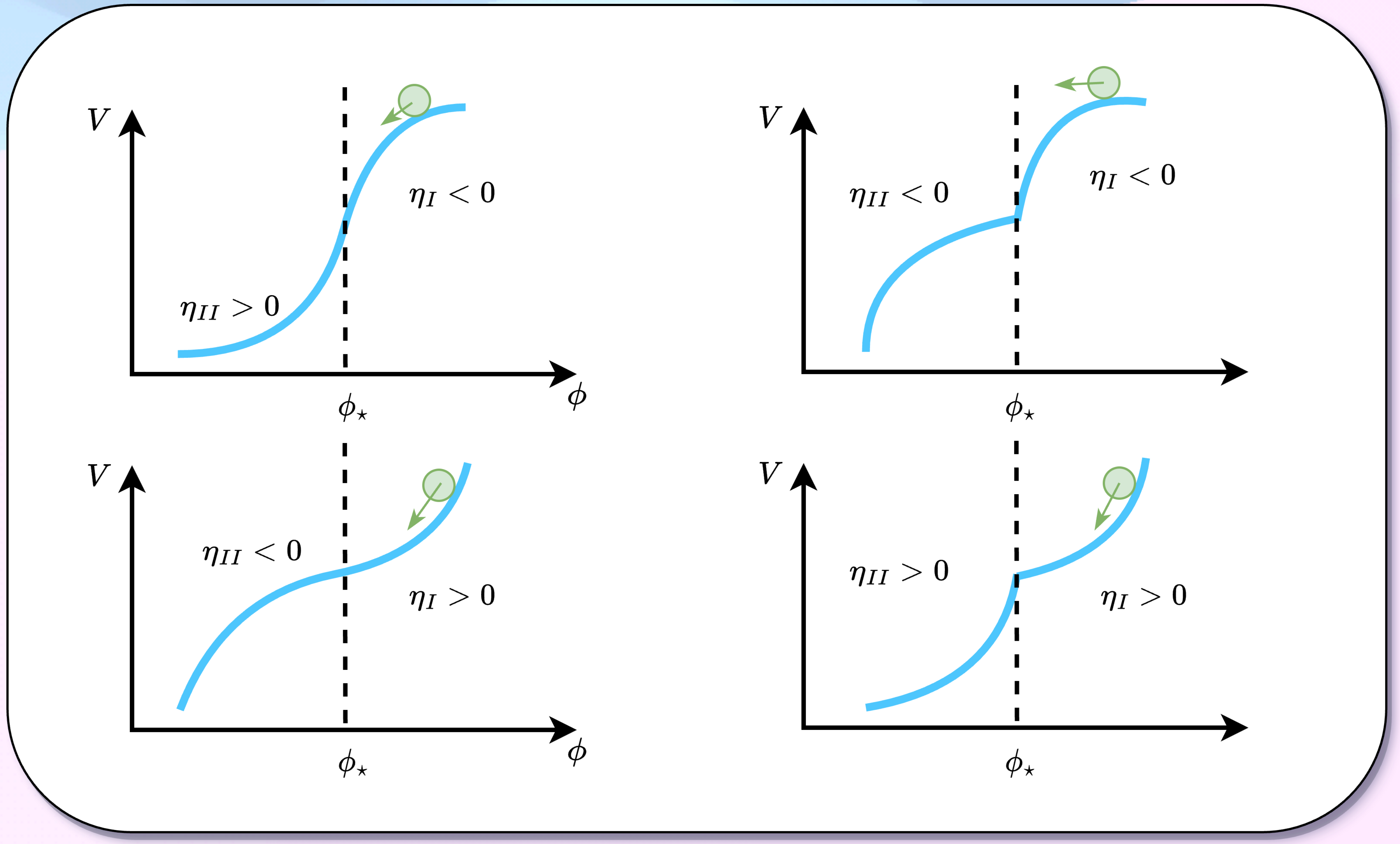
What will happen if we consider a quadratic piecewise potential?

Enhanced $\mathscr{P}_{\mathscr{R}}$?

New shape of spectrum?

Non-Gaussianity?

$$V = \begin{cases} V_\star\left(1+\sqrt{2\epsilon_I}(\phi-\phi_\star)+\frac{1}{2}\eta_I(\phi-\phi_\star)^2\right) & \phi \geq \phi_\star, \\ V_\star\left(1+\sqrt{2\epsilon_{II}}(\phi-\phi_\star)+\frac{1}{2}\eta_{II}(\phi-\phi_\star)^2\right) & \phi < \phi_\star\,. \end{cases}$$



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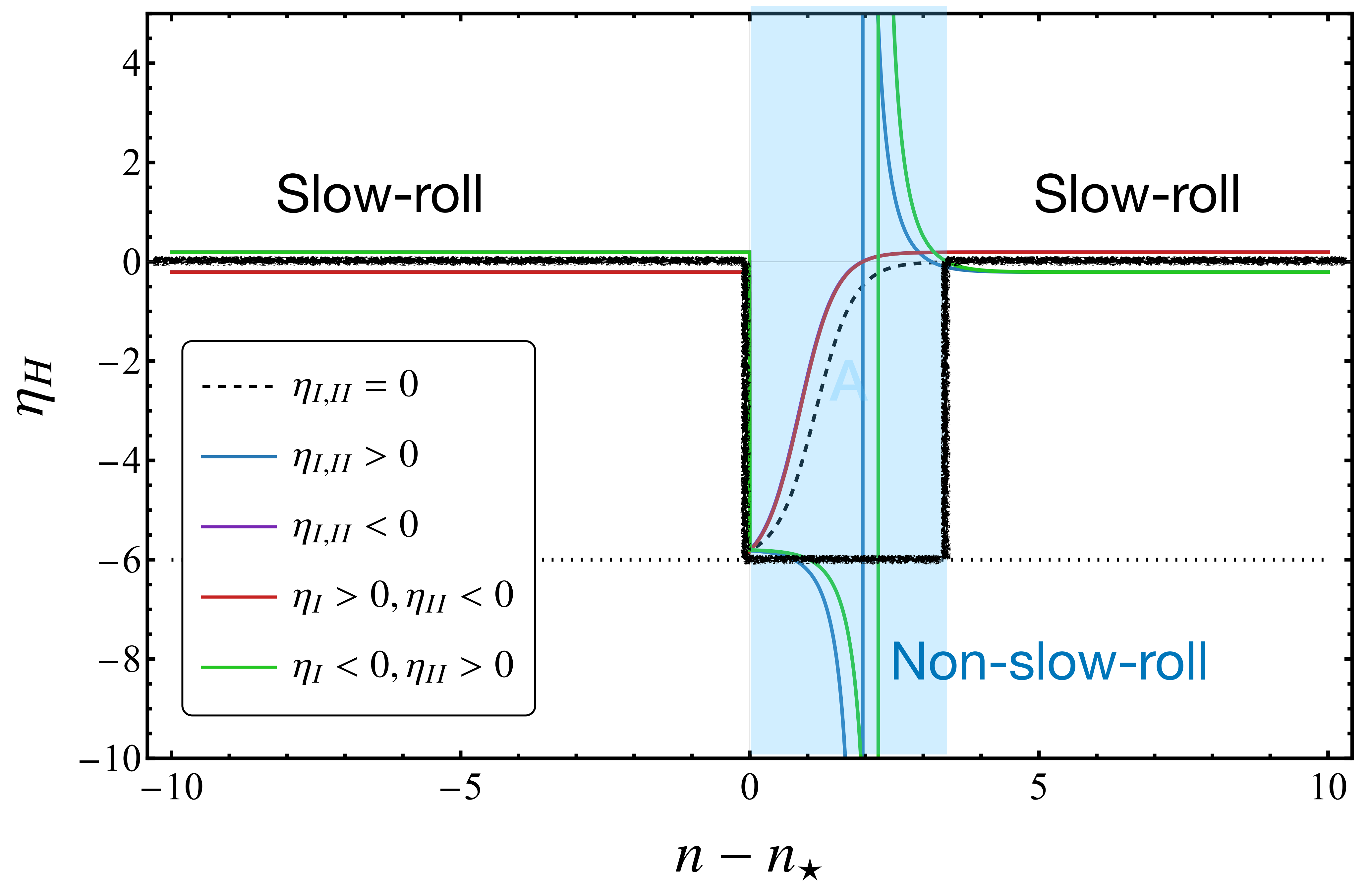


# The Background solution

Non-Decay mode Decay mode

$$\widetilde{\phi}_X = C_{X,+}e^{\lambda_{X,+}(n-n_\star)} + C_{X,-}e^{\lambda_{X,-}(n-n_\star)} \qquad n = Hdt$$

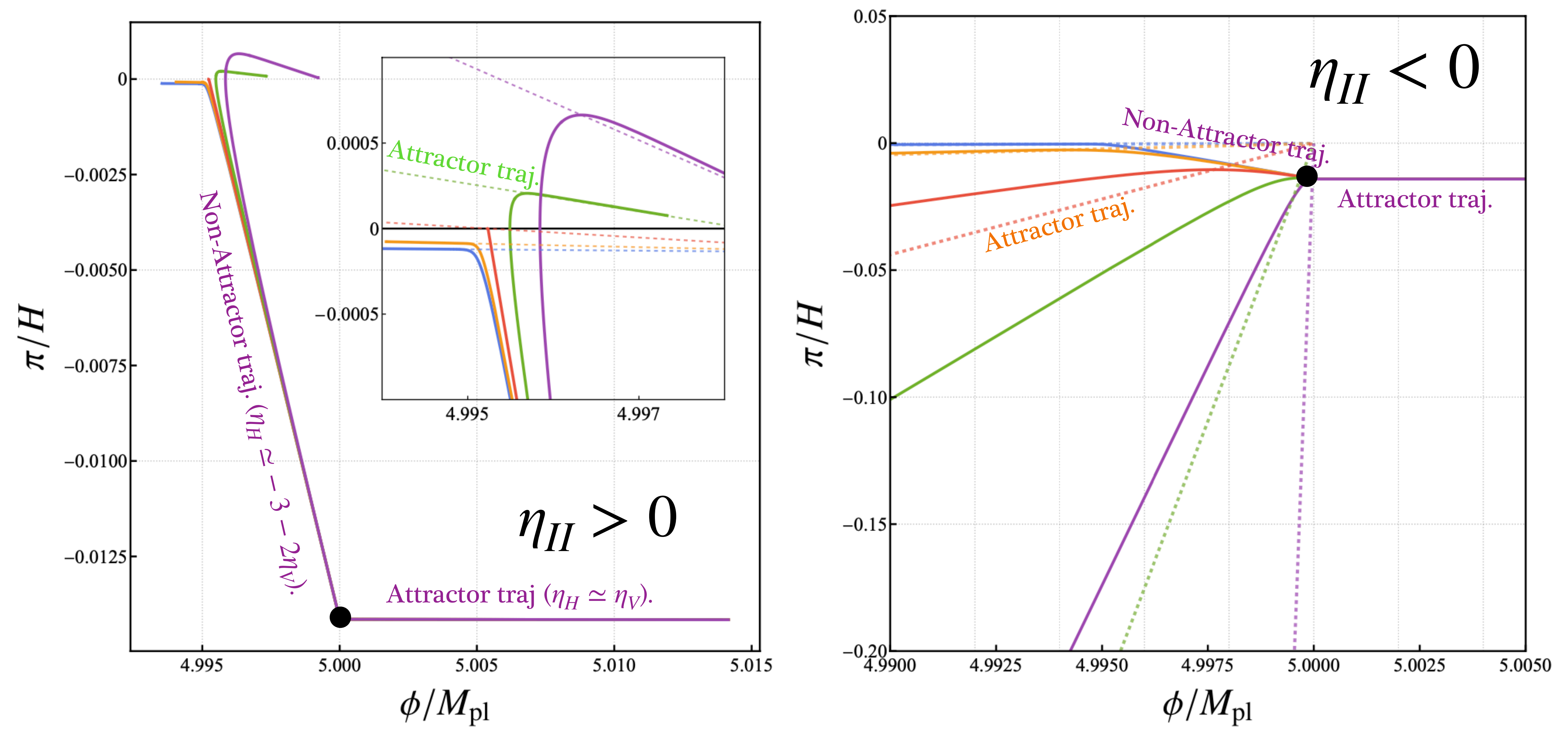
$$\lambda_{X,\pm} = \frac{-3 \pm \sqrt{9-12\eta_X}}{2} = \pm\nu_X - \frac{3}{2}$$

Enters Non-Slow-Roll phase

$(\eta_H = -3 - \sqrt{9-12\eta_X})$

when decay mode dominates,

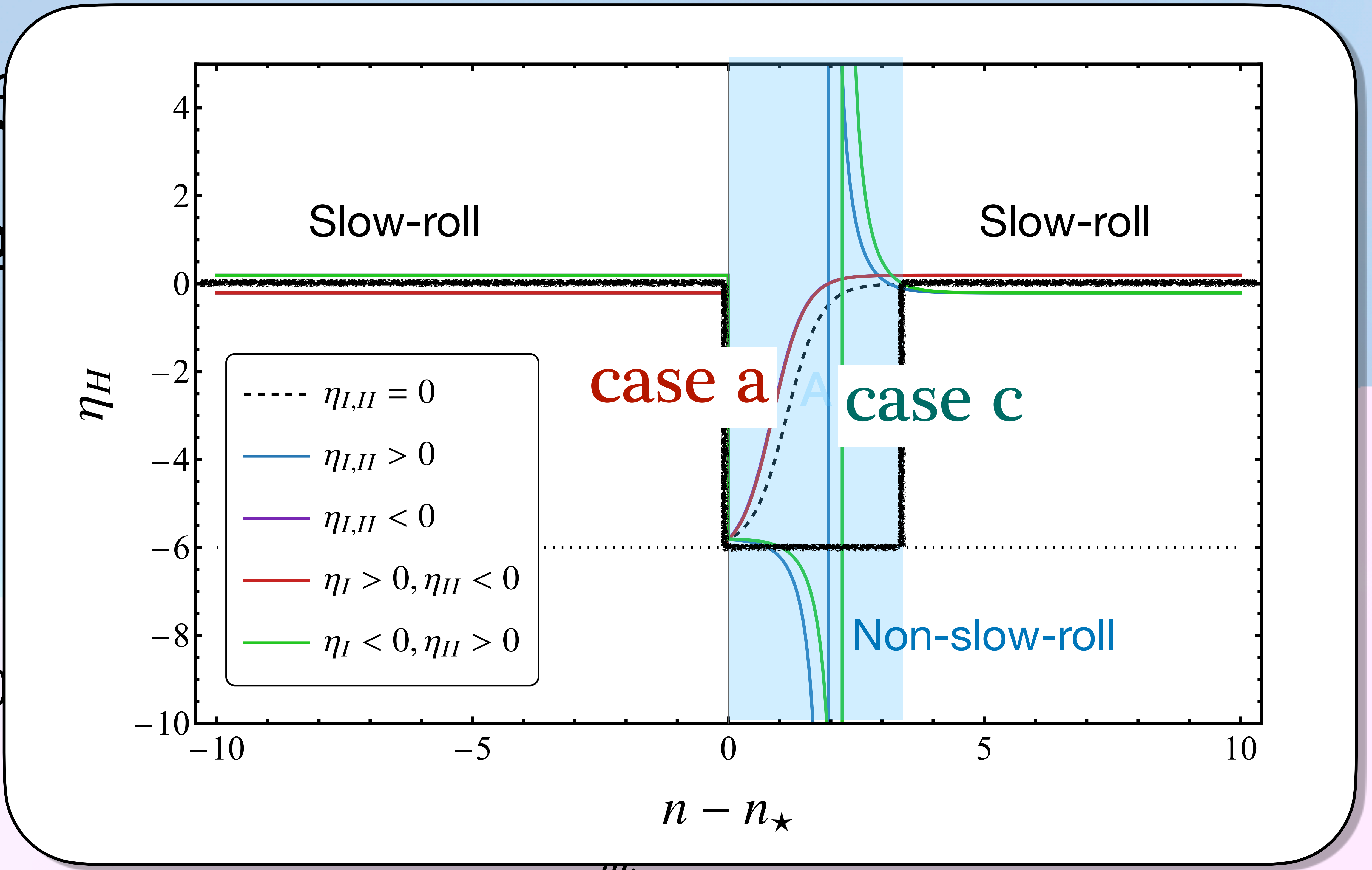
$C_- > C_+$ .



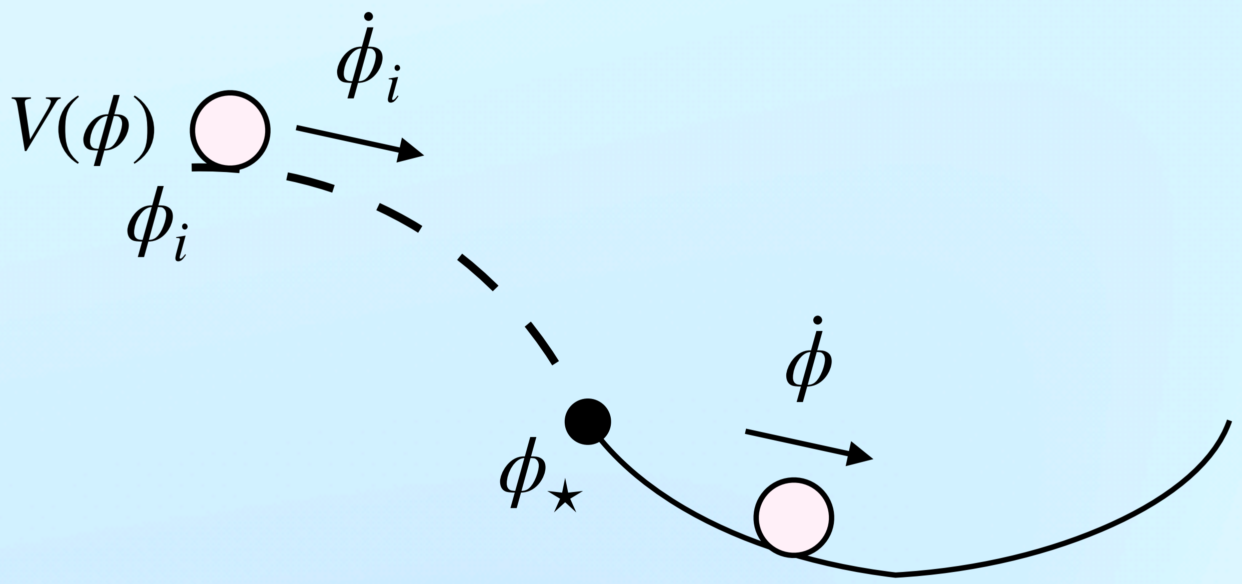
# An estimation of the spe

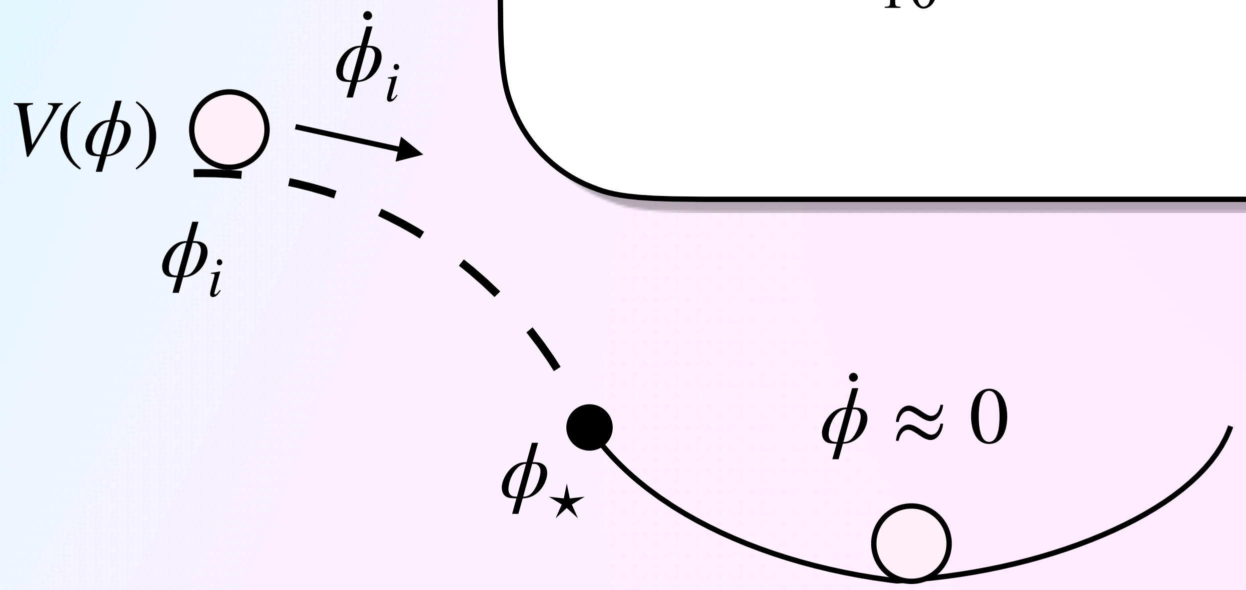
Using $\delta N$ formalism under small $\eta$ app

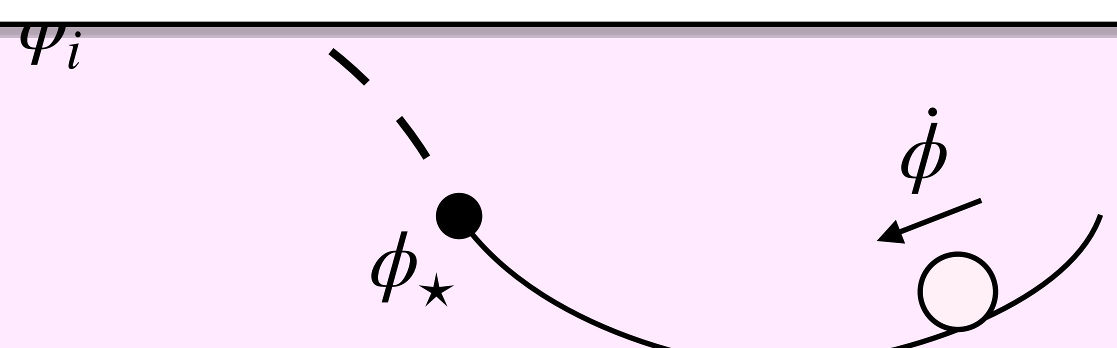
$$\frac{\mathcal{P}_{\mathcal{R}}(k \gg k_\star)}{\mathcal{P}_{\mathcal{R}}(k \ll k_\star)} \approx \frac{\epsilon_H(t \gg t_\star)}{\epsilon_H(t \ll t_\star)} \propto (\alpha - 1)^{-2}$$




Three cases when the second slow roll starts for $\eta_{II} > 0$




a. NOT pass the potential minimum $\alpha < 1$

b. Stop at the potential minimum $\alpha = 1$

c. Pass the potential minimum $\alpha > 1$

# The analytical power spectrum

We solve the Sasaki-Mukhanov equation,

$$u_k'' + \left(k^2 - \frac{z''}{z}\right) u_k = 0, \qquad \frac{z''}{z} \simeq \begin{cases} \tau^{-2}(2 - 3\eta_I) & \phi > \phi_\star, \\ \tau^{-2}(2 - 3\eta_{II}) & \phi < \phi_\star, \end{cases}$$

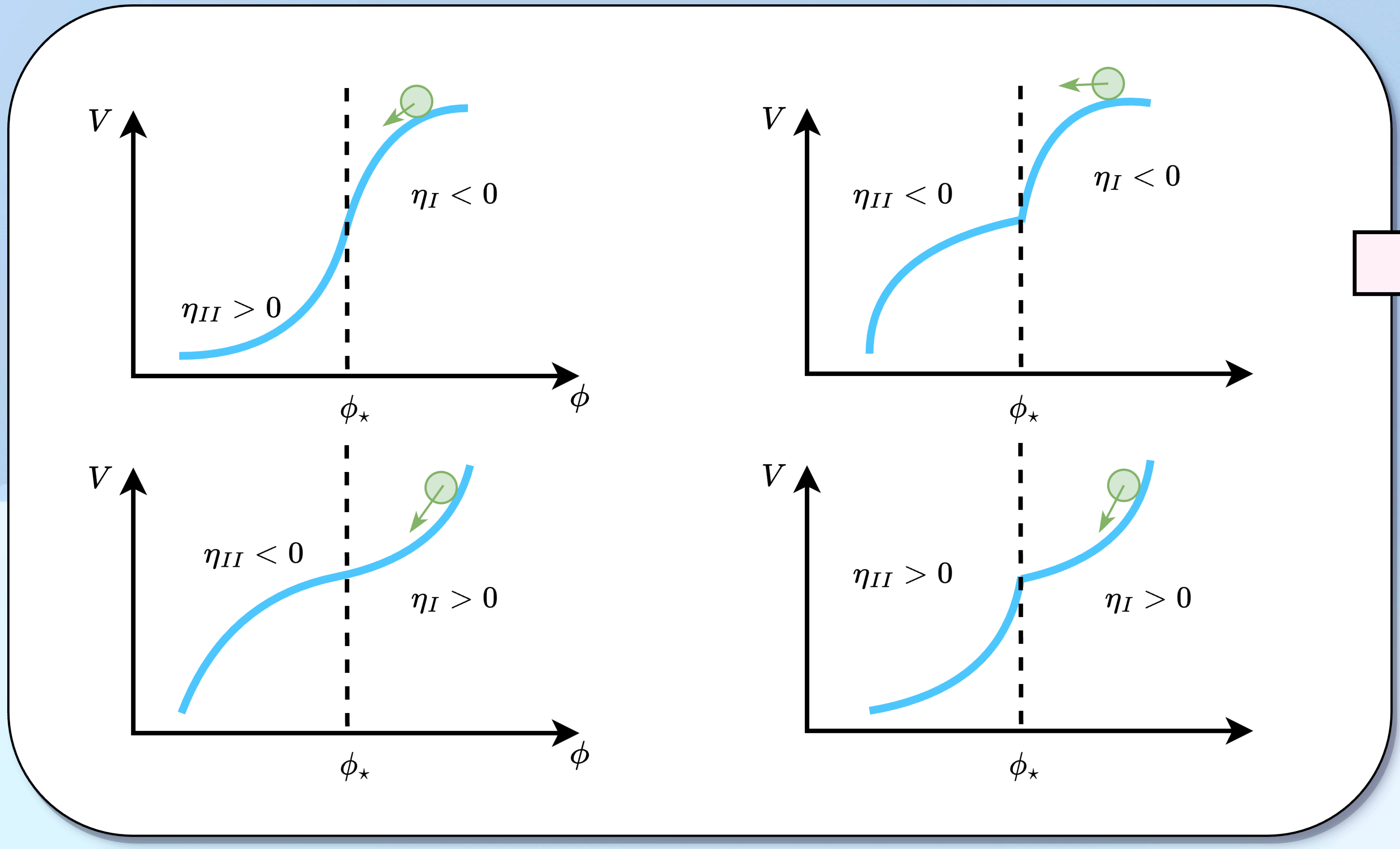
and match at joint point $\phi_\star$ by the continuity of $\mathcal{R}, \mathcal{R}'$,

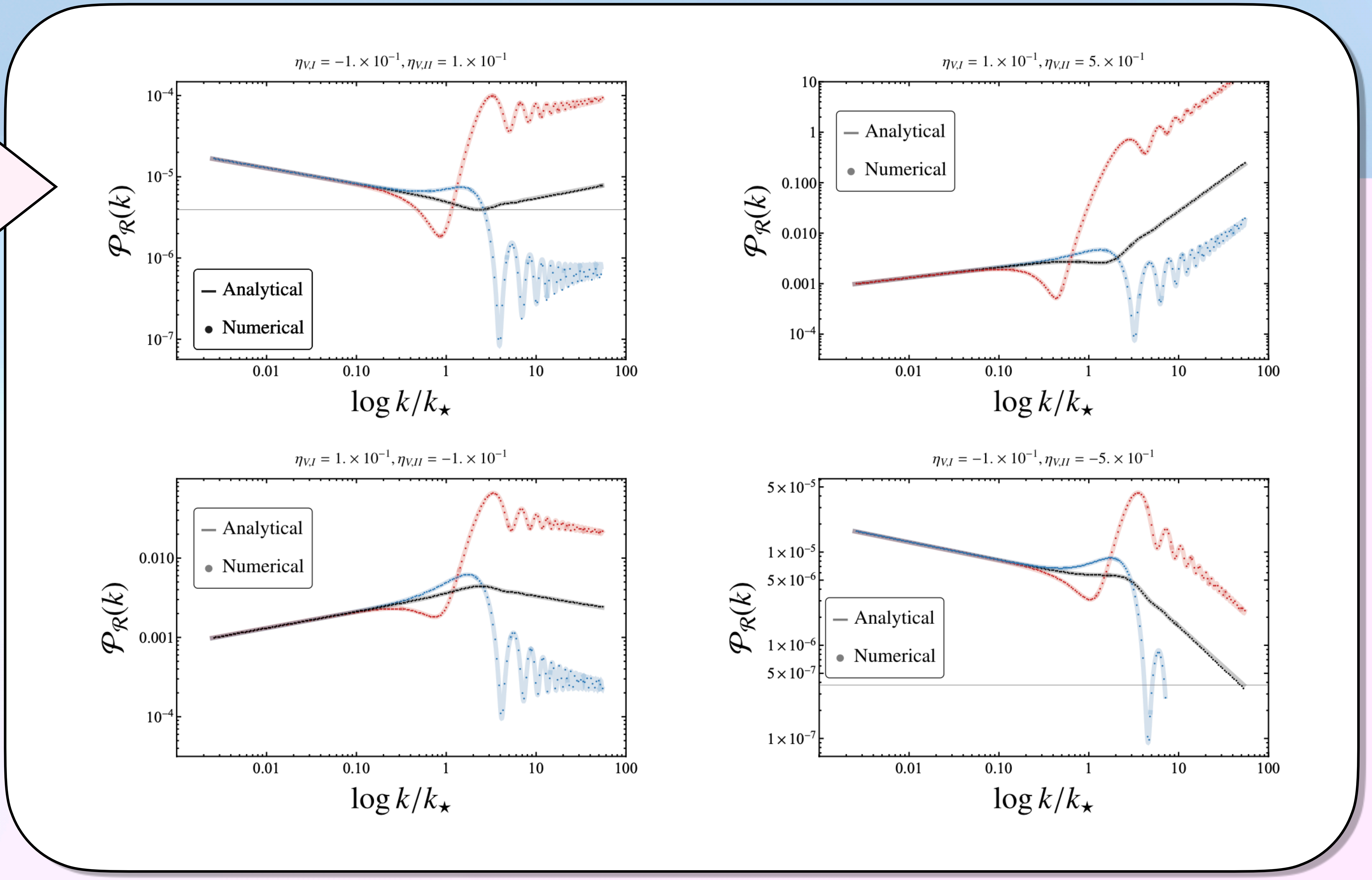
Bogoliubov Coefficients

$$u_{k,X} = C_I\sqrt{-k\tau}H^{(1)}_{\nu_X}(-k\tau) + D_X\sqrt{-k\tau}H^{(2)}_{\nu_X}(-k\tau) \Rightarrow \begin{aligned} C_{II} &= f(C_I, D_I, \epsilon_{I,II}, \eta_{I,II}, k) \\ D_{II} &= g(C_I, D_I, \epsilon_{I,II}, \eta_{I,II}, k) \end{aligned}$$

$-k\tau \to 0$ $\Rightarrow$ The power spectrum at the end of inflation,

# The analytical and numerical power spectrum



# The analytical and numerical power spectrum

$$\frac{\mathcal{P}_{\mathcal{R}}(k \gg k_\star)}{\mathcal{P}_{\mathcal{R}}(k \ll k_\star)} = 2^{2(\nu_{II}-\nu_I)}\left(\frac{\Gamma(\nu_{II})}{\Gamma(\nu_I)}\right)^2 \left(\frac{4\nu_{II}}{(-3+2\nu_{II})+R_\epsilon(3+2\nu_I)}\right)^2$$

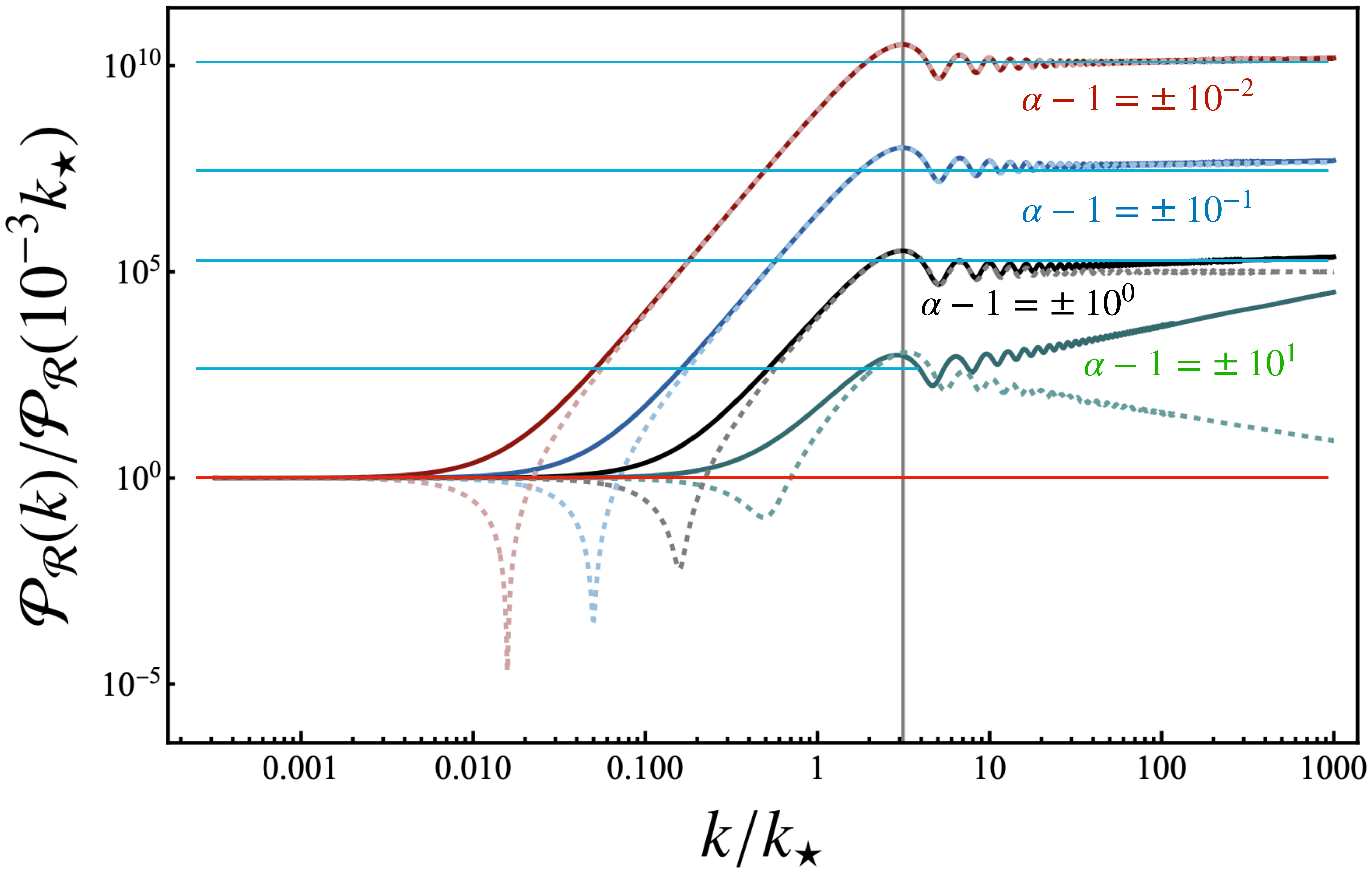
$\propto (\alpha - 1)^{-2}$

$\epsilon_{II}/\epsilon_I = 1, \quad \eta_{I,II} \ll 1$ ➡️ $\alpha - 1 \simeq 1$:

No significant enhancement

$\epsilon_{II}/\epsilon_I < 1, \quad \eta_{I,II} \ll 1$ ➡️ $\alpha \equiv \dfrac{\eta_{II}}{3-\eta_I}\sqrt{\dfrac{\epsilon_I}{\epsilon_{II}}}$

Maximum enhancement when $\eta_{II} \sim 3\sqrt{\epsilon_{II}/\epsilon_I}$



# The growth rate of the power spectrum

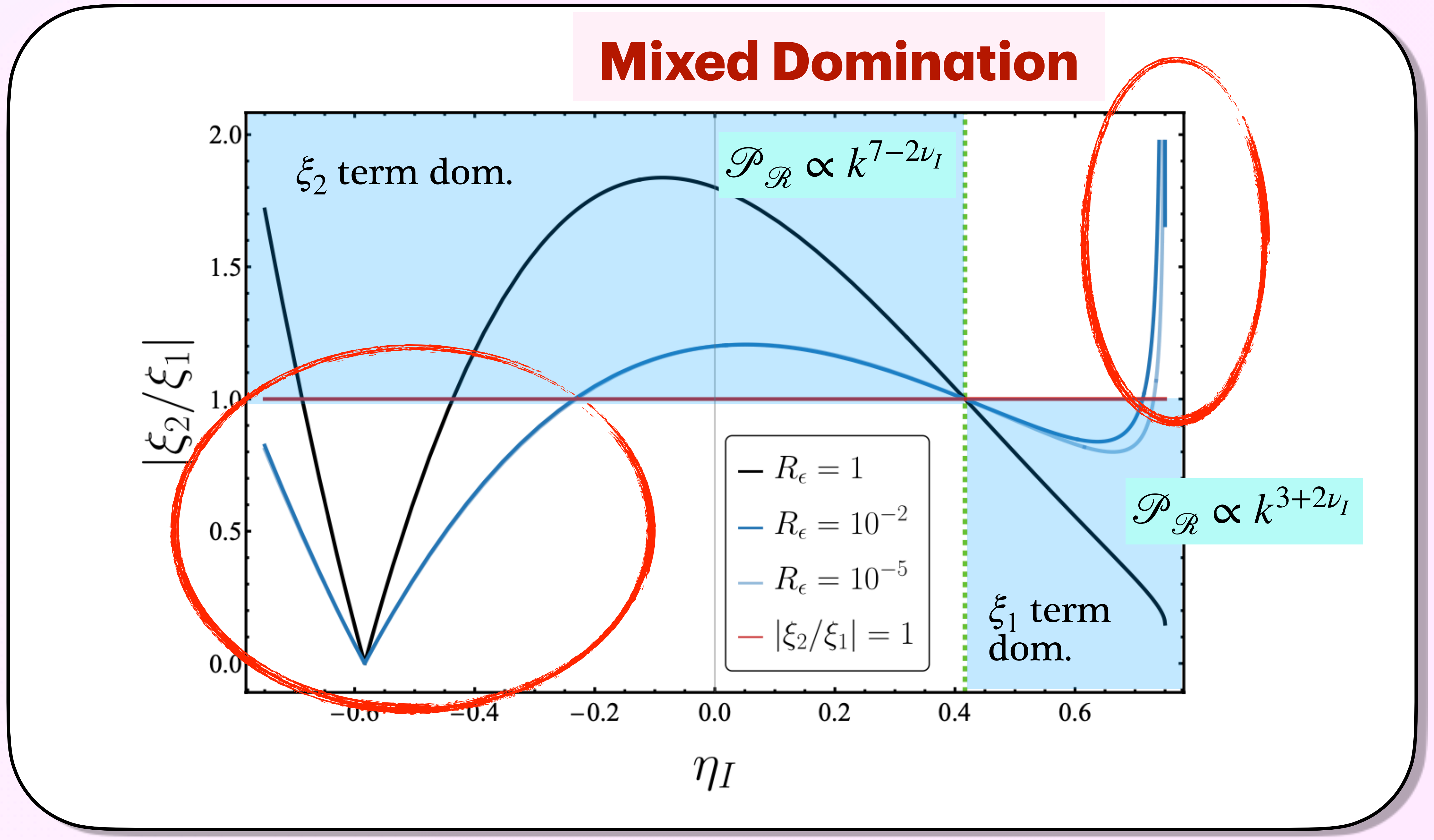
When $x = \dfrac{k}{k_\star} \ll 1$, expand the analytical $\mathscr{P}_\mathscr{R}$ in small $x$ limit

$$(\mathscr{P}_\mathscr{R})^{1/2} = \mathscr{A}^L_\mathscr{R} x^{\frac{3}{2}-\nu_I} \left| 1 + \xi_1 x^{2\nu_I} + \xi_2 x^2 + \mathscr{O}(x^{2+2\nu_I}) \right|$$

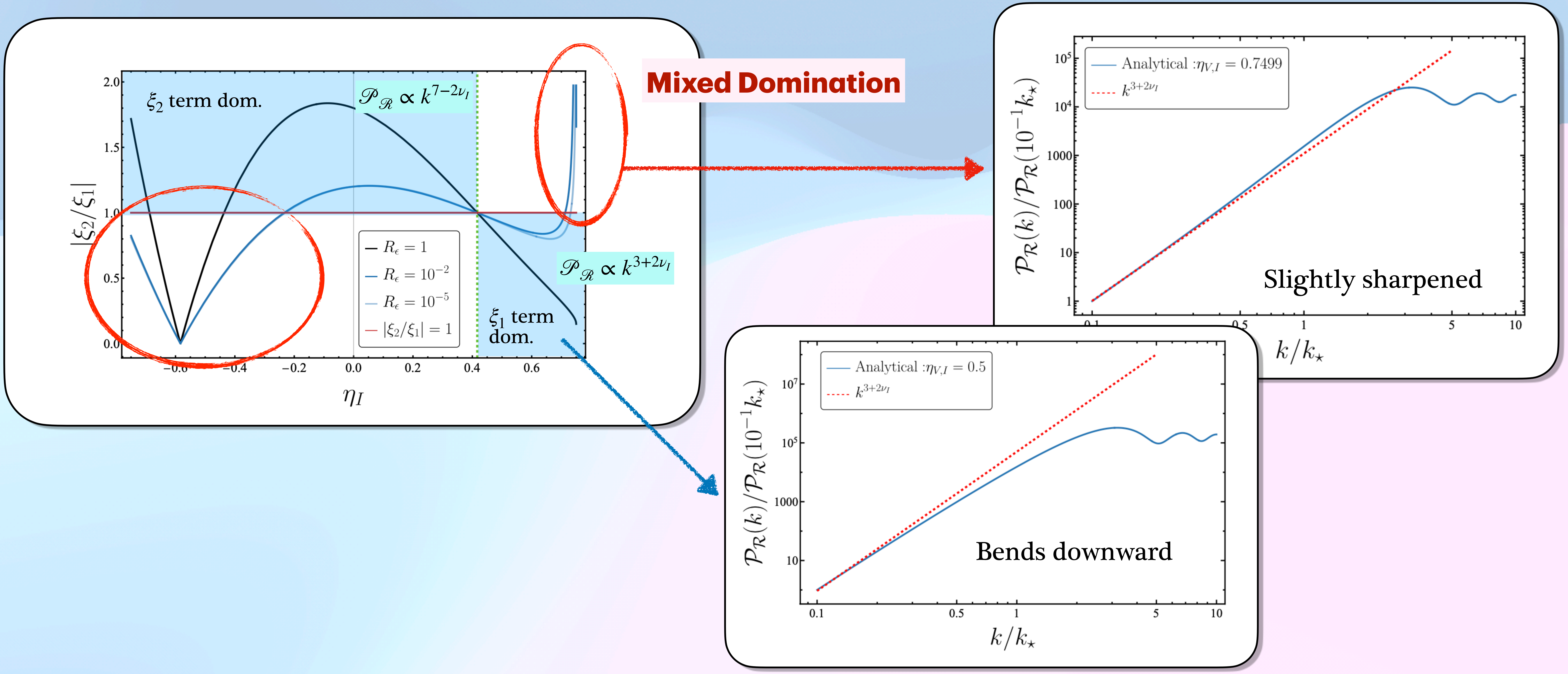
For $k \ll k_\star$ $\qquad \mathscr{P}_\mathscr{R}(k) \propto k^{3-2\nu_I}$

For $k \lesssim k_\star$ $\qquad \mathscr{P}_\mathscr{R}(k) \propto k^{\min(3/2+\nu_I, 5/2-\nu_I)}$

Which is the Next-to-leading-order (NLO) term?



# The growth rate of the power spectrum



# The growth rate of the spectrum

The steepest growth

The steepest growth appears when $\nu_I = 1(\eta_I = 5/12)$

The former steepest growth for single field inflation was $k^4$ for BD init. cond.

$$7 - 2\nu_1 = 3 + 2\nu_1 = 5 \Rightarrow \mathcal{P}_{\mathcal{R}} \propto k^5 (\ln k)^2$$

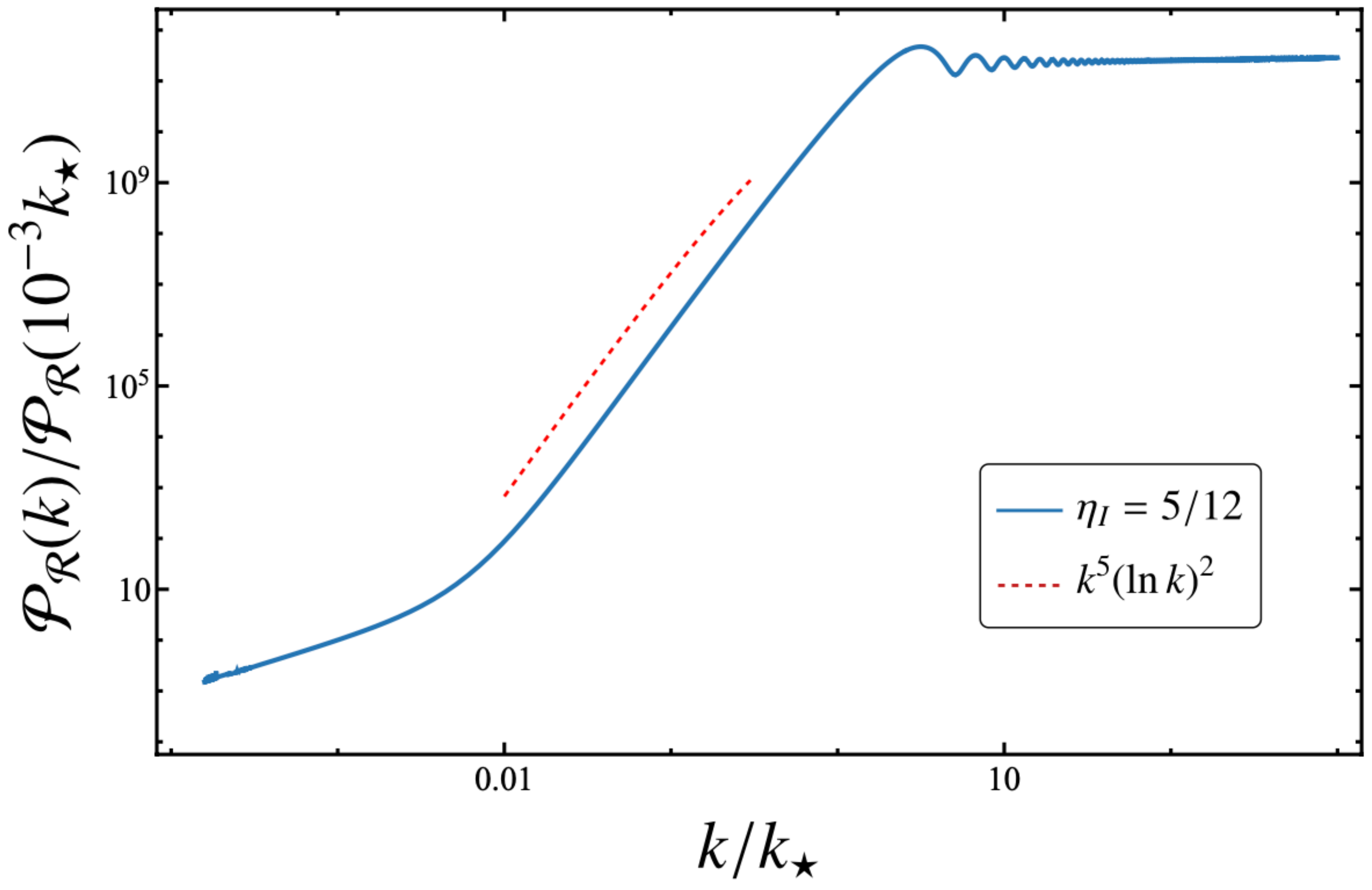
Why $\ln k$?

The Bessel function of second type when $\nu$ is an integer

$$Y_\nu(x) = -\frac{(\nu-1)!}{\pi}\left(\frac{2}{x}\right)^\nu + \ldots + \frac{2}{\pi}\left(\frac{x}{2}\right)^\nu \frac{1}{n!} \ln\left(\frac{x}{2}\right) + \ldots,$$

Thus when the order of the Bessel $\nu = 0$

$$Y_0(x) \approx -0.074 + 0.63662 \ln\left(\frac{x}{2}\right) + \ldots .$$



# The non-Gaussianity

Kodama & Sasaki, 1984    Sasaki & Stewart, 1995 (astro-ph/9507001)

$\delta N$ formalism is used to calculate the full possibility distribution function of $\mathscr{R}$,

*$\mathscr{R} = \delta N$ is based on separate universe approach, leading order of gradient expansion

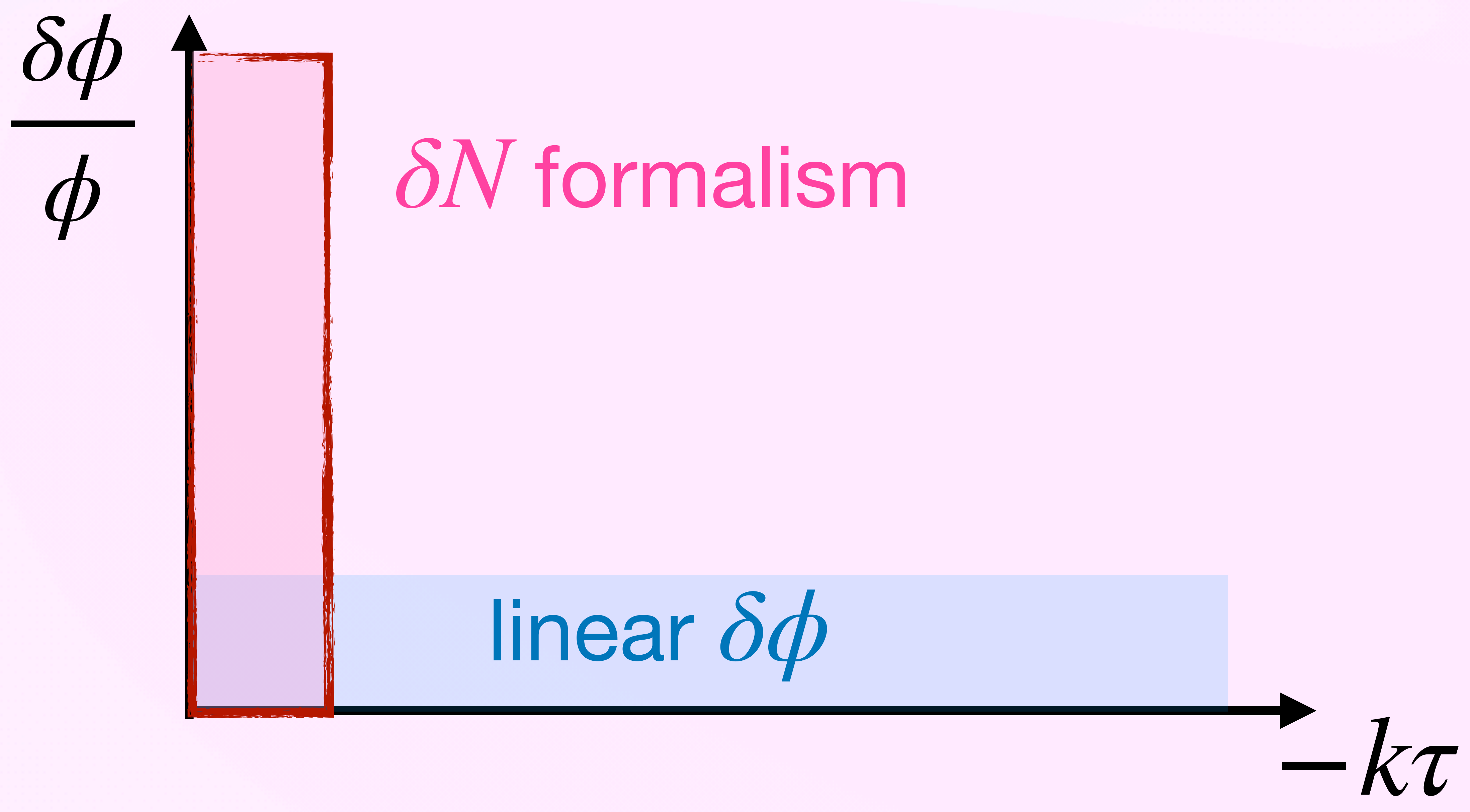
Background solution $\phi = \phi(N)$

⇓

$$\mathscr{R} \equiv \delta N = \tilde{N}_{\rm tot} - N_{\rm tot}$$

⇓

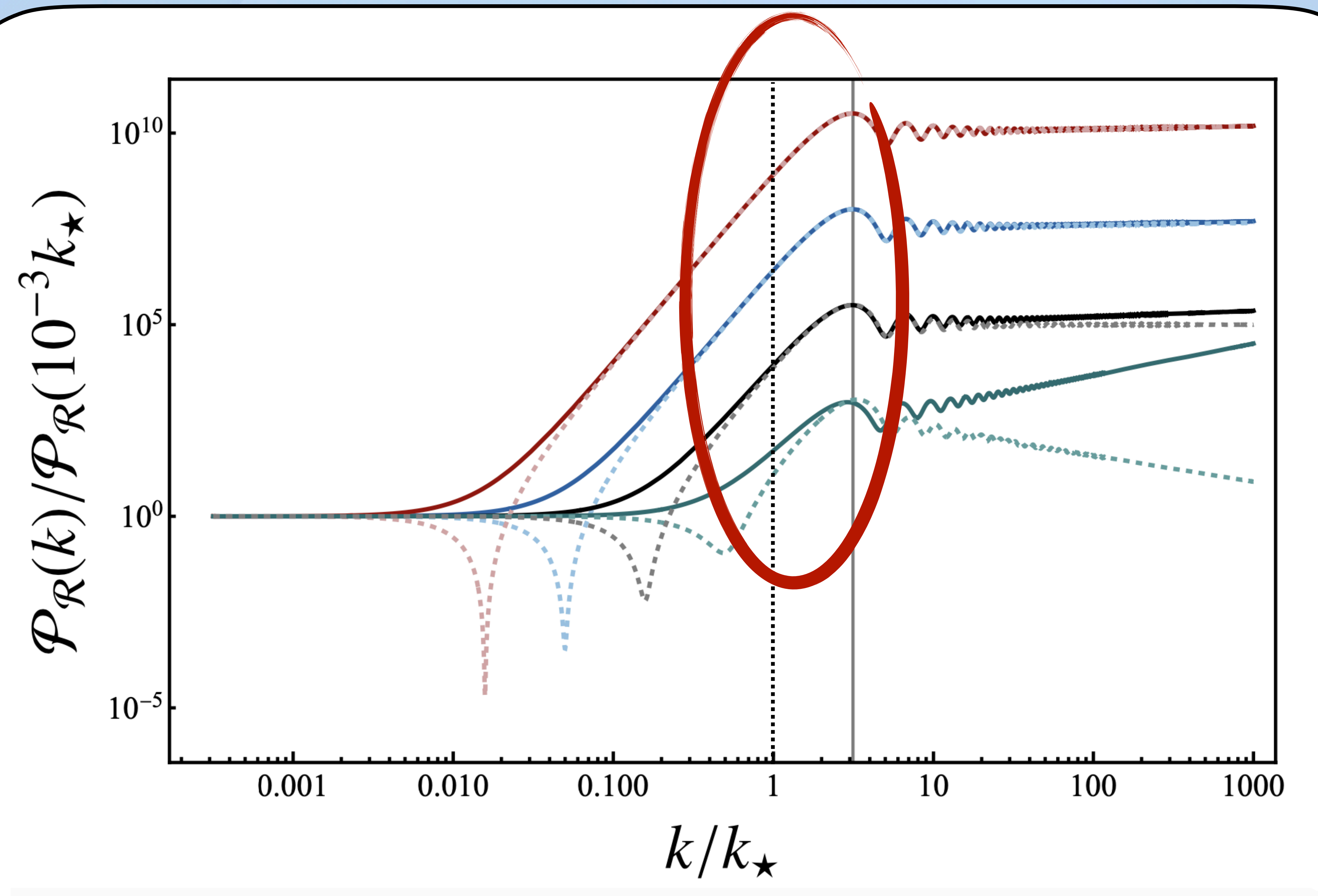
$$P[\mathscr{R}] \approx P[\mathscr{R}^{-1}(\delta\phi)]$$



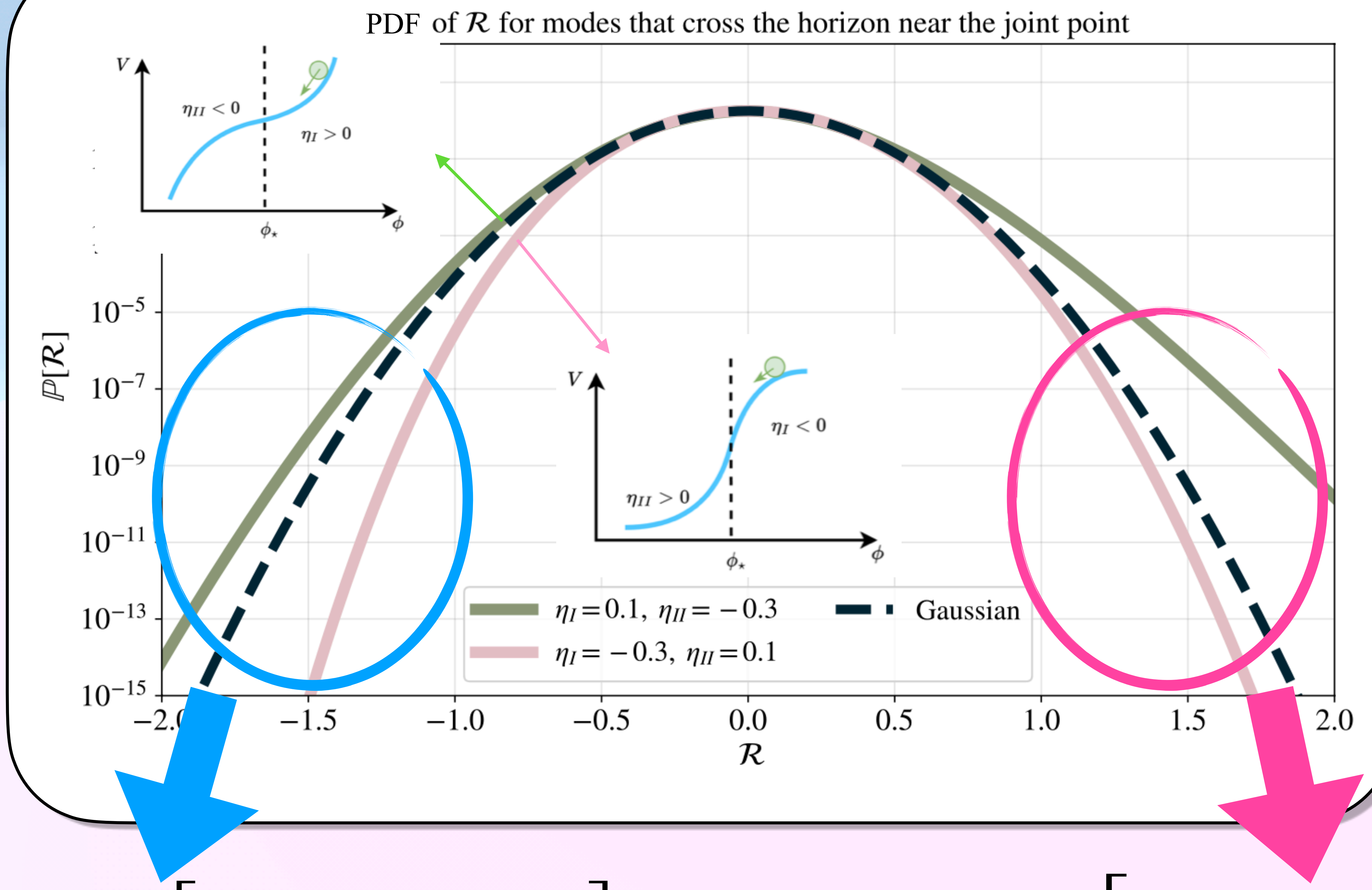
# The non-Gaussianity

S. Pi, M. Sasaki, 2211 (2211.13932)

## Assuming a Gaussian $\delta\phi$




## For modes that exit the horizon close to $n_\star$




$$\mathscr{R} \approx -\frac{1}{\lambda_{II,+}} \ln\left[1 + \frac{\lambda_{I,-}\delta\phi + \delta\pi}{\pi_\star + \lambda_{II,-}\widetilde{\phi}_{II,\star}}\right]$$

$$\mathscr{R} \approx -\frac{1}{\lambda_{II,+}} \ln\left[1 + \frac{\lambda_{II,-}\delta\phi + \delta\pi}{\pi + \lambda_{II,-}\widetilde{\phi}_{II}}\right]$$

The + $\mathscr{R}$ tail is controlled by $\eta_{II}$

# A small trial to PBH formation

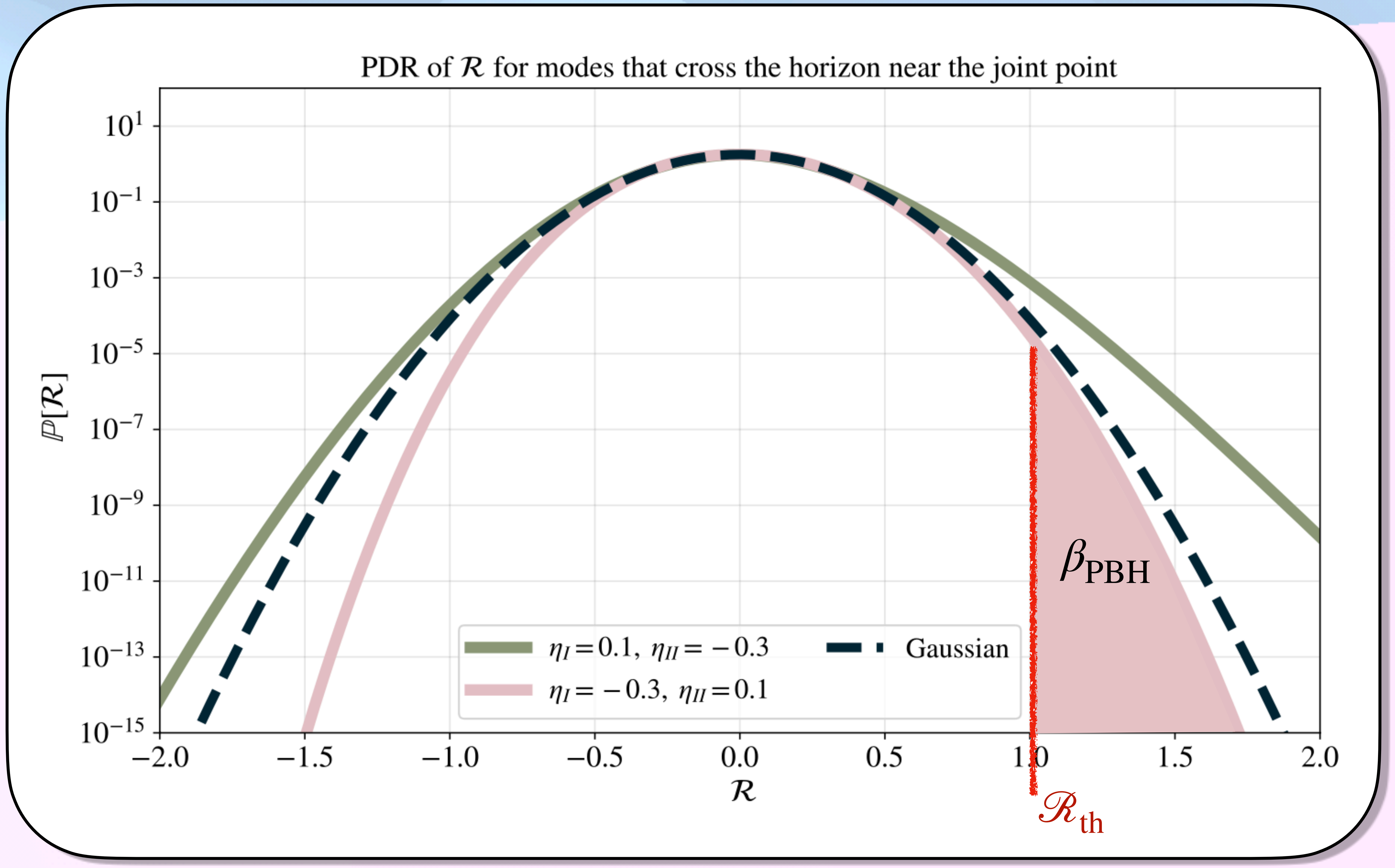
Using Press-Schechter method to calculate the PBH abundance $\beta_{\rm PBH}$

$$\delta \simeq -\frac{2}{3}\frac{3(1+\omega)}{5+3\omega}\left(\frac{1}{aH}\right)^2 \nabla^2 \mathscr{R}\ ,$$

$$\delta\phi^{\rm ng} = \frac{e^{-\lambda_{+,II}\mathscr{R}}-1}{\lambda_{-,II}}(\pi + \lambda_{-,II}\phi)\,.$$

$$\delta_{\rm th} \to \mathscr{R}_{\rm th} \to \delta\phi_{\rm th}$$

NG NG Gaussian




Still a Gaussian Integral!

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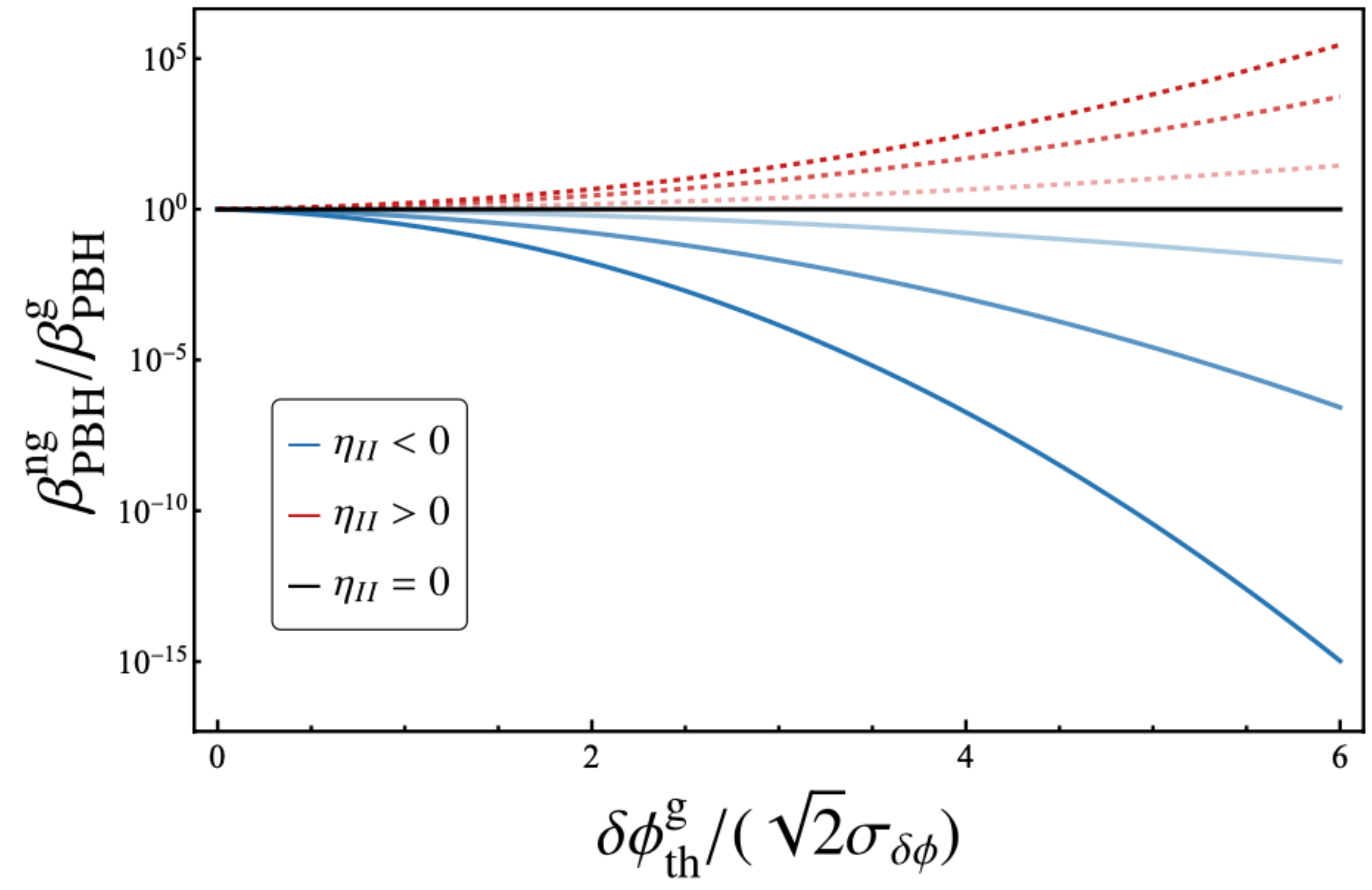


For the same $\eta_{II}$ , the suppression effect is much stronger than enhancement effect.

# Conclusion

1. The super-horizon growth of the curvature power spectrum can be sourced by isocurvature, adiabatic decaying mode and NLO term of the gradient expansion.

2. The involvement of quadratic term (even very small) in the discussion of the SR-USR-SR transition can lead to significant imprints on the shape of power spectrum:

Extracting $\eta$/extra field inform. from future observations...

The tilted IR(UV) end/ the amplitude of enhancement/ the scaling behavior and dip feature in the near IR scale/ the peak position…

and causes exponential tails in the PDF of $\mathscr{R}$ that suppress/enhance the PBH production efficiently.

Thank you for the attention!