Supersymmetric Partition Functions and Central Charges of Topological Branes

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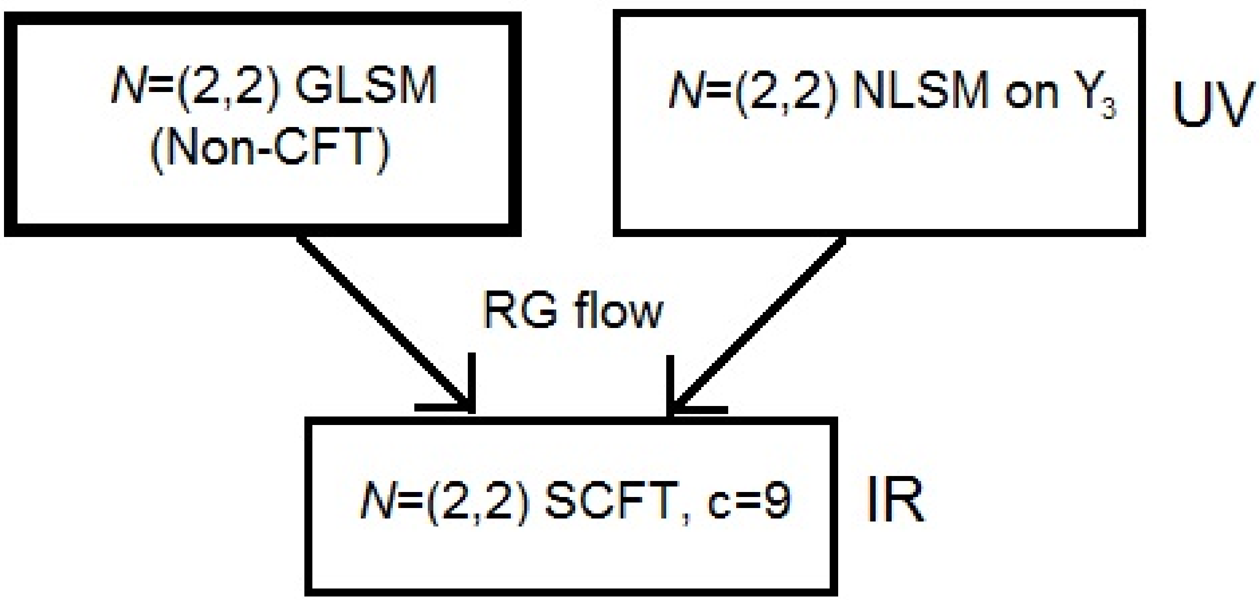
K. Hori and M. R., arXiv:1308.2438 [hep-th]
H. Jockers, V. Kumar, J. Lapan, D. Morrison and M. R., Commun.Math.Phys. **325** (2014)

Big Picture

String compactification on $\mathbb{R}^{1,3} \times Y_3$



$$Z = \int \mathcal{D}\Phi e^{-S_{GLSM}}$$

Moduli of Gauged Linear Sigma Models (GLSM)

GLSMs characterize strings propagating in a compact Calabi-Yau 3-fold X.

There exists 2 types of distinguished moduli on GLSMs

These are related with the moduli of the compactification

Moduli:

- $\mathcal{M}_{CS}$: coefficients of W. Complex structure moduli.

- $\mathcal{M}_K$: $q_l = e^{r_l + i\theta_l} \in \mathbb{C}^*$ Kähler moduli.

$\mathcal{M}_{CS}$ **vs.** $\mathcal{M}_K$

- $\mathcal{M}_{CS}$ and $\mathcal{M}_K$ are Kähler manifolds (indeed special Kähler).

- $Y^\vee$ mirror of Y

$$\mathcal{M}_{CS}(Y^\vee) \cong \mathcal{M}_K(Y)$$

$$\mathcal{M}_{CS}(Y) \cong \mathcal{M}_K(Y^\vee)$$

$$G_{\bar{t}t}(\mathcal{M}) = \partial_t \overline{\partial}_{\bar{t}} \mathcal{K}(t, \bar{t})$$

S^2 partition function of GLSM

Exact partition function for a GLSM on S^2:

$$Z_{S^2} = \frac{1}{|\mathcal{W}|} \sum_{\mathfrak{m} \in \mathbb{Z}^{\mathrm{rank}(G)}} \int \left(\prod_{\mu=1}^{\mathrm{rank}(G)} \frac{d\sigma_\mu}{2\pi} \right) Z_{\mathrm{class}} Z_{\mathrm{gauge}} \prod_A Z_{\Phi_A},$$

where

$$Z_{\mathrm{gauge}} = \prod_{\alpha > 0} \left(\frac{\alpha(\mathfrak{m})^2}{4} + \alpha(\sigma)^2 \right)$$

$$Z_{\Phi_A} = \prod_{\rho \in R_A} \frac{\Gamma\left(\frac{\mathrm{gr}[\phi_A]}{2} - i\rho(\sigma) - \frac{\rho(\mathfrak{m})}{2} \right)}{\Gamma\left(1 - \frac{\mathrm{gr}[\phi_A]}{2} + i\rho(\sigma) - \frac{\rho(\mathfrak{m})}{2} \right)}$$

$$Z_{\mathrm{class}} = \exp\left(-4\pi i r \,\mathrm{Tr}(\sigma) - i\theta \,\mathrm{Tr}(\mathfrak{m}) \right)$$

S^2 partition function of GLSM

So, write

$$z_\ell = e^{-2\pi r_\ell + i\theta_\ell}$$

Then, we conjecture

$$Z_{S^2}(z_\ell, \bar{z}_\ell) = e^{-\mathcal{K}_K(z_\ell, \bar{z}_\ell)}$$

D^2 Partition function of GLSMs

$$\partial D^2 \cong S^1$$

B-branes: objects of $D(X)$, equivalently, holomorphic cycles $C \subset X$ plus data of vector bundle $V \to C$.

$$Z_{D^2}(\mathcal{B})(z_\ell) = Z(B_{IR})(z_\ell)$$

D^2 partition function of GLSM

Exact partition function for a GLSM on D^2:

$$Z_{D^2} = \frac{1}{|\mathcal{W}|} \int \left(\prod_{\mu=1}^{\mathrm{rank}(G)} \frac{d\sigma_\mu}{2\pi} \right) Z_{\mathrm{class}} Z_{\mathrm{gauge}} \prod_A Z_{\Phi_A} f_{\mathcal{B}}(\sigma),$$

where

$$Z_{\mathrm{gauge}} = \prod_{\alpha>0} \alpha(\sigma) \sinh(\pi\alpha(\sigma))$$

$$Z_{\Phi_A} = \prod_{\rho\in R_A} \Gamma\left(i\rho(\sigma) + \frac{R}{2} \right)$$

$$Z_{\mathrm{class}} = e^{it(\sigma)}$$

D^2 partition function of GLSM

Near a Large Volume (LV) point:

$$Z_{D^2}(\mathcal{B}) \sim \int_X e^{B+i\frac{\omega}{2\pi}} \widehat{\Gamma}(X) \mathrm{ch}(B_{LV})$$

$$\widehat{\Gamma}(X) = \prod_i \Gamma\left(1 - \frac{x_i}{2\pi i}\right)$$

$$\widehat{\Gamma}(X)\widehat{\Gamma}^*(X) = \widehat{A}(X)$$

when X is a CY 3-fold and so $c_1(X) = 0$:

$$\widehat{\Gamma}(X) = 1 + \frac{1}{24} c_2(X) + i\frac{\zeta(3)}{(2\pi)^3} c_3(X)$$