Deconfinement Transition As Black Hole Formation By The Condensation Of QCD Strings

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In gauge/gravity duality the deconfinement transition of a gauge theory is dual To the formation of a Black Hole in the gravity bulk [Witten -1998]

We want to describe an intuitive way of understanding this Duality without referring to a sophisticated duality dictionary

Our initial motivation was to study a simple Matrix Model for a Black Hole by looking at the deconfinement transition of 4d $\mathcal{N} = 4$ SYM on an $\mathcal{S}^3$ and the Hawking-Page Transition of the Black hole in the corresponding AdS bulk [Hawking,Page - 1983]

At the Hawking-page transition a small black hole with $R_s \sim R_{AdS}$ forms in the AdS Bulk
Such a black can be modeled as a long and winding string [Susskind,Teitelboim - 1993; Horowitz,Polchinski - 1997]

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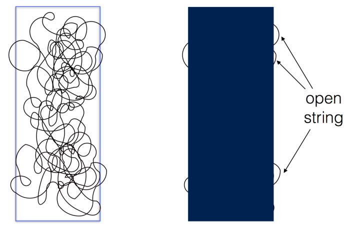
Identifying the long string as a black hole, fluctuations of the string near The horizon are reguarded as open strings attached to the black hole



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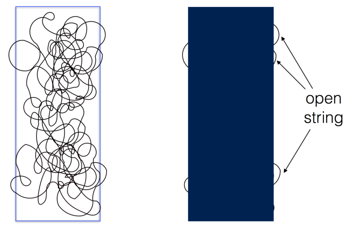

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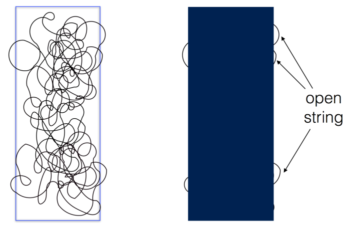

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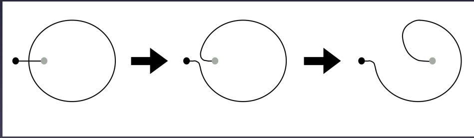
In our picture, the condensation of the polyakov loop is equivalent to the disappearance Of the linear confining potential between pairs of a probe quark and anti-quark

In terms of strings, the linear potential appears in the confineing phase because an open string must be stretched to separate quarks

In the deconfineing phase however, the short open string will intersect with the long closed string many times This allows the quarks to seperated without a linear potential.



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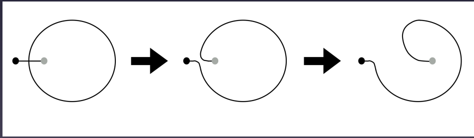

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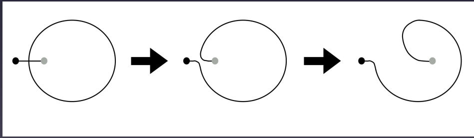

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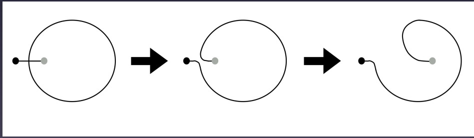

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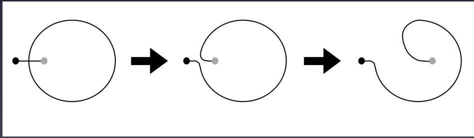

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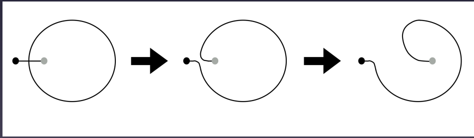

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As concrete example consider $(D+1)$ pure $U(N)$ YM Theory on a discrete lattice

$$H = K + V \qquad\qquad K = \frac{\lambda N}{2} \sum_{\vec{x}} \sum_{\mu} \sum_{\alpha=1}^{N^2} \left(E^{\alpha}_{\mu,\vec{x}} \right)^2$$

$$V = \frac{N}{\lambda} \sum_{\vec{x}} \sum_{\mu<\nu} \left(N - \mathrm{Tr}(U_{\mu,\vec{x}} U_{\nu,\vec{x}+\hat{\mu}} U^{\dagger}_{\mu,\vec{x}+\hat{\nu}} U^{\dagger}_{\nu,\vec{x}}) \right).$$

$$[E^{\alpha}_{\mu,\vec{x}}, U_{\nu,\vec{y}}] = \delta_{\mu\nu}\delta_{\vec{x}\vec{y}} \cdot \tau^{\alpha} U_{\nu,\vec{y}},$$

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When loops don’t self intersect (as is typical in the confining phase) the kinetic operator K acts on a state $|W_c\rangle = W_c|0\rangle$

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$$E = K = \frac{\lambda}{2} L_{total}(T).$$

$$S = L_{total} \log(2D - 1).$$

$$F = L_{total}(T) \left(\frac{\lambda}{2} - T \log(2D - 1) \right)$$

$$T_c = \lambda / (2 \log(2D - 1)).$$

Strictly speaking D+1 YM is dual to a D-dimensional black brane rather then a black hole as the string condensation fills the whole D-dimensional space

In order to describe a black hole 0-brane let us consider two lattice models

First the dimensionally reduced D-matrix model This is the Eguchi-Kawai model with with continuous time direction

At strong coupling the $U(1)^D$ center symmetry is not broken, then this theory Is then know to be equivalent to the D+1 dim. YM at large N. In the sense that translationally Invariant observables are reproduced from the former at leading order

At weak coupling this model is Equivalent to the bosonic part of The BFSS matrix model of M-theory, which is dual to black 0-branes in type IIA supergravity In the t̀ Hooft large N limit.

For $D \geq 2$ this theory exhibits a deconfinement transition, characterized by the non-vanishing expectation value of the absolute value of the Polyakov loop. The energy and entropy are of order N^2 and a typical state contains a long winding string such as $Tr(U_1 U_2 U_1^\dagger U_1^\dagger U_2^\dagger \ldots)$

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At strong coupling the $U(1)^D$ center symmetry is not broken, then this theory Is then know to be equivalent to the D+1 dim. YM at large N. In the sense that translationally Invariant observables are reproduced from the former at leading order

At weak coupling this model is Equivalent to the bosonic part of The BFSS matrix model of M-theory, which is dual to black 0-branes in type IIA supergravity In the ỳ Hooft large N limit.

For $D \geq 2$ this theory exhibits a deconfinement transition, characterized by the non-vanishing expectation value of the absolute value of the Polyakov loop. The energy and entropy are of order N^2 and a typical state contains a long winding string such as $Tr(U_1 U_2 U_1^\dagger U_1^\dagger U_2^\dagger \ldots)$

Strictly speaking D+1 YM is dual to a D-dimensional black brane rather then a black hole as the string condensation fills the whole D-dimensional space

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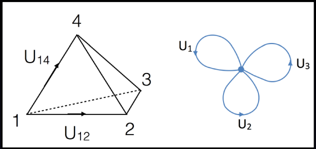
The second Model is the tetrahedron Lattice, here the entropy and temperature scale as

$S = L_{total} \log 2$ and $T_c = \lambda/(2 \log 2)$ This system also possesses a deconfinement transition with a long string described as

$Tr(U_{12} U_{23} U_{31} U_{14} U_{42} \ldots)$



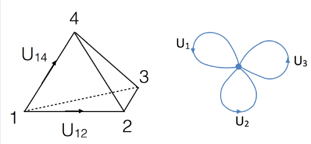
$$
\begin{aligned}
S_{lattice} &= -\frac{N}{2a\lambda}\sum_{\mu,t}\mathrm{Tr}\left(V_t U_{\mu,t+a} V^{\dagger}_{\mu,t} U_{\mu,t} + c.c.\right) \\
&\quad + \frac{aN}{\lambda}\sum_{\mu\neq\nu,t}\left(N - \mathrm{Tr}(U_{\mu,t} U_{\nu,t} U^{\dagger}_{\mu,t} U^{\dagger}_{\nu,t})\right)
\end{aligned}
$$

$$
\begin{aligned}
S_{tet} &= -\frac{N}{2a\lambda}\sum_{t}\sum_{m<n}\left(\mathrm{Tr}(V_{m,t} U_{mn,t+a} V^{\dagger}_{n,t} U_{nm,t}) + c.c.\right) \\
&\quad - \frac{aN}{\lambda}\sum_{t}\sum_{l<m<n}\left((N - \mathrm{Tr}(U_{lm,t} U_{mn,t} U_{nl,t})) + c.c.\right).
\end{aligned}
$$

$$P_{tet} = \frac{1}{4N}\sum_{m=1}^{4}\mathrm{Tr}(V_{m,t=a} V_{m,t=2a} \cdots V_{m,t=n_t a})$$

$$P = \frac{1}{N}\mathrm{Tr}(V_{t=a} V_{t=2a} \cdots V_{t=n_t a}).$$

We use the absolute value of P in order to eliminate the $U(1)$



$$S_{lattice} = -\frac{N}{2a\lambda}\sum_{\mu,t}\mathrm{Tr}\left(V_t U_{\mu,t+a} V^\dagger_{\mu,t} U_{\mu,t} + c.c.\right) + \frac{aN}{\lambda}\sum_{\mu\neq\nu,t}\left(N - \mathrm{Tr}(U_{\mu,t}U_{\nu,t}U^\dagger_{\mu,t}U^\dagger_{\nu,t})\right)$$

$$S_{tet} = -\frac{N}{2a\lambda}\sum_t\sum_{m<n}\left(\mathrm{Tr}(V_{m,t}U_{mn,t+a}V^\dagger_{n,t}U_{nm,t}) + c.c.\right) - \frac{aN}{\lambda}\sum_t\sum_{l<m<n}\left((N - \mathrm{Tr}(U_{lm,t}U_{mn,t}U_{nl,t})) + c.c.\right).$$

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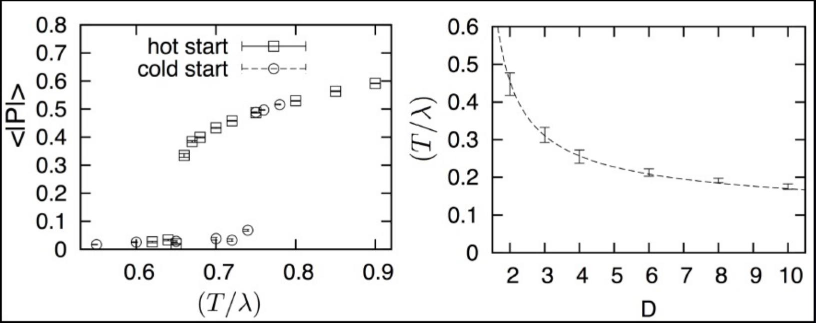


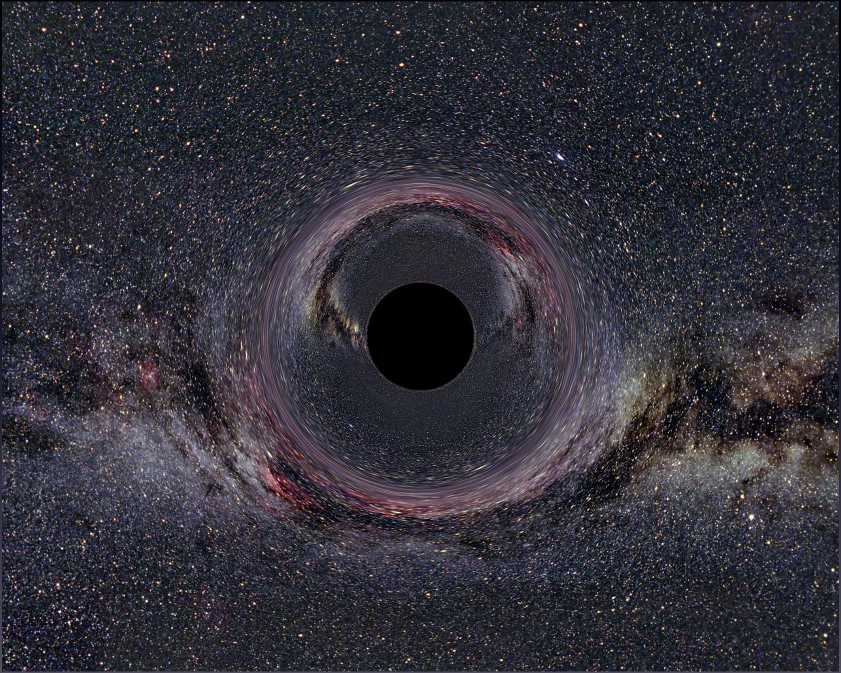
The expectation value of the tetrahedron
With N = 64 nt =12
There strong hysteresis about the
Theoretically predicted critical temperature

$$(T_c/\lambda) = 1/(2\log 2) \simeq 0.721$$

The range for hyteresis of the EK model
with N =64 nt =12
for various dimensions
The dashed curve is the critical temperature

$$(T_c/\lambda) = 1/(2\log(2D-1))$$

The End



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