

Spontaneous Symmetry Breaking and Higgs mechanism with or without Lorentz invariance

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arXiv:1203.0609, 1302.4800, **1303.1527**,
1401.8139, 1402.7066, 1403.3365, 1405.0997

4 PRLs!





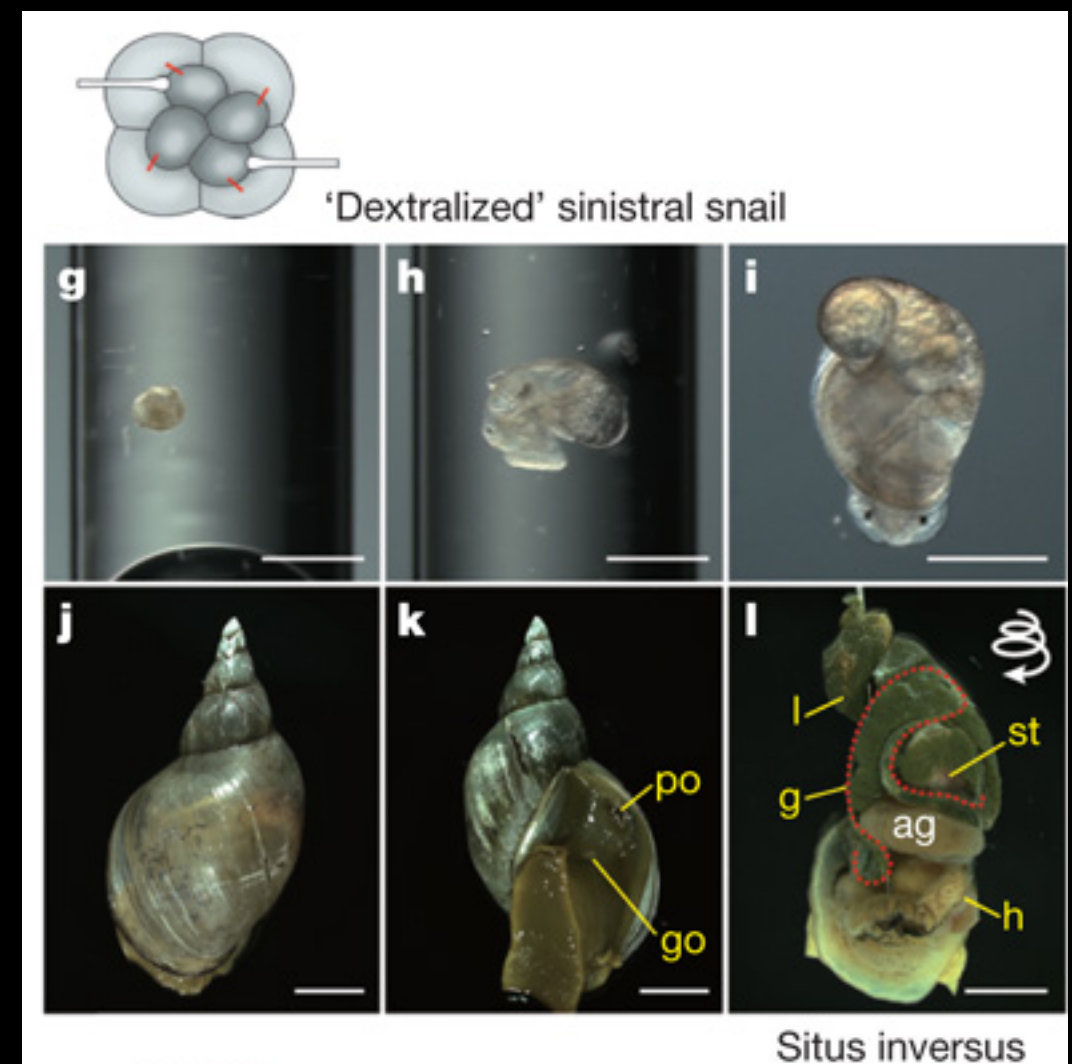
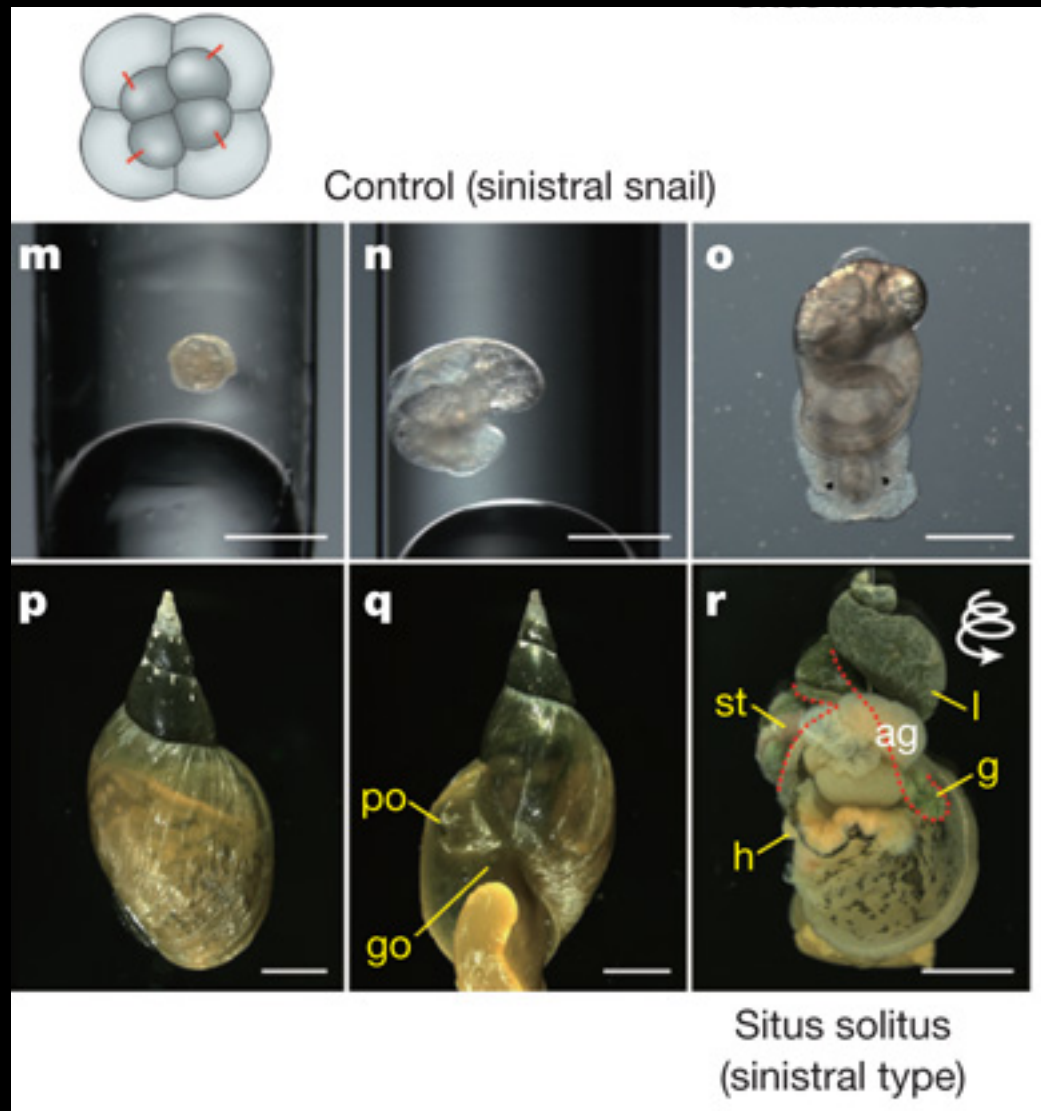
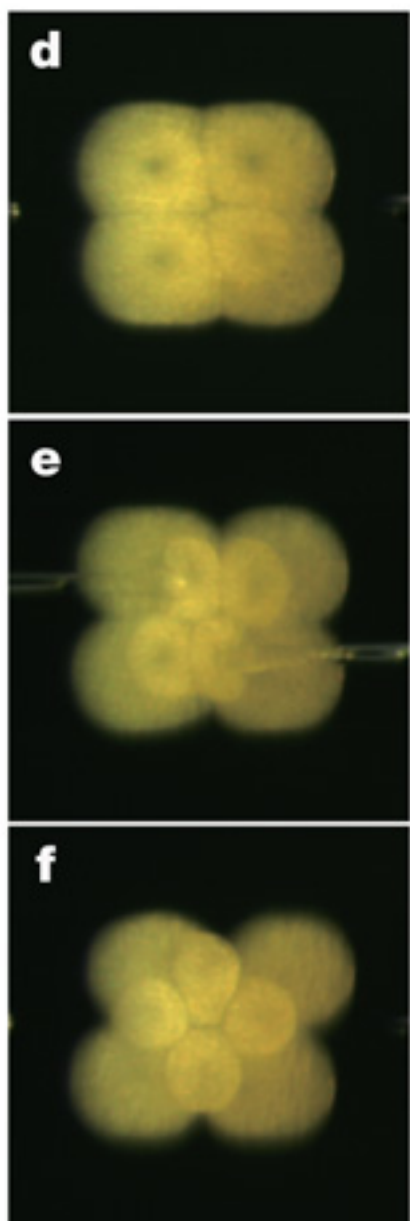


chirality



Reiko Kuroda

Dextralization of
sinistral embryo





Nambu-Goldstone boson



Nambu



Scanned at the American
Institute of Physics

Goldstone

**For every broken symmetry generator,
there is one massless boson
assuming Lorentz invariance**



Englert



Brout

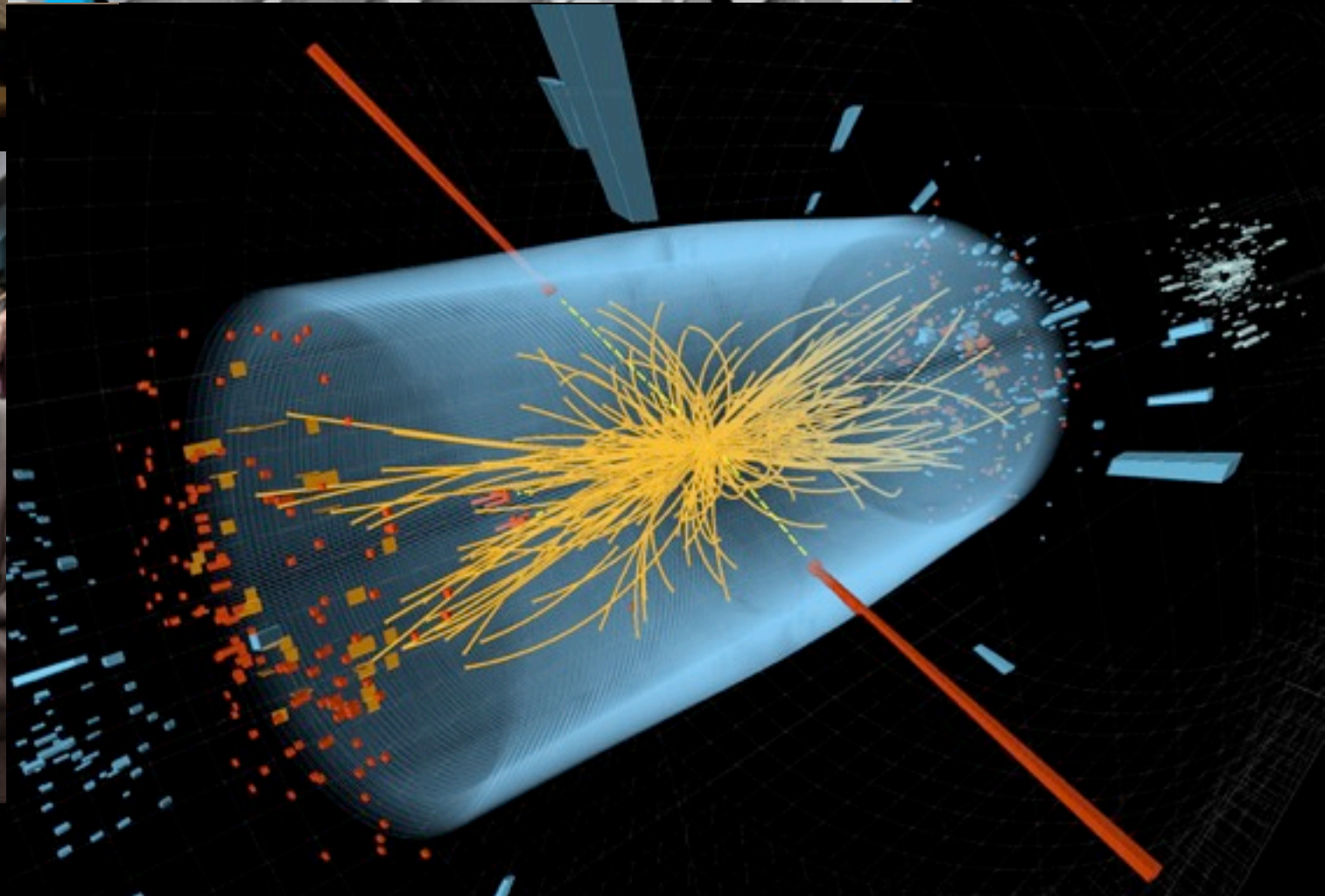
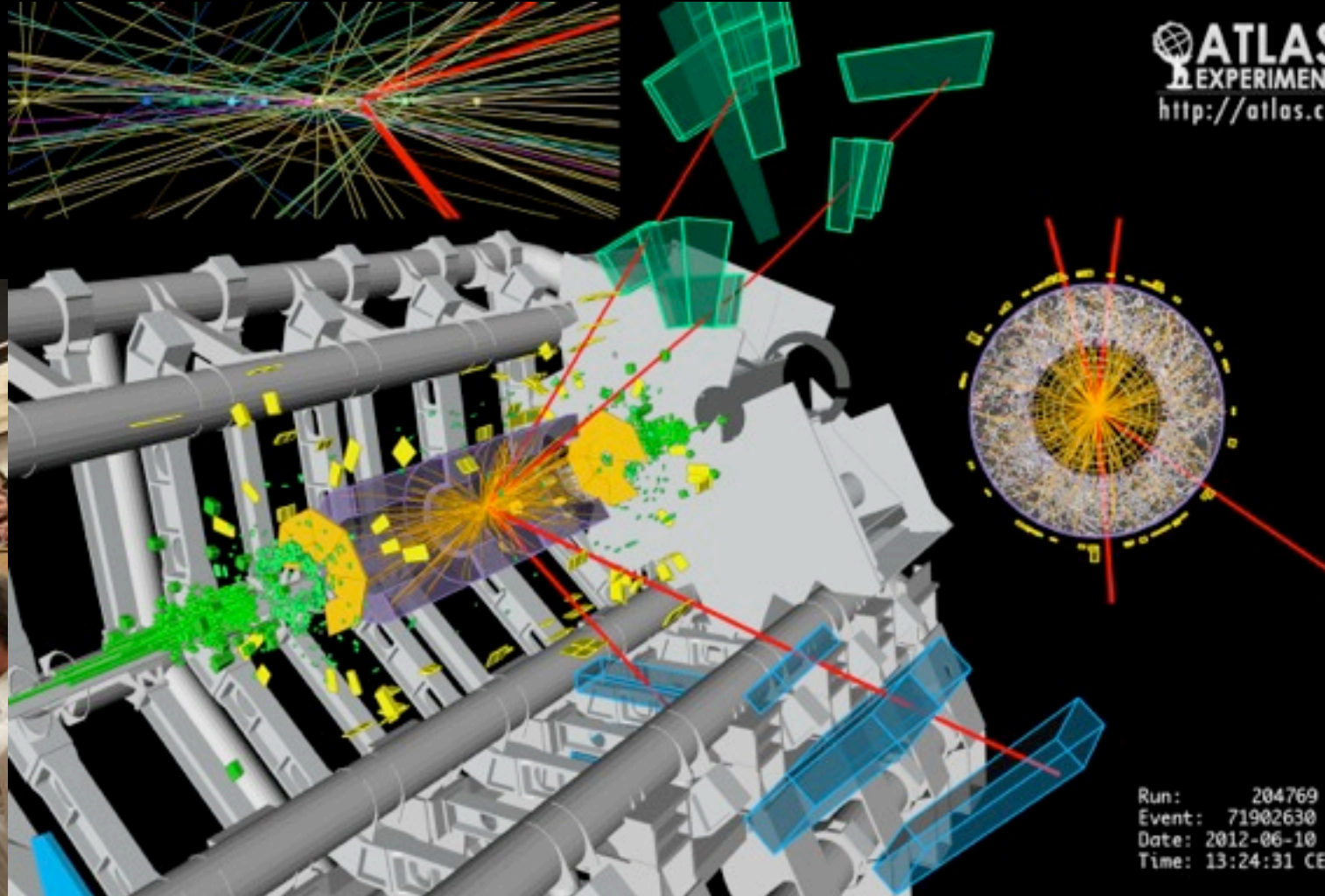


Higgs

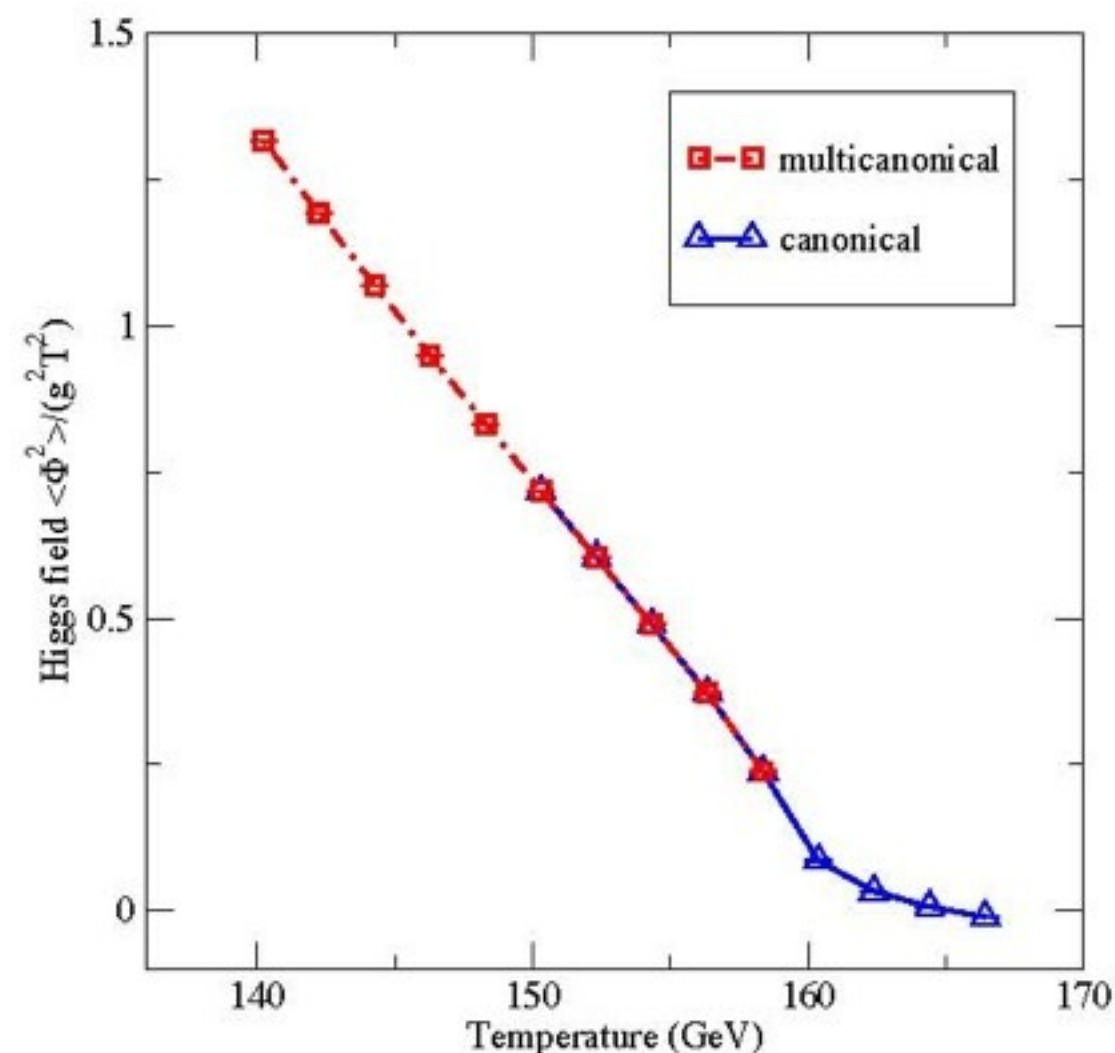
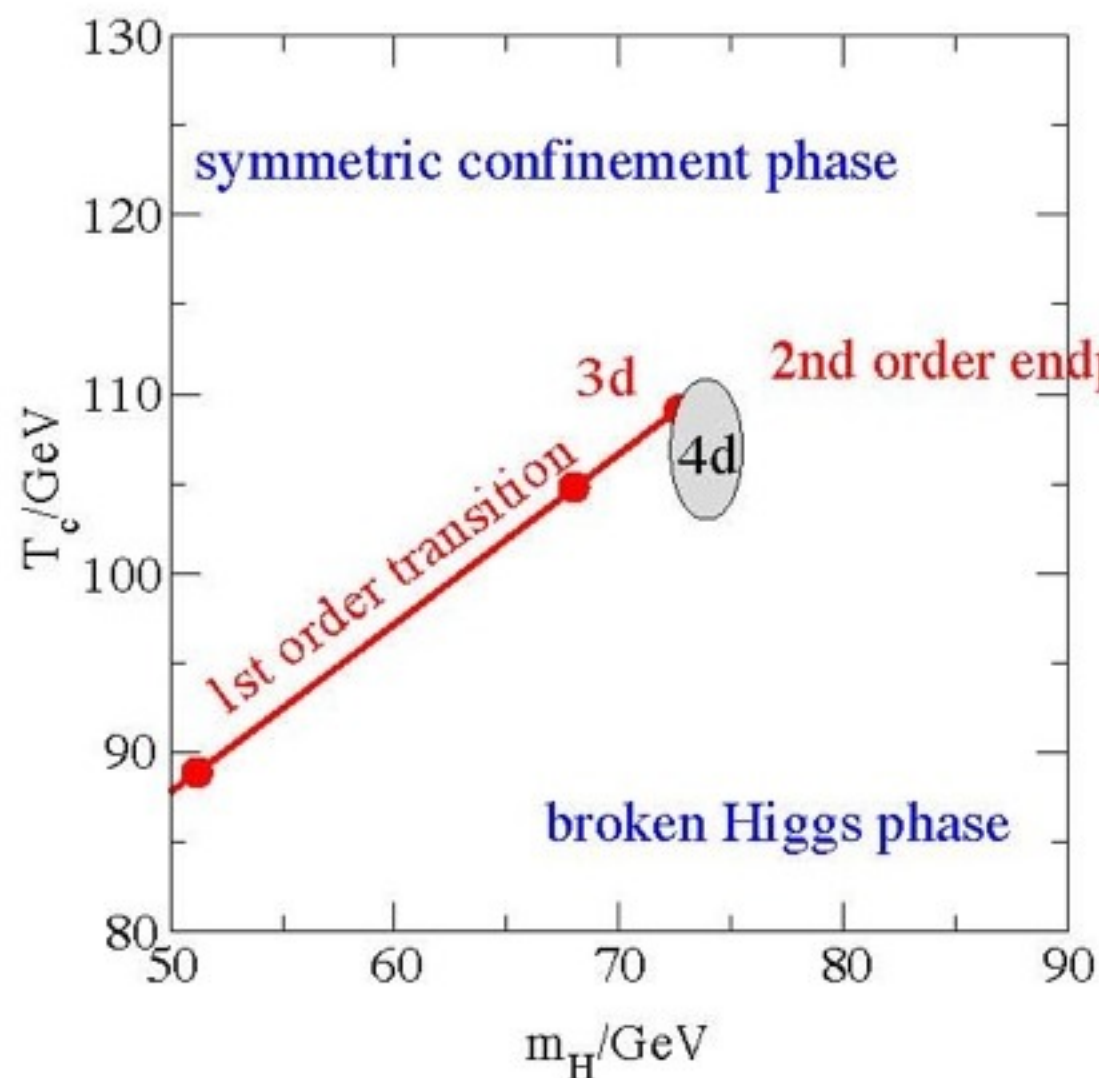
a gauge boson acquires a mass gap
by “eating” a Nambu-Goldstone boson

2012.7.4

discovery of Higgs boson



Phase diagram for the Standard Model:



$\langle H \rangle = 0$ from gauge invariance (Elitzur)

$\langle H^\dagger H \rangle$ is not an order parameter

for $m_h = 126$ GeV, it is crossover

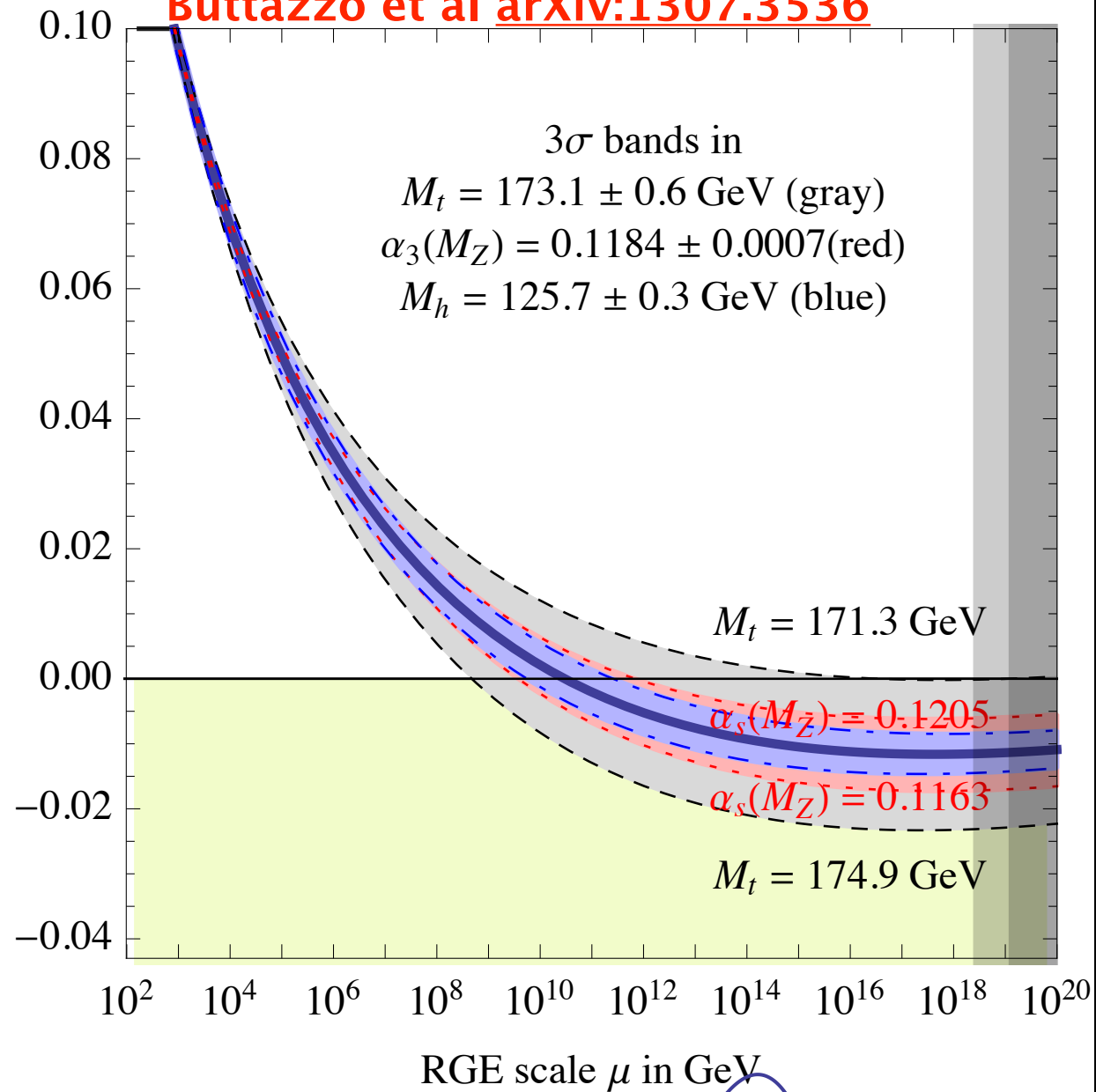
No phase transition in the Minimal Standard Model

No topological defects

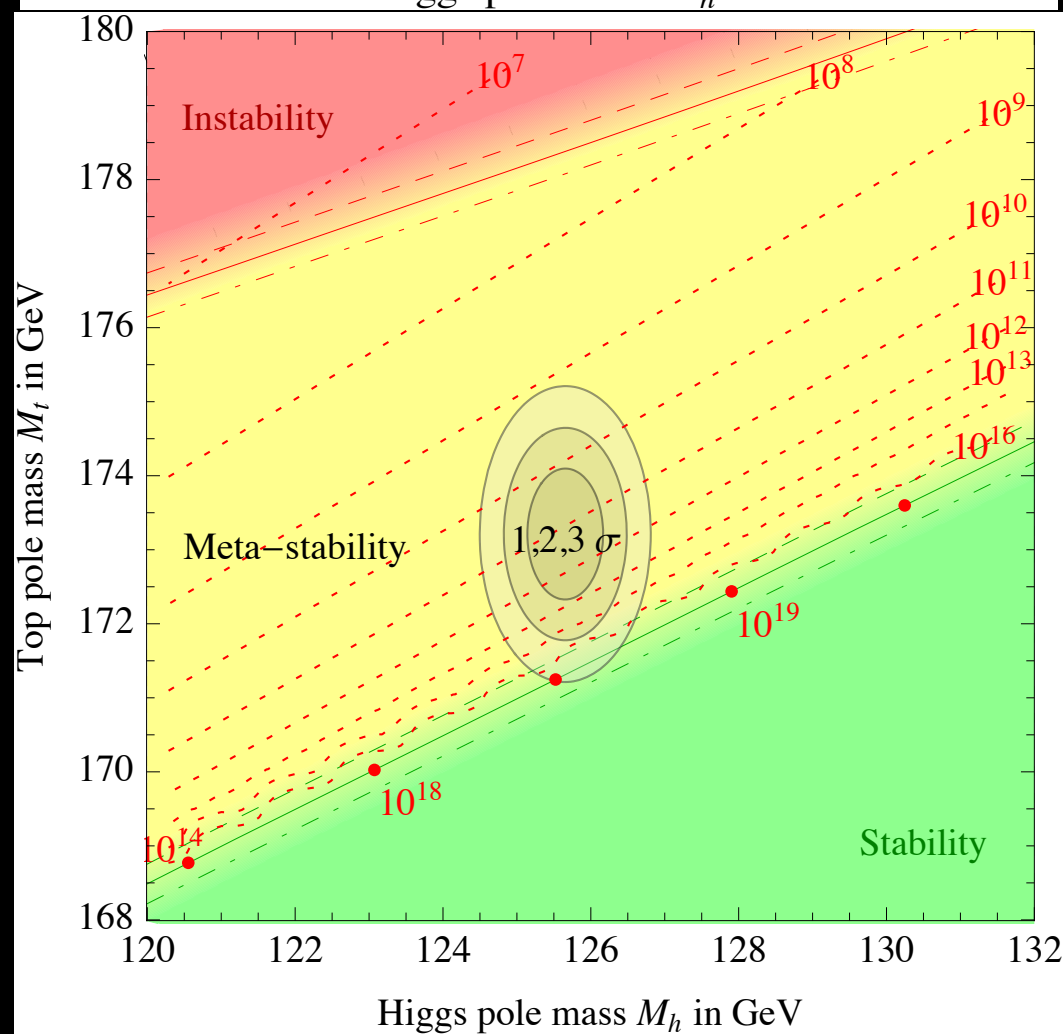
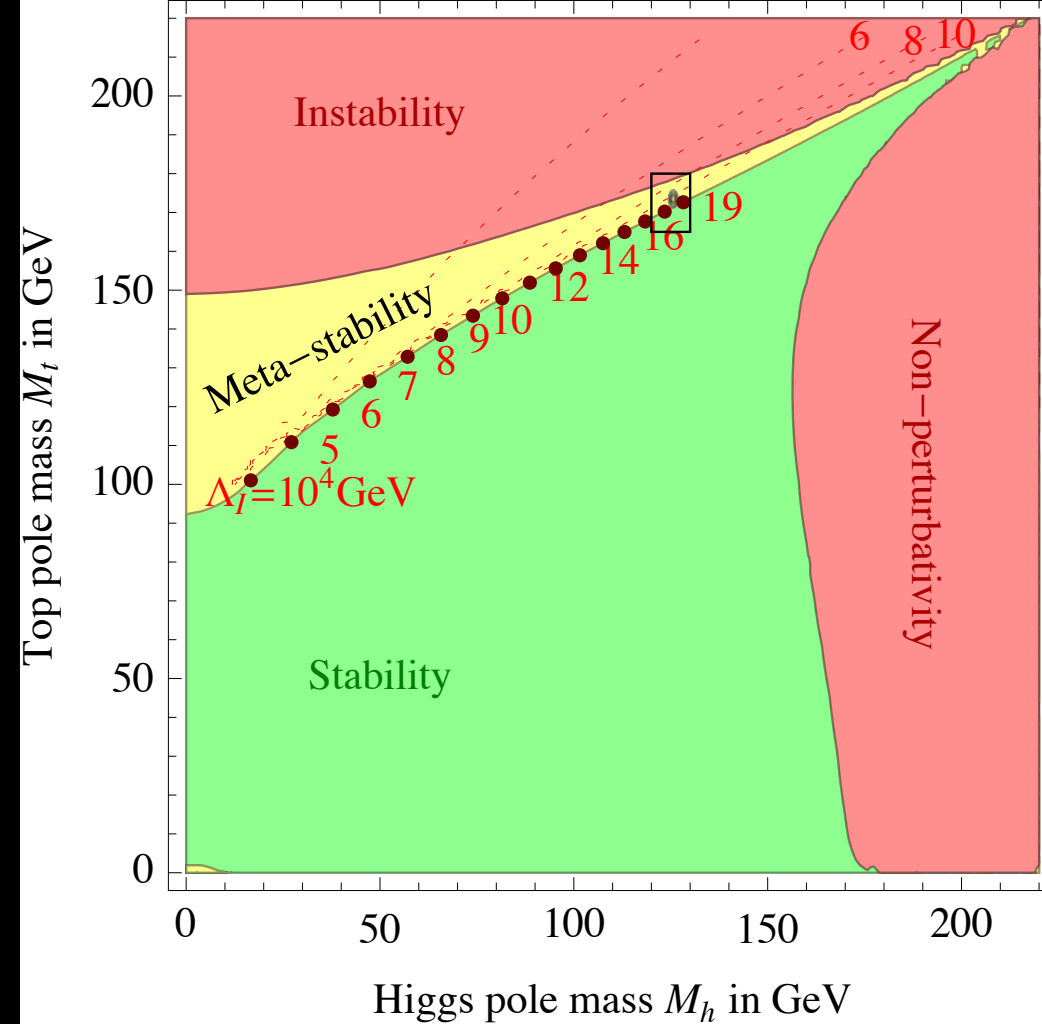
- $G = \text{SU}(2)_L \times \text{U}(1)_Y$
- $H = \text{U}(1)_{\text{EM}}$
- $\pi_1(G/H) = 0$ no Abrikosov vortices
- $\pi_2(G/H) = 0$ no magnetic monopoles
- $\pi_3(G/H) = \mathbb{Z}$ but no θ -vacuum because of the anomaly and trivial topology of $\text{U}(1)_Y$

Buttazzo et al arXiv:1307.3536

Higgs quartic coupling λ



our minimum decays
in about 10^{800} years



longitudinal

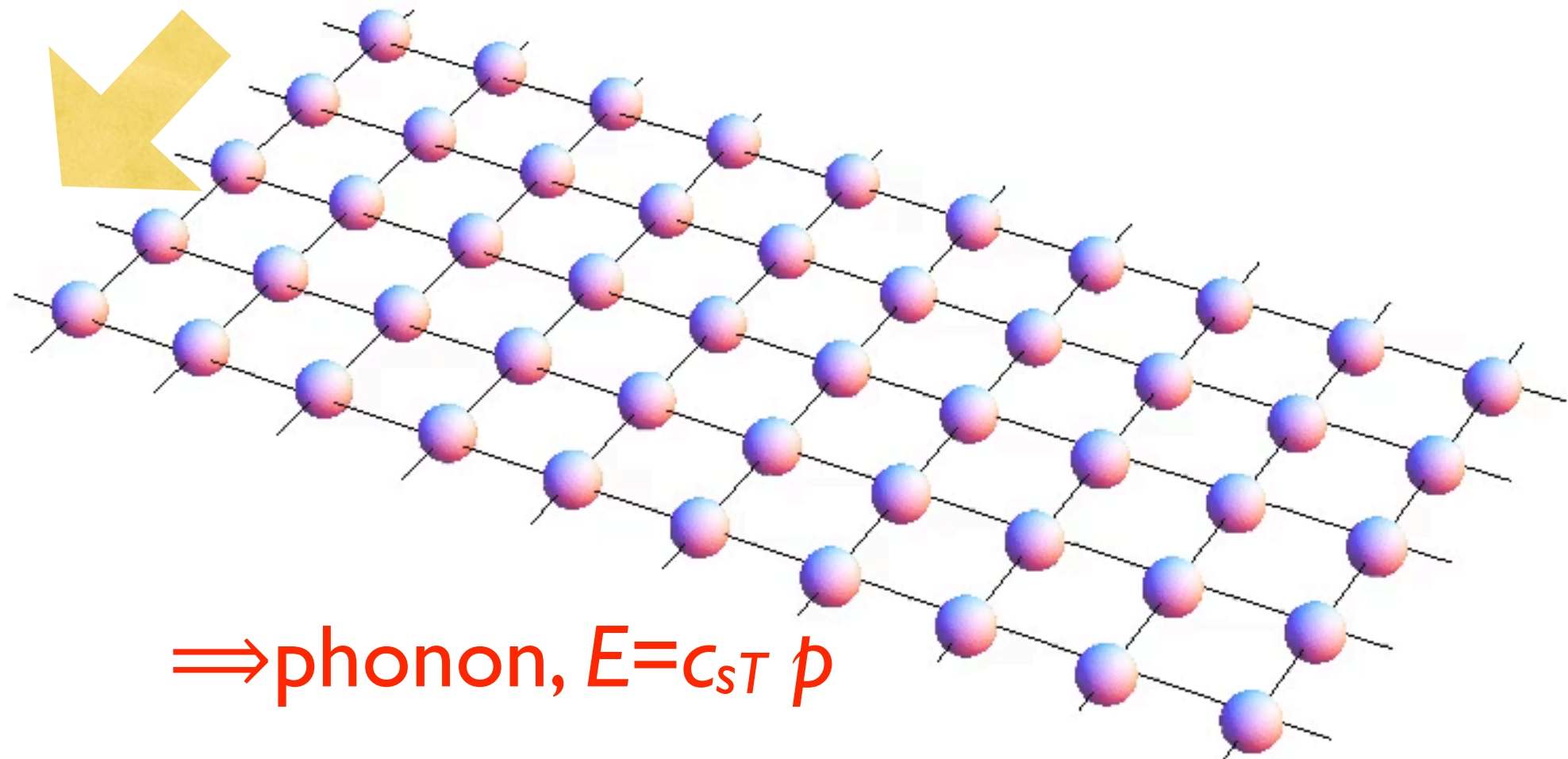
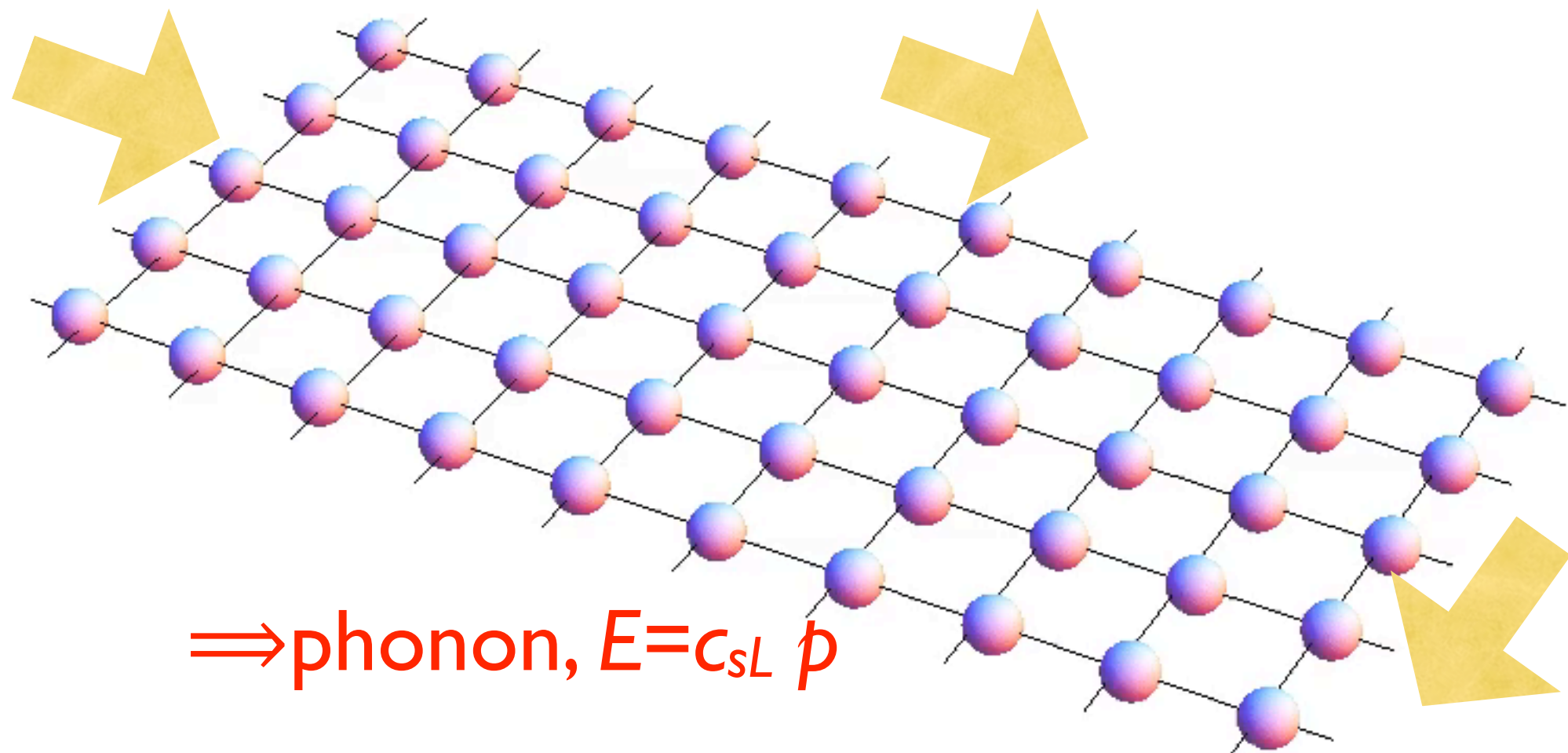
crystal

$$G = \mathbb{R}^2$$

$$H = \mathbb{Z}^2$$

$$G/H = \mathbb{T}^2$$

transverse



Particle numbers

- U(1) symmetry

$$\psi(x) \rightarrow e^{i\theta} \psi(x)$$

$$N = \int dx \psi^* \psi$$

- Ginzburg-Landau theory

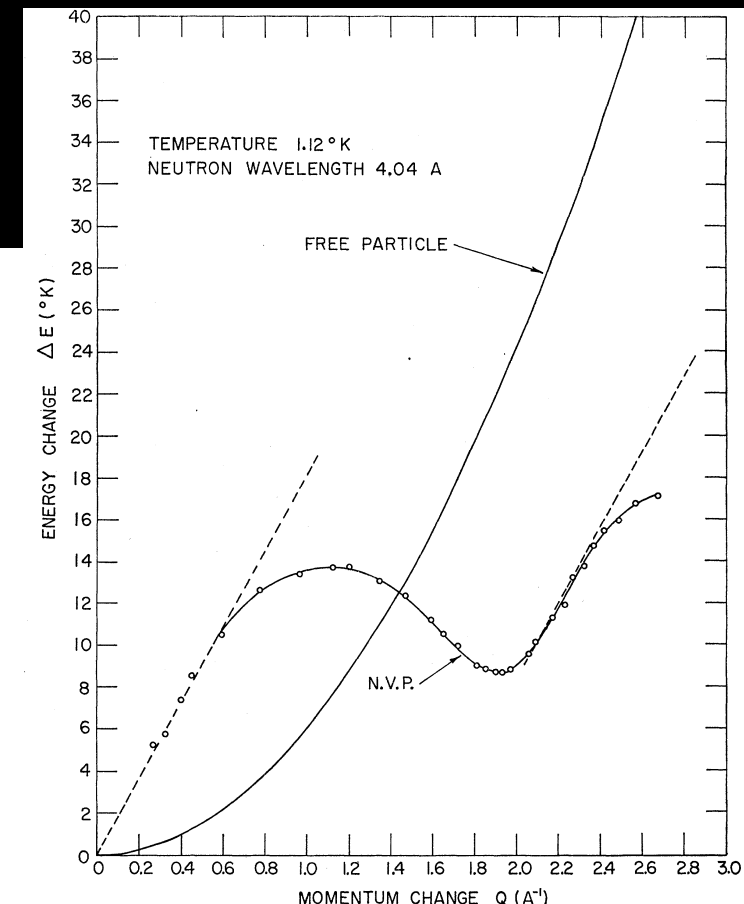
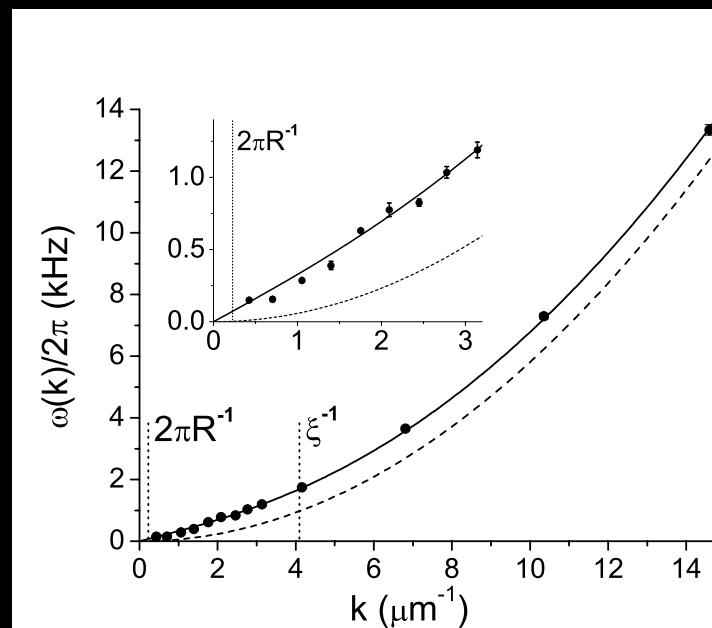
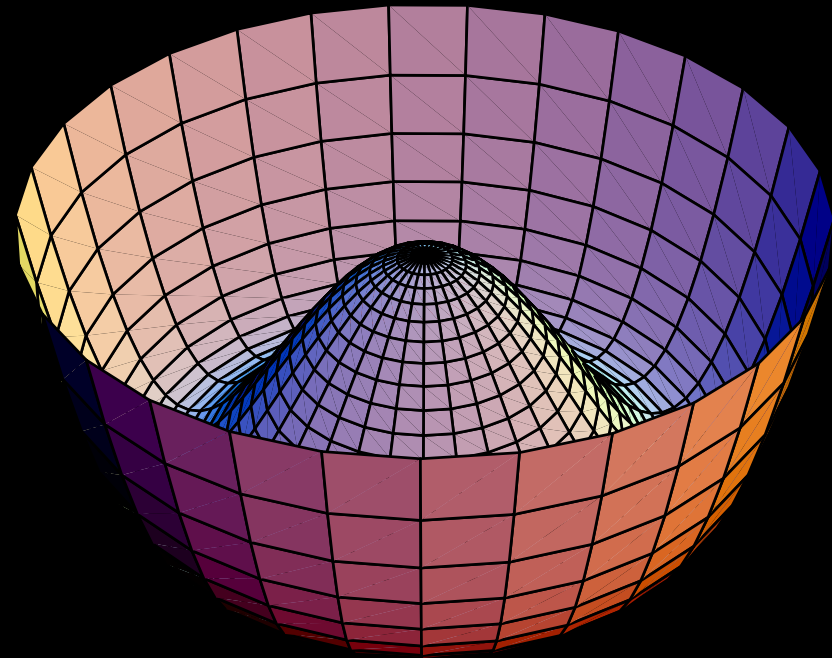
$$V = -\mu \psi^* \psi + \lambda (\psi^* \psi)^2$$

$$\langle 0 | \psi | 0 \rangle \neq 0$$

- $G=U(1), H=0$

- ^4He superfluid

- scalar BEC





Heisenberg models

$$H = +J \sum_{\langle i,j \rangle} \vec{s}_i \cdot \vec{s}_j \quad 2 \text{ NGBs}$$

$E \propto p$



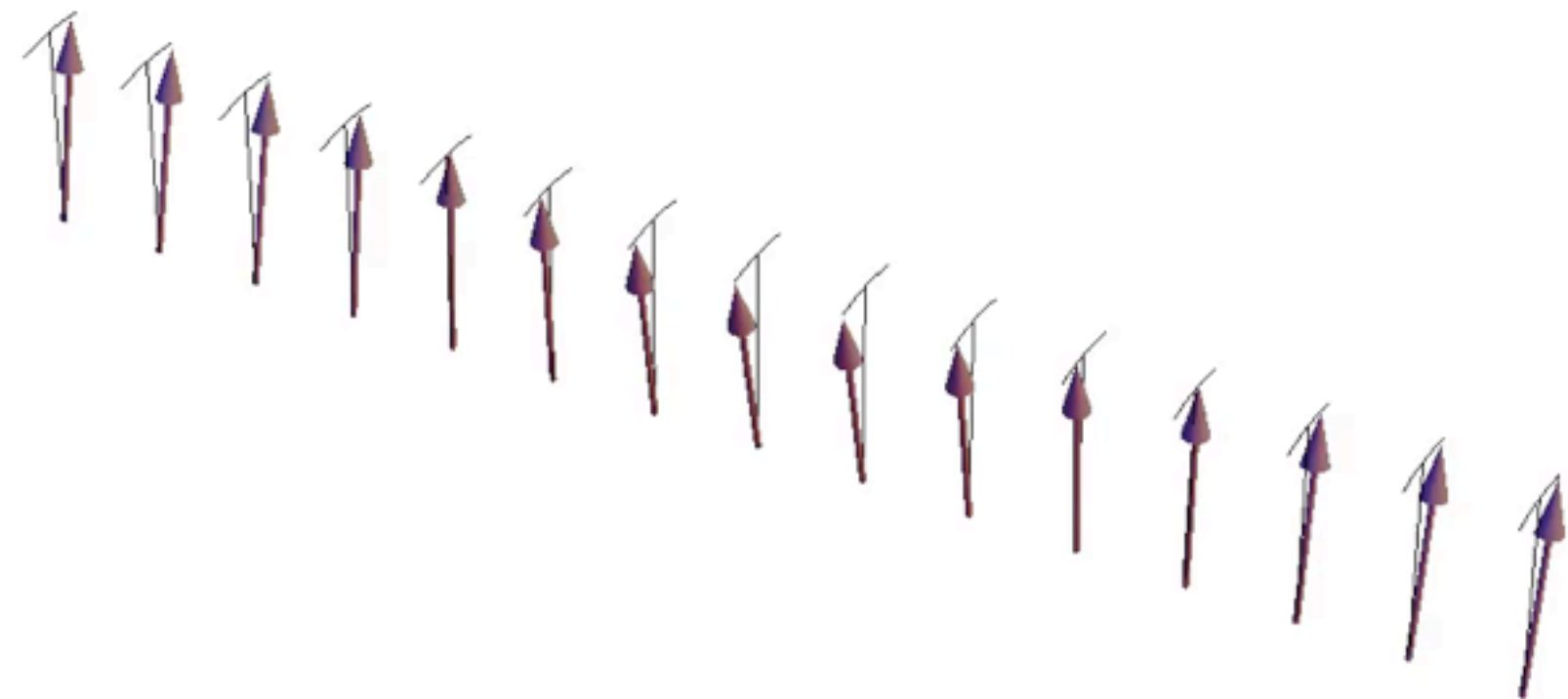
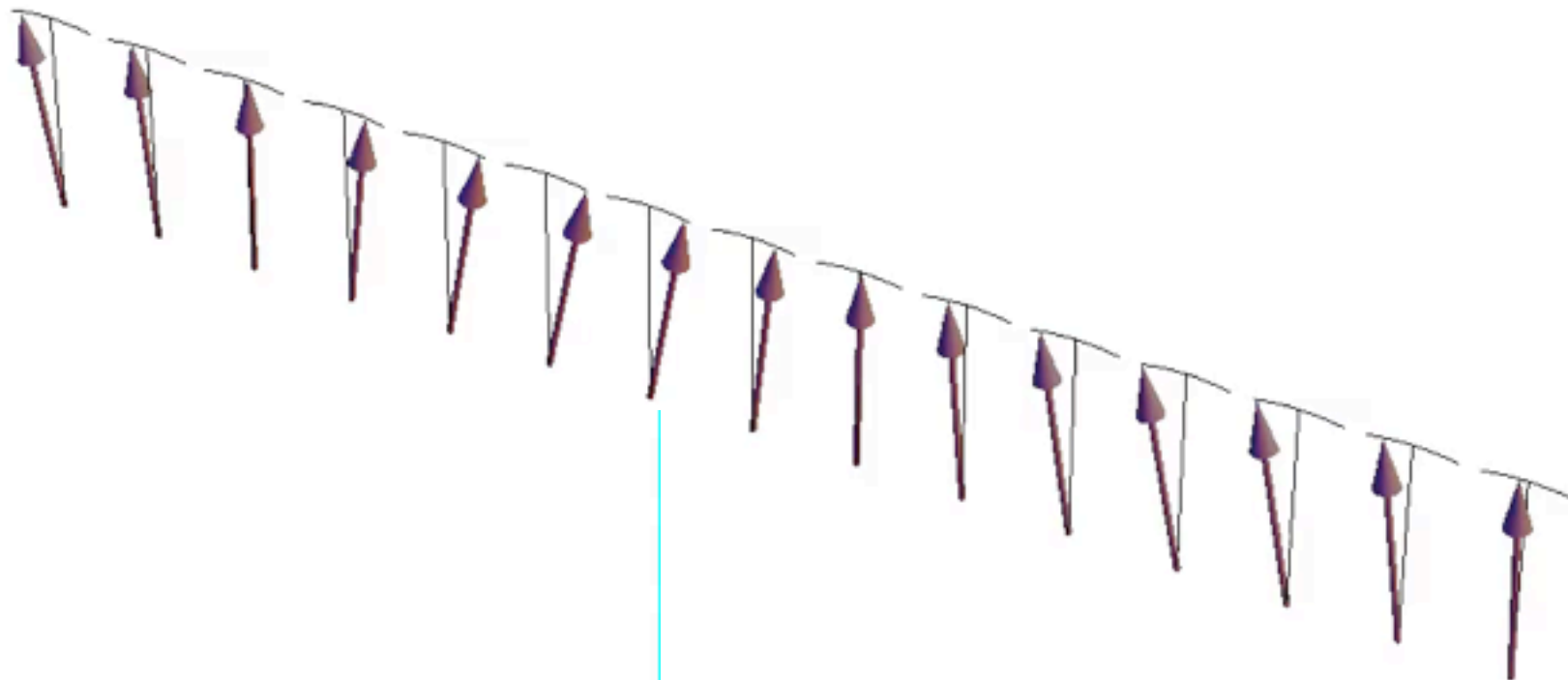
$$H = -J \sum_{\langle i,j \rangle} \vec{s}_i \cdot \vec{s}_j \quad 1 \text{ NGB}$$

$E \propto p^2$



Both $G/H = \text{SO}(3)/\text{SO}(2) = S^2$

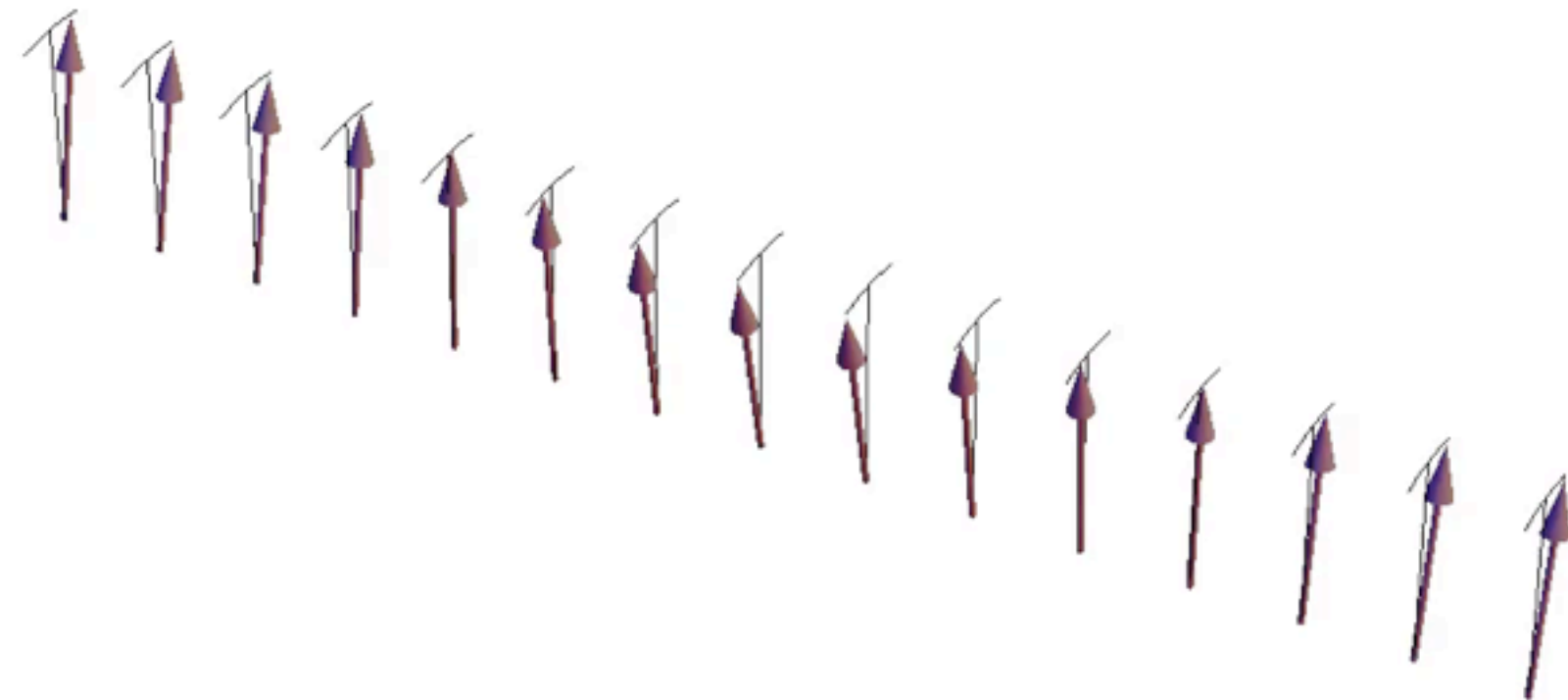
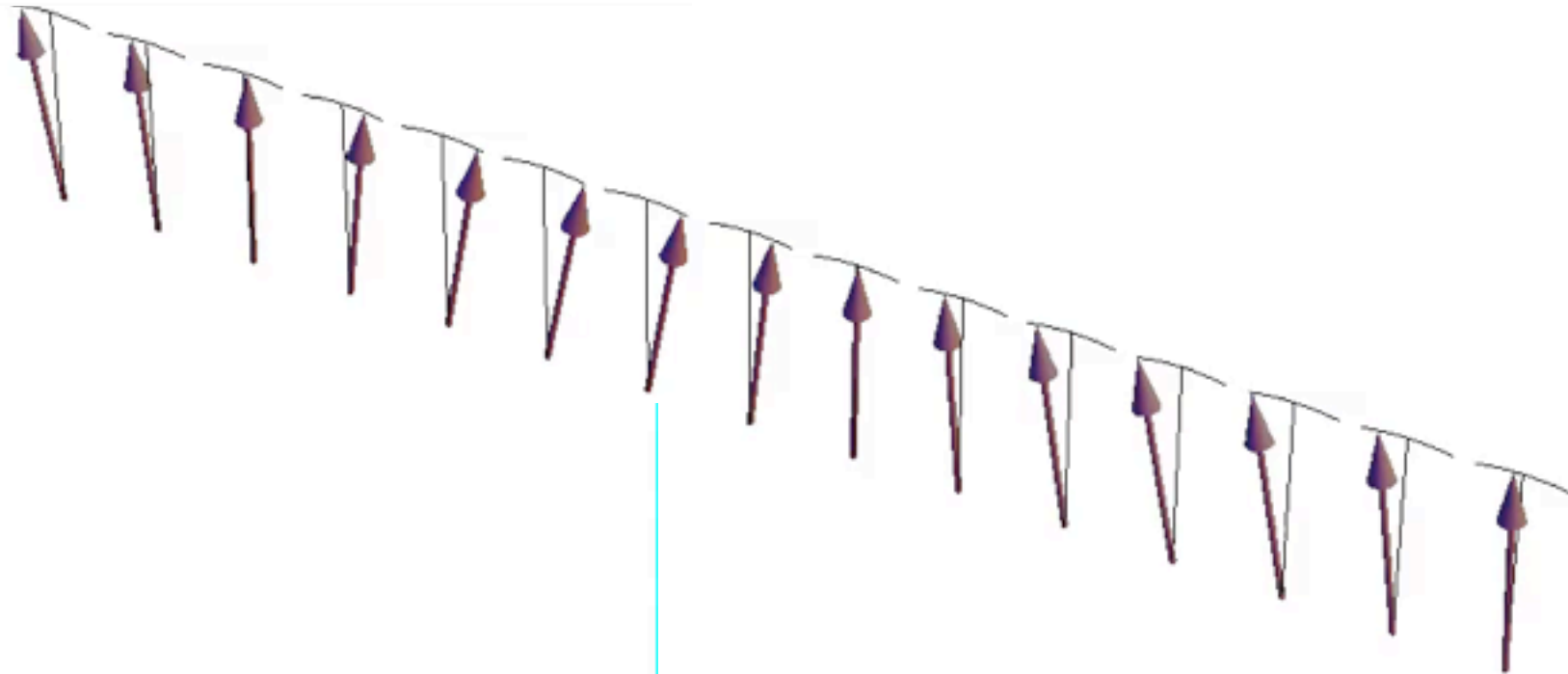
two NGBs?



No!

the only mode

?



No!

experiment

- Dispersion relations can be tested experimentally
- specific heat
 - $E \propto p \Rightarrow C_V \propto T^3$
 - $E \propto p^2 \Rightarrow C_V \propto T^{3/2}$
- Plot $C_V/T^{3/2}$ vs $T^{3/2}$

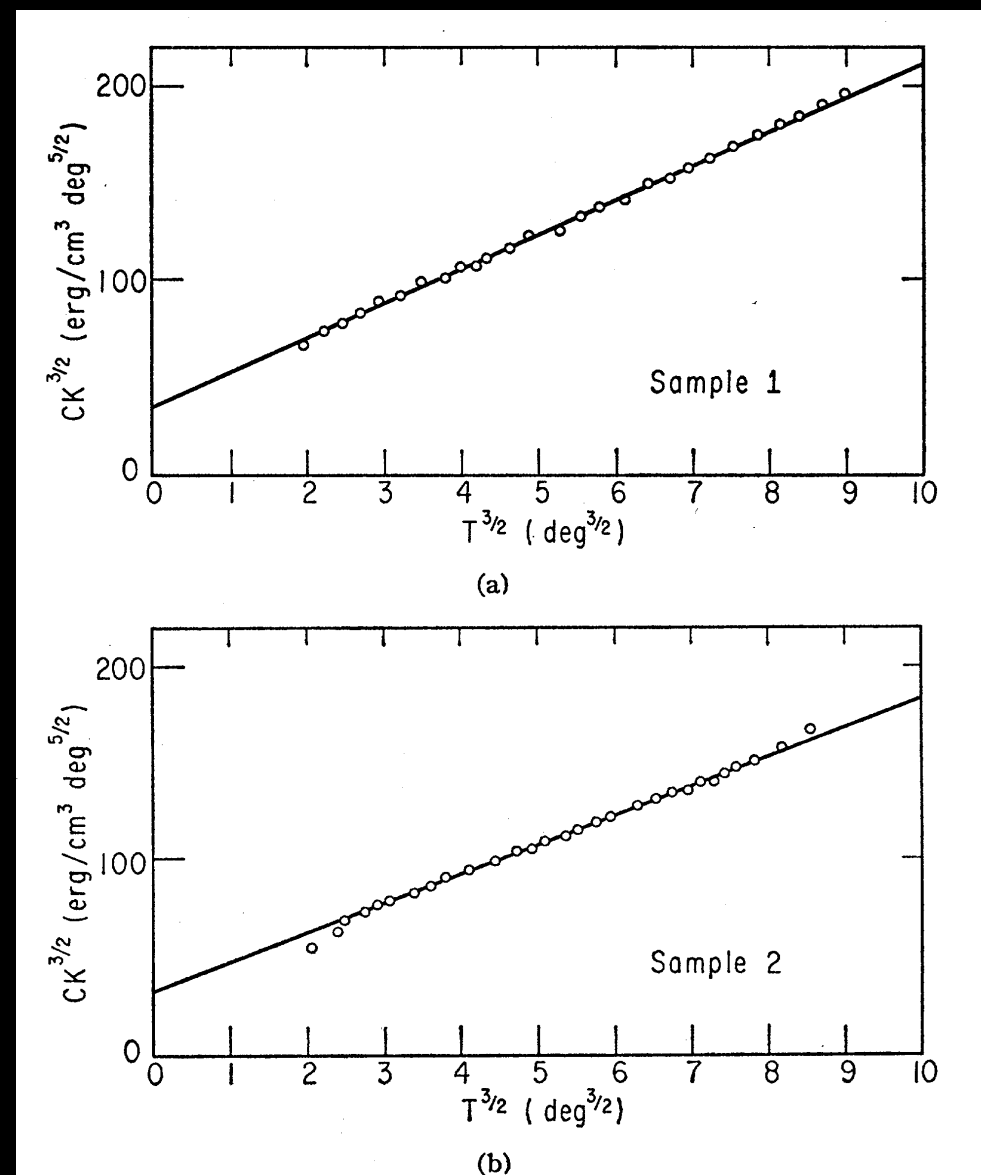
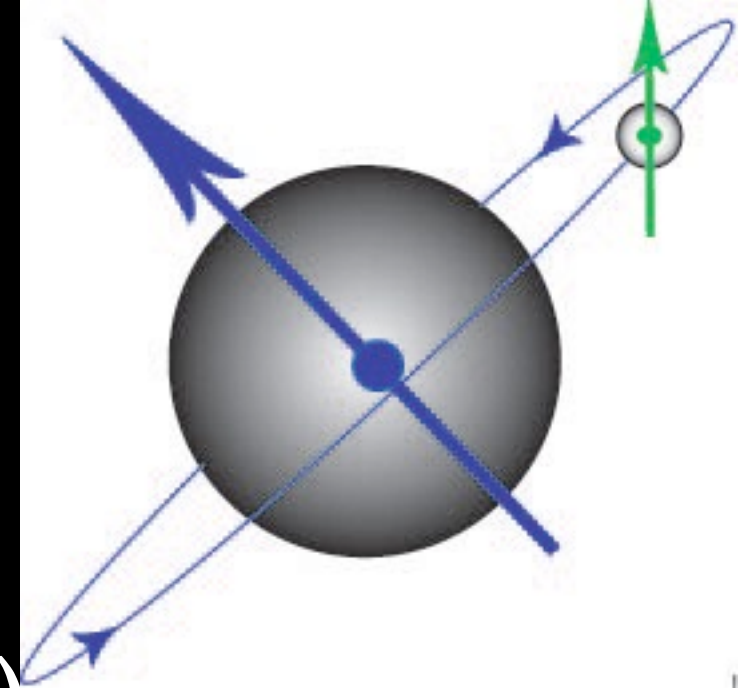


FIG. 1. The specific heat of the two YIG samples. The points for sample 1 give the results obtained in one experiment. The points for sample 2 give the results obtained in two separate experiments.



spinor BEC



- BEC of $F=1$ atoms (ferromagnetic)

- $SO(3) \times U(1) \rightarrow SO(2)$

- $G/H = \mathbb{RP}^3$

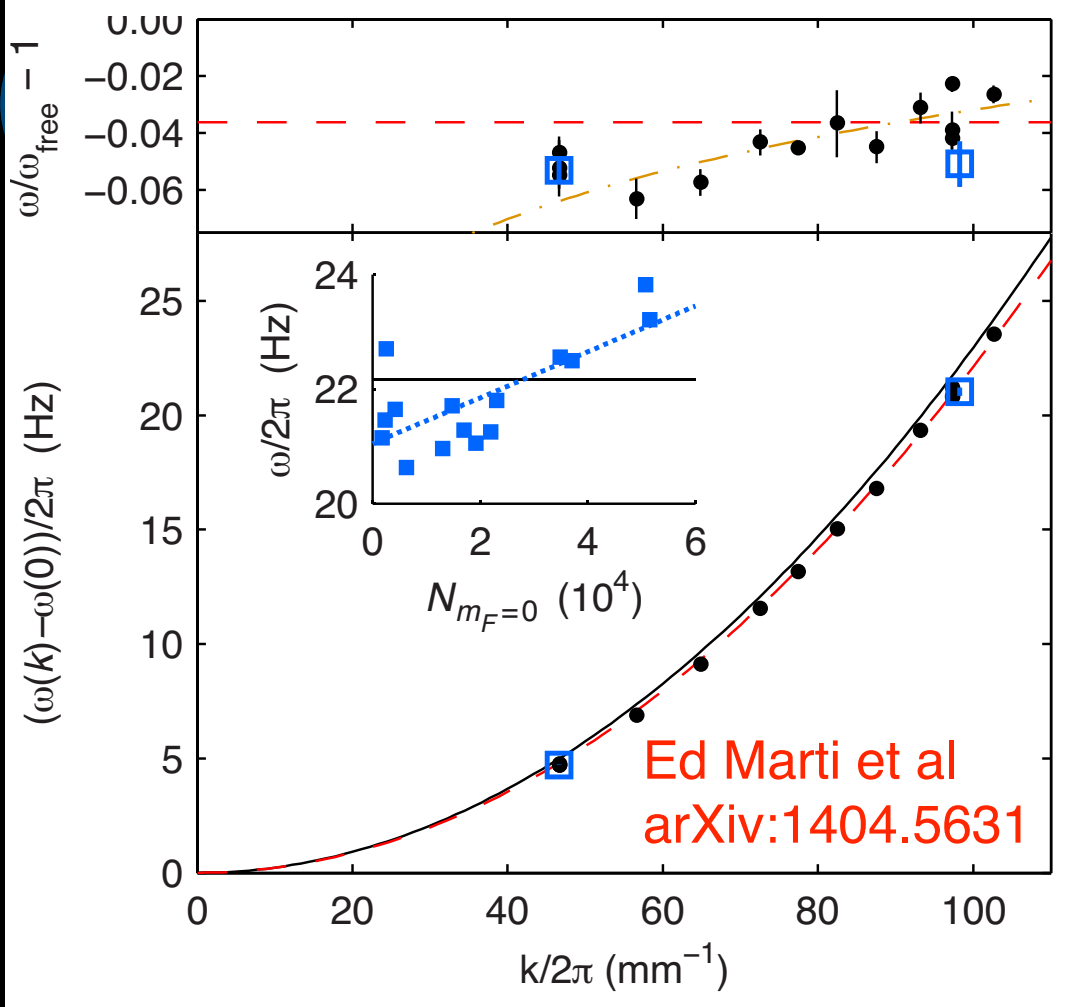
$$\psi = \begin{pmatrix} \psi_x \\ \psi_y \\ \psi_z \end{pmatrix} = \begin{pmatrix} R_x & I_x \\ R_y & I_y \\ R_z & I_z \end{pmatrix}$$

- 3 broken generators

- 1 NGB with $E \propto p$

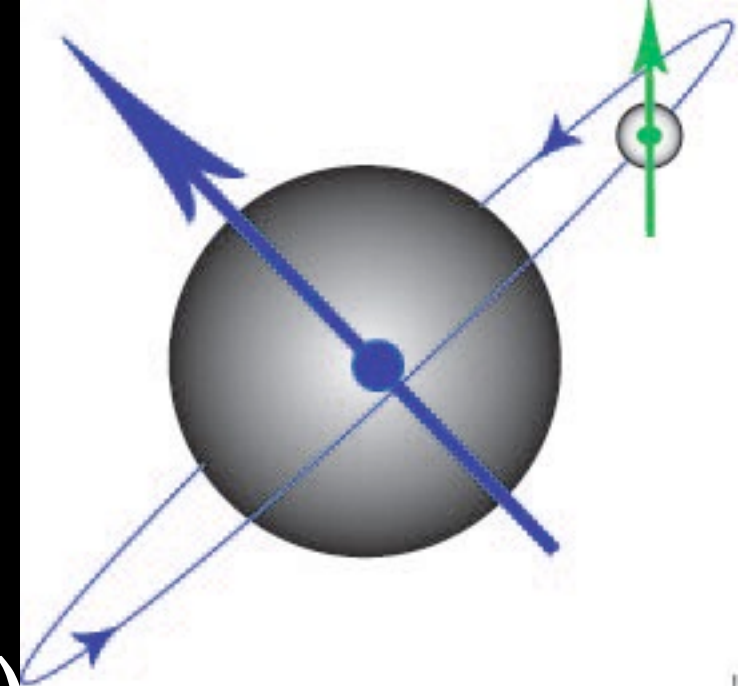
$$\langle \psi \rangle = v \begin{pmatrix} 1 \\ i \\ 0 \end{pmatrix} = v \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}$$

- 1 NGB with $E \propto p^2$



nor BEC

atoms (ferromagnetic)



SO(2)

- $G/H = \mathbb{R}P^3$
- 3 broken generators
- 1 NGB with $E \propto p$
- 1 NGB with $E \propto p^2$

$$\psi = \begin{pmatrix} \psi_x \\ \psi_y \\ \psi_z \end{pmatrix} = \begin{pmatrix} R_x & I_x \\ R_y & I_y \\ R_z & I_z \end{pmatrix}$$

$$\langle \psi \rangle = v \begin{pmatrix} 1 \\ i \\ 0 \end{pmatrix} = v \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}$$

Spontaneous Symmetry Breaking with Abnormal Number of Nambu-Goldstone Bosons and Kaon Condensate

Departme

PHYSICAL REVIEW D **70**, 014006 (2004)

Sch

Abnormal number of Nambu-Goldstone bosons in the color-asymmetric dense color superconducting phase of a Nambu–Jona-Lasinio–type model

We des
breakdown
required
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excitation

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PHYSICAL REVIEW A **74**, 033604 (2006)

Superfluidity in a three-flavor Fermi gas with SU(3) symmetry

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Physics Department, Tsinghua University, Beijing 100084, China

(Received 26 April 2006; published 8 September 2006)

We investigate the superfluidity and the associated Nambu-Goldstone modes in a three-flavor atomic Fermi gas with SU(3) global symmetry. The *s*-wave pairing occurs in flavor antitriplet channel due to the Pauli principle, and the superfluid state contains both gapped and gapless fermionic excitations. Corresponding to the spontaneous breaking of the SU(3) symmetry to a SU(2) symmetry with five broken generators, there are only three Nambu-Goldstone modes, one is with linear dispersion law and two are with quadratic dispersion law. The other two expected Nambu-Goldstone modes become massive with a mass gap of the order of the fermion energy gap in a wide coupling range. The abnormal number of Nambu-Goldstone modes, the quadratic dispersion law, and the mass gap have significant effect on the low-temperature thermodynamics of the matter.

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PACS number(s): 03.75.Ss, 05.30.Fk, 74.20.Fg, 34.90.+q

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Spontaneous Breaking of Lie and Current Algebras

Yoichiro Nambu¹

Received December 26, 2002; accepted January 29, 2003

The anomalous properties of Nambu–Goldstone bosons, found by Miransky and others in the symmetry breaking induced by a chemical potential, are attributed to the SSB of Lie and current algebras. Ferromagnetism, antiferromagnetism, and their relativistic analogs are discussed as examples.²

KEY WORDS: Symmetry breaking; Nambu–Goldstone boson; color superconductivity; chemical potential; ferromagnetism; Lorentz symmetry; current algebra.

1. INTRODUCTION AND SUMMARY

In general the number of the Nambu–Goldstone (NG) bosons associated with a spontaneous symmetry breaking (SSB) $G \rightarrow H$ is equal to the number of symmetry generators Q_i in the coset G/H . In the absence of a gauge field, their energy ω goes as a power k^γ of wave number. In a relativistic theory, $\gamma = 1$ necessarily unless Lorentz invariance is broken.

There are, however, exceptions to the above “theorem.”^(1–5) Recently



$$\mathcal{L}_{\text{eff}} = \frac{s}{2} \frac{n_y \dot{n}_x - n_x \dot{n}_y}{1 + n_z} - c_s^2 \frac{1}{2} \vec{\nabla} n_i \vec{\nabla} n_i \quad n_z = \sqrt{1 - n_x^2 - n_y^2}$$

Heisenberg models

- anti-ferromagnet $H = +J \sum_{\langle i,j \rangle} \vec{s}_i \cdot \vec{s}_j$ 2 NGBs

$$\langle 0 | J_z^0 | 0 \rangle = 0$$

$$E \propto p$$



- ferromagnet $H = -J \sum_{\langle i,j \rangle} \vec{s}_i \cdot \vec{s}_j$ 1 NGB

$$\langle 0 | J_z | 0 \rangle = -i \langle 0 | [J_x, J_y] | 0 \rangle \neq 0$$

$$E \propto p^2$$



J_x and J_y canonically conjugate to each other cf. $[x, p] = i \hbar$
describing a single degree of freedom *together*



General formula

- Define a commutator among broken generators

$$\rho_{ab} = \frac{-i}{V} \langle 0 | [Q^a, Q^b] | 0 \rangle$$

- $n_B = 1/2 \text{ rank } \rho$ counts the number of canonically conjugate pairs (Type-B)

generically

$$E \propto p^2$$

- each pair describes one d.o.f.

- the remainder $n_A = n_{BG} - 2n_B$

- stand-alone NGB d.o.f. (Type-A)

generically

$$E \propto p$$

$$n_{NGB} = n_A + n_B = n_{BG} - \frac{1}{2} \text{rank } \rho$$

conjectured by Watanabe and Brauner



Applications

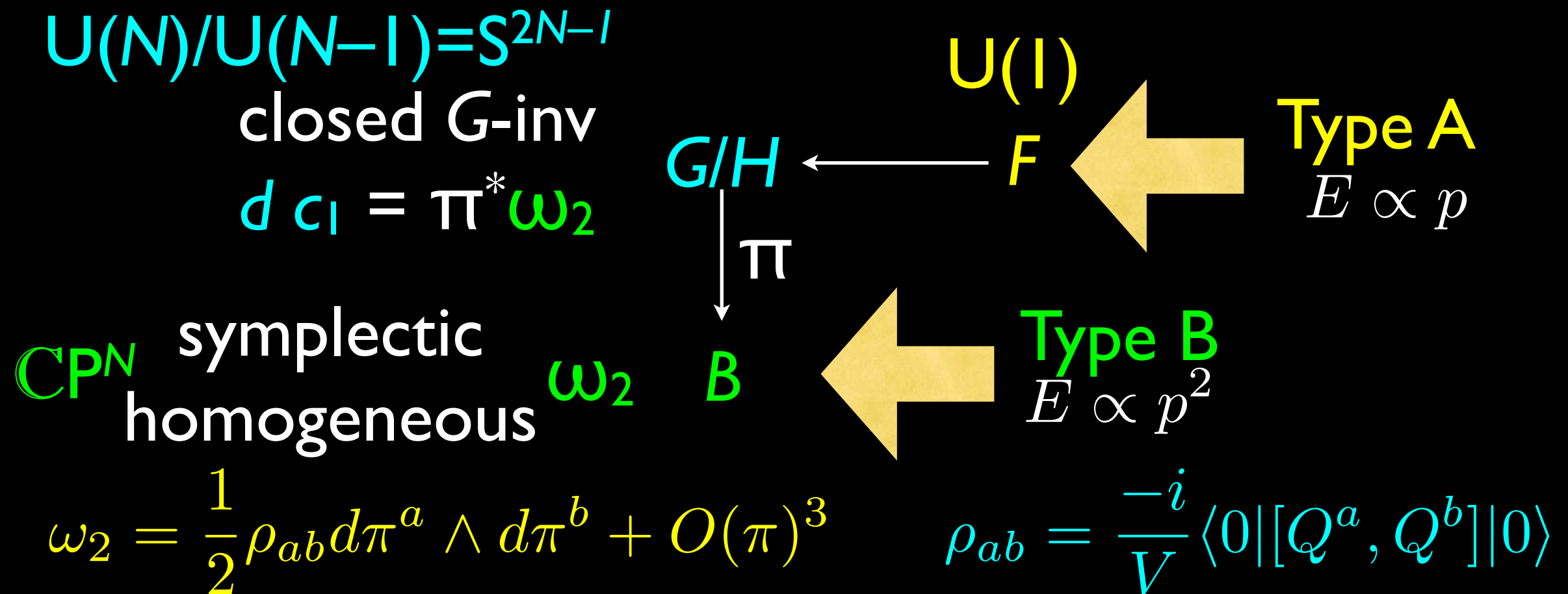
$$n_{NGB} = n_{BG} - \frac{1}{2} \text{rank } \rho$$



example	coset space	BG	NGB	rank ρ	theorem
anti-ferromagnet	$O(3)/O(2)$	2	2	0	$2=2-0$
ferromagnet	$O(3)/O(2)$	2	1	2	$1=2-1$
superfluid ^4He	$U(1)$	1	1	0	$1=1-0$
superfluid ^3He B phase	$O(3) \times O(3) \times U(1)/O(2)$	4	4	0	$4=4-0$
(in magnetic field)	$O(2) \times O(3) \times U(1)/O(2)$	4	3	2	$3=4-1$
BEC ($F=0$)	$U(1)$	1	1	0	$1=1-0$
BEC ($F=1$) polar	$O(3) \times U(1)/U(1)$	3	3	0	$3=3-0$
BEC ($F=1$) ferro	$O(3) \times U(1)/SO(2)$	3	2	2	$2=3-1$
3-comp. Fermi liquid	$U(3)/U(2)$	5	3	4	$3=5-2$
neutron star	$U(1)$	1	1	0	$1=1-0$
kaon cond. ($\mu=0$)	$U(2)/U(1)$	3	3	0	$3=3-0$
kaon cond. ($\mu \neq 0$)	$U(2)/U(1)$	3	2	2	$2=3-1$
crystal	$\mathbb{R}^3/\mathbb{Z}^3$	3	3	0	$3=3-0$
(in magnetic field)	$\mathbb{R}^3/\mathbb{Z}^3$	3	2	2	$2=3-1$



General Geometry



NGBs for generators a and b are symplectic pairs
 and describe a single degree of freedom

$$\dim G - \dim H = n_A + 2n_B$$

projection possible for any compact semi-simple

n_A	n_B	U
30	0	.
20	5	$SU(5) \times U(1)$
14	8	$SU(4) \times SU(2) \times U(1)$
12	9	$SU(4) \times U(1)^2$
12	9	$SU(3)^2 \times U(1)$
8	11	$SU(3) \times SU(2) \times U(1)^2$
6	12	$SU(3) \times U(1)^3$
6	12	$SU(2)^3 \times U(1)^2$
4	13	$SU(2)^2 \times U(1)^3$
2	14	$SU(2) \times U(1)^4$
0	15	$U(1)^5$

TABLE III. Possible number of type-A and type-B NGBs for $SU(6)/U(1)^5$.

n_A	n_B	U
40	0	.
24	8	$SO(8) \times U(1)$
20	10	$U(5)$
14	13	$SO(6) \times U(2)$
12	14	$SO(6) \times U(1)^2$
12	14	$U(4) \times U(1)$
10	15	$SO(4) \times U(3)$
8	16	$U(3) \times U(2)$
6	17	$SO(4) \times U(2) \times U(1)$
6	17	$U(3) \times U(1)^2$
4	18	$SO(4) \times U(1)^3$
4	18	$U(2)^2 \times U(1)$
2	19	$U(2) \times U(1)^3$
0	30	$U(1)^5$

TABLE IV. Possible number of type-A and type-B NGBs for $SO(10)/U(1)^5$.

n_A	n_B	$U \subset SO(11)$	$U \subset Sp(5)$
50	0	.	.
32	9	$SO(9) \times U(1)$	$Sp(4) \times U(1)$
20	15	$SO(7) \times U(2)$	$Sp(3) \times U(2)$
20	15	$U(5)$	$U(5)$
18	16	$SO(7) \times U(1)^2$	$Sp(3) \times U(1)^2$
14	18	$SO(5) \times U(3)$	$Sp(2) \times U(3)$
14	18	$SO(3) \times U(4)$	$Sp(1) \times U(4)$
12	19	$U(4) \times U(1)$	$U(4) \times U(1)$
10	20	$SO(5) \times U(2) \times U(1)$	$Sp(2) \times U(2) \times U(1)$
8	21	$SO(5) \times U(1)^3$	$Sp(2) \times U(1)^3$
8	21	$SO(3) \times U(3) \times U(1)$	$Sp(1) \times U(3) \times U(1)$
8	21	$U(3) \times U(2)$	$U(3) \times U(2)$
6	22	$SO(3) \times U(2)^2$	$Sp(1) \times U(2)^2$
6	22	$U(3) \times U(1)^2$	$U(3) \times U(1)^2$
4	23	$SO(3) \times U(2) \times U(1)^2$	$Sp(1) \times U(2) \times U(1)^2$
4	23	$U(2)^2 \times U(1)$	$U(2)^2 \times U(1)$
2	24	$SO(3) \times U(1)^4$	$Sp(1) \times U(1)^4$
2	24	$U(2) \times U(1)^3$	$U(2) \times U(1)^3$
0	25	$U(1)^5$	$U(1)^5$

TABLE V. Possible number of type-A and type-B NGBs for $SO(11)/U(1)^5$ and $Sp(5)/U(1)^5$.

**List of possible U
for G with rank=5**

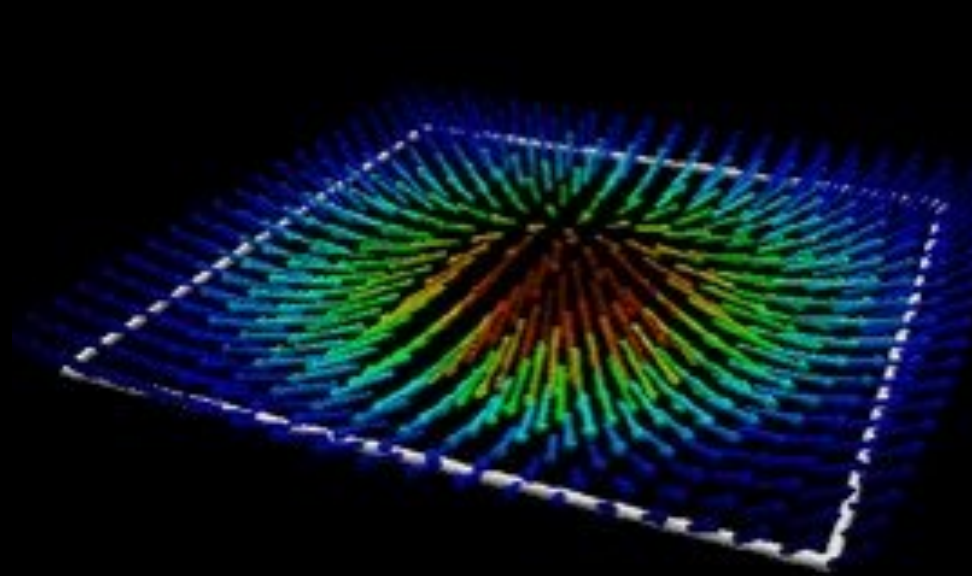


Low-dimensions

- There are additional terms one can write up to two derivatives in low dimensions
- 1+1: $\tilde{c}_a(\pi)\nabla_x\pi^a + \tilde{g}_{ab}(\pi)\dot{\pi}^a\nabla_x\pi^b + \tilde{b}_{ab}(\pi)\dot{\pi}^a\nabla_x\pi^b$
- 2+1: $-\frac{1}{2}b_{ab}(\pi)\epsilon^{rs}\nabla_r\pi^a\nabla_s\pi^b$



skyrmion

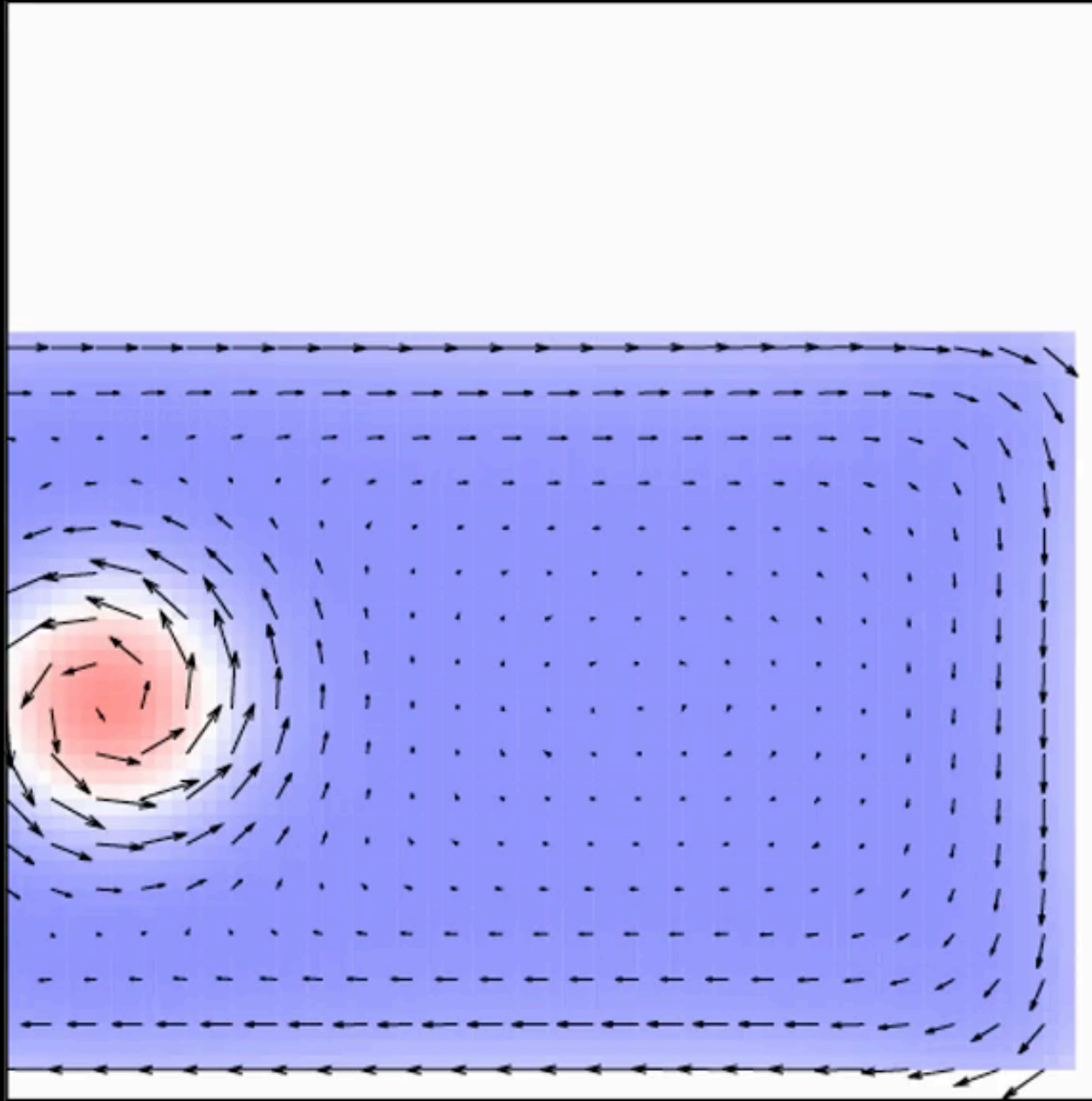


- Consider a Heisenberg ferromagnet
- On a two-dimensional plane, non-trivial maps $\mathbb{R}^2 \rightarrow S^2$ classified by $\pi_2(S^2) = \mathbb{Z}$
- skyrmion has moduli:
 - translations in x and y directions
 - dilation, rotation
- derive effective Lagrangian for moduli
- momenta don't commute!

$$[P_x, P_y] = i\hbar 4\pi s N_{\text{skyrmion}} \quad \dot{y} - F_x = 0$$



$$L = \frac{1}{2}(x\dot{y} - y\dot{x}) - F_x x \qquad \dot{y} - F_x = 0$$



Higgs mechanism

H. Watanabe and HM, arXiv:1405.0997



Anderson-Englert-Brout- Higgs-Guralnik-Hagen-Kibble

- massless vector boson in d dimension has $d-1$ dof
- *massive* vector boson has d dof
- it needs to *eat* massless NGB



mismatch

- Heisenberg model breaks $SO(3)$ to $SO(2)$
- two broken generators, one type-B NGB
- If $SO(3)$ gauged, two broken gauge bosons
- but only one NGB to be “eaten”
- what happens?
- NB type-B NGB comes with charge density



turning on gauge field

- Yang-Mills equation $D_\mu F^{\mu\nu} = ej^\nu$
$$\nabla \cdot \vec{E} = ie[\vec{A}, \vec{E}] + ej^0$$
- thermodynamics limit requires **r.h.s.=0**
- Non-zero charge density j^0 forces A^0 to acquire VEV
- Space-time constant solution requires
$$e[A_i, [A_i, A^0]] = j^0$$
- Spatial components of gauge field also acquire VEVs



charge density

- Noether current for a global gauge transformation: $j^\mu = i[A_\nu, F^{\mu\nu}] + j_{\text{matter}}^\mu$
- matter contribution cancelled by the gauge contribution
- NGBs for global gauge transformation from gauge bosons provide additional dof to be eaten, **become type-A**
- additional NGBs from gauge bosons for broken rotation

$$S^2 = \text{SU}(2)/\text{U}(1)$$

- Heisenberg model with spin density // z
- need a VEV for a spatial component
- other A^i are NGBs for rotation symmetry together with a massless type-A NGB
- A^2 and A^3 acquire mass eating two type-A NGBs

$$D_\mu F^{\mu\nu} = j^\nu$$

$$[A_i, [A_i, A^0]] = j^0$$

$$\langle e A_\mu \rangle = \begin{pmatrix} & 1 & 2 & 3 \\ 0 & 0 & O(e) \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ O(\sqrt{e}) & 0 & 0 \end{pmatrix} \begin{matrix} t \\ x \\ y \\ z \end{matrix}$$



can be tested?

- multi-layer graphene?
- spin-orbit coupling?
- frustrated spin liquid?
- high-density QCD?



NGB dark matter

- NGBs (pions) in QCD-like theories can be dark matter, e.g. $SU(4)/Sp(2)=S^5$
- freeze-out of $3 \rightarrow 2$ process, not $2 \rightarrow 2$
- “SIMP” Strongly Interactive Massive Particle
- to appear soon with Yonit Hochberg, Eric Kuflik, Tomer Volansky, Jay Wacker

