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KAVLI INSTITUTE FOR THE PHYSICS AND MATHEMATICS OF THE UNIVERSE

Hitoshi-fest: from particle physics, cosmology to instrumentation
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Emergence of Gravitational Spacetime from Quantum Information

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YUKAWA INSTITUTE FOR THEORETICAL PHYSICS

Grant-in-Aid for Transformative Research Areas A
Extreme Universe

My wonderful days at IPMU [2008-2012]

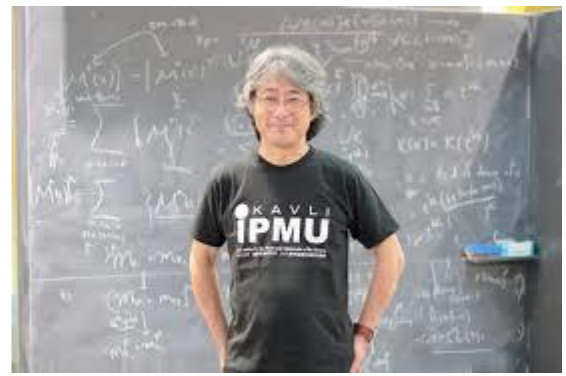
◆ I thank very much

- Great colleagues
- A lot of research time
- Tea time discussions

◆ I learned a lot from opportunities of

- Supervising post-docs and students
- Organizing international workshops
- Watching Murayama-san's effort to make IPMU internationally prominent

The valuable experiences in IPMU have been very helpful for me after I move to YITP, Kyoto.

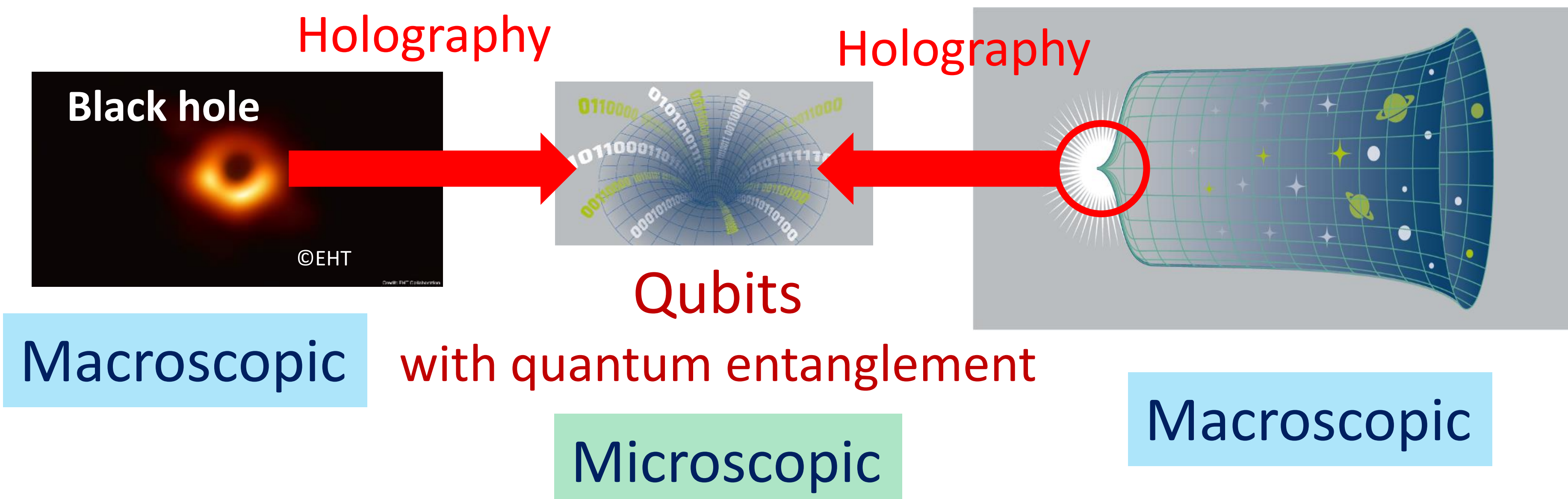


I am extremely grateful to Murayama-san for founding this amazing institute and for providing us with this ideal environment for researchers !

① Introduction

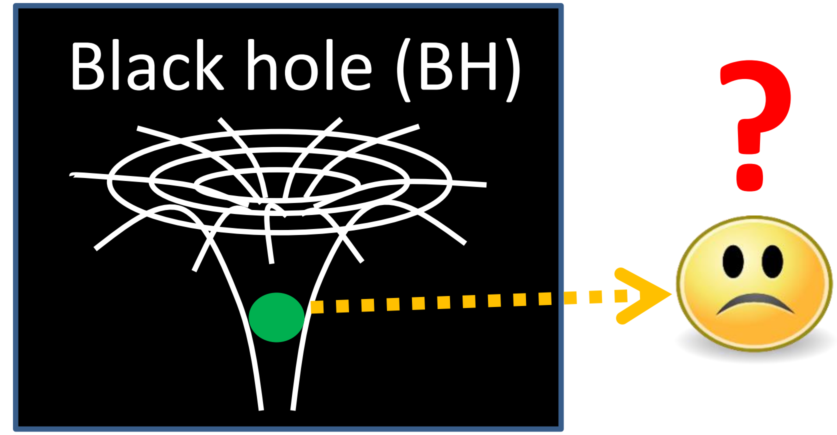
Holography is a very promising approach to quantum gravity, towards understanding of the creation of the Universe.

This is because holography is much like ***a microscope*** in thought experiments of quantum gravity.



Macroscopic Side: Black hole Entropy

A huge amount
of quantum information
is hidden in side black hole !



$$S_{BH} = \frac{k_B c^3}{\hbar} \times \frac{A_{BH}}{4G_N}$$

⇒ BH thermodynamics !

[Bekenstein, Hawking, 1972−1976]

A_{BH}= Surface Area of Black hole ⇒ **Geometry**
G_N=Newton constant ⇒ **Gravity**
$\hbar$=Planck constant ⇒ **Quantum Mechanics**
k_B=Boltzmann const.⇒ **Stat. Mech. , Quantum Info.**

} **Quantum Gravity!**

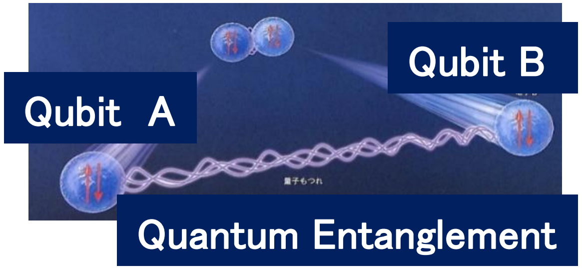
Microscopic Side: Quantum Entanglement (QE)

Quantum Entanglement ⇒ Quantum correlations between A and B.

$$|\Psi\rangle_{AB} \neq |\Psi_1\rangle_A \otimes |\Psi_2\rangle_B \quad .$$

$$\text{e.g. Bell state}: \ |\Psi_{Bell}\rangle = \frac{1}{\sqrt{2}}\left[|\uparrow\rangle_A \otimes |\downarrow\rangle_B + |\downarrow\rangle_A \otimes |\uparrow\rangle_B\right]$$

⇒ **Minimal Unit of Entanglement**
⇔ Planck length

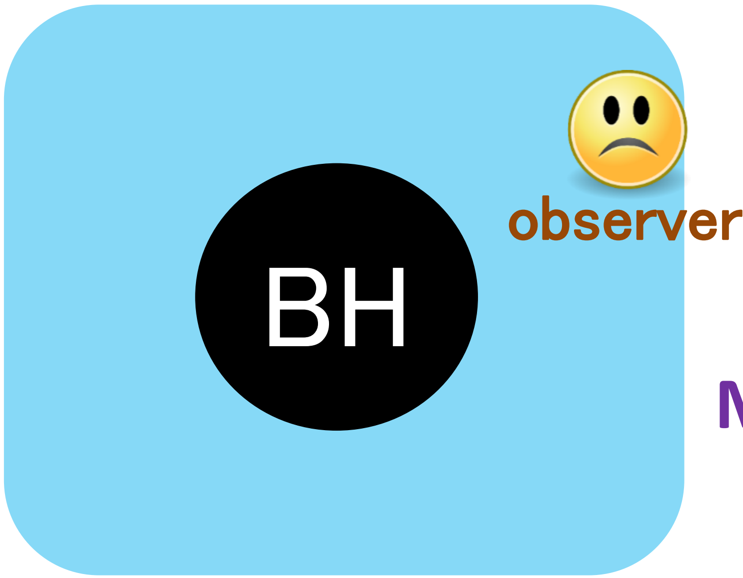


The best (or only) measure of quantum entanglement for pure states is known to be **entanglement entropy (EE)**.

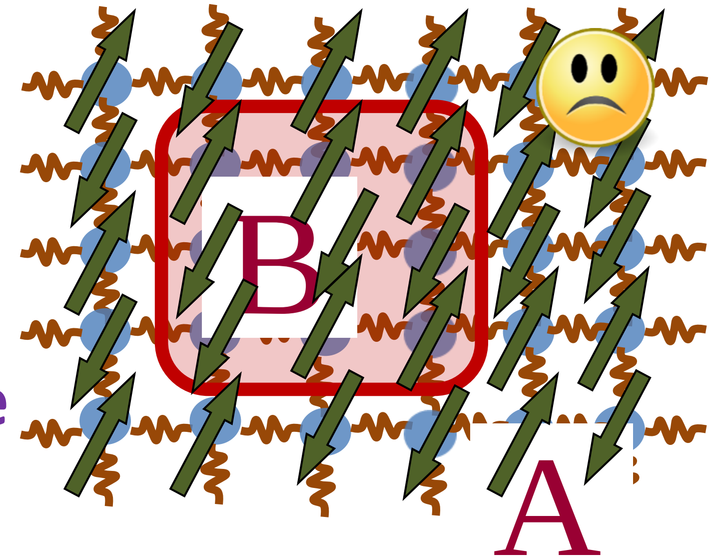
EE ＝S_A= # of Bell states between A and B

From BHs to Qubits

Blackhole Spacetime



Quantum Many-body System



Microscope

BH entropy S_{BH} → Entanglement entropy S_A

Area law | Area law

What is a useful microscope in quantum gravity ?

② Holography and Quantum Entanglement

Holography as microscope in quantum gravity

BH Entropy Formula

$$S_{BH} = \frac{A_{BH}}{4G_N}$$

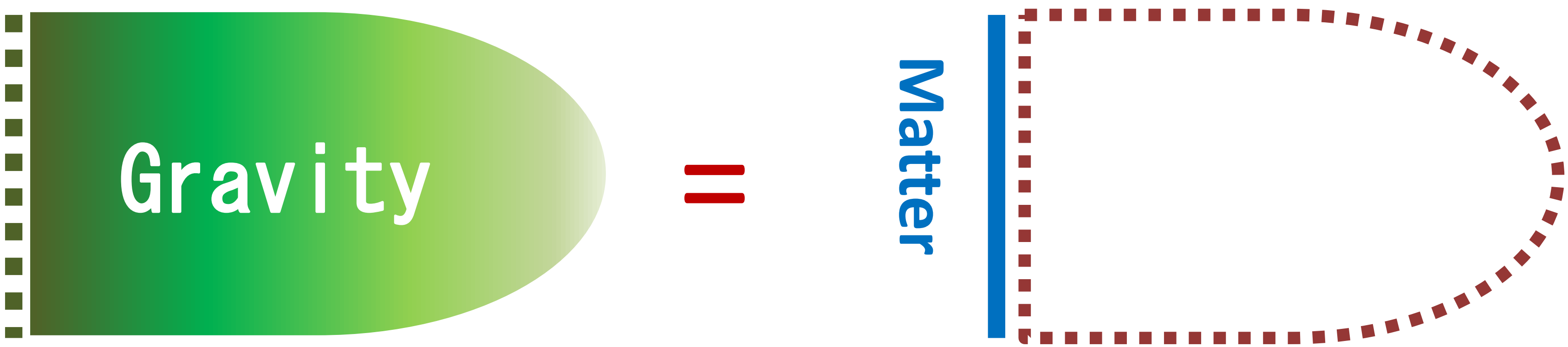
→ **Degrees of freedom in Gravity ∝ Area**

['t Hooft 1993, Susskind 1994]

Holography

Boundary of M

Gravity on M = Quantum Matter on ∂M



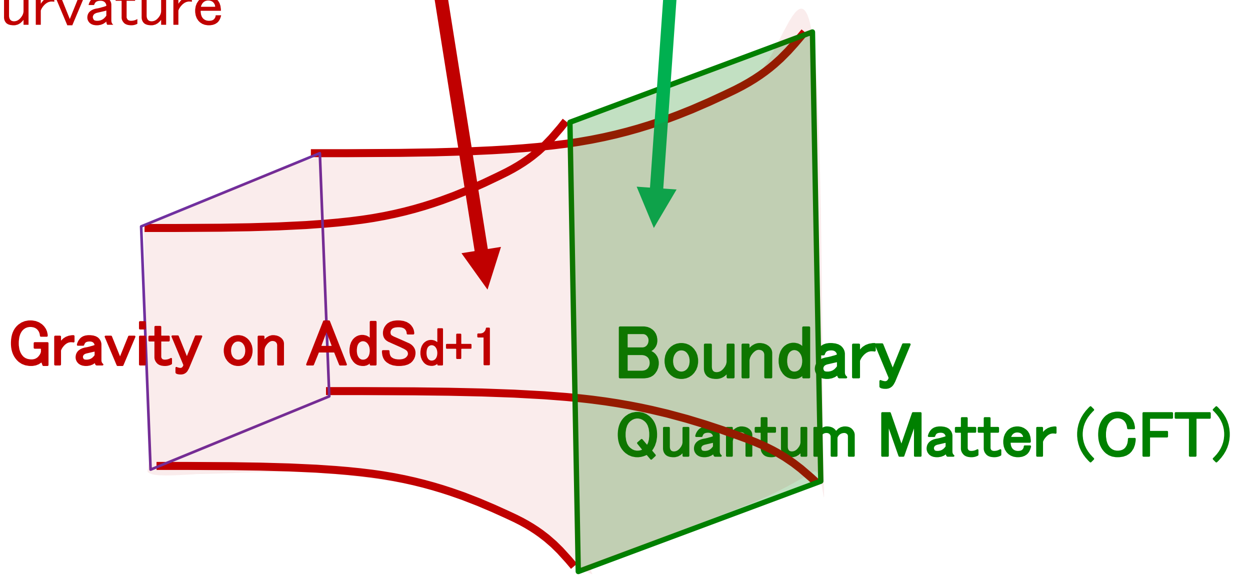
BH entropy(∝Area)= Thermal Entropy of Matter (∝Volume)

Gauge/Gravity Duality (AdS/CFT) – [Maldacena 1997]

(Quantum) Gravity on d+1 dim. Anti-de Sitter Space = **d dim. Gauge theory (or Conformal field theory)**

Anti-de Sitter space (AdS)
➔ Universe with negative curvature
$\Lambda < 0$

Conformal Field Theory(CFT)
➔ quantum field theory with only massless fields



Mining Qubits from Gravity

Holographic Entanglement Entropy (HEE)

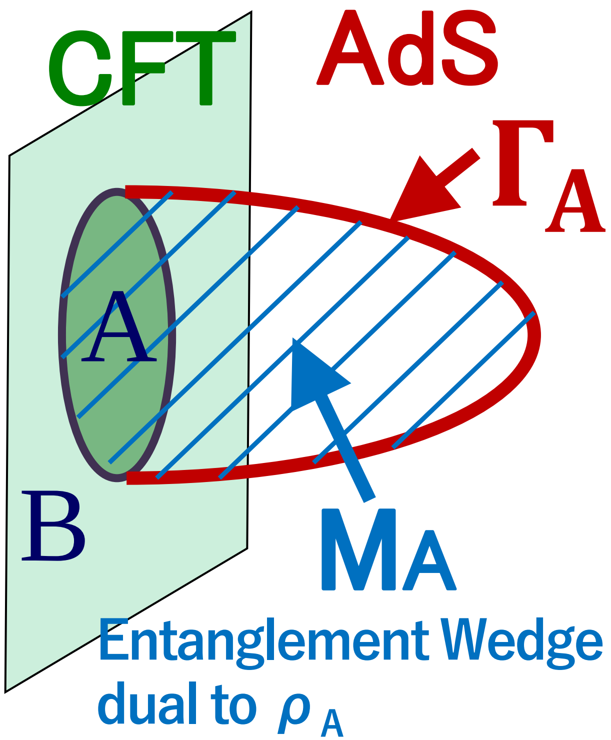
[Ryu–TT 2006, Hubeny–Rangamani–TT 2007]

$$S_A = \underset{\Gamma_A}{\text{Min Ext}} \left[\frac{\text{Area}(\Gamma_A)}{4G_N} \right]$$

This formula can compute entanglement entropy in time-dependent backgrounds.

↓

Real time (Lorentzian) evolution



$$\rho_A = \text{Tr}_B\left[|\Psi_{tot}\rangle\langle\Psi_{tot}| \right].$$

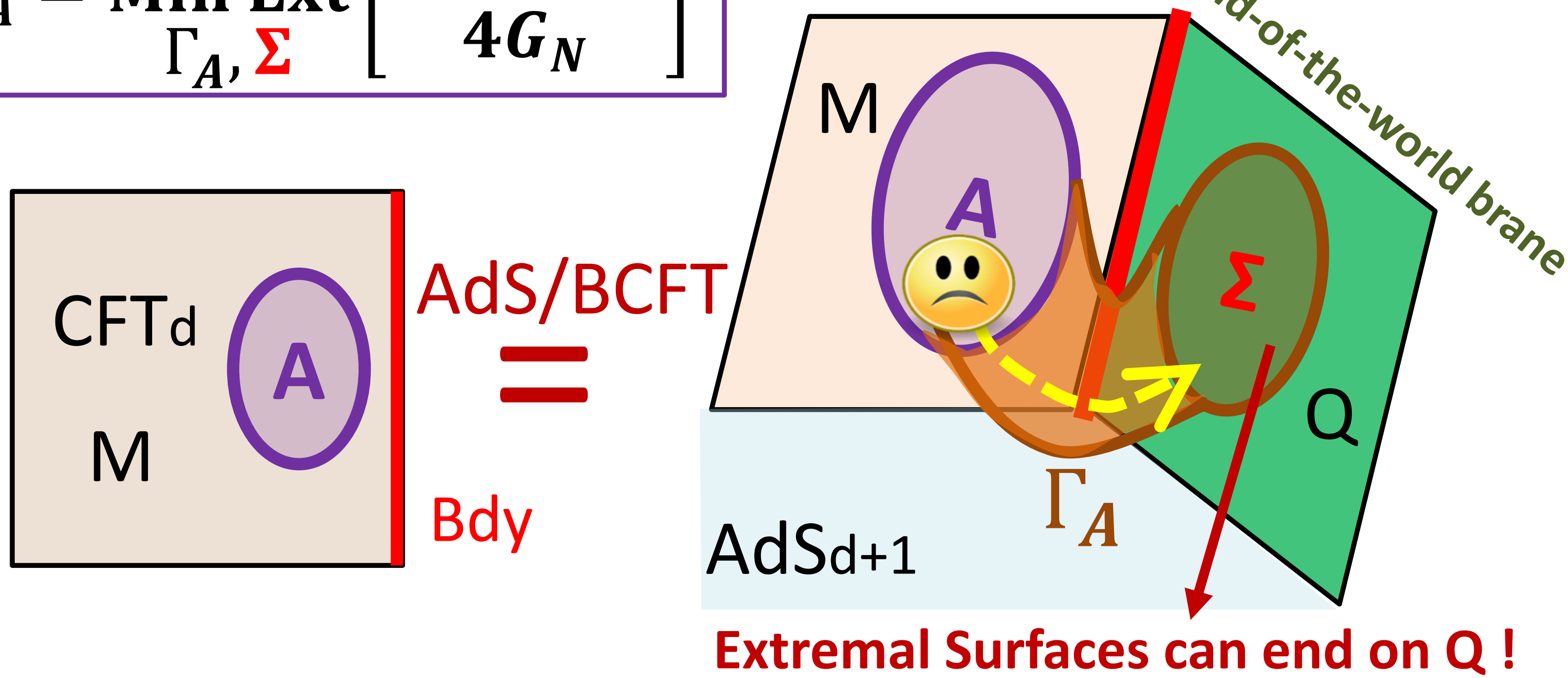
$$S_A = -\text{Tr}_A\, \rho_A \log \rho_A\, .$$

Extension 1: HEE in AdS/BCFT [TT 2011, Fujita-Tonni-TT 2011]

Q. What is holography and HEE for CFT on a space with boundaries ?

$$S_A = \underset{\Gamma_A, \Sigma}{\text{Min Ext}} \left[\frac{\text{Area}(\Gamma_A)}{4G_N} \right]$$

$$\partial\Gamma_A = \partial A \cup \partial\Sigma$$



The region **Σ** is now known as an **Island** ! ➡ A key to resolve BH information paradox

[Penington 2019, Almheiri et.al. 2019]

☹ can see BH interior !

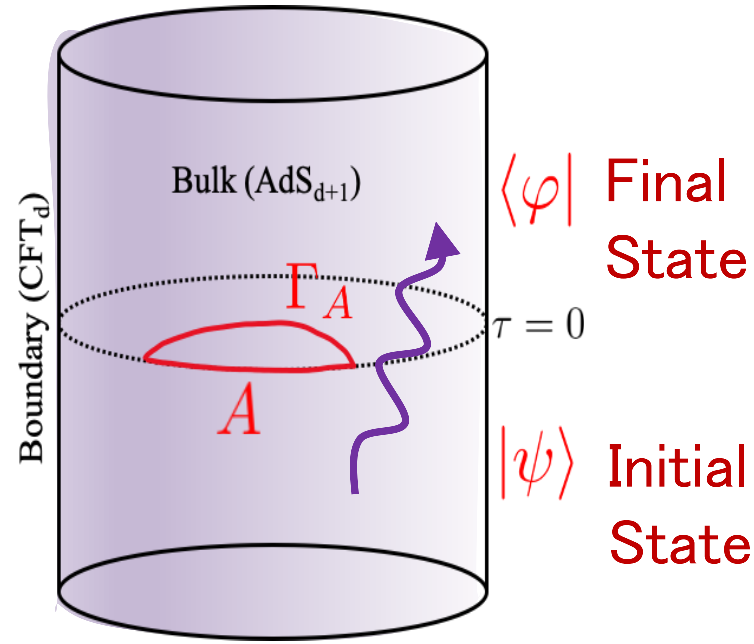
Extension 2: HEE for non-Hermitian density matrices

Q. What is HEE in a *Euclidean time-dependent AdS* geometry ?

Holographic Pseudo Entropy

[Nakata–Taki–Tamaoka–Wei–TT, 2020]

$$S(\mathcal{T}_A^{\psi|\varphi}) = \min_{\Gamma_A} \frac{\text{Area}(\Gamma_A)}{4G_N}$$



$$H_{tot} = H_A \otimes H_B \quad .$$

Transition matrix

$$\tau^{\psi|\varphi} = \frac{|\psi\rangle\langle\varphi|}{\langle\varphi|\psi\rangle}.$$

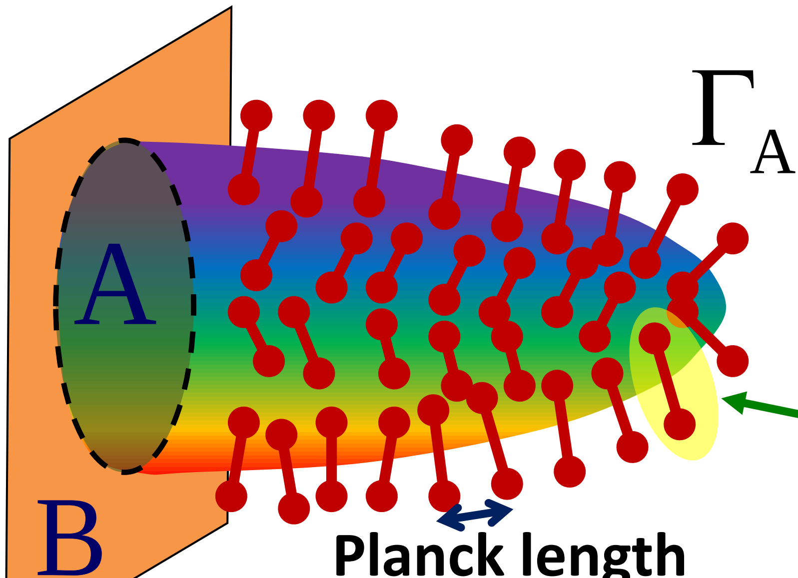
$$\tau_A^{\psi|\varphi} = \mathrm{Tr}_B\left[\tau^{\psi|\varphi}\right]$$

Pseudo entropy:

$$S\left(\tau_A^{\psi|\varphi}\right) = -\mathrm{Tr}\left[\tau_A^{\psi|\varphi}\log\tau_A^{\psi|\varphi}\right].$$

In general, complex valued.

HEE/HPE suggests that in spacetimes with $\Lambda<0$, there is one qubit of entanglement for each Planck length area !

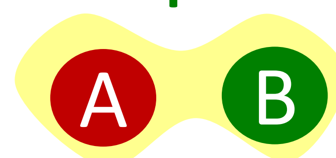


$$l_P = \sqrt{\frac{\hbar G}{c^3}} = 1.6\times 10^{-35}m$$

$$S_A = \frac{\mathrm{Area}(\Gamma_A)}{4 l_{pl}^{D-1}}$$

~10^{65} qubits per 1cm^2 !

Bell pair



= Planck scale mini Universe

$\Lambda<0$ spacetime may emerge from entangled Qubits !

Q. How about de Sitter space $\Lambda>0$?

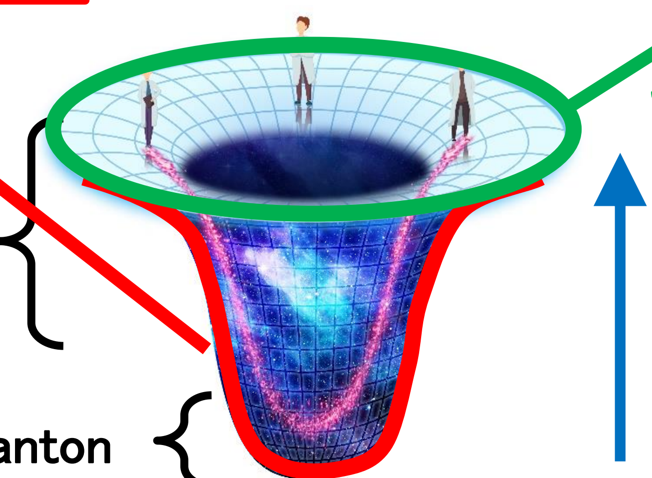
③ De Sitter (dS) Holography and Holographic Entropy

An extension of AdS/CFT to de Sitter space (dS) is known as the dS/CFT correspondence. [Strominger 2001, Maldacena 2002]

A Sketch of dS/CFT

Equivalent
dS_{d+1}/CFT_d

d+1 dim. Lorentzian de−Sitter spacetime	⟺	Euclidean d. dim CFT on S^d



Lorentzian de Sitter space

$$ds^2 = L_{dS}^2(-dt^2 + \mathrm{Cosh}^2 t\, d\Omega^2)$$

Euclidean instanton

Why dS/CFT is much more difficult than AdS/CFT ?

[1] Dual Euclidean CFTs should be exotic and non-unitary !

A "standard" Euclidean CFTs is dual to gravity on hyperbolic space.

e.g. dS3/CFT2 ➔ ***Imaginary valued*** central charge $c \approx i\frac{3L_{dS}}{2G_N}$!

[2] Time should emerge from Euclidean CFT !

From a usual Euclidean CFT, a space-like direction will emerge as RG scale.

How does a ***time-like direction emerge*** from a Euclidean CFT ?

[3] Entanglement entropy becomes complex valued !

Extremal surfaces in dS which end on its boundary are ***time-like*** !

Non-unitary CFT dual of 3 dim. dS

[Hikida-Nishioka-Taki-TT, 2021, 2022]


Physics ABOUT BROWSE PRESS COLLECTIONS

VIEWPOINT

Steps toward Quantum Gravity in a Realistic Cosmos

Jordan Cotler

Society of Fellows, Harvard University, Cambridge, MA, USA

July 18, 2022 • *Physics* 15, 107

Theorists have modeled an expanding spacetime—akin to our Universe—by taking inspiration from a string theory framework in which spacetime is emergent.

Large c limit of SU(2)k WZW model (a 2dim. CFT)
= **Einstein Gravity on 3 dim. de Sitter** (radius L_{dS})

Level

$$k \approx -2 + \frac{4iG_N}{L_{dS}}$$

→ **Central charge**

$$c = \frac{3k}{k+2} \approx i\frac{3L_{dS}}{2G_N}$$

$$Z[S^3, R_j] = |S_j^0|^2 \approx e^{\frac{\pi L_{dS}}{2G_N}\sqrt{1-8G_N E}}$$

CFT partition function — De Sitter Entropy

This non-unitary CFT is equivalent to the Liouville CFT

at $b^{-2} \approx \pm\frac{i}{4G_N}$.

$$I_{CFT}[\phi] = \int d^2x \left[\frac{1}{4\pi}(\partial_a\varphi\partial_a\varphi) + \mu e^{2b\varphi}\right].$$

complex !

[→Reproduced by Verlinde-Zhang 2024 via the Double Scaled SYK]

Holographic Entanglement Entropy in dS3/CFT2 ?

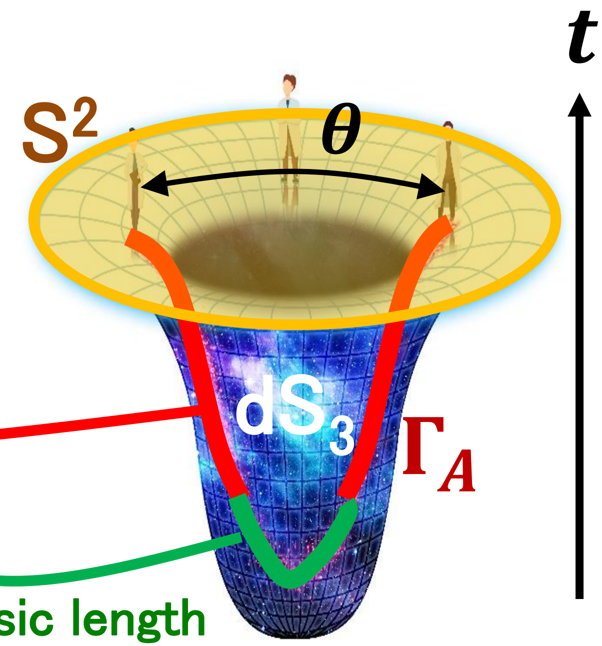
[Doi-Harper-Mollabashi-Taki-TT 2022, 2023]

In dS3/CFT2, the geodesic Γ_A becomes time-like and we find:

$$S_A=\frac{L(\Gamma_A)}{4G_N}=i\frac{c_{ds}}{3}\log\left(\frac{2}{\epsilon}\mathrm{Sin}\frac{\theta}{2}\right)+\underbrace{\frac{c_{dS}}{6}\pi}_{S_{dS}/2}.$$

Time-like geodesics length
→imaginary part !

Space-like geodesic length
→Real part



Agree !

CFT calculation

$$S_A=\frac{c_{CFT}}{6}\log\left[\frac{\mathrm{Sin}^2\frac{\theta}{2}}{\tilde{\varepsilon}^2}\right],$$ by setting

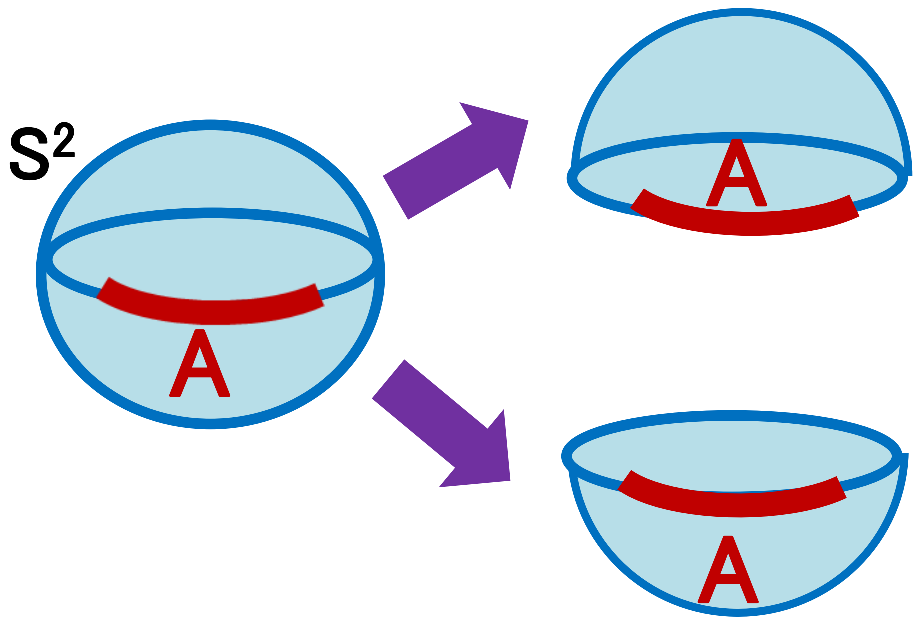
$$c_{CFT}=ic_{dS}$$ and $$\tilde{\varepsilon}=i\varepsilon=ie^{-t_\infty}.$$

→ **Complex valued entropy !**
(should not be EE !)

We argue it is more properly considered as pseudo entropy (PE).

This is because the reduced density matrix **ρ_A is not Hermitian !**

$\rho_A =$



$\langle\varphi| \sim \int_{S^2_+} D\varphi\, e^{-I_{CFT}[\varphi]}$

Different states !

$|\psi\rangle \sim \int_{S^2_-} D\varphi\, e^{-I_{CFT}[\varphi]}$

$\rightarrow \rho_A \neq \rho_A^\dagger$

Note: the emergent time coordinate = imaginary part of PE.

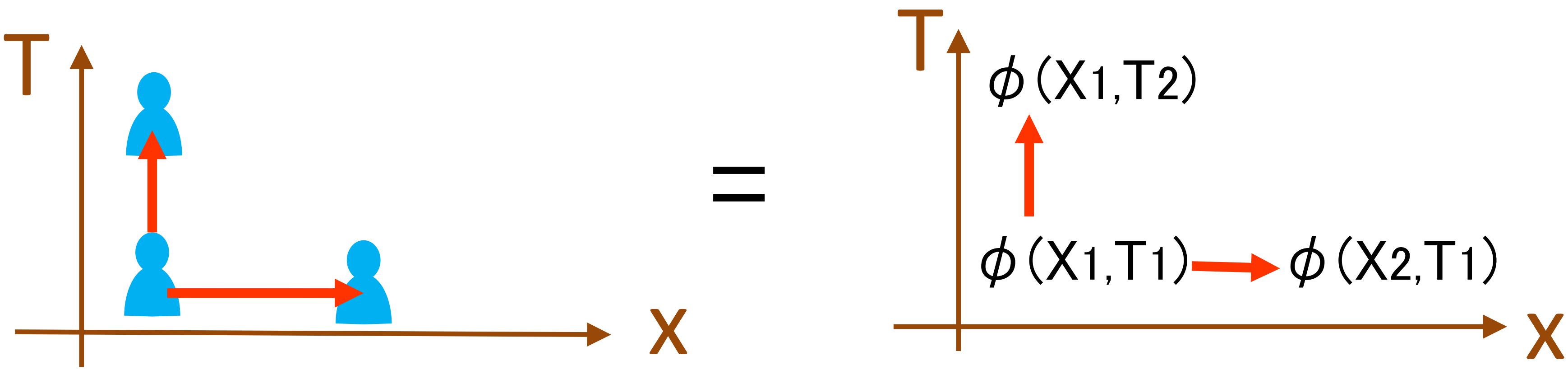
④ Probing De Sitter Space from CFT

Consider an observer in 2d CFT.

➡How does the observer feel that he or she lives in AdS or dS ?

To probe the spacetime, we introduce a local excitation.

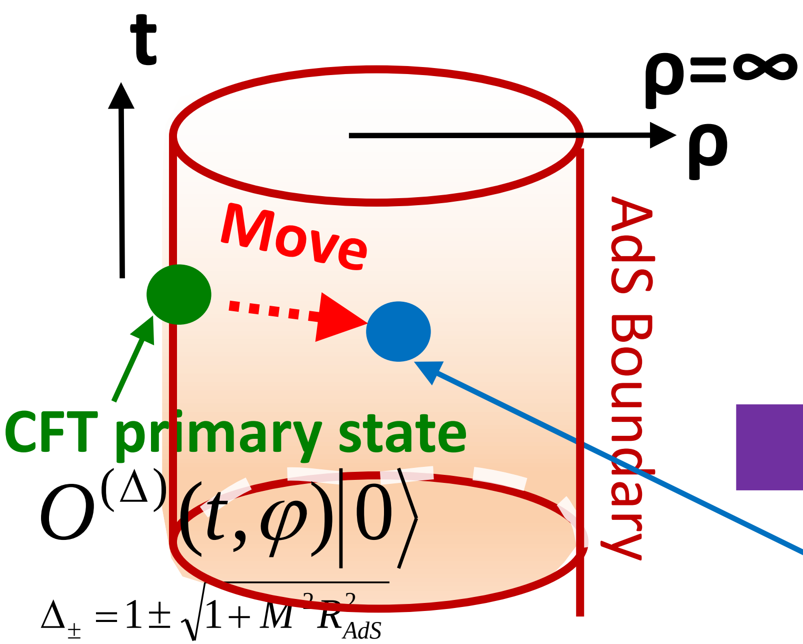
"How many directions can the observer move ?"



In flat spacetime, $\left|\Psi_{\text{Flat}}(t,x)\right\rangle = e^{-iHt} e^{iPx} \hat{\phi}(0)\left|0\right\rangle$

Warming up: Probing AdS from CFT

AdS3 metric $ds^2 = R_{AdS}^2(-\mathrm{Cosh}^2\rho dt^2 + d\rho^2 + \mathrm{Sinh}^2\rho d\varphi^2)$



Geometric Symmetry
$= SL(2,R)_L \times SL(2,R)_R$

= Conformal symmetry of 2d CFT

$L_0, L_{-1}, L_1 \quad \tilde{L}_0, \tilde{L}_{-1}, \tilde{L}_1$

$[L_n, L_m] = (n-m)L_{n+m}$

$\left|\Psi_{AdS}^{(\Delta)}(\rho,t,\varphi)\right\rangle$ CFT dual of bulk local excitation

The CFT dual of a localized excitation in AdS is given by:

$$\left|\Psi_{AdS}^{(\Delta)}(\rho,t,\varphi)\right\rangle = e^{-i(L_0+\tilde{L}_0)t} e^{i\varphi(L_0-\tilde{L}_0)} e^{-\frac{\rho}{2}\left(L_{-1}+\tilde{L}_{-1}-L_1-\tilde{L}_1\right)} e^{\frac{\pi i}{2}(L_0+\tilde{L}_0)} \left|I^{(\alpha)}\right\rangle.$$

[Miyaji–Numasawa–Shiba–Watanabe–TT 2015, equivalent to HLKK]

SL(2,R) Ishibashi state

Main problem: Probing dS from CFT

[Doi–Ogawa–Shinmyo–Suzuki–TT 2024]

First we note the "formal" relation between AdS and dS:

AdS metric
$$ds^2 = R_{AdS}^2(-\mathrm{Cosh}^2\rho dt^2 + d\rho^2 + \mathrm{Sinh}^2\rho d\varphi^2)$$

$$R_{AdS}^2 = -R_{dS}^2$$
$$t = i\tau$$
$$\rho = i\theta$$

$$(L_0)^\dagger = L_0, \quad (L_{\pm1})^\dagger = L_{\mp1}$$

dS metric
$$ds^2 = R_{dS}^2(-\mathrm{Cos}^2\theta d\tau^2 + d\theta^2 + \mathrm{Sin}^2\theta d\varphi^2)$$

Unusual conjugation
$$(L_0)^\dagger = -\tilde{L}_0, \quad (L_{\pm1})^\dagger = \tilde{L}_{\pm1}$$

$$H^\dagger = -H$$

Hamiltonian is anti-Hermitian ! →Non-unitary CFT!

A naïve analytical continuation from AdS leads to the following quantum state for a localized excitation in dS:

$$\left|\tilde{\Psi}_{dS}^{(\Delta_+)}(\tau,\theta,\varphi)\right\rangle = e^{(L_0+\tilde{L}_0)\tau} e^{i\varphi(L_0-\tilde{L}_0)} e^{i\frac{\theta}{2}\left(L_1+\tilde{L}_1-L_{-1}-\tilde{L}_{-1}\right)} e^{\frac{\pi i}{2}(L_0+\tilde{L}_0)} \left|I^{(\alpha)}\right\rangle.$$

Non-unitary evolution
→emergent time

SL(2,R) Crosscap State

Primary operator in CFT

$$O^{(\Delta_+)}(t,\varphi)$$

$$\Delta_\pm = 1 \pm i\sqrt{M^2 R_{dS}^2 - 1}$$

SL(2,R) Ishibashi State

$$(L_n - \tilde{L}_{-n})\left|I^{(\alpha)}\right\rangle = 0$$

However, this state ***leads to the confusing result*** (due to the *unusual conjugation*):

$$\left\langle \tilde{\Psi}_{dS}^{(\Delta_+)}(\tau,\theta,\varphi) \middle| \tilde{\Psi}_{dS}^{(\Delta_+)}(\tau',\theta',\varphi')\right\rangle = 0.$$

Resolution

The correct answer is found by requiring the CPT invariant state:

$$\left|\Psi_E^{(\Delta_+)}(\tau,\theta,\varphi)\right\rangle = \frac{1}{\sqrt{i}}\left|\tilde{\Psi}_{dS}^{(\Delta_+)}(\tau,\theta,\varphi)\right\rangle + CPT\cdot\left(\frac{1}{\sqrt{i}}\left|\tilde{\Psi}_{dS}^{(\Delta_+)}(\tau,\theta,\varphi)\right\rangle\right)$$

$$= \frac{1}{\sqrt{i}}\left|\tilde{\Psi}_{dS}^{(\Delta_+)}(\tau,\theta,\varphi)\right\rangle + \sqrt{i}\left|\tilde{\Psi}_{dS}^{(\Delta_-)}(\underline{\tau,\pi-\theta,\varphi+\pi})\right\rangle.$$

Antipodal map

[cf. Global sym. in gravity: Harlow–Ooguri 2018, Gauging CPT in QG: Harlow–Numasawa 2023]

Indeed, this reproduces the correct dS Green function at Euclidean vacuum

$$\left\langle\Psi_E^{(\Delta_+)}(x)\middle|\Psi_E^{(\Delta_+)}(x')\right\rangle = G_{dS}^E(x,x') = \frac{\sinh\mu(\pi - D_{dS}(x,x'))}{4\pi\sinh\pi\mu\cdot\sin D_{dS}(x,x')}.$$

Summary

- In dS/CFT, the Hamiltonian is anti Hermitian.
  →Emergence of time
- In dS/CFT, we need to gauge the CPT symmetry.

⑤ Conclusions

- Holography has successfully predicted that Λ<0 (AdS) spacetimes emerge from quantum entanglement in CFTs.
- Holography for Λ>0 (dS) spacetimes is still in the development stage due to many exotic properties of dual CFTs.
- Complex valued entanglement entropy in dS/CFT can be interpreted as pseudo entropy. ➡ Emergence of time
- An analysis of CFT dual of bulk local excitation again reveals non-Hermitian nature of the dual CFT.

Future problems

- CFT interpretations of Einstein equation in dS
- QI understandings of pseudo entropy
- CFT Models of creation of the Universe

:

Happy birthday, Murayama-san !

Contents