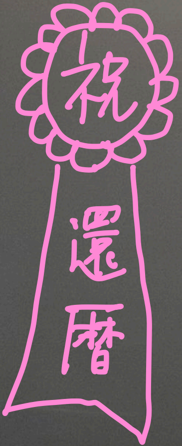
Singularity

Yukari Ito

(Kavli IPMU, UTokyo)

2024.12.16



0. Memorial Facts

① 2017.7.15 @ Yokohama

At an event for female students in Science. 1st time in person.

② 2017.9.1 @ Kavli IPMU

I became a member.

- 10th aniversary "Bethoven's 9th" Chorus
- Art activity Photos

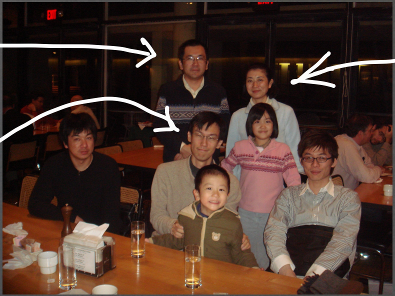
③ 2019.10.28 @ Kavli IPMU

Original & New Tshirts on 12th aniversary

1. Institute for Advanced Study in Princeton ~2007~

Hiraku Nakajima (Kyoto)

Yuji Tachikawa (IAS Postdoc)

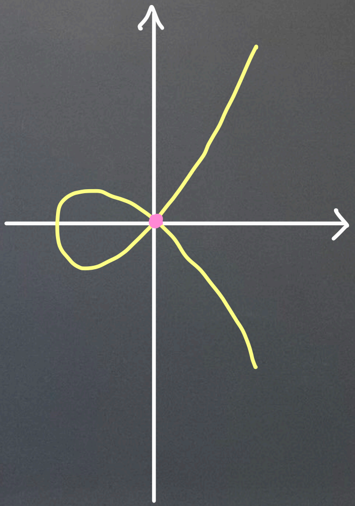


Yukari Ito (Nagoya)

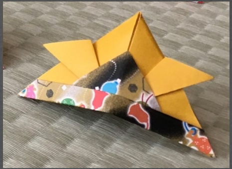
← Picture at IAS restaurant.

We were invited to a special dinner with director of IAS and met President of U.Tokyo Komiyama in October 2007. He asked the most important thing at the Institute, we answered Good Restaurant with Good foods! We are still waiting for it!

2. Singularity



something different from other points.



non-singular

3. Mathematics @ Kavli IPMU

"singularity of Kavli IPMU"

When I came to Kavli IPMU

Prof. Kyoji Saito

who was my host
when I was a PD at RIMS.

Dr. Yusuke Nakajima

Came as Postdoc.

now ass. prof. of
Kyoto-Sangyo Univ.

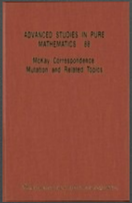
Dr. Yusuke Sato

Came as PhD student

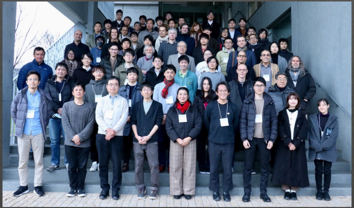
Now, Lecturer of
Osaka Inst. of Tech.

and nice postdocs !!!

"McKay conferences" & Book.



2020

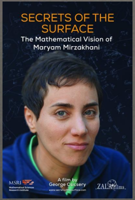


2023

4. Other activities @ Kavli IPMU

- Colloqium Organizer
- Exhibition of Mathematics
- Outreach talks
- Women in Math & Physic
  - Women's Lunch (Every Wed)
  - 3 conferences "The world of…" with Ikkatai, Yamazaki

- Add Japanese suptitles on a movie of a female Fields Louriate. with Ikkatai & Yamazaki.



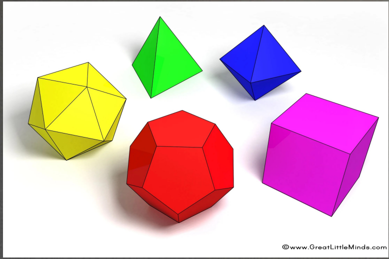
- Internship of undergrate female students.

and Choir, Piano, Ikebana!

5. Finite groups

A finite subgroup of $SL(2, \mathbb{C})$ is isomorphic to one of the following groups:

$$\begin{cases} A_n & \text{cyclic group} \\ D_n & \text{binary dihedral group} \\ E_6 & \text{binary tetrahedral group} \\ E_7 & \text{binary octahedral group} \\ E_8 & \text{binary icosahedral group} \end{cases}$$



Platonic solids
(= regular polyhedron)

$\Updownarrow$

polyhedral group $\subset SO(3)$

$\downarrow$

binary polyhedral group
$E_6, E_7, E_8 \subset SU(2)$

6. McKay quiver

Quiver $= \{Q_0, Q_1\}$

Q_0: vertex, Q_1: arrows

https://www.amazon.ae/JIALUCONG-Arrow-Quiver-Rustic-Leather/dp/B08TWZ6NC3

ρ_i = irreducible representation of $G/\sim$

$\rho_{nat} : G \to GL(n, \mathbb{C})$: natural. rep.

$$\rho_{nat} \otimes \rho_i = \sum a_{ij} \rho_j$$

McKay quiver is given by

$a_{ij} \leftrightarrow$ ⓘ $\rightrightarrows$ ⓙ

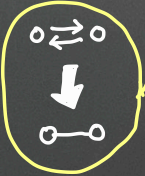
a_{ij} = # arrows from i to j

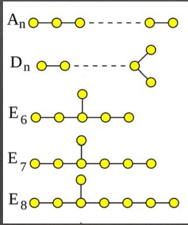
Theorem (J. McKay)

$G \subset SL(2, \mathbb{C})$ finite group

$a_{ij} = a_{ji} = 0$ or 1

McKay quiver $\to$ Dynkin diagram of A. D. E.





Simple Lie algebra $\longleftrightarrow$

7. Quotient Singularity

finite group $G \curvearrowright \mathbb{C}^n$

quotient space $\mathbb{C}^n/G$ may have singularity.

Example $G = \left\langle \begin{pmatrix} \varepsilon & 0 \\ 0 & \varepsilon^{-1} \end{pmatrix} \middle| \ \varepsilon^n = 1 \right\rangle$

invariant ring

$$\mathbb{C}[u,v]^G = \mathbb{C}[\underset{\substack{\parallel \\ x}}{u^n}, \underset{\substack{\parallel \\ y}}{v^n}, \underset{\substack{\parallel \\ z}}{uv}]$$

$$= \mathbb{C}[x,y,z]/(xy - z^n)$$

$\therefore$ $\mathbb{C}^2/G$ is defined by $xy - z^n = 0$ in $\mathbb{C}^3$

$\rightsquigarrow$ A_{n-1} singularity

$G \subset SL(2,\mathbb{C})$: finite.

$\mathbb{C}^2/G$ is defined by the following equations and have a singularity at $(0,0,0) \in \mathbb{C}^3$.

$$A_n \ : \ x^2 + y^2 + z^{n+1} = 0 \quad (n \geq 1),$$

$$D_n \ : \ x^2 + y^2 z + z^{n-1} = 0 \quad (n \geq 4),$$

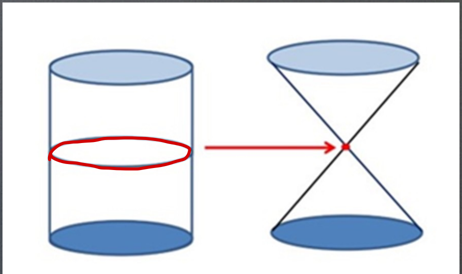
$$E_6 \ : \ x^2 + y^3 + z^4 = 0,$$

$$E_7 \ : \ x^2 + y^3 + yz^3 = 0,$$

$$E_8 \ : \ x^2 + y^3 + z^5 = 0$$

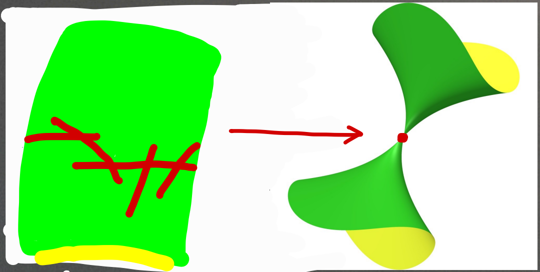
8. Resolution of singularity (blow-up)

The minimal singularity of $\mathbb{C}^2/G$ gives exceptional curves $\cong \mathbb{P}^1$

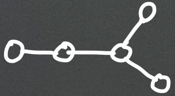


the minimal resolution of A_1 singularity. $x^2+y^2+z^2=0$

D_5 singularity $x^2+y^2z+z^4=0$



$\updownarrow$ dual graph



D_5 Dynkin diagram.

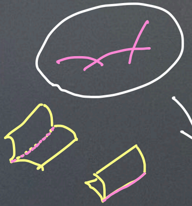
9. McKay correspondence

Theorem 1 (Gonzalez-Sprinberg & Verdier)

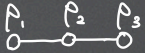
$G \subset SL(2, \mathbb{C})$ finite,

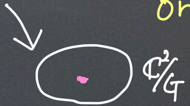
$\exists$ bijection

$\{$irreducible exceptional curve$\} \longleftrightarrow \{$non-trivial irreducible rep.$\}$



$\longleftrightarrow$





cf. Origami

$\rightsquigarrow$ Derived McKay corresp. (Kapranov et. al)

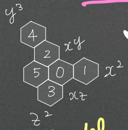
$\to$ 3-dim McKay correspondence

- Construction of crepant resolution (I, Markushevich, Roan)
- McKay corresp. $\begin{cases} \text{I.} - \text{Reid} \\ \text{I.} - \text{Nakajima} \end{cases}$

$\rightsquigarrow$ Derived McKay correspondence

[Bondal et. al] $\to$ Bridgeland-King-Reid

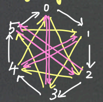
cf. Toda.



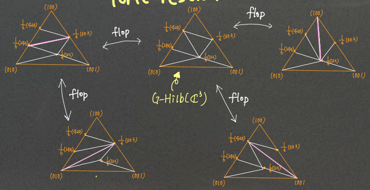
Dimer model

$\rightsquigarrow$ Mirror Symmetry Milanov, Abe

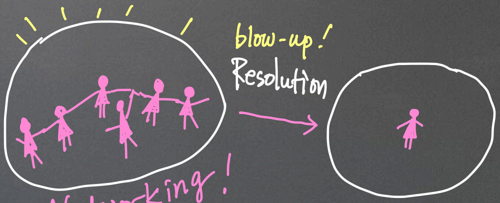
McKay quiver



toric resolution



10. Resolution of Singularity for Women in STEM



★ AOWM
(Asian-Oceanian Women in Math)
EC → President (Aug. 2025)

★ AASSA WISE committee.
(Women In Science & Eng.)
WISE Symposium on DEI in STEM

March 2024 @ Manila

TRANSACTIONSNASTPHL

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Dear Murayama-san,

Thank you and

Happy 60th Birthday!