Shadow Matter

For Hitoshi Fest

A birthday party game for Hitoshi

In 2016 I was interviewing for a faculty position at IPMU

I asked each interviewer (15 faculty) the same question

Match the faculty member to their response

Q: what would happen if Hitoshi stepped down as IPMU director tomorrow?

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Suitably modified to c. 2016

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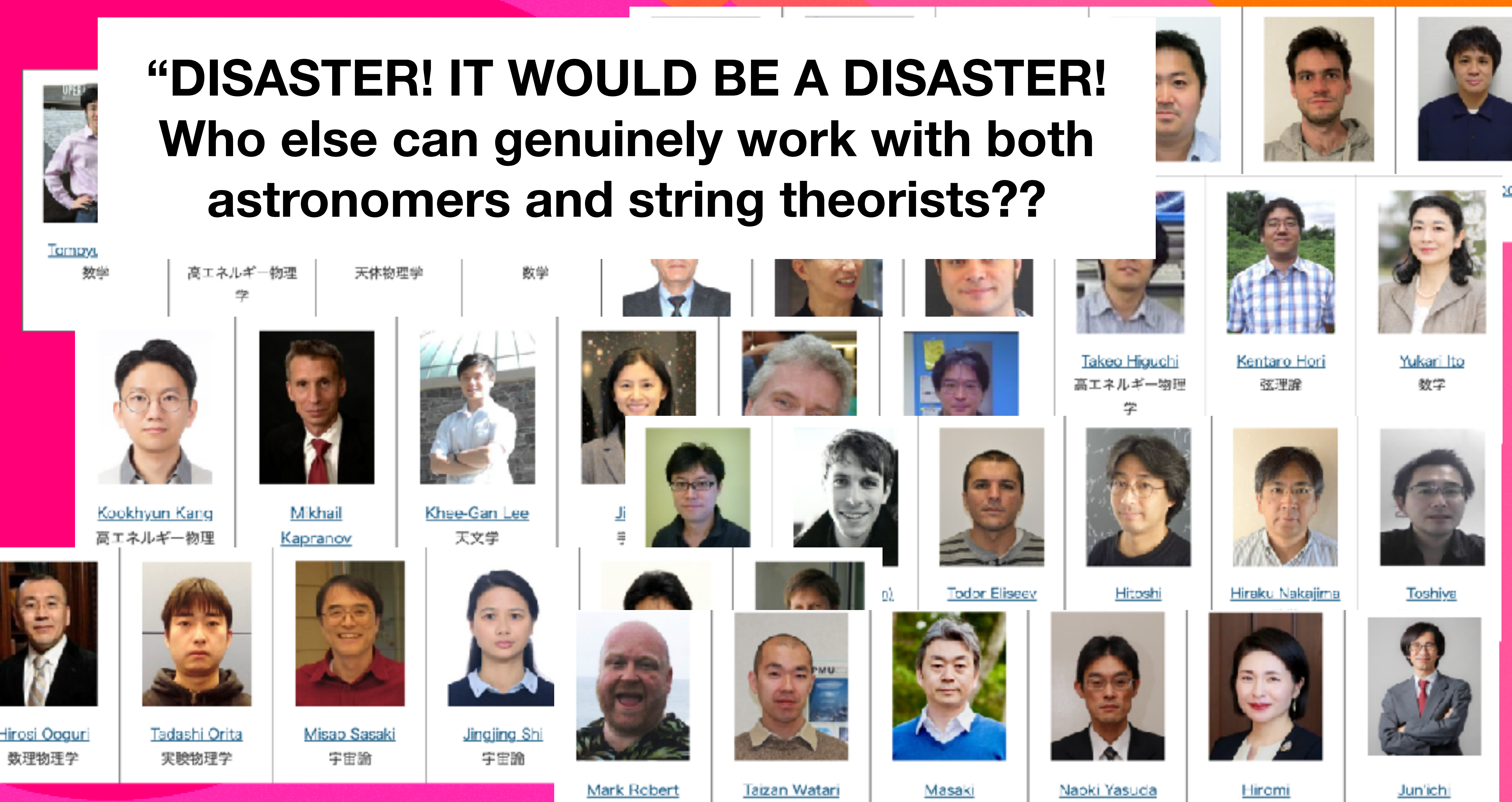

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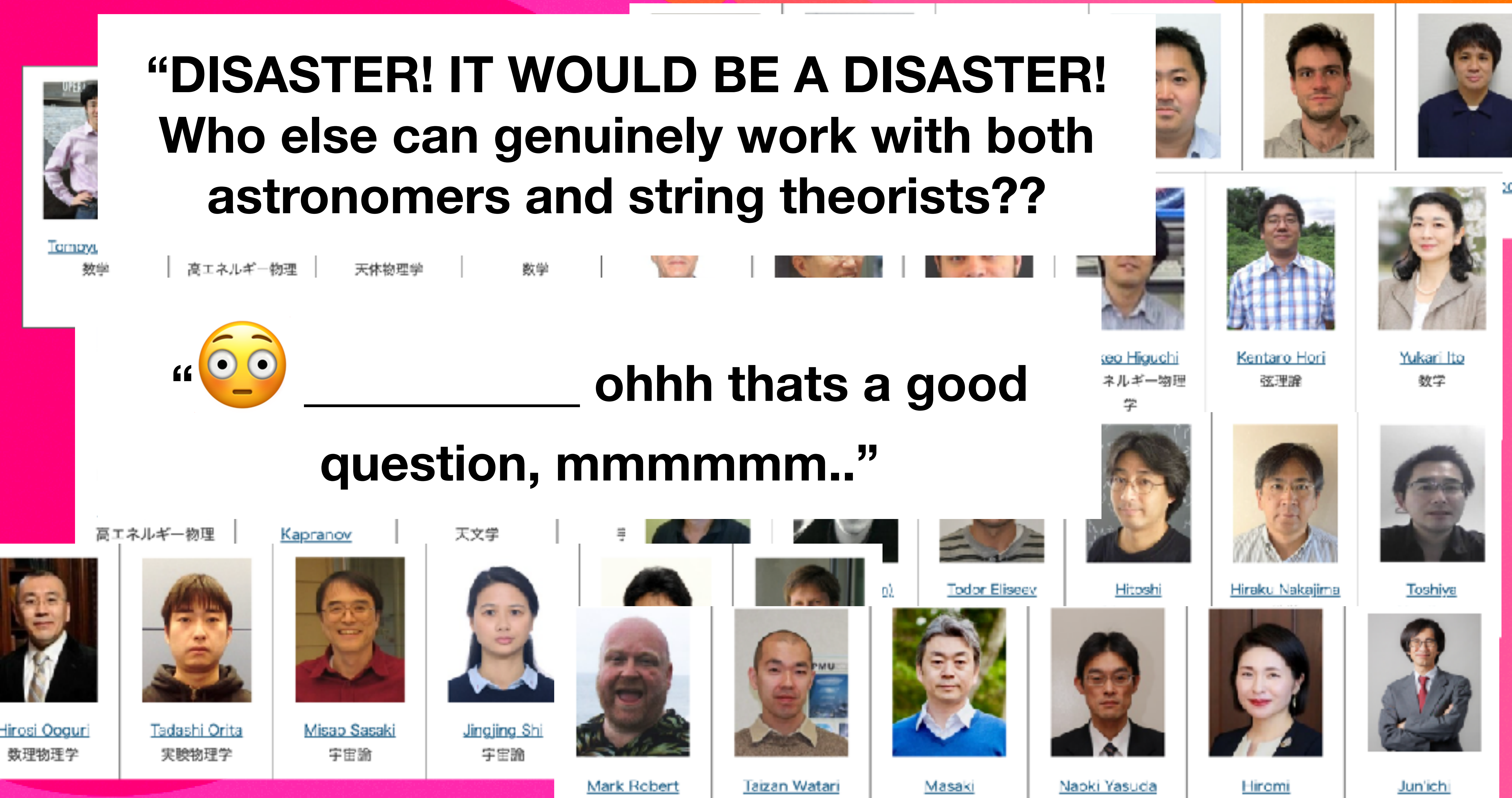

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"He isn't going to leave."

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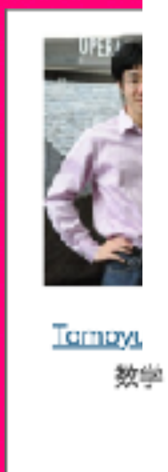

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End of the world

"😳 ___________ ohhh thats a good question, mmmmmm.."

Avoidance

"He isn't going to leave."

Denial

...

Q: what would happen if Hitoshi stepped down as IPMU director tomorrow?

Like any good theorist making a decision in the face of data…

End of the world + # Avoidance + # Denial

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$$\frac{\#\,\text{End of the world} + \#\,\text{Avoidance} + \#\,\text{Denial}}{3}$$

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$$\sqrt{\Delta_{\text{End of the world}}^2 + \Delta_{\text{Avoidance}}^2 + \Delta_{\text{Denial}}^2}$$

Q: what would happen if Hitoshi stepped down as IPMU director tomorrow?

Like any good theorist making a decision in the face of data…

=

IPMU - its science, its culture, its vision - must very deeply reflect those elements of Hitoshi himself

And that must make it a great place to be

Q: what would happen if Hitoshi stepped down as IPMU director tomorrow?

Like any good theorist making a decision in the face of data…

That is exactly what it has been

Thank you, Hitoshi, on your 60th year

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Thank you, Hitoshi, on your 60th year

(Lesson: always just add errors in quadrature)

Cosmological Consequences of Unconstrained Gravity and Electromagnetism

Or: “Shadow Matter”

Tom Melia Kavli IPMU

Based on ***2405.06374*** with Loris Del Grosso, David E Kaplan, TM, Surjeet Rajendran, Vivian Poulin, Tristan L Smith

2305.01798, ***2307.09475*** Kaplan, TM, Rajendran

And ***2204.03043*** Anne-Katherine Burns, Kaplan, TM, Rajendran

Classical physics asserts

Maxwell $$\partial_\mu F^{\mu\nu} = J^\nu$$

Einstein $$G^{\mu\nu} = T^{\mu\nu}$$

Quantum physics described by Schrodinger equation

$$i\frac{\partial}{\partial t}|\psi\rangle = H|\psi\rangle$$

From this equation classical physics follows…

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From this equation classical physics follows…

$$\partial_t\langle\hat{X}\rangle = i\left\langle\left[\hat{H},\hat{X}\right]\right\rangle = \left\langle\frac{\partial\hat{H}}{\partial\hat{P}}\right\rangle$$

$$\partial_t\langle\hat{P}\rangle = i\left\langle\left[\hat{H},\hat{P}\right]\right\rangle = -\left\langle\frac{\partial\hat{H}}{\partial\hat{X}}\right\rangle$$

..in expectation value

A subtlety for gauge theories

Fewer d.o.f. than fields A^μ $g^{\mu\nu}$

1. **Fewer 2nd order equations for evolution**

2. **Additional constraint equations on the dof**

Compare

$$\partial_\mu F^{\mu\nu} = J^\nu$$

$$G^{\mu\nu} = T^{\mu\nu}$$

Both dynamics and Constraints:

$$\partial_\mu F^{\mu 0} = J^0$$

$$G^{0\mu} = T^{0\mu}$$

$$i\frac{\partial}{\partial t}|\psi\rangle = H_{EM}|\psi\rangle$$

$$i\frac{\partial}{\partial t}|\psi\rangle = H_{GR}|\psi\rangle$$

Dynamics only

& Initial conditions

$$|\psi(0)\rangle$$

Compare

$$\partial_\mu F^{\mu\nu} = J^\nu$$

$$G^{\mu\nu} = T^{\mu\nu}$$

Both dynamics and Constraints:

Gauss' law $\nabla \cdot \mathbf{E} = \rho_{ch}$

e.g. First Friedmann $\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho$

$$i\frac{\partial}{\partial t}|\psi\rangle = H_{EM}|\psi\rangle$$

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$$|\psi(0)\rangle$$

Why should they be just so?

Exploring a choice

$$i\frac{\partial}{\partial t}|\psi\rangle = H_{EM}|\psi\rangle$$

$$i\frac{\partial}{\partial t}|\psi\rangle = H_{GR}|\psi\rangle$$

Once we write down a Hamiltonian, we can evolve what we like

$$|\psi(0)\rangle$$

State of lowest energy (highest symmetry) reproduces conventional classical limit

Not the only choice

Electromagnetism

In Weyl gauge $A_0 \ = \ 0 \qquad \Pi_j = \frac{\partial \mathcal{L}_{\mathcal{EM}}}{\partial A_j} = -E_j$

Comm. $$\left[\hat{A}_j\left(\mathbf{x}\right), \hat{E}_{j'}\left(\mathbf{x}'\right)\right] = -i\,\delta\left(\mathbf{x}-\mathbf{x}'\right)\delta_{jj'}$$

Ham. $$\hat{H}_W = \int d^3\mathbf{x}\left(\frac{1}{2}\left(\hat{\vec{E}}\cdot\hat{\vec{E}}+\hat{\vec{B}}\cdot\hat{\vec{B}}\right)+\hat{\vec{J}}\cdot\hat{\vec{A}}+\hat{H}_J\right)$$

SE $$i\frac{\partial|\Psi\rangle}{\partial t} = \hat{H}_W|\Psi\rangle$$

All the above produces Ampere's law in expectation (dynamics)

Gauss' law needs a supplement:

$$\left(\vec{\nabla}\cdot\hat{\vec{E}}-\hat{J}_0\right)|\Psi_{EM}\rangle = \ 0$$

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Gauss' law operator commutes with H

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Electromagnetism

Instead consider

$$\left(\vec{\nabla} \cdot \hat{\vec{E}} - \hat{J}_0\right) |\Psi_{EM}\rangle = J_0^d(\mathbf{x}) |\Psi_{EM}\rangle$$

- **Looks like a fixed background charge, but…**
- **No additional microphysics**
- ***Simply a choice of EM quantum state***

Gravity: Minisuperspace

$$g_{\mu\nu} \to ds^2 = -N(t)^2 dt^2 + a(t)^2 \left(dx^2 + dy^2 + dz^2\right)$$

$$S_{ms} = \int dt \sqrt{-g}(M_{pl}^2 R + \mathcal{L}_\phi)$$

For the quantum theory

$$\pi_N = \frac{\delta \mathcal{L}}{\delta \dot{N}} = 0 \qquad \pi_a = \frac{\delta \mathcal{L}}{\delta \dot{a}} \qquad \pi_\phi = \frac{\delta \mathcal{L}}{\delta \dot{\phi}}$$

Hamiltonian

$$H = \left[\pi \dot{a} + \pi_\phi \dot{\phi} - \mathcal{L}\right]_{\dot{a}=\cdots,\dot{\phi}=\cdots} = -N \frac{\delta S_{ms}}{\delta N} \equiv N\tilde{H}$$

$$\frac{\delta S_{ms}}{\delta N} = -\frac{\pi_a^2}{24 M_{pl}^2 a} + \frac{\pi_\phi^2}{2a^3} + a^3 V(\phi)$$

This is the 1st Friedmann equation

Gravity: Minisuperspace

Schrodinger equation

$$i\partial_t|\psi\rangle = N(t)\hat{\tilde{\hat{H}}}|\psi\rangle \quad \textbf{Or} \quad i\frac{1}{N(t)}\frac{\partial}{\partial t}|\psi\rangle = \hat{\tilde{\hat{H}}}|\psi\rangle$$

Just choose N = choice of time coordinate (=1)

Gravity: Minisuperspace

Schrodinger equation

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Just choose N = choice of time coordinate (=1)

Only 2nd Friedmann eqns in gravitational sector follow (dynamics!)

$$\partial_t\langle\hat{a}\rangle = -i\langle\left[\hat{\tilde{H}},\hat{a}\right]\rangle$$

$$\partial_t\langle\hat{\pi}\rangle = -i\langle\left[\hat{\tilde{H}},\hat{\pi}\right]\rangle$$

Gravity: Minisuperspace

First Friedmann $\langle \hat{\tilde{H}} \rangle = 0$ **not a consequence of the quantum dynamics**

QM only guarantees

$$\partial_t \langle \hat{\tilde{H}} \rangle = i \langle \left[\hat{\tilde{H}}, \hat{\tilde{H}} \right] \rangle = 0$$

i.e. $$\langle \hat{\tilde{H}} \rangle = \mathbb{H}_0$$

Time evolution as in conventional QM systems

c.f. Wheeler-DeWitt

$$\hat{H} |\psi\rangle = 0$$

Gravity: Minisuperspace

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$$a^3 \left(6 M_{pl}^2 \frac{\dot{a}^2}{a^2} - \frac{\dot{\phi}^2}{2} - V(\phi) \right) = \mathbb{H}_0$$

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$$6M_{pl}^2 \frac{\dot{a}^2}{a^2} - \frac{\dot{\phi}^2}{2} - V(\phi) = \frac{\mathbb{H}_0}{a^3}$$

Could **be zero, but generally contributes like matter to expansion**

General(er) relativity

$$G^{\mu\nu} = T^{\mu\nu} + T^{\mu\nu}_{aux}$$

Constraint equations

$$T^{\mu\nu}_{\mathrm{aux}} = \frac{1}{\sqrt{-g}} \begin{pmatrix} \mathbb{H} & \mathbb{P}^1 & \mathbb{P}^2 & \mathbb{P}^3 \\ \mathbb{P}^1 & 0 & 0 & 0 \\ \mathbb{P}^2 & 0 & 0 & 0 \\ \mathbb{P}^3 & 0 & 0 & 0 \end{pmatrix}$$

Dynamical equations

"Shadow matter"

General(er) relativity

$$G^{\mu\nu} = T^{\mu\nu} + T^{\mu\nu}_{aux}$$

$$T^{\mu\nu}_{\rm aux} = \frac{1}{\sqrt{-g}} \begin{pmatrix} \mathbb{H} & \mathbb{P}^1 & \mathbb{P}^2 & \mathbb{P}^3 \\ \mathbb{P}^1 & 0 & 0 & 0 \\ \mathbb{P}^2 & 0 & 0 & 0 \\ \mathbb{P}^3 & 0 & 0 & 0 \end{pmatrix}$$

Bianchi identity implies:

$$\partial_0 \mathbb{H} = -\partial_i \mathbb{P}^i$$

$$\partial_0 \left(g_{ij} \mathbb{P}^j \right) = 0.$$

Can ask what this auxiliary component looks like for certain choices of these functions

General(er) relativity

$$G^{\mu\nu} = T^{\mu\nu} + T^{\mu\nu}_{aux}$$

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Bianchi identity implies:

$$\partial_0 \mathbb{H} = -\partial_i \mathbb{P}^i$$

$$\partial_0 \left(g_{ij} \mathbb{P}^j \right) = 0.$$

Simplest choice $\mathbb{P}^i = 0$

$$T^{\mu\nu}_{\text{aux}} = \rho_{\text{aux}} u^\mu u^\nu$$

(In this gauge)

$$u^\mu = \{1, 0, 0, 0\}$$

$$\rho_{\text{aux}} = \mathbb{H}/\sqrt{-g}$$

Would clump, virialize, behave as exactly as DM

Structures may form down to very small scales, even below free-streaming length of conventional DM candidates e.g. WIMP

Early universe cosmology

The auxiliary shadow matter is *not* dynamical
- relic structure

Period of inflation would dilute it and it could not be the dark matter

Evolution of shadow matter in alternative cosmologies is of interest

These typically need null energy condition to be violated

$$T^{\mu\nu}_{\rm aux} = \frac{1}{\sqrt{-g}} \begin{pmatrix} \mathbb{H} & \mathbb{P}^1 & \mathbb{P}^2 & \mathbb{P}^3 \\ \mathbb{P}^1 & 0 & 0 & 0 \\ \mathbb{P}^2 & 0 & 0 & 0 \\ \mathbb{P}^3 & 0 & 0 & 0 \end{pmatrix}$$

$\mathbb{H} < 0$ **Or** $\mathbb{H}^2 < \mathbb{P}^2$

e.g. [Maeda and Harada, Class Quan Grav 39 195002 (2022)]

(Very possible for this shadow fluid)

Conclusions

Relic structure is a feature of the gauge theories of EM and Gravity: should be experimentally constrained

Comes hand-in-hand with simple(st?) solution to 'the problem of time'

Presence in universe today not consistent with inflation. Conversely, if shadow matter is the DM, this would rule out inflation and require novel early cosmology

Signals include null energy violation (negative mass), or detection of relic structure from EM coupled to gravity - *"Shadow Charge"* - with a potentially much more visible phenomenology

Additional slides

Minisuperspace

$$g_{\mu\nu} \to ds^2 = -N(t)^2 \, dt^2 + a(t)^2 \left(dx^2 + dy^2 + dz^2\right)$$

00 component of metric, lapse function

$$S_{ms} = \int dt \sqrt{-g} \left(M_{pl}^2 R + \mathcal{L}_{matter}\right)$$

$$= \int dt \left(-6M_{pl}^2 \frac{a(t)\dot{a}(t)^2}{N(t)} + \frac{a(t)^3 \dot{\phi}(t)^2}{2N(t)} - N(t)a(t)^3 V(\phi) \right)$$

Minisuperspace

$$S_{ms} = \int dt \left(-6M_{pl}^2 \frac{a(t)\dot{a}(t)^2}{N(t)} + \frac{a(t)^3\dot{\phi}(t)^2}{2N(t)} - N(t)a(t)^3 V(\phi) \right)$$

The equations of motion naively follow

$$\begin{aligned}
\frac{\delta S_{ms}}{\delta N} &= a^3 \left(6M_{pl}^2 \frac{\dot{a}^2}{N^2 a^2} - \frac{\dot{\phi}^2}{2N^2} - V(\phi) \right) = 0 \\
\frac{\delta S_{ms}}{\delta a} &= 3Na^2 \left(4M_{pl}^2 \frac{\ddot{a}}{N^2 a} + 2M_{pl}^2 \frac{\dot{a}^2}{N^2 a^2} - 4M_{pl}^2 \frac{\dot{a}\dot{N}}{N^3 a} + \frac{\dot{\phi}^2}{2N^2} - V(\phi) \right) = 0 \\
\frac{\delta S_{ms}}{\delta \phi} &= -\frac{a^3}{N} \left(\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} - \frac{\dot{N}\dot{\phi}}{N} + N^2 \frac{\partial V(\phi)}{\partial \phi} \right) = 0
\end{aligned}$$

Minisuperspace

$$S_{ms} = \int dt \left(-6M_{pl}^2 \frac{a(t)\dot{a}(t)^2}{N(t)} + \frac{a(t)^3\dot{\phi}(t)^2}{2N(t)} - N(t)a(t)^3 V(\phi) \right)$$

The equations of motion naively follow

But this variation is suspect!

$$\frac{\delta S_{ms}}{\delta N} = a^3 \left(6M_{pl}^2 \frac{\dot{a}^2}{N^2 a^2} - \frac{\dot{\phi}^2}{2N^2} - V(\phi) \right) = 0$$

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If $N(t) \rightarrow N(t) + \delta N(t)$

Redefine $dt' = (1 + \frac{\delta N}{N})dt$

Then $S_{ms}[N + \delta N, a, \phi] = S_{ms}[N, a, \phi]$

Analysing this auxiliary fluid

$$T^{\mu\nu}_{\rm aux} = \frac{1}{\sqrt{-g}} \begin{pmatrix} \mathbb{H} & \mathbb{P}^1 & \mathbb{P}^2 & \mathbb{P}^3 \\ \mathbb{P}^1 & 0 & 0 & 0 \\ \mathbb{P}^2 & 0 & 0 & 0 \\ \mathbb{P}^3 & 0 & 0 & 0 \end{pmatrix}$$

Bianchi identity implies

$$\partial_0 \mathbb{H} = -\partial_i \mathbb{P}^i$$

$$\partial_0 \left(g_{ij} \mathbb{P}^j \right) = 0.$$

More general cosmological ansatz

$$\mathbb{H}(x) \equiv \mathbb{H} + \delta\mathbb{H}(x) \qquad \mathbb{P}^i(x) \equiv \delta\mathbb{P}^i(x)$$

Linear perturbations about homogeneous isotropic background $g_{ij} \sim a(t)^2 \delta_{ij}$

$$\delta\mathbb{P}^i / \sqrt{-g} \sim a^{-5}$$

This contribution redshifts very quickly even outside horizon

BUT: may be important for non-linear small scales

Early times would be a source of anisotropy

The case of 'shadow charge'

$$\nabla_\mu F^{\mu\nu} = (J^\nu + J^\nu_{\rm aux})$$

$$J^\mu_{\rm aux} = \rho^{ch}_{\rm aux} v^\mu \qquad \rho^{ch}_{\rm aux} = \mathbb{J}/\sqrt{-g}$$

$$\mathbb{J} = \mathbb{J}(\mathbf{x})$$

A charge density that follows geodesics, does not respond to electromagnetic forces $v^\mu \nabla_\mu v^\nu = 0$

Modes outside the horizon redshift like matter

Inside the horizon, rich pheno with complex dynamics