

SRF Cavity Search for Dark Photon



Jing Shu (Peking University, Beijing)

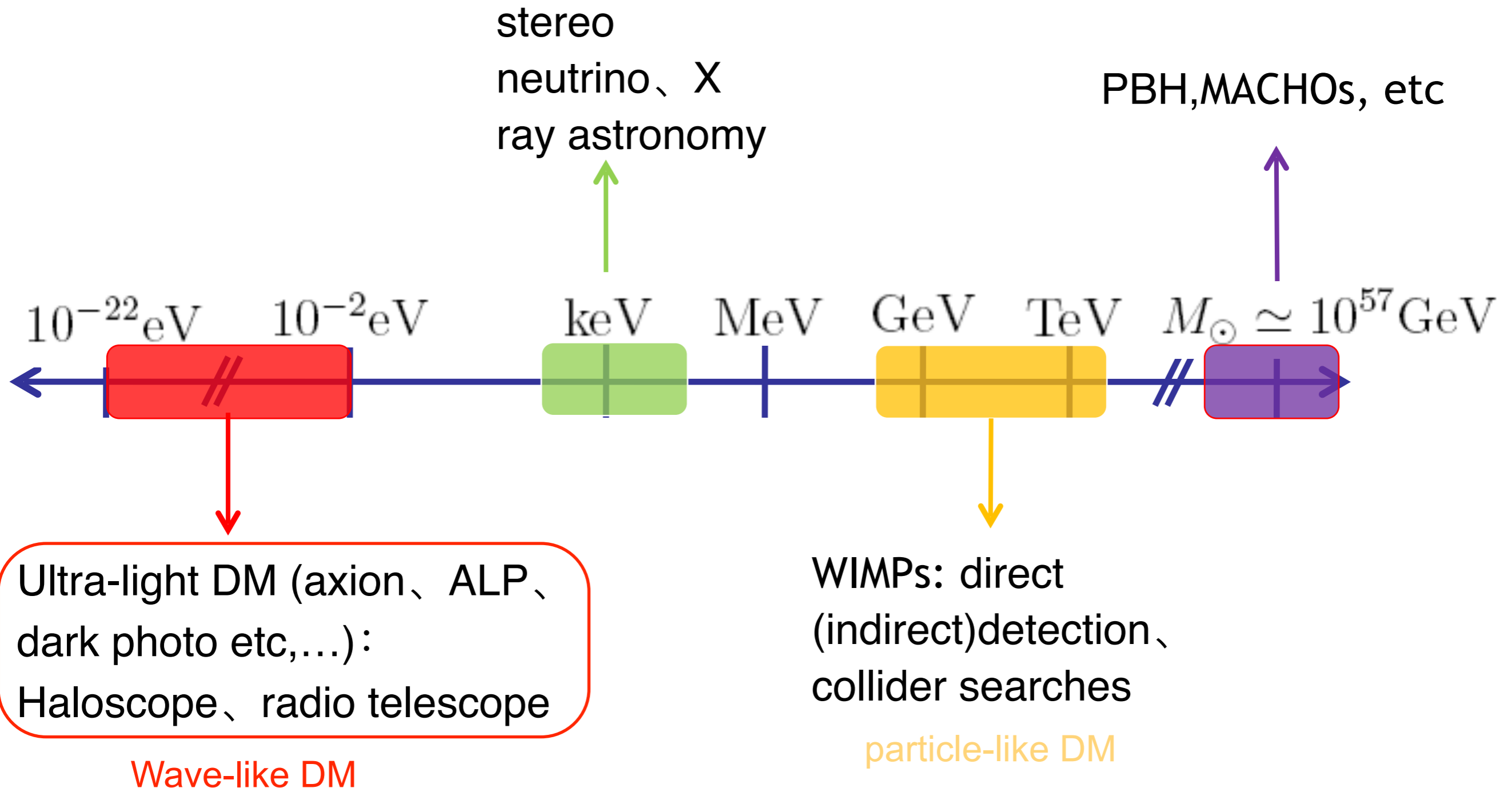
Outline

- Motivation of ultra-light dark matter search using Superconducting Radio Frequency (SRF) Cavity
- SRF Cavity Project for DPDM search
- SRF Cavity Project for cosmic DP
- Experimental group
- SRF with quantum quits
- Arrays to eliminate all the background noises
- Summary and Outlook

A decorative graphic on a blue background. It features a central white rounded rectangle containing the text. To the left, there is a large orange circle, a smaller white circle, and a green circle. To the right, there is a green circle and a large blue circle. All circles are connected to the central text box by thin white lines.

Motivation of ultra- light dark matter

Various DM candidate



There's a broad spectrum of possible particles with varied masses and interaction strengths, making experimental searches challenging.

The ultra-light DM

QM: All matter exhibits both particle and wave properties.



($m \sim 10^{-22}$ eV)

The de Broglie wavelength:
galactic scales(kpc)

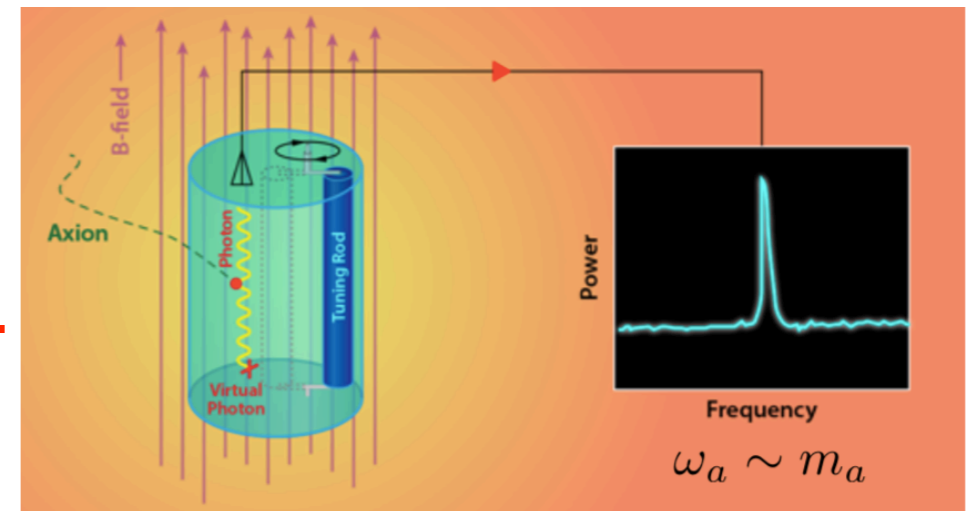
- Astronomical observation
(time, position, velocity,
polarization, etc)

Wavelengths at
macroscopic scales,
manifesting as a wave-
like background field

Distinct from traditional
dark matter detection
(particle scattering)

enormous potential for
development in this field

similar as the GWs detection



$$m_a \sim \text{GHz} \sim 10^{-6} \text{ eV}$$

Compton wave length (m)

Haloscope, Quantum
amplifier

New search methods!!!

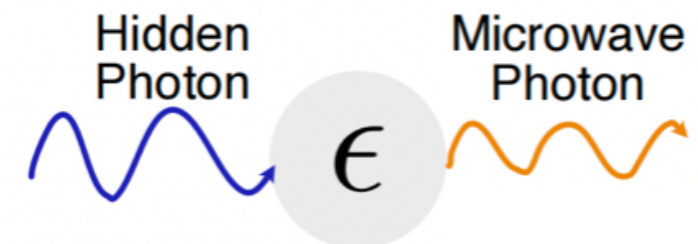
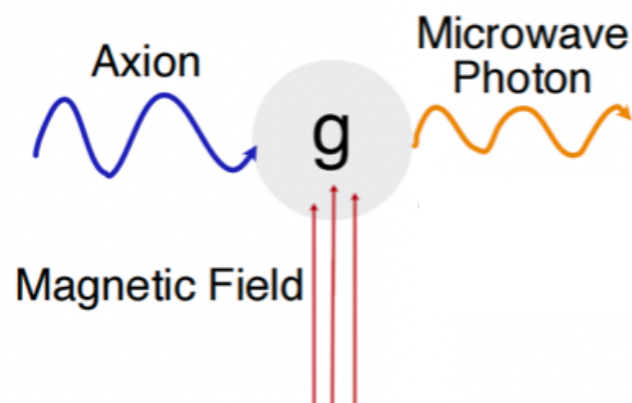
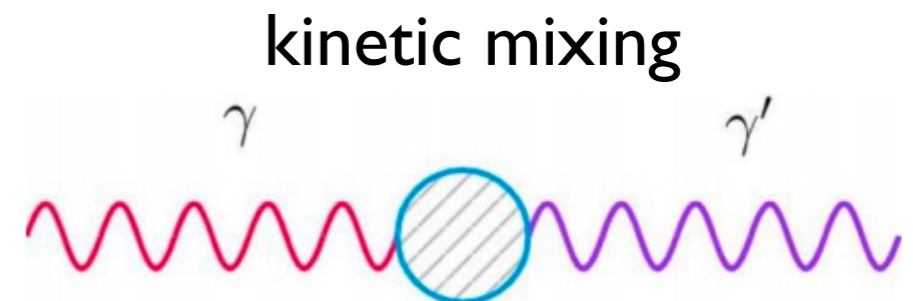
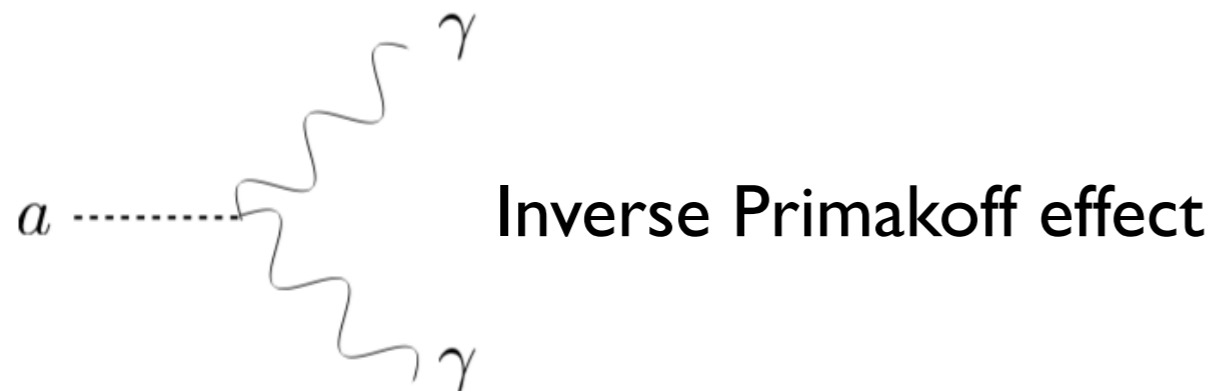
Quantum sensor

Ultra-light DM candidate

Axion (ALP): spin 0, CP odd

Dark photon: spin 1

mili-charge particles?



$$\nabla \times \mathbf{B} \simeq \partial_t \mathbf{E} + \mathbf{J} + \underline{g_{a\gamma\gamma} \mathbf{B} \partial_t a}$$

induces an effective current under strong **magnetic field**.

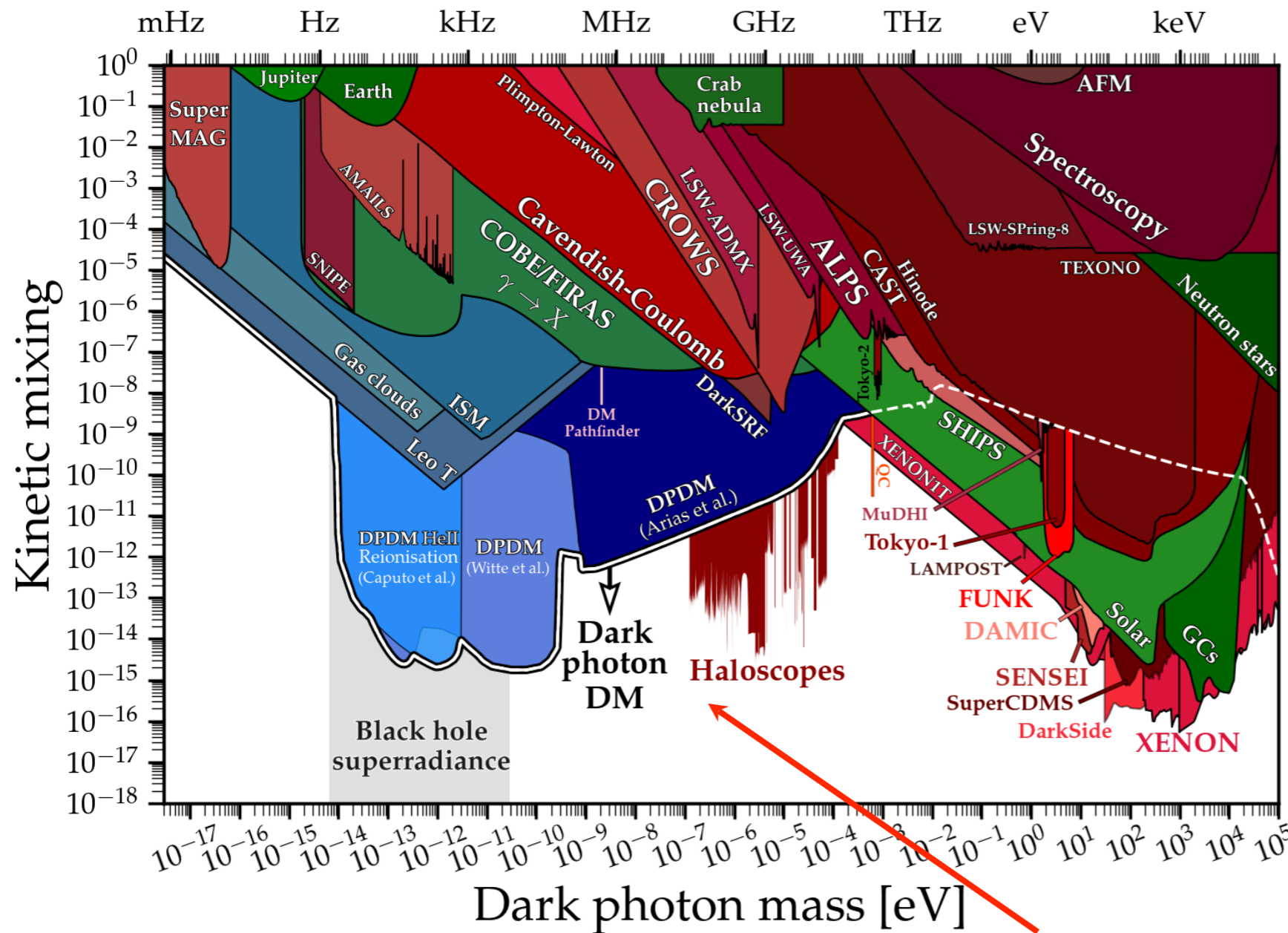
$$\vec{J}_{\text{eff}}^a = g_{a\gamma} \omega_a a \vec{B}_0.$$

$$\square \mathcal{L} \supset -\tilde{A}_\mu (eJ_{EM}^\mu - \epsilon m_{A'}^2 \tilde{A}'^\mu)$$

induces an effective current **anyway**.

$$J_{\text{eff}}^{A'\mu} = \epsilon m_{A'}^2 A'^\mu;$$

Current DPDM search



Haloscope sensitivity largely depends on Q :
 Superconducting cavity has $Q \sim 10^{10}$

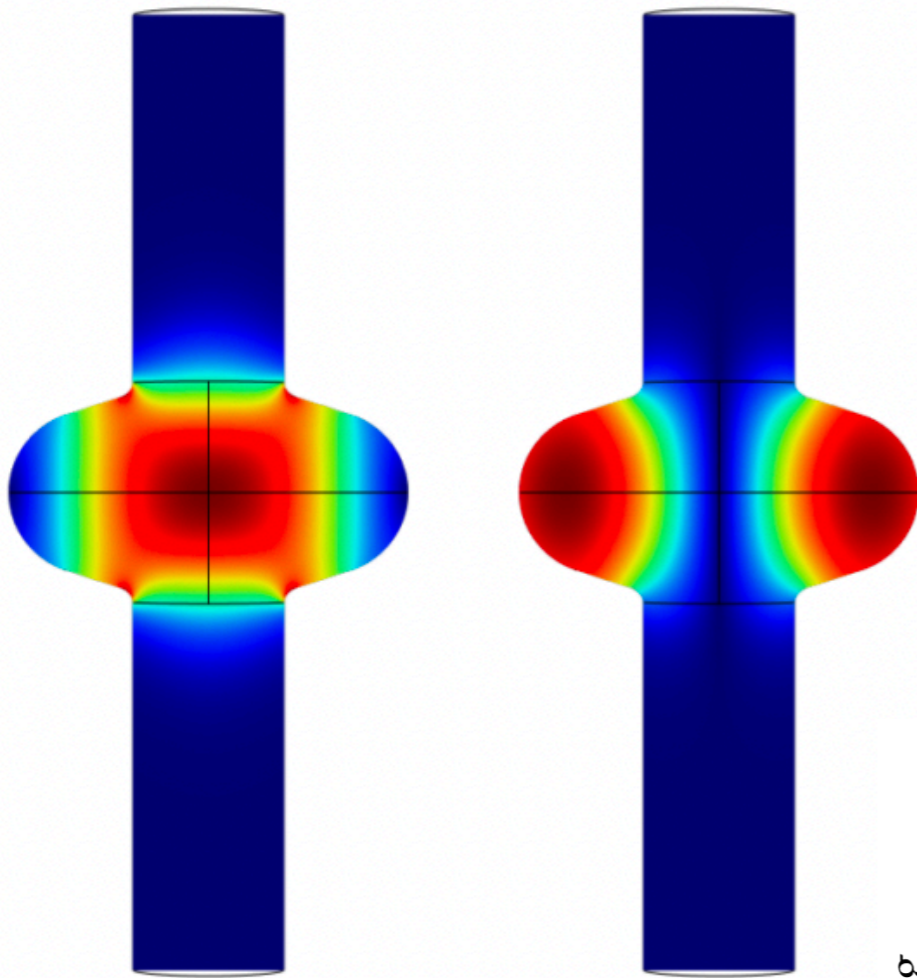


Still a lot of room to detect

how to make use it?
 5 orders more than traditional cavity.

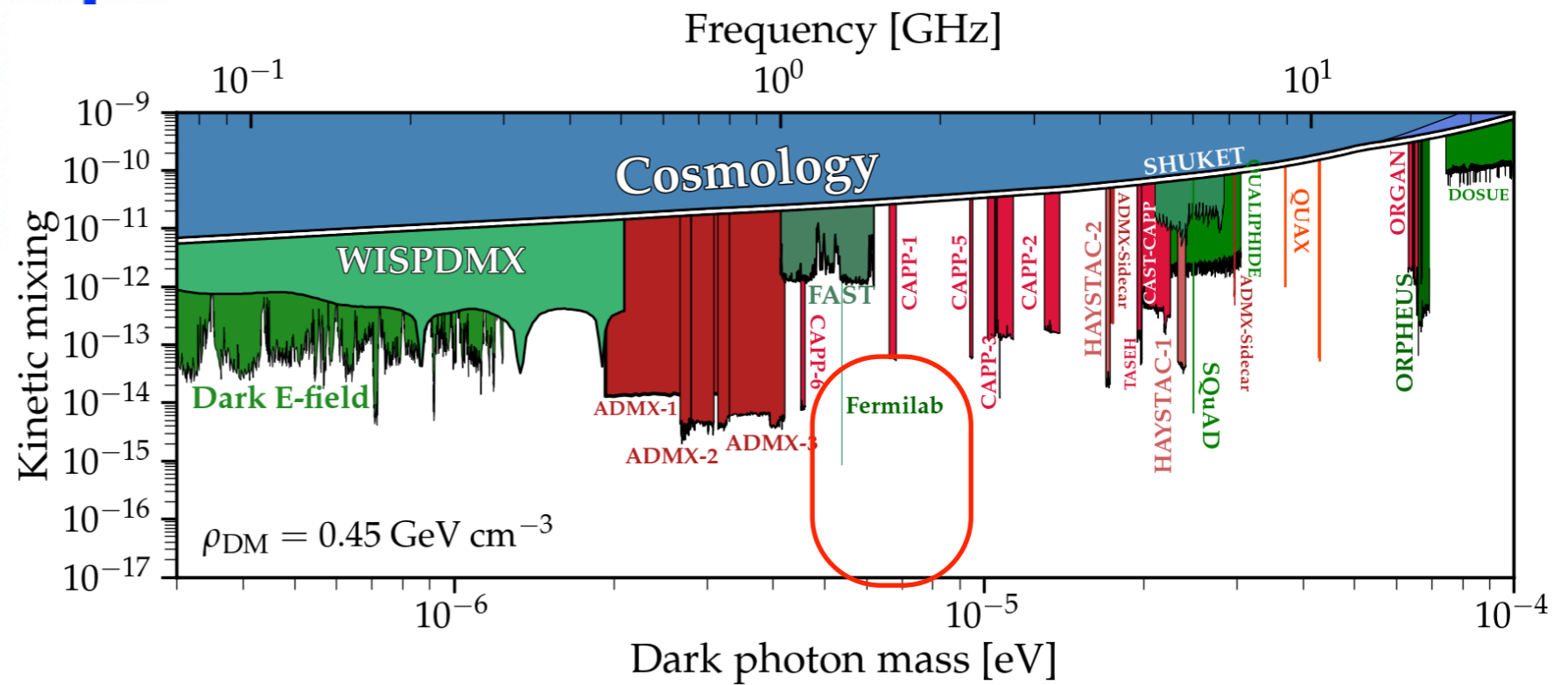
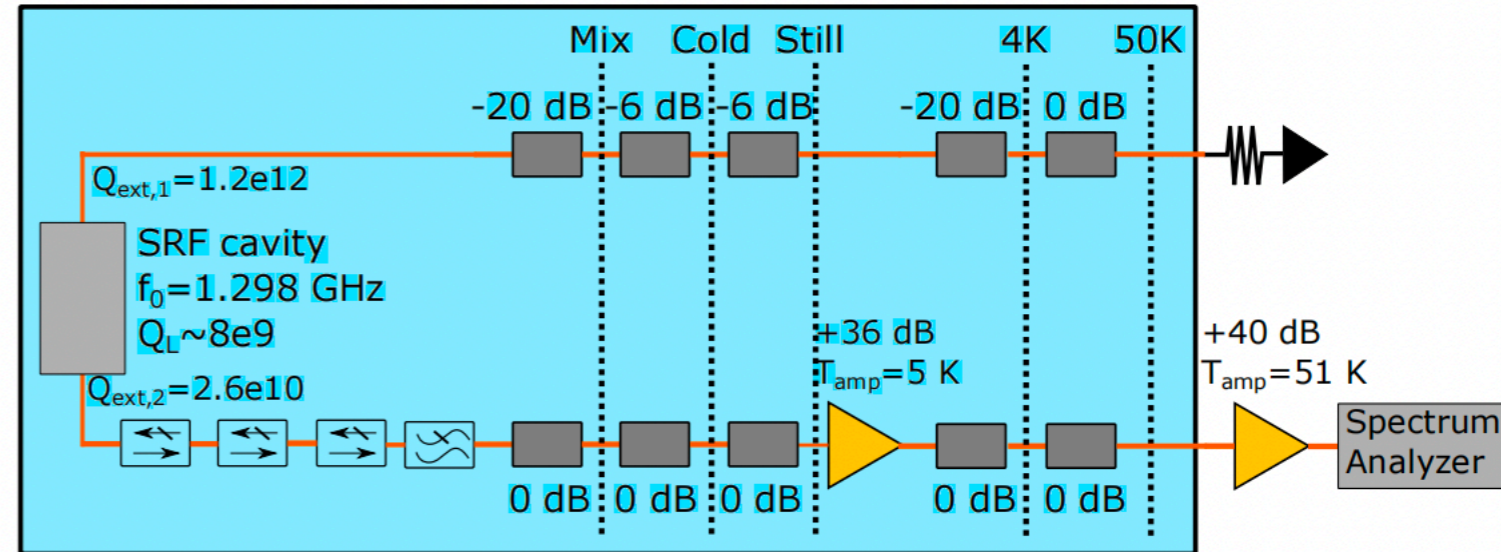
Axion limit webpage: <https://github.com/cajohare/AxionLimits/blob/master/docs/dp.md>

Result from Fermilab



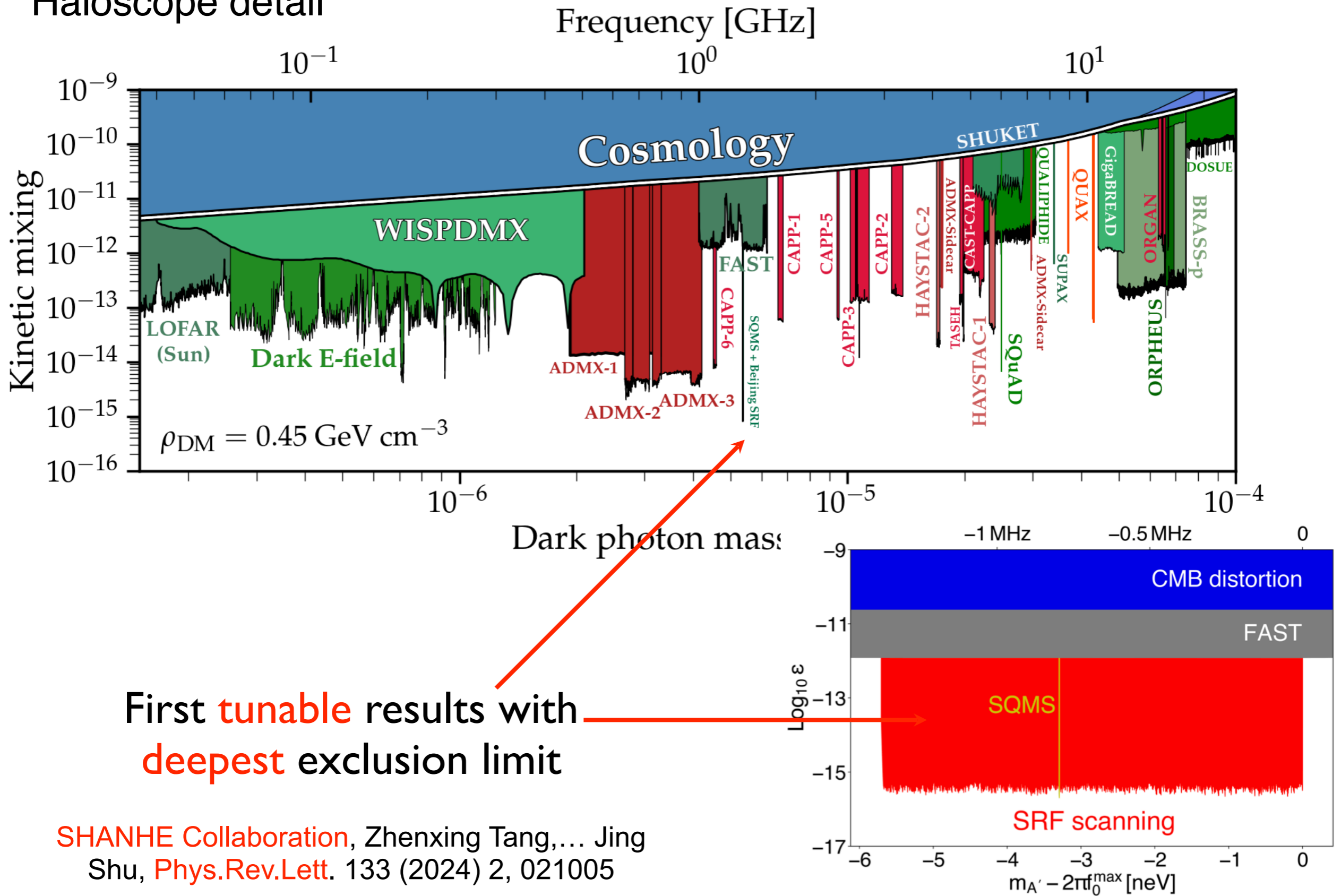
Deepest
constrain

$$Q \sim 4.7 \times 10^9$$



DPDM search

Haloscope detail



First **tunable** results with **deepest** exclusion limit

SHANHE Collaboration, Zhenxing Tang, ... Jing Shu, *Phys.Rev.Lett.* 133 (2024) 2, 021005

Spectrum of Ultra-light Dark Matter

The Virial Theorem: the velocity of dark matter near Earth is approximately 10^{-3} boosted by gravity.

$$a(t) = \frac{\sqrt{2\rho_{\text{DM}}}}{m_a} \cos(m_a t + \phi)$$

Frequency: $\omega_a \simeq \text{GHz} \frac{m_a}{10^{-6} \text{ eV}}$

Coherence: $\tau_a \simeq \text{ms} \frac{10^{-6} \text{ eV}}{m_a}$

Max Exp. Size: $\lambda_a \simeq 200 \text{ m} \frac{10^{-6} \text{ eV}}{m_a}$

Axion **DM** as an example, same for other kinds (DPDM, etc)

$$\tau_a \sim 1/m_a \langle v_{\text{DM}}^2 \rangle \sim Q_a/m_a \sim 10^6/m_a$$

Bandwidth of axion DM is 10^{-6}

**Detector bandwidth $< 10^{-6}$
accelerate the scan rate**

$$\lambda_a \sim 1/m_a \sqrt{\langle v_{\text{DM}}^2 \rangle} \sim 10^3/m_a$$

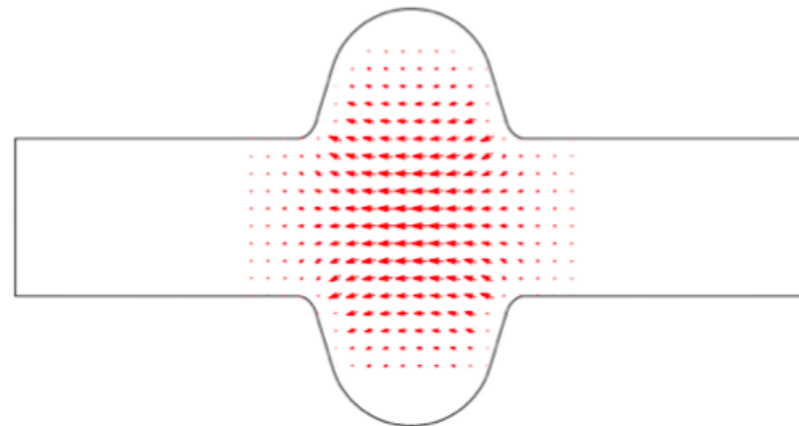
Momentum width 10^{-3}

A decorative graphic on a blue background. It features a central white rounded rectangle containing the text. To the left, there is a large orange circle and a smaller green circle, both connected to the white box by white lines. To the right, there is a green circle and a large blue circle, also connected to the white box by white lines. The text is in a blue, sans-serif font.

SRF Cavity Project for DPDM

SRF Cavity

- ▶ Significant $Q_0 > 10^{10}$ compared to copper cavity with $Q_0 \leq 10^6$.
- ▶ Superconducting Radio-Frequency (SRF) Cavities:
extremely high $Q_0 \approx 10^{10} \rightarrow$ improve SNR $\propto Q_0^{1/4}$
- ▶ I-cell elliptical niobium cavity with mechanical tuner, immersed in liquid helium at $T \sim 2$ K
- ▶ TM₀₁₀ mode: z-aligned \vec{E} , maximizes the overlap for dark photon dark matter (DPDM)



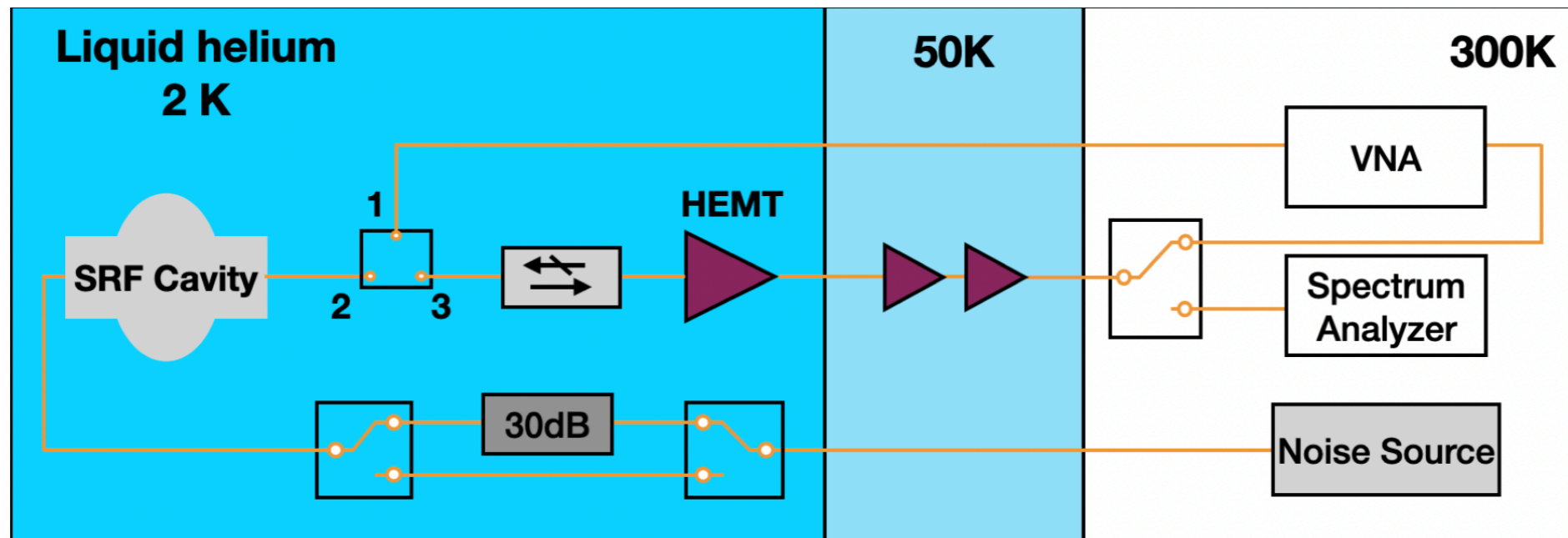
$$\epsilon \approx 10^{-16} \left(\frac{10^{10}}{Q_0} \right)^{\frac{1}{4}} \left(\frac{4 \text{ L}}{V} \right)^{\frac{1}{2}} \left(\frac{0.5}{C} \right)^{\frac{1}{2}} \left(\frac{100 \text{ s}}{t_{\text{int}}} \right)^{\frac{1}{4}} \left(\frac{1.3 \text{ GHz}}{f_0} \right)^{\frac{1}{4}} \left(\frac{T_{\text{amp}}}{3 \text{ K}} \right)^{\frac{1}{2}}$$

Experimental operation

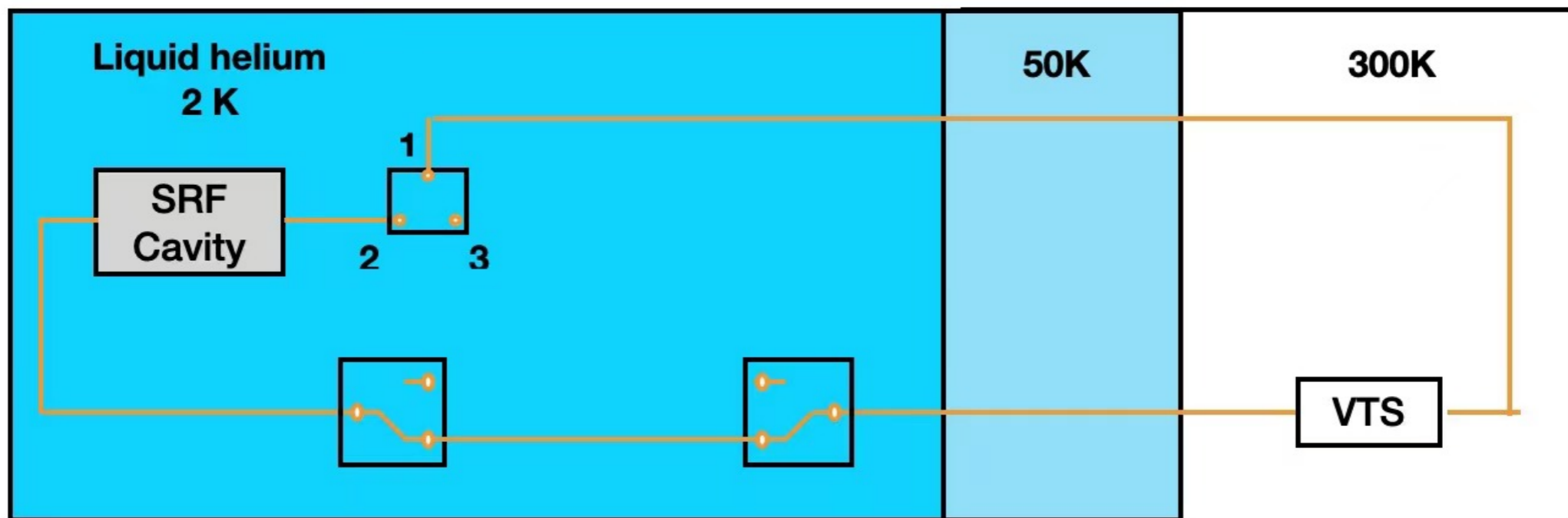
Parameters

	Value	Fractional Uncertainty
$V_{\text{eff}} \equiv V C/3$	693 mL	< 1%
β	0.634 ± 0.014	1.4%
G_{net}	(57.30 ± 0.14) dB	3.1%
Q_L	$(9.092 \pm 0.081) \times 10^9$	/
f_0^{max}	1.2991643795 GHz	/
Δf_0	11.5 Hz	/
t_{int}	100 s	/

microwave electronics for DPDM searches

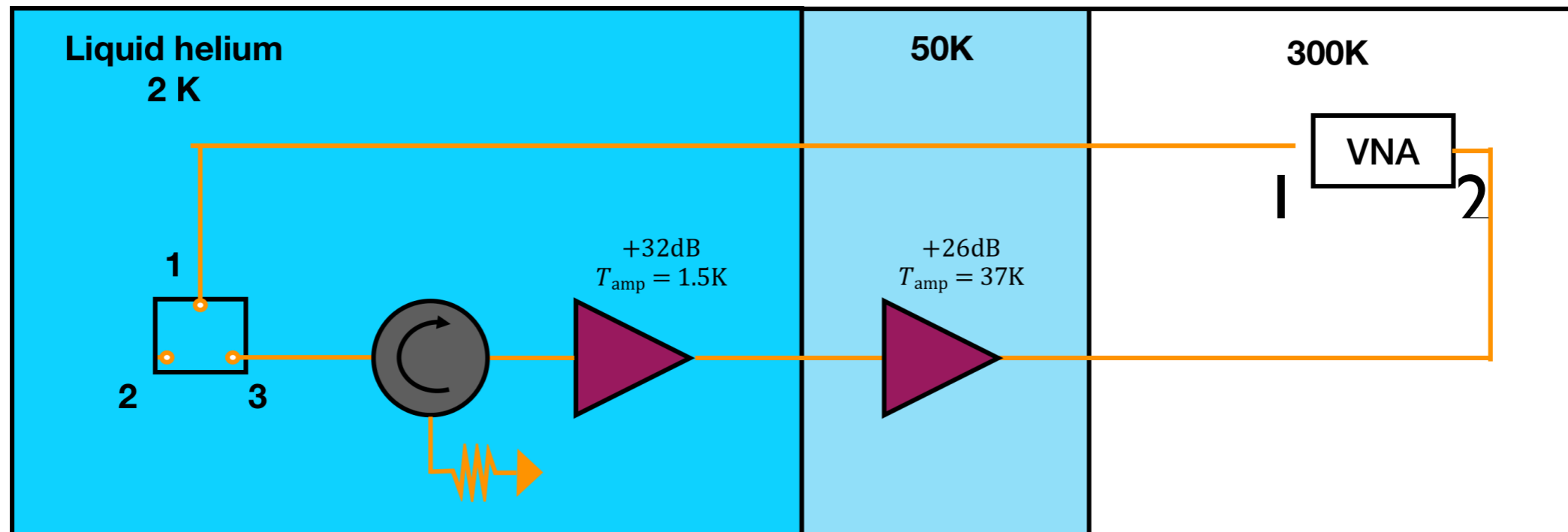


Step 1: Measure Cavity property



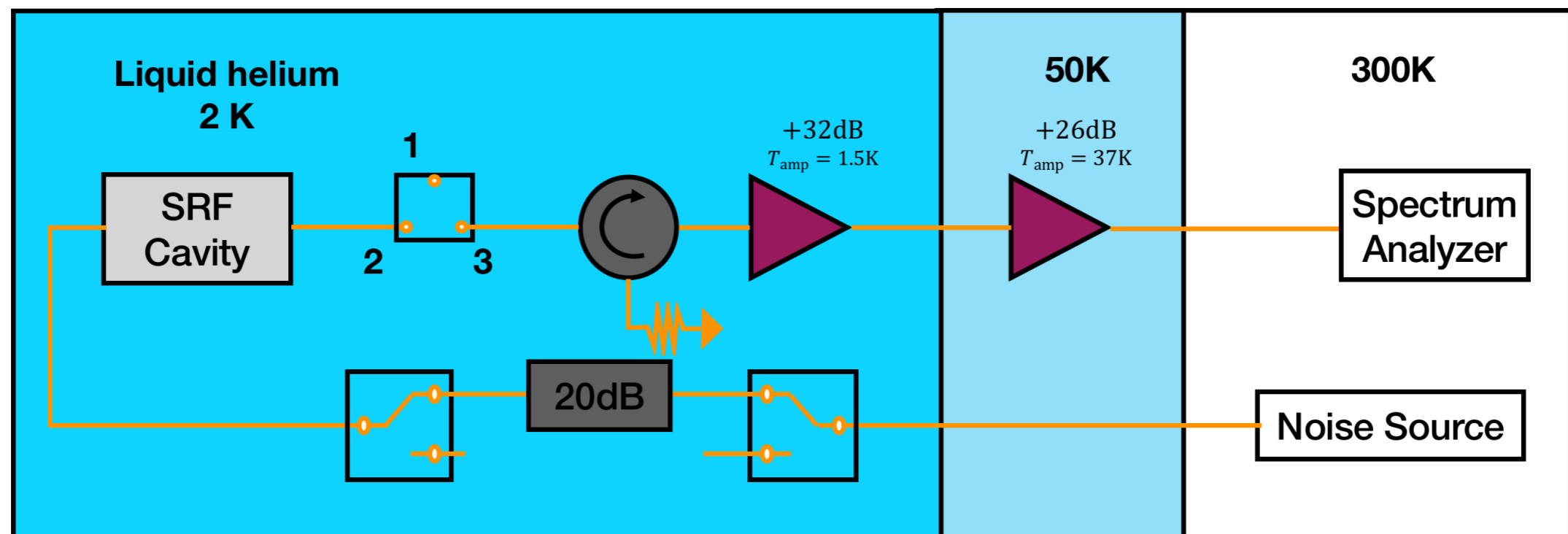
1-2 connection: VTS measurement for the cavity property.

Step 2: calibration



I-3 connection: calibration by subtracting the line loss to get the total gain G_{net} .

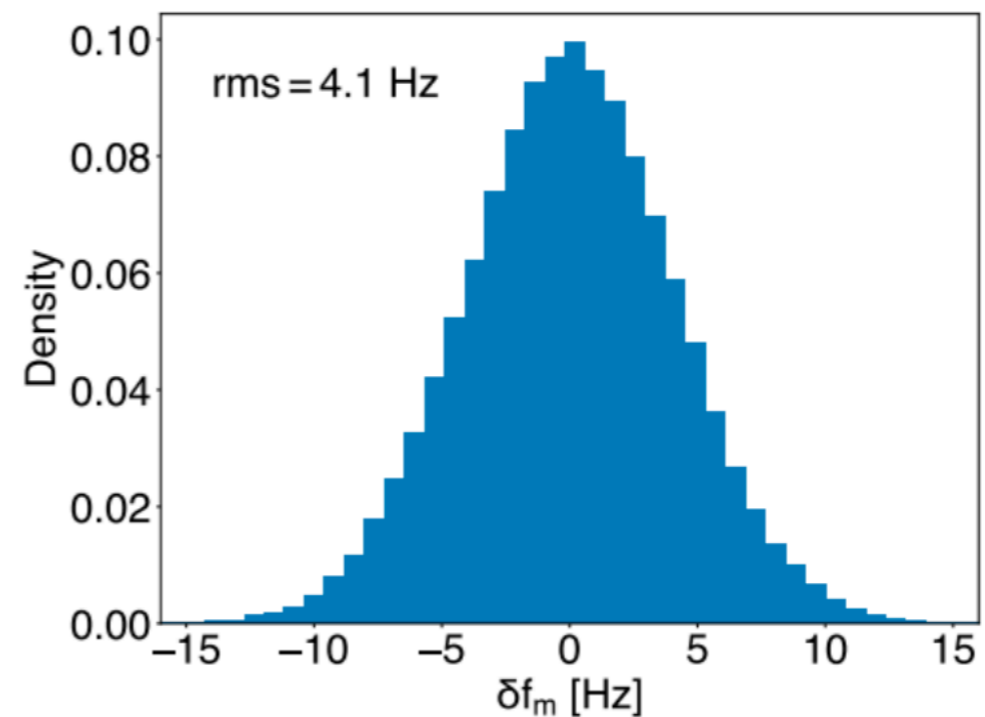
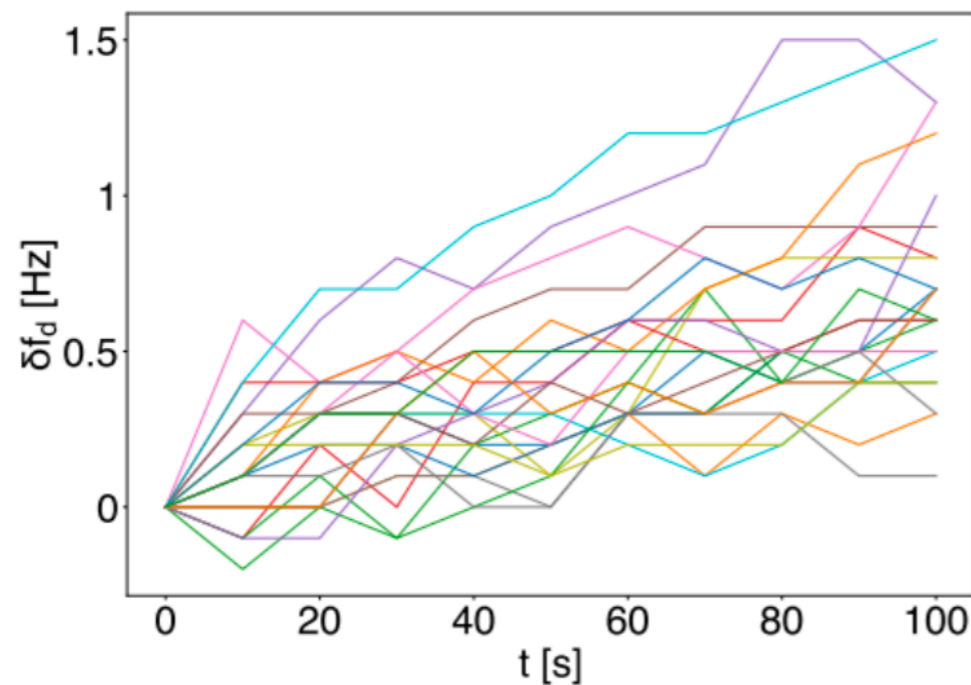
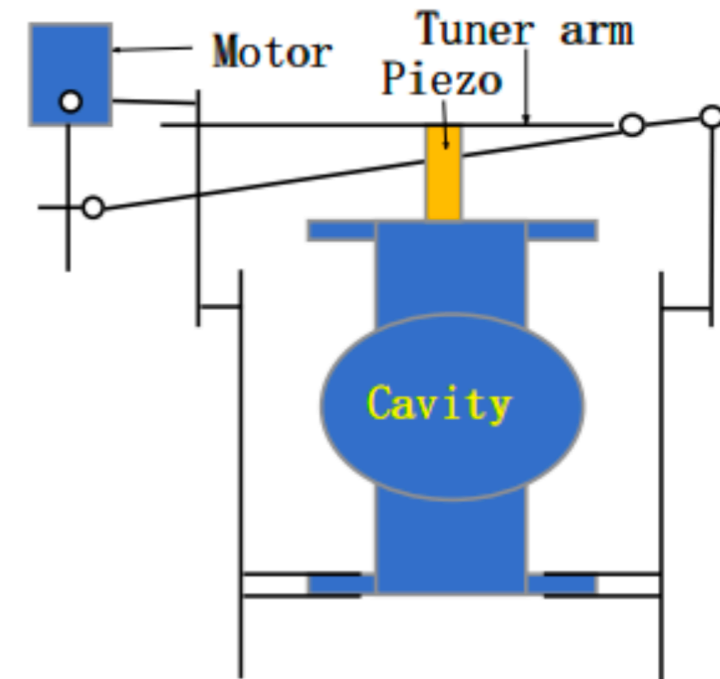
Step 3: Do experiment



2-3 connection: tune the cavity resonant frequency to do the experiment

Scan Search with Mechanical Tuning

- ▶ **Mechanical tuner** scans resonant frequency f_0 with the step $\sim f_0/Q_{DM}$
- ▶ Calibrate f_0 and its stability range Δf_0 **in each scan**
- ▶ Frequency drift $\delta f_d \leq 1.5\text{Hz}$ and microphonics effect $\sigma_{f_0} \approx 4\text{Hz}$

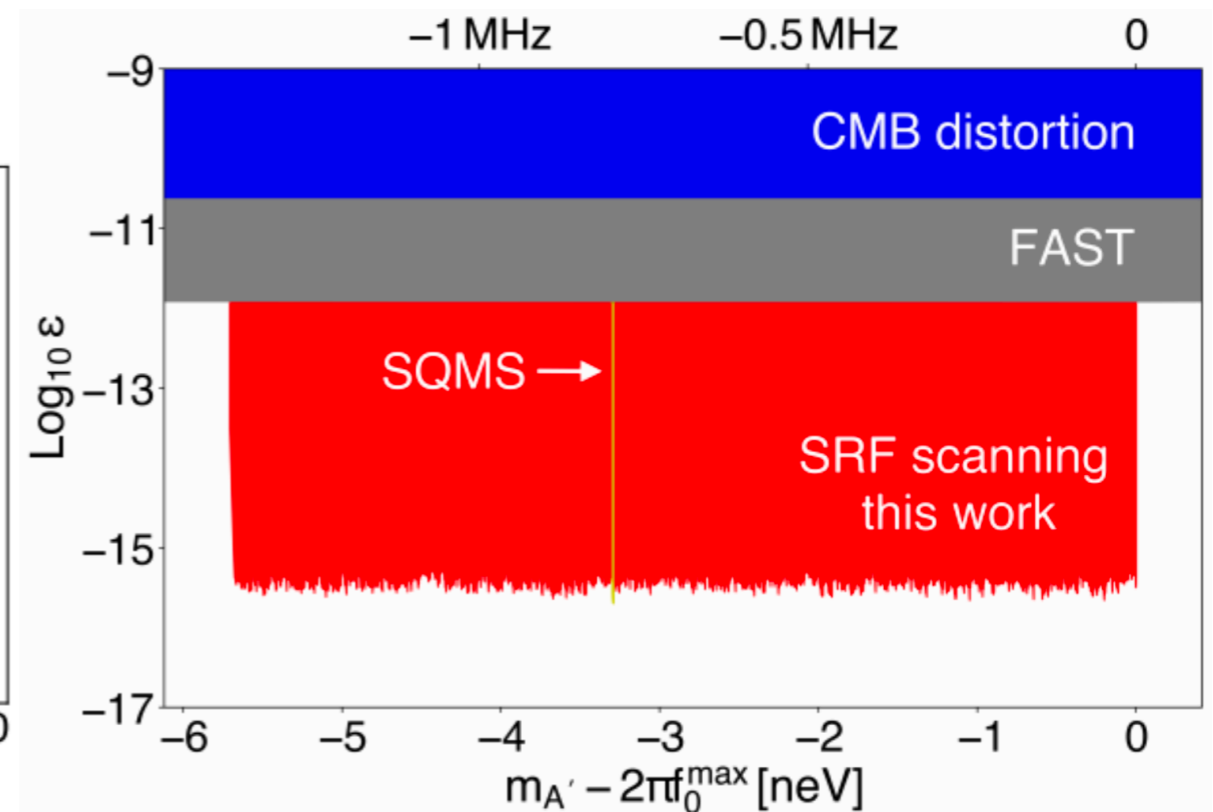
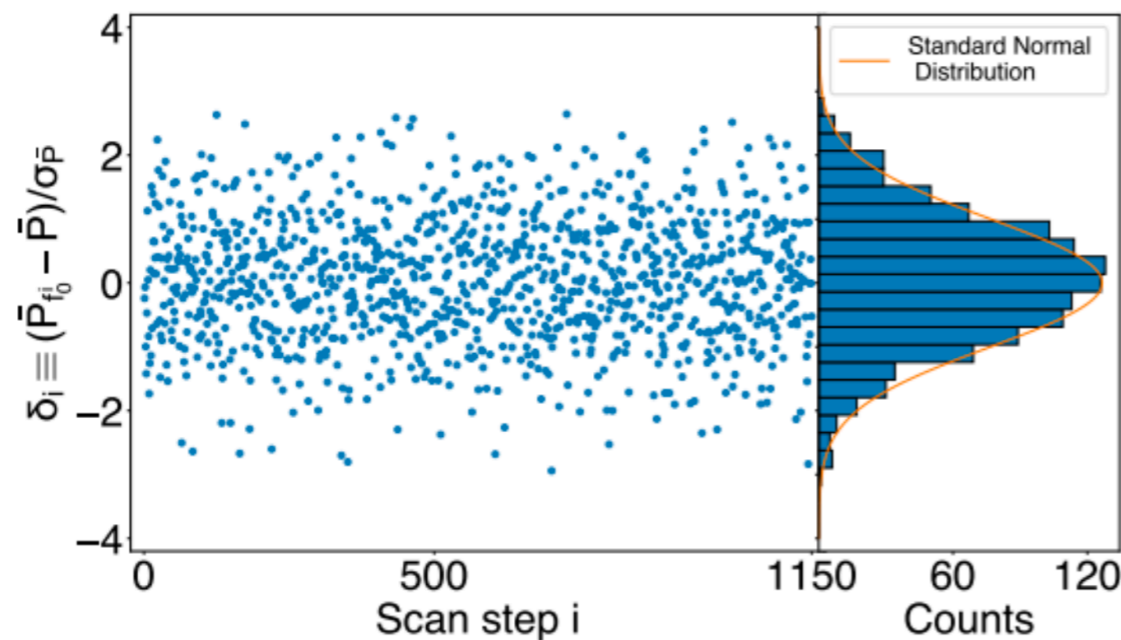


- ▶ **Conservatively** choose $\Delta f_0 \approx 10\text{Hz}$

Data analysis and constraints

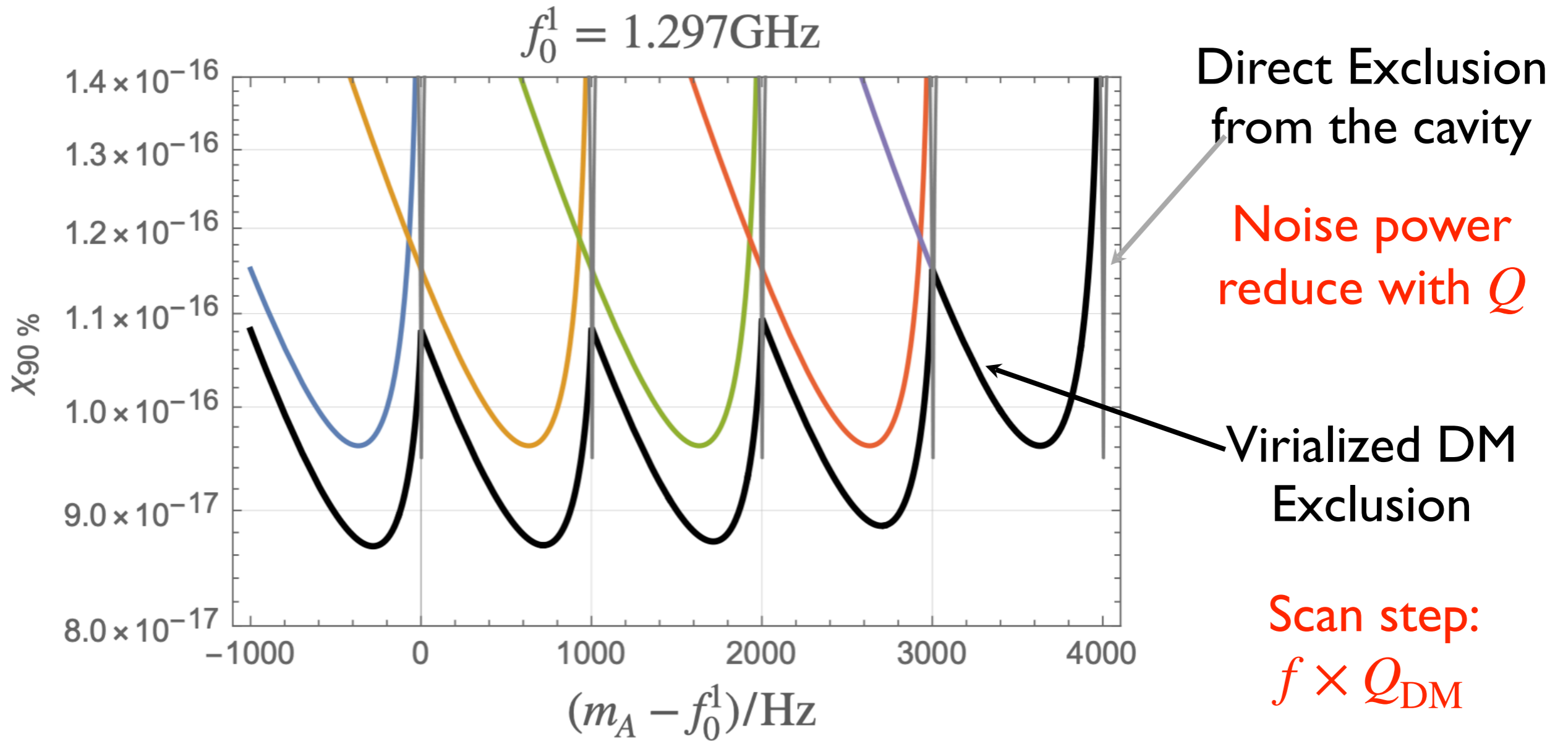
- ▶ Total 1150 scan steps with each 100 s integration time
- ▶ Group every 50 adjacent bins and perform a constant fit to address small helium pressure fluctuation.
- ▶ Normal power excess shows **Gaussian distribution**:

SHANHE Collaboration, Zhenxing Tang, ... Jing Shu, *Phys.Rev.Lett.* 133 (2024) 2, 021005



- ▶ First scan search with SRF and most stringent constraints in most exclusion space.

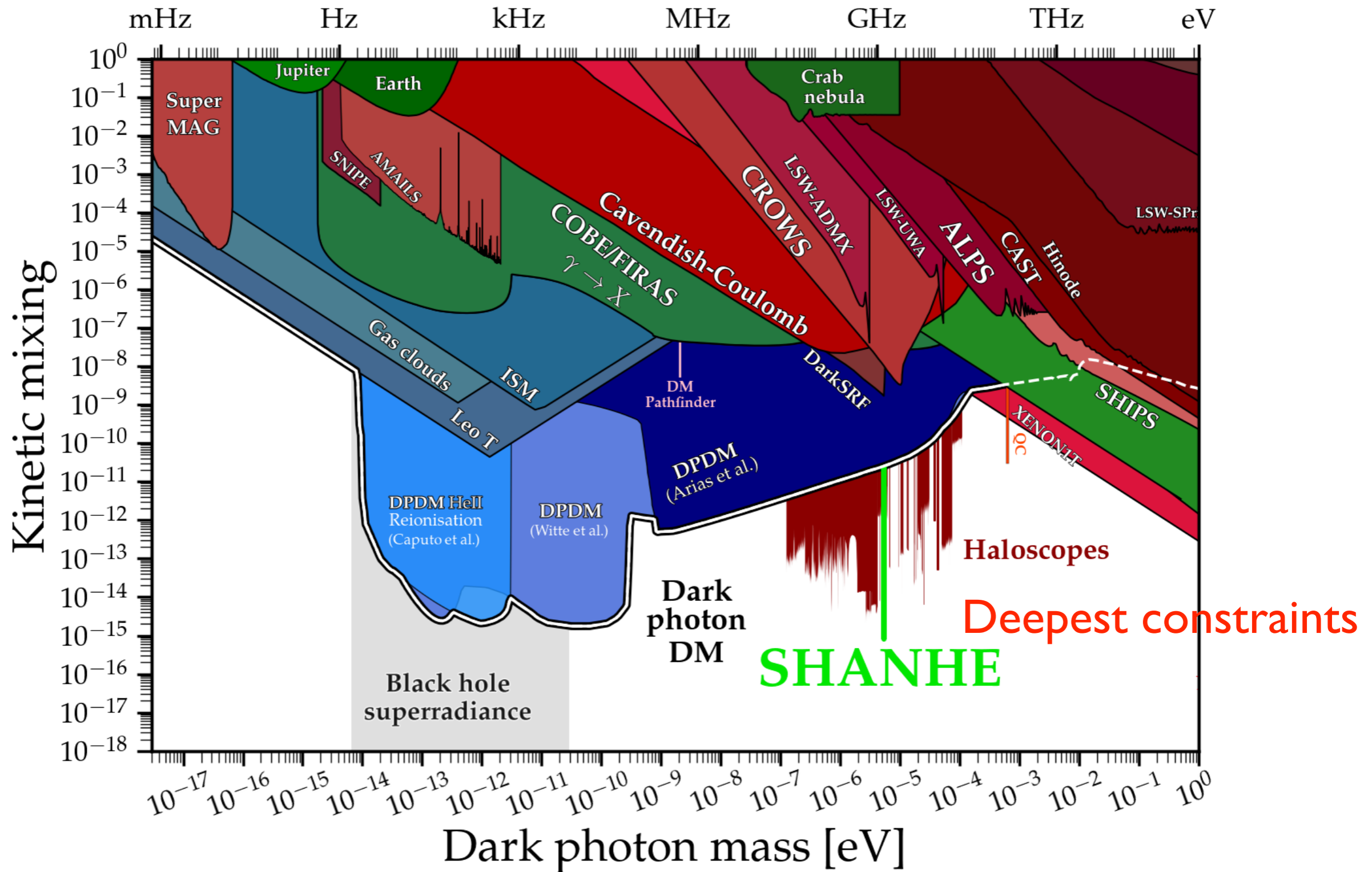
Few comment on $Q \gg Q_{\text{DM}}$

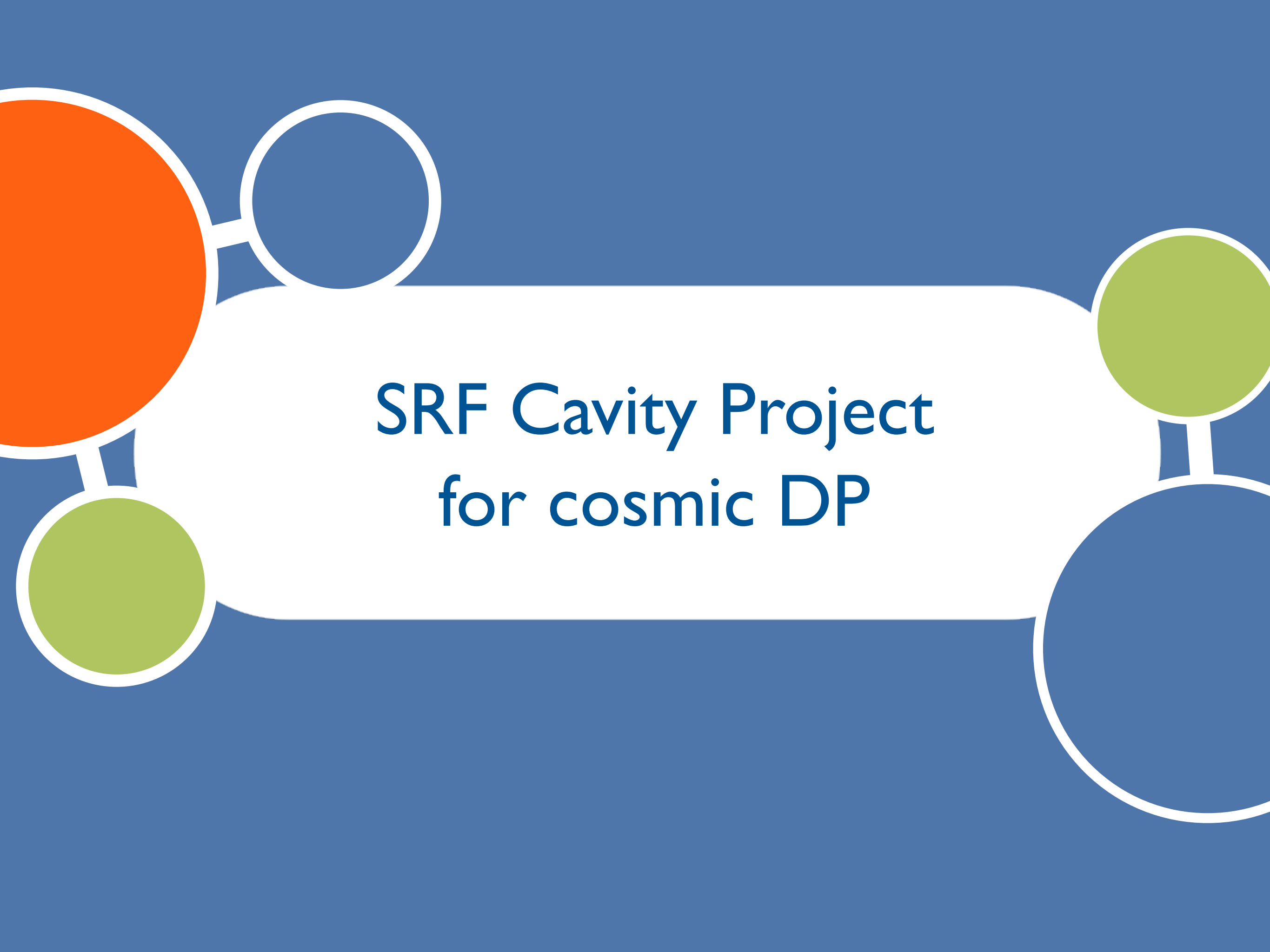


simple fit function (constant):
attenuation factor almost 1

different from ADMX

Data analysis and constraints



A decorative graphic on a blue background. It features a central white rounded rectangle containing the text. To the left of the rectangle is a large orange circle, and below it is a smaller green circle. To the right of the rectangle is a green circle above a larger blue circle. A white outline of a circle is positioned above the rectangle. All circles are connected to the central area by thin white lines.

SRF Cavity Project for cosmic DP

Cosmic DP backgrounds

The Cosmic Axion Background

Jeff A. Dror,^{1,2,3,*} **Hitoshi Murayama,^{2,3,4,†}** and Nicholas L. Rodd^{2,3,‡}

¹*Department of Physics and Santa Cruz Institute for Particle Physics,
University of California, Santa Cruz, CA 95064, USA*

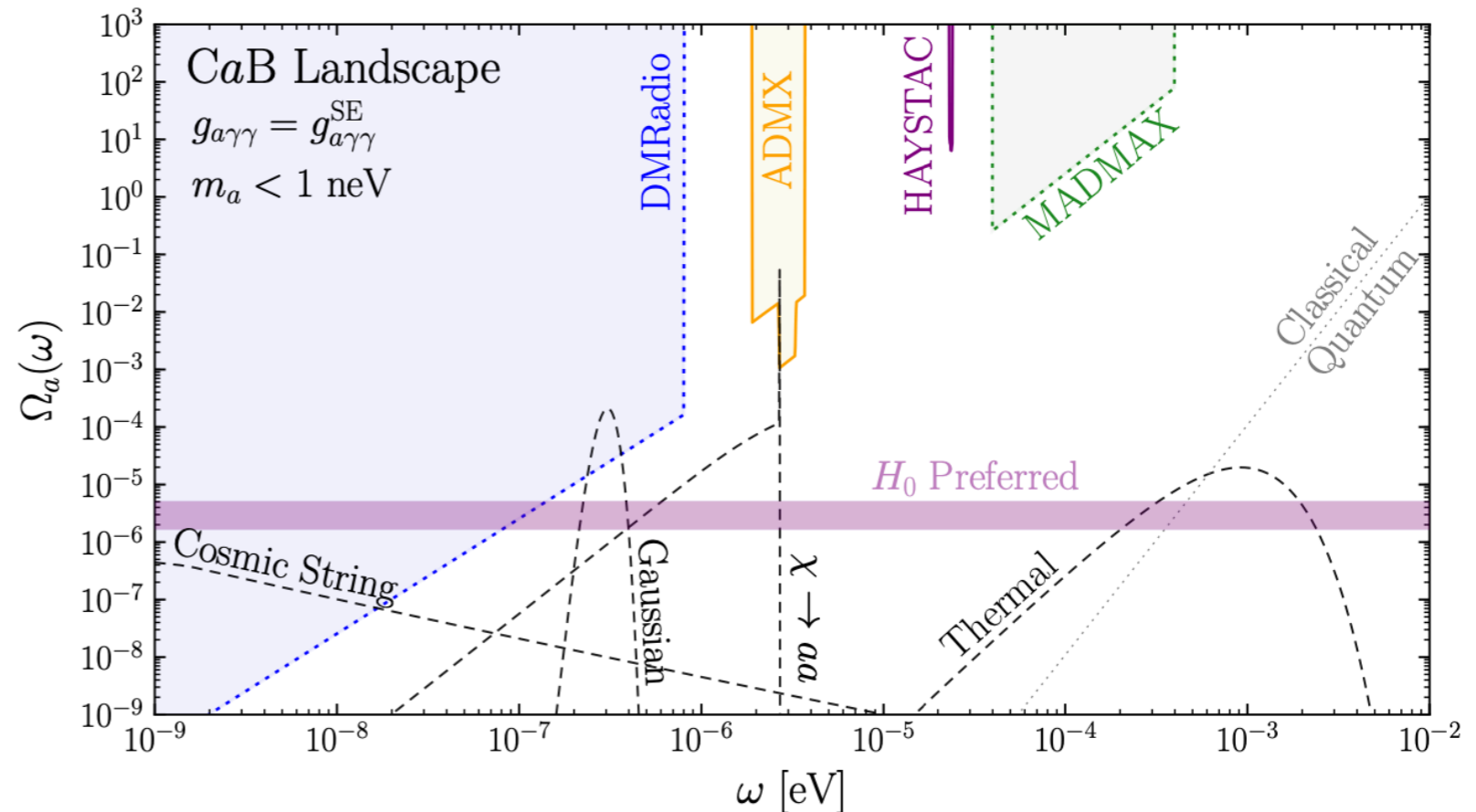
²*Berkeley Center for Theoretical Physics, University of California, Berkeley, CA 94720, USA*

³*Theory Group, Lawrence Berkeley National Laboratory, Berkeley, CA 94720, USA*

⁴*Kavli Institute for the Physics and Mathematics of the Universe (WPI), University of Tokyo, Kashiwa 277-8583, Japan*

New particles can also
be served as the cosmic
backgrounds.

- Relativistic
- Anisotropic



Modulated Signal from Galactic Dark Photons

- How about galactic DP backgrounds? (Anisotropic backgrounds, from annihilation or decay?)

Perturbative cascade decay (broad 4-body spectrum)

Parametric resonance decay (relative sharp 2-body spectrum)

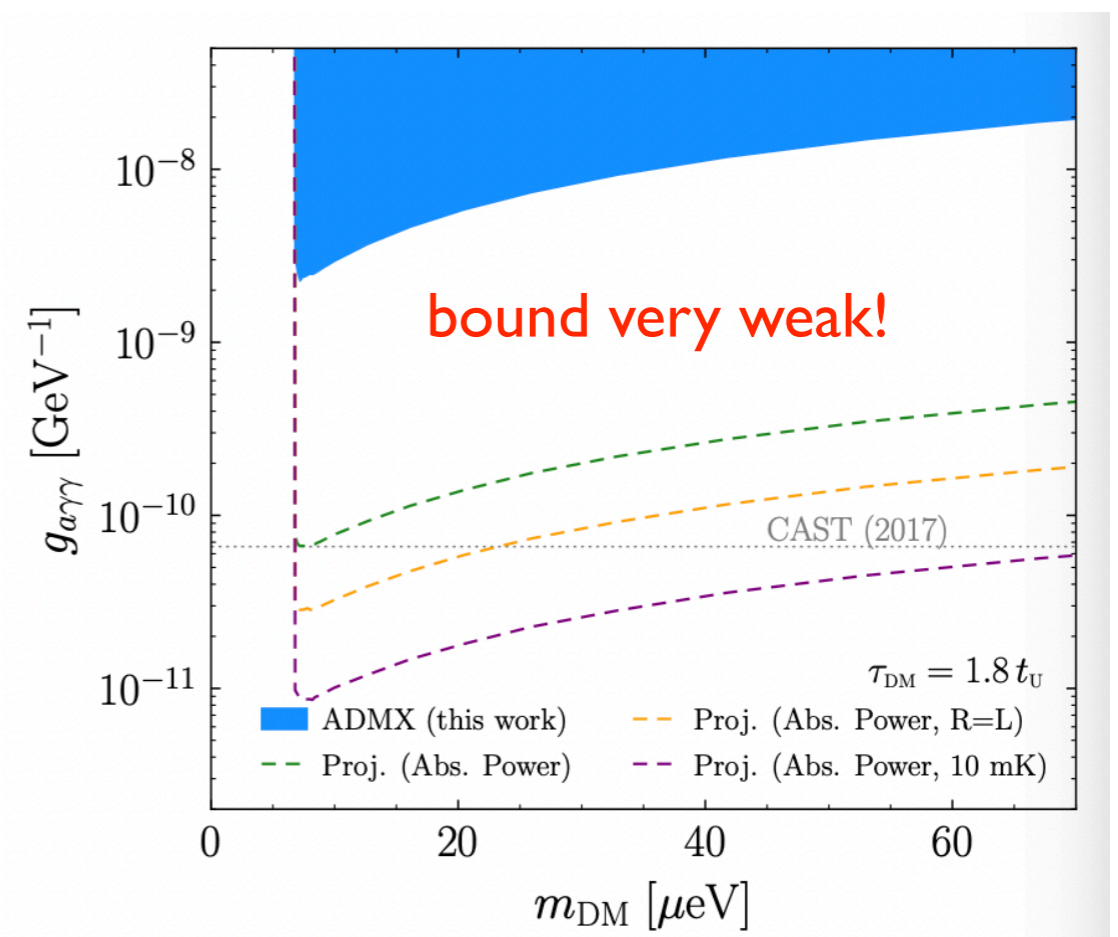
- ADMX experiment (axion)

The very deep constrains for DP would give us much stringent constrains

- Polarization

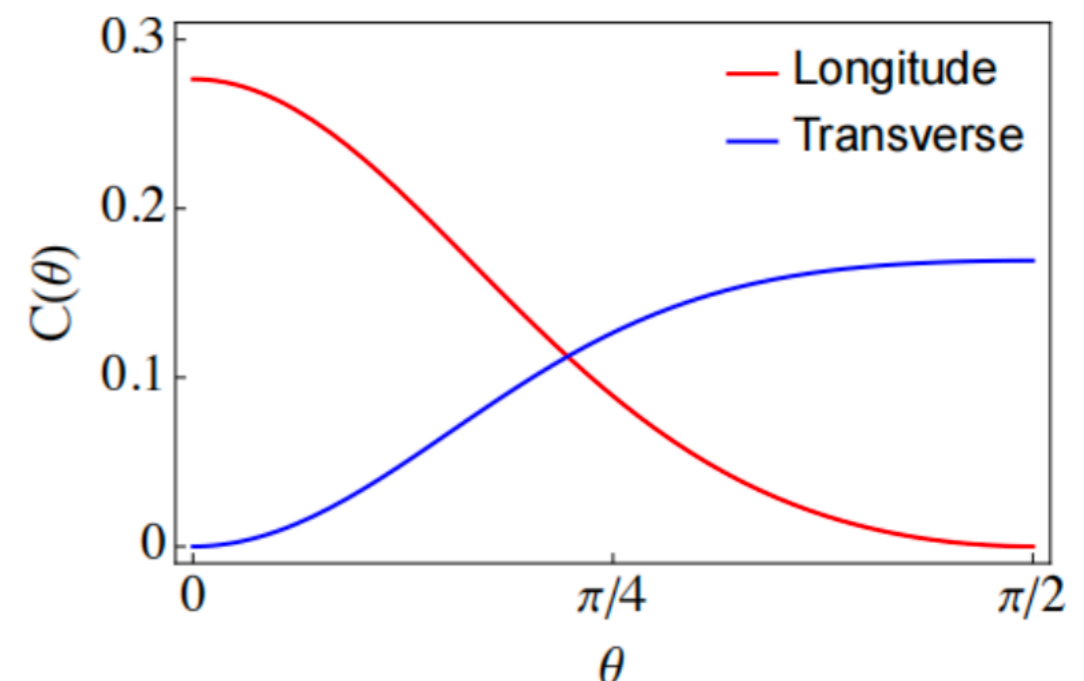
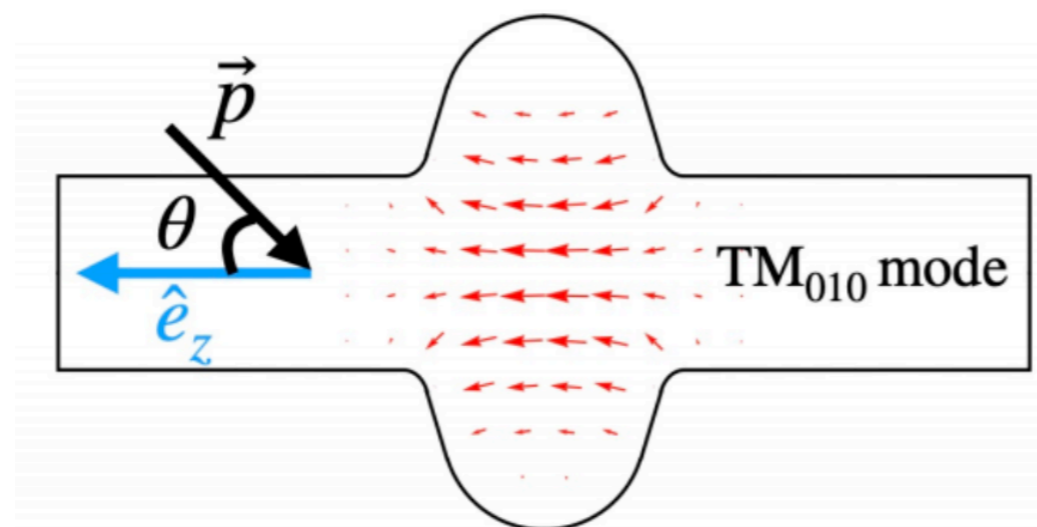
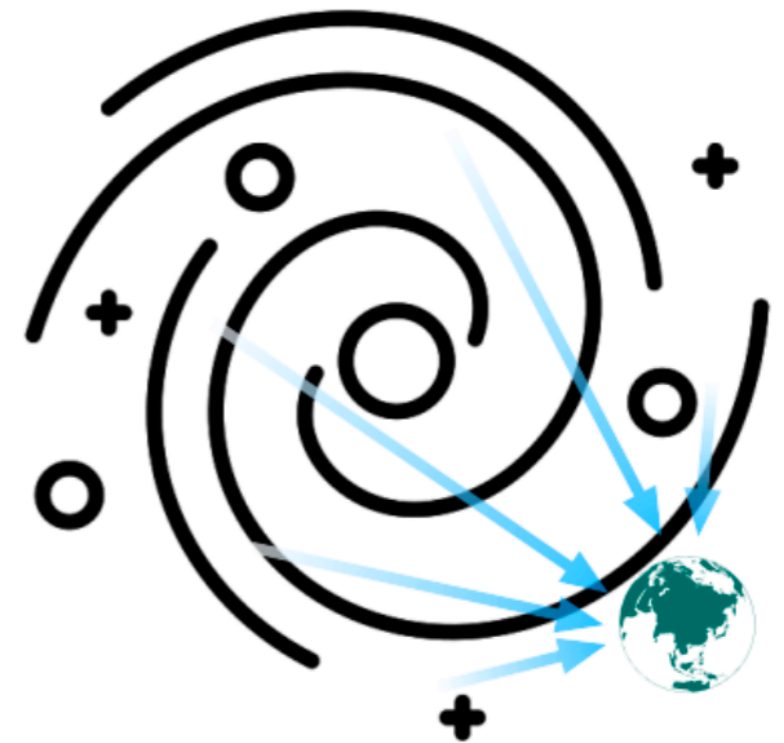
Longitudinal: from a heavy dark Higgs decay

Transverse: axion-DP coupling



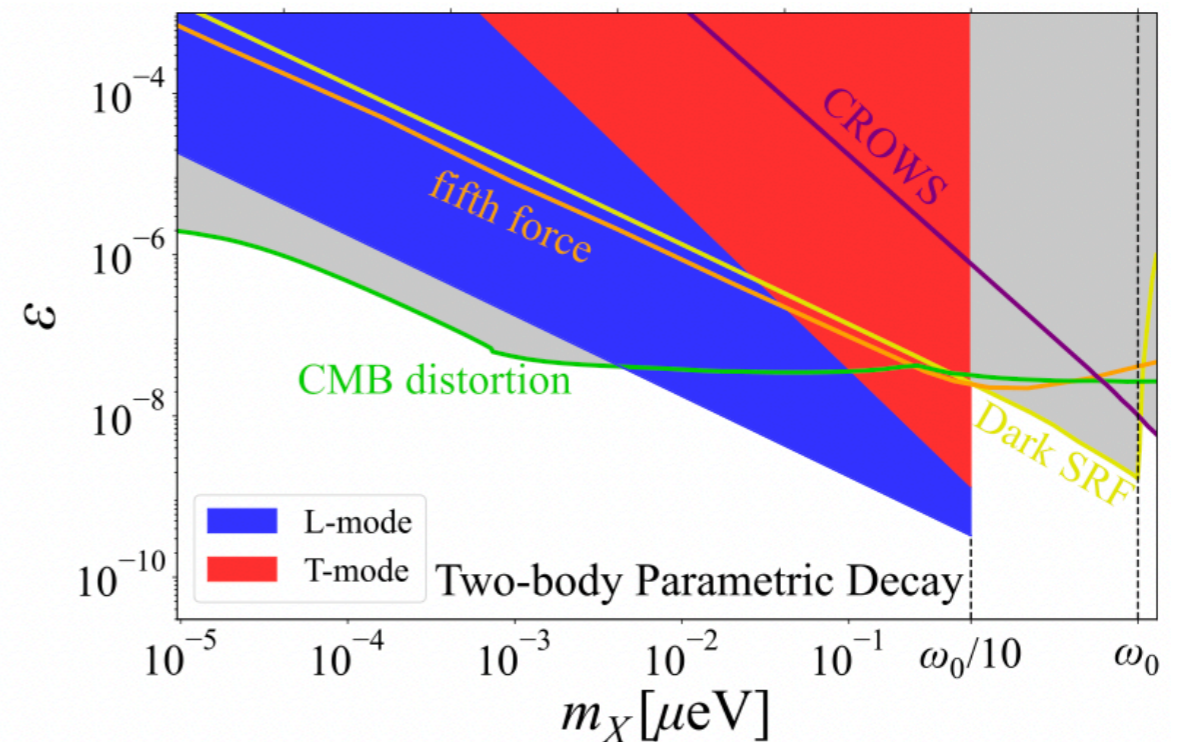
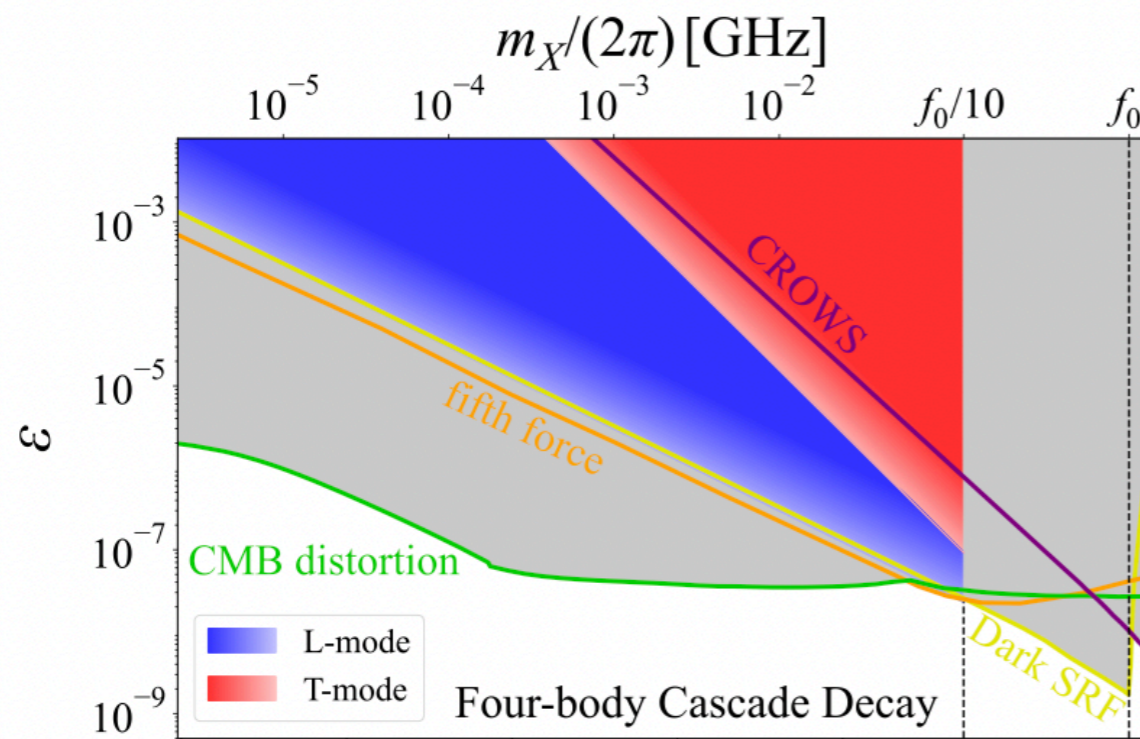
Modulated Signal from Galactic Dark Photons

- ▶ Galactic dark photons from DM decay, e.g.:
cascade decay from DM halo
- ▶ **Vectorial** observable $\propto \vec{A}'$
 - angular-dependent signal $\propto C(\theta)$
 - modulation as the Earth rotates
- ▶ Production is **polarization-dependent**,
modulations for longitude and transverse
modes are **opposite**



SRF Constraints for Galactic Dark Photons

- ▶ **Same dataset** as DPDM search
- ▶ Scanned range within galactic dark photon bandwidth → **combine all scan steps** to analyze
- ▶ **Longitude** mode has **better sensitivity** because of the larger spatial wave function $\sim \omega_{A'}/m_{A'}$



- ▶ Gradient color region represents exclusion for different DM mass

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**A brief introduction to the
team member**

SHANHE collaboration

SHANHE
(mountains and rivers)

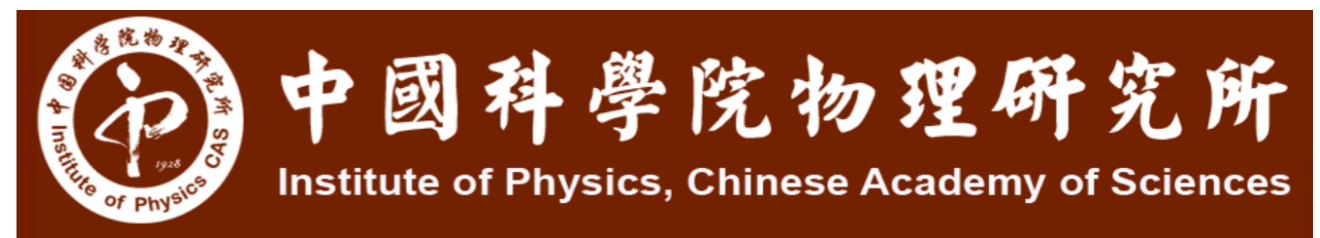


Superconducting cavity for High-frequency gravitational wave, Axion, and other New particles in High Energy physics

Main collaboration



Supportive collaboration



北京量子信息科学研究院
Beijing Academy of Quantum Information Sciences



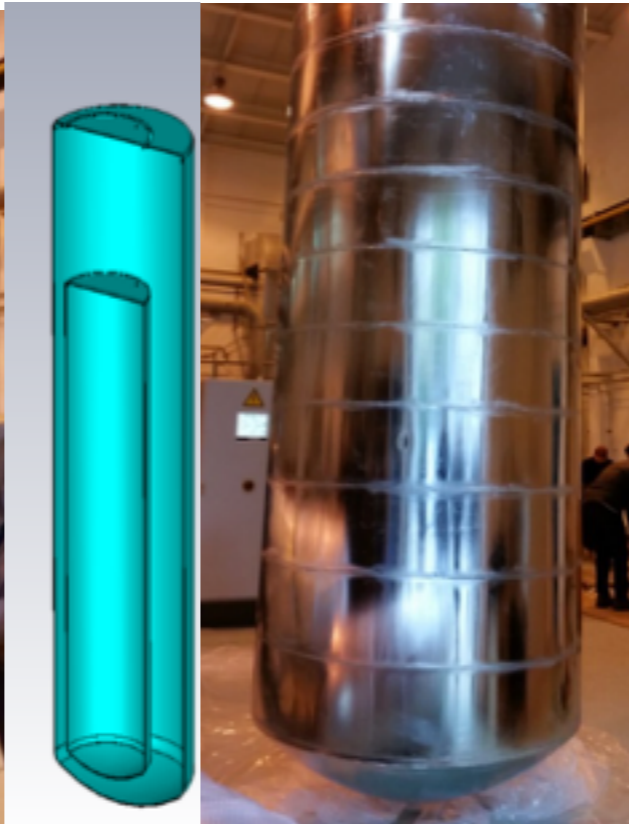
Experimental facilities



Liquid helium system



2K pumping system



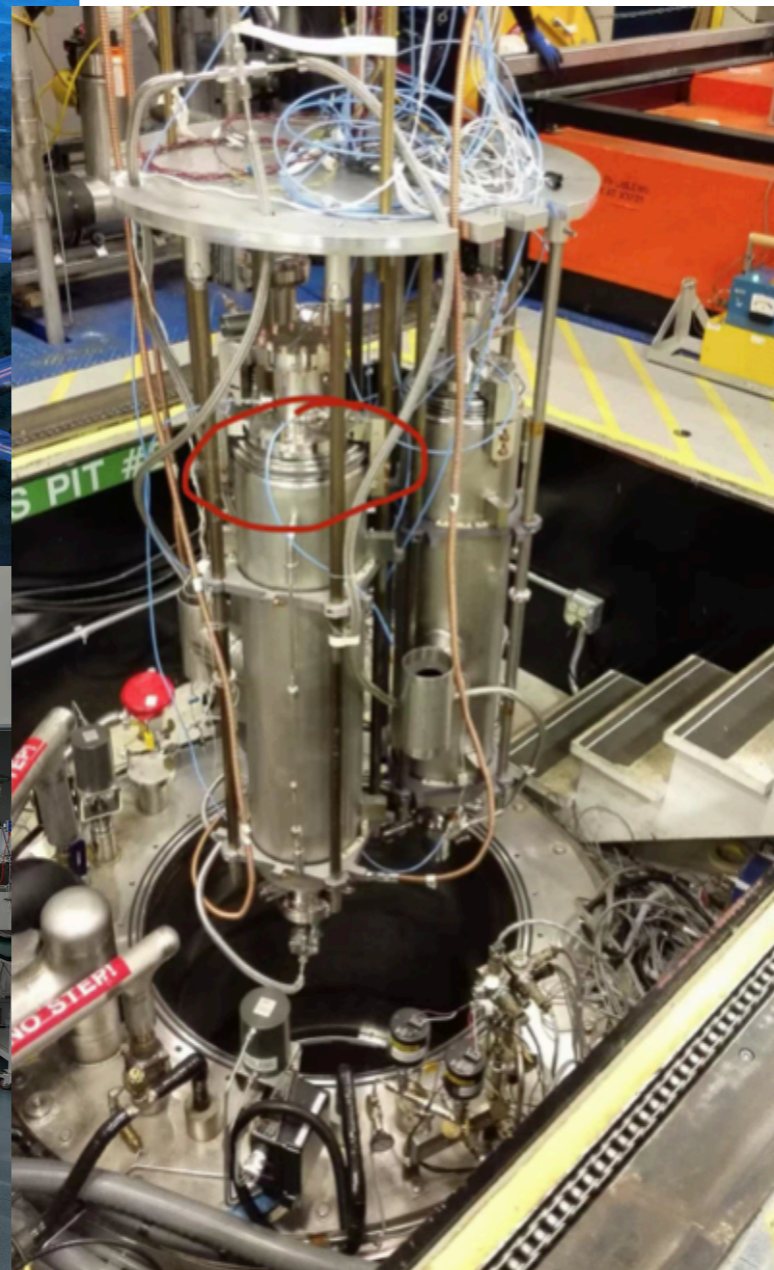
Vertical Dewar Cavity suspension Magnetic shielding

- residual magnetism < 10 mGs
- Static heat leak: < 1 W
- Cooling power: > 200 W @ 2K

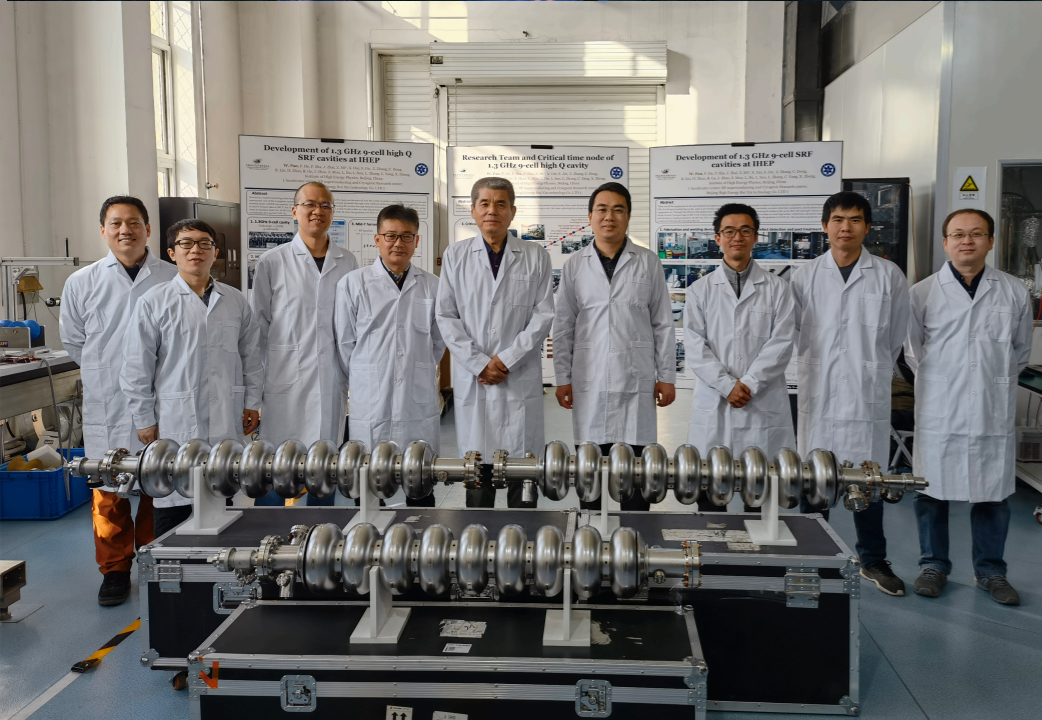
SRF in IHEP



中国科学院高能物理研究所
Institute of High Energy Physics Chinese Academy of Sciences



SRF used for Beijing & Shanghai Synchrotron Radiation Facility and future CEPC



ENDCAP

International SRF Campaigns

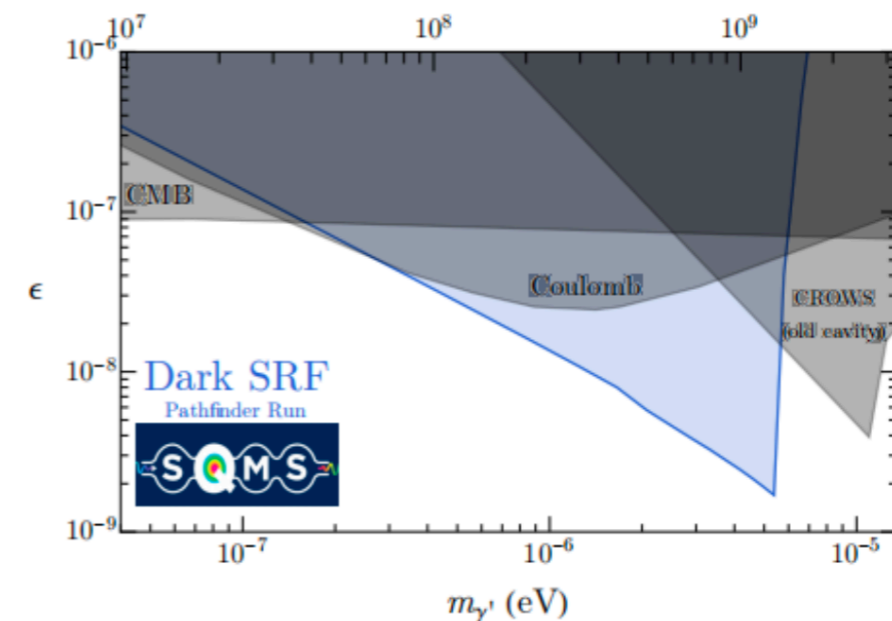
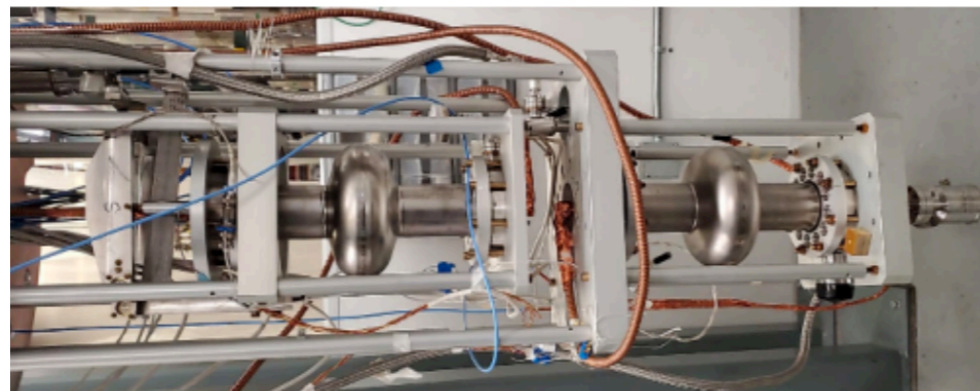
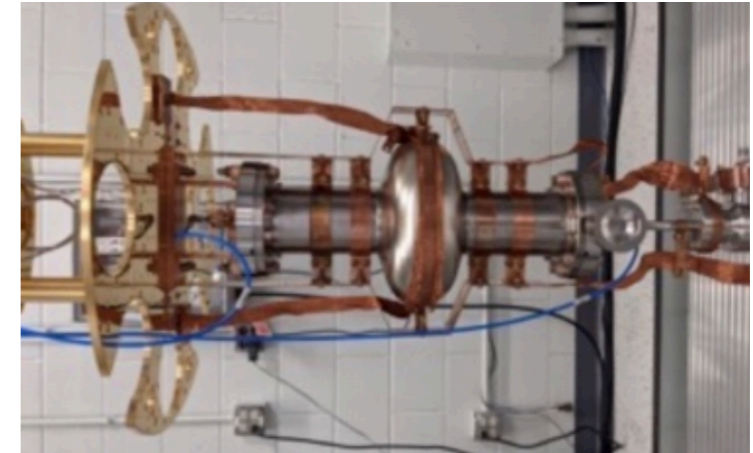
► Fermi-Lab SQMS

- SERAPH:

Single-bin search and ongoing scan searches

- Dark SRF:

Light-shining-wall search for dark photon



► DESY:

- MAGO 2.0

Mode transition from GW-induced cavity deformation



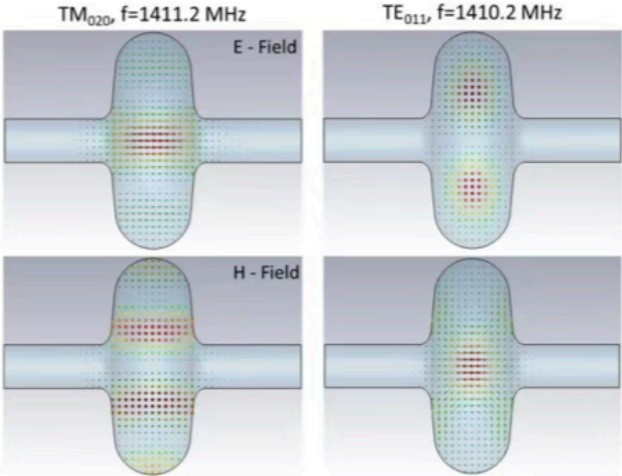
Axion search

TDR like

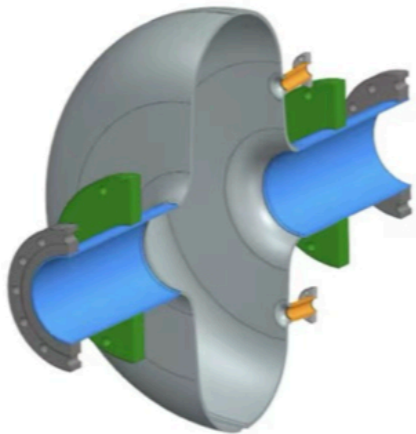
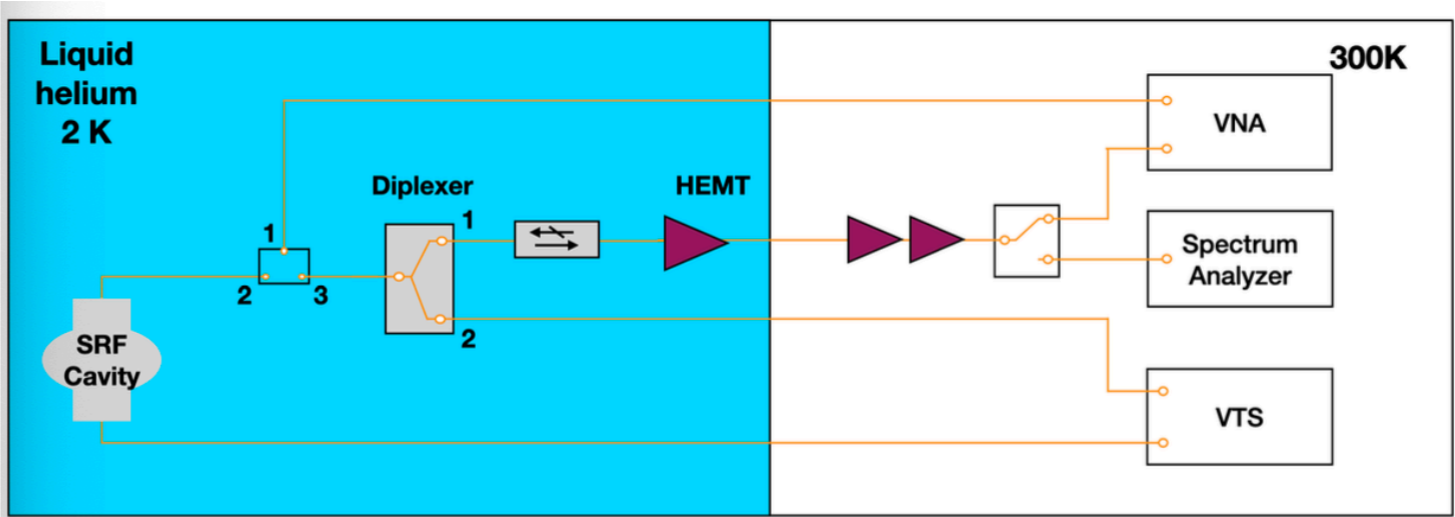
SHANHE collaboration



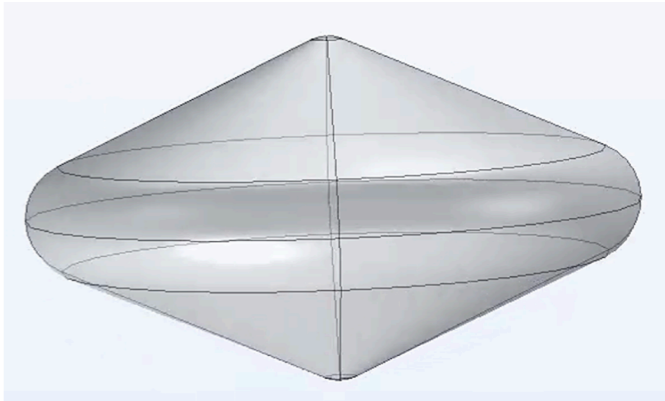
arXiv:2207.11346



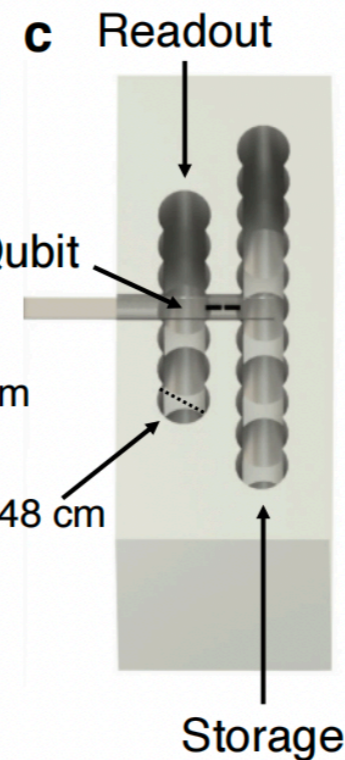
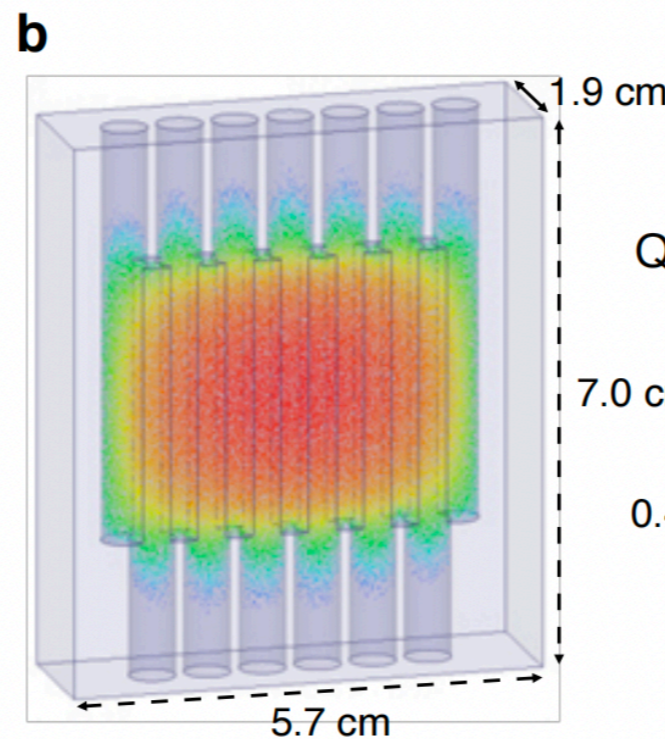
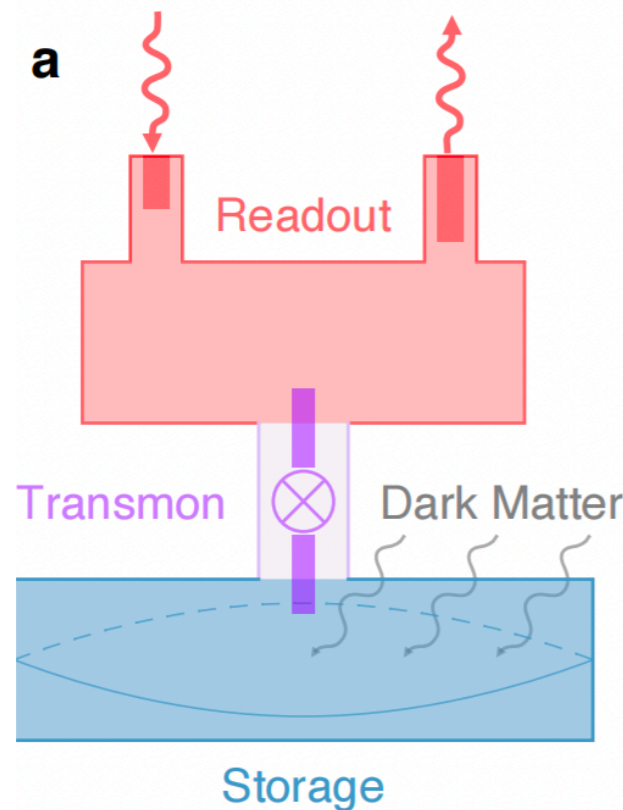
Using the existing 1.3G cavity as a pathfinder



New designed cavity will be operated in the future.



Quantum qubits measure DPDM



Storage 6.011 GHz
Readout 8.052 GHz
Qubit 4.749 GHz

$$\mathcal{H} = \omega_c a^\dagger a + \frac{1}{2} \omega_q \sigma_z + 2\chi a^\dagger a \frac{1}{2} \sigma_z$$

$$\mathcal{H} = \omega_c a^\dagger a + \frac{1}{2} \omega_q \sigma_z + 2\chi a^\dagger a \frac{1}{2} \sigma_z$$

Measuring **only** the photon number while completely ignore the phase (quantum non-demolition measurements).

$$\begin{aligned} \mathcal{H}_{int} &= \vec{d} \cdot \vec{E} \\ &= g(\sigma_+ + \sigma_-)(a + a^\dagger) \\ &\sim 2\chi a^\dagger a \frac{1}{2} \sigma_z \end{aligned}$$

Quantum qubit (two level system)
as a single photon detector

Using Ramsey interferometry to measure the photon number parity

Quantum qubits measure DPDM

- Wave-like dark matter background field will change the time evolution operator. For a coherent state, can be modeled as a displacement operator.

$$\hat{D}(\beta) = e^{\beta a^\dagger - \beta^* a}, \text{ where } |\beta| = \sqrt{\rho_{DM} m_{DM} G V \epsilon \tau}$$

- Acting on the cavity prepared in the vacuum state

$$\left| \langle 1 | \hat{D}(\beta) | 0 \rangle \right|^2 \approx \left| \langle 1 | \beta a^\dagger | 0 \rangle \right|^2 = |\beta|^2$$

- For a large n Fock state, signal enhanced by (n+1).

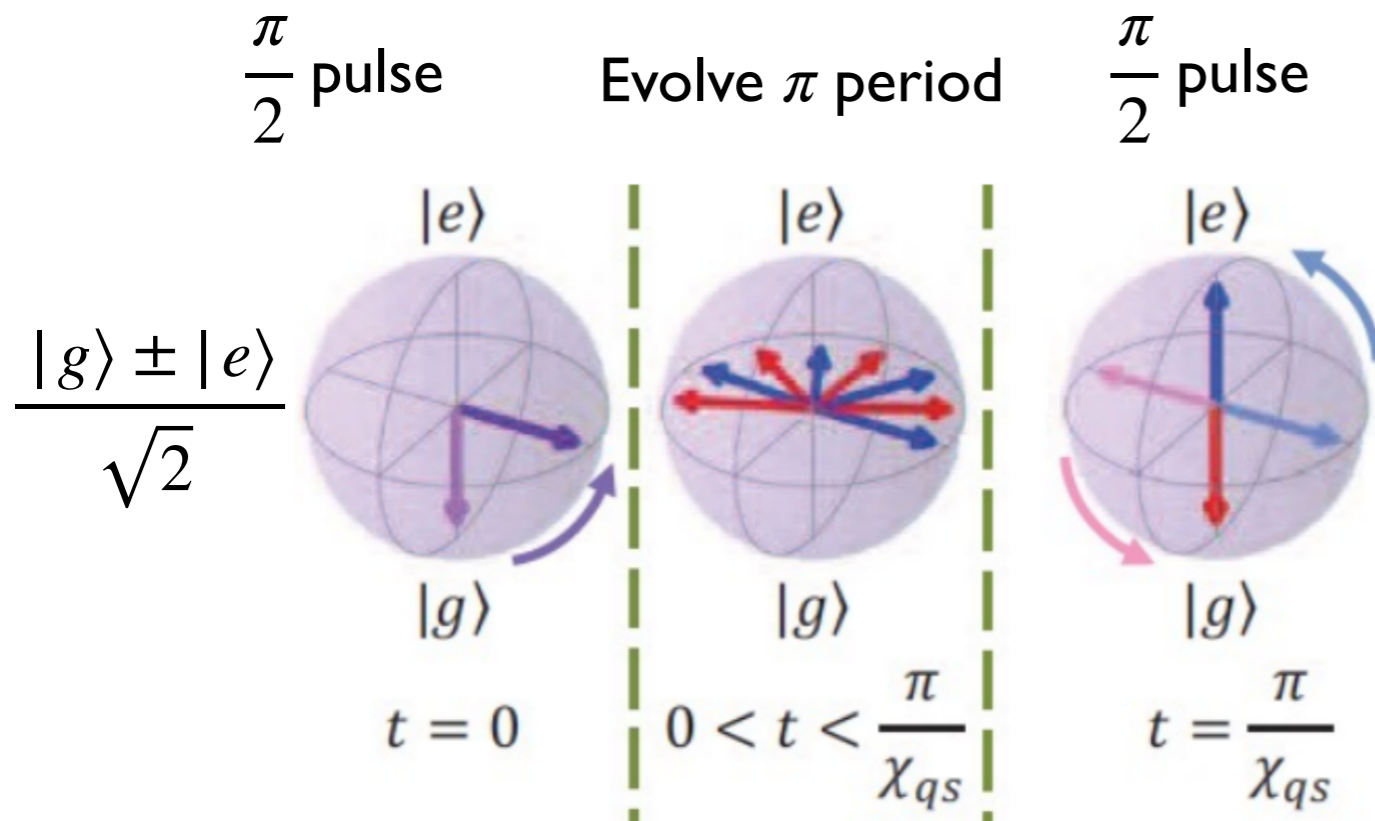
$$\left| \langle n + 1 | \hat{D}(\beta) | n \rangle \right|^2 \approx \left| \langle n + 1 | \beta a^\dagger | n \rangle \right|^2 = |\beta|^2 (n + 1)$$

- Shorter coherence time $T_n = T_{cavity}^1 / n$, More difficult to prepare

- Sensitivity mainly given by the background photon number \bar{n}_c

Quantum parity measurement

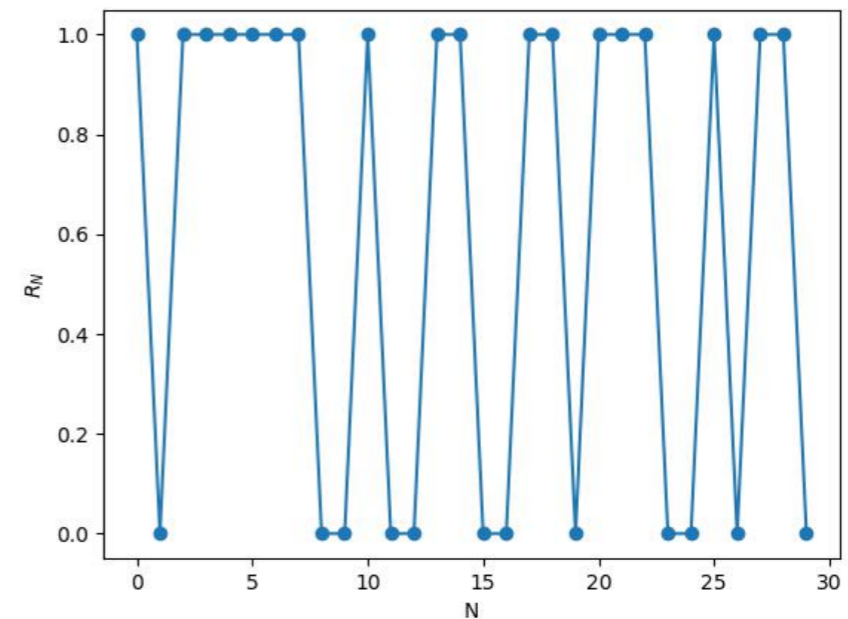
Ramsey interferometry in Bloach sphere



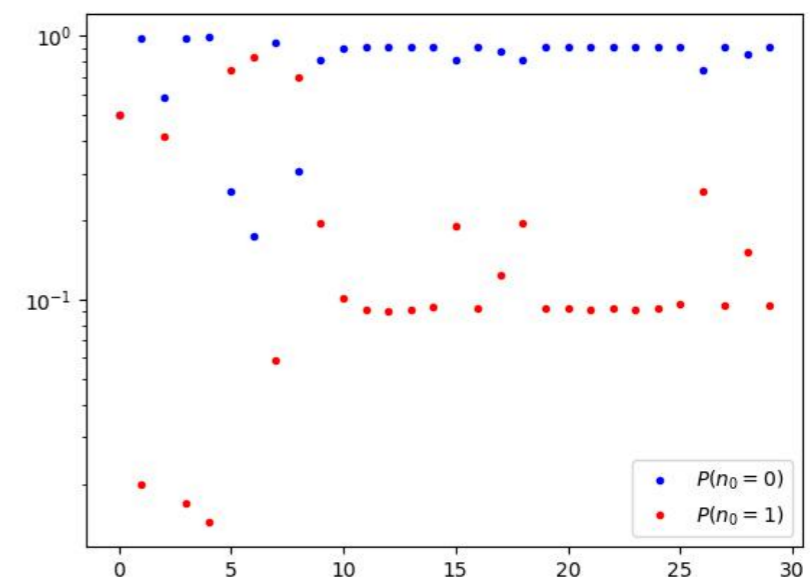
$$H_{int} = 2\chi a^\dagger a \frac{\sigma_z}{2}, \quad t = \frac{\pi}{\chi}$$

The qubit state is flipped if there is odd number of photons in the cavity.

Excited state probability



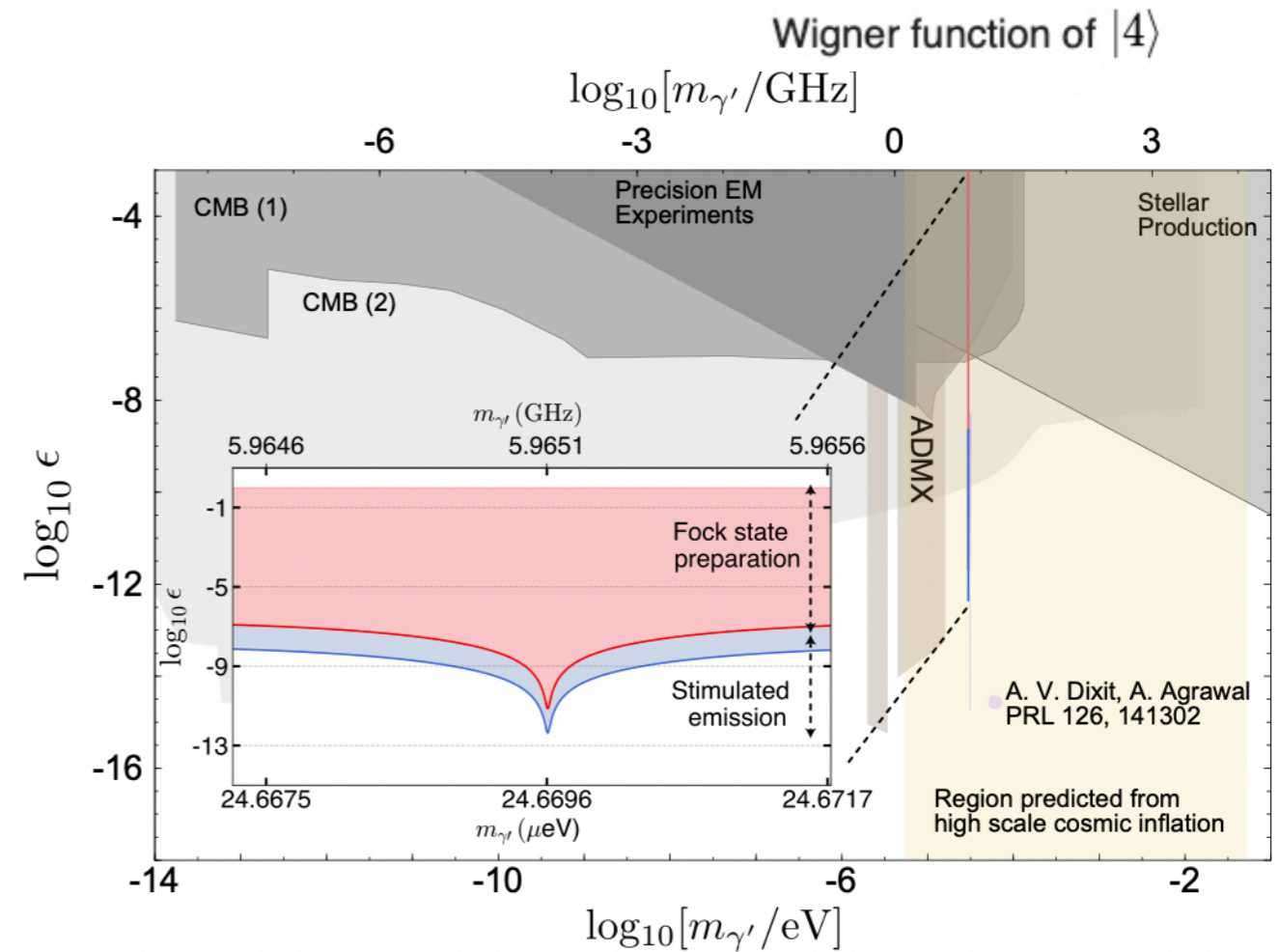
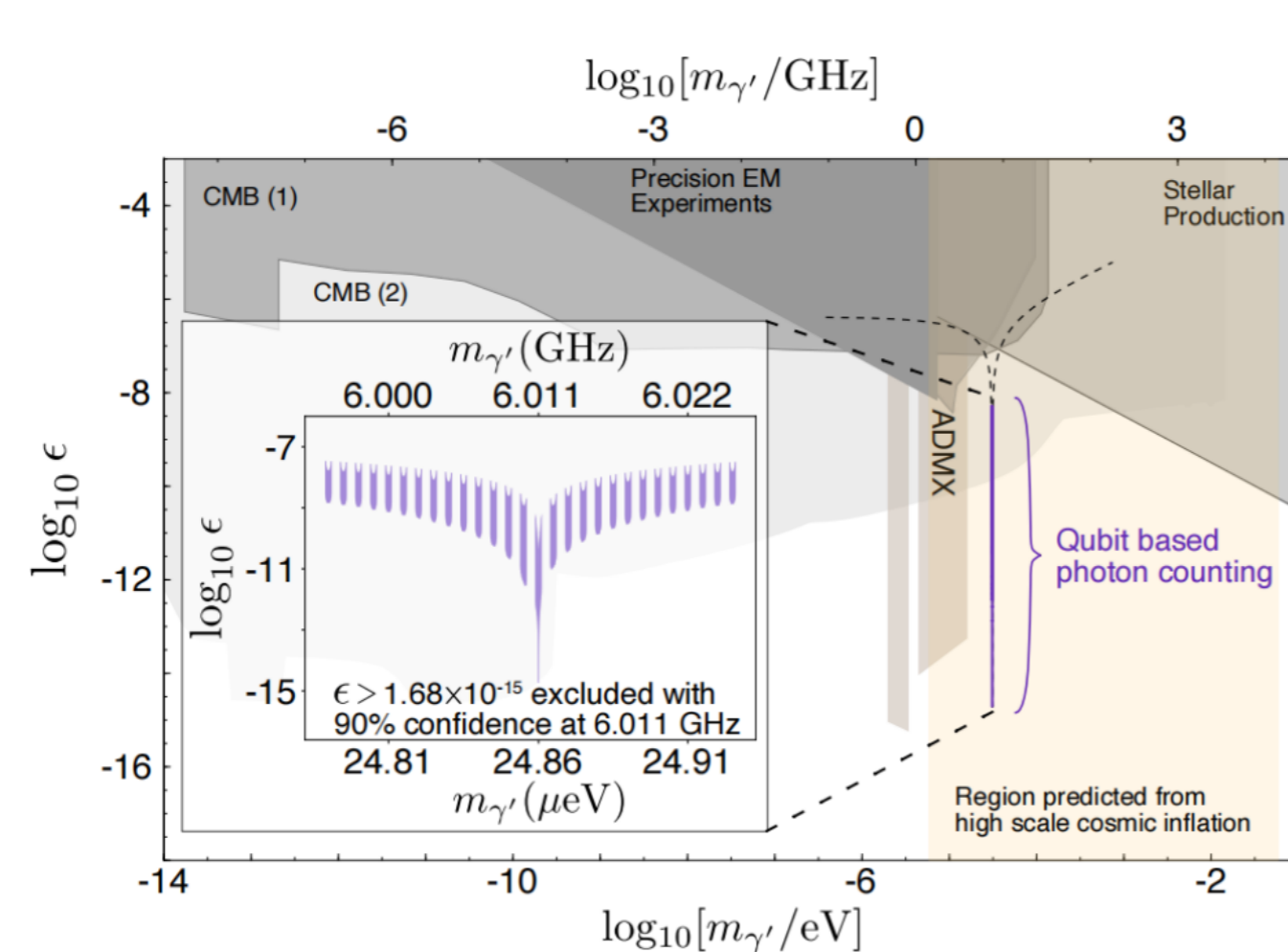
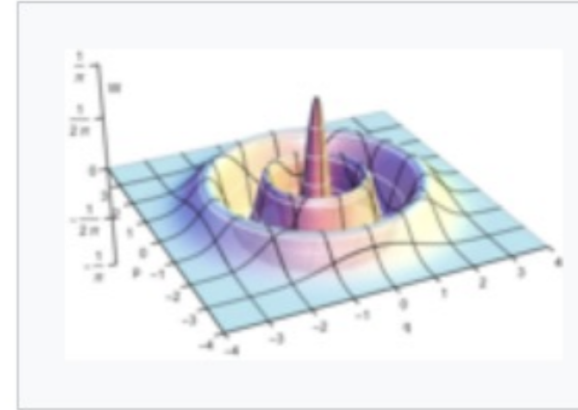
hidden Markov model to specify the probability



Quantum qubits measure DPDM

DPDM: Using the Vacuum state to measure

DPDM: Using the Fock state (N=4) to measure



$$\epsilon > 1.68 \times 10^{-15}$$

$$\epsilon \geq 4.35 \times 10^{-13}$$

How to eliminate the noise?

Even we use various clever ways to background noises, even beyond standard quantum limit, can we really to further suppress them?

One observation is that when using multiple sensors, the DM signals can be **coherent** among multiple sensors, while most backgrounds are not!

Consider the **2-level systems readout**

- The displacement operator

$$\hat{D}(\beta) = e^{\beta a^\dagger - \beta^* a} \text{ where } \beta = \frac{1}{2} \eta_{DM} e^{i\phi_{DM} t}$$

- Restricting to the two-level subspace

$$D(\beta) |g\rangle \simeq |g\rangle + \beta |e\rangle$$

- Dark matter signal

$$\left| \langle e | \hat{D}(\beta) | g \rangle \right|^2 \approx \left| \langle e | \beta a^\dagger | g \rangle \right|^2 = |\beta|^2$$

- Sensitivity mainly given by the background occupation number \bar{n}_c

Can generalize

Can apply to various systems

- SRF cavity

$$\eta_{DM} = \epsilon_{DM} \sqrt{\rho_{DM} \omega G V}$$

- Transmon qubits

$$\eta_{DM} = \frac{1}{2} \epsilon_{DM} \kappa d \sqrt{C \omega \rho_{DM}}$$

- Trapped ions

$$\eta_{DM} = e \epsilon_{DM} \sqrt{\frac{\rho_{DM}}{m_{ION} \omega}}$$

How to eliminate the noise?

Density matrix

$$\rho = |\beta\rangle\langle\beta| \simeq \begin{pmatrix} 1 - |\beta|^2 & \beta^* \\ \beta & |\beta|^2 \end{pmatrix}$$

Density matrix with error

What you can measure is the population measurements (diagonal term) ?

$$\rho \rightarrow \mathcal{E}(\rho) = \begin{pmatrix} p_{T_1}\rho_{gg} + p_{\text{reset}}p_0 & p_{T_2}\rho_{ge} \\ p_{T_2}\rho_{eg} & p_{T_1}\rho_{ee} + p_{\text{reset}}p_1 \end{pmatrix}$$

Relax and dephasing

$$p_{T_1} = e^{-\tau/T_1} \text{ and } p_{T_2} = e^{-\tau/T_2}$$

$$SNR = \frac{S}{B} = \frac{p_{T_1}|\beta|^2}{p_{\text{reset}}p_1}$$

Thermal population

$$p_0 \text{ and } p_1 = 1 - p_0$$

$$\frac{S}{\sqrt{B}} = \frac{p_{T_1}|\beta|^2}{\sqrt{p_{\text{reset}}p_1}}$$

How to eliminate the noise?

Two detectors

$$|\Psi\rangle = \hat{D}(\beta) |gg\rangle \simeq |gg\rangle + \beta |ge\rangle + \beta |eg\rangle$$

- Density matrix

$$\rho^{(2)} = |\Psi\rangle\langle\Psi| = \begin{pmatrix} 1 - 2\beta^2 & \beta & \beta & 0 \\ \beta & \beta^2 & \beta^2 & 0 \\ \beta & \beta^2 & \beta^2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} + O(\beta^3) + \begin{pmatrix} 1 - \epsilon_A - \epsilon_B & 0 & 0 & 0 \\ 0 & \epsilon_B & 0 & 0 \\ 0 & 0 & \epsilon_A & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

- Dark matter signal

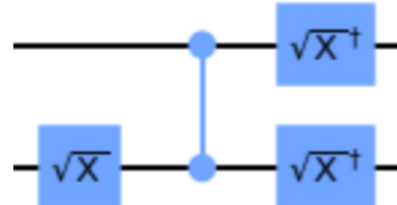
$$\langle eg | \rho^{(2)} | eg \rangle + \langle ge | \rho^{(2)} | ge \rangle = \rho_{11}^{(2)} + \rho_{22}^{(2)} \sim 2\beta^2 + \epsilon_A + \epsilon_B$$

increases with N_q

How to eliminate the noise?

Coherent signals are **off-diagonal**, how to measure them?

- Define

$$\mathcal{U}_c = \left(\sqrt{X}^\dagger \otimes \sqrt{X}^\dagger \right) ZZ \left(\sqrt{X} \otimes I \right) =$$


$$\text{Where } \sqrt{X} = \frac{1}{2} \begin{pmatrix} 1+i & 1-i \\ 1-i & 1+i \end{pmatrix}, ZZ = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

Use an unitary transformation realized by quantum gates

- Applying \mathcal{U}_c

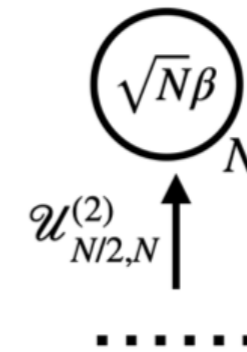
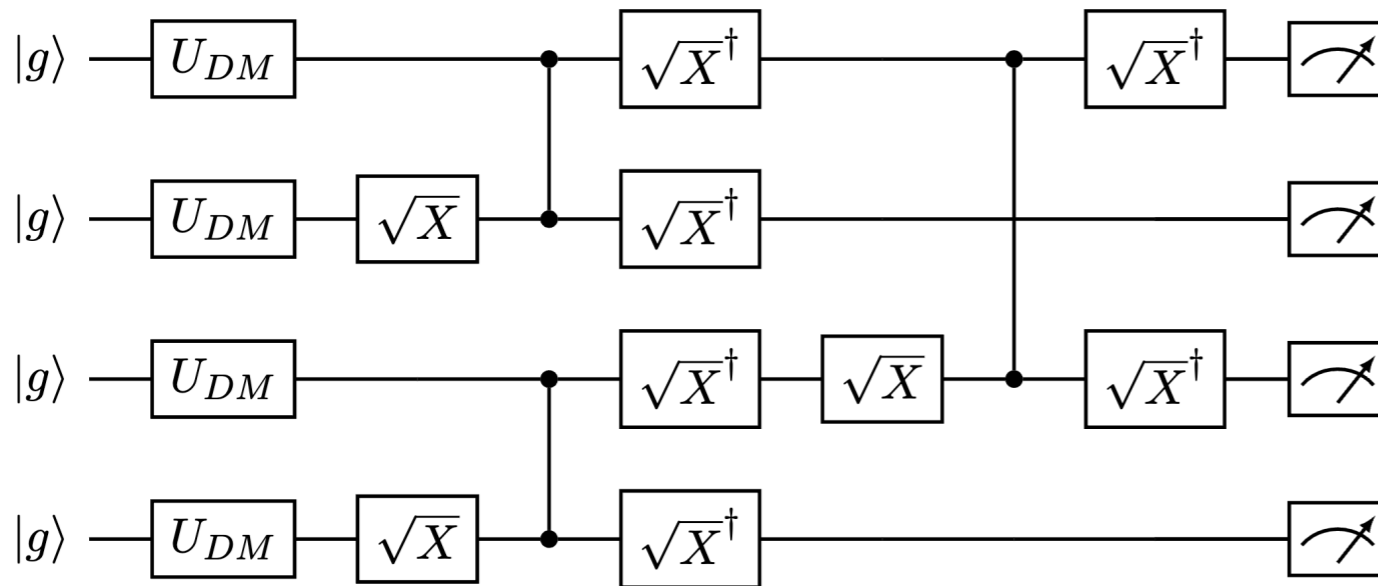
$$\mathcal{U}_c \rho^{(2)} \mathcal{U}_c^\dagger = \begin{pmatrix} \frac{1}{2} & -\frac{i}{2} + i\beta^2 & 0 & -i\beta \\ \frac{i}{2} - i\beta^2 & \frac{1}{2} - 2\beta^2 & 0 & \beta \\ 0 & 0 & 0 & 0 \\ i\beta & \beta & 0 & 2\beta^2 \end{pmatrix} + O(\beta^3) + \begin{pmatrix} \frac{1}{2}(1 - \epsilon_A - \epsilon_B) & \frac{i}{2}(\epsilon_A + \epsilon_B - 1) & 0 & 0 \\ -\frac{i}{2}(\epsilon_A + \epsilon_B - 1) & \frac{1}{2}(1 - \epsilon_A - \epsilon_B) & 0 & 0 \\ 0 & 0 & \frac{1}{2}(\epsilon_A + \epsilon_B) & \frac{i}{2}(\epsilon_A - \epsilon_B) \\ 0 & 0 & -\frac{i}{2}(\epsilon_A - \epsilon_B) & \frac{1}{2}(\epsilon_A + \epsilon_B) \end{pmatrix}$$

- Pure signal given by

$$\rho'_{44} - \rho'_{33} = \left(2\beta^2 + \frac{\epsilon_A + \epsilon_B}{2} \right) - \frac{\epsilon_A + \epsilon_B}{2} = 2\beta^2 \quad \text{No incoherent noise!!!}$$

Generalize to N nodes

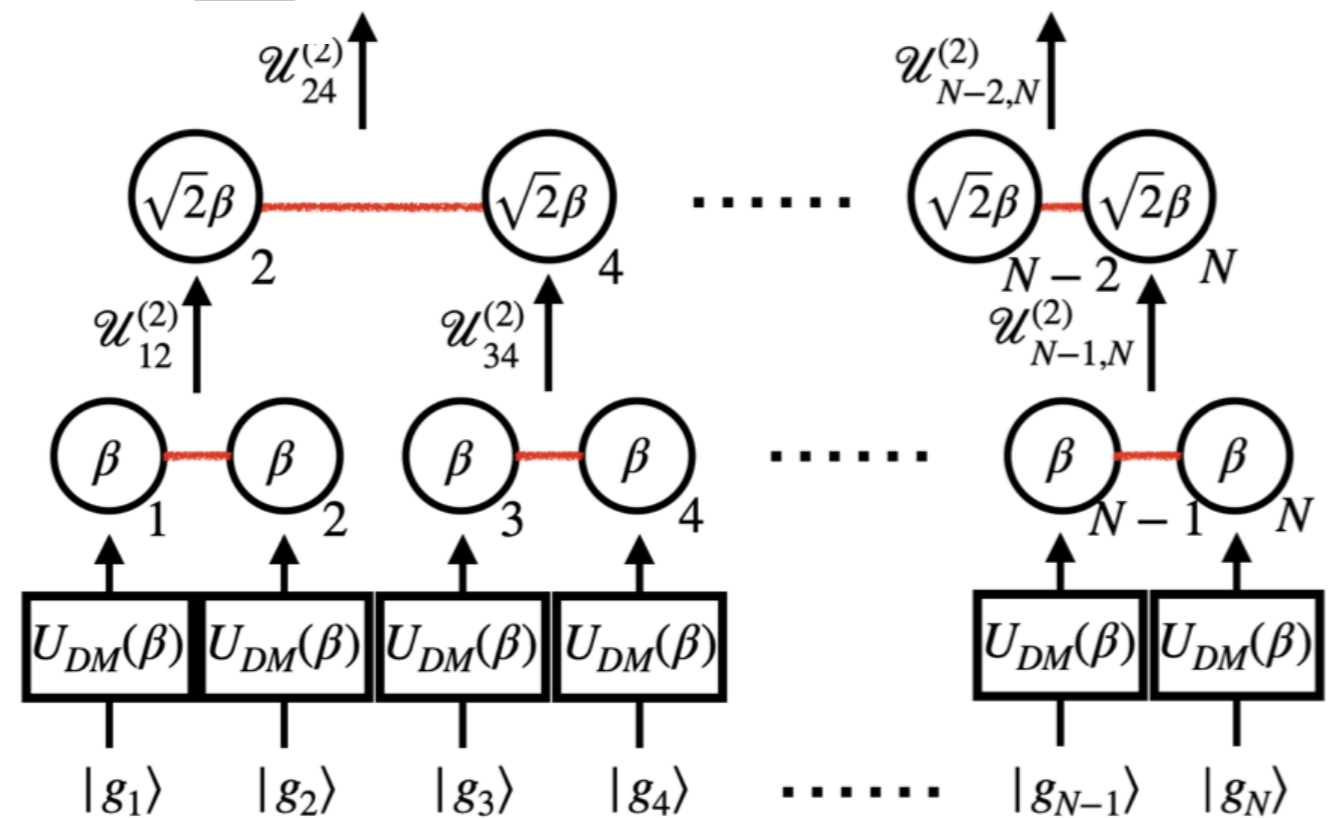
Four nodes



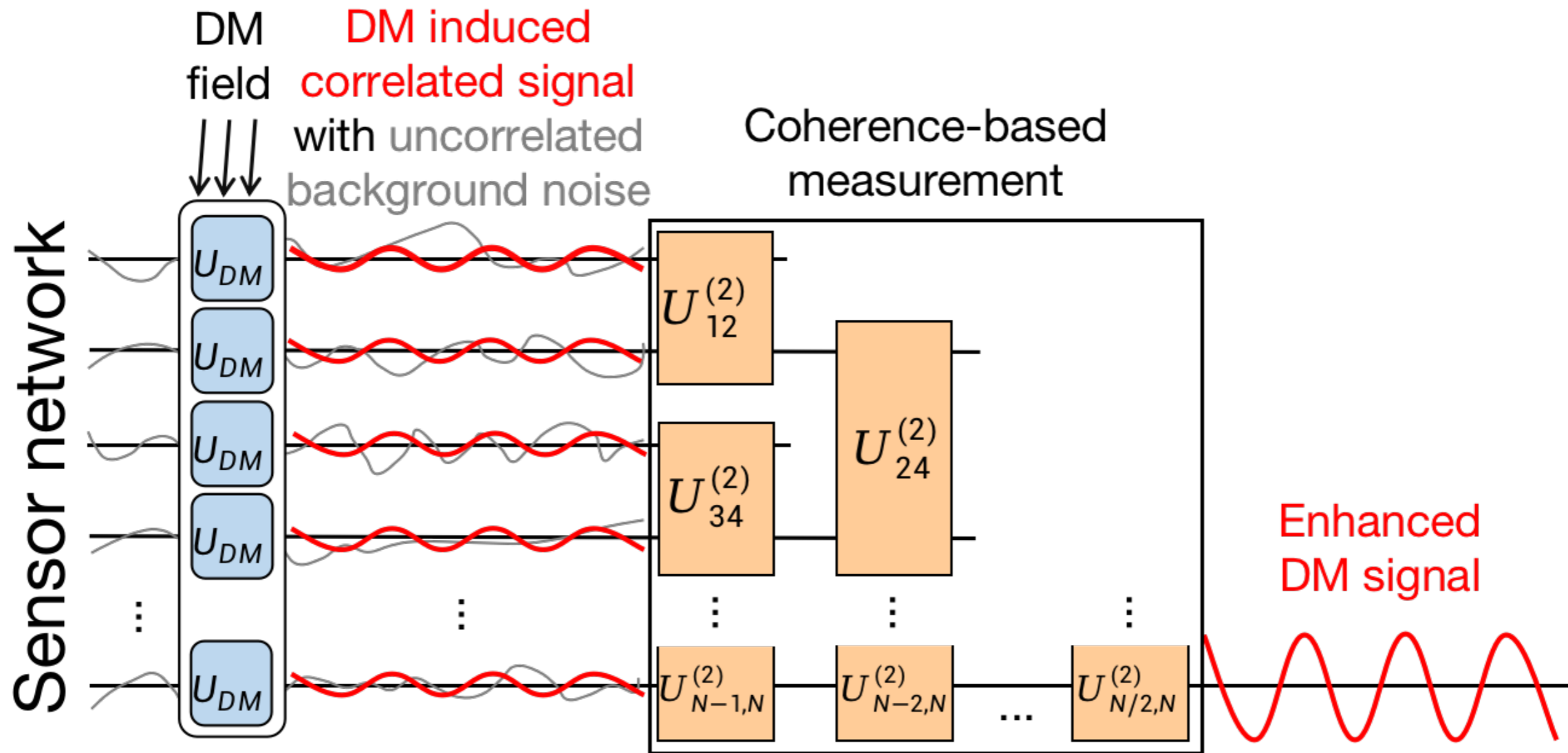
N nodes

Signal (off-diagonal)
goes like N^2

Backgrounds go like N



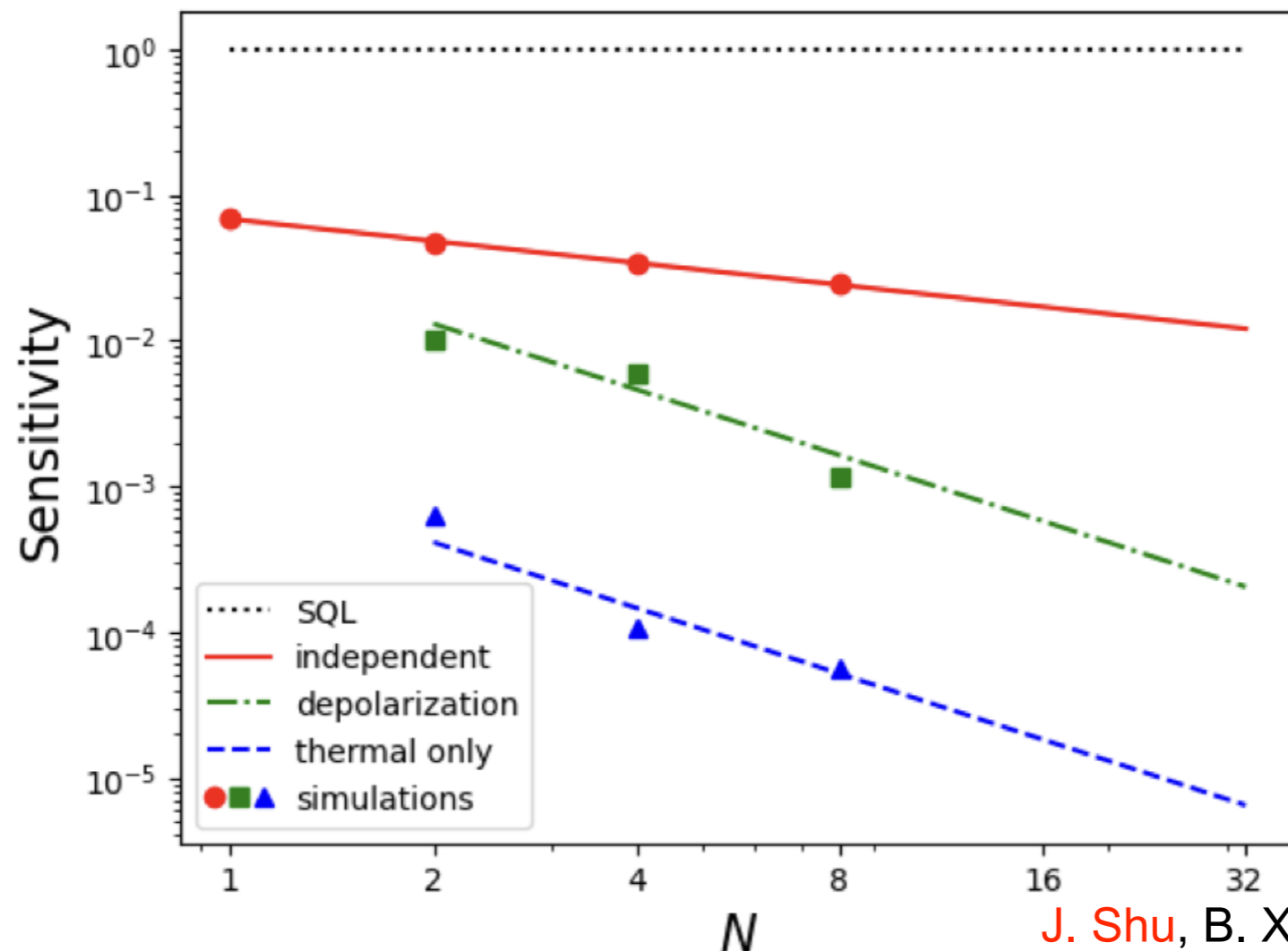
How to eliminate the noise?



How to eliminate the noise?

The gates have infidelities, can be much smaller

Sensor type	\bar{n}_{r0}	Two-qubit gate infidelity $\bar{\mathcal{F}}$
Cavity	10^{-3} [46]	10^{-4} [47]
Transmon	10^{-3} [48, 49]	10^{-3} [50, 51]
Ions	10^{-2} [52]	10^{-3} [53, 54]



A decorative graphic on a blue background. It features a central white rounded rectangle containing the text 'Summary and outlook'. Surrounding this rectangle are several circles of different colors (orange, green, blue) and sizes, connected by thin white lines, resembling a network or a stylized logo.

Summary and outlook

Summary and outlook

- High-Q SRF is extremely interesting in Haloscope wave-like DM searches (get deepest constraints).
- DP backgrounds has rich information (polarization & angular distribution).
- When combined with QND methods, very powerful
- Can be extended to sensors arrays (quantum 2.0)
- In the future (axion, GWs, quantum qubit, etc), much more can be done .

A decorative graphic on a blue background. It features a central white rounded rectangle containing the text "Thank you!". Surrounding this rectangle are several circles of different colors (orange, green, blue) and sizes, connected by white lines, resembling a network or a stylized map. The circles are positioned at the corners and along the sides of the central text box.

Thank you!

A decorative graphic on a blue background. It features a central white rounded rectangle containing the text "Backup slides" in red. Surrounding this rectangle are several circles of different colors (orange, green, blue) and sizes, connected by white lines, resembling a network or a stylized logo.

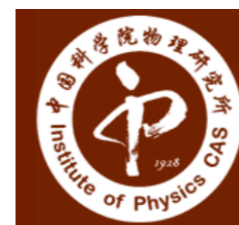
Backup slides

Myself and other collaborations



中國科學院高能物理研究所
Institute of High Energy Physics Chinese Academy of Sciences

supportive collaborations



中國科學院物理研究所
Institute of Physics, Chinese Academy of Sciences

Ph.D.: UChicago

Boya distinguished professor: PKU

Both theory and experimentalist

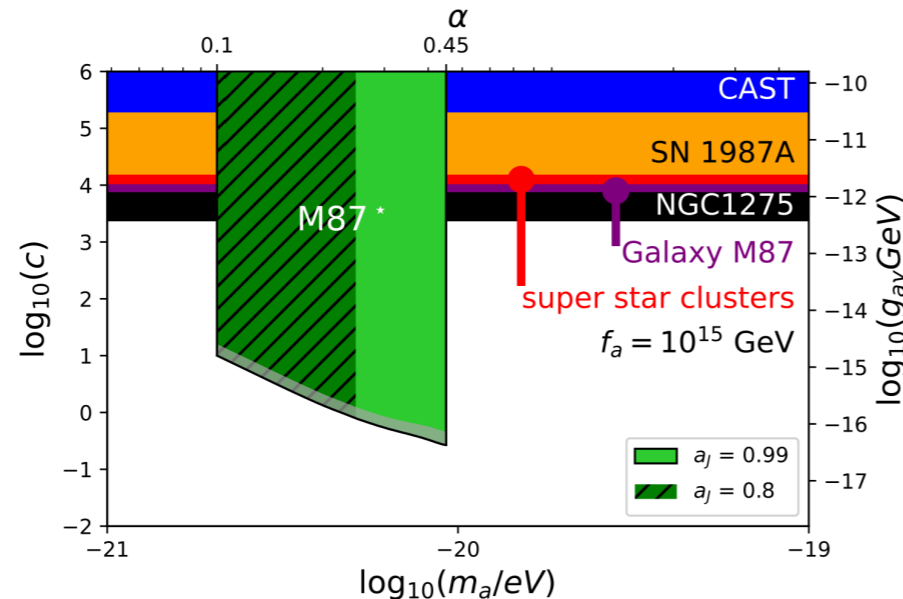
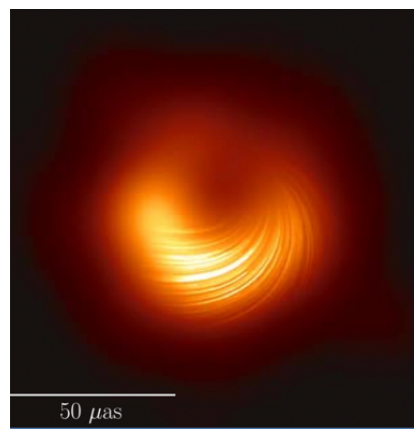
Top quark, Composite Higgs, EFT,
DM, baryon asymmetry, PT, etc.

Wave-like DM & GWs

Job opening for people how has
experience in axion DM searches
(electronics like JPA, quantum qubit, etc)

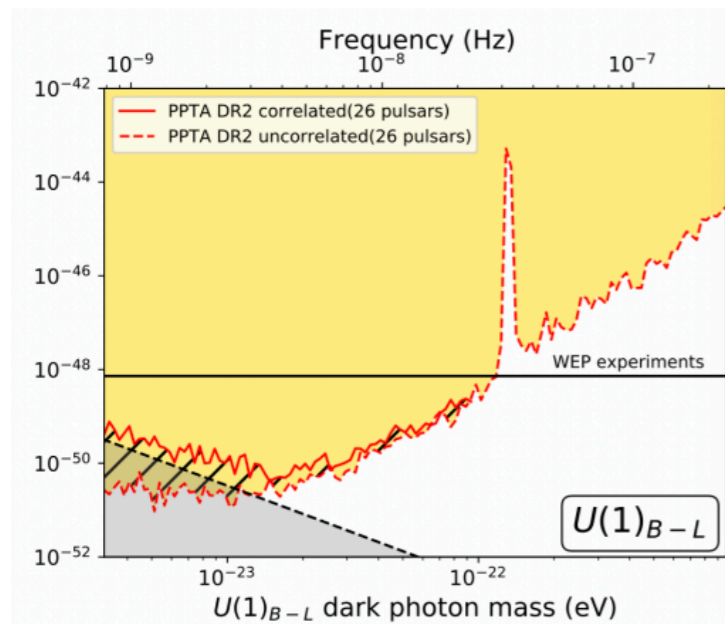
Myself and other collaborations

EHT probe axion (birefringence)



Y.F. Chen, **J. Shu**, X. Xue, Q. Yuan, Y. Zhao,
Phys. Rev. Lett. 124 (2020) no6, 061102

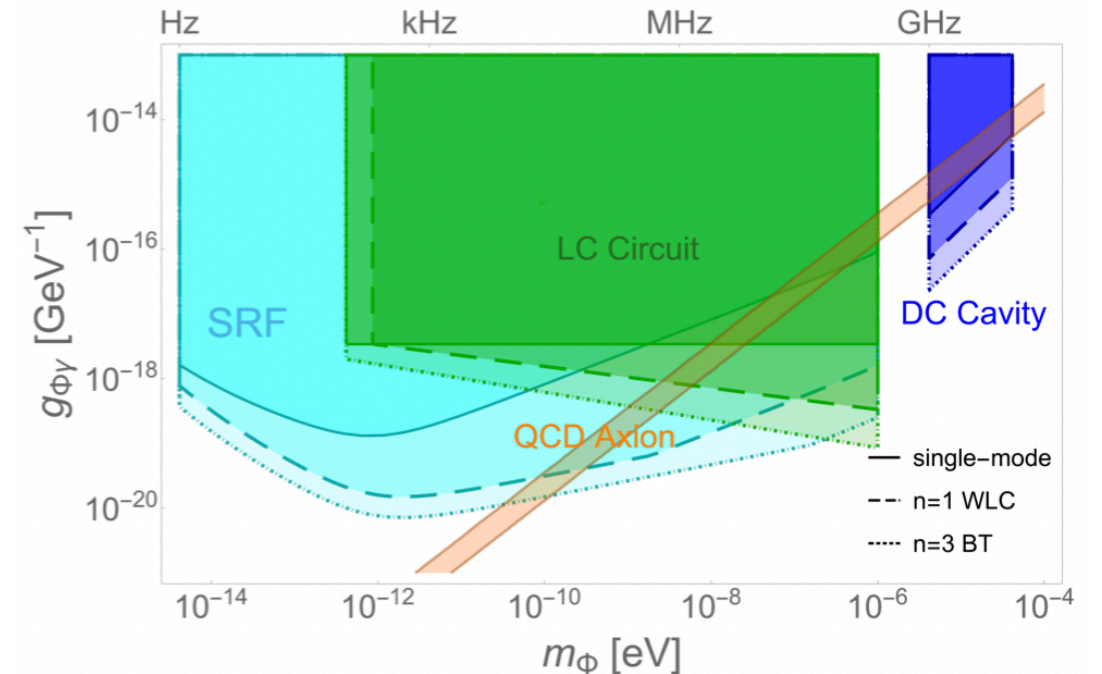
Y.F. Chen, ..., **J. Shu**, ..., Y. Zhao, Nature Astron. 6 (2022) 5, 592-598



PTA probe DPDM

X. Xiao, ..., **J. Shu**, Y. Qiang, ..., Phys.Rev.Res. 4 (2022) 4, L012022

Beyond SQL wave-like DM searches



Y-f. Chen, M-y. Jiang, **J. Shu.**, Y-t. Yang,
Phys.Rev.Res. 4 (2022) 2, 023015
(arxiv time before Haystack)

Y-f. Chen, C-L. Li, Y.X. Liu, **J. Shu.**, Y-t. Yang, Y.-J. Zeng, arxiv: 2309.12387

K. Wurtz, B. M. Brubaker, Y. Jiang, E. P. Ruddy, D. A. Palken
and K. W. Lehnert, PRX Quantum 2 (2021) 4, 040350
Y. Jiang, K. O. Quinlan, M. Malnou, N.E. Frattini, and K. W.
Lehnert, PRX Quantum 4 (2023) 4, 020302

