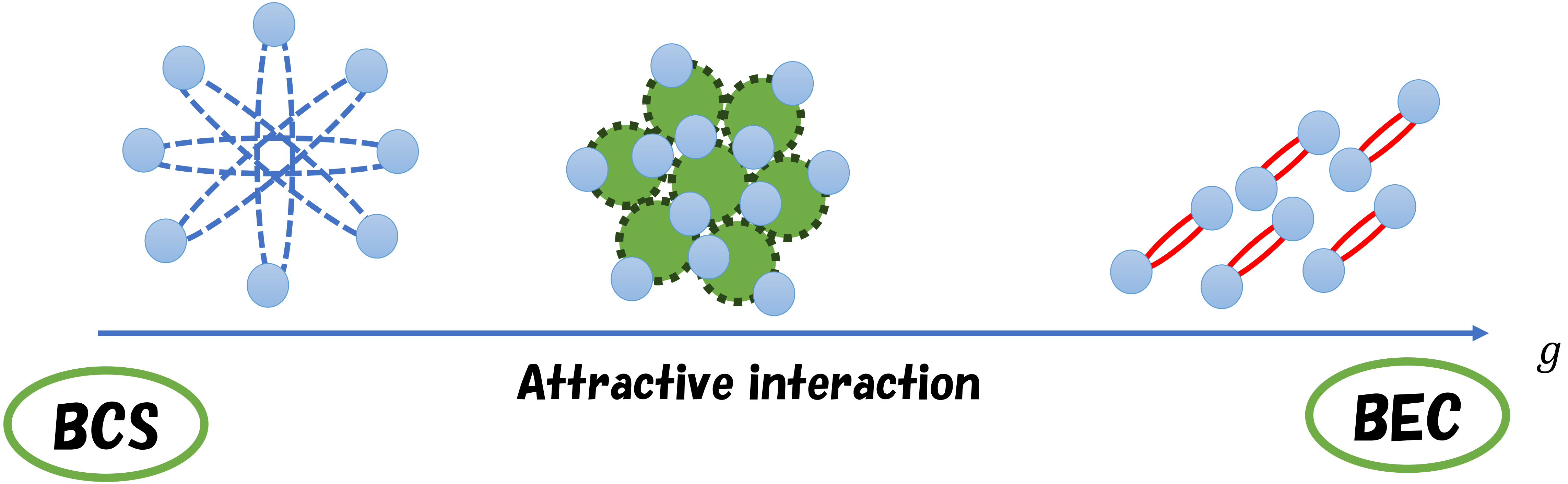
Bridge for understanding from SQCD to QCD

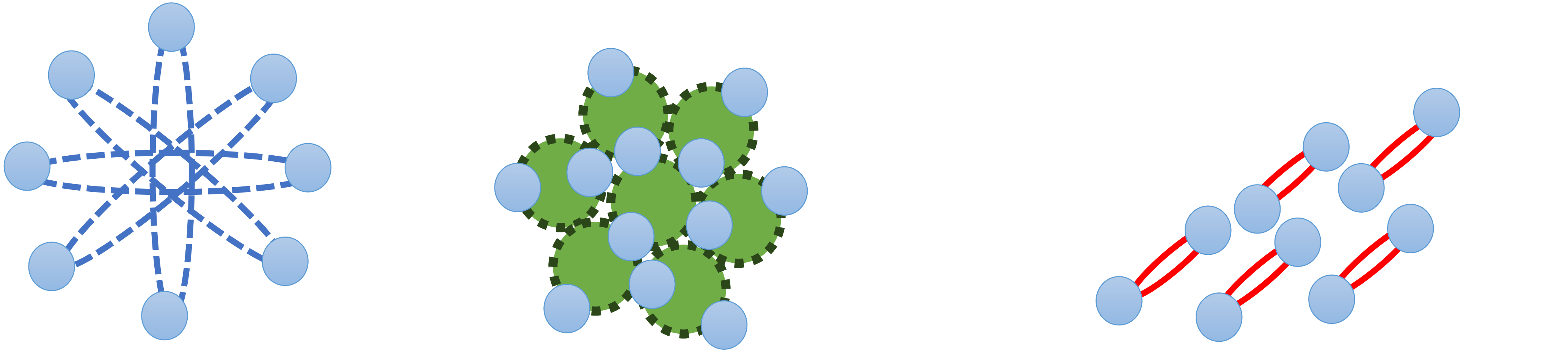
Dan Kondo (IPMU)
with Hitoshi Murayama and Bea Noether

12/17 Hitoshi Fest at IPMU

BCS-BEC cross over



BCS-BEC cross over



g

BCS

Attractive interaction

BEC

SQCD&AMSB (ASQCD)

SQCD+deformation

QCD?

m

Soft mass

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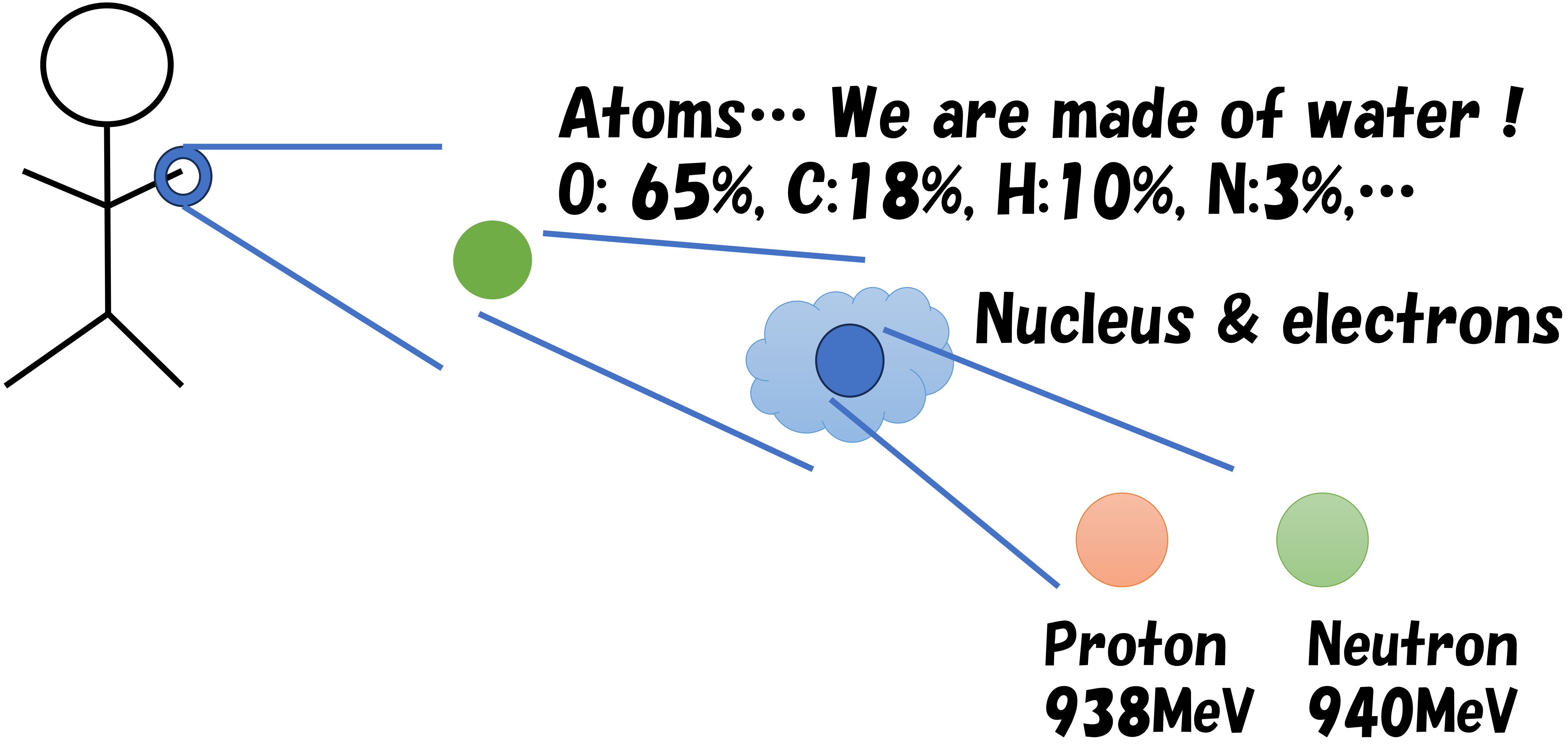
What are we made of ?



Proton
938MeV

…uud

Neutron
940MeV

…udd

Very close !

$m_p \simeq m_n \simeq$ 1GeV ??? $\gg$

In nature
(current quark mass)
m_u =2.16MeV
m_d =4.70MeV
(by pdg)

$m_u \simeq m_d \simeq$ 300MeV ?
(constituent quark mass)

Also,

…uud

Proton
938MeV

$m_p \simeq m_n \simeq$ 1GeV $\gg$??? In nature $m_\pi =$ 100MeV (by pdg)

…udd

Neutron
940MeV

Same quarks different mass ???

Also,

Proton
938MeV ···uud

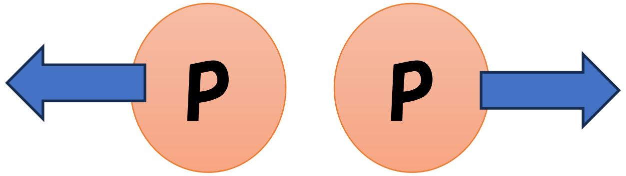
Neutron
940MeV ···udd

$m_p \simeq m_n \simeq$ 1GeV $\gg$??? In nature $m_\pi =$ 100MeV (by pdg)

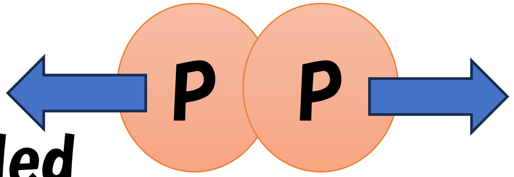
Same quarks different mass ???

m_π light is important

If m_π too heavy···
EM can win.



m_π is light
Nucleons are bounded.



Let us look at QCD Lagrangian !

The chiral quark field… defined as eigenstates of γ_5

$\gamma_5\psi_L(x) = -\psi_L(x),\ \gamma_5\psi_R(x) = \psi_R(x)$

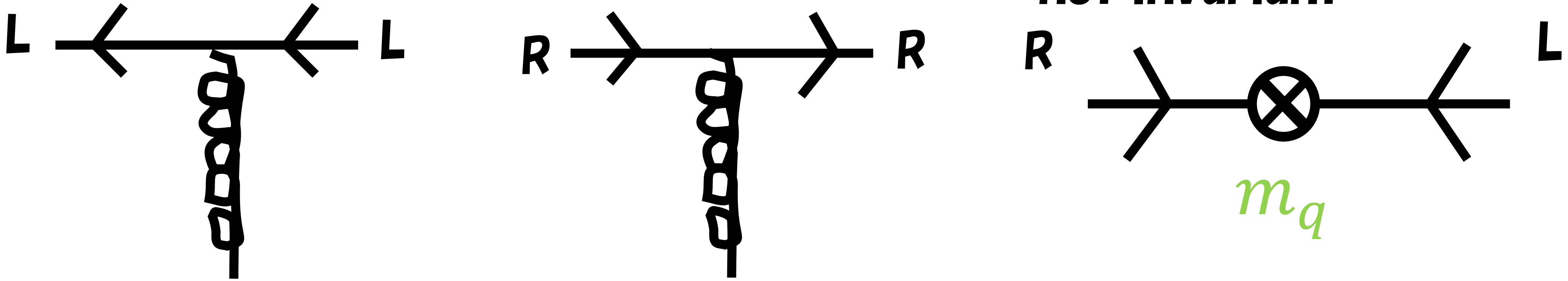
$$L_{QCD} = \sum_q [\bar{\psi}_L^{(q)} i\, D_\mu \gamma^\mu \psi_L^{(q)} + \bar{\psi}_R^{(q)} i D_\mu \gamma^\mu \psi_R^{(q)} - m_q \left(\bar{\psi}_L^{(q)} \psi_R^{(q)} + \bar{\psi}_R^{(q)} \psi_L^{(q)}\right)] - \frac{1}{4} G_{\mu\nu}^a G^{\mu\nu a}$$

$D_\mu = \partial_\mu - igA_\mu$: **covariant derivative**

Chiral transformation: $\psi_L \to e^{i\theta_L}\psi_L,\ \psi_R \to e^{i\theta_R}\psi_R$

Chiral transformation: $\psi_L \to e^{i\theta_L}\psi_L,\ \psi_R \to e^{i\theta_R}\psi_R$

$$\sum_q [\underbrace{\bar{\psi}_L^{(q)} i\, D_\mu\gamma^\mu\psi_L^{(q)} + \bar{\psi}_R^{(q)} iD_\mu\gamma^\mu\psi_R^{(q)}}_{\text{invariant}} - m_q \underbrace{\left(\bar{\psi}_L^{(q)}\psi_R^{(q)} + \bar{\psi}_R^{(q)}\psi_L^{(q)}\right)}_{\text{not invariant}}]$$



In QCD, it is believed that even if $m_q \to 0$, hadrons are massive.
The order parameter of chiral symmetry remains non-zero.

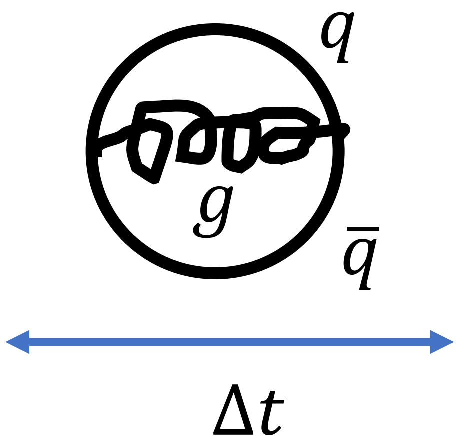
$$\langle 0|\,\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L|0\rangle \neq 0$$

Qualitative picture

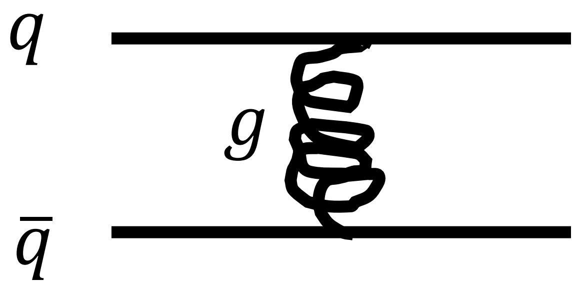
Ref: arxiv:1411.7853 (lecture of Yukinari Sumino)

Start: perturbative vacuum state $|0\rangle$ (no quarks no gluons)

QCD interaction generates quantum fluctuations.
For short time Δt, uncertainty ΔE with $\Delta t \Delta E \sim 1$



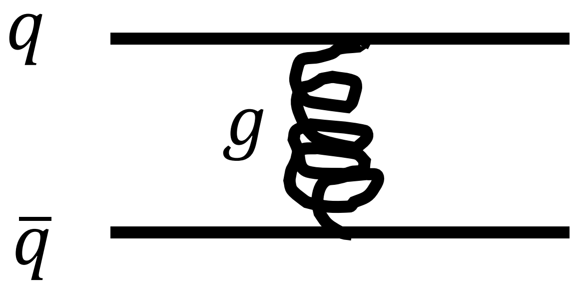
$$\Delta E \geq 2m_q - E_{bind}$$



$\Delta E > 0$ The smaller ΔE the longer the lifetime of $q\bar{q}g$

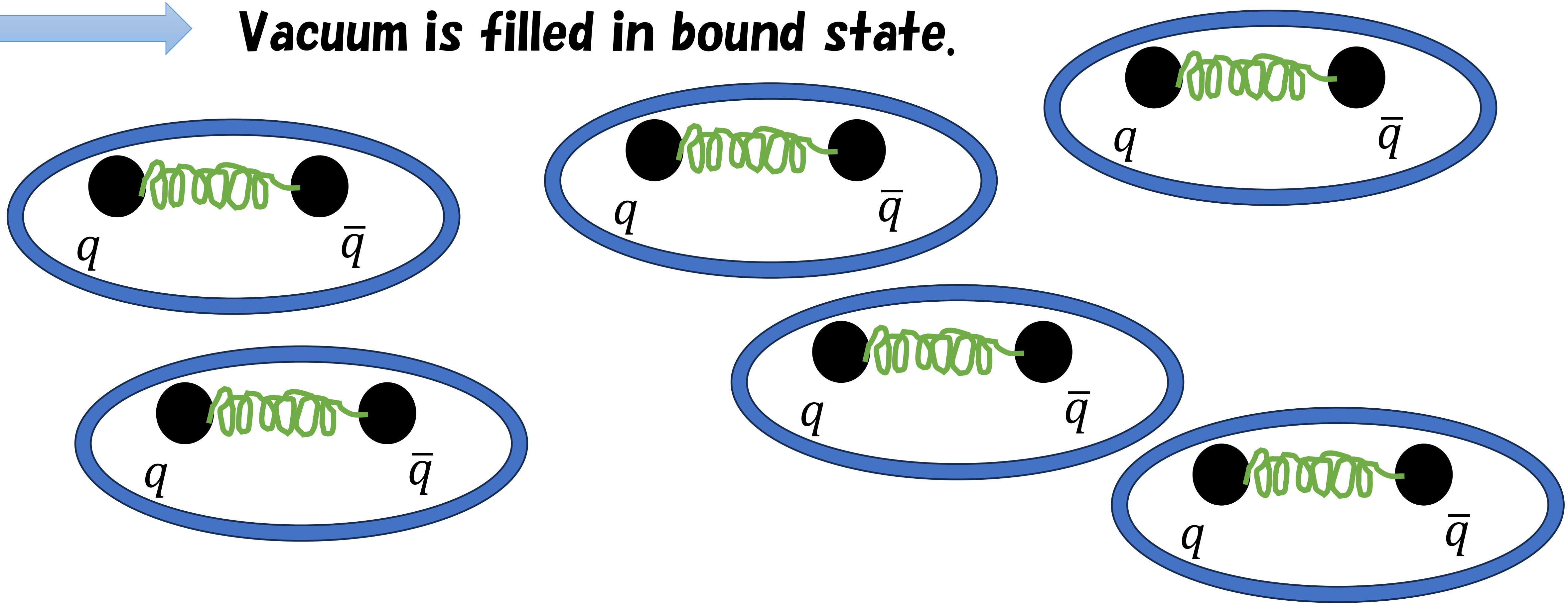
$\Delta E < 0$ The vacuum prefers to make bound state.

$$\Delta E \geq 2m_q - E_{bind}$$

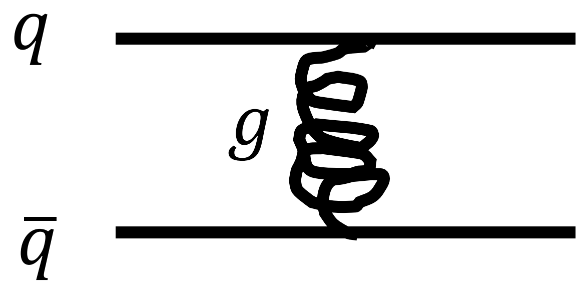


$\Delta E < 0$ **The vacuum prefers to make bound state.**

Vacuum is filled in bound state.

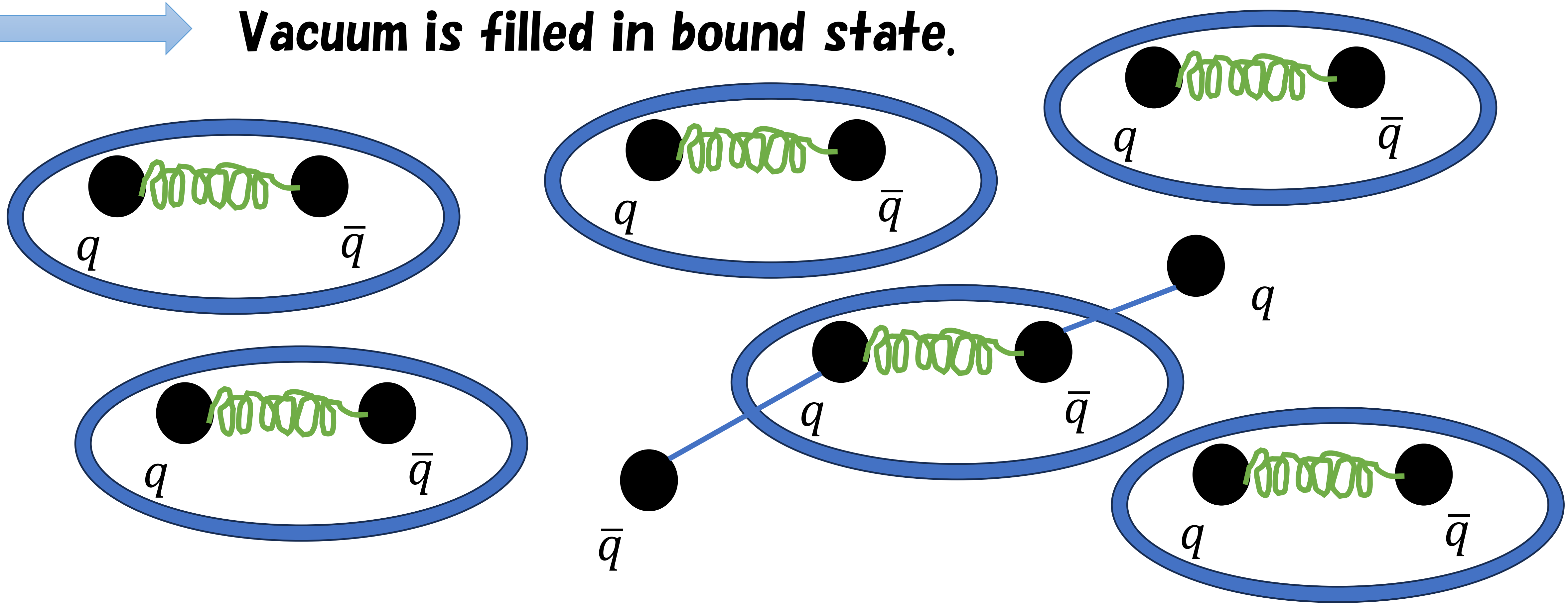


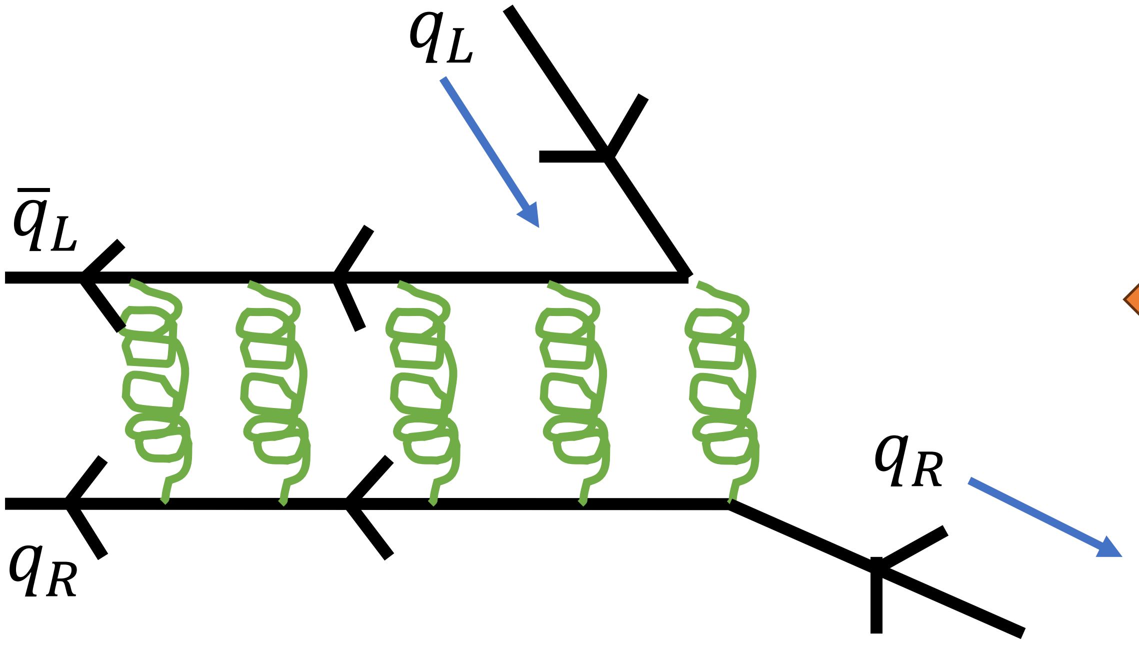
$$\Delta E \geq 2m_q - E_{bind}$$

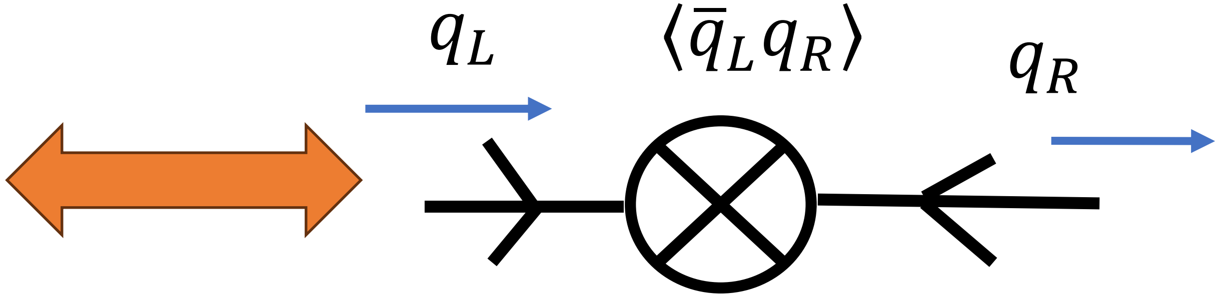


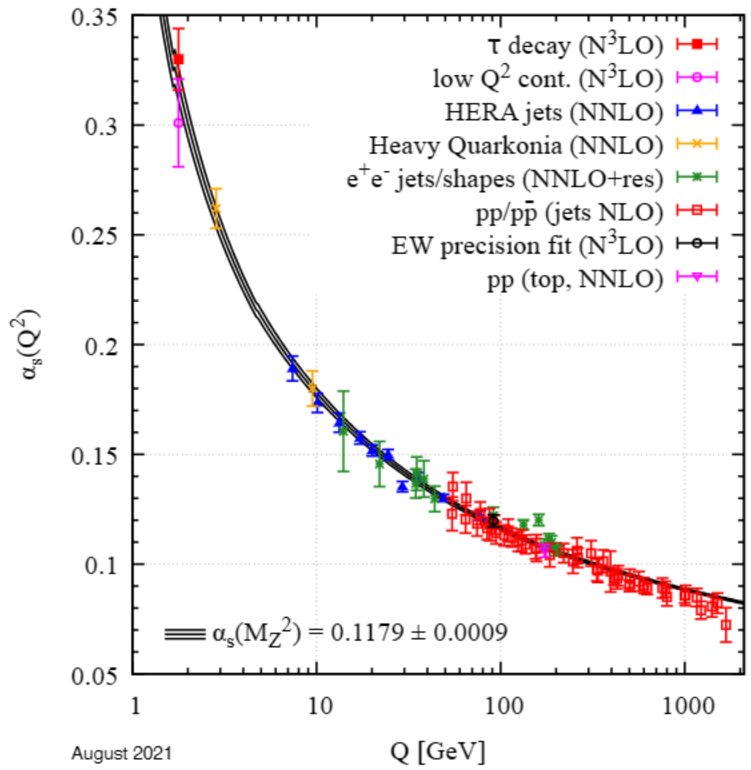
$\Delta E < 0$ The vacuum prefers to make bound state.

Vacuum is filled in bound state.









Chirality violation by quark condensate

Can we understand ?

Ref: PDG QCD review

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SUSY QCD (SQCD)

Super partners
Scalar quark $\tilde{q}$
Gluino λ

If they are decoupled,
particle contents are identical.

$SU(N_c)$ flavor N_f SQCD Lagrangian

``Quark superfield'' $Q = Q + \theta\psi_Q + \theta^2 F,\ \tilde{Q} = \tilde{Q} + \theta\tilde{\psi}_{\tilde{Q}} + \theta^2\tilde{F}$

ψ_Q Left-handed quark

$i\sigma^2\tilde{\psi}_{\tilde{Q}}$ Right-handed quark

Gauge invariant and Supersymmetric

$$L = tr\int d^4\theta\left(Q^\dagger e^V Q + \tilde{Q}e^{-V}\widetilde{Q^\dagger}\right) + \frac{1}{8\pi}Im\left[\tau\int d^2\theta W^{a\alpha}W_\alpha^a\right] + \int d^2\theta \underbrace{W(Q,\tilde{Q})}_{\text{Super potential}} + h.c$$

$SU(N_c)$ flavor N_f SQCD Lagrangian

"Quark superfield" $Q = Q + \theta\psi_Q + \theta^2 F,\ \tilde{Q} = \tilde{Q} + \theta\tilde{\psi}_{\tilde{Q}} + \theta^2 \tilde{F}$

ψ_Q **Left-handed quark**

$i\sigma^2\tilde{\psi}_{\tilde{Q}}$ **Right-handed quark**

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$$L = tr \int d^4\theta \left(Q^\dagger e^V Q + \tilde{Q} e^{-V}\, \widetilde{Q^\dagger}\right) + \frac{1}{8\pi} Im\left[\tau \int d^2\theta W^{a\alpha} W^a_\alpha\right] + \int d^2\theta W(Q,\tilde{Q}) + h.c$$

SUSY … vacuum energy zero

$$\boxed{V = \frac{g^2}{8}\left(Q^\dagger t^a Q - \tilde{Q} t^a \tilde{Q}\right)^2}$$

should be zero in SQCD dynamics

Weyl compensator…encapsulate AMSB process

$\Phi = 1 + \theta^2 m$ …non dynamical spurion of scale invariance

SUSY breaking m is encoded

$$L = \int d^4\theta\ \Phi^*\Phi K + \int d^2\theta\ \Phi^3 W + c.c$$

Effects Tree level $L_{tree} = -m\left(\phi_i \frac{\partial W}{\partial \phi_i} - 3W\right) + c.c$

Induced mass $M_i^2 = \frac{\beta_i(g^2)}{2g_i^2}\underline{m}$, $m_i^2 = -\frac{\dot{\gamma}_i}{4}\underline{m^2}$, $A_{ijk} = \frac{\gamma_i + \gamma_j + \gamma_k}{2}\underline{m}$

AMSB depends on energy scale of our interest only,
UV insensitive.

Ref: Pomarol et.al JHEP05(1999)013,arxiv:9903448

$SU(N_c)$ SQCD

$$L_{tree} = m\left(\phi_i \frac{\partial W}{\partial \phi_i} - 3W\right) + c.c$$

When $W = 0\cdots$ only loop effects

$$m_Q^2 = m_{\tilde{Q}}^2 = \frac{g^4}{(8\pi)^2} 2C_i(3N_c - N_f)m^2$$

$$m_\lambda^2 = \frac{g^2}{16\pi^2}(3N_c - N_f)m$$

- **As long as the theory is asymptotically free $N_f < 3N_c$, the squarks and gauginos have positive mass.**
- **(Integrating them out) the light particle contents are same as those of non-SUSY QCD!**

Effects

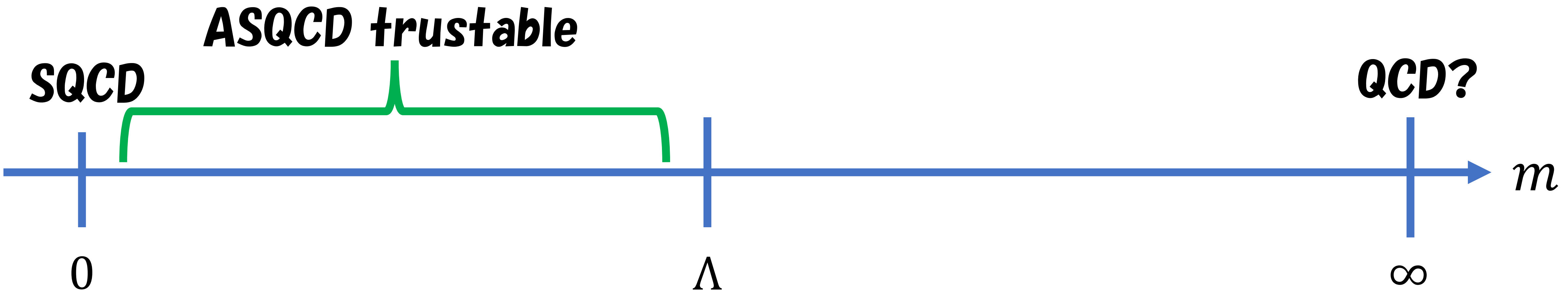
Tree level

$$L_{tree} = -m\left(\phi_i \frac{\partial W}{\partial \phi_i} - 3W\right) + c.c$$

Induced mass

$$M_i^2 = \frac{\beta_i(g^2)}{2g_i^2} m, \quad m_i^2 = -\frac{\dot{\gamma}_i}{4} m^2, \quad A_{ijk} = \frac{\gamma_i + \gamma_j + \gamma_k}{2} m$$

Naively $m \to \infty$, super partner decouples. Seemingly back to original QCD.



$0 \le N_f < N_c$ ADS case

Refs: Affleck et.al Phys.Rev.Lett.51,1026(1983)
Seiberg Nucl.Phys.B435,129(1995); Phys.Rev.D49,6857(1994)
Intriligator et.al Nucl.Phys.B431,551(1994)

SQCD dynamics$\cdots V = \frac{g^2}{8}\left(Q^\dagger t^a Q - \tilde{Q} t^a \tilde{Q}\right)^2 = 0$

Q_α^f (α: **color**, f: **flavor**)$\cdots QQ^\dagger$**can be diagonalized by unitary matrix**

t^a: **generator** $tr\, t^a = 0$

D-flat direction is $Q = \tilde{Q} = \begin{array}{cc} \overbrace{}^{N_f} & \\ \begin{pmatrix} \phi & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \phi \\ 0 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 0 \end{pmatrix} & \begin{array}{l} \left.\begin{array}{c} \\ \\ \\ \end{array}\right\} N_f \\ \\ \\ \\ \end{array} \end{array}$

$$M_{ij} = \delta_{ij}\phi^2$$

Affleck-Dine-Seiberg (ADS) superpotential

$$W = (N_c - N_f)\left(\frac{\Lambda^{3N_c - N_f}}{\det M}\right)^{\frac{1}{N_c - N_f}} ;\ M_{\tilde{i}j} = \widetilde{Q}_{\tilde{i}} Q_j$$

The potential is

$$V = \frac{1}{2N_f}\left| 2N_f \frac{1}{\phi}\left(\frac{\Lambda^{3N_c-N_f}}{\phi^{2N_f}}\right)^{1/(N_c-N_f)} \right|^2 - (3N_c - N_f)m\left(\frac{\Lambda^{3N_c-N_f}}{\phi^{2N_f}}\right)^{\frac{1}{N_c-N_f}} + c.c.$$

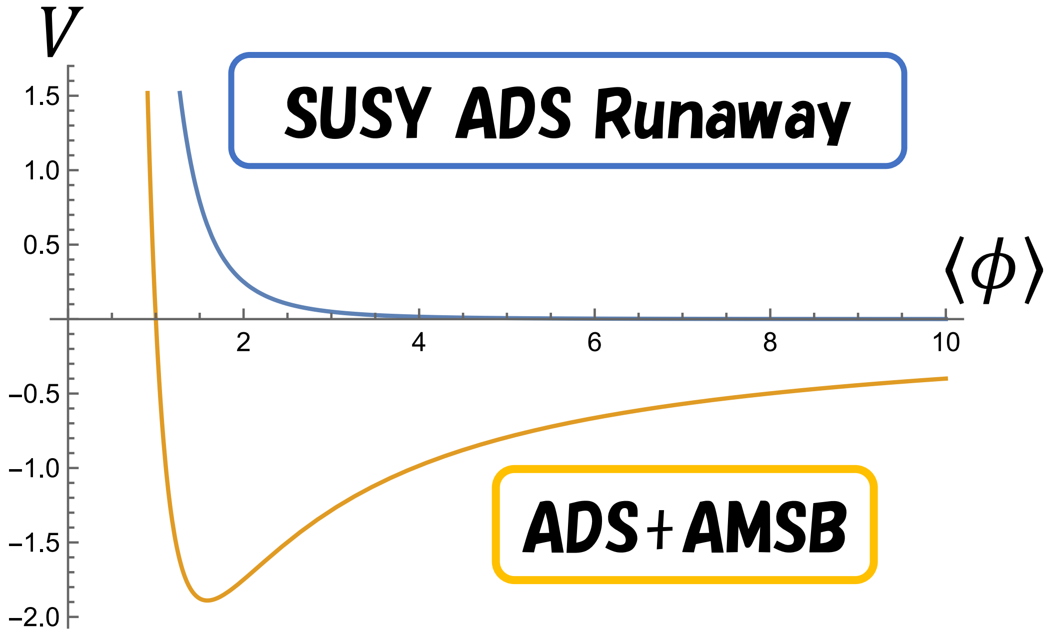
ADS **AMSB**

There is a minimum with AMSB !

$$M_{ij} = \delta_{ij}\Lambda^2\left(\frac{4\,(N_c+N_f)}{3N_c-N_f}\frac{\Lambda}{m}\right)^{(N_c-N_f)/N_c}$$

$$SU(N_f)_L \times SU(N_f)_R \to SU(N_f)_V$$

…chiral symmetry breaking



Massless particle spectrum is Nambu-Goldstone bosons

Scalar and fermion partners have mass $\propto m$

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Let us see the fluctuation.

$$Q = \begin{pmatrix} v\xi^T \\ 0 \end{pmatrix}, \qquad \tilde{Q} = \begin{pmatrix} v\xi \\ 0 \end{pmatrix} \qquad \xi \to h(\xi, g_L, g_R)\xi g_L^T, \xi^T \to h^*(\xi, g_L, g_R)\xi g_R^T, U = \xi^2$$

Kinetic term : $\left|D_\mu Q\right|^2 + \left|D_\mu \tilde{Q}\right|^2 = \frac{v^2}{2} Tr\left[\partial_\mu U^\dagger \partial^\mu U\right] + \frac{1}{2}\mathrm{v}^2 \mathrm{Tr}\left(2g\rho_\mu + i\left(-\xi^\dagger\partial_\mu\xi - \xi\partial_\mu\xi^\dagger\right)\right)^2$

$\longrightarrow$ **Decay constant** : $f_\pi^2 = 8\Lambda^2\left(\frac{N_c + N_f}{3N_c - N_f}\frac{\Lambda}{m}\right)^{(N_c - N_f)/N_c}$

$$F_\pi^2 \propto \Lambda_{\mathrm{eff}}^2, \tag{3.8}$$

$$F_T \propto \Lambda_{\mathrm{eff}}^3 \tag{3.9}$$

so that $m_\pi^2 \propto \Lambda_{\mathrm{eff}} M_q$. Unfortunately, it is easy to check that Eqs. (3.8) and (3.9) do not have the same scaling behavior with m_{soft} in the formal decoupling limit as the weak-coupling results Eqs. (3.3) and (3.4), no matter which of Eqs. (3.5), (3.6), or (3.7) applies. Furthermore, the large N_c scaling of the large-m_{soft} chiral Lagrangian does not conform with expectations from ordinary QCD. Nonsupersymmetric chiral perturbation theory implies [18] that $F_\pi^2 \propto N_c$ and $F_T \propto N_c$, but the formal decoupling limit of the weakly-coupled SQCD chiral Lagrangian scales as $F_\pi^2 \propto N_c^0$ and $F_T \propto N_c^0$.

$O(N_c^0)\cdots$ **glueball contribution ?**

Martin & Wells

Phys.RevD58 (1998)115013

Rescaling anomaly N.Arkani-Hamed & H. Murayama Phys.RevD57(1998)6638-6648

$$\frac{8\pi^2}{g_h^2} = \frac{8\pi^2}{g_c^2} + \frac{N_c}{8\pi^2}\log g_c^2$$

With t' Hooft coupling $N_c g_c^2 = g_t^2$

Scaling relation:

$$\Lambda^{3N_c - N_f} = \mu^{3N_c - N_f} e^{-8\pi^2/g_h^2} = \mu^{3N_c - N_f} e^{-8\pi^2/g_c^2} (g_c^2)^{-N_c} = \mu^{3N_c - N_f} \underline{N_c^{N_c}} e^{-8\pi^2/g_t^2} (g_t^2)^{-N_c}$$

$$f_\pi^2 = 8\left(\frac{N_c + N_f}{3N_c - N_f} \frac{\Lambda^{3N_c - N_f}}{m^{N_c - N_f}}\right)^{\frac{1}{N_c}}$$

$$f_\pi^2 = O(N_c)$$

Not necessarily disconnected !

ρ meson

Kawarabayashi and Suzuki Phys.Rev.Lett 16(1966)255
Riazuddin and Fayazuddin Phys.Rev. 147 (1966)1071
Bando et al Phys.Rev.Lett .54(1985) 1215
Bando et al Nucl.Phys.B259 (1985) 493

Traditionally… ρ meson as dynamical gauge boson of Hidden Local symmetry (HLS)

$G_{global} \times H_{local}$ **with eg.** $\mathrm{G}_{global} = SU(2)_L \times SU(2)_R$, $H_{local} = SU(2)_V$

$$L = L_A + aL_V$$

L_V **is an auxiliary field and** a **is arbitrary parameter.**

If L_{kin} **is generated (quantum mechanically),** $L_{HLS} = L_V + aL_A + L_{kin}$
successful phenomenological results can be derived (in particular with $a = 2$**).**

- **Universality of ρ meson coupling (ρ universality)** $g_{\rho\pi\pi} = g_{\rho NN} = \cdots$
- **KSRF relation** $m_\rho^2 = ag_{\rho\pi\pi}^2 f_\pi^2$
- **Vector meson dominance**

Our case…

Kinetic term : $\left|D_\mu Q\right|^2 + \left|D_\mu \tilde{Q}\right|^2 = \frac{v^2}{2} Tr\left[\partial_\mu U^\dagger \partial^\mu U\right] + \frac{\mathrm{v}^2}{2}\mathrm{Tr}\left(2g\rho_\mu + i(-\xi^\dagger\partial_\mu\xi - \xi\partial_\mu\xi^\dagger)\right)^2$

$\longrightarrow$ $a = 1$

Wess-Zumino-Witten (WZW) term

Effective action $W = -i \log\left[\int dQ\, d\tilde{Q}\, dV \exp\left[i \int d^4x\, L\right]\right]$

Transformation: $Q \to \xi^* Q,\ \tilde{Q} \to \xi^\dagger \tilde{Q}$

$\longrightarrow$ $DQD\tilde{Q} \to DQJ(\xi^T)D\tilde{Q}J(\xi)$

Jacobians arise

$$J(\xi) = \exp\left[iN_c \int Tr\left(\xi^\dagger d\xi\right)^5\right] \qquad J(\xi^T) = \exp\left[-iN_c \int Tr\left(\xi d\xi^\dagger\right)^5\right]$$

$$W' = W - i \log J(\xi) - i \log J(\xi^T)$$

$\longrightarrow$ $N_c \int Tr\left[\left(\xi^\dagger d\xi\right)^5 - \left(\xi d\xi^\dagger\right)^5\right] = N_c \int Tr\left[\left(\xi^\dagger d\xi - \xi d\xi^\dagger\right)^5\right] = N_c \int Tr\left[\left(U^\dagger dU\right)^5\right]$

(up to local counter term)

N_c **comes from** N_f **block of fermions and** $N_c - N_f$ **block of fermions**

Ref: D' Hoker & Farhi, NuclB248(1984)77-89

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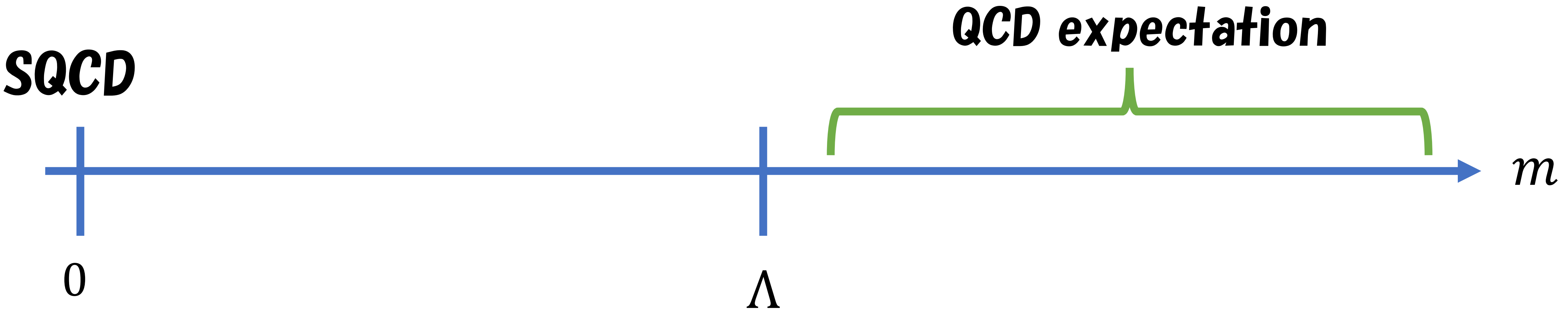
For $m \gg \Lambda$ expectation from QCD

$$\langle q\bar{q}\rangle \propto \Lambda_{QCD}^3$$
$$\langle GG\rangle \propto \Lambda_{QCD}^4$$



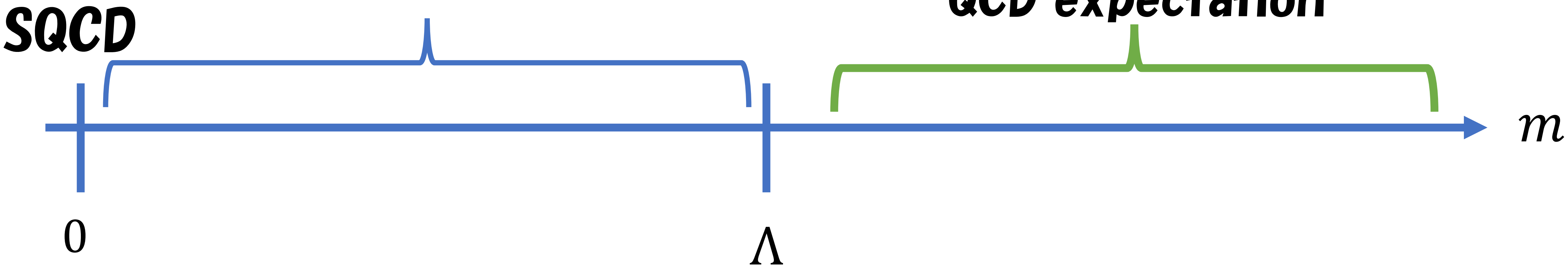
For $m \gg \Lambda$ expectation from QCD

$$\langle q\bar{q} \rangle \propto \Lambda_{QCD}^3$$
$$\langle GG \rangle \propto \Lambda_{QCD}^4$$

Nearly QCD expected

ASQCD → **Connected or disconnected?**

QCD expectation



How to calculate for $m \ll \Lambda$ (ASQCD) ?

Quark / Squark condensate

As a chiral superfield

$$\langle M \rangle = \langle \tilde{q}\tilde{q}^* \rangle + \theta^2 \langle q\bar{q} \rangle$$

Potential minimum

F-component by equation of motion

$$\langle q\bar{q} \rangle = \langle F_M \rangle = -\left\langle \frac{\partial W}{\partial M} \right\rangle$$

How to calculate for $m \ll \Lambda$ (ASQCD) ?

Gluon/Gluino condensate

Generating functional

$$\log Z \supset \frac{1}{16\pi i}\int d^2\theta\, \tau W W$$

Promote to spurion, which sources $\langle GG \rangle$, F-component sources $\langle \lambda\lambda \rangle$

$$\langle GG \rangle = 16\pi i \frac{\partial}{\partial \tau}\log Z = 16\pi i \left\langle \frac{\partial V}{\partial \tau} \right\rangle$$

$$\langle \lambda\lambda \rangle = 16\pi i \frac{\partial}{\partial F_\tau}\log Z = 16\pi i \left\langle \frac{\partial W}{\partial \tau} \right\rangle$$

$$\langle GG \rangle = 16\pi i \frac{\partial}{\partial \tau} \log Z = 16\pi i \left\langle \frac{\partial V}{\partial \tau} \right\rangle$$

$$\langle \lambda\lambda \rangle = 16\pi i \frac{\partial}{\partial F_\tau} \log Z = 16\pi i \left\langle \frac{\partial W}{\partial \tau} \right\rangle$$

Remember the dynamical scale

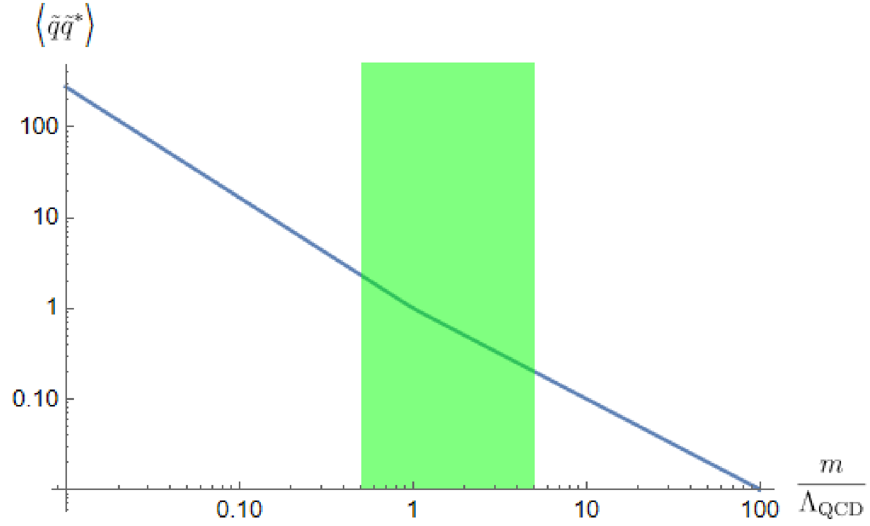
$$\Lambda = \mu \exp\left[\frac{2\pi i}{3N_c - N_f}\tau\right] \longrightarrow \frac{\partial}{\partial \tau} = \frac{2\pi i}{3N_c - N_f}\Lambda\frac{\partial}{\partial \Lambda}$$

$$\boxed{\begin{aligned} \langle GG \rangle &= -\frac{32\pi^2}{3N_c - N_f}\Lambda \left\langle \frac{\partial V}{\partial \tau} \right\rangle \\ \langle \lambda\lambda \rangle &= -\frac{32\pi^2}{3N_c - N_f}\Lambda \left\langle \frac{\partial W}{\partial \tau} \right\rangle \end{aligned}}$$

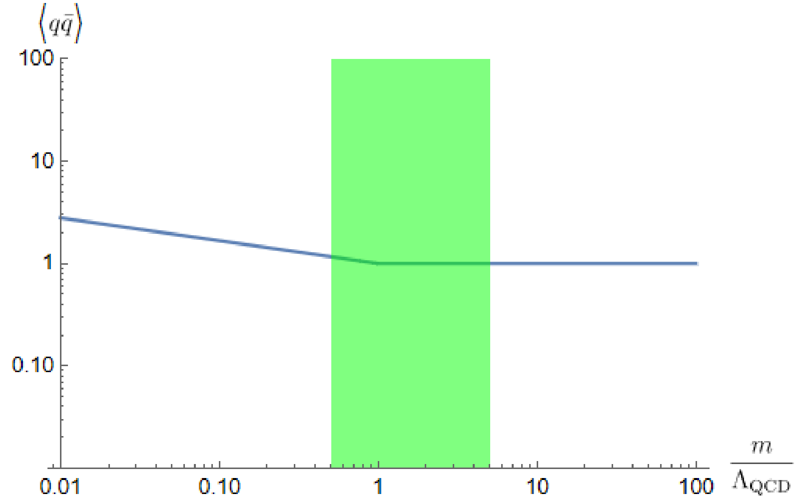
ADS $0 \leq N_f < N_c$

Quark condensate

$$\langle \tilde{q}^* \tilde{q} \rangle = 4 N_f \Lambda^2 \left(\frac{3N_c - N_f}{N_c + N_f} \frac{m}{\Lambda} \right)^{\frac{N_f}{N_c}}$$



$$\langle \bar{q} q \rangle = 4 N_f \Lambda^3 \left(\frac{3N_c - N_f}{N_c + N_f} \frac{m}{\Lambda} \right)^{\frac{N_f}{N_c}}$$

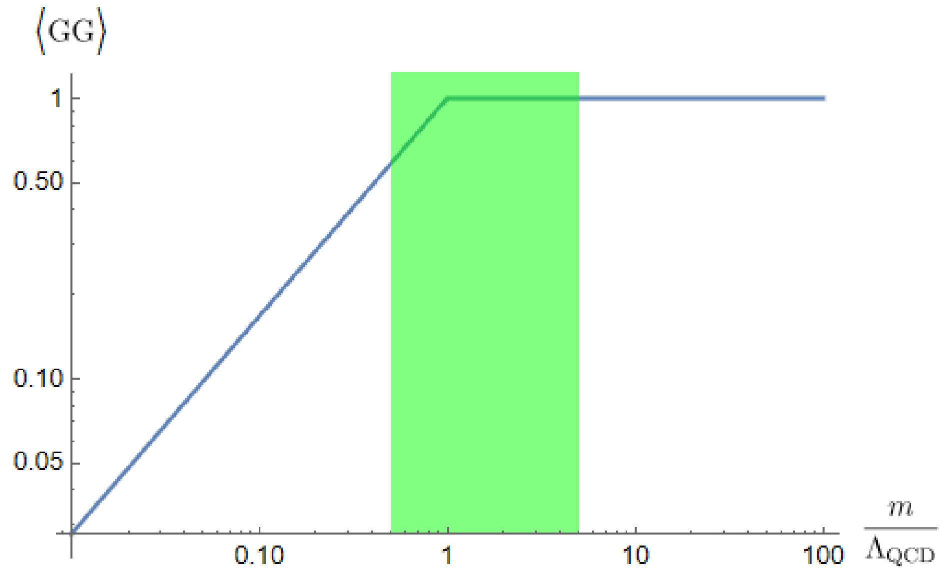


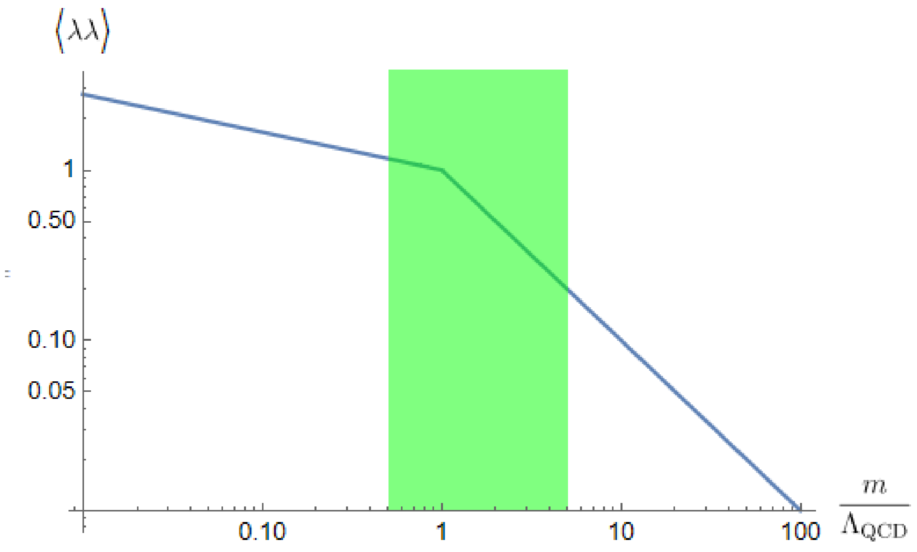
ADS $0 \leq N_f < N_c$

Gluon condensate

$$\langle GG \rangle = 128\pi^2 N_f \Lambda^4 \left(\frac{3N_c - N_f}{N_c + N_f} \frac{m}{\Lambda} \right)^{\frac{N_f}{N_c}}$$

$$\langle \lambda\lambda \rangle = 32\pi^2 N_f \Lambda^3 \left(\frac{3N_c - N_f}{N_c + N_f} \frac{m}{\Lambda} \right)^{\frac{N_f}{N_c}}$$





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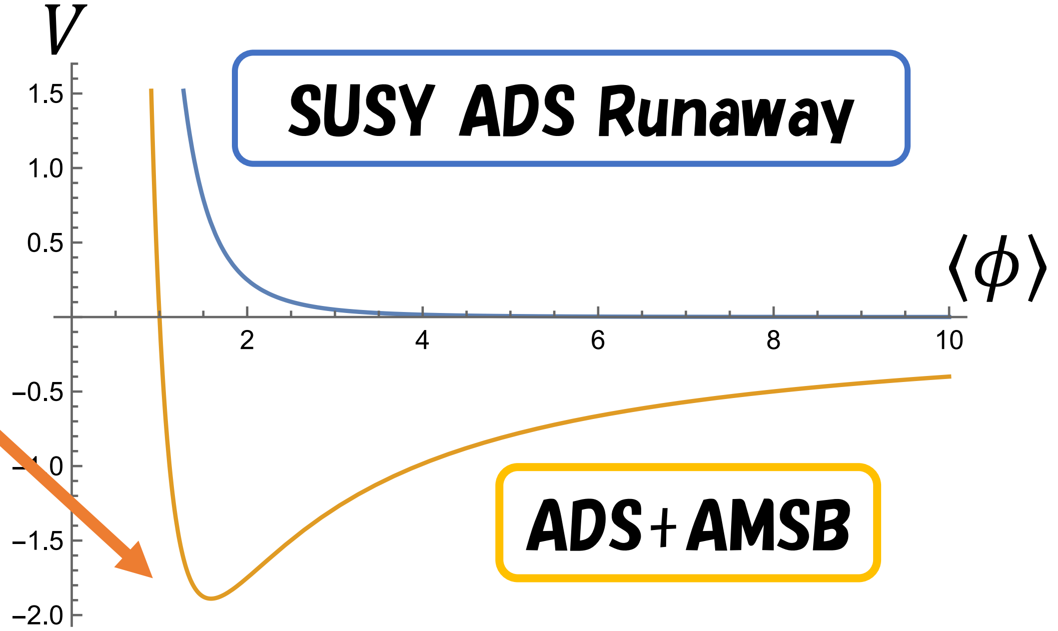
How to calculate ?



Potential → We know the vacuum.

Consider fluctuation… $M = \phi_0 + \frac{1}{2}(\sigma + i\pi)$

Mass matrix is $\begin{pmatrix} \frac{\partial^2 V}{\partial \sigma^2} & \frac{\partial^2 V}{\partial \sigma \partial \pi} \\ \frac{\partial^2 V}{\partial \pi \partial \sigma} & \frac{\partial^2 V}{\partial \pi^2} \end{pmatrix}\Big|_{M=\phi_0}$

Example

ADS $N_f = 3, N_c = 4$

Massless case $m_Q = 0$

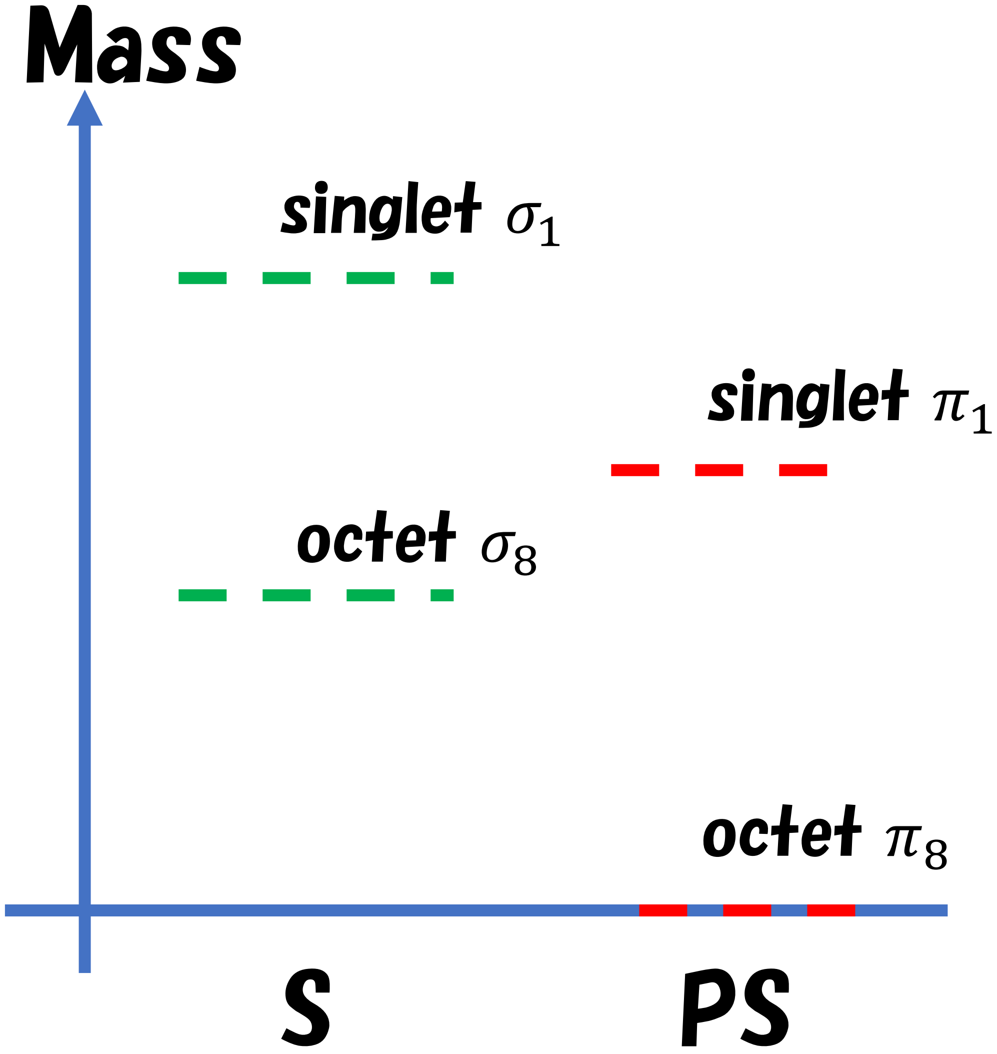
Singlet scalar : $\frac{648}{7}m^2$

Octet scalar : $\frac{162}{49}m^2$

Singlet pseudo-scalar : $\frac{486}{7}m^2$

Octet pseudo-scalar : 0

Pion is Nambu-Goldstone boson



Example

ADS $N_f = 3, N_c = 4$

Add quark mass perturbation

$$W \supset Tr\ [m_Q M]$$

Singlet scalar :

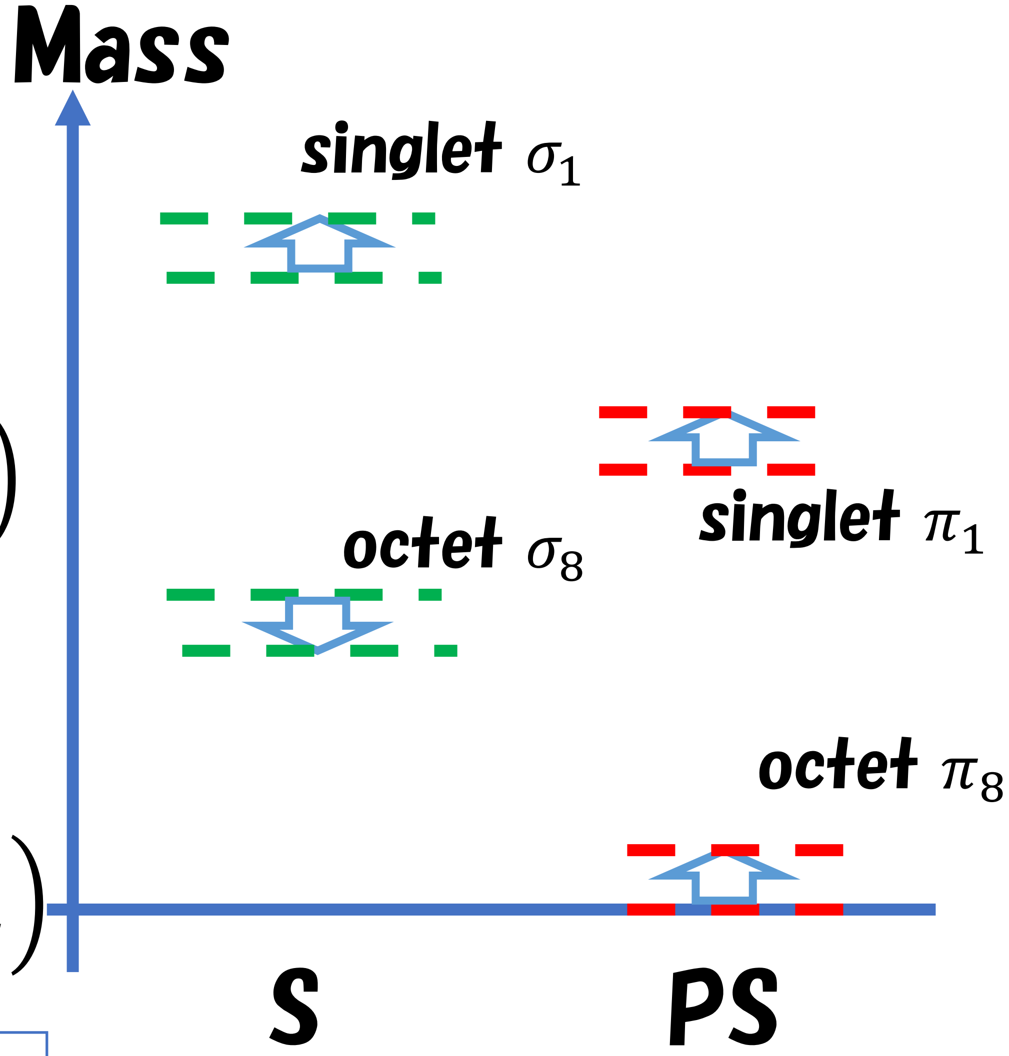
$$\frac{648}{7}m^2 + \frac{1496}{49}m(m_{q_1} + m_{q_2} + m_{q_3})$$

Octet scalar : $\frac{162}{49}m^2 - 3m\left(m_{q_i} + m_{q_j}\right)$

Singlet pseudo-scalar:

$$\frac{486}{7}m^2 + \frac{872}{21}m(m_{q_1} + m_{q_2} + m_{q_3})$$

Octet pseudo-scalar : $\frac{25}{7}m\left(m_{q_i} + m_{q_j}\right)$



Pion is pseudo-Nambu-Goldstone boson

Example

ADS $N_f = 3, N_c = 4$

Compare

Singlet scalar :

$$\frac{648}{7}m^2 + \frac{1496}{49}m\left(m_{q_1} + m_{q_2} + m_{q_3}\right)$$

Octet scalar : $\frac{162}{49}m^2 - 3m\left(m_{q_i} + m_{q_j}\right)$

Singlet pseudo-scalar:

$$\frac{486}{7}m^2 + \frac{872}{21}m\left(m_{q_1} + m_{q_2} + m_{q_3}\right)$$

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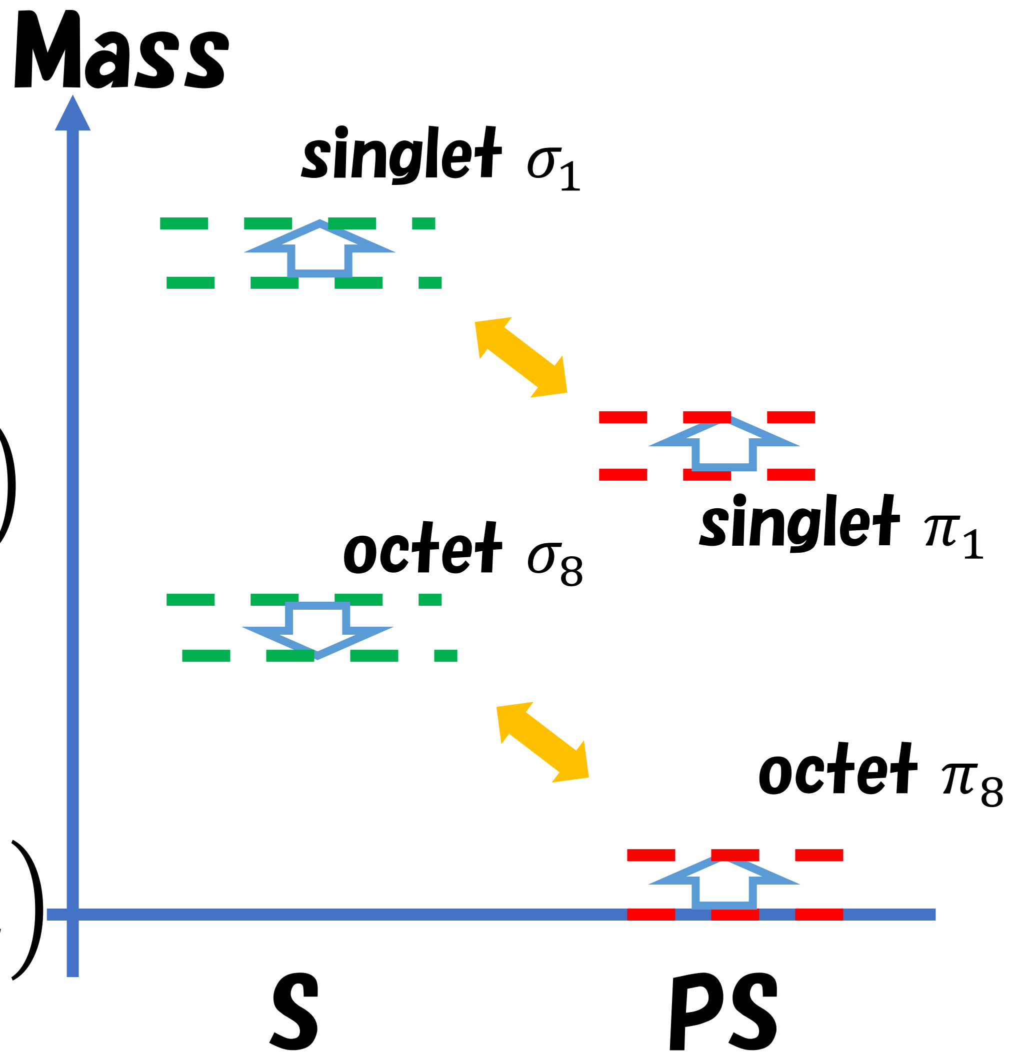
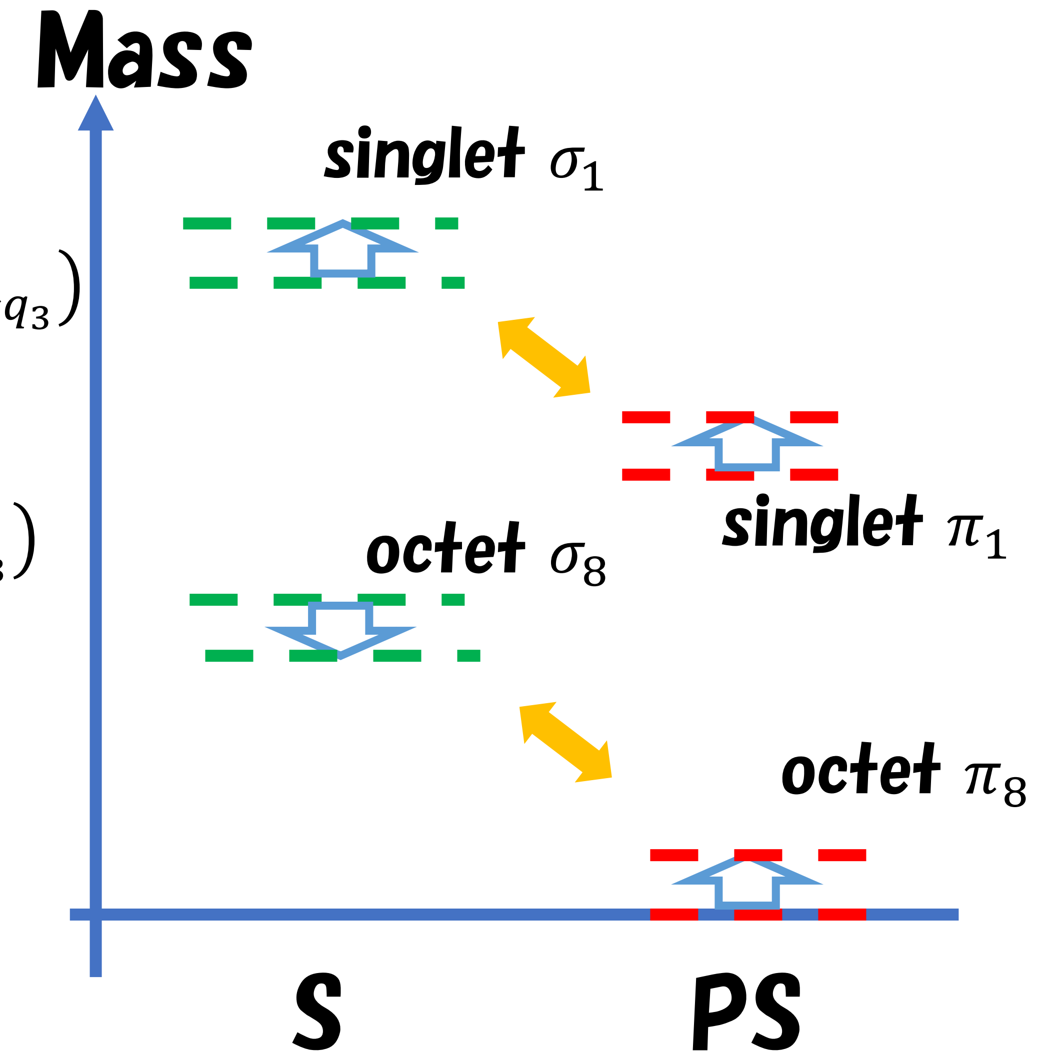


Example

ADS $N_f = 3, N_c = 4$

Compare

$$m_\sigma^2 + \frac{21}{25} m_\pi^2 = \begin{cases} \dfrac{20406}{175} m^2 + \dfrac{11904}{175} m(m_{q_1} + m_{q_2} + m_{q_3}) & \textbf{Singlet} \\ \dfrac{162}{49} m^2 + \dfrac{128}{49} m(m_{q_1} + m_{q_2} + m_{q_3}) & \textbf{Octet} \end{cases}$$



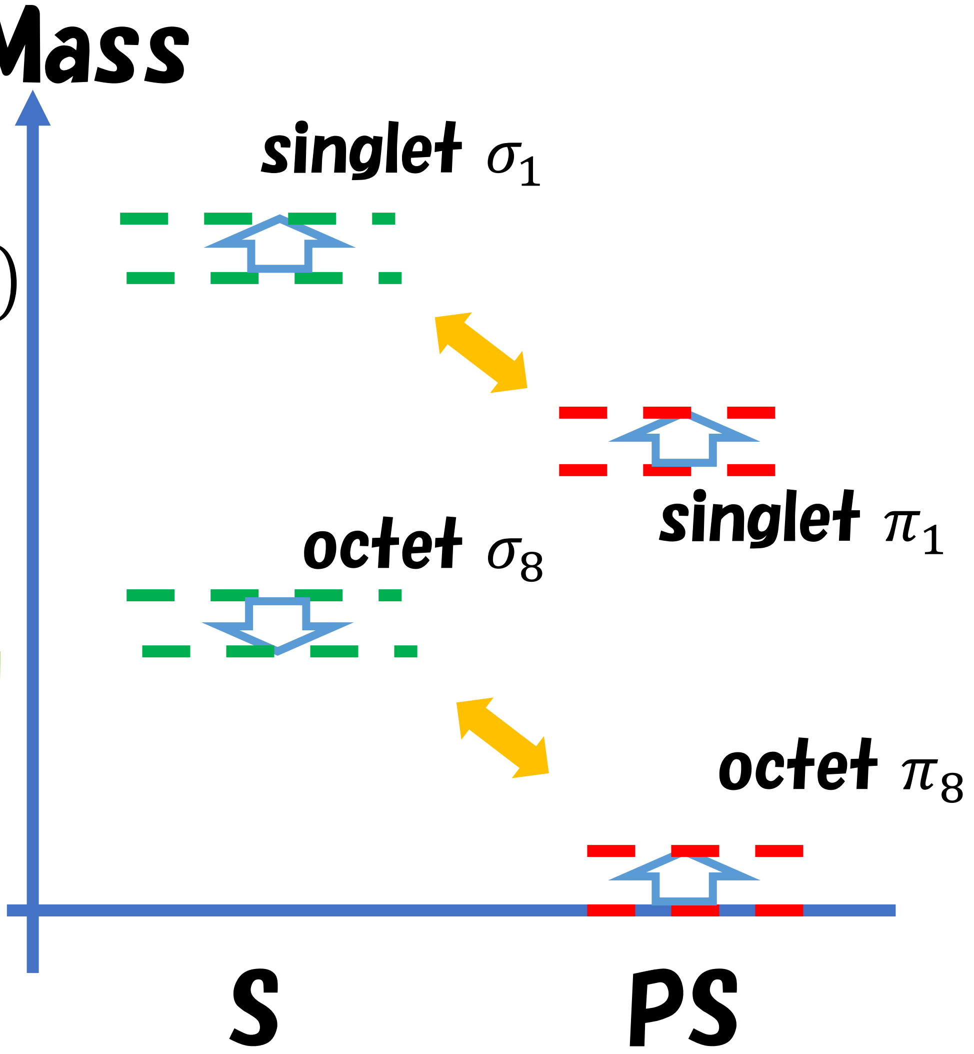
Example

ADS $N_f = 3, N_c = 4$

Compare

$$\underbrace{m_\sigma^2 + \frac{21}{25} m_\pi^2 = \begin{cases} \dfrac{20406}{175} m^2 + \dfrac{11904}{175} m\left(m_{q_1} + m_{q_2} + m_{q_3}\right) & \textbf{Singlet} \\[2ex] \dfrac{162}{49} m^2 + \dfrac{128}{49} m\left(m_{q_1} + m_{q_2} + m_{q_3}\right) & \textbf{Octet} \end{cases}}_{\equiv\, m_0^2 \text{ constant to be renormalized}}$$

$$\Rightarrow \quad m_\sigma^2 = m_0^2 + c\, m_\pi^2 \,;\; c < 0$$

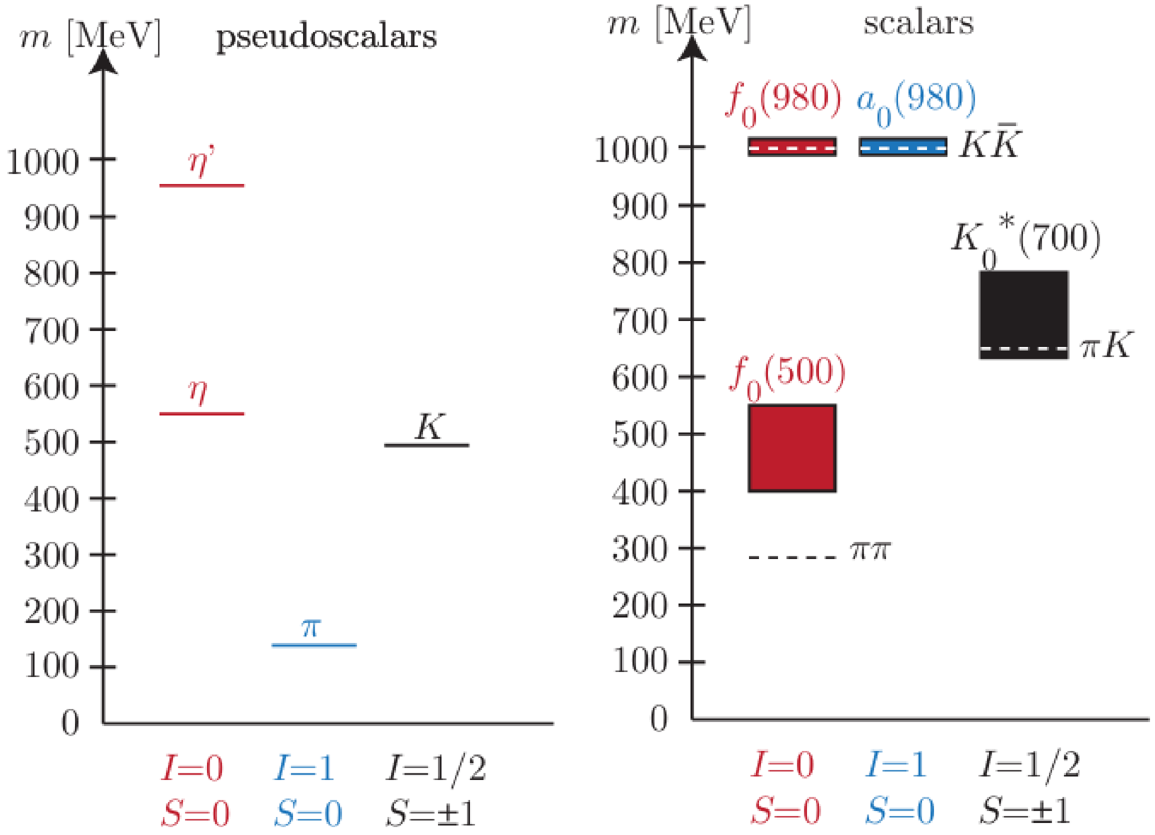


Previous implication $m_\sigma^2 - m_0^2 = c\, m_\pi^2 \quad c < 0$

When we studied SIDM…

PDG data (real)

$f_0(980), a_0(980)$ … close to $K\bar{K}$
Possibly $f_0(500)$…close to $\pi\pi$
if m_u, m_d is large.



<u>Resonance or bound state (molecule)</u>

Kondo et.al JHEP09(2022)041, arxiv:2205.08088

Conclusion

From χSB ASQCD vaccum,

We derived the chiral Lagrangian and WZW term.

We studied the condensation of quarks and gluons.

We studied the mass spectrum.

ASQCD can be a bridge toward understanding QCD.