

# Ensemble Averaging in Quantum Gravity

Masahito Yamazaki



Dec. 19, 2024 @ Hitoshi Fest

**“Life is full of (pleasant) surprises.”**

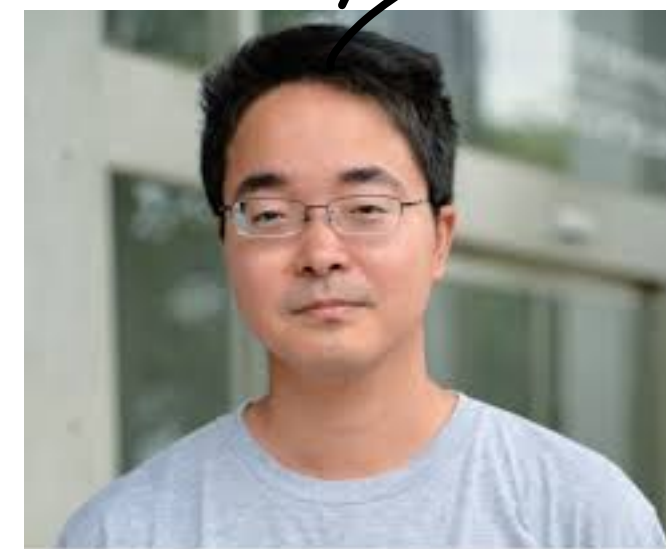
2007

IPMU!



Tsutomu

???



Masahito

## High Energy Physics – Theory

[Submitted on 28 Sep 2007 (v1), last revised 9 Apr 2008 (this version, v3)]

# Moduli Stabilization in Stringy ISS Models

Yu Nakayama, Masahito Yamazaki, T. T. Yanagida

We present a stringy realization of the ISS metastable SUSY breaking model with moduli stabilization. The moduli are stabilized by gauging of a U(1) symmetry and its D-term potential. The SUSY is broken both by F-term and D-term. We obtain de-Sitter vacua with a vanishingly small cosmological constant by an appropriate fine-tuning of the parameters.

Comments: 14 pages, v2: minor corrections, refereces added, v3: better parameters and more figures, published

Subjects: **High Energy Physics – Theory (hep-th)**; High Energy Physics – Phenomenology (hep-ph)

Report number: UCB-PTH-07/19, UT-07-28, IPMU 07-0001

Cite as: [arXiv:0710.0001 \[hep-th\]](https://arxiv.org/abs/0710.0001)  
(or [arXiv:0710.0001v3 \[hep-th\]](https://arxiv.org/abs/0710.0001v3) for this version)

<https://doi.org/10.48550/arXiv.0710.0001> 

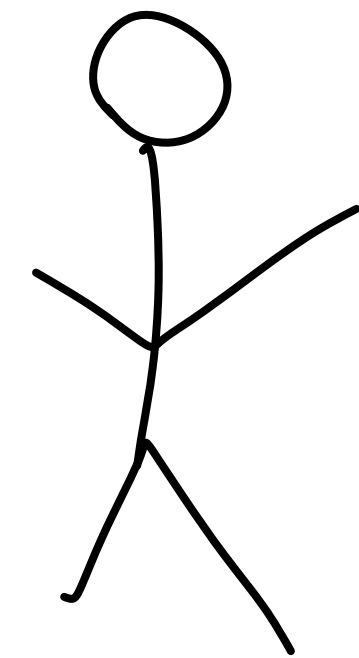
**Many people were skeptical back then.**

**People say many new centers are “only for funding, just a name card.”**

**“It does not work if you try to bring the  
US system into Japan. This is Japan.**

**Hitoshi is a Superman but this does not work.”**

Professor X

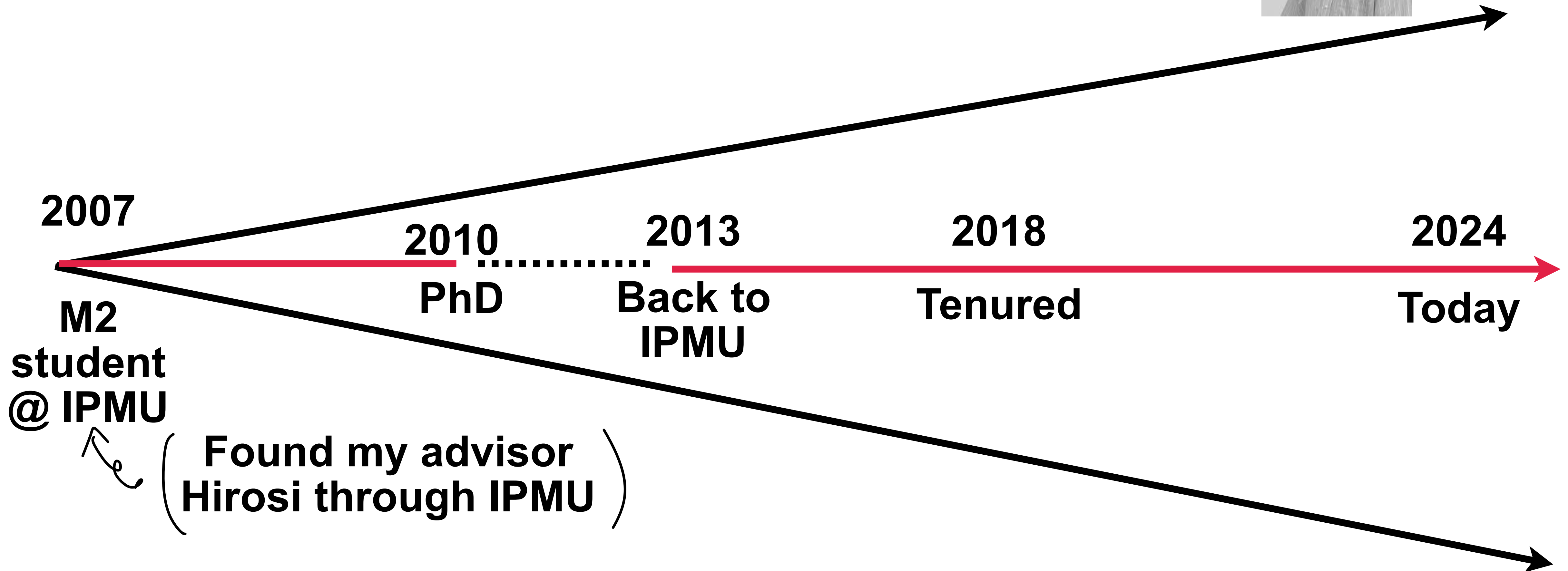


**Of course, Professor X underestimated Superman's ability**



**I started coming to Kashiwa,  
and I immediately fell in love with the institute...**

I was among the **very first generation of students**  
who **grew up together with Kavli IPMU**



**This was impossible without Kavli IPMU, and hence without Hitoshi !!!**

# I later had an opportunity to collaborate with Hitoshi

## High Energy Physics – Theory

*[Submitted on 3 Sep 2018 (v1), last revised 10 Oct 2018 (this version, v2)]*

## Do We Live in the Swampland?

[Hitoshi Murayama](#), [Masahito Yamazaki](#), [Tsutomu T. Yanagida](#)

A low-energy effective theory is said to be in the swampland if it does not have any consistent UV completion inside a theory of quantum gravity. The natural question is if the standard model of particle physics, possibly with some minimal extensions, are in the swampland or not. We discuss this question in view of the recent swampland conjectures. We prove a no-go theorem concerning the modification of the Higgs sector. Moreover, we find that QCD axion is incompatible with the recent swampland conjectures, unless some sophisticated possibilities are considered. We discuss the implications of this result for spontaneous breaking of CP symmetry. We comment on dynamical supersymmetry breaking as well as the issue of multi-valuedness of the potential.

Comments: 31 pages, 1 figure

Subjects: **High Energy Physics – Theory (hep-th)**; High Energy Physics – Phenomenology (hep-ph)

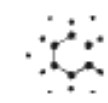
Report number: IPMU-18-0143

Cite as: [arXiv:1809.00478 \[hep-th\]](#)



# 2018: de Sitter swampland conjecture

[Obied, Ooguri, Spodyneiko, Vafa ('18)]



Quantamagazine

Physics

Mathematics

Biology

Computer  
Science

Topics

Archive



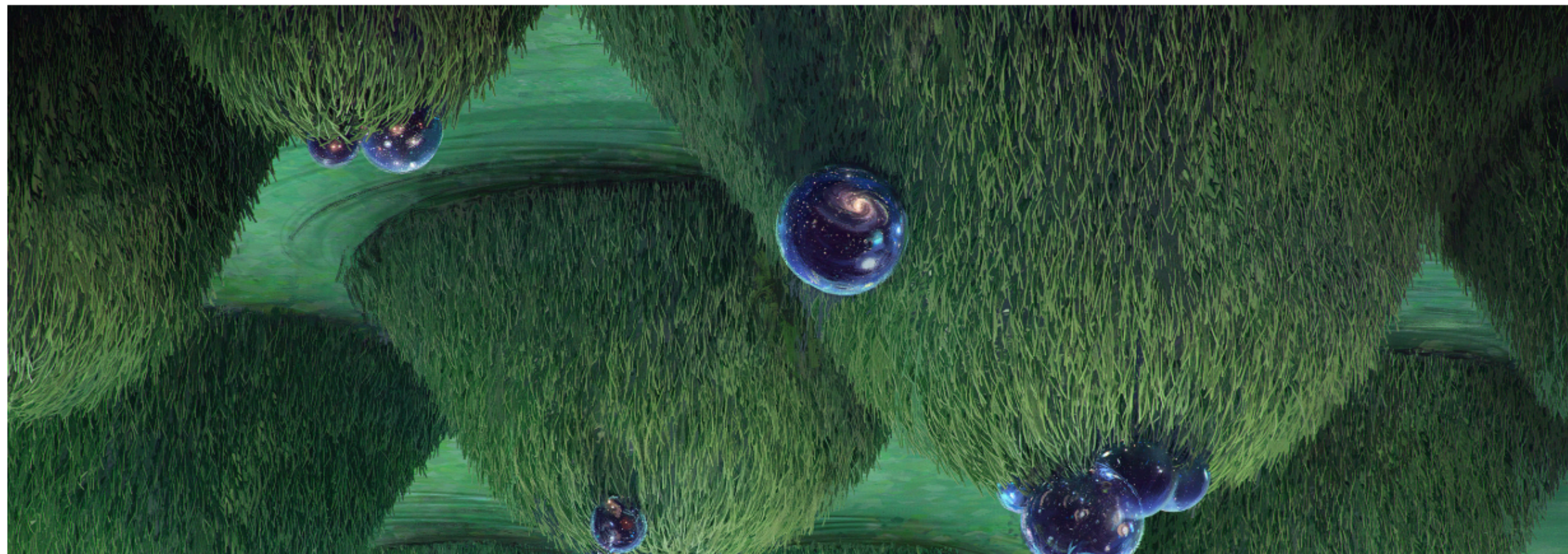
THEORETICAL PHYSICS

## Dark Energy May Be Incompatible With String Theory

133



A controversial new paper argues that universes with dark energy profiles like ours do not exist in the “landscape” of universes allowed by string theory.



# We pointed out **problems of the swampland de Sitter conjecture** and **proposed a new refinement (before Ooguri-Palti-Shiu-Vafa)**

## High Energy Physics – Theory

*[Submitted on 3 Sep 2018 (v1), last revised 10 Oct 2018 (this version, v2)]*

## Do We Live in the Swampland?

Hitoshi Murayama, Masahito Yamazaki, Tsutomu T. Yanagida

A low-energy effective theory is said to be in the swampland if it does not have any consistent UV completion inside a theory of quantum gravity. The natural question is if the standard model of particle physics, possibly with some minimal extensions, are in the swampland or not. We discuss this question in view of the recent swampland conjectures. We prove a no-go theorem concerning the modification of the Higgs sector. Moreover, we find that QCD axion is incompatible with the recent swampland conjectures, unless some sophisticated possibilities are considered. We discuss the implications of this result for spontaneous breaking of CP symmetry. We comment on dynamical supersymmetry breaking as well as the issue of multi-valuedness of the potential.

Comments: 31 pages, 1 figure

Subjects: **High Energy Physics – Theory (hep-th)**; High Energy Physics – Phenomenology (hep-ph)

Report number: IPMU-18-0143

Cite as: [arXiv:1809.00478](https://arxiv.org/abs/1809.00478) [hep-th]

# 2020

## The String Swampland and Quantum Gravity Constraints on Effective Theories

**Coordinators:** Hiroshi Ooguri, Gary Shiu, Cumrun Vafa, and Irene Valenzuela

The idea that the string landscape is too large to lead to concrete predictions has been countered by the idea that most of the naively consistent effective theories of gravity coupled to matter are actually inconsistent and belong to the swampland. The identification of criteria distinguishing the true string landscape from the swampland, which has been studied for more than a decade now, is beginning to reach a more mature stage with the developments of the last few years. In particular a conjectured consistency condition for quantum gravity known as the Weak Gravity Conjecture (WGC), which postulates that gravity is always the weakest force among all the forces, has found an unexpectedly broad range of applications.

The WGC on the one hand has been used to constrain cosmological models of inflation including scenarios being tested by the present



### DATES

Feb 18, 2020 - Mar 13, 2020



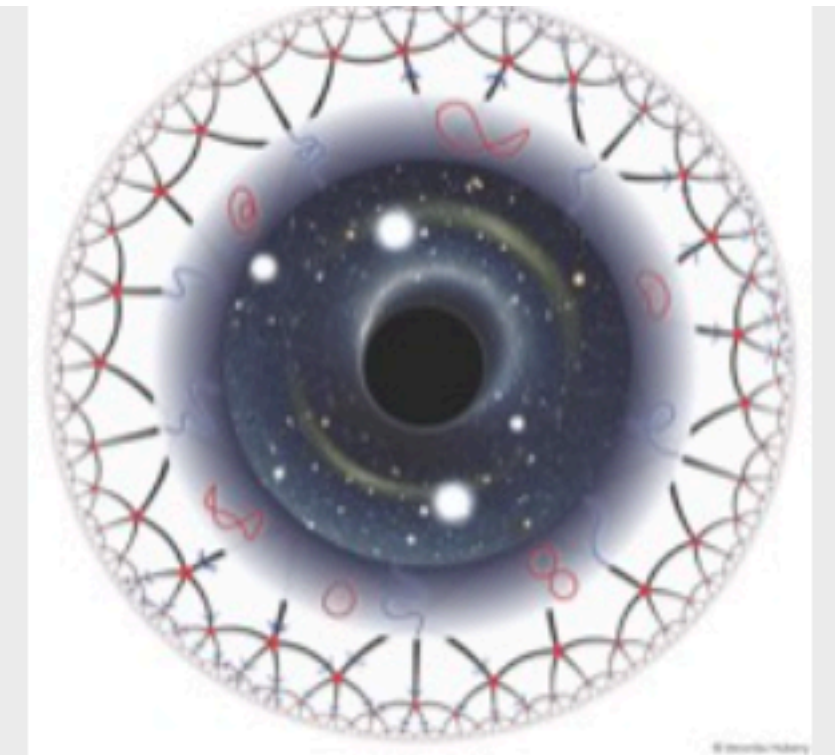
# 2020

## Gravitational Holography

**Coordinators:** Xi Dong, Tom Faulkner, and Veronika Hubeny

**Scientific Advisors:** Juan Maldacena and Eva Silverstein

Understanding quantum gravity has been a long-standing goal of theoretical physics and is essential for solving challenging problems about black holes and cosmology. String theory provides a well-motivated approach to the subject which is still under rapid development. Using string theory and other insights from black hole thermodynamics, it has become increasingly clear that spacetime and gravity should be viewed as an emergent phenomenon arising from the complicated dynamics of some underlying quantum system 'holographically', in the sense that the fundamental degrees of freedom should be described by a lower-dimensional theory. There are by now many different approaches for studying this dynamics and generalizing the framework of holography to a wide range of quantum gravitational theories. A major goal of this program is to build on the recent developments to further explore the connections between these various ideas.



### DATES

Jan 6, 2020 - Mar 26, 2020

### QUICK LINKS



Swampland

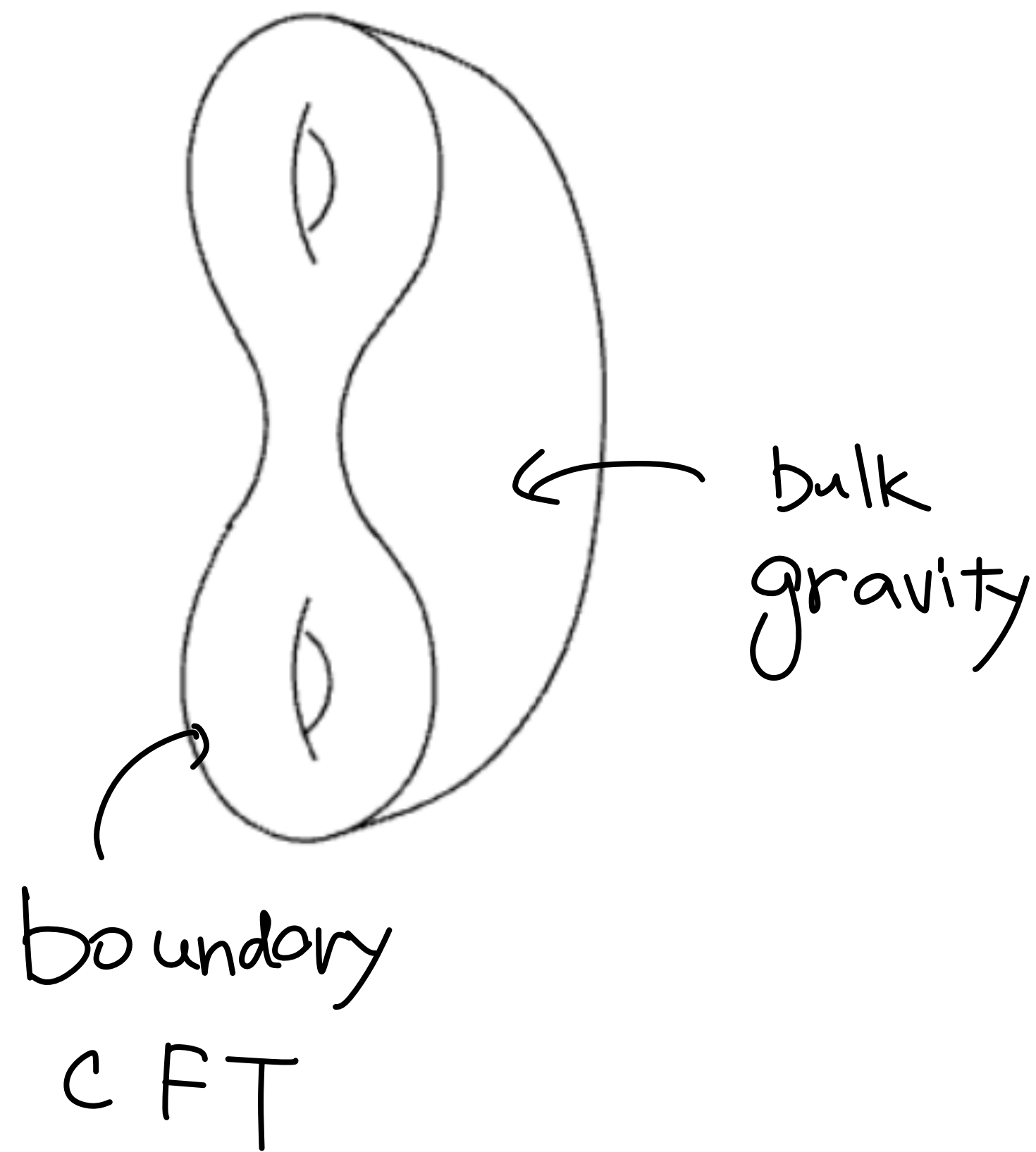
v.s.

Ensemble ?



# Factorization Puzzle

[Maldacena-Maoz '04]



$$\mathbb{Z}_{\text{CFT}} = \mathbb{Z}_{\text{AdS}}$$

# Factorization Puzzle

[Maldacena-Maoz '04]



CFT<sub>1</sub>

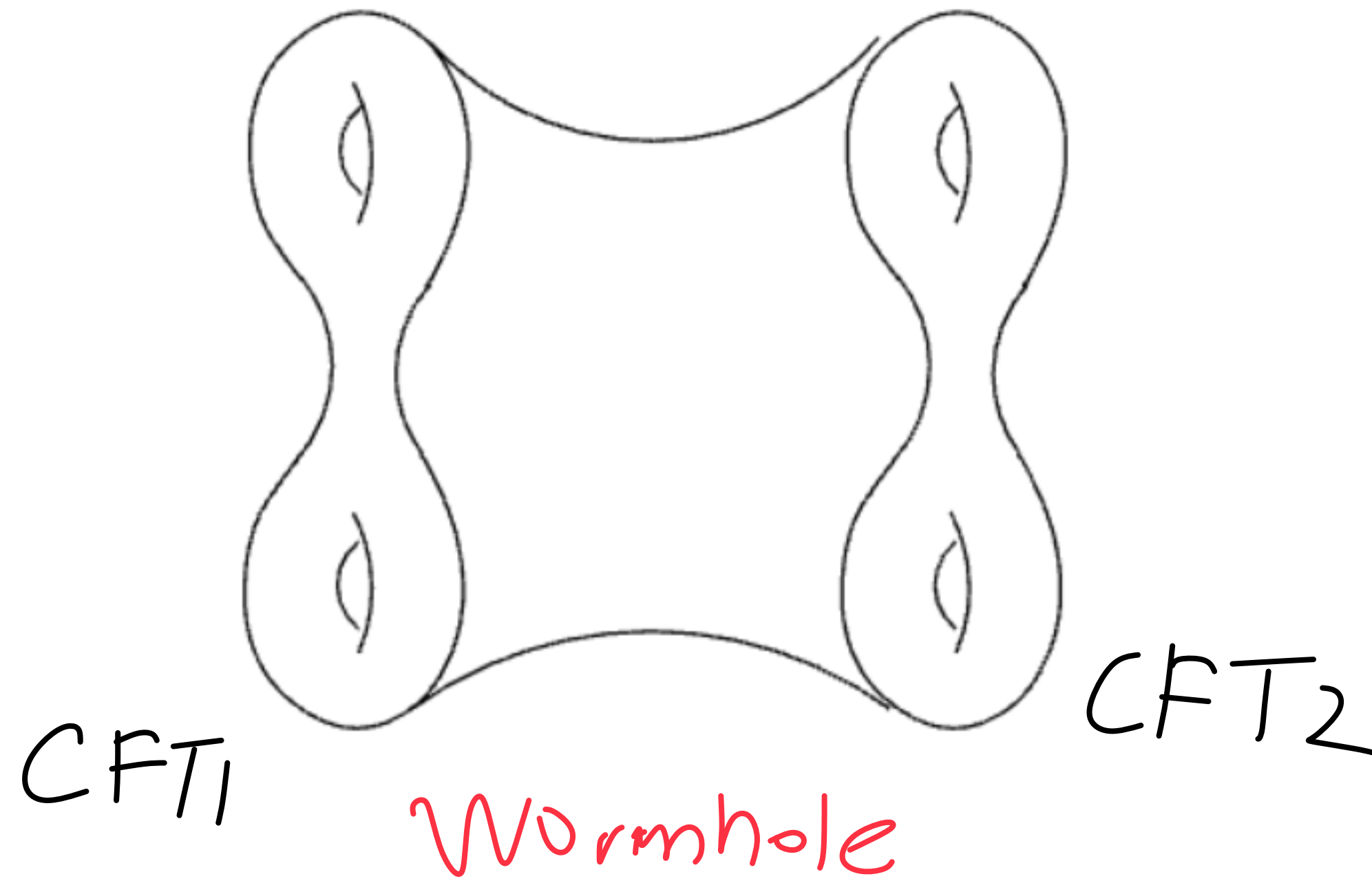


CFT<sub>2</sub>

$$\mathcal{Z}_{\text{tot}} = \mathcal{Z}_{\text{CFT}_1} * \mathcal{Z}_{\text{CFT}_2} = \mathcal{Z}_{\text{AdS}_1} * \mathcal{Z}_{\text{AdS}_2}$$

# Factorization Puzzle

[Maldacena-Maoz '04]



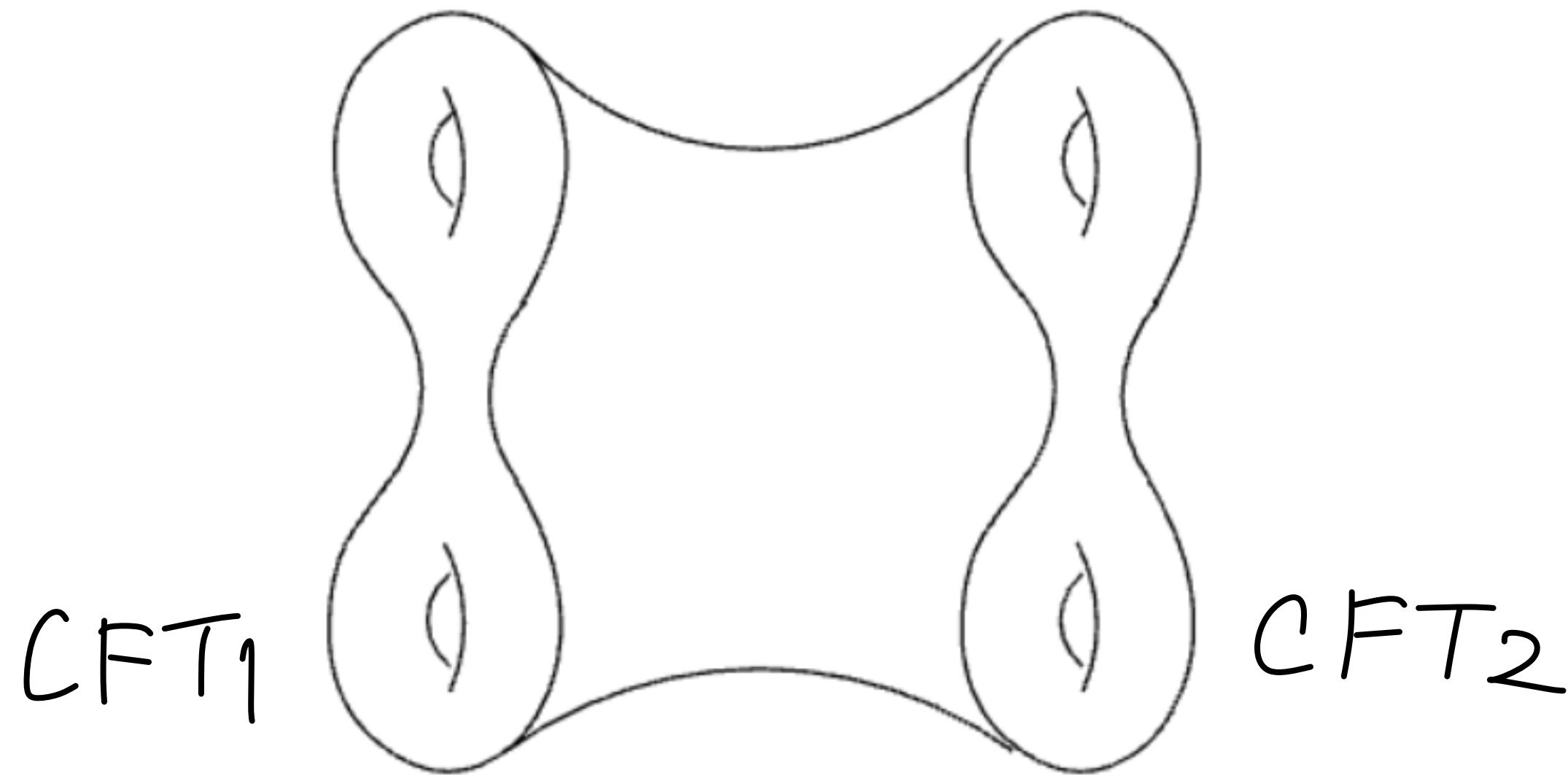
$$Z_{\text{CFT}_1} \cdot Z_{\text{CFT}_2} \neq Z_{\text{AdS}}$$

does not  
factorize



# Factorization Puzzle

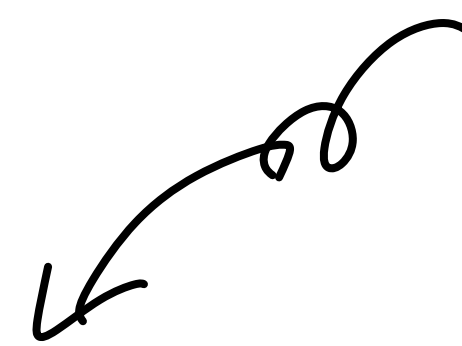
[Maldacena-Maoz '04]



$\langle \rightarrow \rangle$ : ensemble average

ensemble?

$$\langle Z_1 \rangle \langle Z_2 \rangle \neq \langle Z_1 \cdot Z_2 \rangle$$

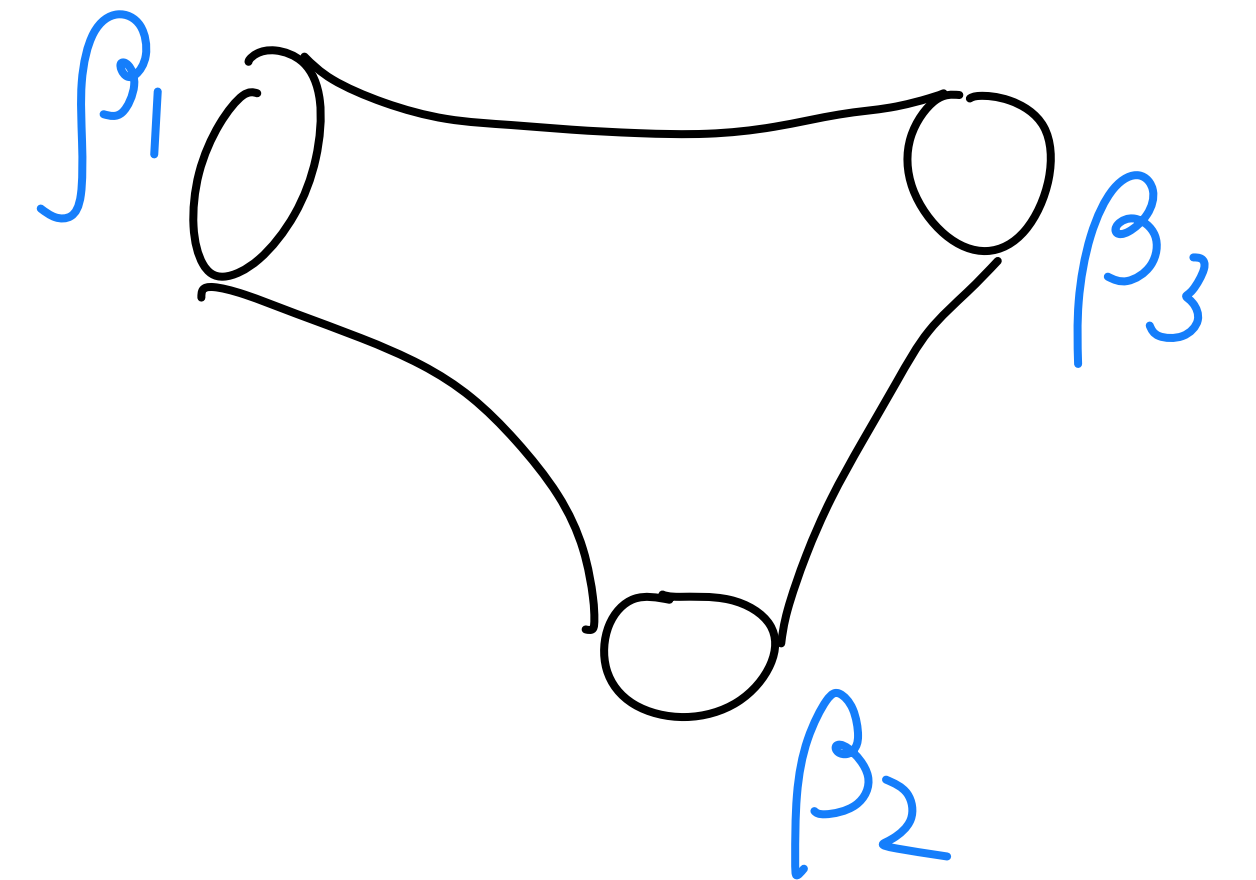


[Saad-Shenker-Stanford ('19)]:

Random matrix ensemble for 2d Jackiw-Teitelboim gravity

$$\langle Z(\beta_1) \cdots Z(\beta_n) \rangle$$

||



$$\frac{1}{N} \int dH e^{-\text{Tr} V(H)} Z(\beta_1) \cdots Z(\beta_n)$$

Technically: { Mirzakhani's recursion for WP volumes  
Eynard-Orantín topological recursion

While ensembles inside a theory is common

e.g. in stat-mech,

here we consider ensembles of THEORIES

$$\langle \hat{O} \rangle = \int_{\mathcal{M}} [dm] O(m)$$

↑  
moduli space of theories

↑  
measure

↑  
parameters of theories

Precise case study:  $CFT_2 / AdS_3$

[ Ashwinkumar - Dodelson - Kidambi - Leedom - MY ('21)  
Ashwinkumar - Kidambi - Leedom - MY ('23) ]



See also

[ Afkhami - Jeddi, Cohen, Hartman, Tajdini ('20)  
Maloney - Witten ('20) ]

# Data

of signature  $(p, g)$

•  $Q$ : even quadratic form

lattice  $\Lambda$

•  $H$ : Hamiltonian

moduli dependent

$$\begin{cases} Q = P_L^2 - P_R^2 \\ H = P_L^2 + P_R^2 \end{cases}$$

•  $M_Q$ : CFT moduli space

(T-duality group)  $\rightsquigarrow M_Q = \underbrace{O(p, g; \mathbb{Z})}_{O_Q(p+g)} / O(p) \times O(g)$

# Theta functions

$$\mathcal{D}_{Q,\alpha}(\tau, \bar{\tau}; m) = \sum_{\ell \in \Lambda} e^{\pi i \tau_1 Q(\ell+\alpha) - \pi \tau_2 H(\ell+\alpha)}$$

$\overbrace{m}$  - dependence

$$\left( \begin{array}{l} m: \text{CFT moduli} \\ \tau: \text{spacetime torus moduli} \\ \alpha \in \mathcal{D} := \Lambda^* / \Lambda \text{ (discriminant)} \end{array} \right) \quad \tau = \tau_1 + i\tau_2$$

# Ensemble Average

Haar measure / Zamolodchikov metric

$$\frac{1}{\text{Vol}(\mathcal{M}_{g,\alpha})} \int_{\mathcal{M}_{g,\alpha}} [dm] \mathcal{D}_{g,\alpha}(\tau, \bar{\tau}; m) = ??$$

T-duality should  
preserve  $\alpha$

Siegel-Weil formula  $\left[ \begin{array}{l} \text{Siegel ('51)} \\ \text{Weil ('64)} \end{array} \right]$

$$\underbrace{\frac{1}{\text{Vol}(\mu_{Q,\alpha})} \int_{\mu_{Q,\alpha}} [dm] \vartheta_{Q,\alpha}(\tau, \bar{\tau}; m)}_{\substack{\text{ensemble average} \\ \text{over CFT moduli space}}} = \underbrace{E_{Q,\alpha}(\tau, \bar{\tau})}_{\substack{\text{non-hol.} \\ \text{Eisenstein series}}}$$



Poincaré sum

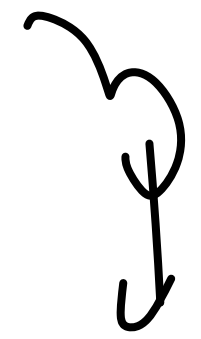
$$\begin{pmatrix} * & * \\ c & d \end{pmatrix} : SL(2, \mathbb{Z}) / \begin{matrix} \Gamma_\infty \\ \mathbb{Z} \end{matrix}$$

$$E_{Q, \alpha}(\tau) := \int_{\alpha \in \Lambda} + \sum_{\substack{(c,d)=1 \\ c > 0}} \frac{\gamma_{Q, \alpha}(c, d)}{(c\bar{\tau} + d)^{\frac{p}{2}} (c\tau + d)^{\frac{q}{2}}}$$

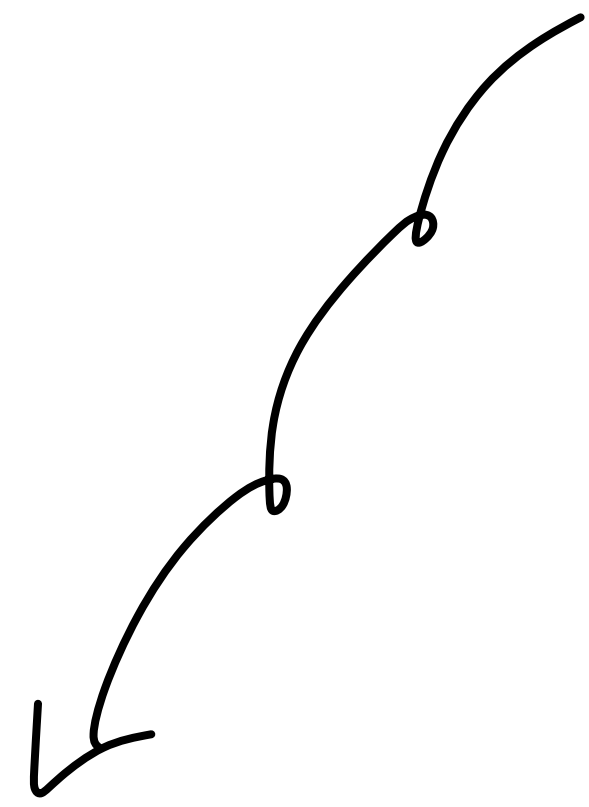
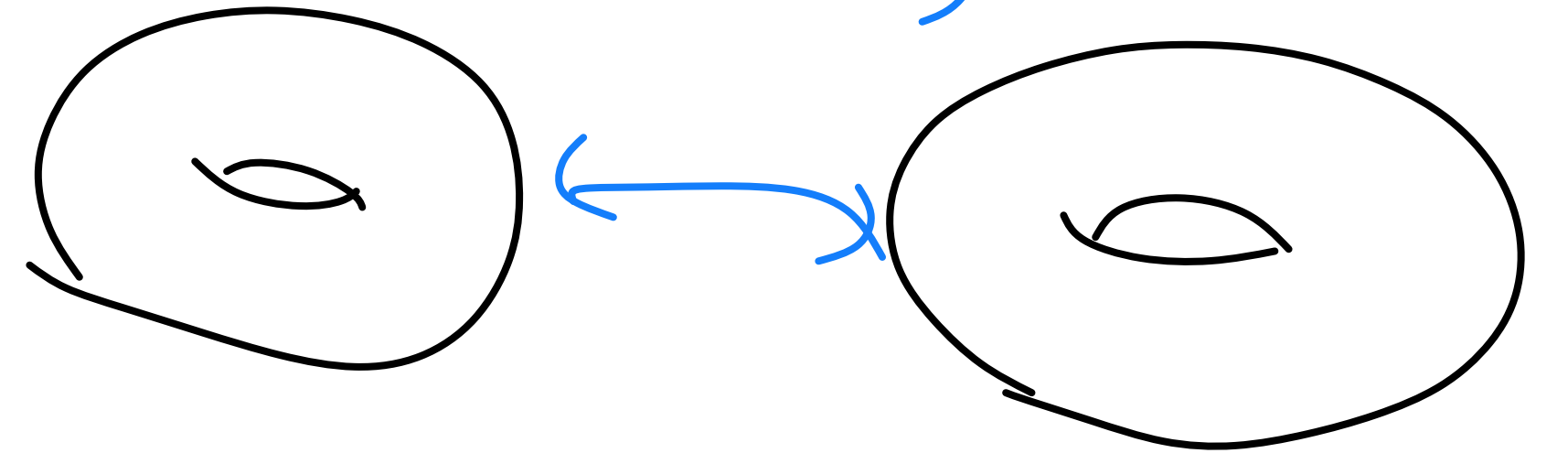
$$\gamma_{Q, \alpha}(c, d) = \frac{e^{\frac{\pi i (p-q)}{4}}}{\sqrt{|\det Q|}} c^{-\frac{p+q}{2}} \sum_{\alpha \in \Lambda/c\Lambda} \exp\left[-\pi i \frac{d}{c} Q(\alpha + d)\right]$$

# lens space partition function

Sum over  
 $PSL(2, \mathbb{Z})$  BH



$$\langle \psi_0 | \begin{pmatrix} * & * \\ c & d \end{pmatrix} | \psi_\alpha \rangle$$



$$E_{Q, \alpha}(\tau) := \int_{d \in \Lambda} + \sum_{\substack{(c, d) = 1 \\ c > 0}} \frac{\gamma_{Q, \alpha}(c, d)}{(c\bar{\tau} + d)^{\frac{p}{2}} (c\tau + d)^{\frac{q}{2}}}$$

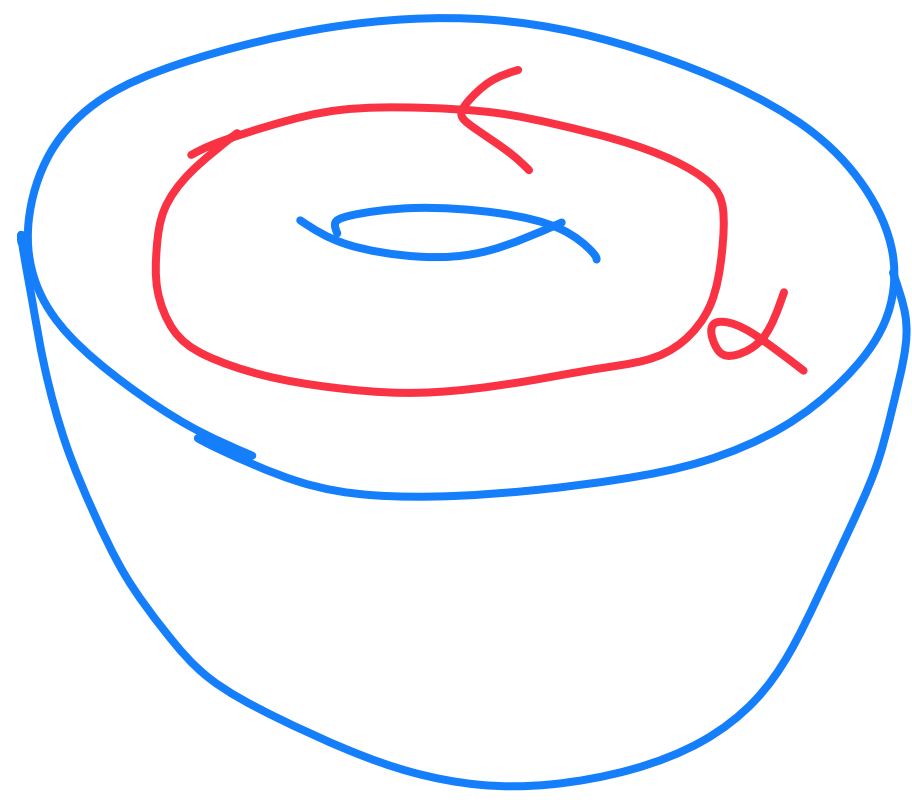
$$\gamma_{Q, \alpha}(c, d) = \frac{e^{\frac{\pi i (p-q)}{4}}}{\sqrt{|\det Q|}} c^{-\frac{p+q}{2}} \sum_{Q \in N_{c\Lambda}} \exp\left[-\pi i \frac{d}{c} Q(l+d)\right]$$

Holographic dual after averaging:

Abelian CS

$$S_{CS} = \sum_{I, J=1}^{p+8} \frac{1}{4\pi} Q_{IJ} \int A_I \wedge dA_J$$

\*  $|E_{Q,\alpha}(\tau)\rangle$  : wave function on  $T^2$



anyon  
insertion

$\alpha \in \frac{\Lambda^*}{\Lambda}$   
↑ charge      ↑ gauge equiv.

# Emergent Global Symmetries

---

[Ashwinkumar, Leedom, MY ('23)]

QG: "No exact global symmetries"

However, ensemble-averaged theories

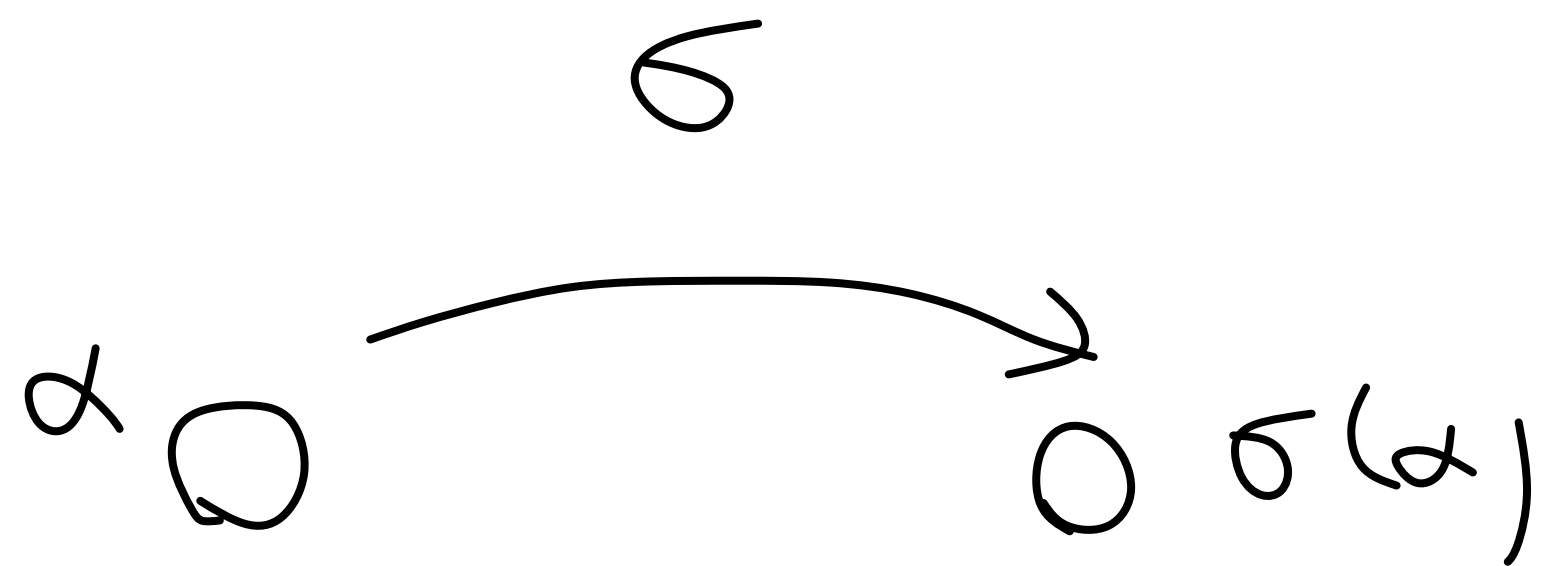
can have emergent global symmetries

# Symmetries of anyons

•  $\alpha \in \mathcal{D} = \Lambda^* / \Lambda$  ; anyons w/ spin  $\theta(\alpha) = e^{\pi i Q(\alpha)}$

• Symmetry:  $\sigma \in \text{Aut}(\mathcal{D})$

$\alpha \mapsto \sigma \cdot \alpha$  s.t.  $\theta(\alpha) = \theta(\sigma \cdot \alpha)$



$\sigma \in \text{Aut}(\mathcal{D})$  consistent with spins

After average

•  $E_{Q, \sigma, \alpha}(\tau, \bar{\tau}) = E_{Q, \alpha}(\tau, \bar{\tau})$  

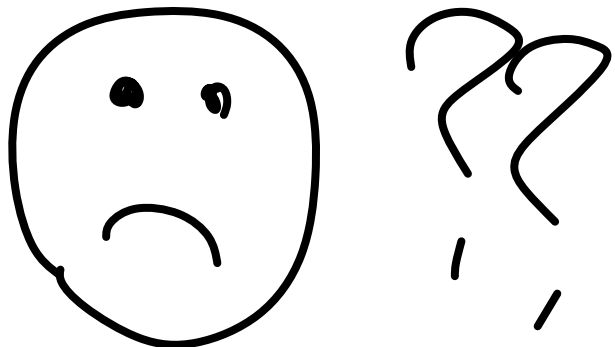


$\sigma \in \text{Aut}(\mathcal{D})$  then

After average

•  $E_{Q, \sigma \cdot \alpha}(\tau, \bar{\tau}) = E_{Q, \alpha}(\tau, \bar{\tau})$  

Before average

$\mathcal{D}Q_{\sigma \cdot \alpha}(\tau, \bar{\tau}; m) \neq \mathcal{D}Q_{\alpha}(\tau, \bar{\tau}; m)$  

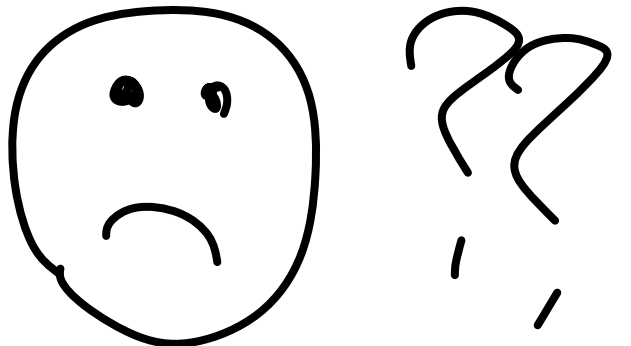
NOT a symmetry of a given CFT

$\sigma \in \text{Aut}(\mathcal{D})$  then

After average

•  $E_{Q, \sigma \cdot \alpha}(\tau, \bar{\tau}) = E_{Q, \alpha}(\tau, \bar{\tau})$  

Before average

$\mathcal{D}Q_{\sigma \cdot \alpha}(\tau, \bar{\tau}; m) \neq \mathcal{D}Q_{\alpha}(\tau, \bar{\tau}; m)$  

$\mathcal{D}Q_{\sigma \cdot \alpha}(\tau, \bar{\tau}; \sigma \cdot m) = \mathcal{D}Q_{\alpha}(\tau, \bar{\tau}; m)$

if  $\nearrow$  T-duality origin relation between different theories

In general,

"ensemble sym."

average over  $\mathcal{M} \subset \sigma \in G$  s.t.  $[d(\sigma(m))] = [dm]$   
of  $\Theta(m, \alpha)$  s.t.  $\Theta(\sigma \cdot m, \sigma \cdot \alpha) = \Theta(m, \alpha)$

$$\rightsquigarrow \langle \Theta \rangle(\alpha) = \int [dm] \Theta(m, \alpha) = \langle \Theta \rangle(\sigma \cdot \alpha)$$

$$\langle \theta \rangle(\alpha) = \int [dm] \theta(m, \alpha)$$

$$= \int [dm] \theta(\sigma \cdot m, \sigma \cdot \alpha)$$

$$= \int [d(\sigma^{-1}m)] \theta(m, \sigma \cdot \alpha)$$

$$= \int [dm] \theta(m, \sigma \cdot \alpha)$$

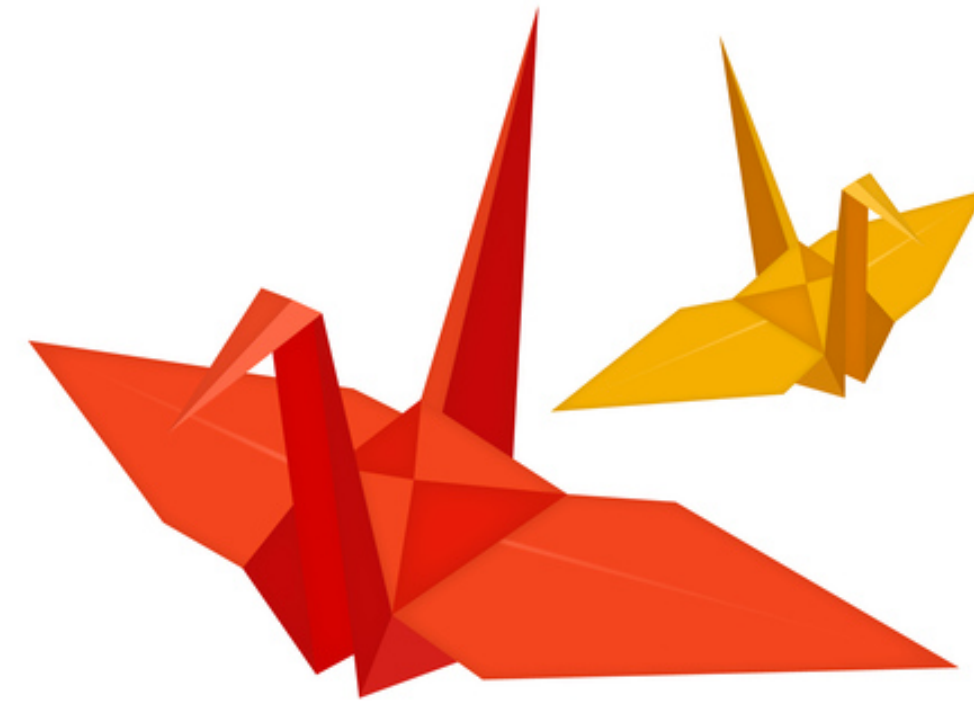
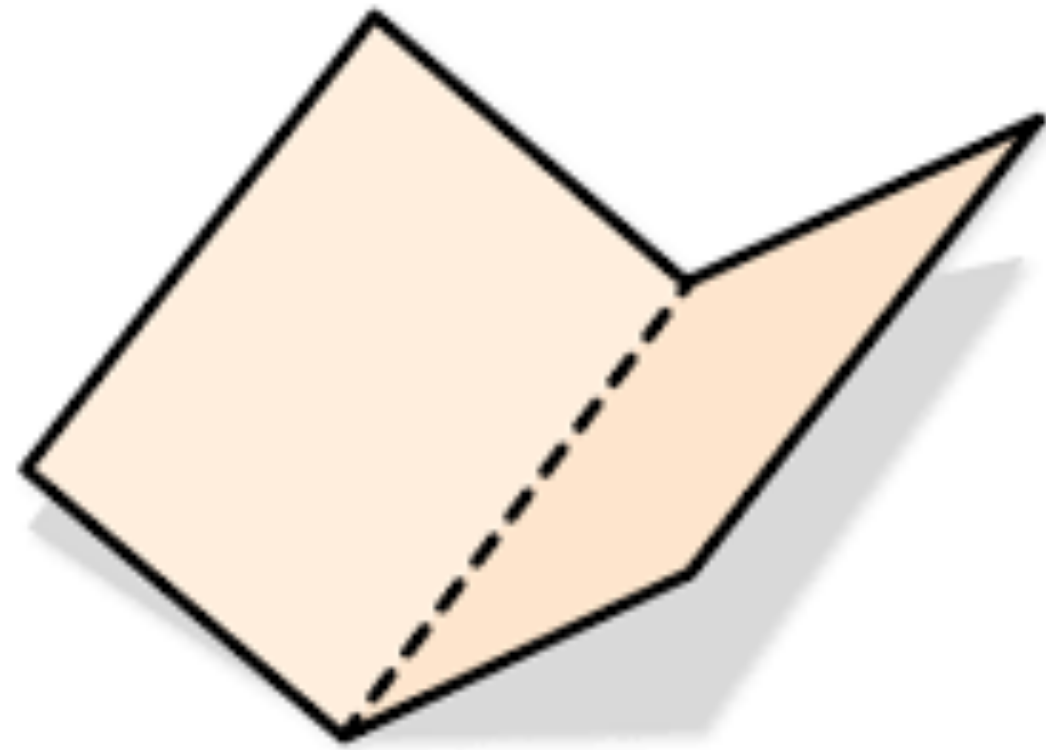
$$= \langle \theta \rangle(\sigma \cdot \alpha)$$

covariance of  $\theta$

change of variable

invariance of measure

# "duality origami"



ensemble symmetry



global symmetry

(e.g. T-duality)

ensemble average

asymptotic at cusps as  $\tau \rightarrow -d/c$

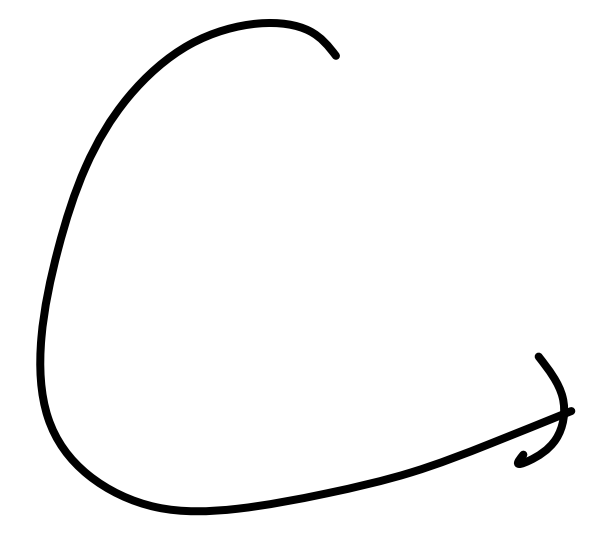
$\vartheta_{Q,\alpha}(\tau)$

$\infty$ -distance

lens inv.

$\gamma_{Q,\alpha}(c,d)$

ensemble average

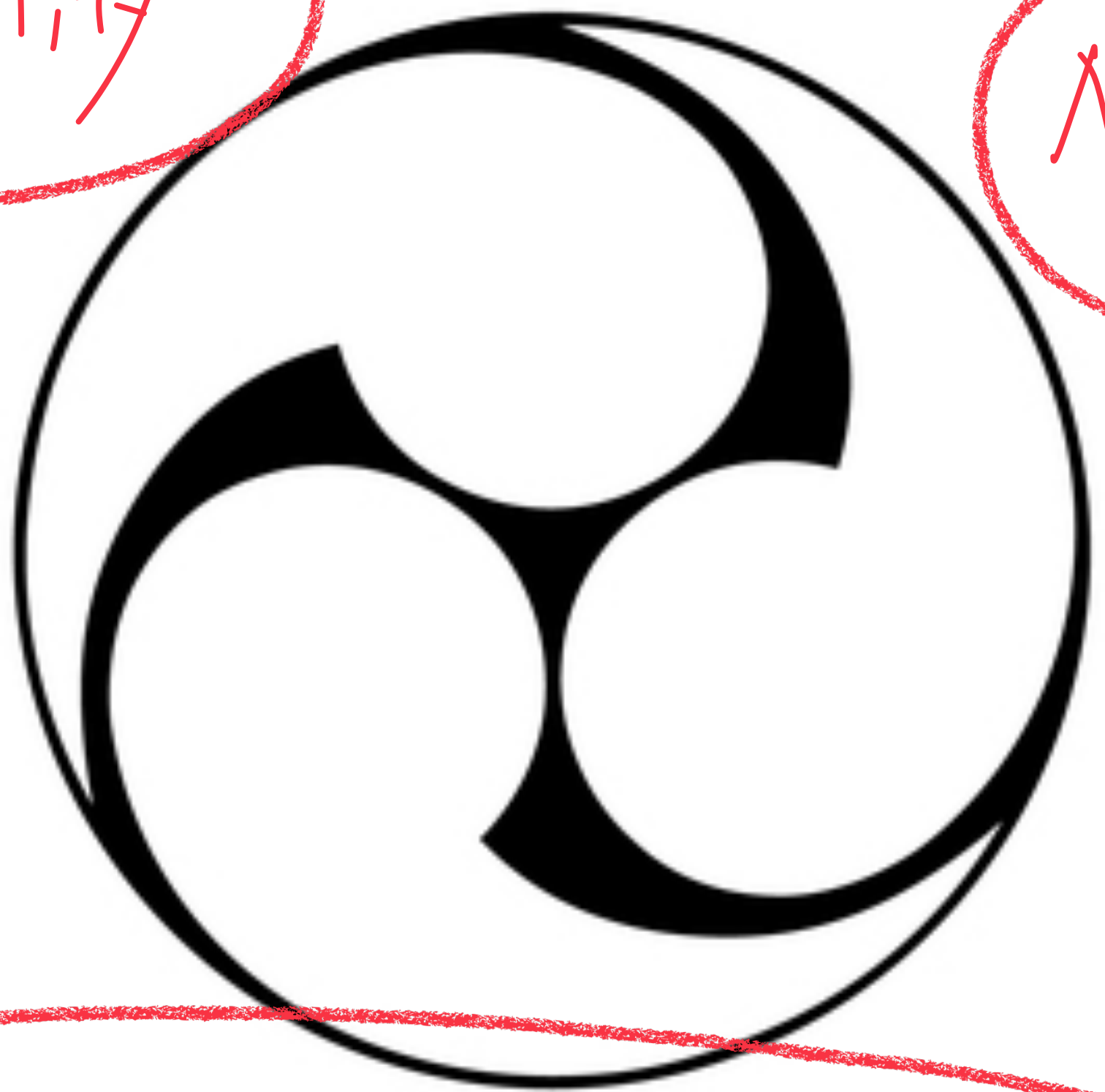


$$\langle \vartheta_{Q,\alpha} \rangle(\tau) = \delta_{\alpha \in \Lambda} + \sum_{\substack{(k,d)=1 \\ c > 0}} \frac{\gamma_{Q,\alpha}(c,d)}{(c\tau+d)^{\frac{p}{2}} (c\bar{\tau}+d)^{\frac{q}{2}}}$$

building block

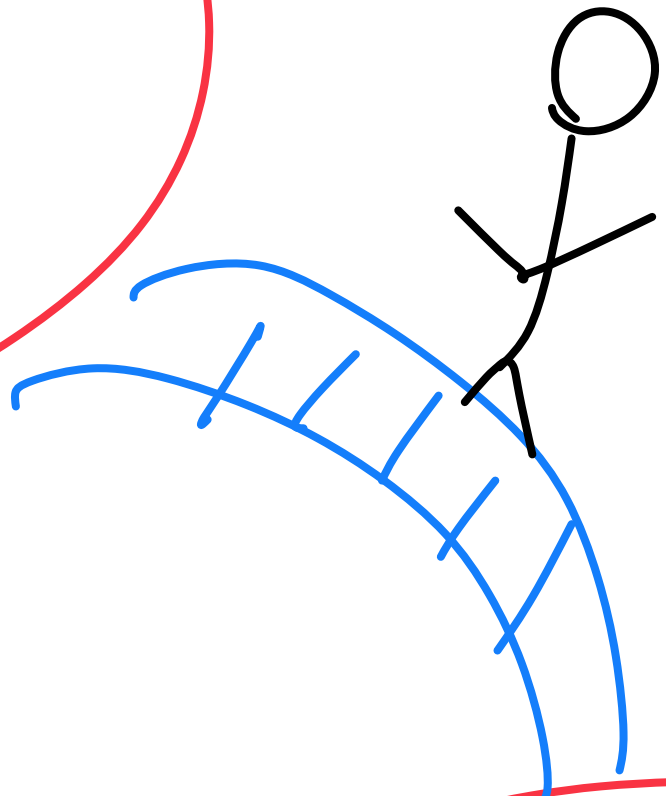
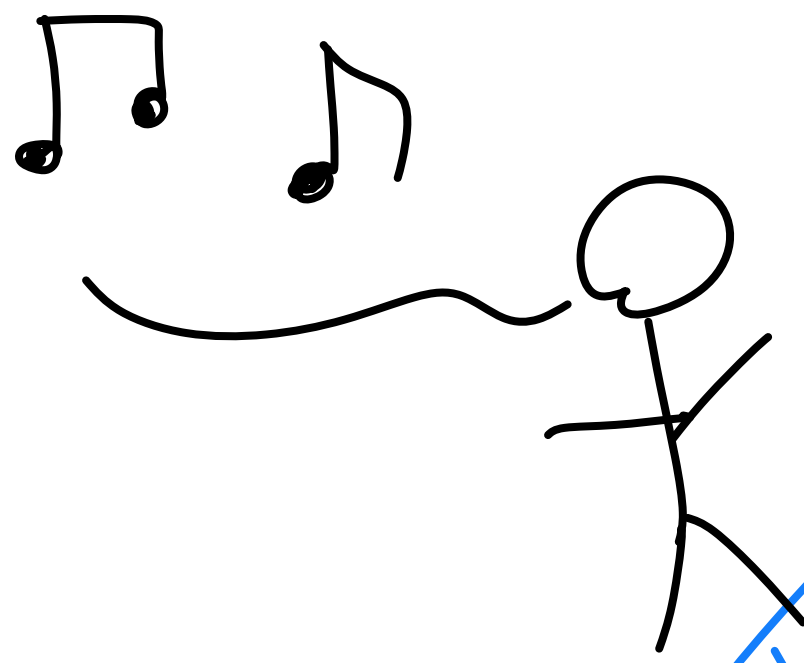
T-duality

No global sym,



distance conjecture

Number Theory



Ensemble Average

Swampland Conjectures

