QED 5-loop

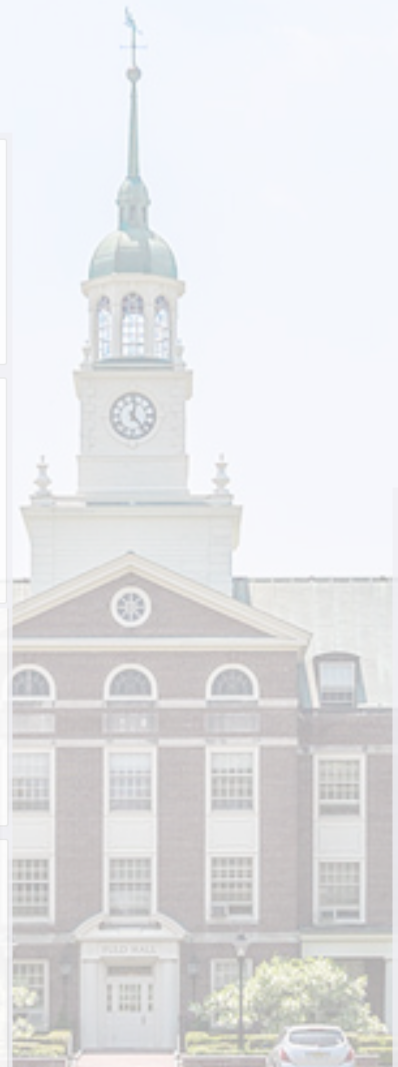
Ryuichiro Kitano (KEK)

Reference:
Kitano, 2411.11554 [hep-lat]

Hitoshi Fest@Kavli IPMU, December 16-20, 2024

Happy birthday, Hitoshi!

I am one of thousands of physicists who are pretty much influenced by his physics works. I was so lucky that I was at IAS when Hitoshi stayed there for a year.



Enhanced Higgs Mass in Compact Supersymmetry #2

Kohsaku Tobioka (Tel Aviv U. and Weizmann Inst. and KEK, Tsukuba), Ryuichiro Kitano (Tokyo U., IPMU and KEK, Tsukuba and Sokendai, Tsukuba), Hitoshi Murayama (LBL, Berkeley and Tokyo U., IPMU and UC, Berkeley) (Nov 12, 2015)

Published in: *JHEP* 04 (2016) 025 • e-Print: 1511.04081 [hep-ph]

pdf DOI cite claim reference search 5 citations

Unified origin of baryons and dark matter #3

Ryuichiro Kitano (Los Alamos), Hitoshi Murayama (Tokyo U., Math. Sci. and UC, Berkeley and LBL, Berkeley), Michael Ratz (Munich, Tech. U.) (Jul, 2008)

Published in: *Phys.Lett.B* 669 (2008) 145-149 • e-Print: 0807.4313 [hep-ph]

pdf DOI cite claim reference search 124 citations

Natural soft leptogenesis #4

Yuval Grossman (Technion and Boston U. and Harvard U., Phys. Dept.), Ryuichiro Kitano (Princeton, Inst. Advanced Study), Hitoshi Murayama (UC, Berkeley and LBL, Berkeley) (Apr, 2005)

Published in: *JHEP* 06 (2005) 058 • e-Print: hep-ph/0504160 [hep-ph]

pdf DOI cite claim reference search 40 citations

Models of neutrino mass with a low cutoff scale #5

Hooman Davoudiasl (Wisconsin U., Madison and Princeton, Inst. Advanced Study), Ryuichiro Kitano (Princeton, Inst. Advanced Study), Graham D. Kribs (Princeton, Inst. Advanced Study and Oregon U.), Hitoshi Murayama (UC, Berkeley and LBL, Berkeley and Princeton, Inst. Advanced Study) (Feb, 2005)

Published in: *Phys.Rev.D* 71 (2005) 113004 • e-Print: hep-ph/0502176 [hep-ph]

pdf DOI cite claim reference search 51 citations

A Viable supersymmetric model with UV insensitive anomaly mediation #6

Masahiro Ibe (Tokyo U.), Ryuichiro Kitano (Princeton, Inst. Advanced Study), Hitoshi Murayama (Princeton, Inst. Advanced Study and UC, Berkeley and LBL, Berkeley) (Dec, 2004)

Published in: *Phys.Rev.D* 71 (2005) 075003 • e-Print: hep-ph/0412200 [hep-ph]

pdf DOI cite claim reference search 42 citations

The New minimal standard model #7

Hooman Davoudiasl (Princeton, Inst. Advanced Study), Ryuichiro Kitano (Princeton, Inst. Advanced Study), Tianjun Li (Princeton, Inst. Advanced Study), Hitoshi Murayama (Princeton, Inst. Advanced Study) (May, 2004)

Published in: *Phys.Lett.B* 609 (2005) 117-123 • e-Print: hep-ph/0405097 [hep-ph]

pdf DOI cite claim reference search 370 citations

Gravitational baryogenesis #8

Hooman Davoudiasl (Princeton, Inst. Advanced Study), Ryuichiro Kitano (Princeton, Inst. Advanced Study), Graham D. Kribs (Princeton, Inst. Advanced Study), Hitoshi Murayama (Princeton, Inst. Advanced Study), Paul J. Steinhardt (Princeton, Inst. Advanced Study) (Mar, 2004)

Published in: *Phys.Rev.Lett.* 93 (2004) 201301 • e-Print: hep-ph/0403019 [hep-ph]

pdf DOI cite claim reference search 239 citations

Viable supersymmetry and leptogenesis with anomaly mediation #9

Masahiro Ibe (Tokyo U.), Ryuichiro Kitano (Princeton, Inst. Advanced Study), Hitoshi Murayama (Princeton, Inst. Advanced Study), Tsutomu Yanagida (Tokyo U.) (Mar, 2004)

Published in: *Phys.Rev.D* 70 (2004) 075012 • e-Print: hep-ph/0403198 [hep-ph]

pdf DOI cite claim reference search 75 citations

Electroweak symmetry breaking via UV insensitive anomaly mediation #10

Ryuichiro Kitano (Princeton, Inst. Advanced Study), Graham D. Kribs (Princeton, Inst. Advanced Study), Hitoshi Murayama (Princeton, Inst. Advanced Study) (Feb, 2004)

Published in: *Phys.Rev.D* 70 (2004) 035001 • e-Print: hep-ph/0402215 [hep-ph]

pdf DOI cite claim reference search 49 citations

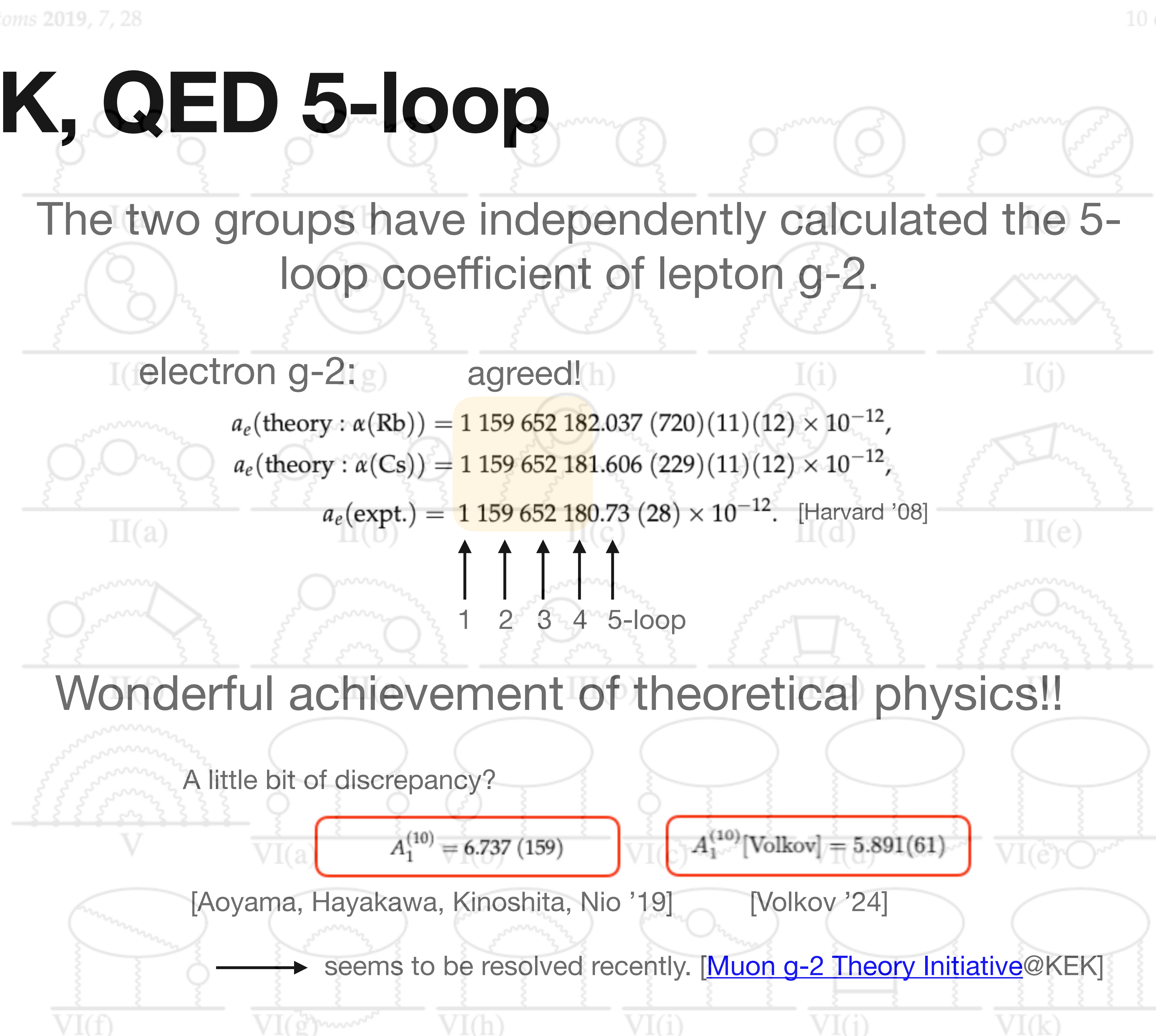
I'm proud of many papers which we've written together at IAS!

I was like a kid when we were collaborating. I was always excited to talk to Hitoshi who has been the world leader of theoretical physics.

→ now the director of the Universe

Today, I would like to be back to a kid and try to let Hitoshi say **“good job!”**

OK, QED 5-loop



The two groups have independently calculated the 5-loop coefficient of lepton g-2.

electron g-2: agreed!

$$a_e(\text{theory} : \alpha(\text{Rb})) = 1\,159\,652\,182.037\,(720)(11)(12) \times 10^{-12},$$

$$a_e(\text{theory} : \alpha(\text{Cs})) = 1\,159\,652\,181.606\,(229)(11)(12) \times 10^{-12},$$

$$a_e(\text{expt.}) = 1\,159\,652\,180.73\,(28) \times 10^{-12}.$$ [Harvard '08]

1 2 3 4 5-loop

Wonderful achievement of theoretical physics!!

A little bit of discrepancy?

$A_1^{(10)} = 6.737\,(159)$ [Aoyama, Hayakawa, Kinoshita, Nio '19]

$A_1^{(10)}[\text{Volkov}] = 5.891(61)$ [Volkov '24]

→ seems to be resolved recently. [Muon g-2 Theory Initiative@KEK] by the way,

Speaking of g-2…

electron g-2: agreed!

$$a_e(\text{theory} : \alpha(\text{Rb})) = 1\,159\,652\,182.037\,(720)(11)(12) \times 10^{-12},$$
$$a_e(\text{theory} : \alpha(\text{Cs})) = 1\,159\,652\,181.606\,(229)(11)(12) \times 10^{-12},$$
$$a_e(\text{expt.}) = 1\,159\,652\,180.73\,(28) \times 10^{-12}.$$ [Harvard '08]

This is great. It seems that we understood particle physics very **deeply**.

But, here is a question.
What's the size of the (quantum) gravity correction to it?

standard answer： $O(m_e^2/M_{Pl}^2) \sim 10^{-43}$ very small.

possible answer （Cohen, Kaplan, Nelson)： $O((m_e/M_{Pl})^{1/2}) \sim 10^{-11}$

This could be within the future reach!

The latter estimate is based on a very deep Volume vs. Area discussion.

I think this is really important!

…

Today, I'll talk about nothing deep, just QED.
I try to develop a numerical method to evaluate the perturbative coefficients in QED

on the **lattice**.

Path integral

throw dice many times and take an average.

For example, two point functions of electrons:

$$\langle \psi(x)\bar{\psi}(0)\rangle = \int [dA] \det D \, D^{-1}_{(x,0)} e^{-S[A]}$$

Dice part

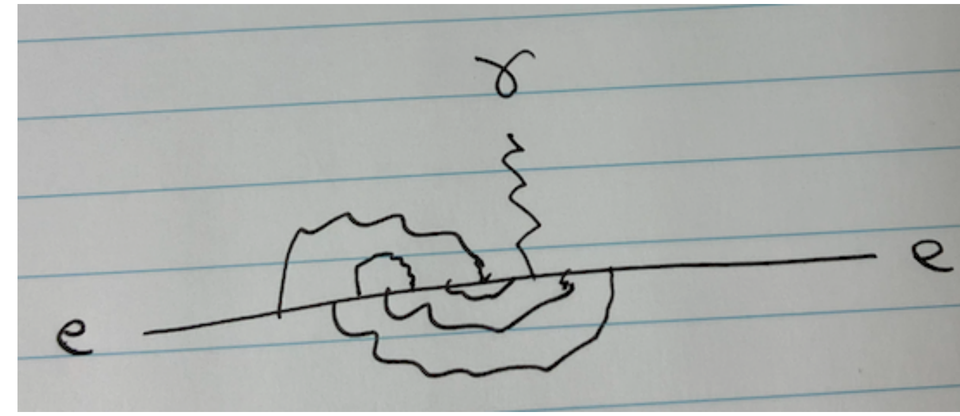
take average of this

Lattice people do this everyday.

No Feynman diagram needed.

Simple!

Even simpler



The most difficult and the most important part of the contributions are from diagrams with no lepton loops.

That's actually the easiest part for lattice.

$$\langle \psi(x)\bar{\psi}(0)\rangle = \int [dA]\,\det D D^{-1}_{(x,0)} e^{-S[A]}$$

Ignore lepton loops

$$\langle \psi(x)\bar{\psi}(0)\rangle = \int [dA]\,\cancel{\det D} D^{-1}_{(x,0)} \boxed{e^{-S[A]}}$$

In QED, this part is **free** theory!

throwing the dice part is trivially done!

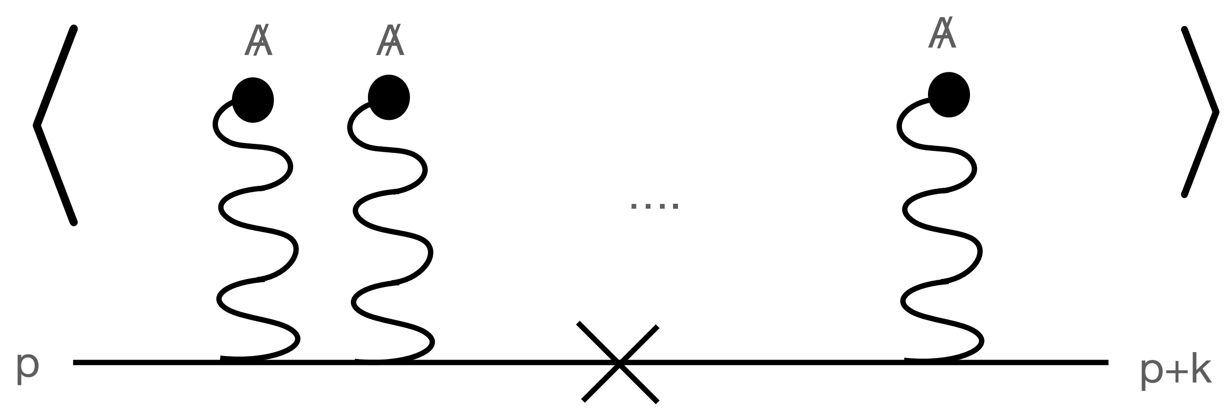
(Gaussian noise)

Perturbative calculations on the lattice

[Di Renzo, Scorzato '00]

generated gauge field configurations

only one diagram to calculate:



statistic average

Sink $t=t_{snk}$ D^{-1} γ_μ **Vertex** D^{-1} **Source** $t=0$

diagonal in the position space.

$$\eta(t_{\rm snk}) \ \underset{x\leftarrow p}{\text{FFT}} \ \boxed{\frac{1}{i\not{p}+m}} \ \underset{p\leftarrow x}{\text{FFT}} \ \boxed{ie\not{A}} \ \underset{x\leftarrow p}{\text{FFT}} \ \cdots\cdots \gamma_\mu \cdots\cdots \ \boxed{\frac{1}{i\not{p}+m}} \ \underset{p\leftarrow x}{\text{FFT}} \ \boxed{ie\not{A}} \ \underset{x\leftarrow p}{\text{FFT}} \ \boxed{\frac{1}{i\not{p}+m}} \ \underset{p\leftarrow x}{\text{FFT}} \ \eta(0)$$

diagonal in the momentum space

Sequence of multiplying diagonal matrices and FFT.

Very effectively done on computers.

We simply store the values at **each order** in the perturbation.
Averaging process adds up all the diagrams at each order **automatically**.

Renormalization?

This is the three-point function I can calculate perturbatively. This is a **divergent** quantity.

$$G_\mu(t) = \left\langle \sum_{\mathbf{p}'} D^{-1}(t_{\rm sink}, t; \mathbf{p}, \mathbf{p}')\gamma_\mu D^{-1}(t, t_{\rm src}; \mathbf{p}' + \mathbf{k}, \mathbf{p} + \mathbf{k}) \right\rangle$$

But anyway, by separating $t_{\rm sink}$, t, $t_{\rm src}$, this quantity is dominated by the contributions from **on-shell fermion states**.

Electric and Magnetic projections: $G_E(t) = {\rm tr}\left[\frac{1+\gamma_4}{2} G_4(t)\right], \quad G_M(t) = i\sum_{i,j,k} \epsilon_{ijk} {\rm tr}\left[\frac{1+\gamma_4}{2}\gamma_5\gamma_i G_j(t)\right]\mathbf{k}_k,$

Repeat the same calculations for $G^{\rm norm}_\mu(t) = \sum_{\mathbf{p}'} \left\langle D^{-1}(t_{\rm sink}, t; \mathbf{p}, \mathbf{p}') \right\rangle \gamma_\mu \left\langle D^{-1}(t, t_{\rm src}; \mathbf{p}' + \mathbf{k}, \mathbf{p} + \mathbf{k}) \right\rangle$

and normalize $$F_E(t) = \frac{G_E(t)}{G^{\rm norm}_E(t)}, \quad F_M(t) = \frac{G_M(t)}{G^{\rm norm}_M(t)},$$

now **external legs are taken away**. We get form factors.

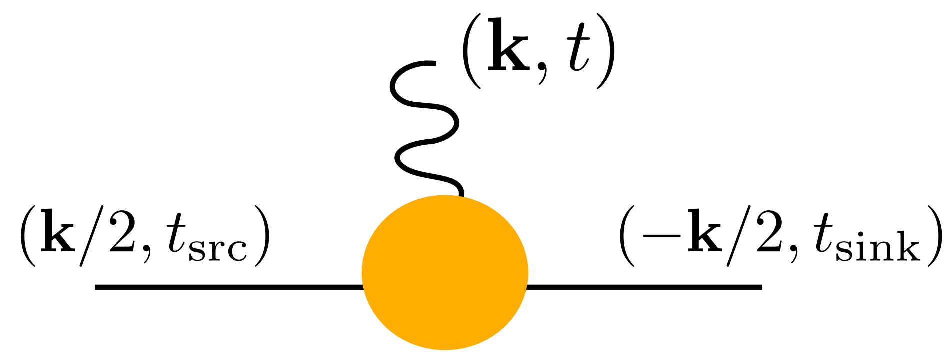
Finally, we get the g-factor $$\frac{g(t)}{2} = \frac{F_M(t)}{F_E(t)},$$ perturbatively.

All the divergence is gone, because this is a physical quantity!

Done!

Of course, the life is not so easy.

Limit, limit, limit…



We need to take the limits of

IR cutoff $m_\gamma \to 0$

continuum limit $m \to 0$
(fermion mass parameter)

infinite volume $L \to \infty$

(in the lattice unit, a=1)

while keeping

$$1/L \ll m_\gamma \ll m$$

The strategy is to keep $m_\gamma L \gg 1$,
and take the double limit, $m_\gamma/m \to 0$ and $m \to 0$.

We need a large volume!!

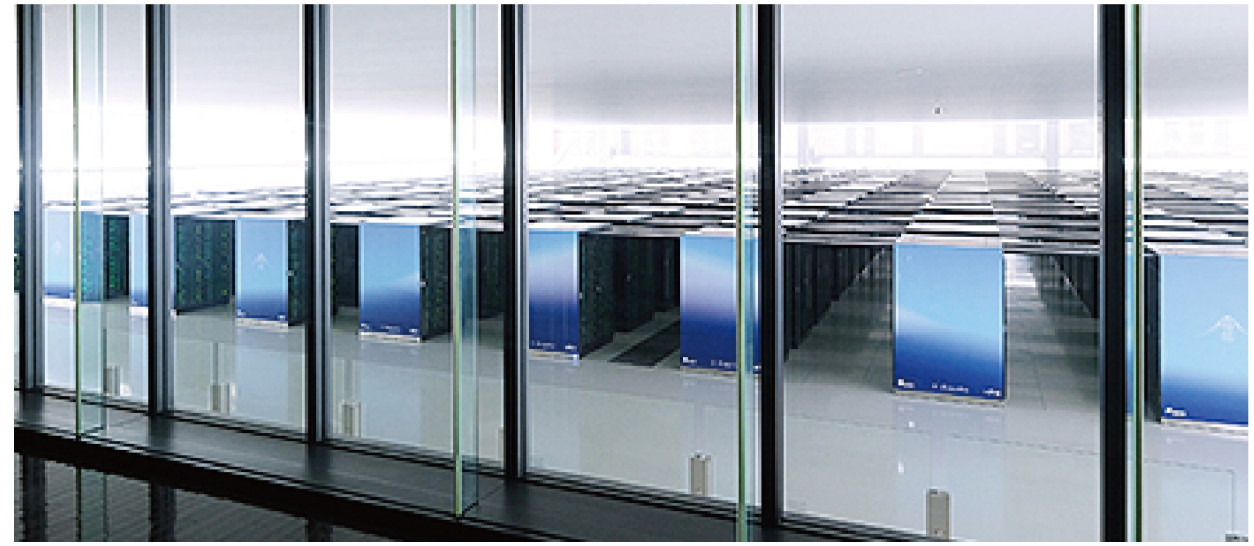
For example, if we want $m_\gamma L \sim 4$, $m_\gamma/m \sim 0.1$, and $m^2 \sim 0.1$, we need $L \sim 100$!

We need a supercomputer.

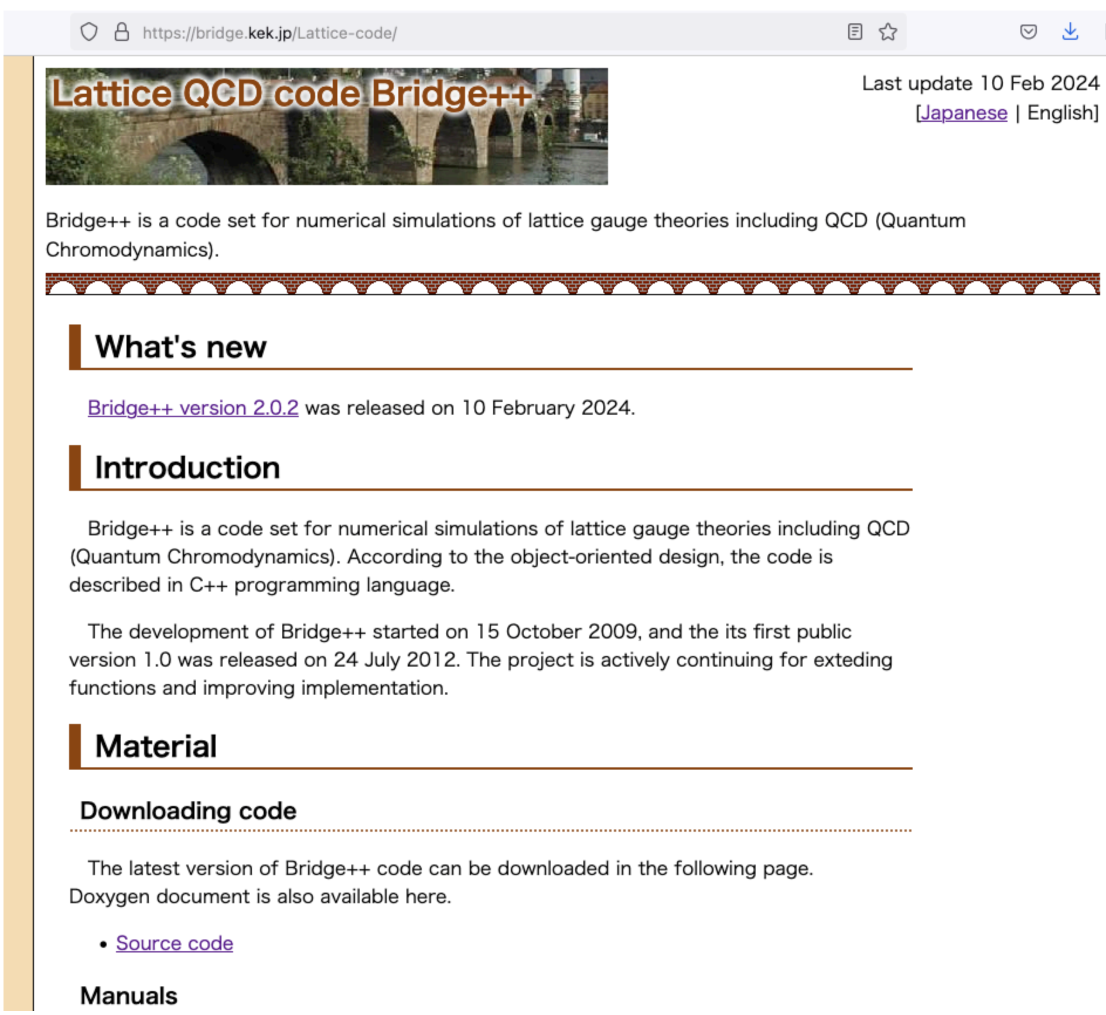
Supercomputer and code:

We had a good one in the next building. (-2024)

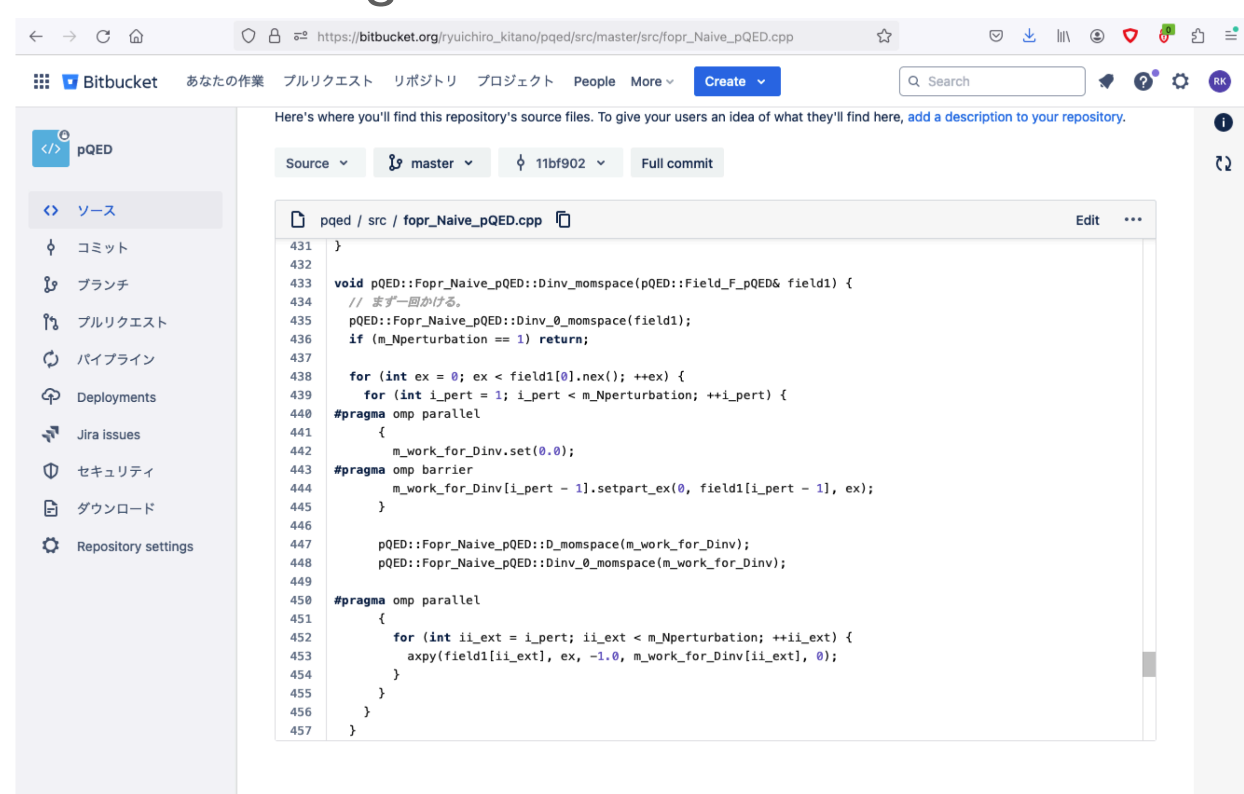
FUGAKU is also open for researchers.



Matsufuru-san in the next building has been developing a user friendly open lattice codes:
(Thanks, Matsufuru san!)

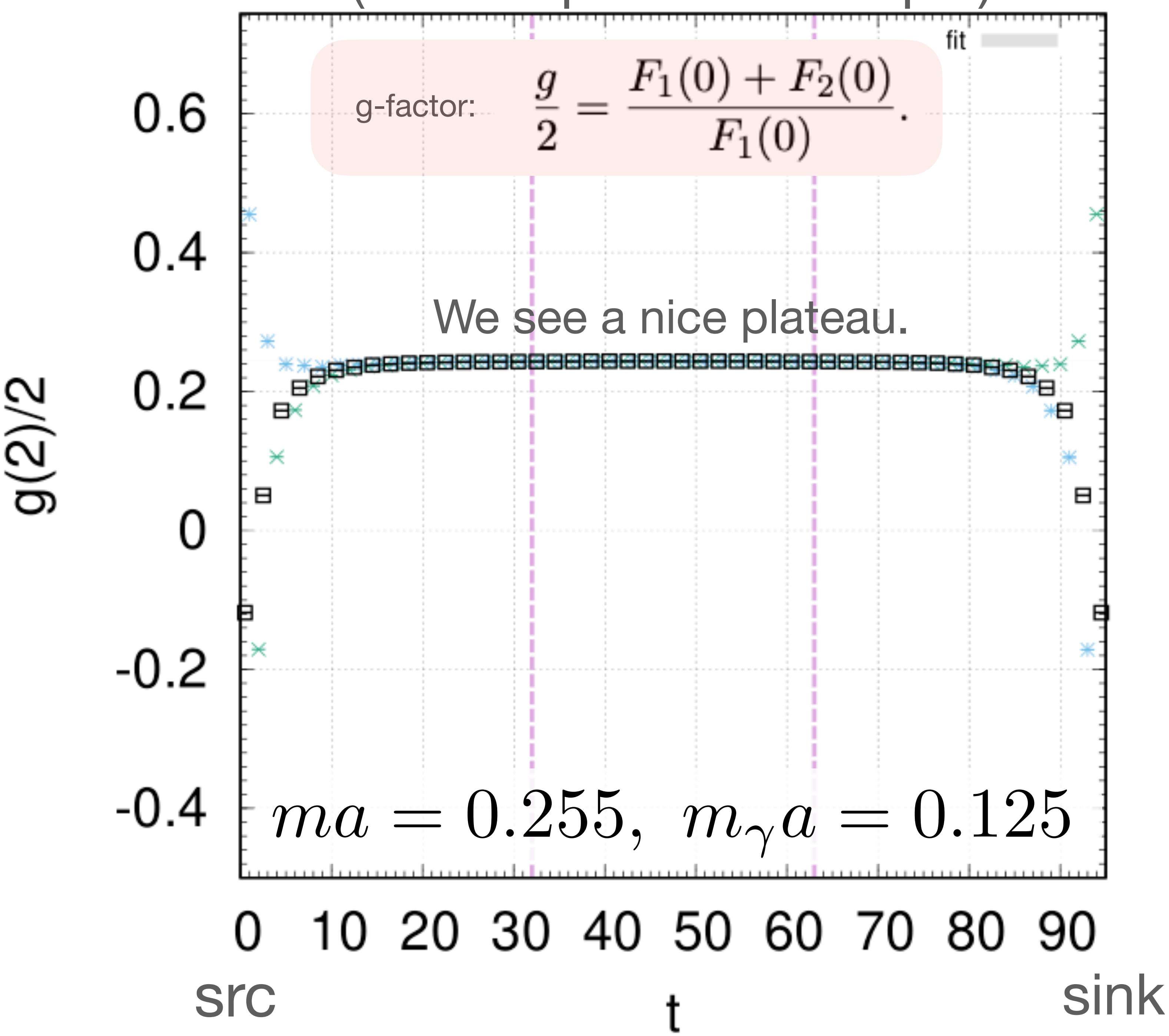


It is a good summer homework!



64^3x128 lattice results:

(one-loop order example)



location of vertex

O(200,000) configurations.

This is a result of FUGAKU 3days.

plateau ~ on-shell

$$m_\gamma L = 8$$

finite volume effect under control.

I'm actually using a trick to make T-direction larger by averaging periodic and anti-periodic boundary conditions. No worry about backward propagation.

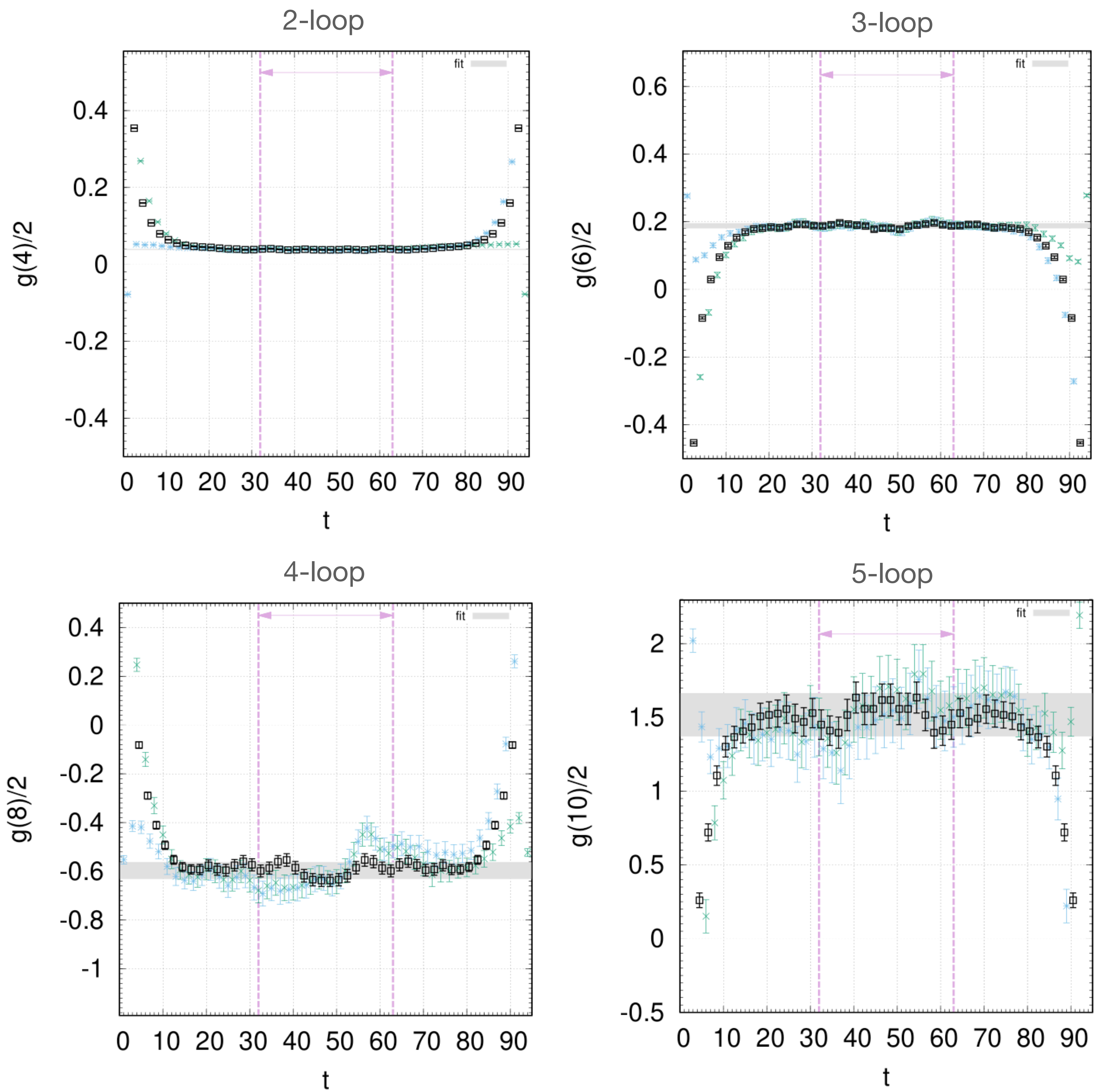
64^3x128 lattice results:

O(200,000) configurations.

This is a result of FUGAKU 3days.

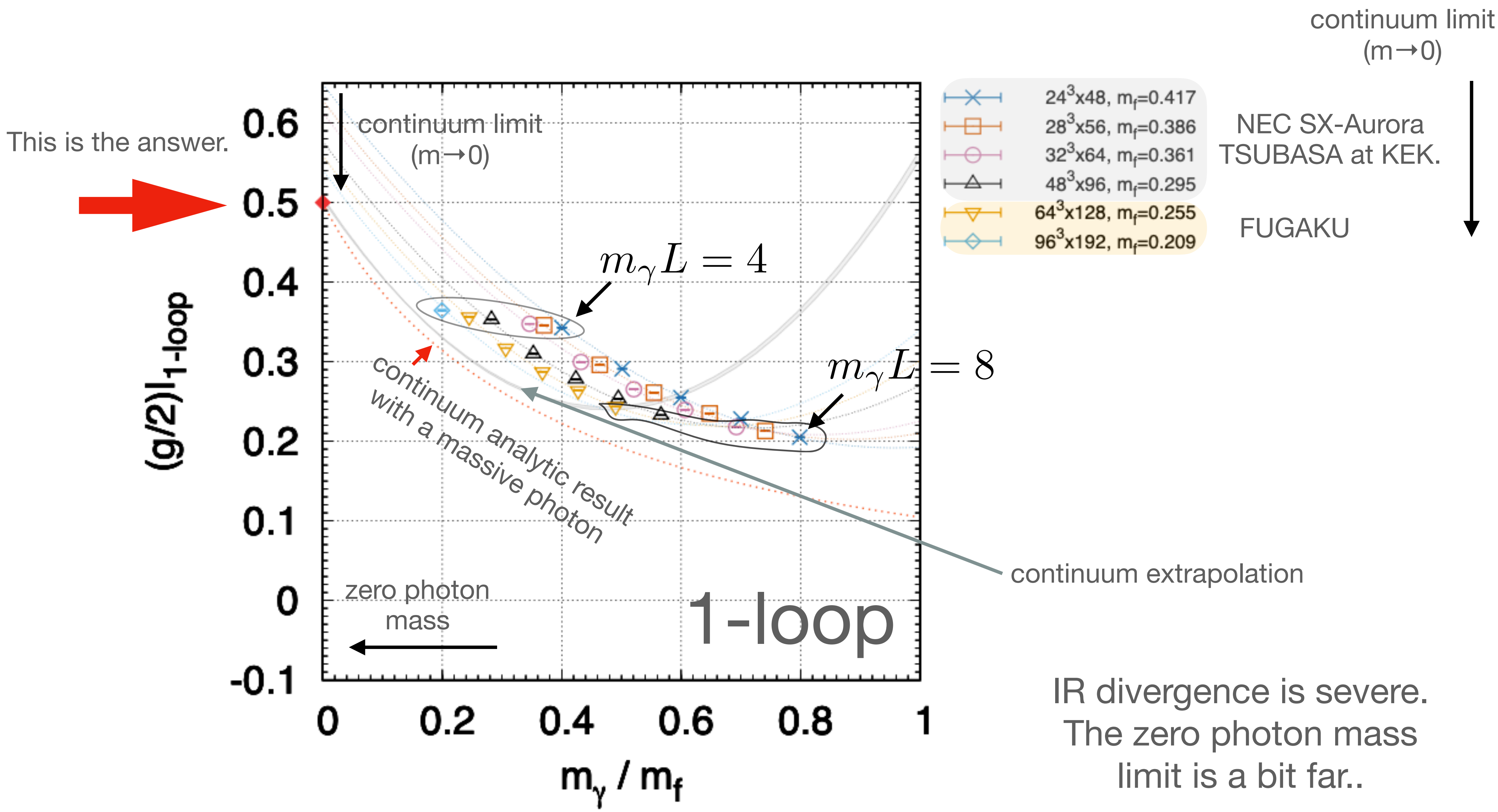
$$ma = 0.255,\ m_\gamma a = 0.125$$

But the photon mass is still big.



Fitting:

$$A^{(2n)}(m, m_\gamma) = a_0^{(2n)}\left(1 + b_0^{(2n)}(ma)^2\right) + a_1^{(2n)}\frac{m_\gamma}{m}\left(1 + b_1^{(2n)}(ma)\right) + a_2^{(2n)}\left(\frac{m_\gamma}{m}\right)^2\left(1 + b_2^{(2n)}(ma)\right)$$



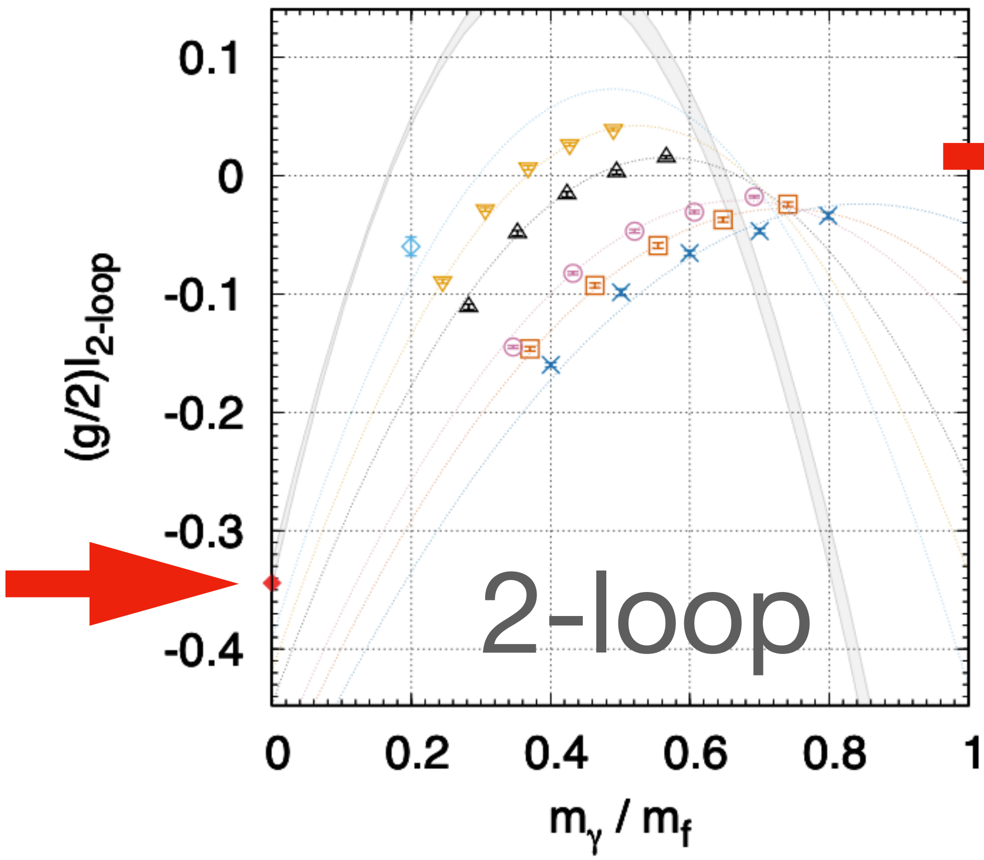
IR divergence is severe. The zero photon mass limit is a bit far..

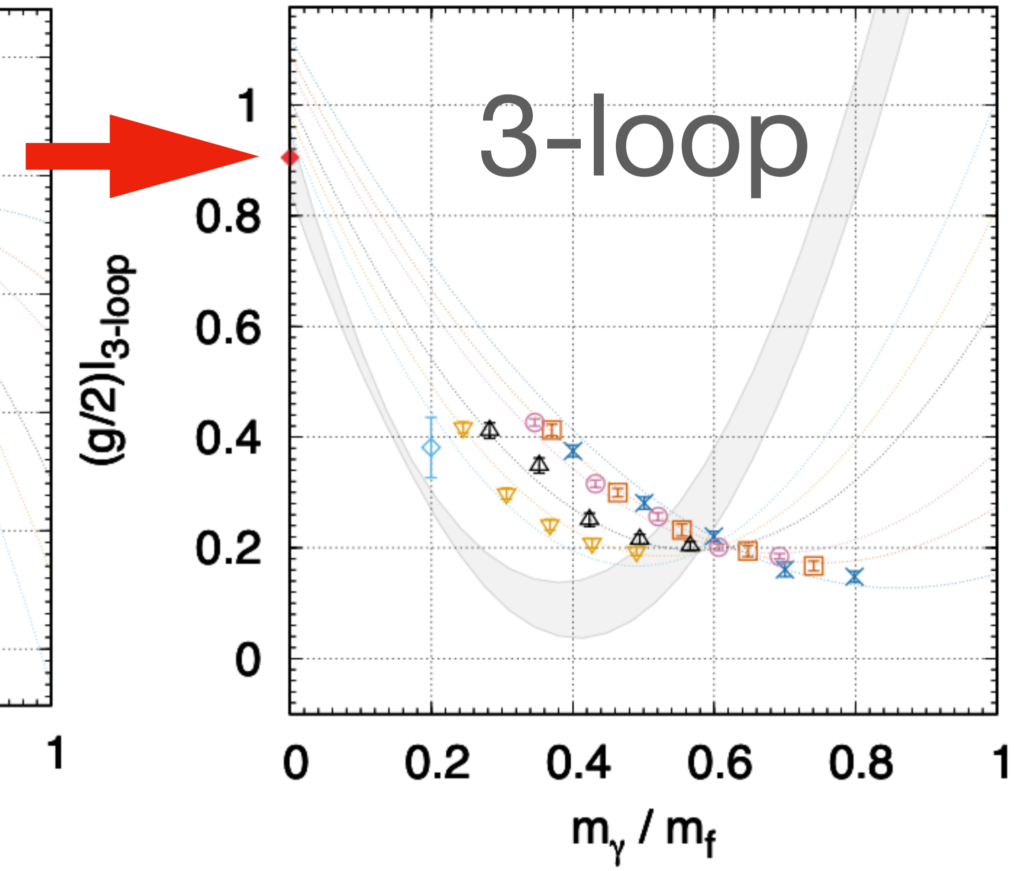
Looks like we could reproduce α/π.

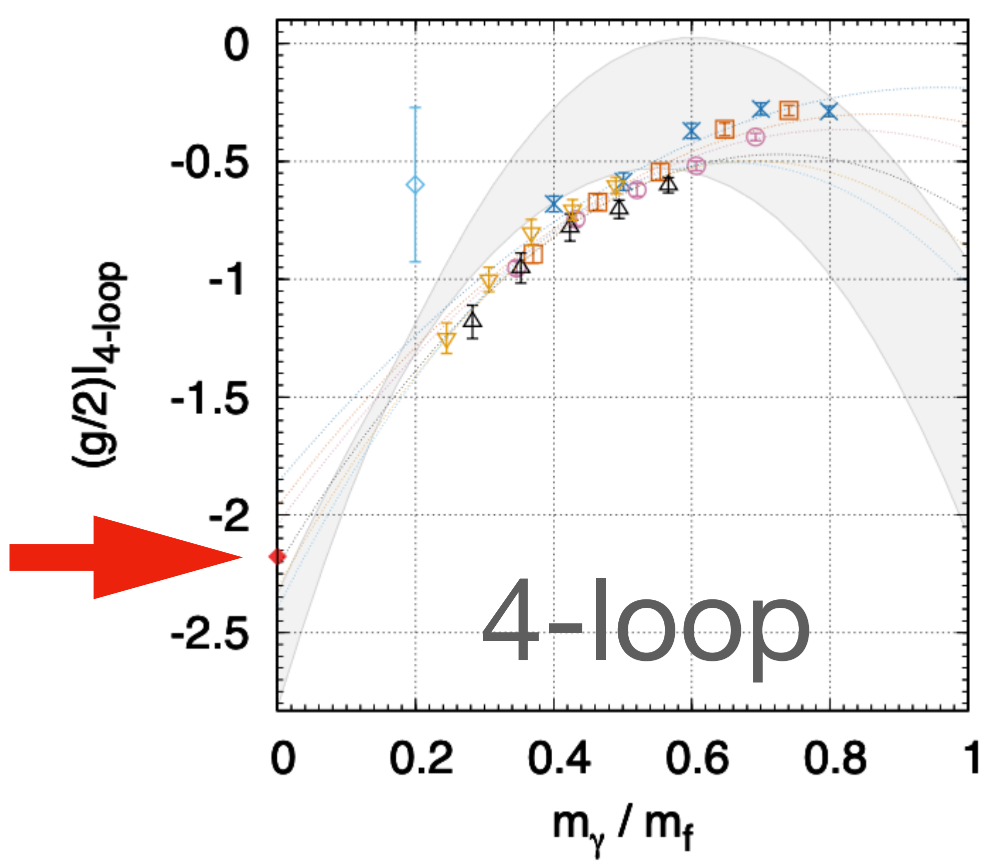
systematic error (including fitting, finite volume etc.) is a percent level. (hopefully)

higher loops:

fitting with quadratic functions.



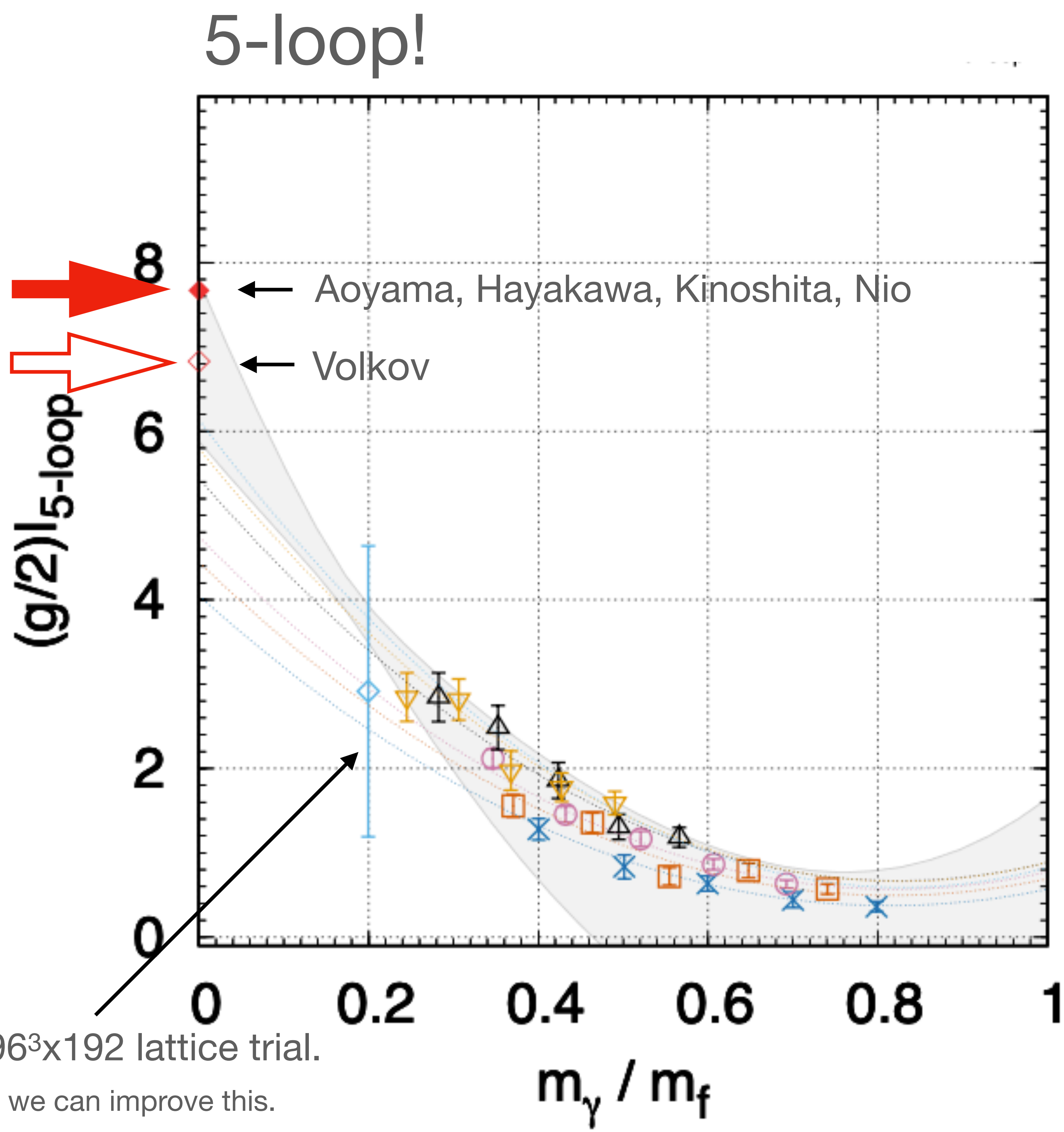




Looks like I'm doing all right.

5-loop results:

fitting with quadratic functions.



I guess I could give an independent confirmation.

This is 96^3x192 lattice trial.

Maybe we can improve this.

My estimate:

$$A^{(10)}(\text{no lepton loop}) = 7.0 \pm 0.9$$

to be compared with

$$7.668 \pm 0.159 \quad \text{(AHKN)}$$

$$6.828 \pm 0.060 \quad \text{(Volkov)}$$

I guess this would be a totally independent check of the 5-loop coefficient.

Summary

I tried.

I couldn’t quite reach the precision of the Feynman diagram method, but at least this gives a totally independent calculation/confirmation.

Happy birthday, Hitoshi!