19th December, 2024 @IPMU

String-wall composites from a scalar dark matter model motivated by grand unification

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This talk is based on ...

▪#1 ***“GUTs, hybrid topological defects, and gravitational waves”***, David I. Dunsky, Anish Ghoshal, **Hitoshi Murayama**, Yuki Sakakihara, Graham White, PhysRevD.106.075030 (2022) + APPC15 Proceedings

Modeling composite defects and the gravitational waves from them

▪#2 ***“String-wall composites winding around a torus knot vacuum in an axionlike model”***, Minoru Eto, Takashi Hiramatsu, Izumi Saito, Yuki Sakakihara, PhysRevD.108.116004 (2023)

Structures of the composite defects in field theoretical ground

▪#3 ***“Three-dimensional simulation of string-wall composites in an axionlike model”***, Minoru Eto, Takashi Hiramatsu, Izumi Saito, Yuki Sakakihara, Number: 86, pp. 154-161, Proceedings of JSST 2024 and AsiaSim 2024.

A benchmark simulation of the composite defects

Supports

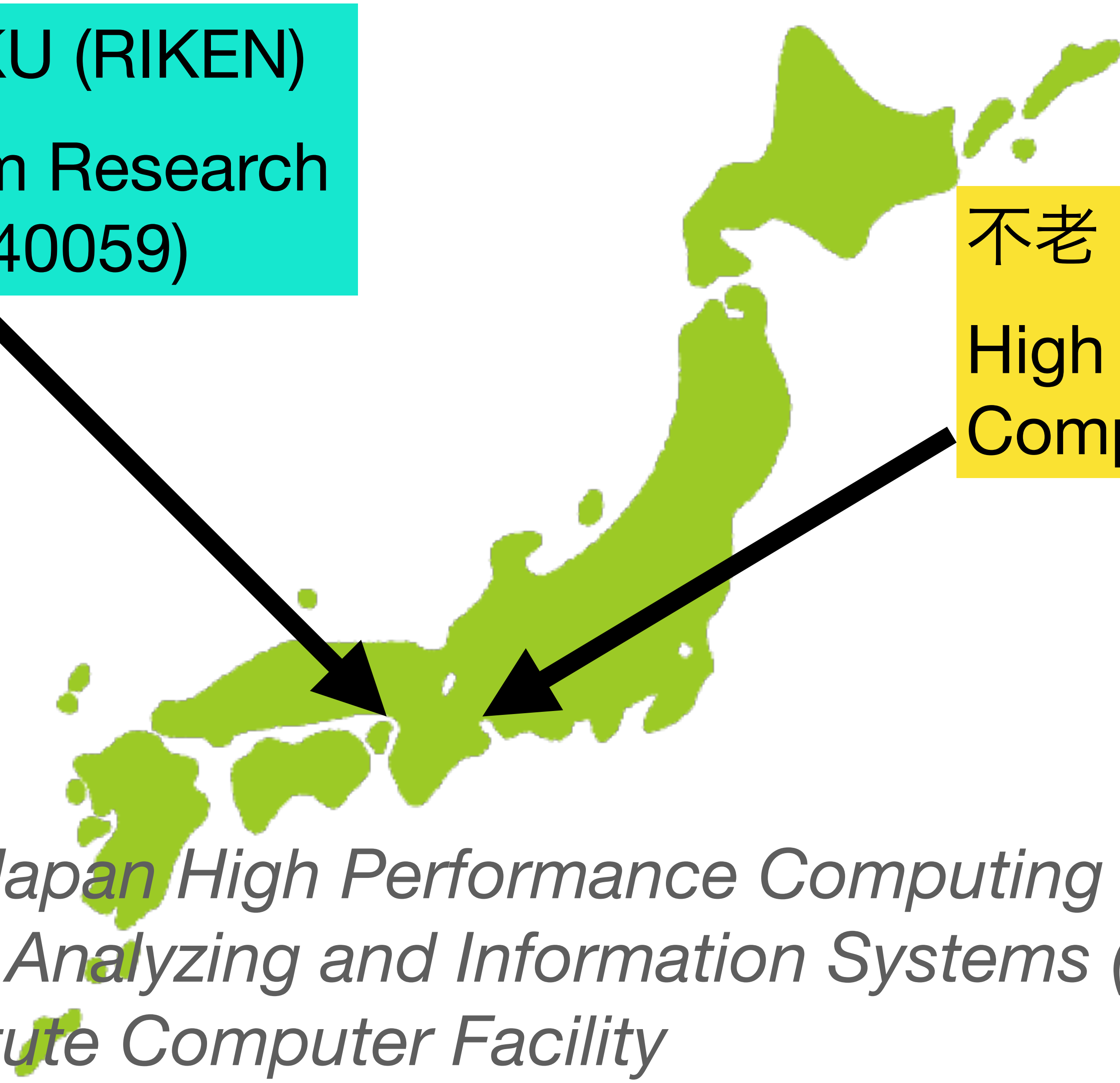
JSPS KAKENHI: JP22K20365 (YS)

富岳 FUGAKU (RIKEN)

HPCI System Research project (hp240059)

不老 Flow (Nagoya U.)

High Performance Computing (HPC2024)



Other supports: Japan High Performance Computing and Networking plus Large-scale Data Analyzing and Information Systems (jh240020), and Yukawa Institute Computer Facility

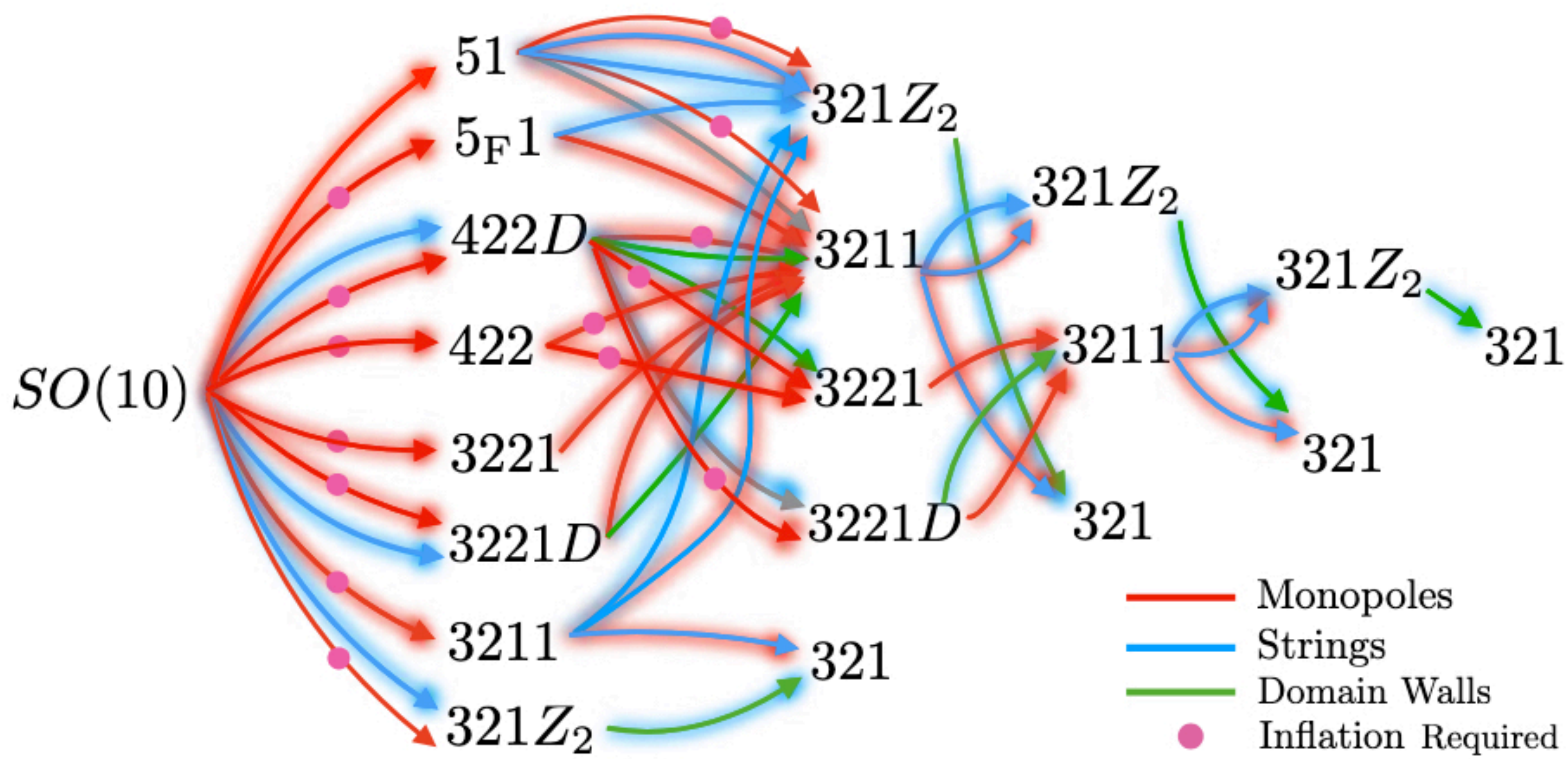
Grand unification and solitons

- Unification of three forces
  ⇒ embedding SM into a larger gauge group, such as,

$$SO(10) \to \cdots \to SU(3) \times SU(2) \times U(1)_Y$$

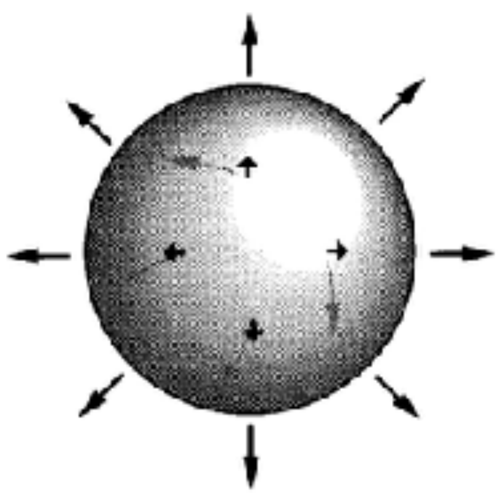
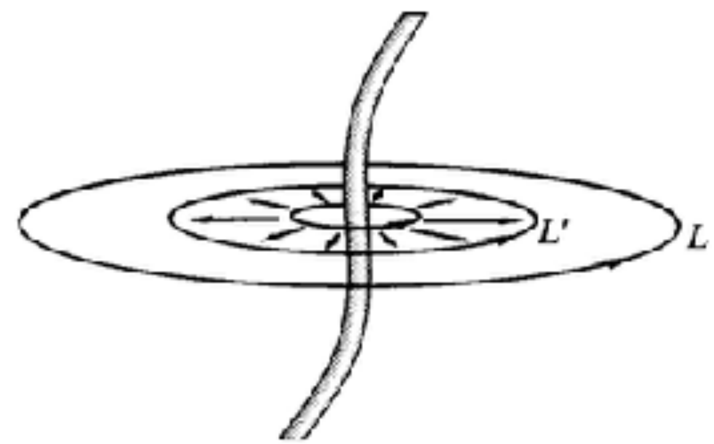
Avoiding the current proton lifetime constraint.

A variety of topological solitons appears at each symmetry breaking scale.



Dunsky, Ghoshal, Murayama, YS, White, 2022

Topological solitons

Monopoles	Cosmic strings	Domain walls
		
0-dim obj.	1-dim obj.	2-dim obj.

- G/H is disconnected: $\pi_0(G/H) \neq I \ \rightarrow$ Domain wall
- $G/H \sim S^1 : \ \pi_1(G/H) \neq I \ \rightarrow$ Cosmic String
- $G/H \sim S^2 : \ \pi_2(G/H) \neq I \ \rightarrow$ Monopoles

Dangerous solitons to cosmology

- Monopoles dominates the universe quickly

$$\rho_{\text{monopole}} \sim 10^{13}\rho_0 \left(\frac{T_{\text{form}}}{10^{15}\text{GeV}}\right)^4$$

- Domain walls dominates the universe quickly

$$R \sim t,\ H \sim t^{-1}$$

$$\rho_{\text{DW}} \sim \sigma R^2/H^{-3} \sim R^2/t^3 \sim 1/t$$

$$\rho_{\text{rad}} \sim 1/t^2$$

- (Exception) cosmic string is not dangerous

$$\rho_{\text{DW}} \sim \mu R/H^{-3} \sim R/t^3 \sim 1/t^2$$

the fraction to the total energy of the universe does not change

Grand unification and solitons

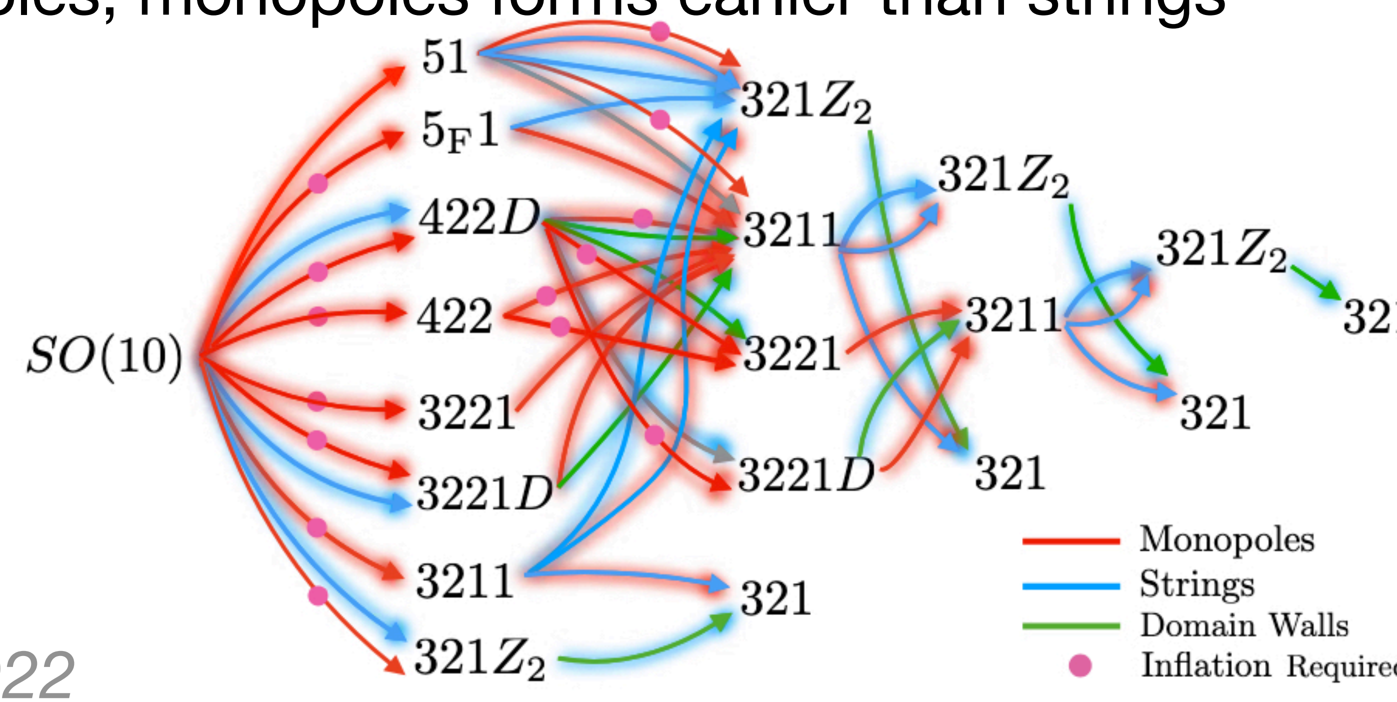
- Many of the topological solitons formed in SO(10) are composites of monopoles, strings, and walls

  - Walls bounded by strings,
  - Strings bounded by monopoles, ...

■ For composite topological solitons from symmetry breakings, *a lower dimensional topological solitons forms earlier than the one dimensional higher topological solitons*

  - For walls bounded by strings, strings forms earlier than walls
  - For strings bounded by monopoles, monopoles forms earlier than strings

$$
\begin{aligned}
51 &= SU(5)\times U(1)_X/\mathbb{Z}_5 \ ,\\
5_{\rm F}1 &= SU(5)_{\rm flipped}\times U(1)_{\rm flipped}/\mathbb{Z}_5 \ ,\\
422 &= SU(4)_c\times SU(2)_L\times SU(2)_R/\mathbb{Z}_2 \ ,\\
3221 &= SU(3)_c\times SU(2)_L\times SU(2)_R\times U(1)_{B-L}/\mathbb{Z}_6 \ ,\\
3211 &= SU(3)_c\times SU(2)_L\times U(1)_Y\times U(1)_X/\mathbb{Z}_6 \ ,\\
321 &= SU(3)_c\times SU(2)_L\times U(1)_Y/\mathbb{Z}_6 . \qquad (20
\end{aligned}
$$



Dunsky, Ghoshal, Murayama, YS, White, 2022

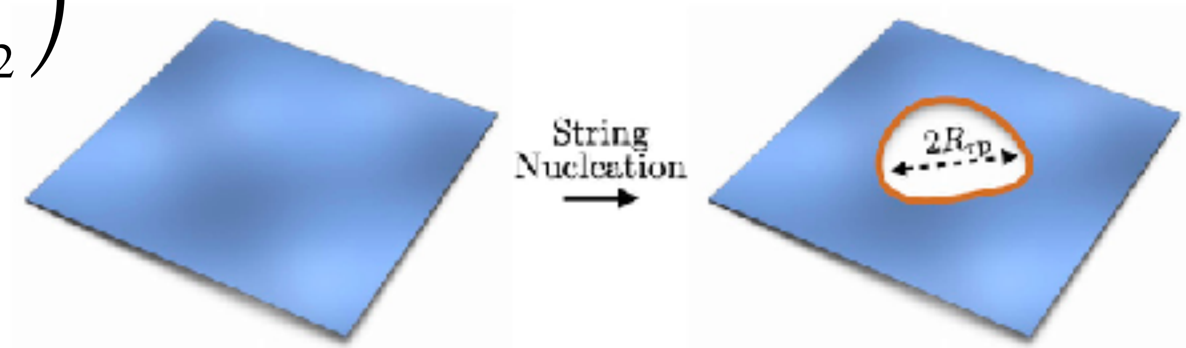
Grand unification and solitons

- Inflation can occur at some stage
  (If stable monopoles formed at a symmetry breaking,
  they need to be inflated away after the formation.)

【Walls bounded by strings】

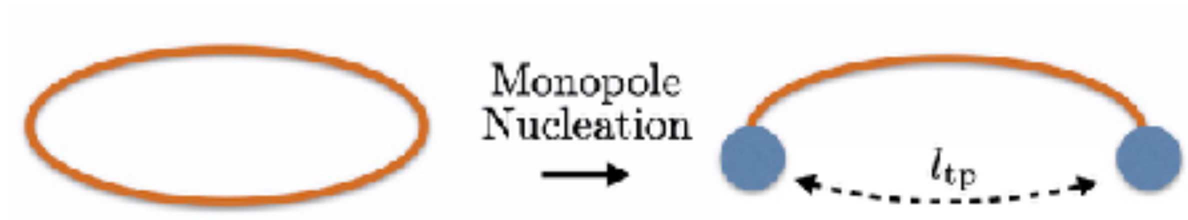
If inflation occurs after the string formation,
walls are unstable toward the nucleation of string on them

$$\Gamma_s \sim \sigma \exp(-\frac{16\pi}{3}\kappa_s) \quad \kappa_s = \frac{\mu^3}{\sigma^2} \sim \left(\frac{v_1}{v_2}\right)^6$$



【Strings bounded by monopoles】

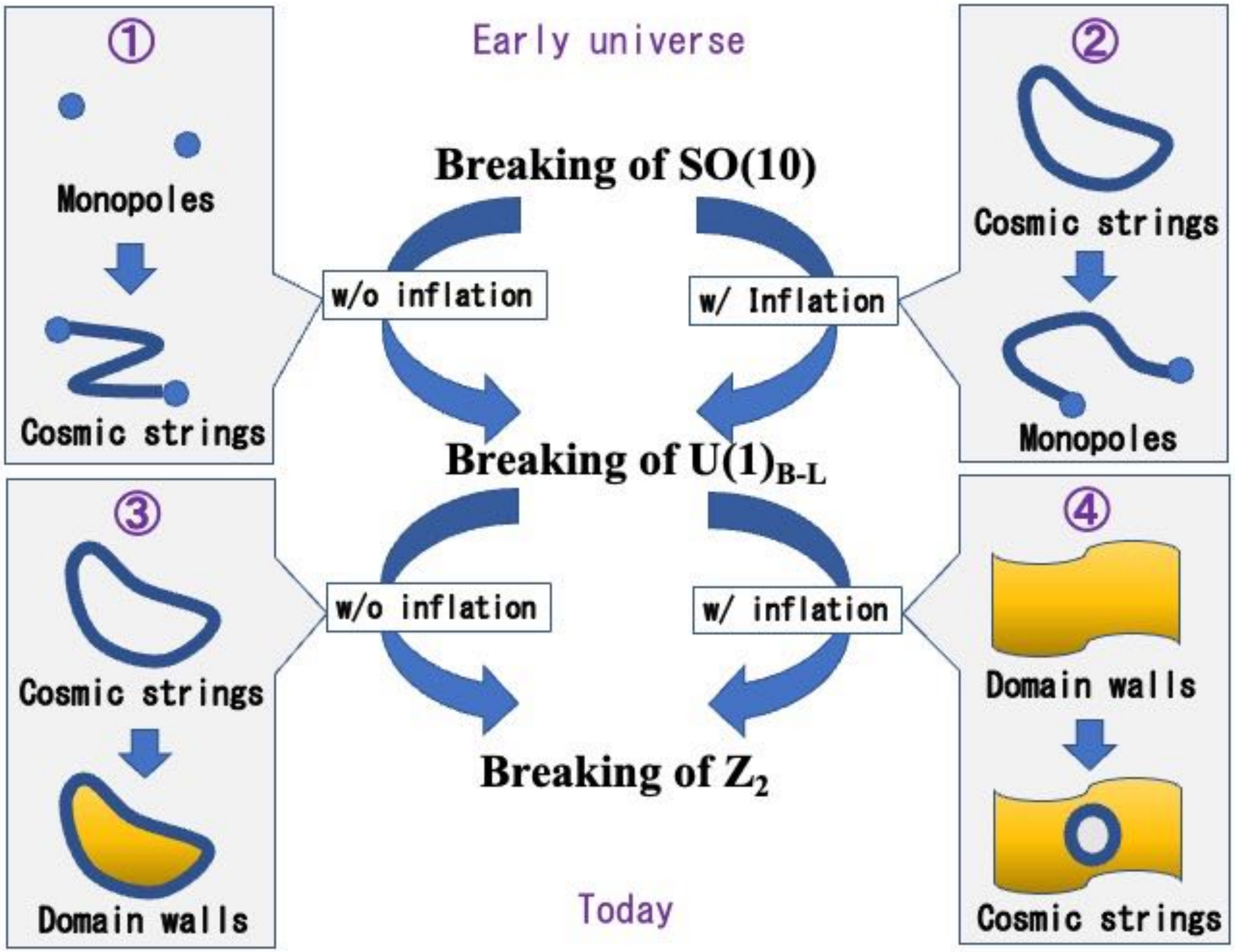
$$\Gamma_d = \frac{\mu}{2\pi}\exp(-\pi\kappa_m) \quad \kappa_m = \frac{m^2}{\mu} \sim \left(\frac{v_1}{v_2}\right)^2$$



Dunsky, Ghoshal, Murayama, YS, White, 2022

Grand unification and solitons

- As for composites made of two different dimensional topological solitons, there are four types

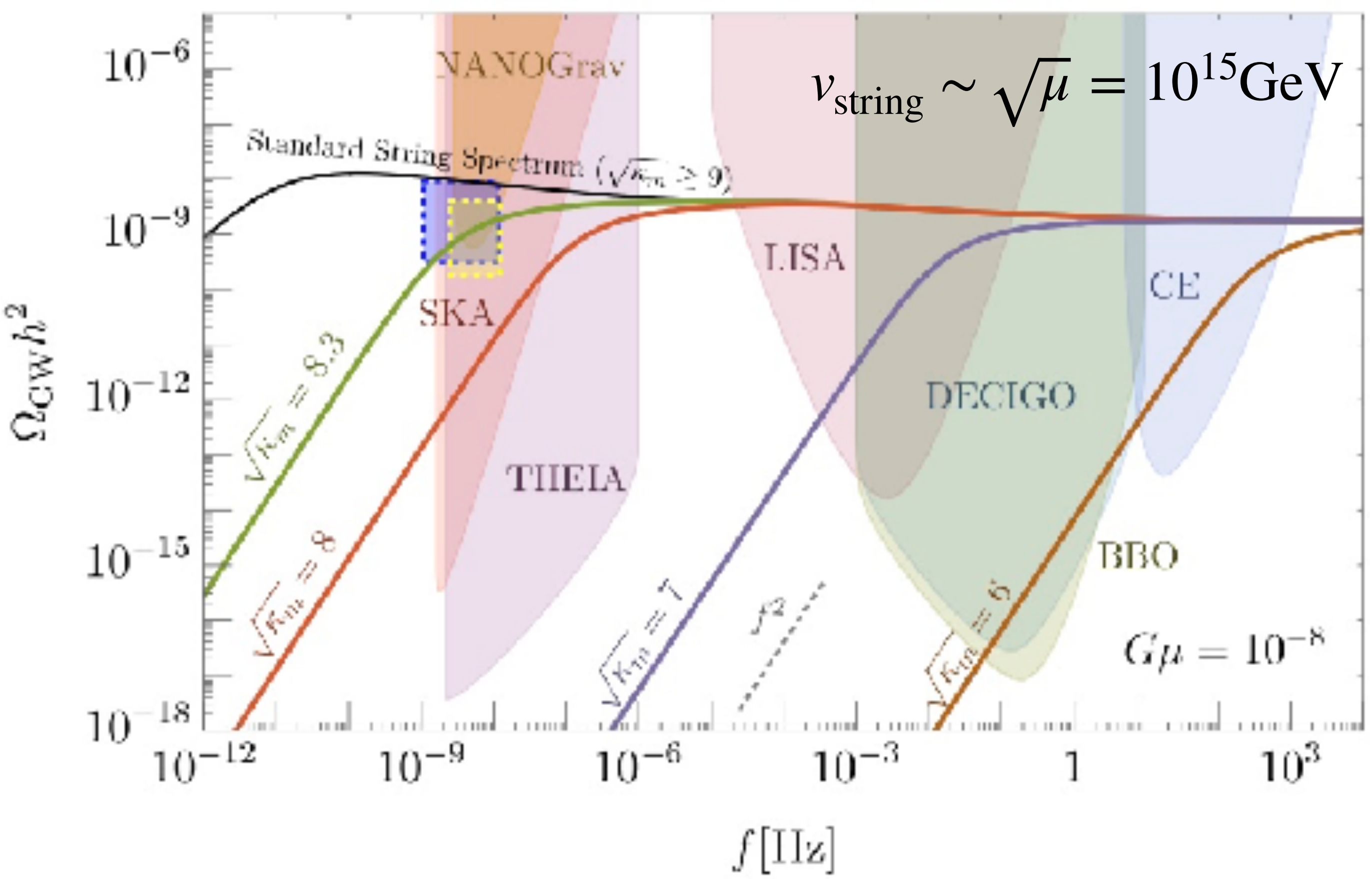


Dunsky, Ghoshal, Murayama, YS, White, 2022

Gravitational waves from the topological solitons $\mu R < \sigma R^2$

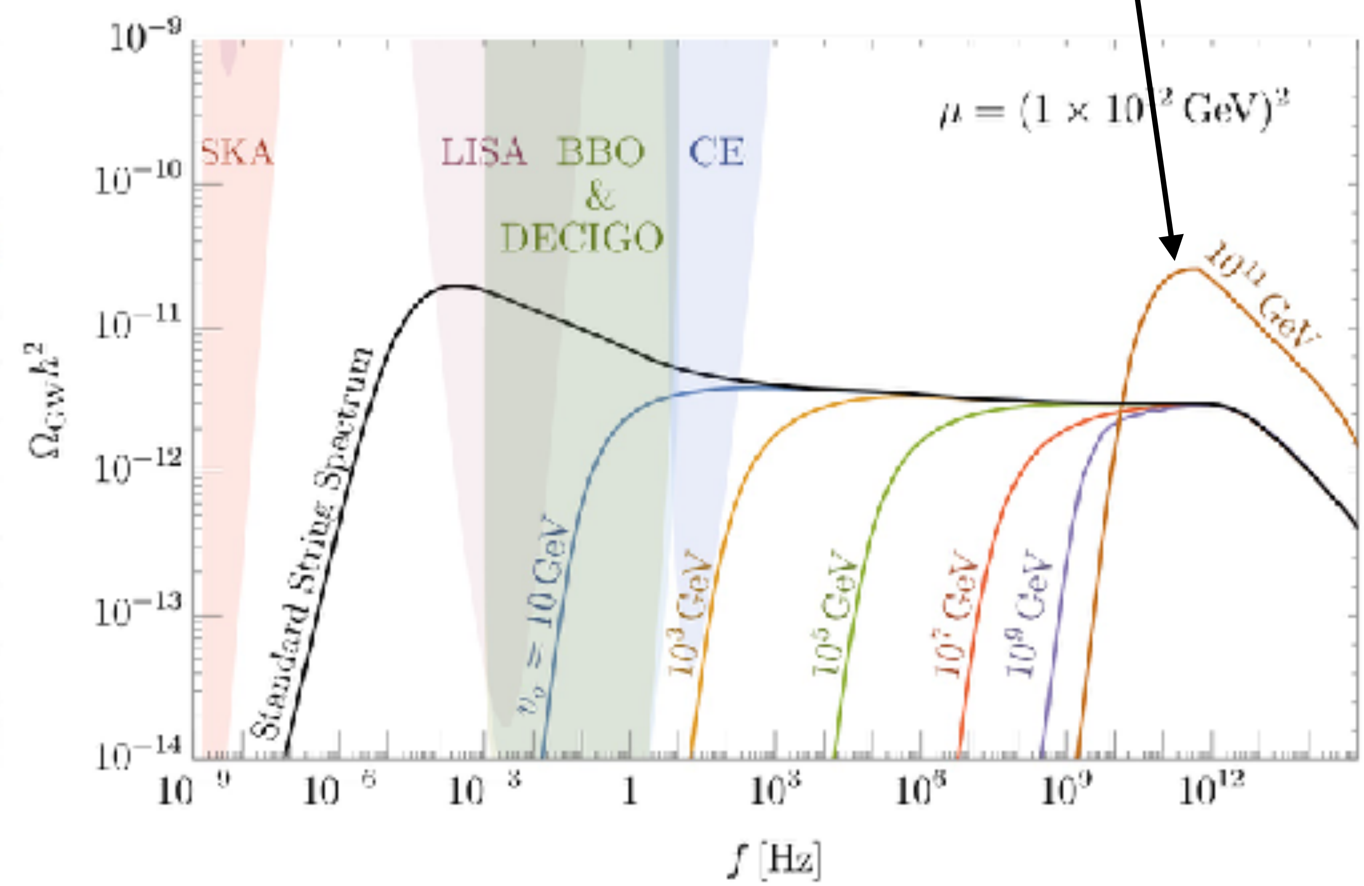
Wall energy is larger than string energy at the wall formation

② Monopole nucleation on strings



High frequency region: f^0
Low frequency region: f^2

③ Wall bounded by strings



High frequency region: f^0
Low frequency region: f^3

Dunsky, Ghoshal, Murayama, YS, White, 2022

Gravitational wave spectra

① Strings bounded by monopoles

② Monopole nucleation on strings

③ Walls bounded by strings

④ String nucleation on walls

	Low freq.	Middle freq.	High freq.
①	f^3	log f	f^{-1}
②	f^2	–	f^0
③	f^3	–	f^0
④	f^3	–	f^{-1}

- Four types of composite topological solitons are distinguishable by multi-band gravitational wave observations (IR and UV)

- These spectra are different from the spectra from stable strings at the low frequency region f^n ($n<2$)

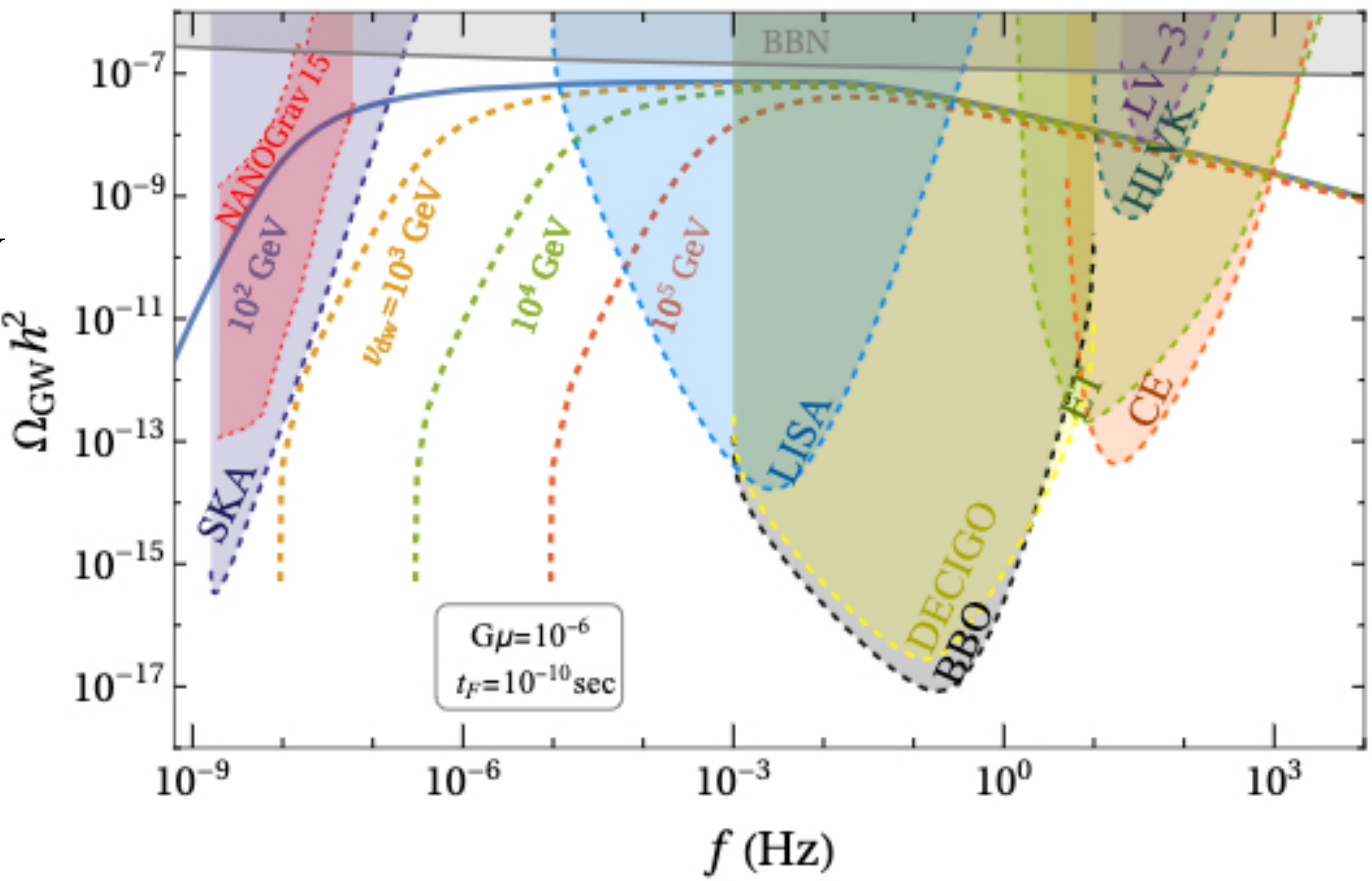
Dunsky, Ghoshal, Murayama, YS, White, 2022

Comparison with NANOGrav signal

Our analytical model for string-walls has been used for the comparison with the NANOGrav 15yr signal

- Walls bounded by strings can explain the NANOGrav signal

$$v_1 = 10^{-3} M_{\rm pl} \sim 10^{16}\text{GeV}$$
$$v_2 = 10^2\text{GeV}$$



Lazarides, Maji, Shafi 2023

Numerical approach

- For evaluating the gravitational waves from the composite solitons, we used Nambu-Goto description to model the composites

$$L = -2\pi\mu|\mathbf{r}_s(\eta)|a^2(\eta)\sqrt{1-\left(\frac{d\mathbf{r}_s}{d\eta}\right)^2} - \sigma\pi\mathbf{r}_s(\eta)^2a^3(\eta),$$

  - Suppose that we can explain the stochastic gravitational waves spectrum observed in near feature with our analytical model
  - We will be able to extract the energy scale of the symmetry breaking
  - But, it is still hard to extract the detailed information of the fundamental theory coming from SO(10) GUT because of the simplicity of the model

→ Field theoretical treatment is required

Numerical approach

- We also imposed several assumptions to be confirmed

  【Walls bounded by strings】

  - There is no equilibrium (stable) configuration of string-walls
  - String-wall network vanishes at the very instance when the walls form

➡ Confirmation of the assumption for concrete models is needed

Numerical simulations of a field theoretic model

Our model for string-walls

$$\mathrm{SM} \times U(1) \to \mathrm{SM} \times Z_2 \to \mathrm{SM}$$

$$\langle |\zeta| \rangle = v_1 \qquad\qquad \langle |\phi| \rangle = v_2$$

- Beyond SM by two SM singlet complex scalar (Higgs-like) fields with new U(1) charges

Charge assignment

$$\zeta \to e^{i2\theta}\zeta\,, \qquad \phi \to e^{i\theta}\phi\,,$$

$$\mathscr{V}_{\rm int} = \frac{m}{2}(\zeta\phi^{*2} + \zeta^*\phi^2) \to \frac{mv_1}{2}(\tilde{\phi}^{*2} + \tilde{\phi}^2)$$

$$\tilde{\phi} := \phi e^{-i\alpha}$$

- Walls bounded by strings form

$$\pi_1(U(1)/Z_2) = \boldsymbol{Z}$$ Strings

$$\pi_0(Z_2/I) = Z_2$$ Walls

Eto, Hiramatsu, Saito, YS, 2023, 2024

Our model for string-walls

$$\mathrm{SM} \times U(1) \to \mathrm{SM} \times Z_2 \to \mathrm{SM}$$

$$\langle |\zeta| \rangle = v_1 \qquad \langle |\phi| \rangle = v_2$$

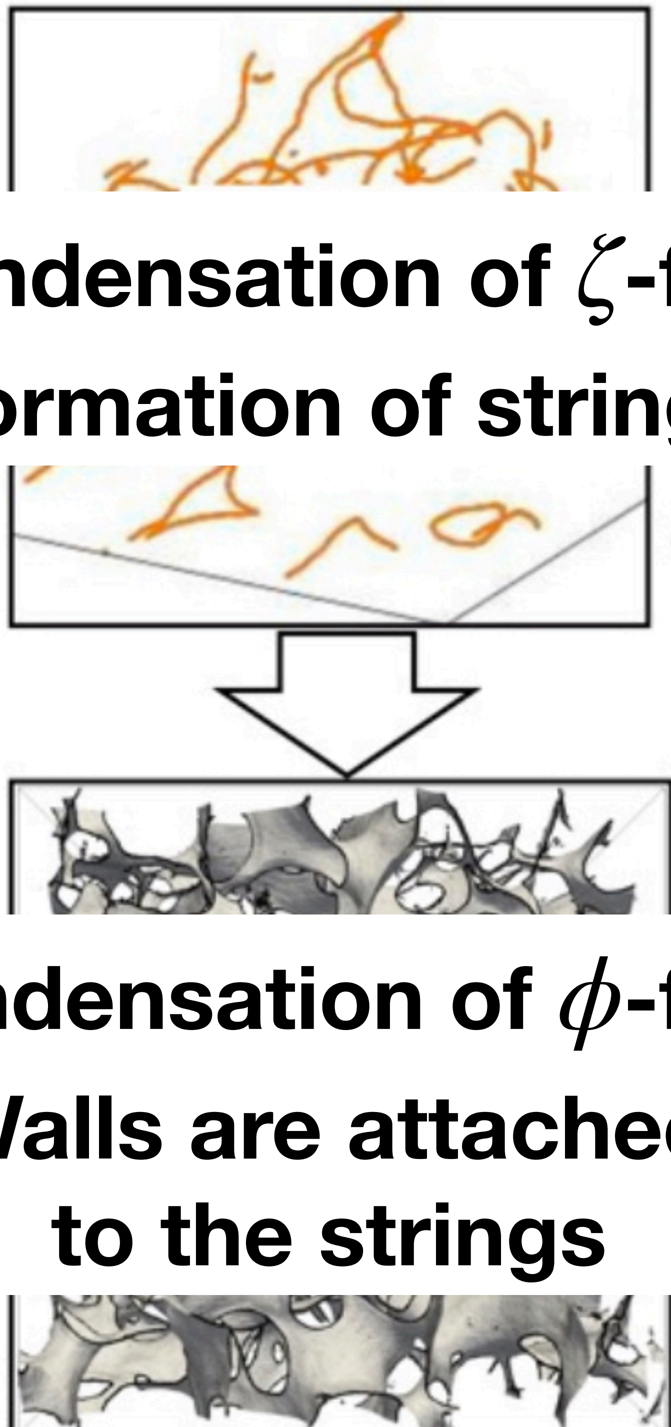
- If we regard the new U(1) as U(1)B-L,
  - **Neutrino mass** from seesaw mechanism $m_\nu \sim 0.05\mathrm{eV}$
  - **Matter-antimatter asymmetry** from leptogenesis $\Omega_b h^2 \sim 0.022$
  - From the seesaw scale, $v_1 \lesssim 10^{15}\mathrm{GeV}$
- Phase difference $2\arg(\phi) - \arg(\zeta)$ is a pseudo Nambu-Goldstone boson of the approximate symmetry $U(1)_\zeta \times U(1)_\phi$

⇒ **Dark matter candidate**

Eto, Hiramatsu, Saito, YS, 2023, 2024

String-walls in our model

Composite topological defects appear in the early universe

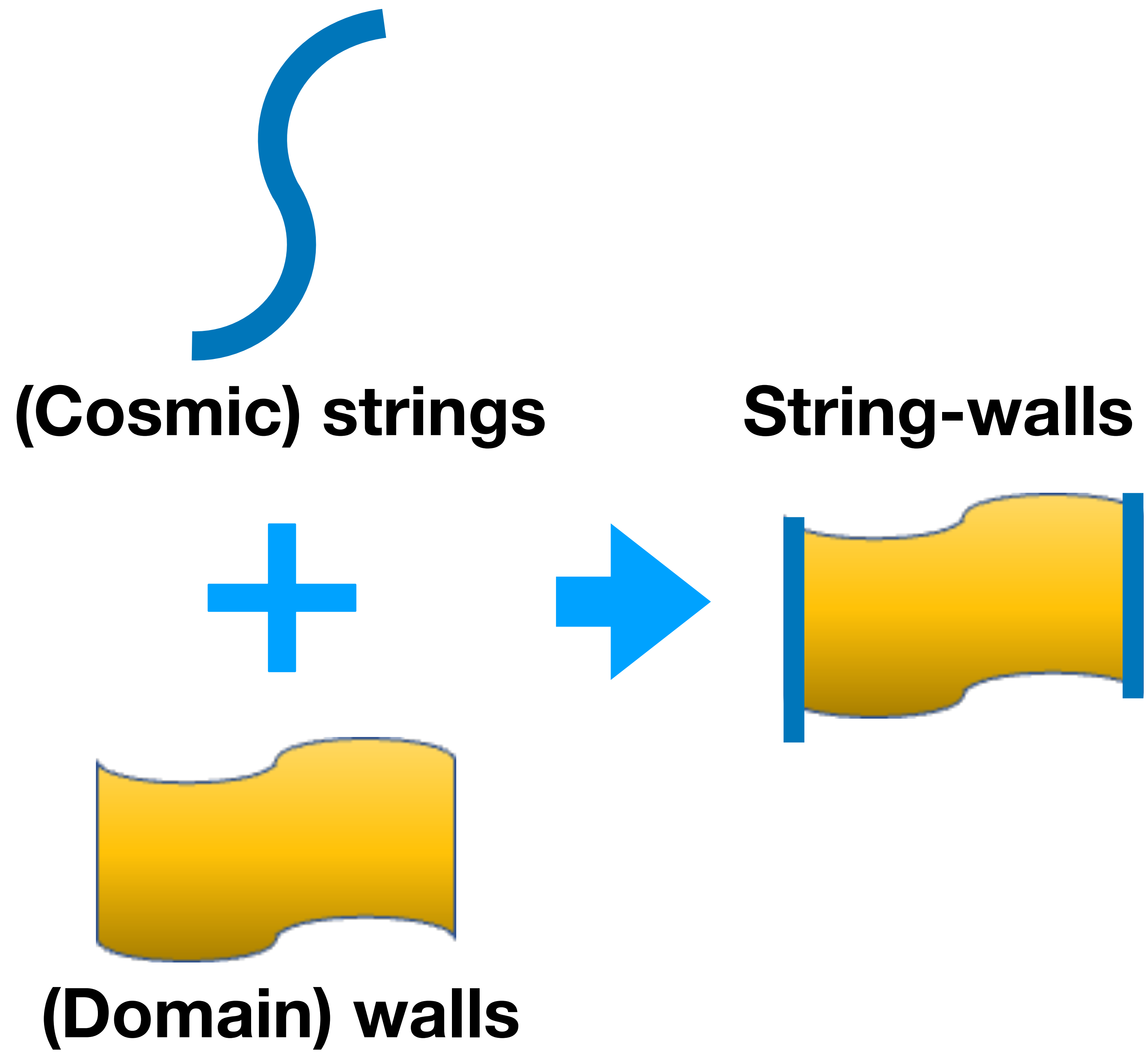


Condensation of ζ-field

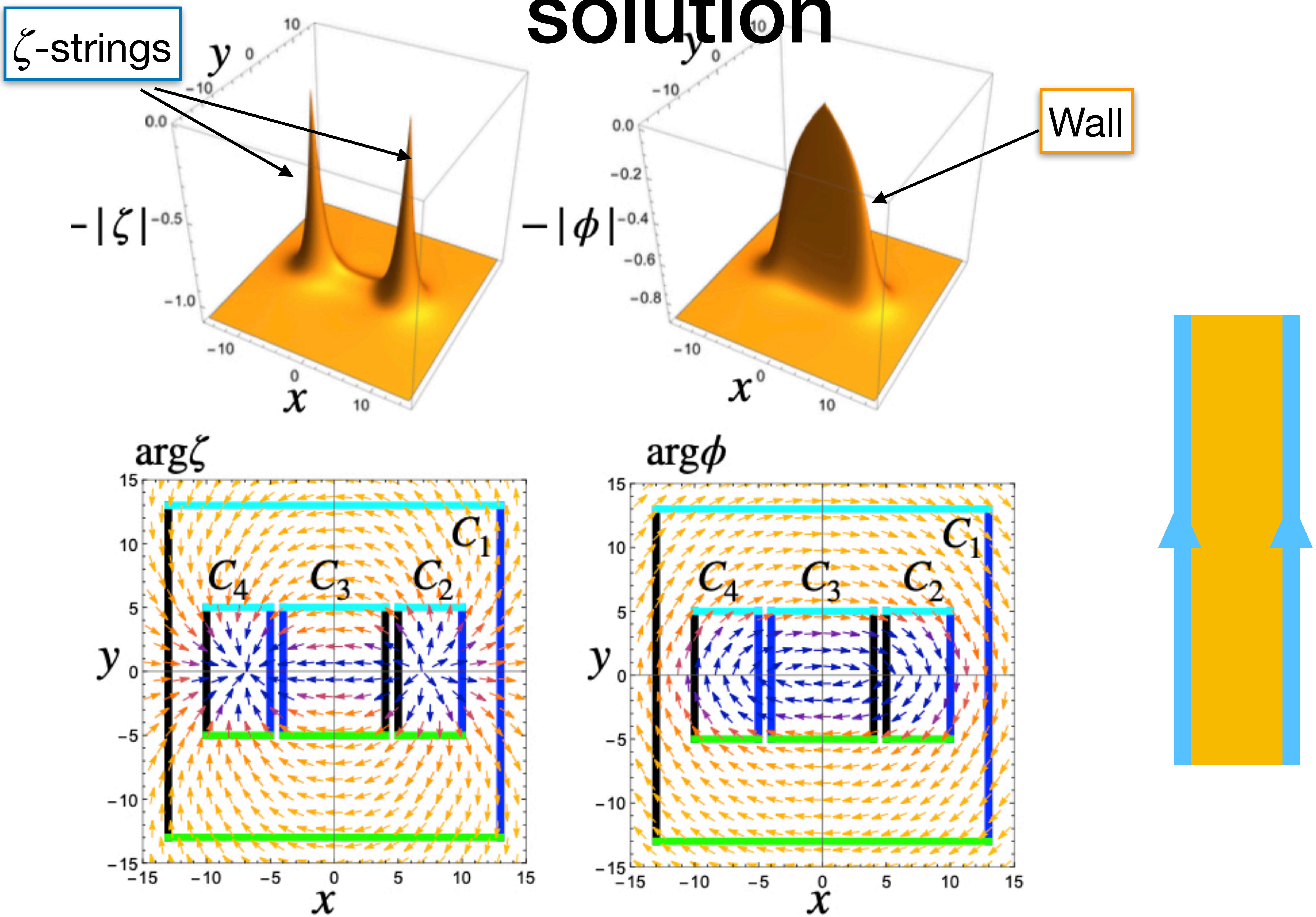
Formation of strings

Condensation of ϕ-field

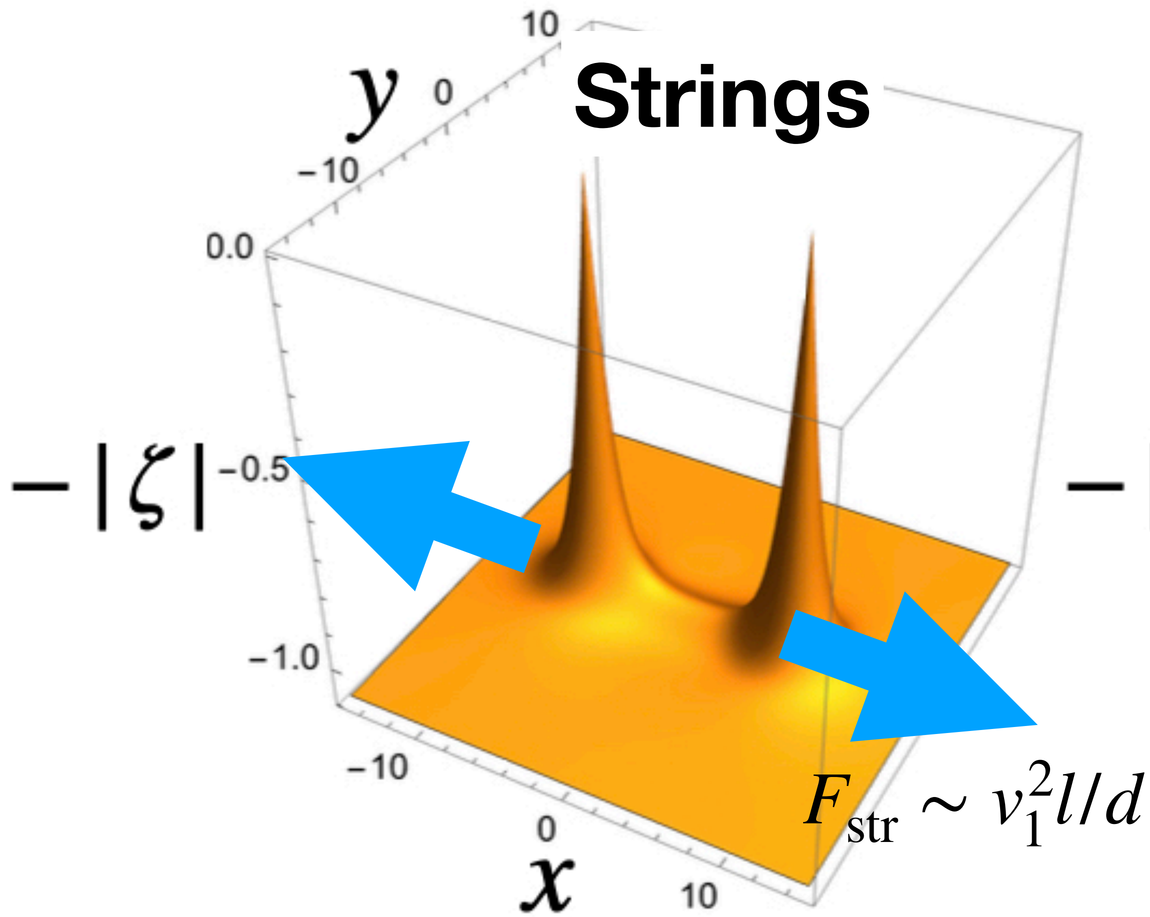
Walls are attached to the strings

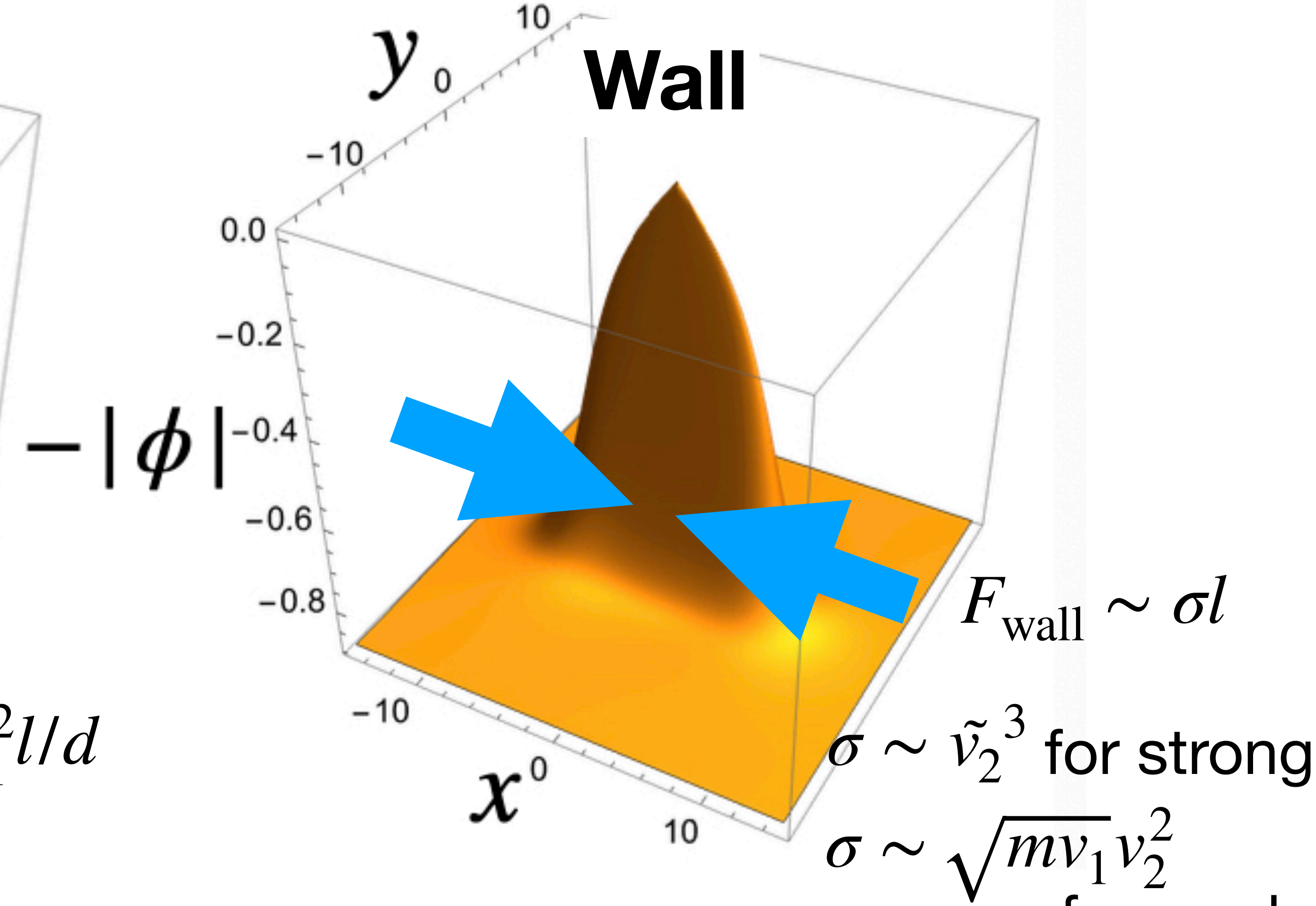


Two dimensional numerical solution



Equilibrium configuration





Repulsive force between the strings

Wall tension = pull the strings

At $d \sim v_1^2/\sigma$, the repulsive/attracting forces balance

⇒ Remaining string-walls are expected to become narrow strips

Eto, Hiramatsu, Saito, YS, 2023, 2024

Equilibrium configuration

In three dimensional space,

Equilibrium configuration

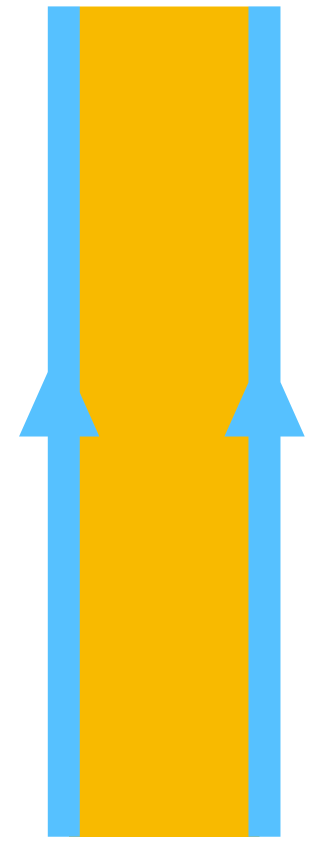
If a string-wall is unclosed,

narrow strips

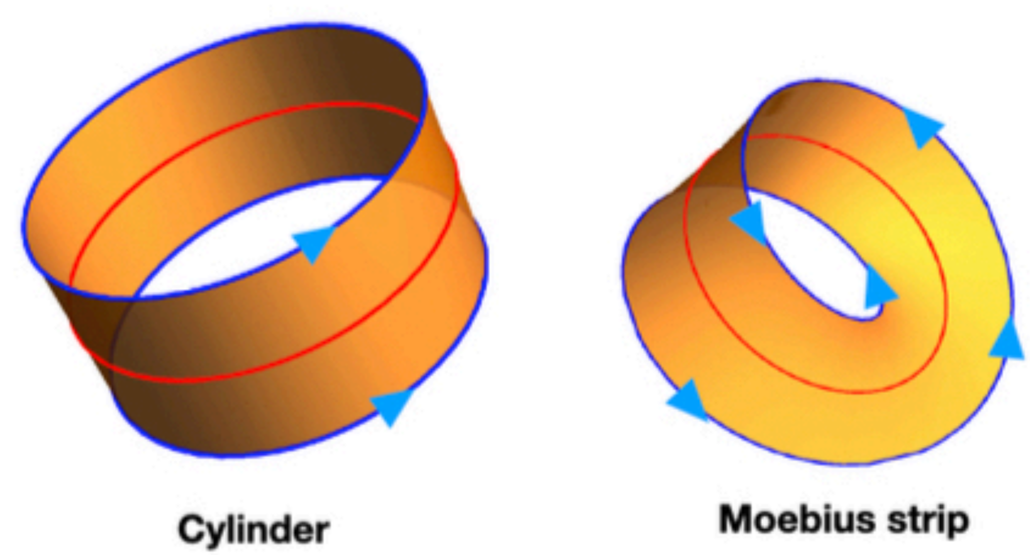
Kishimen-like structure

If a string-wall is closed,

cylinders and moebius strips

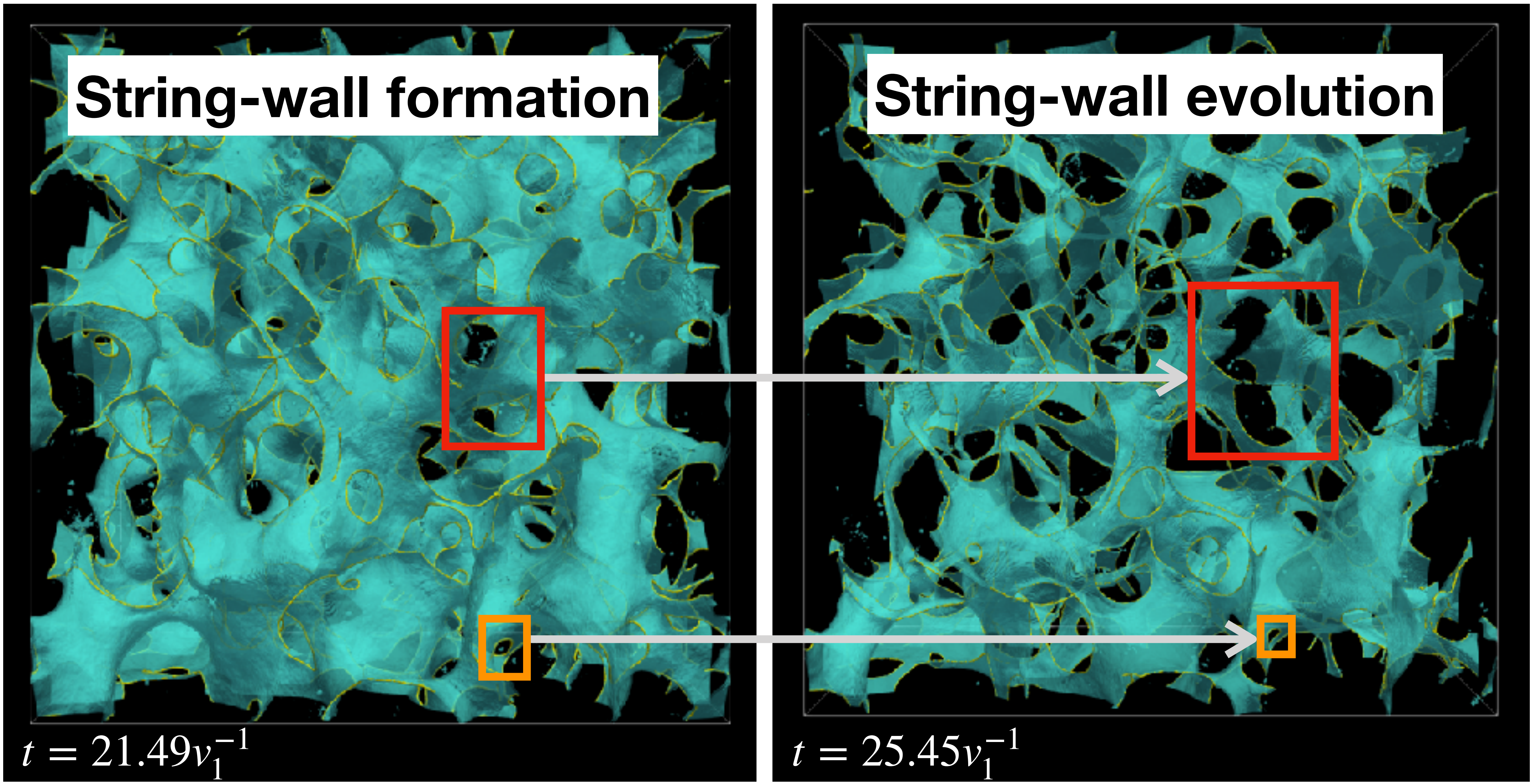


Kishimen is a kind of Japanese noodle



Eto, Hiramatsu, Saito, YS, 2023, 2024

Three dimensional simulation



Yellow: Strings

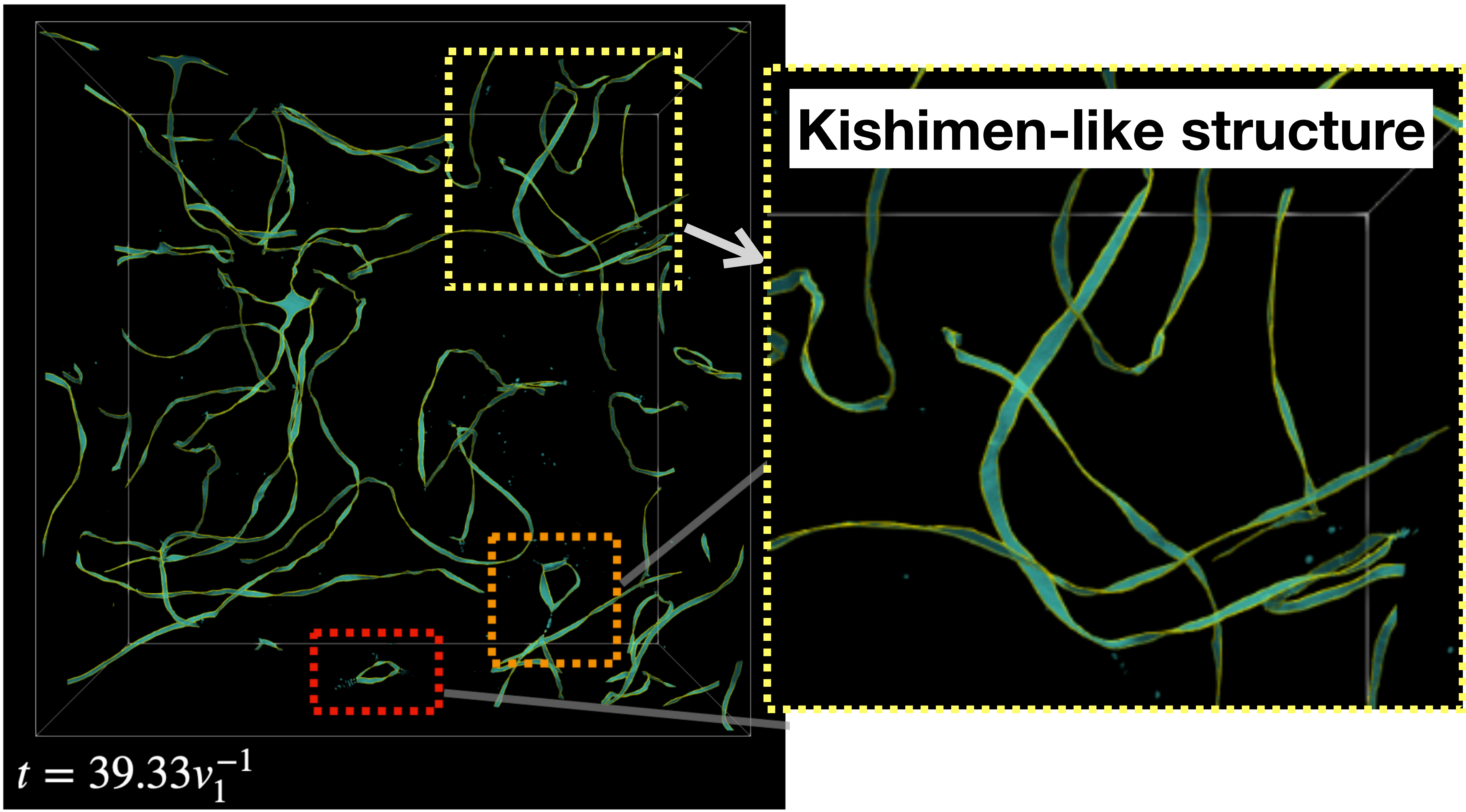
Green: Walls

Number of grids: 2048^3

64 nodes on Flow@Nagoya

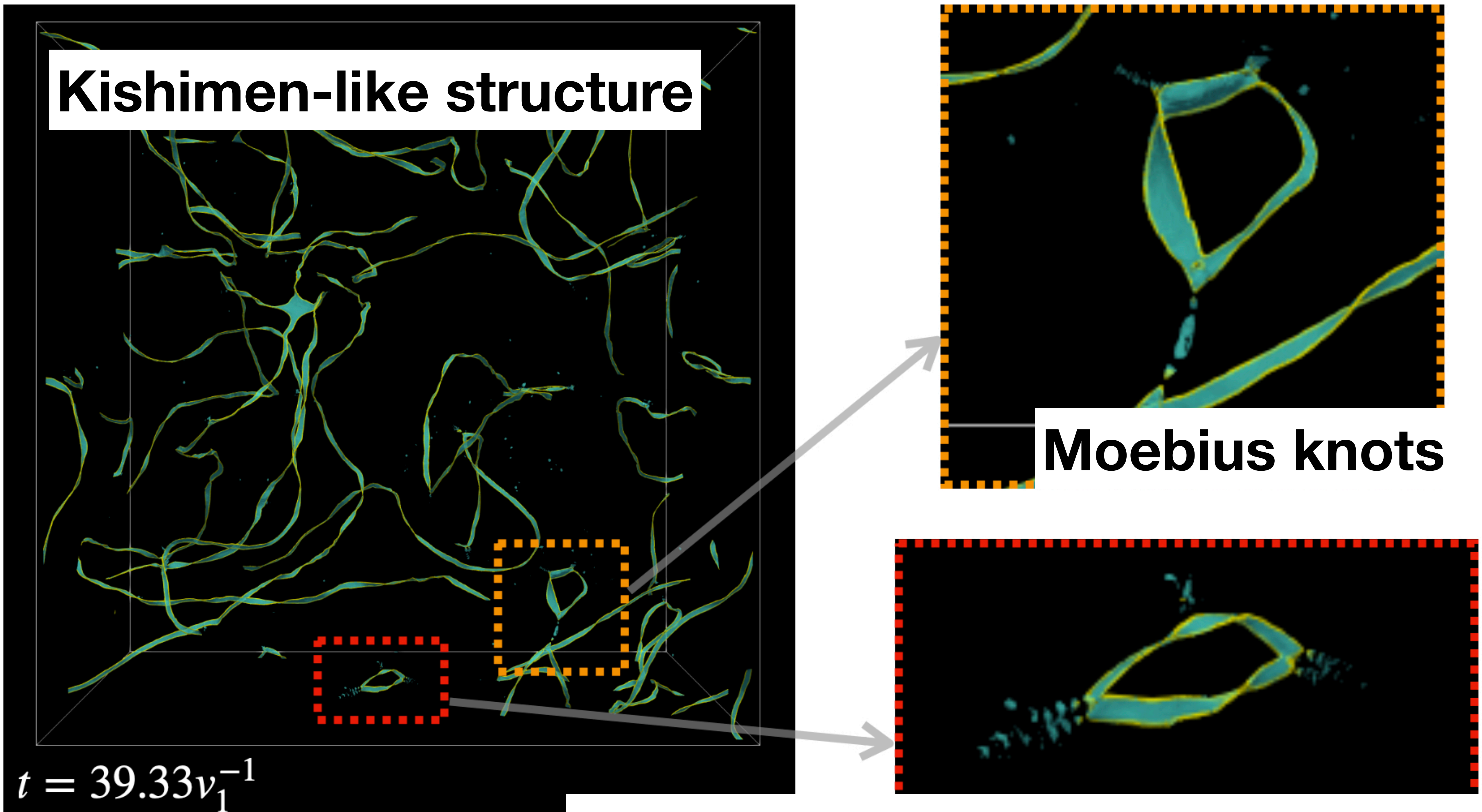
30 mins

Late time behavior



Eto, Hiramatsu, Saito, YS, 2023, 2024

Late time behavior



Scalar wave emission from loops

Eto, Hiramatsu, Saito, YS, 2023, 2024

Numerical approach

- We also imposed several assumptions to be confirmed

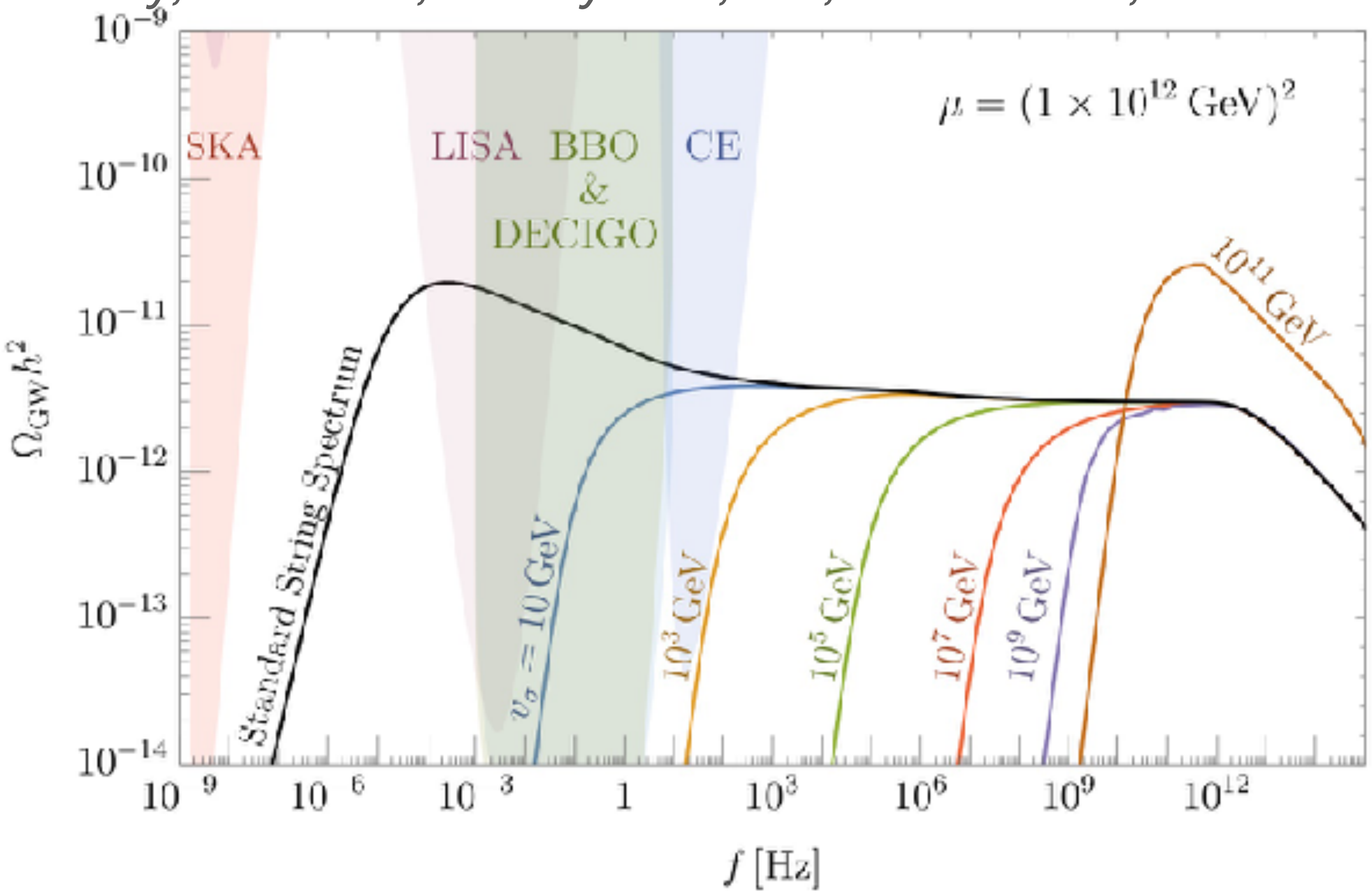
  【Walls bounded by strings】

  - ~~There is no equilibrium (stable) configuration of string-walls~~
    Equilibrium configurations exist
  - ~~String-wall network vanishes at the very instance when the walls form~~
    No destruction of the string-wall network at the wall formation

➡ **<u>Gravitational wave signature will be changed!</u>**

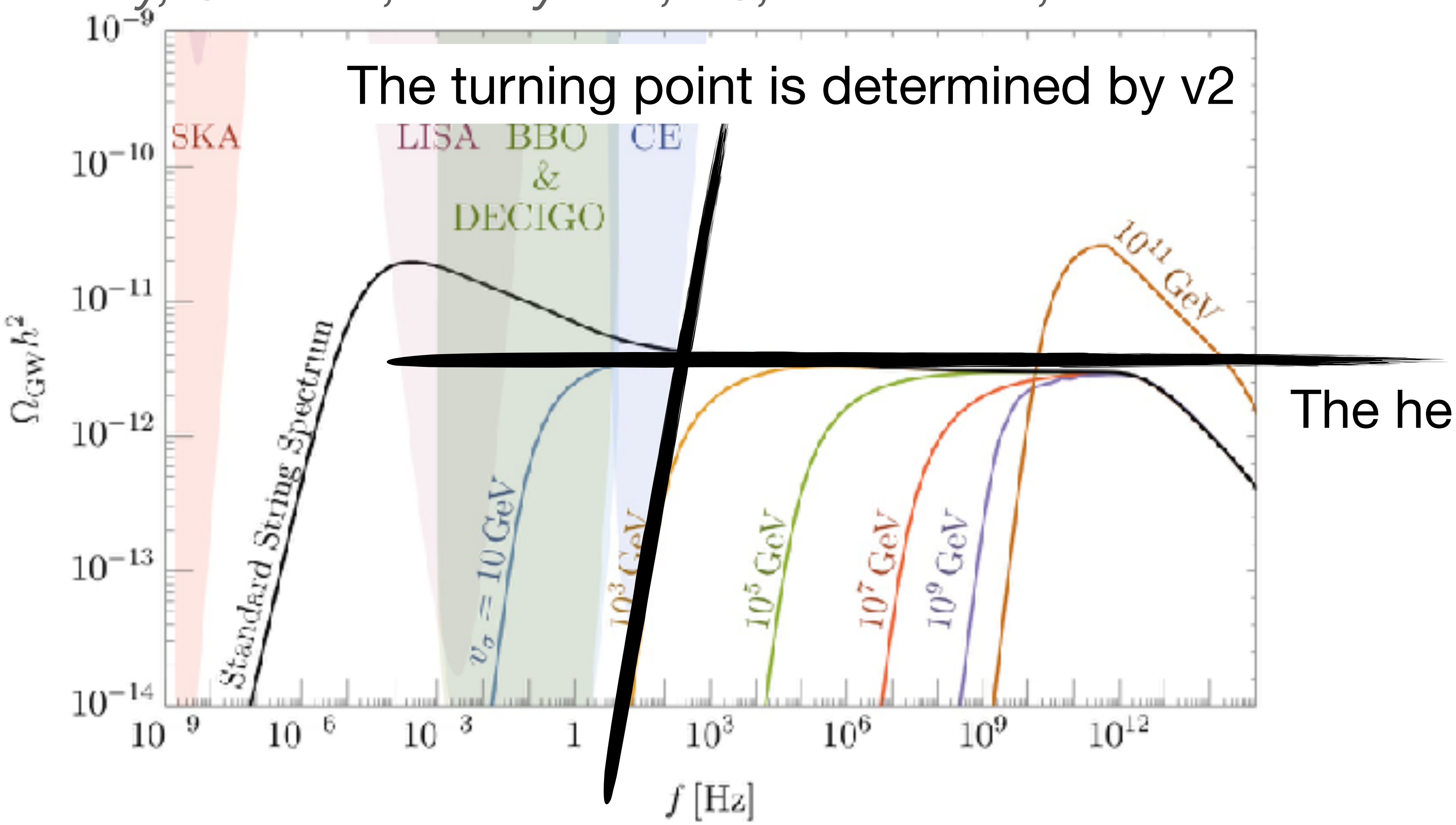
Gravitational waves revisited

Dunsky, Ghoshal, Murayama, YS, White et al, 2022



Gravitational waves revisited

Dunsky, Ghoshal, Murayama, YS, White et al, 2022



The height is determined by v1

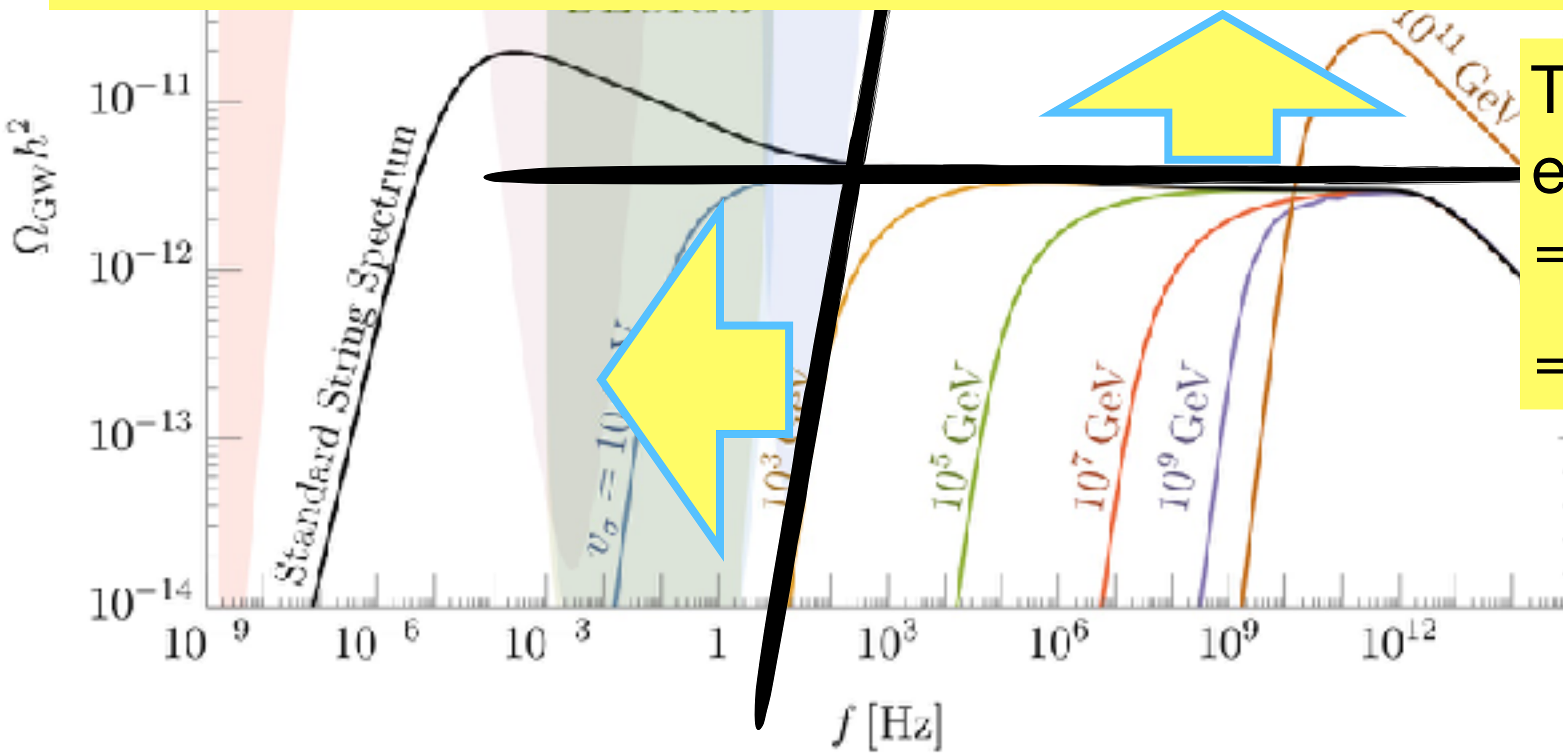
Gravitational waves revisited

The vanishment of string-wall network was assumed
⇒ The network survives as a network of Kishimen-like strips

⇒ The turning point moves to a lower frequency and

the slope becomes more gradual

⇒ v2 ↗



The winding number of strings is effectively doubled
⇒ The height increases

⇒v1 ↘

Taking into account the Kishimen-like structure found in the numerical simulations, the modification tend to well explain the neutrino masses, i.e., $v_1 \lesssim 10^{15}\text{GeV}$.

Summary

- SO(10)GUT leads to the formation of a variety of composite topological solitons

- The gravitational wave signatures are distinguishable from each other even from the non-composite ones

- Numerical simulations are helpful to improve the analytical modeling of topological solitons

I would like to thank Hitoshi for involving me in such a quite exciting research field!!

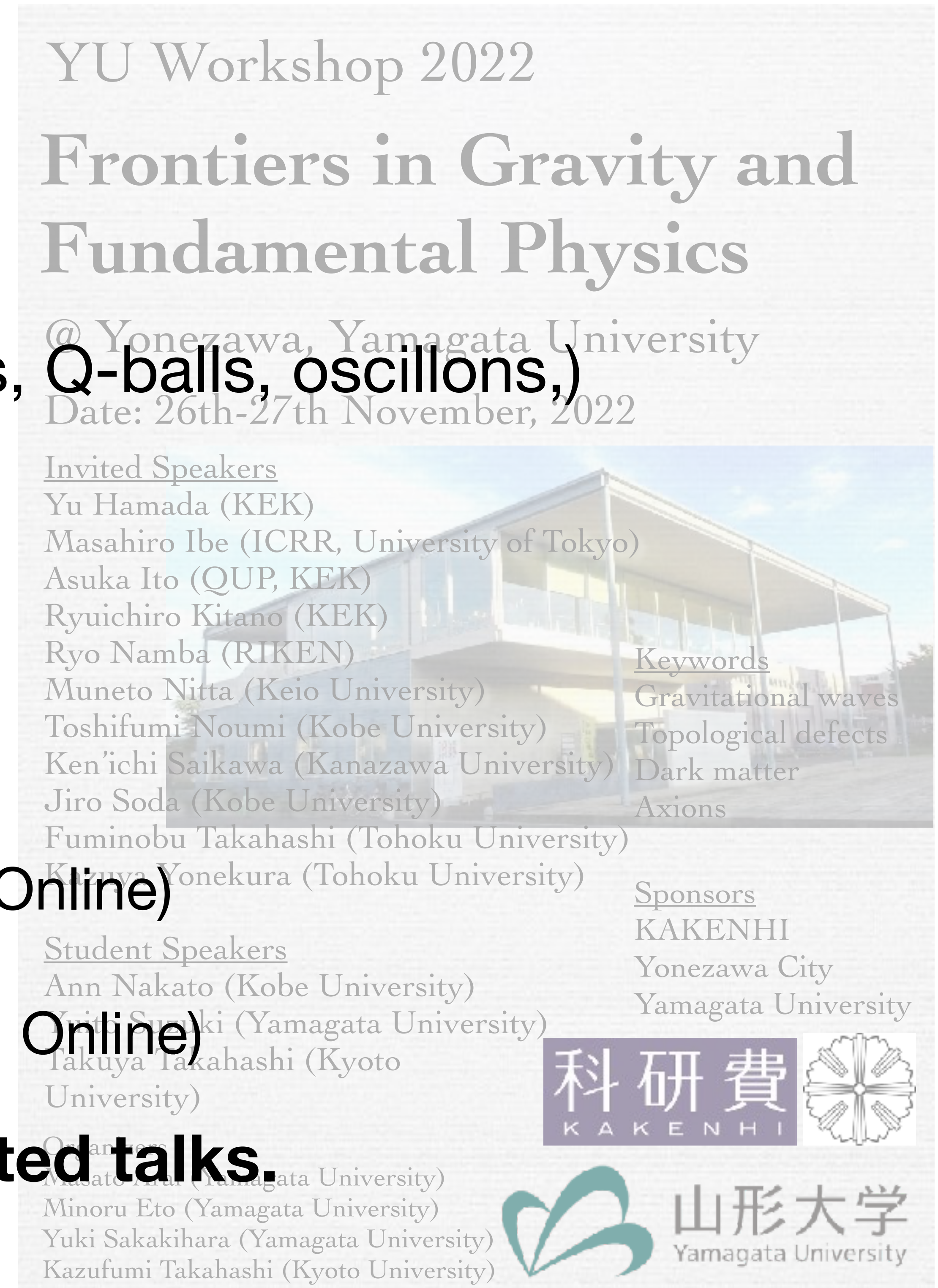
"Frontiers in Gravity and Fundamental Physics (YU Workshop 2025)" @Yonezawa, Yamagata + Zoom, on March 3-4, 2025

- Keywords:
  dark matter,
  axions,
  phase transitions,
  solitons, (including topological defects, Q-balls, oscillons,)
  gravitational waves.

- Invited speakers
  Junseok Lee (Tohoku University)
  Kaloian Lozanov (IPMU)
  Wakutaka Nakano (KEK)
  Nobuchika Okada (University of Alabama, Online)
  Takashi Toma (Kanazawa University)
  Graham White (University of Southampton, Online)

5 slots are still available for contributed talks.

Please join us!



Congratulations
on your 60th birthday!