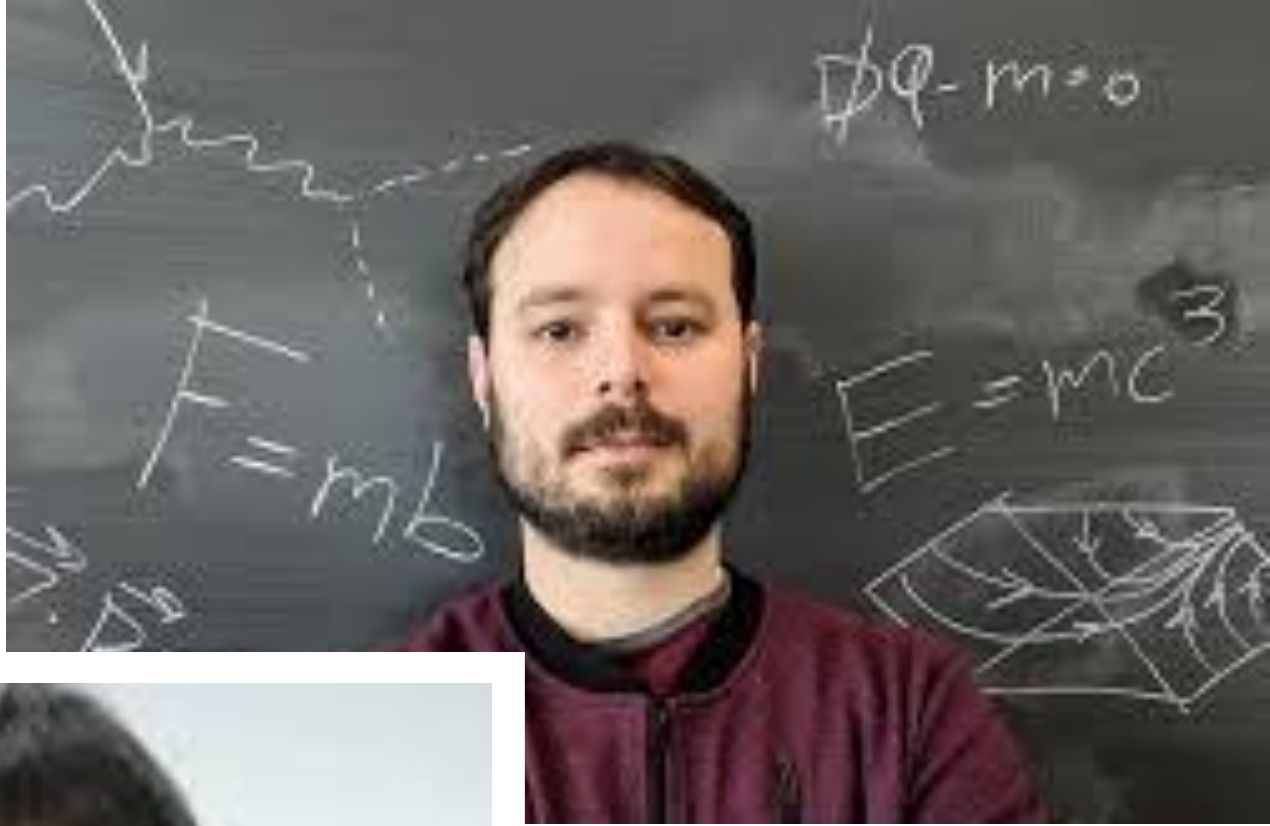
Using AMSB to Understand Strongly Coupled Gauge Theories

Bea Noether

Work with Hitoshi Murayama, Csaba Csáki, Dan Kondo, Digvijay Roy Varier, Ofri Telem, Andrew Gomes

Happy Birthday, Hitoshi! Thank you for all your guidance!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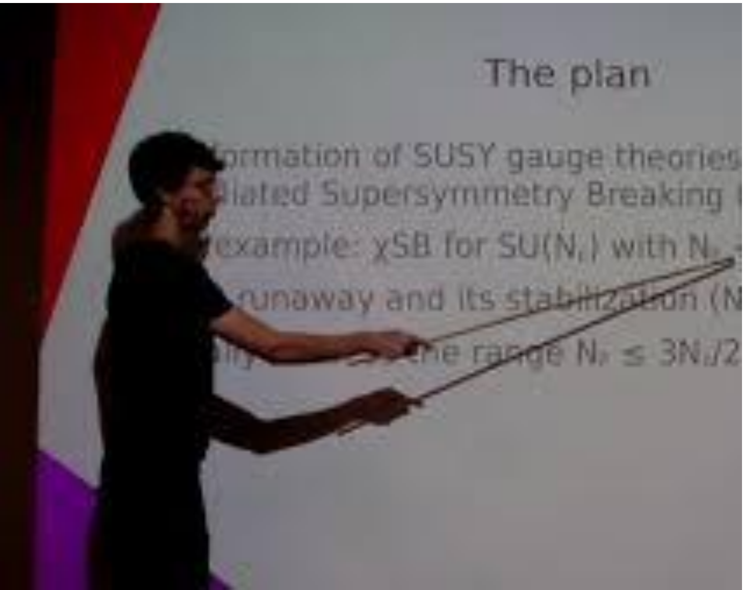
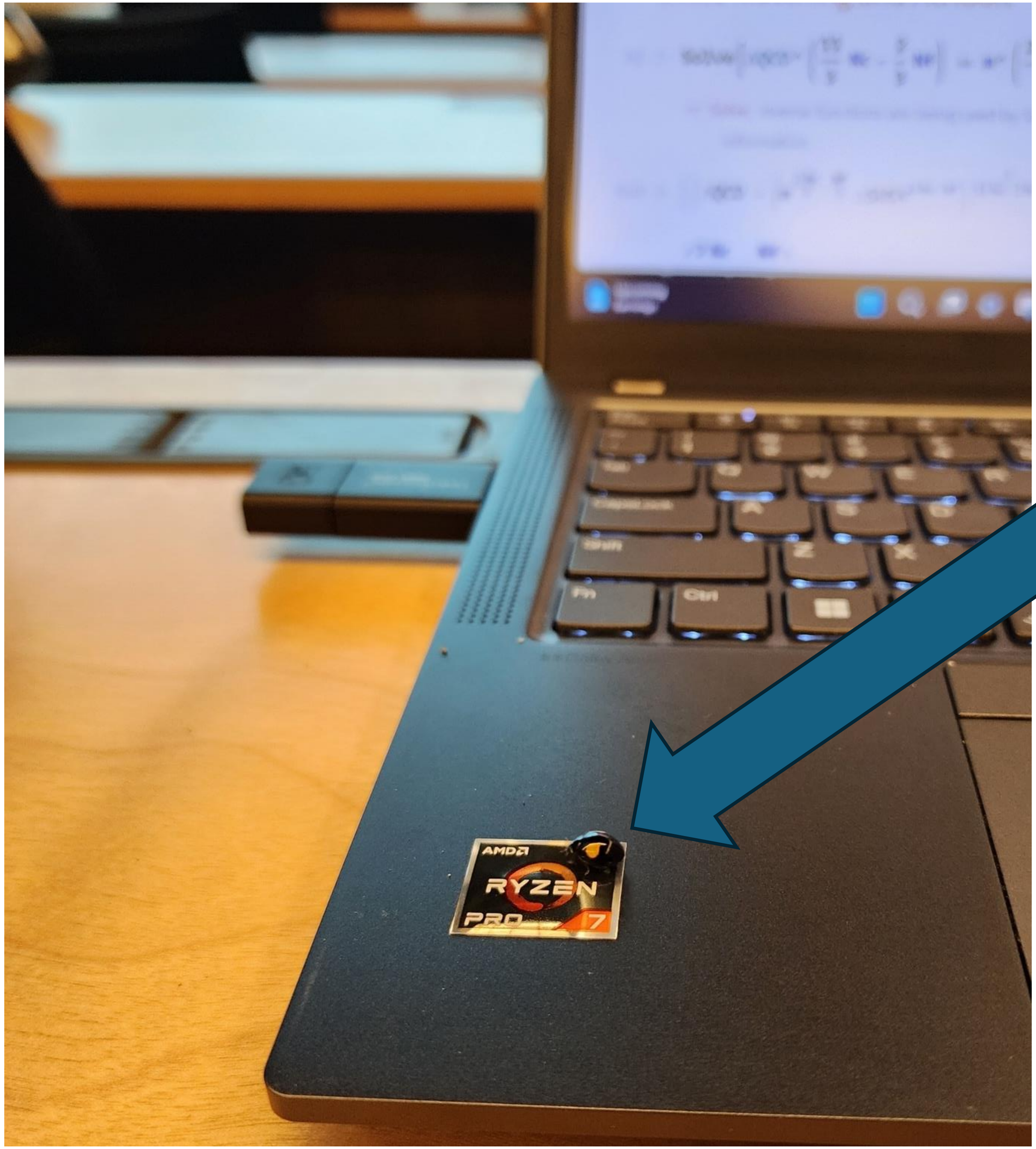

The plan
Supersymmetry Breaking



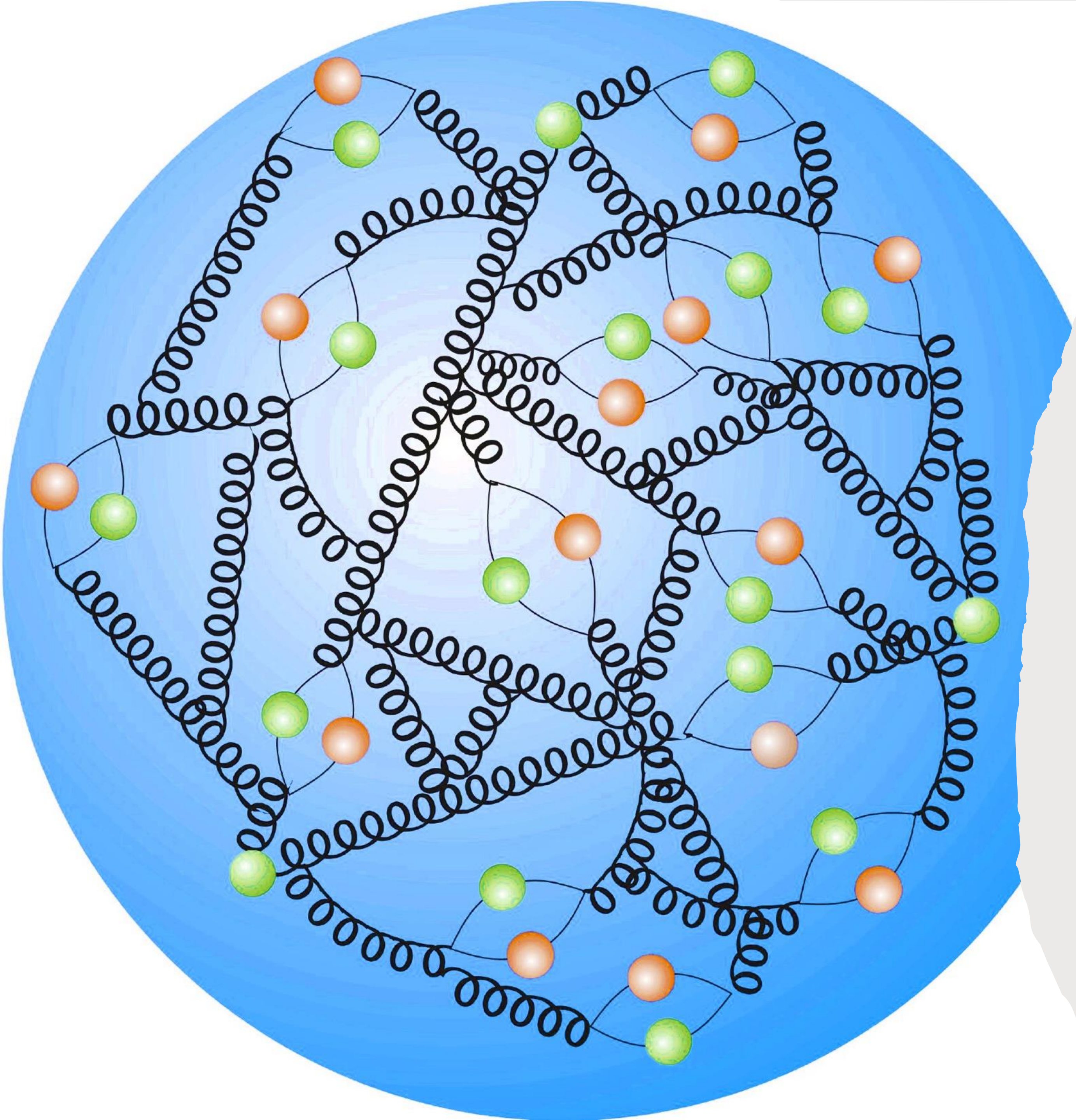
Special thanks to my unexpected collaborator yesterday!

And to my family, I’ll be back in time for the Holidays!

Outline

- Motivation
- SUSY and Strong Coupling
- AMSB Method
- Chiral Symmetry Breaking in QCD
  - “Lower # of Flavors”
  - Conformal Window

Motivations: Understanding Strong Phenomena in QCD



- Quarks and gluons bind together into hadrons

- It’s a big hot QCD soup, strongly coupled, and we can’t calculate it.

- We’re able to infer things about it from observation.
  - Confinement
  - Chiral symmetry breaking
  - Etc.

- But we still lack an analytic demonstration that QCD actually does this at low energies.

SUSY and Strong Coupling

Seiberg Phys.Rev.D 49 (1994) 6857-6863

Seiberg, Leigh, Intriligator, Phys.Rev.D 50 (1994) 1092-1104

Seiberg, Witten, Nucl.Phys.B 426 (1994) 19-52, Nucl.Phys.B 430 (1994) 485-486

Seiberg, Intriligator, Nucl.Phys.B 431 (1994) 551-568

Seiberg, Witten, Nucl.Phys.B 431 (1994) 484-550

Seiberg, Nucl.Phys.B 435 (1995) 129-146

etc. etc. etc.

Non-perturbative Methods for SUSY

- Superpotential is holomorphic
- Non-renormalization
- Non-anomalous symmetries
- ‘t Hooft Anomaly Matching

All together can sometimes fully determine low energy effective theory!

Affleck-Dine-Seiberg Superpotential: $N_f < N_c$

Strong scale is a spurion superfield of anomalous axial symmetry (b is beta function coefficient)

Gauge invariants	$U(1)_A$	$U(1)_R$
$W^a W^a$	0	2
Λ^b	$2N_f$	0
$\det M$	$2N_f$	$2(N_f - N_c)$

$$M_i^j = \bar{Q}^{jn} Q_{ni}$$

Only consistent holomorphic term is

$$W = (N_c - N_f) \left(\frac{\Lambda^{3N_c - N_f}}{\det M} \right)^{\frac{1}{N_c - N_f}}$$

Quantum Modified Moduli Space: $N_f = N_c$

Moduli space in terms of gauge invariant mesons and baryons

$$M^i_{\tilde{\imath}} = Q^i \tilde{Q}_{\tilde{\imath}} \ ,$$

$$B_{i_{N_c+1}, i_{N_c+2}, \ldots, i_{N_f}} = \frac{1}{N_c!} \epsilon_{i_1, \ldots, i_{N_f}} Q^{i_1} Q^{i_2} \cdots Q^{i_{N_c}}$$

$$\tilde{B}^{\tilde{\imath}_{N_c+1}, \tilde{\imath}_{N_c+2}, \ldots, \tilde{\imath}_{N_f}} = \frac{1}{N_c!} \epsilon^{\tilde{\imath}_1, \ldots, \tilde{\imath}_{N_c}} \tilde{Q}_{\tilde{\imath}_1} \tilde{Q}_{\tilde{\imath}_2} \ldots \tilde{Q}_{\tilde{\imath}_{N_c}} \ ,$$

Classical constraint $\det M - B\tilde{B} = 0$

Consistency with mass deformations requires modified constraint:

$$\det M - B\tilde{B} = \Lambda^{2N_c}$$

s-Confining Phase $N_f = N_c + 1$

- Confinement without chiral symmetry breaking
- Unbroken global symmetries at origin of moduli space
- Smooth interpolation between Higgs and confining phase
- Dynamically generated IR Superpotential:

$$W = \lambda \frac{\det M}{\Lambda^{N_f - 3}} - \kappa \tilde{B} M B$$

Free Magnetic $N_c + 2 \leq N_f \leq \frac{3N_c}{2}$

Electric Theory

$SU(N_c) + N_f \times Q + N_f \times \tilde{Q}$

Weakly coupled UV
Strongly coupled IR

Magnetic Theory

$SU\left(N_f - N_c\right) + N_f \times q + N_f \times \tilde{q} + N_f^2 \times M_{ij}$

Superpotential $W = \lambda q M \tilde{q}$

Strongly coupled UV
Weakly coupled IR

Supposed to
describe IR phase

Conformal Window $\frac{3N_c}{2} < N_f < 3N_c$

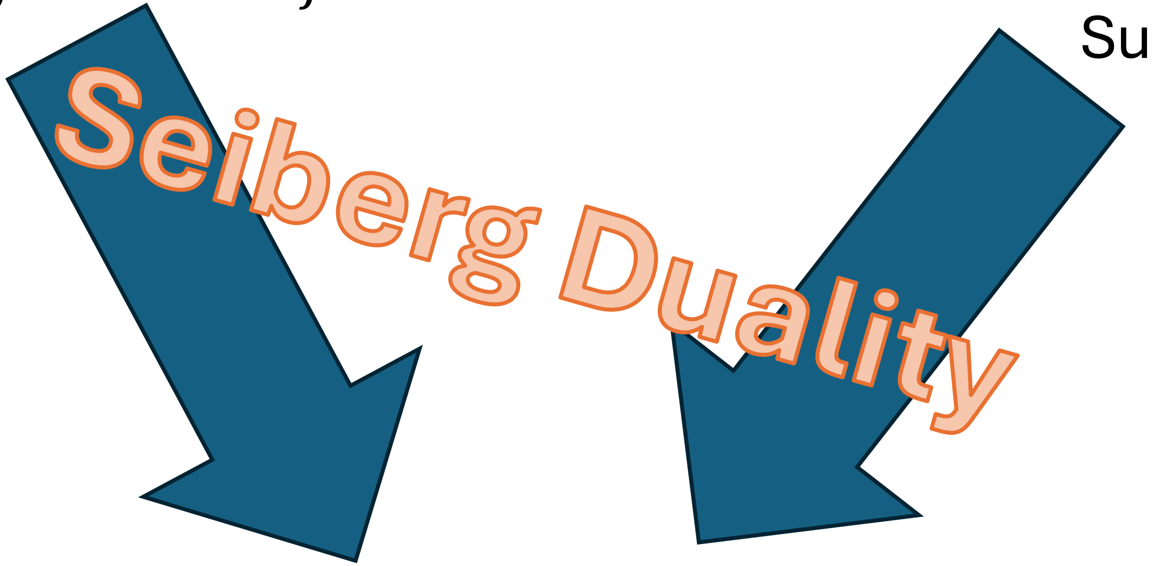
Electric Theory

$SU(N_c) + N_f \times Q + N_f \times \tilde{Q}$

Magnetic Theory

$SU(N_f - N_c) + N_f \times q + N_f \times \tilde{q} + N_f^2 \times M_{ij}$

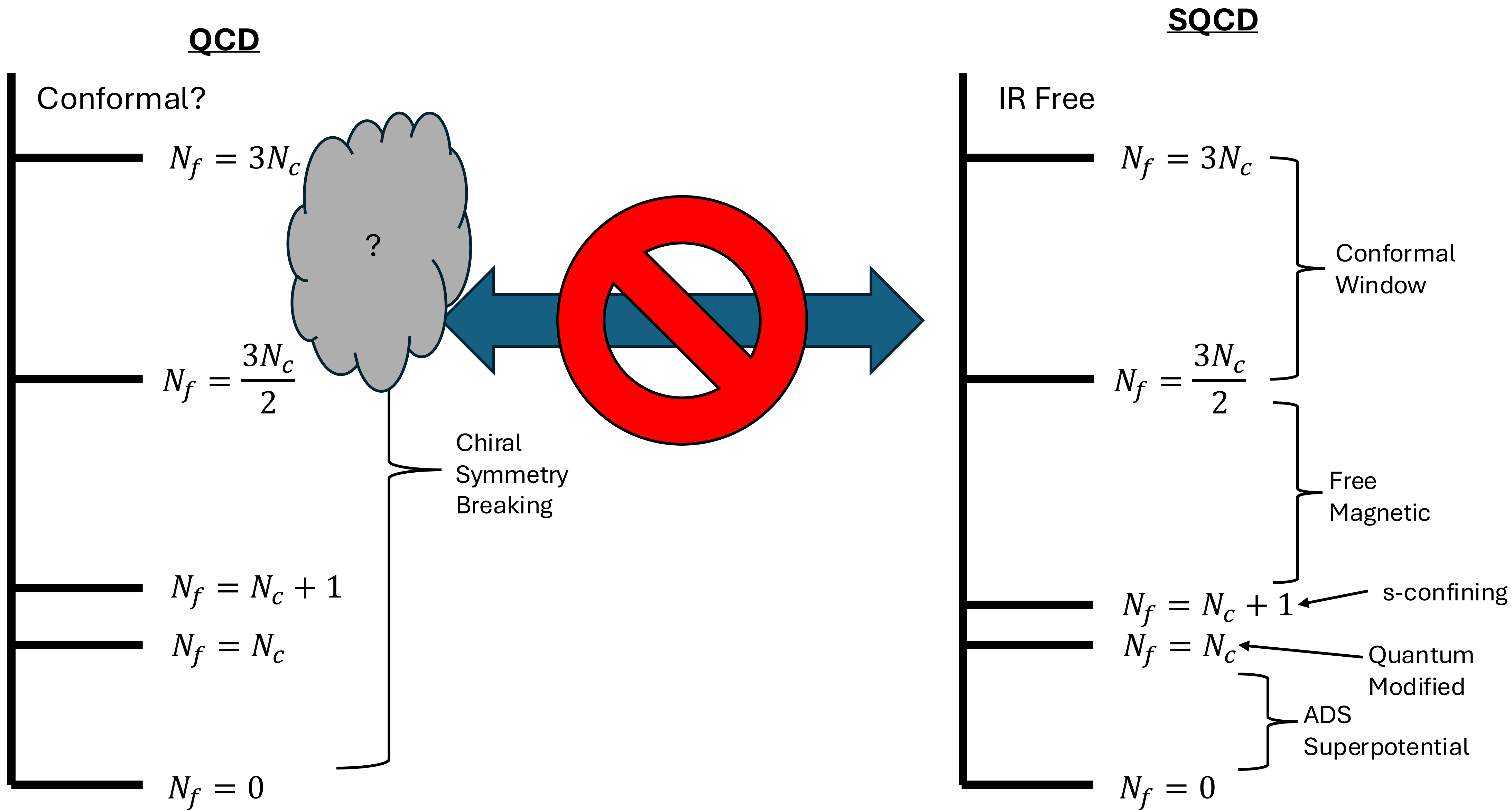
Superpotential $W = \lambda q M \tilde{q}$



Same SCFT fixed point in IR

Could we use SQCD to understand the IR behavior of regular QCD?

There's an issue ➔ The IR of SQCD looks very different!



Anomaly Mediated SUSY Breaking

Murayama, Giudice, Luty, Rattazzi, JHEP 12 (1998) 027

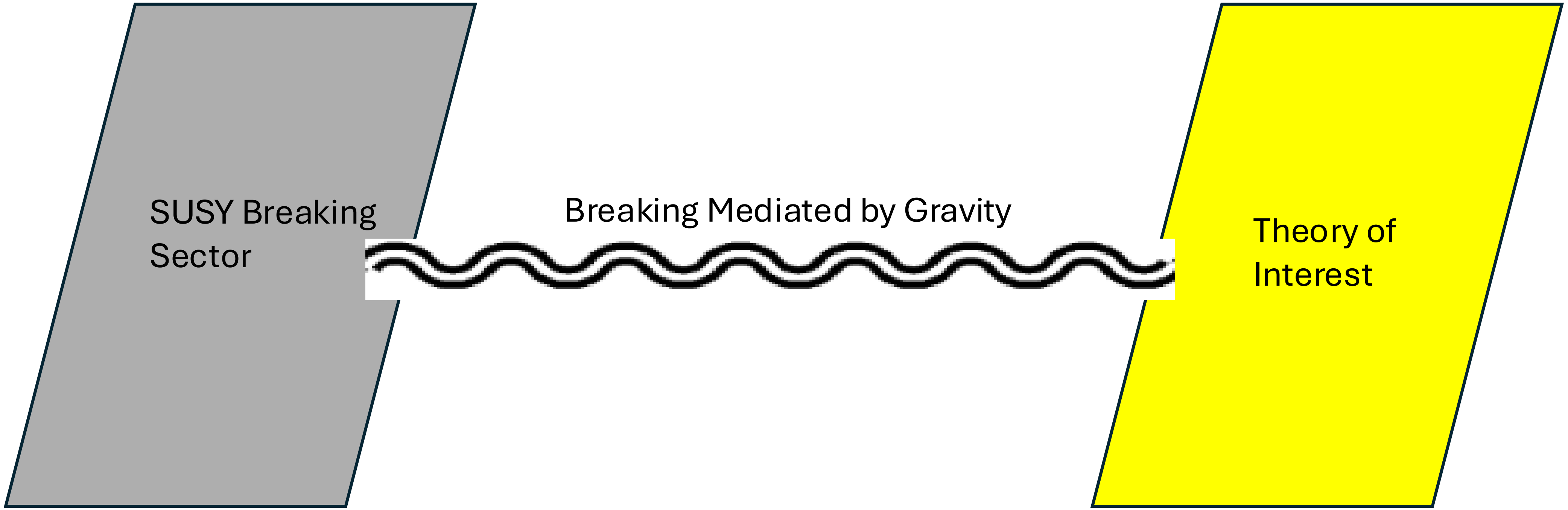
Randall, Sundrum, Nucl.Phys.B557:79-118,1999

Fun fact!

In the summer of 1998, Hitoshi and collaborators were inventing AMSB.

That same summer, I was born!

It took me a while of reading, but I get it now!



Universal coupling of gravity at all scales
➔ **UV insensitivity** of SUSY breaking mechanism

We can express everything just in terms of d.o.f. at that energy scale

We can determine the IR of a SUSY theory
➔ We can determine the IR of the AMSB theory

Weyl Compensator Formalism

Spurion superfield for superconformal anomaly

$$\Phi = 1 + \theta^2 m$$

$$\mathcal{L} = \int d^4\theta \Phi^* \Phi K + \int d^2\theta \Phi^3 W + c.c.$$

And there are loop level masses and trilinear couplings

$$A_{ijk}(\mu) = -\frac{1}{2}(\gamma_i + \gamma_j + \gamma_k)(\mu)m,$$

$$m_i^2(\mu) = -\frac{1}{4}\dot{\gamma}_i(\mu)m^2,$$

$$m_\lambda(\mu) = -\frac{\beta(g^2)}{2g^2}(\mu)m.$$

$\gamma_i = \mu\frac{d}{d\mu}\ln Z_i(\mu)$, $\dot{\gamma} = \mu\frac{d}{d\mu}\gamma_i$, and $\beta(g^2) = \mu\frac{d}{d\mu}g^2$

Shows up at tree level if there are dimensionful parameters

$$\mathcal{L}_{\text{tree}} = m\left(\phi_i \frac{\partial W}{\partial \phi_i} - 3W\right) + c.c.$$

Superpartner Decoupling

Sfermions

$$m_i^2(\mu) = -\frac{1}{4}\dot{\gamma}_i(\mu)m^2,$$

When theory is asymptotically free, sfermion masses are positive and vacuum is stabilized

Gauginos

$$m_\lambda(\mu) = -\frac{\beta(g^2)}{2g^2}(\mu)m.$$

$SU(N_c)$ with N_f flavors

$$m_Q^2 = m_{\tilde{Q}}^2 = \frac{g^4}{(8\pi^2)^2}2C_i(3N_c - N_f)m^2,$$

$$m_\lambda = \frac{g^2}{16\pi^2}(3N_c - N_f)m.$$

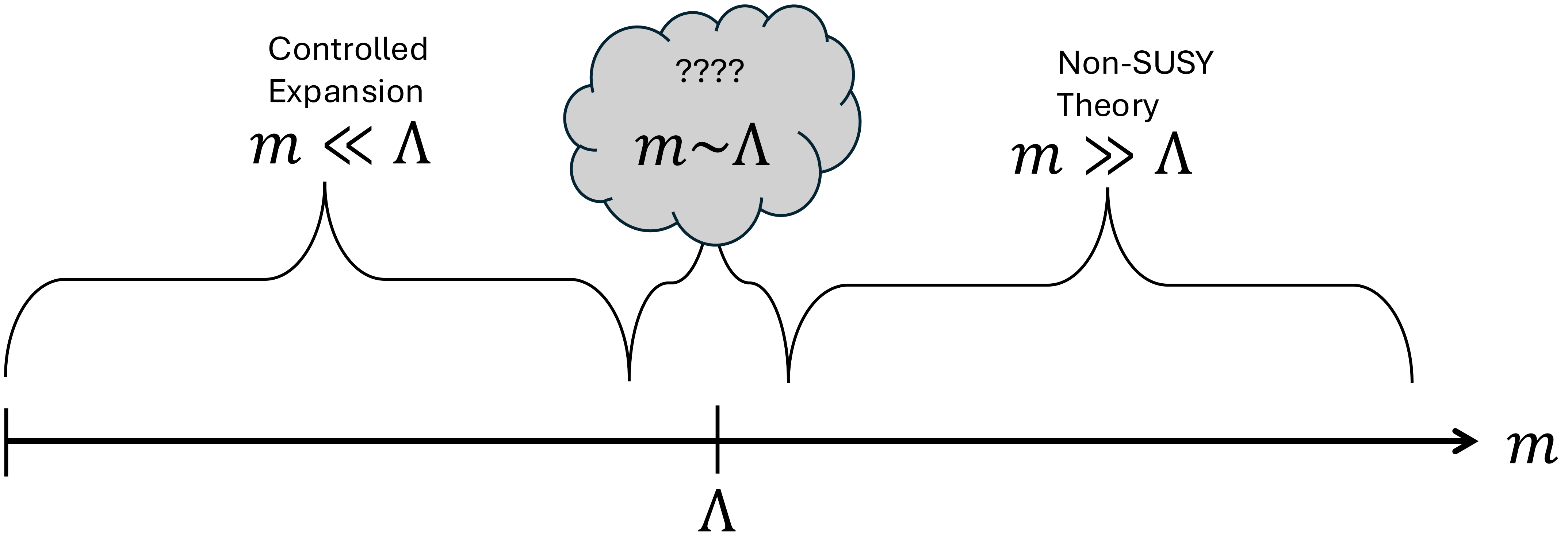
When $N_f < 3N_c$ we get positive mass superpartners

Integrate them out ➔ vacuum looks like non-SUSY theory!

AMSB provides a **UV insensitive** way of breaking SUSY to obtain an IR theory with the right degrees of freedom to match onto the non-SUSY theory!

Let’s try to use it to make conclusions about QCD!

The question of a Phase Transition



“The AMSB Conjecture”

There exists **some class** of strongly coupled theories that are continuously connected to their AMSB counterparts such that they share the same **vacuum** universality class.

When and how this conjecture fails is itself an active area of research!

Some Hints of a Failure Condition

So far there are three examples:

- $SU(2) + 3$ flavors — de Lima, Stolarski 2307.13154
- $SU(5)$ with 3 10's and 3 $\bar{5}$'s — Bai, Stolarski 2111.11214
- $SU(N_c) + \frac{3N_c}{2}$ flavors

All three cases have a classically conformal dynamical Superpotential.

This is a hint at what the conditions of the "AMSB Conjecture" should be.

Chiral Symmetry Breaking Minima

Murayama, Phys. Rev. Lett. 126, 251601 (2021)

Murayama, Csáki, ***BN***, Gomes, Telem, Roy Varier, Phys.Rev.D 107 (2023) 5, 054015

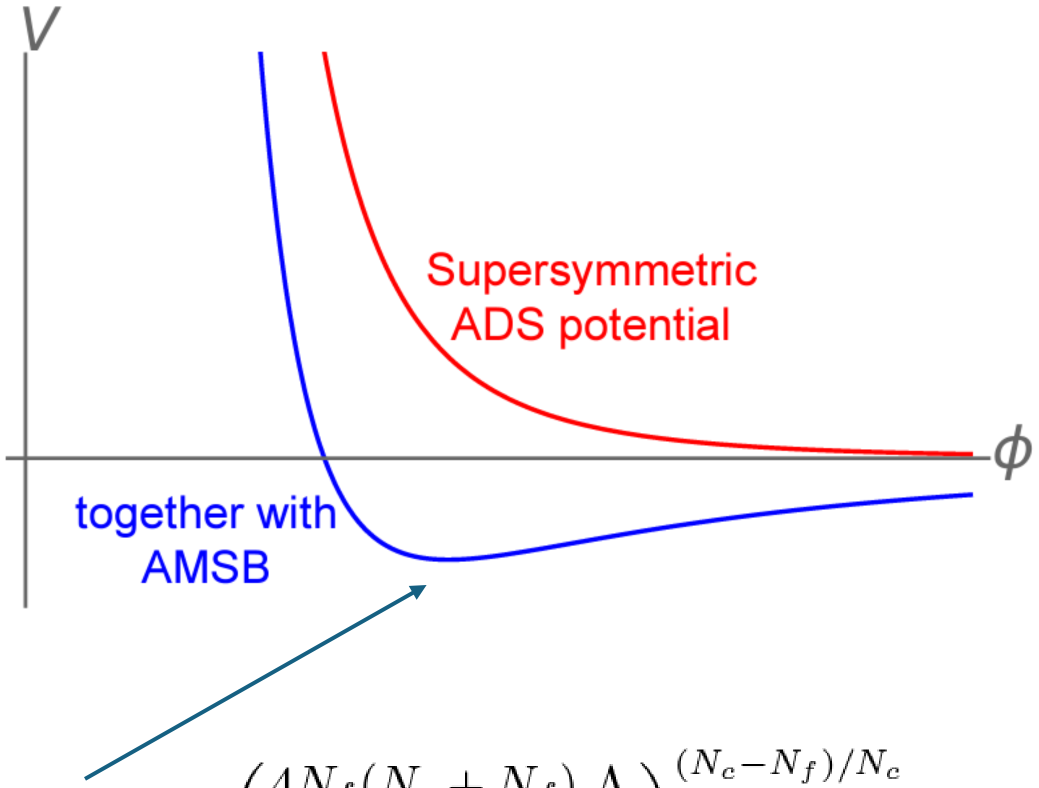
Murayama, ***BN***, Roy Varier, Kondo 2501.XXXX

$N_f < N_c$: ADS

$$Q = \bar{Q} = \begin{pmatrix} 1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1 \\ \hline 0 & \cdots & 0 \\ \vdots & \vdots & \vdots \\ 0 & \cdots & 0 \end{pmatrix} \phi, \qquad M = \phi^2.$$

$$W = (N_c - N_f)\left(\frac{\Lambda^{3N_c - N_f}}{\det M}\right)^{1/(N_c - N_f)}$$

$$V = \left| 2N_f \frac{1}{\phi}\left(\frac{\Lambda^{3N_c - N_f}}{\phi^{2N_f}}\right)^{1/(N_c - N_f)} \right|^2 - (3N_c - N_f)m\left(\frac{\Lambda^{3N_c - N_f}}{\phi^{2N_f}}\right)^{1/(N_c - N_f)} + c.c.$$



$$M_{ij} = \Lambda^2 \left(\frac{4N_f(N_c + N_f)}{3N_c - N_f}\frac{\Lambda}{m}\right)^{(N_c - N_f)/N_c} \delta_{ij}.$$

Quantum Modified Moduli Space $N_f = N_c$

IR moduli space in terms of mesons and baryons, with "quantum modified" constraint: $\det M - B\bar{B} = \Lambda^{2N_c}$

[Case of N=2 is equivalent to an Sp theory, can show stable minimum]

Meson point, unbroken Baryon#

$$M = \mathbf{1}, B = \bar{B} = 0$$

$$M = (1+B\bar{B})^{1/N_c} e^{\Pi} = \mathbf{1} + \frac{1}{N_c} B\bar{B} + \Pi + \frac{1}{2}\Pi^2 + \cdots$$

Unclear

$$K \supset \alpha(B^\dagger B + \bar{B}^\dagger \bar{B}) + \frac{\beta}{2}(B\bar{B} + c.c.)$$

Stabilized

$$\mathrm{Tr}\, M^\dagger M \supset \mathrm{Tr}\, \Pi^\dagger \Pi + \frac{1}{2}\mathrm{Tr}\, \Pi^2 + \frac{1}{2}\mathrm{Tr}\, \Pi^{\dagger 2}.$$

Baryon point, unbroken flavor

$$M = 0, B = -\bar{B} = 1$$

$$B = (1 - \det M)^{1/2} e^{b}$$
$$\bar{B} = -(1 - \det M)^{1/2} e^{-b}.$$

Unclear

Stabilized

$$B^\dagger B + \bar{B}^\dagger \bar{B} = 2 + (b + b^\dagger)^2 + \cdots$$

Quantum Modified Moduli Space $N_f = N_c$

Cannot identify minima directly – can only conjecture based on surrounding cases.

Implement constraint with Lagrange Multiplier

$$W = X\left(\lambda \frac{\det M}{\Lambda^{N_c-2}} - \kappa \tilde{B}B - \Lambda^2\right)$$

Identify a chiral symmetry breaking minimum (leading order)

$$M^{ij} = \lambda^{-1/N_c}\Lambda\delta^{ij}, \qquad B = \tilde{B} = 0,$$
$$X = \lambda^{-2/N_c}m, \qquad V = -N_c\lambda^{-2/N_c}m^2\Lambda^2$$

There is another potential minimum, shallower by naïve dimensional analysis

Since fields are order Λ, one worries about higher order Kahler terms!

$N_f = N_c + 1$: s-confining phase

$$W = \lambda \frac{\det M}{\Lambda^{N_f - 3}} - \kappa \tilde{B} M B.$$

$$V = N_f \lambda^2 \frac{|\phi|^{2N_f - 2}}{\Lambda^{2N_f - 6}} - \lambda (N_f - 3) m \phi^{N_f} + c.c.$$

$$\phi = \Lambda \left(\frac{(N_f - 3)m}{(N_f - 1)\lambda \Lambda} \right)^{1/(N_f - 2)} \ll \Lambda,$$

$$W = \alpha B M \bar{B} - \beta \det M$$

$$\begin{aligned} V_{\rm SUSY} &= \alpha^2 (|(M\bar{B})_a|^2 + |(BM)_a|^2) \\ &\quad + |\alpha \bar{B}_a B_b - \beta \det M (M^{-1})_{ab}|^2 \\ V_{\rm AMSB} &= -(N_c - 2)\beta m \det M + c.c. \end{aligned}$$

$$B = \begin{pmatrix} b \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \bar{B} = \begin{pmatrix} \bar{b} \\ 0 \\ \vdots \\ 0 \end{pmatrix}, M = \begin{pmatrix} x & & & \\ & v & & \\ & & \ddots & \\ & & & v \end{pmatrix}.$$

$$\begin{aligned} V &= 2\alpha^2 x^2 b^2 + (\alpha b^2 - \beta v^{N_c})^2 + N_c \beta^2 x^2 v^{2(N_c - 1)} \\ &\quad - 2(N_c - 2)\beta m x v^{N_c}. \end{aligned}$$

$$v = x = \left(\frac{(N_c - 2)m}{N_c \beta} \right)^{\frac{1}{N_c - 1}}, V_{\rm min} = -\mathcal{O}(m^{2N_c/(N_c - 1)}).$$

Nc = 2 is special.
See Stolarski paper

Not disturbed by higher-order effects. Runaway direction stabilized by loop effects

$N_c + 2 \leq N_f \leq \frac{3Nc}{2}$: Free Magnetic Phase

$$W = \lambda \, \mathrm{Tr} \, q_i M_{ij} \bar{q}_j$$

For $N_f < 1.43 \, N_c$ we find stable chiral symmetry breaking minimum

$$m_q^2 = \frac{(-\widetilde{b})g^4}{(16\pi^2)^2} \frac{N_f^2 - 3N_f \widetilde{N}_c - \widetilde{N}_c^2 + 1}{2N_f + \widetilde{N}_c} m^2$$

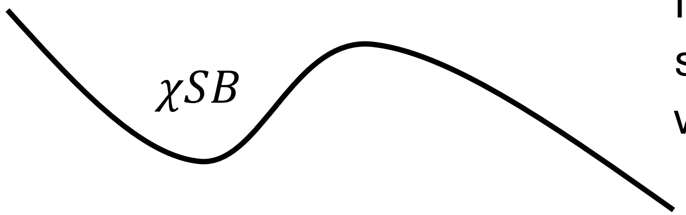
$$m_M^2 = \frac{(-\widetilde{b})\widetilde{N}_c \lambda^2 g^2}{(16\pi^2)^2} m^2$$

$$\Lambda_L^{3\widetilde{N}_c} = \widetilde{\Lambda}^{3\widetilde{N}_c - N_f} \det M. \quad W = \widetilde{N}_c \Lambda_L^3 = \widetilde{N}_c (\det M)^{1/\widetilde{N}_c}$$

$$V = N_f |v^{N_f/\widetilde{N}_c - 1}|^2 + (N_f - 3\widetilde{N}_c) m v^{N_f/\widetilde{N}_c} + c.c.$$

$$v = \left(\frac{(3\widetilde{N}_c - N_f)m}{N_f - \widetilde{N}_c} \right)^{\frac{\widetilde{N}_c}{N_f - 2\widetilde{N}_c}}, V_{\min} = -\mathcal{O}\left(m^{2\frac{N_f - \widetilde{N}_c}{N_f - 2\widetilde{N}_c}} \right)$$

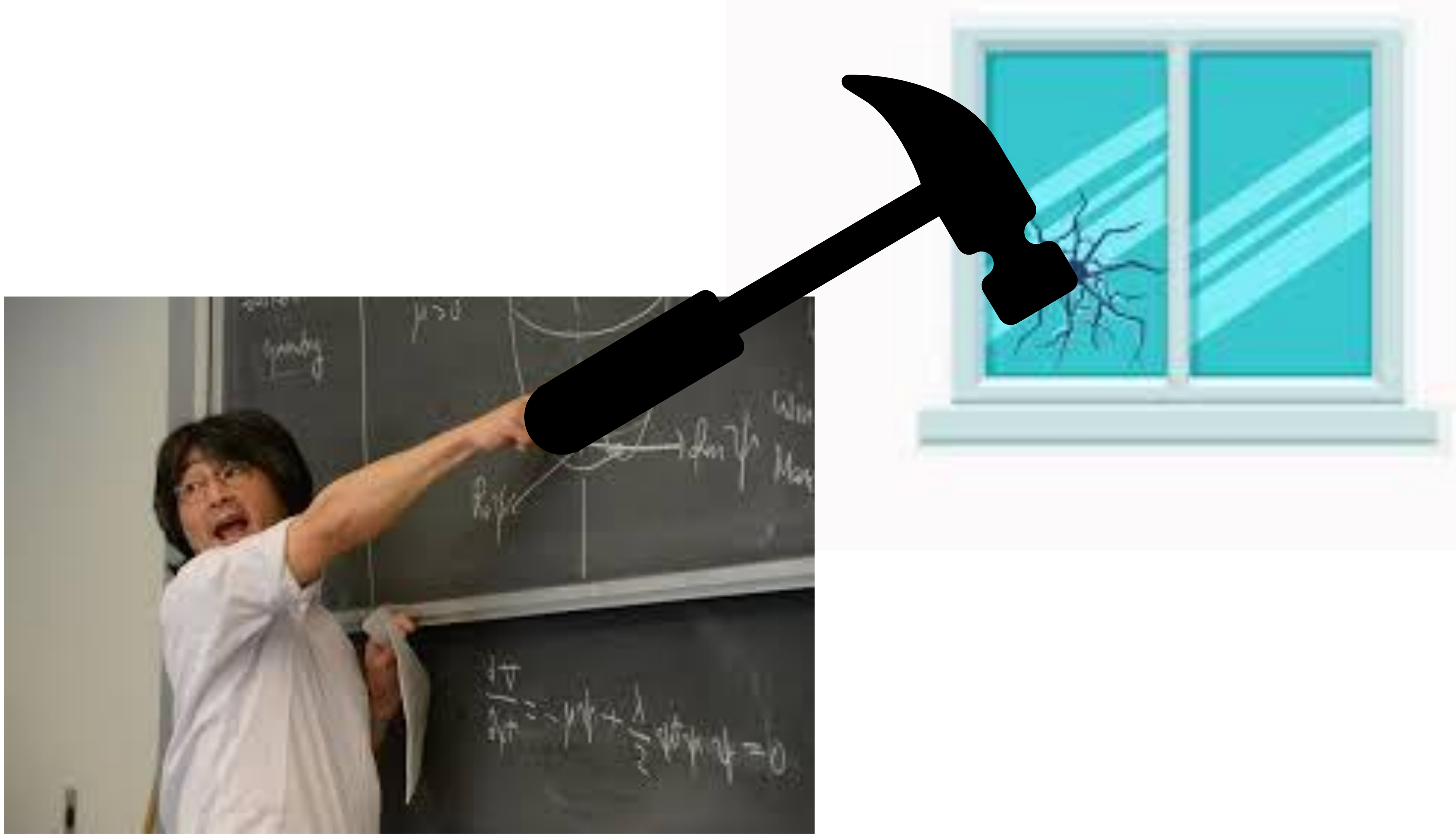
Once $N_f > 1.43 \, N_c$ the dual squarks become tachyonic and we expect baryonic runaways



Metastable minima could still be connected to QCD, we don't know!

Right at $\frac{3N_c}{2}$ the Superpotential is classically conformal and there is no chiral symmetry breaking

Breaking the Conformal Window



$\frac{3N_c}{2} < N_f < 3N_c$: Conformal Window

Because the un-perturbed theories flow to a SCFT in the IR, it is non-trivial whether AMSB changes the vacuum.

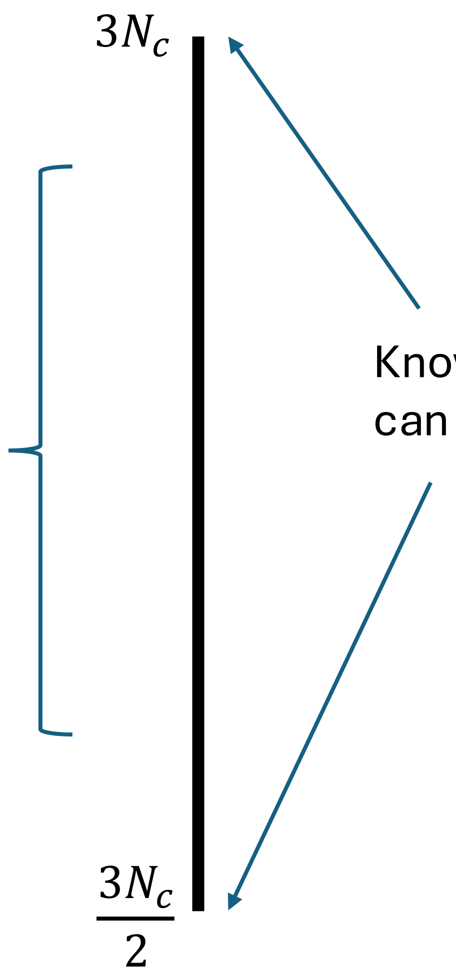
Remember: AMSB vanishes when conformal symmetry is restored.

We can quantitatively answer this question if we can calculate the RG flow of the perturbed theories near the un-perturbed fixed points.

$\frac{3N_c}{2} < N_f < 3N_c$: Conformal Window

Perturb dual theories by AMSB, hope that they remain dual

Fixed points conjectured,
Can perturbatively determine and see whether higher loop contributions appear to converge



Known BZ fixed points,
can perturb around them

Find that linear deviations from fixed point flow too slowly
➔ AMSB remains ***relevant***

Magnetic Theory: Chiral Symmetry Breaking Minimum

Exact non-perturbative Superpotential after integrating out dual squarks

$$W = \tilde{N}_c \left(\Lambda_m^{3\tilde{N}_c - N_f} \det \frac{M}{\mu_{\rm m}} \right)^{1/\tilde{N}_c}$$

$$M_{ij} = \lambda \mu_m \phi \delta_{ij}$$

$$\tilde{\phi} = (c_m Z_m)^{1/2} \phi$$

$$V[\tilde{\phi}] = N_f^2 (\lambda/\sqrt{c_m})^{2N_f/\tilde{N}_c} \tilde{\phi}^4 - 2m[\frac{N_f}{\tilde{N}_c}(\frac{\gamma_M}{2} - 1) + 3]\tilde{N}_c (\lambda/\sqrt{c_m})^{N_f/\tilde{N}_c} \tilde{\phi}^3 - \frac{1}{4}\dot{\gamma}_M m^2 \tilde{\phi}^2$$

Near conformal fixed point, deviations in anomalous dimensions can be parametrized by:

$$\gamma_M \simeq -2\frac{3\tilde{N}_c - N_f}{N_f} + c_\gamma e^{\alpha t}$$

Know how to determine if fixed point is perturbative

Potential with $\mu \simeq \tilde{\phi}$ (e.g. Coleman-Weinberg)

$$V[\tilde{\phi}] = N_f^2(\lambda/\sqrt{c_m})^{2N_f/\tilde{N}_c}\tilde{\phi}^4 - 2mN_f\frac{c_\gamma}{2}(\lambda/\sqrt{c_m})^{N_f/\tilde{N}_c}\Lambda_m^{-\alpha}\tilde{\phi}^{3+\alpha} - \frac{1}{4}c_\gamma \alpha m^2 \Lambda_m^{-\alpha}\tilde{\phi}^{2+\alpha}$$

Chiral-symmetry breaking minimum will exist as long as $c_\gamma > 0$

This term is less relevant than this term

As $N_f \rightarrow 3N_c$ SQCD becomes IR free. This motivates a conjecture that $\alpha \rightarrow 2$ in that same limit.

Get a χ_{SB} minimum that scales like:

$$\tilde{\phi} \propto \Lambda_m \left(\frac{m}{\Lambda_m}\right)^{\frac{2}{2-\alpha}}$$

What about the upper edge of the conformal window?

Electric theory is naively a non-starter, since it has no superpotential

But Seiberg duality would suggest that we could equivalently work in the theory dual to the Magnetic theory!

AMSB Minimum of Twice-Dual

Normally N serves as a Lagrange multiplier superfield

But with AMSB it can get a full rank VEV, giving a mass to the quarks

Note that M here receives no wavefunction renormalization

Integrate out Quarks

$$W = N^{ij}(Q_i\tilde{Q}_j - M_{ij}) \longrightarrow W = N_c\left(\Lambda^{3N_c - N_f}\det N\right)^{1/N_c} - \Lambda N M$$

Mass term breaks conformal invariance

Treat as a deformation from the conformal fixed point (valid for small M VEV)

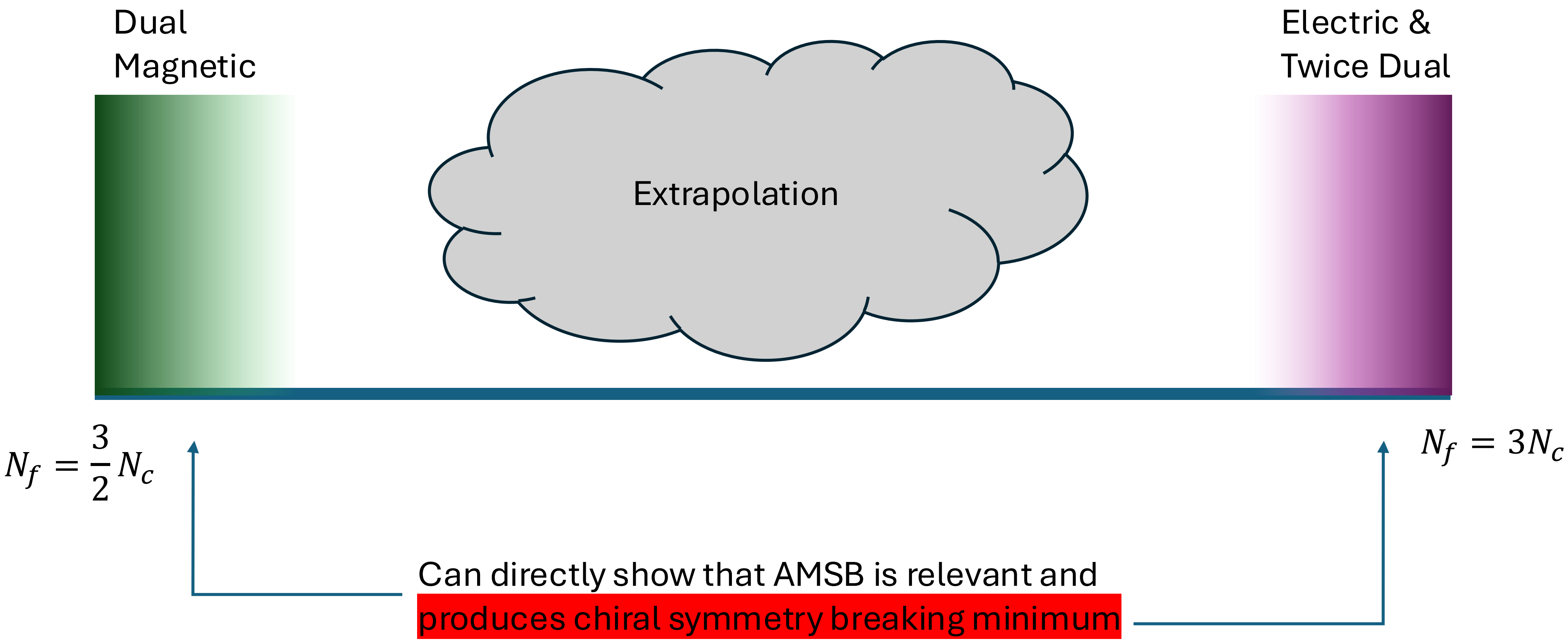
Following the same procedure, we get a potential dominated by:

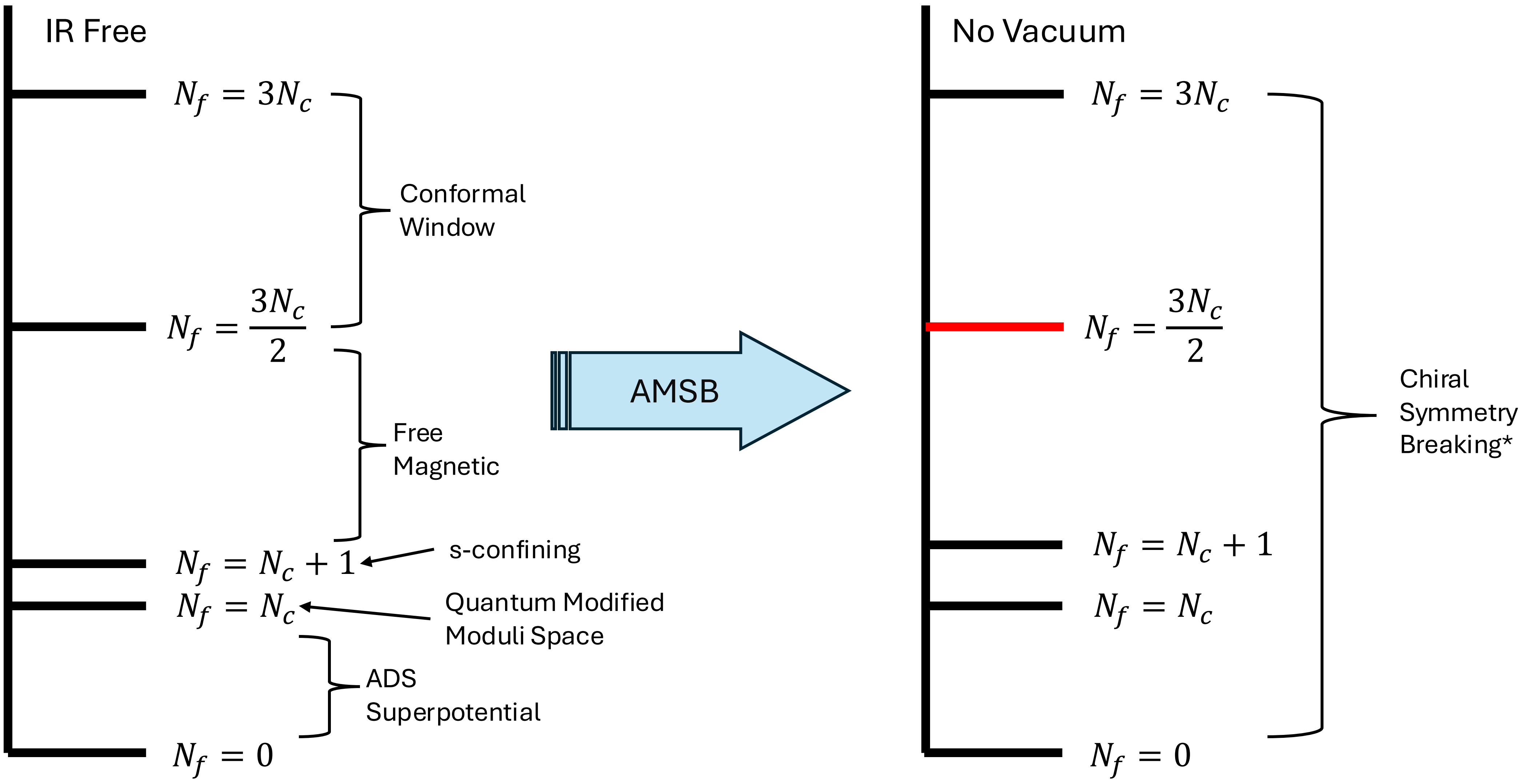
$$V = \frac{2}{c_N}\Lambda^{2-\frac{6N_c}{Nf}}\phi^{\frac{3N_c}{Nf}}(-3\sqrt{c_N}m(N_c - N_f)\Lambda^{\frac{3N_c}{N_f}}\phi + N_f^2\Lambda^2\phi^{\frac{3N_c}{N_f}}$$

We get a minimum: $$\phi \propto \Lambda\left(\frac{m}{\Lambda}\right)^{\frac{N_f}{3N_c - N_f}}$$

Note that if we equate this power to that of the condensate in the magnetic picture, we get $\alpha = 2\frac{2N_f - 3N_c}{N_f}$ which goes to 2 as $N_f \to 3N_c$, confirming what we previously had conjectured!

What we've learned* about the Conformal Window in ASQCD




IR Free
$N_f = 3N_c$
Conformal Window
$N_f = \frac{3N_c}{2}$
Free Magnetic
s-confining
$N_f = N_c + 1$
$N_f = N_c$
Quantum Modified Moduli Space
ADS Superpotential
$N_f = 0$
AMSB
No Vacuum
$N_f = 3N_c$
$N_f = \frac{3N_c}{2}$
Chiral Symmetry Breaking*
$N_f = N_c + 1$
$N_f = N_c$
$N_f = 0$

Conclusions

- AMSB is a promising candidate for a way to deform SUSY to Non-SUSY
- In the case of QCD, we see lots of evidence for continuous connection
- We can reproduce canonical results from first principles

Thank you!